

Independent Variable: A variable whose variation does not depend on another

Dependent Variable: A variable depending on independent variable(s)

General ODE:

$$F(x, y, y', y^2, \dots, y^{(n)}) = 0$$

Order of the ODE: n^{th} order if the highest derivative involved is n .

$$F(x, y, y', y^2, \dots, y^{(n)}) = 0$$

First order ODE:

$$F(x, y, y') = 0$$

Second order ODE:

$$F(x, y, y', y'') = 0$$

Linear:

A transformation, $L: V \rightarrow W$ is said to be linear if $L(\alpha x_1 + \beta x_2) = \alpha L(x_1) + \beta L(x_2)$, for all $\alpha, \beta \in R$ or $C, x_i \in V$

Linear:

A transformation, $L: V_1 \times V_2 \rightarrow W_1 \times W_2$ is said to be linear in x if $L(\alpha x_1 + \beta x_2, y) = \alpha L(x_1, y) + \beta L(x_2, y)$, and linear in y if $L(x, \alpha y_1 + \beta y_2) = \alpha L(x, y_1) + \beta L(x, y_2)$, for all $\alpha, \beta \in R$ or C

Linear first order ODE:

$L(y, y') = F(x, y, y')$ is linear if it is linear in both y and y'

Linear second order ODE:

$L(y, y', y'') = F(x, y, y', y'')$ is linear if it is linear in both y, y' and y'' .

Linear n^{th} order ODE:

$L(y, y', y'', \dots, y^{(n)}) = F(x, y, y', y'')$ is linear if it is linear in each of the dependent variables $y, y', \dots, y^{(n)}$.

Implicit Form:

$$F(x, y, y') = 0$$

Explicit Form:

$$y' = f(x, y)$$

Initial Value Problem (Explicit Form):

$$y' = f(x, y) \text{ and } y(x_0) = y_0$$

Solution: A value or function that makes the equation valid. A function $h: (a, b) \rightarrow R$ is said to be a solution of $y' = f(x, y)$ if

1. h is differentiable in (a, b)
2. h satisfies $h' = f(x, h)$.

The graph of the curve is called solution curve.

Fundamental theorem of calculus: If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t)dt$ is continuous on $[a, b]$ and differentiable on (a, b) and its derivative is

$$f(x) = \frac{d}{dx} \int_a^x f(t)dt = F'(x)$$

If F is any derivative of f on $[a, b]$, then

$$\int_a^b f(t)dt = F(b) - F(a)$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int \frac{1}{1+x^2} dx = \tan(x) + c$$

$$\int dx = x + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

General First Order Linear ODE:

$$\frac{dy}{dx} + p(x)y = r(x)$$

Homogeneous First Order Linear ODE or NULL:

$$\frac{dy}{dx} + p(x)y = 0$$

Velocity:

Rate of change of displacement of an object.

$$v = \frac{dx}{dt}$$

Acceleration:

Rate of change of velocity of an object.

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Jerk:

Rate of change of acceleration of an object.

$$j = \frac{da}{dt} = \frac{d^3x}{dt^3}$$

Jounce or Snap:

Rate of change of jerk of an object.

$$s = \frac{dj}{dt} = \frac{d^4x}{dt^4}$$

Crackle:

Rate of change of snap of an object.

$$c = \frac{ds}{dt} = \frac{d^5x}{dt^5}$$

Pop:

Rate of change of crackle of an object.

$$p = \frac{dc}{dt} = \frac{d^6x}{dt^6}$$

Momentum:

$$p = mv = m \frac{dx}{dt}$$

Force:

$$F = ma = \frac{dp}{dt} = m \frac{dv}{dt} = m \frac{d^2x}{dt^2}$$

Yank:

Rate of change of force.

$$Y = \frac{dF}{dt} = m \frac{d^3x}{dt^3} = mass \times jerk$$

Tug:

Rate of change of yank.

$$T = \frac{dY}{dt} = m \frac{d^4x}{dt^4} = mass \times snap$$

Snatch:

Rate of change of force.

$$S = \frac{dT}{dt} = m \frac{d^5x}{dt^5} = mass \times Crackle$$

Shake:

Rate of change of force.

$$Sh = \frac{dS}{dt} = m \frac{d^6x}{dt^6} = mass \times Pop$$