# A Hybrid Learning RBF Neural Network For Human Face Recognition with Pseudo Zernike Moment Invariant

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Abstract - This paper introduces a method for the recognition of human faces in 2-Dimensional digital images using a new Hybrid Learning Algorithm (HLA) for Radial Basis Function (RBF) neural network as classifier and Pseudo Zernike Moment Invariant (PZMI) as face feature. Also we evaluate the effect of orders of The PZMI on recognition rate, in the proposed technique. Simulation has been carried out on the face database of Olivetti Research Laboratory (ORL) and recognition rate of 98.7% is obtained using this proposed technique.

## I. INTRODUCTION

Computer based face recognition system is still a challenging and difficult task. The difficulties arise due to large variations in facial appearance, head size, orientation and changes in environmental conditions. Such difficulties make face recognition one of the fundamental problems in pattern analysis. In recent years there has been a growing interest in machine recognition of faces due to potential commercial applications such as film processing, law enforcement, person identification, access control systems, etc. A recent survey of the face recognition systems can be found in references [1]-[2].

A complete face recognition system should include two stages. The first stage is detecting the location of face, which is difficult and complicated because of the unknown position, orientation and scaling of face in an arbitrary image [3]-[5] and extraction of pertinent features from the localized image. The second stage involves classification of facial images based on the derived feature vector obtained in the previous stage.

In the second stage, the classifier plays a crucial role in the face recognition process. Neural networks have been employed and compared to conventional classifiers for a number of classification problems. The results have shown that the accuracy of the neural network approaches equivalent to, or slightly better than, other methods. Also, due to the simplicity, generality and good learning ability of the neural networks, these types of

classifiers are found to be more efficient [6]. The Neural Network approaches take the feature vectors as input for training and to learn.

Radial Basis Function (RBF) neural networks have found to be very attractive for many engineering problem because: (1) they are universal approximators, (2) they have a very compact topology and (3) their learning speed is very fast because of their locally tuned neurons [7]-[8]. An important property of RBF neural networks is that they form a unifying link between many different research fields such as function approximation, regularization, noisy interpolation and pattern recognition. Therefore RBF neural networks serve as an excellent candidate for pattern applications and attempts have been carried out to make the learning process in this type of classification faster than normally required for the multilayer feed forward neural networks [9].

In this paper we have used RBF neural network classifier with a new Hybrid Learning Algorithm (HLA) for human face recognition. Also in the first stage of face recognition system that involves face feature extraction PZMI was used to generate the feature vector which yielded the best results for human face recognition in comparison with other moments [10].

The rest of this paper is organized as follows. Section 2 present RBF neural network structure. Classifier design based on RBF neural network is introduced in section 3. Feature extraction technique is given in section 4 and section 5 describes the experimental results and conclusion is described in section 6.

### II. RBF NEURAL NETWORK STRUCTURE

An RBF neural network structure is shown in Fig. (1) which has an architecture similar to that of a traditional three-layer feed forward neural network. The construction of the RBF neural network involves three different layers with feed forward architecture. The input layer of this network is a set of n units, which accept the elements of an n dimensional input feature vector. The

input units are fully connected to the hidden layer with r hidden units. Connections between the input and hidden layers have unit weights and, as a result, do not have to be trained. The goal of the hidden layer is to cluster the data and reduce its dimensionality. In this structure hidden layer is named RBF units. The RBF units are also fully connected to the output layer. The output layer supplies the response of neural network to the activation pattern applied to the input layer. The transformation from the input space to the RBF-unit space is nonlinear (nonlinear activation function), whereas the transformation from the RBF-unit space to the output space is linear (linear activation function).

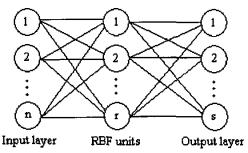


Figure 1: RBF neural network structure

The RBF neural network is a class of neural networks, where the activation function of the hidden units is determined by the distance between the input vector and a prototype vector. The activation function of the RBF units is expressed as follow [8]:

$$R_{i}(x) = R_{i}(\frac{\|x - c_{i}\|}{\sigma_{i}})$$
,  $i=1,2,...,r$  (1)

Where x is an n-dimensional input feature vector,  $\mathbf{c}_i$  is a n-dimensional vector called the center of the RBF unit,  $\sigma_i$  is the width of RBF unit and r is the number of the RBF units. Typically the activation function of the RBF units is chosen as a Gaussian function with mean vector  $\mathbf{c}_i$  and variance vector  $\sigma_i$  as follows:

$$R_{i}(x) = \exp(-\frac{\|x - c_{i}\|^{2}}{\sigma_{i}^{2}})$$
 (2)

Note that  $\sigma_i^2$  represents the diagonal entries of covariance matrix of Gaussian function. The output units are linear and therefore the response of the j-th output unit for input x is given as:

$$y_j(x) = b(j) + \sum_{i=1}^r R_i(x) w_2(i, j)$$
 (3)

Where  $w_2(i, j)$  is the connection weight of the i-th RBF unit to the j-th output node and b(j) is the bias of the j-th output. The bias is omitted in this network in order to reduce network complexity. Therefore:

$$y_{j}(x) = \sum_{i=1}^{r} R_{i}(x) \times w_{2}(i, j)$$
 (4)

# III. RBF BASED CLASSIFIER DESIGN

RBF neural network classifier can be viewed as a function mapping interpolant that tries to construct hypersurfaces, one for each class, by taking a linear combination of the RBF units. These hypersurfaces can be viewed as discriminant functions, where the surface has a high value for the class it represents and a low value for all others. An unknown input feature vector is classified as belonging to class associated with the hypersurface with the largest output at that point. In this case the RBF units' serve as components in a finite expansion of the desired hypersurface where the component coefficients (the weights) have to be trained [11].

For designing a classifier based on RBF neural network, we have set the number of input nodes in the input layer of neural network equal to the number of feature vector elements. The number of nodes in the output layer is set to the number of image classes. The RBF units are selected using the following steps:

Step1: Initially the RBF units are set equal to the number of outputs.

Step2: For each class k, (that is k=1,2,...,s) the center of RBF units is selected as the mean value of the sample features belonging to the same class, i.e:

$$C^{k} = \frac{\sum_{i=1}^{N^{k}} p^{k}(n, i)}{N^{k}}$$
 (5)

Where  $p^k(n,i)$  is the i-th sample with n as the number of features, belonging to the class k and  $N^k$  is the number of images in the same class.

Step3: For any class k, compute the distance  $d_k$  from the mean  $C^k$  to the furthest point  $p_f^k$  belonging to the class k:

$$d_k = |p_f^k - C^k| \tag{6}$$

Step 4: For any class k, compute the distance dc(k, j) between the mean of class k and the mean of other classes as follows:

$$dc(k, j) = |C^k - C^j|$$
 (7)

Where j=1,2,...,s and  $j \neq k$ .

Then find  $d_{min}(k,l) = min(dc(k,j))$  and check the relationship between  $d_{min}(k,l)$ ,  $d_k$ , and  $d_1$ . If  $d_k + d_1 \le d_{min}(k,l)$  then the class k is not overlapping with other classes. Otherwise the class k is overlapping with other classes and misclassifications may occur in this case.

Step5: For all the training data within the same class, check how they are classified.

Step6: The mean values of the classes are selected as the centers of RBF units.

## A. Hybrid Learning Algorithm

Training RBF neural network can be made faster than the methods used to train multi-layer neural networks. This is based on the properties of the RBF units, and lead to a two stage training procedure. The first stage of training involves the determination of the output connection weights, which requires the solution of a linear problem, which is fast. In the second stage, the parameters governing the basis function (corresponding to the RBF units) are determined using unsupervised method that is also relatively fast.

Training of the RBF neural network involves estimating output connection weights, centers and widths of RBF units. Dimensionality of the input vector and the number of classes set the number of input and output units respectively. A Hybrid Learning Algorithm (HLA), which combines the gradient method and the Linear Least Squared (LLS) method is proposed for training neural network in this paper. This is done in two steps. In the first step, the neural network connection weights in the output of the RBF units (  $w_{\,2}(i,j)$  ) are adjusted that is shown in Fig. (1) with this assuming that the centers and widths of the RBF units are known. In the second step, the centers and widths (c and  $\sigma$ ) of the RBF units are estimated.

1) Computing Connection Weights: Let r and s be the number of inputs and outputs respectively, and we assume that the number of u RBF outputs are generated for all training face patterns. For any input  $P_i(p_{1i}, p_{2i},..., p_{ri})$ , the j-th output  $y_j$  of the RBF neural network in equation (4) can be calculated in more compact form as follows:

$$W_2 \times R = Y \tag{8}$$

Our objective is the followings:

Given  $R \in \Re^{u \times N}$  that contains c and  $\sigma$  of RBF units and  $T = (t_1, t_2, ..., t_s)^T \in \Re^{s \times N}$ , where N is the total number of sample face patterns and T is the target matrix consisting of 1's and 0's with each column having only one nonzero element. This matrix identifies the processing pattern to which the given exemplar belongs. Furthermore, given the following relationship for error:

$$E = ||T - Y|| \tag{9}$$

We find an optimal coefficient matrix  $W_2 \in \Re^{s \times u}$  such that the  $E^T E$  to be minimized. This is done by the well-known LLS method [7] as follows:

$$W_2' \times R = Y \tag{10}$$

The optimal  $W_2$  is given by:

$$W_2' = T \times R^+ \tag{11}$$

Where R + is the pseudo inverse of R and is given by:

$$R^{+} = (R^{T}R)^{-1}R^{T}$$
 (12)

We can compute the connection weights using equation (11) and (12) by knowing R matrix as follows:

$$W_2' = T(R^T R)^{-1} R^T$$
 (13)

2) Defining Center and Width of RBF Units: Here, the center and width of RBF units (R matrix elements) are adjusted by taking the negative gradient of the error function, E<sup>n</sup>, which is given by [7]:

$$E^{n} = \frac{1}{2} \sum_{k=1}^{s} (t_{k}^{n} - y_{k}^{n})^{2}$$
, n=1,2,...,N (14)

Where  $y_k^n$  and  $t_k^n$  represent the k-th real output and target output of the n-th face pattern respectively.

For the RBF units with the center C and the width  $\sigma$ , the error rate of the center  $\Delta C^n(i,j)$  can be derived from equation (14) by the chain rule as follows:

$$\Delta C^{n}(i,j) = -\xi \frac{\partial E^{n}}{\partial C^{n}(i,j)}$$
 (15)

$$\Delta C^{n}(i,j) = 2\xi \sum_{k=1}^{s} y_{k}^{n}.w_{2}^{n}(k,j).R_{j}^{n}.(P_{i}^{n} - C^{n}(i,j))/(\sigma_{j}^{n})^{2}$$

and the error rate of the width  $\Delta\sigma_{\ j}^{\ n}$  are computed as follow:

$$\Delta \sigma_{j}^{n} = -\xi \frac{\partial E^{n}}{\partial \sigma_{i}^{n}}$$
 (16)

$$\Delta \sigma_{j}^{n} = 2\xi \sum_{k=1}^{s} y_{k}^{n}.w_{2}^{n}(k,j).R_{j}^{n}.(P_{i}^{n} - C^{n}(i,j))/(\sigma_{j}^{n})^{3}$$

Where i=1,2,...,r and j=1,2,...,u and also  $P_i^n$  is the i-th input variable of the n-th face pattern and  $\xi$  is the learning rate.  $\Delta C^n$  (i,j) is the error rate of the center of the i-th input variable of the j-th prototype of the n-th training pattern.  $\Delta \sigma_j^n$  is the error rate of the width of the j-th prototype at the face pattern.

Now we can combine the linear least square and the gradient method to update the parameters in RBF neural network. Each epoch of this hybrid learning algorithm is composed of forward pass and a backward pass. In the forward pass, we calculate the W<sub>2</sub> matrix. In the backward pass, the RBF parameters are obtained.

#### IV. FEATURE EXTRACTION

Statistical-based approaches for feature extraction such as moment invariants [12] have received considerable attention in recent years for their invariance property. The term invariant denotes an image feature remains unchanged if that image undergoes one or a combination of the changes such as: change of size (scale), change of position (translation), change of orientation (rotation), and reflection. The invariant properties of moments are utilized as pattern sensitive features in classification and recognition applications [13].

#### A. Pseudo Zernike Moment Invariant

Pseudo Zernike polynomials are well known and widely used in the analysis of optical systems. Pseudo Zernike polynomials are orthogonal set of complex-valued polynomials defined as [12]:

$$V_{nm}(x,y) = R_{nm}(x,y) \exp(jm tan^{-1}(\frac{y}{x}))$$
 (17)

Where  $x^2 + y^2 \le 1$ ,  $n \ge 0$ ,  $|m| \le n$  is even and Radial polynomials  $R_{nm}$  are defined as:

$$R_{nm}(x,y) = \sum_{s=0}^{n-|m|} D_{n,|m|,s}(x^2 + y^2)^{\frac{n-s}{2}}$$
 (18)

Where:

$$D_{n,|m|,s} = (-1)^{S} \frac{(2n+1-s)!}{s!(n-|m|-s)!(n-|m|-s+1)!}$$
(19)

The PZMI can be computed using the scale invariant central moments and the radial geometric moments that defined in reference [12] as follows:

PZMI 
$$_{nm} = \frac{n+1}{\pi} \sum_{(n-m-s)\text{ even}, s=0}^{n-|m|} D_{n,|m|,s} \sum_{a=0}^{k} \sum_{b=0}^{m} \binom{k}{a} \binom{m}{b}$$

$$(-j)^{b} S_{n,|m|,s} CM_{2k+m-2a-b,2a+b}$$

$$+ \frac{n+1}{\pi} \sum_{(n-m-s)\text{ odd}, s=0}^{n-|m|} D_{n,|m|,s} \sum_{a=0}^{d} \sum_{b=0}^{m} \binom{d}{a} \binom{m}{b}$$

$$(-j)^{b} RM_{2d+m-2a-b,2a+b}$$
(20)

Where k=(n-s-m)/2, d=(n-s-m+l)/2,  $S_{n,|m|,s}$ ,  $CM_{i,j}$  is the Central moments and RM<sub>i,j</sub> is the Radial moments are as follow:

$$S_{n,|m|,s} = (-1)^{S} \frac{(n-s)!}{s!(\frac{n+|m|}{2}-s)!(\frac{n-|m|}{2}-s)!}$$
(21)

$$CM_{pq} = \frac{\mu_{pq}}{M_{00}^{(p+q+2)/2}}$$
 (22)

$$RM_{pq} = \frac{\sum_{x} \sum_{y} f(x,y) (\hat{x}^2 + \hat{y}^2)^{1/2} \hat{x}^p \hat{y}^q}{M_{00}^{(p+q+2)/2}}$$
(23)

Where  $\hat{x}=x-x_0$ ,  $\hat{y}=y-y_0$  and  $x_0$ ,  $y_0$ ,  $\mu_{pq}$  and  $M_{pq}$  are defined as follow:

$$M_{pq} = \sum_{x} \sum_{y} f(x,y) x^{p} y^{q}$$
 (24)

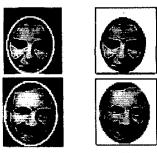
$$\mu_{pq} = \sum_{x} \sum_{y} f(x, y) (x - x_0)^p (y - y_0)^q$$
 (25)

$$x_0 = M_{10} / M_{00} (26)$$

$$y_0 = M_{01} / M_{00} \tag{27}$$

## B. Feature Vector Creation

In this study the feature extraction is done in two steps. In the first step, considering the elliptical shape of a face in general, it is convenient to search for connected component of region using a region-growing algorithm and fit an ellipse to every connected component of nearly elliptical shape [3]. After that we create a subimage, which contains information needed for recognition algorithm. The subimage encloses the pertinent information around the face in the best-fit ellipse while pixel value outside the ellipse is set to zero. This is a basic preprocessing step, because it disregards irrelevant data to facial portion such as hair, shoulders and background and therefore the speed of computing PZMI is increased due to smaller pixels content of the subimages [3]. Figure 2 shows sample of selecting of face location and creating of subimage in feature extracting respectively.



face location subimage
Figure 2: Creating subimages from face images

In the second step, the feature vector is obtained through calculating of the PZMI of the derived subimage. For selecting PZMI as face feature we define four categories of feature vectors based on the order (n) of the PZMI. In the first category that is n=1,2,...6, all moments of PZMI are considered as feature vectors elements. The number of the feature vector elements in this category is 26. In the second category, n=4,5,6,7 is chosen. All moments of each order included in this category are summed up to create a feature vectors of size 26. In the third category, n=6,7,8 is considered. The feature vectors for this category has 24 element. Finally in the last cate-

gory that n=9,10 is considered with 21 feature element. These elements are shown in table 1.

Our experimental study indicates that this categorizing process allows the feature extractor to have a lower dimensional vector while maintaining a good discrimination capability. Also the lower dimensionality of the feature vector lowers computational burden of the recognition algorithm [10].

TABLE | FEATURE VECTORS ELEMENTS BASED ON PZMI

Category no. PZMI feature elements					
PZMI feature elements					
n=1	m=0,1				
n=2	m=0,1,2				
n=3	m=0,1,2,3				
n=4	m=0,1,2,3,4				
n=5	m=0,1,2,3,4,5				
n=6	m=0,1,2,3,4, <u>5,</u> 6				
n≃4	m=0,1,2,3,4				
n=5	m=0,1,2,3,4,5				
n=6	m=0,1,2,3,4,5,6				
n=7	m=0,1,2,3,4,5,6,7				
n=6	m=0,1,2,3,4,5,6				
n=7	m=0,1,2,3,4,5,6,7				
n=8	m=0,1,2,3,4,5,6,7,8				
n=9	m=0,1,2,3,4,5,6,7,8,9				
n=10	m=0,1,2,3,4, <u>5,6,7,8,9,10</u>				
	n=1 n=2 n=3 n=4 n=5 n=6 n=4 n=5 n=6 n=7 n=6 n=7 n=8 n=9				

#### V. EXPRIMENTAL RESULTS

Experimental studies are carried out on the ORL database images of Cambridge University. 400 face images from 40 individuals in different states from the ORL database have been used to evaluate the performance of the proposed method. None of the 10 samples are identical to each other. They vary in position, rotation, scale and expression. In this database each person has changed his face expression in each of 10 samples (open/close eye, smiling/not smiling). For some individuals, the images were taken at different times, varying facial details (glasses/no glasses). Samples of database used are shown in Fig. (3).



Figure 3: Sample of face database

A total of 200 images are used to train and another 200 are used to test. Each training set consists of 5 randomly chosen images from the same class in the training stage. Simulation has been done in four steps based on the order (n) of the PZMI as described in section III.B. Number of elements in the feature vectors in each step depends on the number of moments in each category as was shown in table 1. Neural network classifier is trained in each category based on training images. The outcome of experimental results is shown in the table 2.

TABLE 2 EXPRIMENTAL RESULTS

Features vectors		Training	Testing Phase		
Category	NFE <sup>1</sup>	No. of Epochs	RMSE	NM <sup>2</sup>	ER <sup>3</sup>
n=1,2,,6	27	80 ~ 100	0.09~ 0.06	15	7.5%
n=4,5,6,7	26	60 ~ 80	0.06 ~ 0.04	9	4.5%
п=6,7,8	24	45 ~ 60	0.04 ~ 0.02	6	3%
n=9,10	21	30~45	0.04 ~ 0.01	3	1.3%

- 1: Number of Feature Element
- 2: Number of Misclassification
- 3: Error Rate = NM/ Number of total testing images

This table indicates that in the training phase of the RBF neural network classifier, number of epochs decrease when the moments orders increase. On the other hand, training phase in RBF neural network with HLA learning method has converged faster when higher orders of PZMI are used in comparison with the lower orders of PZMI. Also this table indicate that HLA method in training phase has a lower error (RMSE) with good capability to discriminate classes.

Table (2) shows the recognition rate has increased in spite of the decrease in the number of feature elements of the feature vector. This observation is interesting since the improvement in recognition rate is obtained while the number of moment calculation is decreased.

To compare the HLA learning algorithm with other learning algorithms, we develop the k-mean clustering algorithm for training RBF neural network that very popular method for training RBF neural network [14]. We have applied it to our database with same feature extraction technique. Table 3 shows the outcome of this comparative study.

TABLE 3
COMPARISON BETWEEN TWO METHOD

Feature	K-mean clustering		HLA method	
category	No. of Epochs	RMSE	No. of Epochs	RMSE
1	135 ~ 120	0.12~ 0.09	80 ~ 100	0.09~ 0.06
2	120 ~ 95	0.09 ~ 0.06	60 ~ 80	0.06 ~ 0.04
3	95 ~ 70	0.06 ~ 0.04	45 ~ 60	0.04 ~ 0.02
4	70 ~ 50	0.04 ~ 0.02	30 ~ 45	0.04 ~ 0.01

As table 3 indicates the HLA method has converged faster than the k-mean clustering. Also RMSE during the training phase for HLA method is smaller than k-mean clustering learning algorithm.

Also to compare the effectiveness of our proposed method in comparison with k-mean clustering in overall human face recognition system, we have chosen the PZMI of orders 9 and 10 for feature extraction, which yielded the best result in our study. The feature vector in this case has 21 elements. Also we consider RBF neural network as classifier with HLA and k-mean clustering techniques as learning algorithm. In this study an average error rate is defined as:

$$E_{ave} = \frac{\sum_{i=1}^{m} E_{NM}(i)}{mN_{t}}$$
 (28)

Where m is the number of experimental runs, each being performed on random partition of the database into sets,  $E_{NM}(i)$  is the number of misclassification for the ith run, and  $N_{\rm t}$  is the number of total testing images for each runs. Table 4 shows the result of this comparative.

TABLE 4
ERROR RATE IN DIFFERENT METHOD

ERROR RATE IN DIFFERENT METHOD					
Learning Methods	No. of Experimental (m)	E <sub>ave</sub> %			
k-mean clustering [14]	4	1.723			
HLA	4	1.323			

# VI. CONCLUSION

This paper presents a novel technique for the RBF neural network learning algorithm in recognition of human facial images. This algorithm improves faster convergence of the learning phase of the RBF neural network classifier. A Hybrid Learning Algorithm that was presented in this paper substantially decreases the dimensions of the search space in the gradient method, which is crucial in optimization of high-dimensional problem such as human face recognition. A comparative study with one of the more popular learning techniques demonstrates the superiority of the proposed algorithm. Also In this paper we have shown the relationship between the order of the PZMI and convergence speed of RBF neural network with HLA technique. It is shown while higher orders PZMI contain more information about facial image and hence improves the recognition rate the speed of convergence in learning phase is increased.

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