

Independent Variable: A variable whose variation does not depend on another

Dependent Variable: A variable depending on independent variable(s)

General ODE:

$$F(x, y, y', y^2, \dots, y^{(n)}) = 0$$

Order of the ODE:  $n^{th}$  order if the highest derivative involved is  $n$ .

$$F(x, y, y', y^2, \dots, y^{(n)}) = 0$$

First order ODE:

$$F(x, y, y') = 0$$

Second order ODE:

$$F(x, y, y', y'') = 0$$

Linear:

A transformation,  $L: V \rightarrow W$  is said to be linear if  $L(\alpha x_1 + \beta x_2) = \alpha L(x_1) + \beta L(x_2)$ , for all  $\alpha, \beta \in R$  or  $C$ ,  $x_i \in V$

Linear:

A transformation,  $L: V_1 \times V_2 \rightarrow W_1 \times W_2$  is said to be linear in  $x$  if  $L(\alpha x_1 + \beta x_2, y) = \alpha L(x_1, y) + \beta L(x_2, y)$ , and linear in  $y$  if  $L(x, \alpha y_1 + \beta y_2) = \alpha L(x, y_1) + \beta L(x, y_2)$ , for all  $\alpha, \beta \in R$  or  $C$

Linear first order ODE:

$L(y, y') = F(x, y, y')$  is linear if it is linear in both  $y$  and  $y'$

Linear second order ODE:

$L(y, y', y'') = F(x, y, y', y'')$  is linear if it is linear in both  $y, y'$  and  $y''$ .

Linear  $n^{th}$  order ODE:

$L(y, y', y'', \dots, y^{(n)}) = F(x, y, y', y'')$  is linear if it is linear in each of the dependent variables  $y, y', \dots, y^{(n)}$ .

Implicit Form:

$$F(x, y, y') = 0$$

Explicit Form:

$$y' = f(x, y)$$

Initial Value Problem (Explicit Form):

$$y' = f(x, y) \text{ and } y(x_0) = y_0$$

**Solution:** A value or function that makes the equation valid. A function  $h: (a, b) \rightarrow R$  is said to be a solution of  $y' = f(x, y)$  if

1.  $h$  is differentiable in  $(a, b)$
2.  $h$  satisfies  $h' = f(x, h)$ .

The graph of the curve is called solution curve.

**Fundamental theorem of calculus:** If  $f$  is continuous on  $[a, b]$ , then  $F(x) = \int_a^x f(t)dt$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$  and its derivative is

$$f(x) = \frac{d}{dx} \int_a^x f(t)dt = F'(x)$$

If  $F$  is any derivative of  $f$  on  $[a, b]$ , then

$$\int_a^b f(t)dt = F(b) - F(a)$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int \frac{1}{1+x^2} dx = \tan(x) + c$$

$$\int dx = x + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

**General First Order Linear ODE:**

$$\frac{dy}{dx} + p(x)y = r(x)$$

Homogeneous First Order Linear ODE or NULL:

$$\frac{dy}{dx} + p(x)y = 0$$

Velocity:

Rate of change of displacement of an object.

$$v = \frac{dx}{dt}$$

Acceleration:

Rate of change of velocity of an object.

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Jerk:

Rate of change of acceleration of an object.

$$j = \frac{da}{dt} = \frac{d^3x}{dt^3}$$

Jounce or Snap:

Rate of change of jerk of an object.

$$s = \frac{dj}{dt} = \frac{d^4x}{dt^4}$$

Crackle:

Rate of change of snap of an object.

$$c = \frac{ds}{dt} = \frac{d^5x}{dt^5}$$

Pop:

Rate of change of crackle of an object.

$$p = \frac{dc}{dt} = \frac{d^6x}{dt^6}$$

Momentum:

$$p = mv = m \frac{dx}{dt}$$

Force:

$$F = ma = \frac{dp}{dt} = m \frac{dv}{dt} = m \frac{d^2x}{dt^2}$$

Yank:

Rate of change of force.

$$Y = \frac{dF}{dt} = m \frac{d^3x}{dt^3} = mass \times jerk$$

Tug:

Rate of change of yank.

$$T = \frac{dY}{dt} = m \frac{d^4x}{dt^4} = mass \times snap$$

Snatch:

Rate of change of force.

$$S = \frac{dT}{dt} = m \frac{d^5x}{dt^5} = mass \times Crackle$$

Shake:

Rate of change of force.

$$Sh = \frac{dS}{dt} = m \frac{d^6x}{dt^6} = mass \times Pop$$