OPTIMISATION TECHNIQUES (OT)

Individual Assignment

Prepared By: Nikhil Panchal

greatlearning
Power Ahead

GREAT LAKES
INSTITUTE OF MANAGEMENT

Submission Dt:

September 19th,2021

About ME:

This section will cover brief introduction about me.

Overall **14+ years of cross functional experience** in FMCG and Alcobev industry. Working with **Diageo PLC** since 2017, looking after **Data Analytics CoE for Global Audit & Risk department**. Before to that worked with **Coca-Cola for 10 years** and completed stint in operations, supply chain (direct & indirect), Master data maintenance (SAP cross functional role) and Data analytics (Internal Audit).

Table of Contents

Note: Each problem statement's solution completed using **excel solver**. Refer excel file "OT_assignment". It contains below index as well.

#	Problem statement	Reference Link
P1	The U-save company.... *(expected return maximisation)*	Problem statement
		Solution
		Sensitivity Report
P2	The Crazy Nut company.... *(profit maxmisation)*	Problem statement
		Solution
P3	Candy Business... *(revenue maximisation)*	Problem statement
		Solution
		Sensitivity Report
P4	Staffing problem... *(minimisation drivers count)*	Problem statement
		Solution
		Solution Matrix

Question1: The U-save company…

1) The U-save company is planning its operations for the next year. The company is considering investing in four types of securities. The company has $1 million available for investment. The expected annual return and the “risk index” of each security are as follows:

	Expected return	Risk index (%)
Long-term Bonds	15%	3
Medium-term Bonds	12%	4
Government Bonds	9%	7
Short-term Bonds	10%	9

The company wants to maximize the expected return from its bond investments, subject to the following restrictions:

- The average risk index of the portfolio should not exceed 6
- At most 45% of the total amount (1 million) can be invested in any single bond
- The expected return of the Government bond portfolio should be at least 1.2 times the return of the Long term and medium term bond portfolio.

Formulate the problem as a LP problem. Define the decision variables carefully. Use any software at your disposal to obtain an optimal solution.

Answer1:

Let’s solve the problem. First, we must identify the variables, define the objectives and the constraints:

Step1: The Decision Variables:

There are four decision variables are described in this problem:

- X_{LTB} = Investment to be done in the **long term bond**
- X_{MTB} = Investment to be done in the **medium term bond**
- X_{GB} = Investment to be done in the **government bond**
- X_{STB} = Investment to be done in the **short term bond**

Step2: The Objective Function:

The objective consists of maximising the expected return from its bond investments. The objective function is described by the expression as below:

Maximise: $15\% * X_{LTB} + 12\% * X_{MTB} + 9\% * X_{GB} + 10\% * X_{STB}$

Step3: Constraints:

There are mainly five constraints in the problem statement including non-negativity constraints are mentioned below:

Constraint1: The average risk index of the portfolio should not exceed 6

Total Portfolio

$$3\% * X_{LTB} + 4\% * X_{MTB} + 7\% * X_{GB} + 9\% * X_{STB} <= 6\% * (X_{LTB} + X_{MTB} + X_{GB} + X_{STB})$$

Constraint2: At most 45% of the total amount (1 million) can be invested in any single bond

$X_{LTB} <= 45\%$, $X_{MTB} <= 45\%$, $X_{GB} <= 45\%$ and $X_{STB} <= 45\%$

Constraint3: The expected return of the Government bond portfolio should be at least 1.2 times the return of the Long term and medium term bond portfolio

$$9\% * X_{GB} >= 1.2\,(15\% * X_{LTB} + 12\% * X_{MTB})$$

Constraint4: Company has total investment upto 1 million. Let's take 1 million as 100% for same unit scaling.

Total Portfolio → $(X_{LTB} + X_{MTB} + X_{GB} + X_{STB}) <= 100\%$

Constraint5: Non negativity constraints.

$X_{LTB} >= 0\%, X_{MTB} >= 0\%$, $X_{GB} >= 0\%$ and $X_{STB} >= 0\%$

Now, we have defined the linear equation. Let's decode the same using **Excel Solver**.

- **To each variable we have to attribute a position on the worksheet:** the cells D6, E6, F6 and G6 have been chosen to represent the variables X_{LTB}, X_{MTB}, X_{GB} and X_{STB} respectively.

1 The Decision Variables

	Long term Bonds	Medium term Bonds	Government Bonds	Short term Bonds
Decision Variables	X_{LTB}	X_{MTB}	X_{GB}	X_{STB}

- **Define the objective function:** In cell D17, the objective is defined in function of the variables with subproduct of expected return and decision variables *{i.e., SUMPRODUCT(D6 to G6,D15 to G15)}*.

2 The Objective Function

	Long term Bonds	Medium term Bonds	Government Bonds	Short term Bonds
	LTB	MTB	GB	STB
Risk Index	3%	4%	7%	9%
Expexted Return	15%	12%	9%	10%
Objective Function *(Maximise)*	0%	SUMPRODUCT(D6:G6,D15:G15)		

- **Define all constraints:** The constraints are defined a bit differently than the objective function. A constraint is a relation linking two expressions. Refer Cell

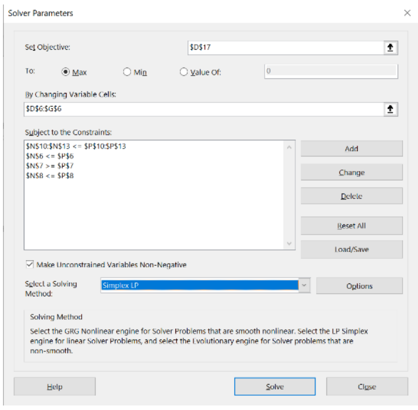
3 Constraints

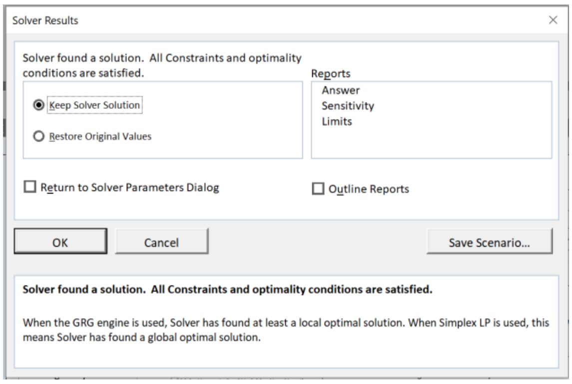
		Left hand side	(<=,>= or =)	Right hand side	
#	Constraints	LHS	sign	RHS	Remarks
1	Risk index constraint	0%	<=	6%	The average risk index of the portfolio should not exceed 6
2	GB to 1.2* (LTB + MTB)	0%	>=	0%	The expected return of the GB should be at least 1.2 times higher than return of LTB and MTB
3	Bonds investment	0%	<=	100%	Total bond investment is upto 1 million (let's consider 1million as 100%)
4	Bond constraint (LTB)	0%	<=	45%	At most 45% of the total amount (1 million) can be invested in any single bond
	Bond constraint (MTB)	0%	<=	45%	
	Bond constraint (GB)	0%	<=	45%	
	Bond constraint (STB)	0%	<=	45%	
5	Non-negativity constraint	Included in Solver using ☑ Make Unconstrained Variables Non-Negative			The bond investment is greater than or equals to ZERO.

The variables, the objective and the constraints having been inserted, we are ready to solve the problem with the help of the Solver:

- Set objective: Cell D17 contains the objective,
- To: we are looking to maximise the objective,
- By changing variable cells: D6 to G6 represent the variables,
- Subject to the constraints: by selecting the Add button, the two constraints of the problem can be dictated to the Solver.
- Use the checkbox: make unconstrained variables non-negative

Then, by clicking on **Solve**, the Solver will give the solution to the problem.





Mentioned below screenshot says that, **Solver has found the optimal solution and values are mentioned below**:

1 The Decision Variables

	Long term Bonds	Medium term Bonds	Government Bonds	Short term Bonds
Decision Variables	X_{LTB}	X_{MTB}	X_{GB}	X_{STB}
	23%	**0%**	**45%**	**7%**
Amt in $	**$2,25,000**	**$0**	**$4,50,000**	**$75,000**

2 The Objective Function

	Long term Bonds	Medium term Bonds	Government Bonds	Short term Bonds
	LTB	**MTB**	**GB**	**STB**
Risk Index	*3%*	*4%*	*7%*	*9%*
Expexted Return	*15%*	*12%*	*9%*	*10%*
Objective Function *(Maximise)*	**8.18%**	**$81,750**		

3 Constraints

		Left hand side	(<=,>= or =)	Right hand side	
#	**Constraints**	**LHS**	**sign**	**RHS**	**Remarks**
1	Risk index constraint	*4.5%*	*<=*	*4.5%*	*The average risk index of* ***the portfolio should not exceed 6%***
2	GB to 1.2* (LTB + MTB)	*4.1%*	*>=*	*4.1%*	*The expected return of the GB should be at least 1.2 times higher than return of LTB and MTB*
3	Bonds investment	*75.0%*	*<=*	*100%*	*Total bond investment is upto 1 million (let's consider 1million as 100%)*
4	Bond constraint (LTB)	*23%*	*<=*	*45%*	*At most 45% of the total amount (1 million) can be invested in any single bond*
	Bond constraint (MTB)	*0%*	*<=*	*45%*	
	Bond constraint (GB)	*45%*	*<=*	*45%*	
	Bond constraint (STB)	*7%*	*<=*	*45%*	
5	Non-negativity constraint	*Included in Solver using* ☑ Make Unconstrained Variables Non-Negative			*The bond investment is greater than or equals to ZERO.*

Solution Interpretations and Insights:

- Out of given four investment bonds optimiser has suggested to invest in only three bonds (i.e., not suggested to invest in medium term bonds)
- Out of total $1 million investment amount optimiser has suggested to invest $0.75 million.
- As per average risk constraint, it should not exceed more than 6% of portfolio. (i.e., 4.5% -> 6% of 75%)
- Further, optimiser has suggested, out of total $1 million:
  - Invest 23% (i.e., $0.225 million) into long term bonds, 45% (i.e., $0.45 million) into government bonds and 7% (i.e., $0.07 million) into short term bonds
- The maximum expected return coming out based on above investment is 8.18% (i.e., $81,750).
- Further, looked out the sensitivity report and found more insights like, solution will not be changed even if we change the excepted return upto that range:
  - Long term bond expected return change from 15% upto 13.33% (i.e., 1.67% decrease)
  - Medium term bond expected return change from 12% to 13.33% (i.e., 1.33% increase)
  - For Government bond and short term bond expected returns are in right mix.
- In Sensitivity report, Shadow price (opportunity cost) of
  - Government bond is 0.1817 and allowable increase is upto 60% (but constraint here is 45%).
  - Risk index is 3.33 and allowable range is 4.2% to 5.25% of total portfolio.
  - GB to 1.2 times of (LTB and MTB) is -1.389. so currently it not worth using due to -ve value.

  So, in future these options to be looked at to increase the return.

Question2: The Crazy Nut company…

2) The Crazy Nut company wishes to market two special nut mixes during the holiday season. Every pound of mix 1 contains 0.5 pound of peanuts and 0.5 pound of cashews; Every pound of Mix 2 contains 0.6 pound of peanuts, 0.25 pound of cashews, and 0.15 pound of almonds. Mix 1 sells for $1.49 per pound; Mix 2 sells for $1.69 per pound. The data pertinent to the raw ingredients appear in the table

Ingredient	Amount available (lb)	Cost per lb. ($)
Peanuts	30,000	$0.35
Cashews	12,000	$0.50
Almonds	9,000	$0.60

Assuming that Crazy can sell all cans of either mix that it produces, formulate an LP model to determine how much of mixes 1 and 2 to produce. Use any software at your disposal to obtain an optimal solution.

Answer2:

Step1: The Decision Variables:

There are two decision variables described in this problem:

- X_{M1} = No. of cans to be produced for **mix1 product**
- X_{M2} = No. of cans to be produced for **mix2 product**

Step2: The Objective Function:

The objective consists of maximising the profit by selling all the can of the give mix of products.

In the dataset, profit value of mix1 and mix2 is not given. So, it is required to formulate. Let's find out.

Profit = selling cost – material cost. Selling cost is given. To get material cost, refer below table to calculate the same. It is the **sum product of material required and cost of each material. Now, we have material cost and selling cost. So, profit per mix product could be found.**

1		*Mix1*	*Mix2*
		X_{M1}	X_{M2}
cost per lb	**Ingredients/Material required**		
$0.35	***Peanuts***	*0.50*	*0.60*
$0.50	***Cashews***	*0.50*	*0.25*
$0.60	***Almonds***		*0.15*
	Material Cost	*$0.43*	*$0.43*
2	**Selling Cost**	*$1.49*	*$1.69*
3	**Profit** *(selling less material cost)*	*$1.07*	*$1.27*

The objective function is described by the expression as below:

$$\textbf{Maximise: Profit } X_{M1\ (\text{selling less material cost})} * X_{M1} + \textbf{Profit } X_{M2\ (\text{selling less material cost})} * X_{M2}$$

Step3: Constraints:

There are mainly five constraints in the problem statement including non-negativity and integer constraints are mentioned below:

Constraint1: Peanut ingredient availability is 30,000

Peanuts reqd. in Mix1 * X_{M1} + Peanuts reqd. in Mix2 * X_{M2} <= 30,000

Constraint2: Cashew ingredient availability is 12,000

Cashews reqd. in Mix1 * X_{M1} + Cashews reqd. in Mix2 * X_{M2} <= 12,000

Constraint3: Almond ingredient availability is 9,000

Almonds reqd. in Mix1 * X_{M1} + Almonds reqd. in Mix2 * X_{M2} <= 9,000

Constraint4: Non negativity constraints

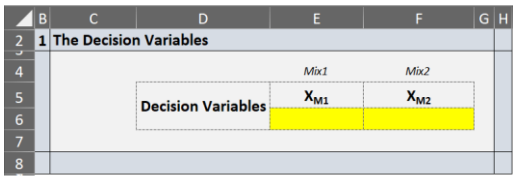
X_{M1} >= 0 and X_{M2} >= 0

Constraint5: Consider decision variables as an integer.

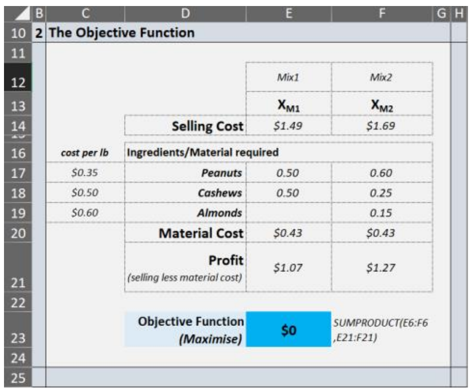
X_{M1} and X_{M2} are an integer.

Now, we have defined the linear equation. Let's decode the same using **Excel Solver**.

- **To each variable we have to attribute a position on the worksheet:** the cells E6 and F6 have been chosen to represent the variables **X_{M1}, X_{M2}** respectively.



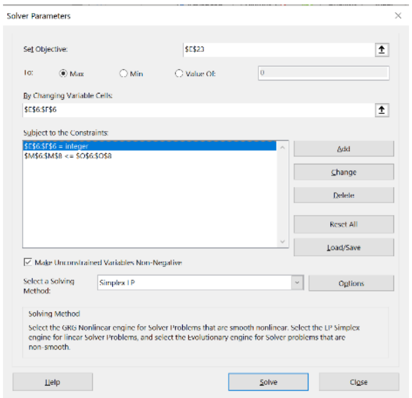
- **Define the objective function:** In cell E23, the objective is defined in function of the variables with subproduct of profit value and decision variables *{i.e., SUMPRODUCT(E6 to F6,E21 to F21)}.*

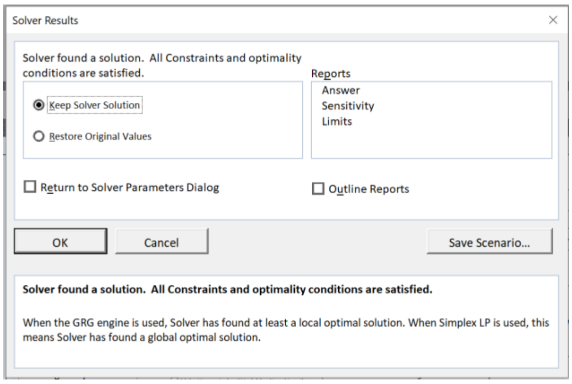


- **Define all constraints:** The constraints are defined a bit differently than the objective function. A constraint is a relation linking two expressions. Refer Cell

3	Constraints				
		Left hand side	*(<=,>= or =)*	*Right hand side*	
#	**Constraints**	**LHS**	**sign**	**RHS**	**Remarks**
1	Peanut constraints	*0*	*<=*	*30,000*	*The peanut ingredient availablity is 30,000*
2	Cashew constraints	*0*	*<=*	*12,000*	*The cashew ingredient availablity is 12,000*
3	Almond constraints	*0*	*<=*	*9,000*	*The almond ingredient availablity is 9000*
4	Non-negativity constraint	*Included in Solver using* ☑ Make Unconstrained Variables Non-Negative			*cans quantity greater than or equals to ZERO.*
5	Integer constraints				*Cans decision variables to be cosidered as an integer.*

The variables, the objective and the constraints having been inserted, we are ready to solve the problem with the help of the Solver:





Mentioned below screenshot says that, **Solver has found the optimal solution and values are mentioned below**:

1	The Decision Variables		
		Mix1	*Mix2*
	Decision Variables	**X_{M1}**	**X_{M2}**
		0	**48,000**

2	The Objective Function		
		Mix1	*Mix2*
		X_{M1}	**X_{M2}**
	Selling Cost	*$1.49*	*$1.69*
cost per lb	**Ingredients/Material required**		
$0.35	***Peanuts***	*0.50*	*0.60*
$0.50	***Cashews***	*0.50*	*0.25*
$0.60	***Almonds***		*0.15*
	Material Cost	*$0.43*	*$0.43*
	Profit *(selling less material cost)*	*$1.07*	*$1.27*
	Objective Function ***(Maximise)***	**$60,720**	

3	Constraints				
		Left hand side	*(<=,>= or =)*	*Right hand side*	
#	**Constraints**	**LHS**	**sign**	**RHS**	**Remarks**
1	Peanut constraints	*28,800*	*<=*	*30,000*	*The peanut ingredient availablity is 30,000*
2	Cashew constraints	*12,000*	*<=*	*12,000*	*The cashew ingredient availablity is 12,000*
3	Almond constraints	*7,200*	*<=*	*9,000*	*The almond ingredient availablity is 9000*
4	Non-negativity constraint	*Included in Solver using* ☑ Make Unconstrained Variables Non-Negative			*cans quantity greater than or equals to ZERO.*
5	Integer constraints				*Cans decision variables to be cosidered as an integer.*

Solution Interpretations and Insights:

- Out of given two products, optimiser has suggested to produce in only mix2 cans to get the maximum profit. (i.e., not suggested to produce mix1 cans)
- Optimiser has suggested to produce 48,000 of mix2 cans which gives the maximise profit of $60,720 assuming all the cans are sold.
- Further, checking on the ingredients found:
  - Peanuts are 1,200 still available out of total 30,000. So, it is utilised 96% and 4% left out.
  - Cashews are fully utilised (i.e., 12,000 out of total 12,000)
  - Almonds are 1,800 still available out of total 9,000. So, it is utilised 80% and 20% left out.

Question3: Candy Business...

3) You have decided to enter the candy business. You are considering producing two types of candies: Slugger Candy and Easy Out Candy, both of which consist solely of sugar, nuts, and chocolate. At present, you have in stock 100 oz of sugar, 20 oz of nuts, and 30 oz of chocolate. The mixture used to make Easy Out candy must contain at least 20% nuts. The mixture used to make Slugger Candy must contain at least 10% nuts and 10% chocolate. Each ounce of Easy Out Candy can be sold for 25 cents, and each ounce of Slugger Candy can be sold for 20 cents. Formulate an LP that will enable you to maximize your revenue from candy sales. Use any software at your disposal to obtain an optimal solution.

Answer3:

Step1: The Decision Variables:

There are six decision variables (also refer as decision matrix) are described in this problem:

- X_{11} = No. sugar oz(ounce) required in **Slugger candy**
- X_{12} = No. sugar oz(ounce) required in **Easy out candy**
- X_{21} = No. nuts oz(ounce) required in **Slugger candy**
- X_{22} = No. nuts oz(ounce) required in **Easy out candy**
- X_{31} = No. chocolate oz(ounce) required in **Slugger candy**
- X_{32} = No. chocolate oz(ounce) required in **Easy out candy**

Step2: The Objective Function:

The objective consists of maximising the revenue by selling slugger candy and easy out candy. In the dataset, selling cost is given. The objective function is described by the expression as below:

$$\text{Maximise: } \$0.20 * (X_{11} + X_{21} + X_{31}) + \$0.25 * (X_{12} + X_{22} + X_{32})$$

Step3: Constraints:

There are mainly seven constraints in the problem statement including non-negativity constraints are mentioned below:

Constraint1: sugar availability is 100 oz (ounce)

$X_{11} + X_{12} <= 100$

Constraint2: nuts availability is 20 oz (ounce)

$X_{21} + X_{22} <= 20$

Constraint3: chocolate availability is 30 oz (ounce)

$X_{31} + X_{32} <= 30$

Constraint4: Easy out candy must contain at least 20% nuts

$X_{22} >= 20\% * (X_{12} + X_{22} + X_{32})$

Constraint5: Slugger candy must contain at least 10% nuts

$X_{21} >= 10\% * (X_{11} + X_{21} + X_{31})$

Constraint6: Slugger candy must contain at least 10% chocolates

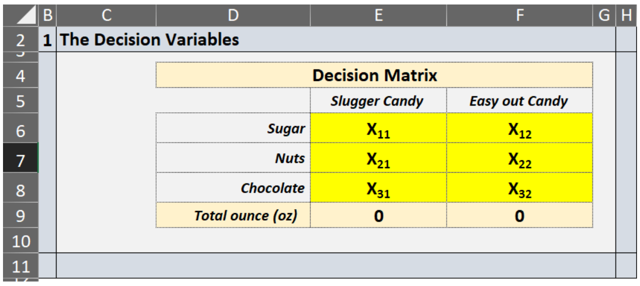
$X_{31} >= 10\% * (X_{11} + X_{21} + X_{31})$

Constraint7: Non negativity constraints

$X_{11} >= 0$, $X_{12} >= 0$, $X_{21} >= 0$, $X_{22} >= 0$, $X_{31} >= 0$, $X_{32} >= 0$

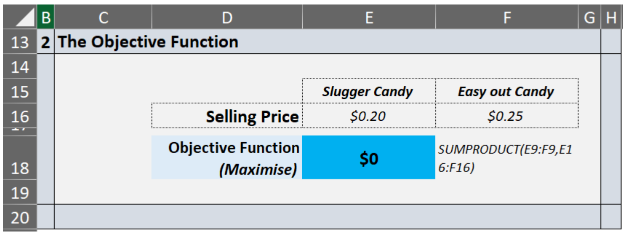
Now, we have defined the linear equation. Let's decode the same using **Excel Solver**.

- **To each variable (here decision matrix) we have to attribute a position on the worksheet:** the cells E6:F8 to represent the variables $\mathbf{X_{11}, X_{12}, X_{21}, X_{22}, X_{31}, X_{32}}$ respectively.



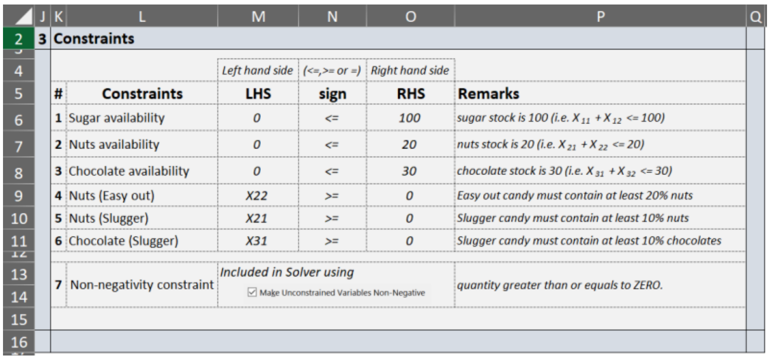
1 The Decision Variables		
	Decision Matrix	
	Slugger Candy	*Easy out Candy*
Sugar	X_{11}	X_{12}
Nuts	X_{21}	X_{22}
Chocolate	X_{31}	X_{32}
Total ounce (oz)	0	0

- **Define the objective function:** In cell E18, the objective is defined in function of the variables with subproduct of profit value and decision variables *{i.e., SUMPRODUCT(E9:F9,E16:F16) }.*



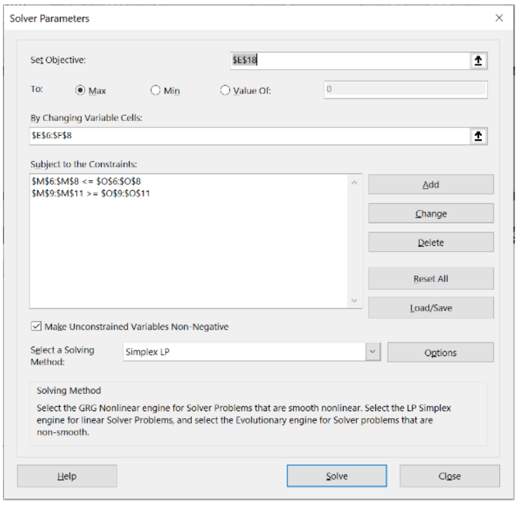
2 The Objective Function			
	Slugger Candy	*Easy out Candy*	
Selling Price	*$0.20*	*$0.25*	
Objective Function *(Maximise)*	**$0**		*SUMPRODUCT(E9:F9,E16:F16)*

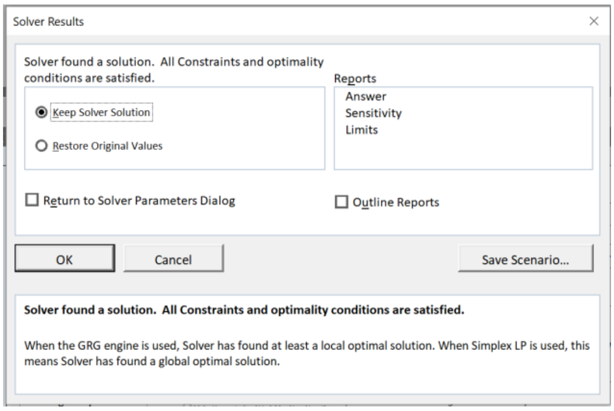
- **Define all constraints:** The constraints are defined a bit differently than the objective function. A constraint is a relation linking two expressions. Refer Cell



3 Constraints		*Left hand side*	*(<=,>= or =)*	*Right hand side*	
#	**Constraints**	**LHS**	**sign**	**RHS**	**Remarks**
1	Sugar availability	*0*	*<=*	*100*	*sugar stock is 100 (i.e. X_{11} + X_{12} <= 100)*
2	Nuts availability	*0*	*<=*	*20*	*nuts stock is 20 (i.e. X_{21} + X_{22} <= 20)*
3	Chocolate availability	*0*	*<=*	*30*	*chocolate stock is 30 (i.e. X_{31} + X_{32} <= 30)*
4	Nuts (Easy out)	*X22*	*>=*	*0*	*Easy out candy must contain at least 20% nuts*
5	Nuts (Slugger)	*X21*	*>=*	*0*	*Slugger candy must contain at least 10% nuts*
6	Chocolate (Slugger)	*X31*	*>=*	*0*	*Slugger candy must contain at least 10% chocolates*
7	Non-negativity constraint	*Included in Solver using* ☑ Make Unconstrained Variables Non-Negative			*quantity greater than or equals to ZERO.*

The variables, the objective and the constraints having been inserted, we are ready to solve the problem with the help of the Solver:





Mentioned below screenshot says that, **Solver has found the optimal solution and values are mentioned below**:

1 The Decision Variables

Decision Matrix	Slugger Candy	Easy out Candy
Sugar	80	20
Nuts	10	10
Chocolate	10	20
Total ounce (oz)	100	50

2 The Objective Function

	Slugger Candy	Easy out Candy
Selling Price	$0.20	$0.25
Objective Function (Maximise)	$32.50	

3 Constraints

#	Constraints	LHS (Left hand side)	sign (<=,>= or =)	RHS (Right hand side)	Remarks
1	Sugar availability	100	<=	100	sugar stock is 100 (i.e. $X_{11} + X_{12} <= 100$)
2	Nuts availability	20	<=	20	nuts stock is 20 (i.e. $X_{21} + X_{22} <= 20$)
3	Chocolate availability	30	<=	30	chocolate stock is 30 (i.e. $X_{31} + X_{32} <= 30$)
4	Nuts (Easy out)	10	>=	10	Easy out candy must contain at least 20% nuts
5	Nuts (Slugger)	10	>=	10	Slugger candy must contain at least 10% nuts
6	Chocolate (Slugger)	10	>=	10	Slugger candy must contain at least 10% chocolates
7	Non-negativity constraint	Included in Solver using ☑ Make Unconstrained Variables Non-Negative			quantity greater than or equals to ZERO.

Solution Interpretations and Insights:

- Out of given decision matrix for slugger and easy out candy, optimiser has suggested to produce both the candy to get the maximum revenue.
- Optimiser has suggested to produce 100 ounce of slugger candy and 50 ounce of easy out candy which gives the maximise revenue of $32.50.
- Further, checking on the ingredients (i.e., sugar, nuts, and chocolate) found all of them are fully utilised.
- In Sensitivity report, Shadow price (opportunity cost) of
    - Sugar is 0.15 and allowable increasing more 33.33 oz (but constraint here is 100).
    - Nuts is 0.65 and allowable increasing more 12.5 oz (but constraint here is 20).
    - Chocolate is 0.15 and allowable increasing more 12.5 oz (but constraint here is 30).

    So, in future these options to be looked at to increase the revenue.

Question4: Staffing problem…

4) Consider the staffing problem faced by the Great Lakes Bus Company that requires the following number of drivers on each day.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number needed	19	16	14	16	19	14	10

Each driver works for 5-consecutive days in a week and then receives two days off. The company assumes that if more than required number of drivers are scheduled to work on any given day, then the extra drivers will be given a paid day off. The company solves the following linear program to determine the optimal staffing plan

MIN Z = X1 + X2 + X3 + X4 + X5 + X6 + X7

Subject to

X1 + +X4 + X5 + X6 + X7 $\geq$ 19
X1 + X2 + X5 + X6 + X7 $\geq$ 16
X1 + X2 + X3 + X6 + X7 $\geq$ 14
X1 + X2 + X3 + X4 + X7 $\geq$ 16
X1 + X2 + X3 + X4 + X5 $\geq$ 19
+ X2 + X3 + X4 + X5 + X6 $\geq$ 14
X3 + X4 + X5 + X6 + X7 $\geq$ 10
X1, X2, X3, X4, X5, X6, X7 $\geq$ 0

Using a linear programming software, the optimal solution was found to be:
X1 = 8; X2 = 3; X3 = 0; X4 = 5; X5 = 3; X6 = 3; X7 = 0

Answer the following questions based on the above information.

Answer4:

In this problem the optimise solution is already given and asked the specific questions around the same. Let's answer the same.

a) What is the definition of the decision variables in the linear program used by the company?

Step1: The Decision Variables:

There are seven decision variables described in this problem statement and are mentioned below:

1 The Decision Variables

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
	X1	*X2*	*X3*	*X4*	*X5*	*X6*	*X7*
No. of Drivers							

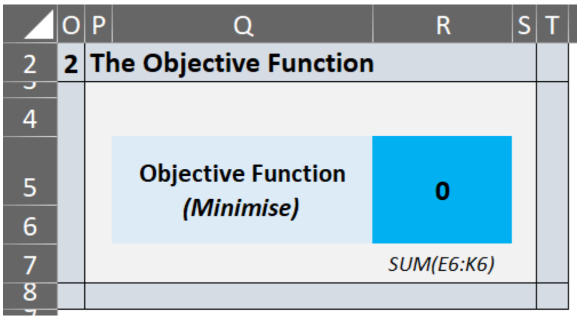
Variable	Decision Variable Definition
X1	No of drivers starting their shift **from Monday** and working 5 consecutive days **(i.e., Monday to Friday)**
X2	No of drivers starting their shift **from Tuesday** and working 5 consecutive days **(i.e., Tuesday to Saturday)**
X3	No of drivers starting their shift **from Wednesday** and working 5 consecutive days **(i.e., Wednesday to Sunday)**
X4	No of drivers starting their shift **from Thursday** and working 5 consecutive days **(i.e., Thursday to Monday)**
X5	No of drivers starting their shift **from Friday** and working 5 consecutive days **(i.e., Friday to Tuesday)**
X6	No of drivers starting their shift **from Saturday** and working 5 consecutive days **(i.e., Saturday to Wednesday)**
X7	No of drivers starting their shift **from Sunday** and working 5 consecutive days **(i.e., Sunday to Thursday)**

b) Describe the objective function (in words) that the company uses in the linear program?

Step2: The Objective Function:

The objective consists of minimising the driver count. So, the objective function that company uses in the linear program is **to minimise the number of drivers working in every shift**

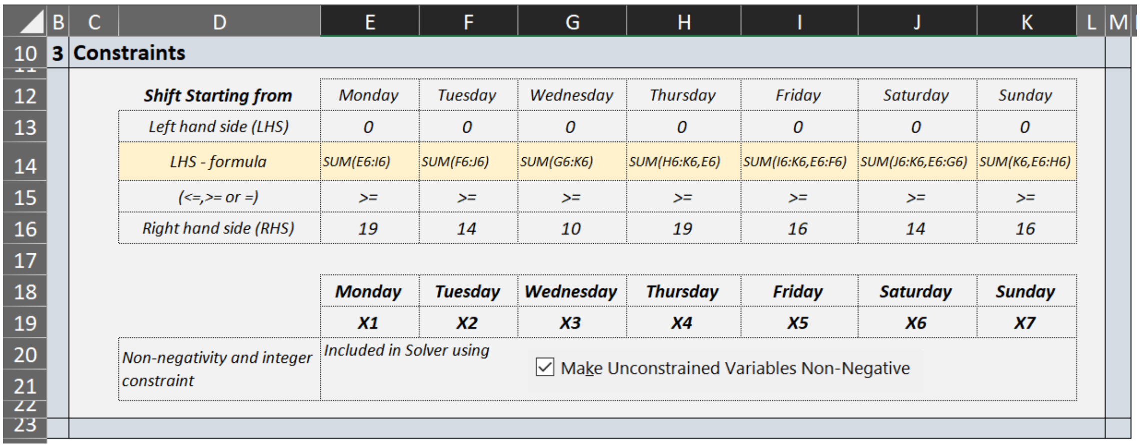
Minimise : X1 + X2 + X3 + X4 + X5 + X6 + X7



c) Determine the optimal value of the objective function?

Step3: Constraints:

There are below constraints given in the problem statement including non-negativity and integer constraints.

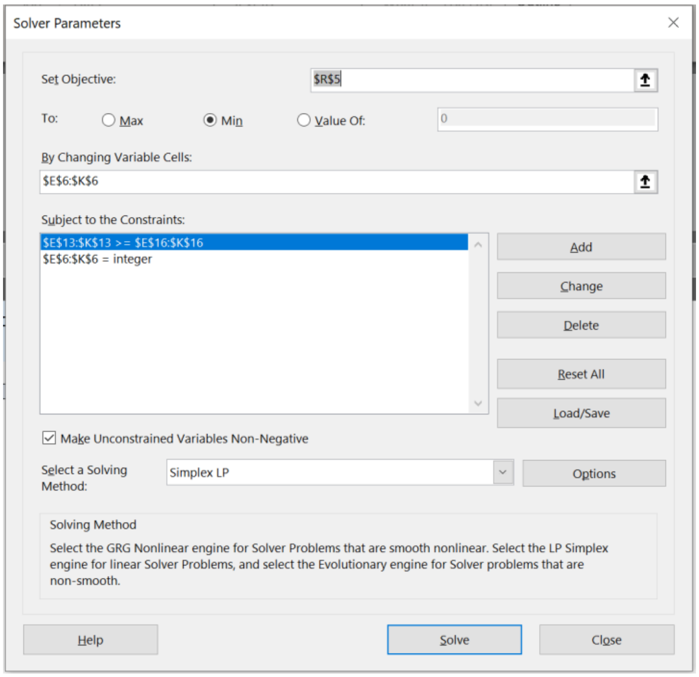


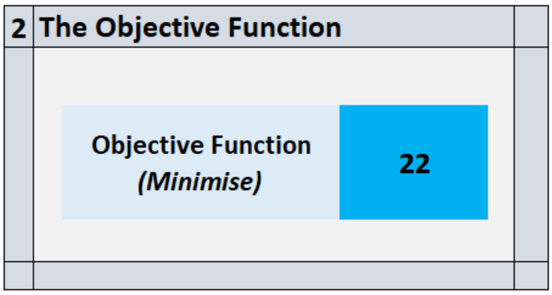
3 Constraints

Shift Starting from	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Left hand side (LHS)	0	0	0	0	0	0	0
LHS - formula	SUM(E6:I6)	SUM(F6:J6)	SUM(G6:K6)	SUM(H6:K6,E6)	SUM(I6:K6,E6:F6)	SUM(J6:K6,E6:G6)	SUM(K6,E6:H6)
(<=,>= or =)	>=	>=	>=	>=	>=	>=	>=
Right hand side (RHS)	19	14	10	19	16	14	16

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**
	X1	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**
Non-negativity and integer constraint	*Included in Solver using* ☑ Make Unconstrained Variables Non-Negative						

Now, we have defined the linear equation. Let's decode the same using **Excel Solver** to get the optimal value.





d) Based on the optimal solution, how many drivers will be scheduled to work on Monday?

Optimisation solution is already given. If I consider the same as the final optimal solution, then the no of drivers scheduled to work on Monday are : X1 + X4 + X5 + X6 + X7 (i.e., 8 + 3 + 0 + 5 + 3) = **19**

1 **The Decision Variables (Optimal Solution) Given**

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
	X1	X2	X3	X4	X5	X6	X7
No. of Drivers	8	3	0	5	3	3	0

e) Based on the optimal solution, how many drivers will be scheduled to work on Tuesday?

Optimisation solution is already given. If I consider the same as the final optimal solution, then the no of drivers scheduled to work on Tuesday are : X1 + X2 + X5 + X6 + X7 (i.e., 8 + 3 + 3 + 3 + 0) = **17**

f) Suppose the drivers were paid $ 50 per weekday and $ 80 for Saturday or Sunday, what is the total money paid to the drivers if the optimal plan determined above was implemented?

Let's prepare a solution matrix which helps to find out:

- No. of drivers scheduled to work for the day
- The payment to the drivers is $50 per weekday and $80 for Saturday or Sunday.

Solution Matrix (Drivers required)

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Total Drivers	Cost ($)
	X1	X2	X3	X4	X5	X6	X7		
Monday	8			5	3	3	0	19	$50
Tuesday	8	3			3	3	0	17	$50
Wednesday	8	3	0			3	0	14	$50
Thursday	8	3	0	5			0	16	$50
Friday	8	3	0	5	3			19	$50
Saturday		3	0	5	3	3		14	$80
Sunday			0	5	3	3	0	11	$80
								Total Amount	$6,250

As per above table, the total money paid to the drivers are sum product of total drivers and cost ($) per day which comes out to be **$6,250.**