MATHEMATICS FOR ENGINEERS

MA5101-DIFFERENTIAL EQUATIONS

PANCHATCHARAM MARIAPPAN
panch.m@iittp.ac.in

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Course outline

1. Grading
2. Course Contents
3. Introduction

Grading Pattern

Grading

- Assignment - 5 Marks
- Surprise Test/Seminar - 5 Marks
- Final Exams - 40 Marks

Course Contents

ODE

ODE

- Linear Equations - First order, Second Order
- Variation of Constants
- Power-Series Solutions
- Legendre and Bessel Functions
- Fourier Series and Integrals
- Sturm-Liouville BVP

PDE

PDE

- Wave Equations
- Heat Equations
- Laplace Equations

Reference Books

References

- E. Kreyszig, Advanced Engineering Mathematics,10th Edition, John Wiley & Sons, 2010.
- C. Roberts, Ordinary differential equations: applications, models and computing, Chapman & Hall/CRC, 2010
- C. Chicone, Ordinary differential equations with applications, Springer, 1999
- C. Ray Wylie, Advanced Engineering Mathematics, 6th Edition, McGraw-Hill Higher Education, 1995.

Lecture Notes

Lecture Notes, Assignments and Presentation

`https://github.com/panchiittp/MA5101.git`

Problems

Problem 1: Is Arjun a Criminal?

There was a murder in a hotel at room number 315 at 4:30PM. Police arrested Arjun, who was in the next room at 5:00PM. But, Arjun claims that he was not in his room for at least half an hour. The police check the water temperature of his tea kettle in his room at the instant of arrest and again 30 minutes later, obtaining the values $87^{\circ}C$ and $43^{\circ}C$, respectively. Can you investigate the case as an inspector and report whether Arjun is a criminal or not?

Problems

Problem 1: Newton's Law of Cooling

$$\frac{dT}{dt} = k(T - T_A) \tag{1}$$

Problems

Problem 2: Outflow of water Through a hole

In a hostel, there is a cylindrical water tank of diameter 2m and height 2.25m. On a fine day, when Ragu was the first person to take the shower at 7AM in the hostel, the tank was empty. After an inspection by the hostel warden, it was found that there is a circular hole in the water tank. The hostel watchman claims that he filled the water completely at 1AM. Without manually measuring the diameter of the hole, could you calculate the diameter of the hole from given information?

Problems

Problem 2: Torricelli' law

$$v(t) = 0.600\sqrt{2gh(t)} \qquad (2)$$

$$\begin{aligned} \Delta V &= Av\Delta t \\ \Delta V^* &= -B\Delta h \\ \Delta V &= \Delta V* \\ \frac{dh}{dt} &= -26.56\frac{A}{B}\sqrt{h} \end{aligned}$$

$A =$ Area of hole, $B =$ Cross-Sectional Area of the tank,
$V =$ Outflow volume, $V^* =$ volume of the water in the tank.

Problems

Problem 3: Drug Injection

One hour before a surgery, certain drug at a constant amount was injected to the patient's blood stream. Certain amount of drug is simultaneously to avoid over dosage of drugs which is proportional to the amount of the drug present at time t.

Problems

Problem 3: Drug Injection

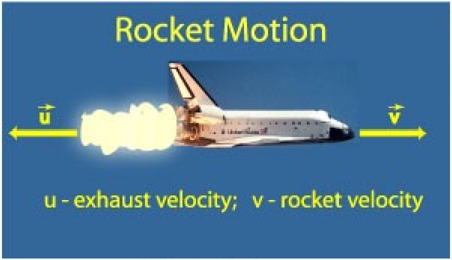
$$\frac{dy}{dt} = A - ky \tag{3}$$

Problems

Problem 4: Rocket Equation

Assume that you are a rocket scientist. Assume the following: initial mass of the rocket $= m$, initial velocity $= v$. In certain time dt, the mass of the rocket decreases by dm due to fuel combustion. So, velocity increases by dv. Conservation of momentum to the system of the rocket and gas flow. Initial momentum of the system is equal to $p = mv$. In a small time dt the momentum of the rocket becomes $p_1 = (m - dm)(v + dv)$. Momentum of the exhaust gases, $p_2 = dm(v - u)$.

Problems



Problem 4: Hormone Level

$$p = p_1 + p_2 or \frac{dv}{dm} = \frac{u}{m} \tag{4}$$

Problems

Problem 5: Hormone Level

It was found that hormone level of a patient varies w.r.to time. The rate of change of the hormone w.r.to time is the difference between the sinusoidal input of a 24-hour period from thyroid gland and a continuous removal rate proportional to the level. What is the hormone model?

Problems

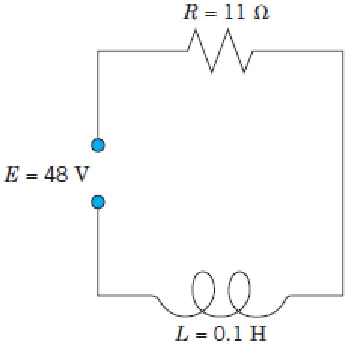
Problem 5: Hormone Level

$$y'(t) = A + B sin\omega t - Ky(t) \tag{5}$$

Problems

Problem 6: Electric Circuit

Model the following RL-Circuit for the current under the assumption that the initial current is zero.



Circuit

Problems

Problem 6: Ohm's Law

$$V = IR \tag{6}$$

Kirchhoff's Law

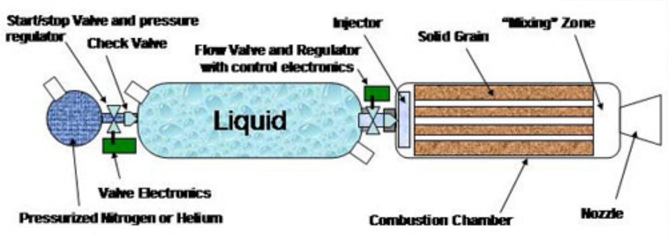
$$LI' + IR = E \tag{7}$$

Problems

Problem 7: Mixing in a Tank

A hybrid fuel tank in a rocket works on the principle of mixing two different fuel substance for combustion which in turn produces fuel supply for the throttle. The first tank contains 2 million litres of fuel in which another substance of 0.18 million kg of solid fuel substance is dissolved. Each 50 litre of the fuel fed into the throttle after mixing contains $(1 + cost)$ kg of the dissolved solid fuel substance. The mixture is uniform and runs to the throttle at the same rate. What is the amount of solid fuel substance at any time t

Problems



Problem 7

$$y' = 50(1 + cost) - 0.000025y \tag{8}$$

Problems

Problem 8: Pied Piper

In the city of Hamelin, the rat population was a big problem. It was initially assumed that the rate of change rat population w.r.to time is equal to twice its population at any given day. Until the pied piper arrived, people killed the rats and hence the growth rate of rat population decreased in proportion to the population every day. How many years will be required to have a rat-free city if the initial and 1 year population of the rat are respectively 2 million and 1.5 million?

Problems

Problem 8

$$y' = (2 - ay)y \tag{9}$$

Problems

Problem 9: Epidemics

A model for the spread of contagious diseases is obtained by assuming that the rate of spread is proportional to the number of contacts between infected and non-infected persons, who are assumed to move freely among each other.

Problems

Problem 9

$$y' = ky(1 - y) \tag{10}$$

Problems

Problem 10: Fish Harvesting

Suppose that the population of a certain kind of fish is given by the logistic equation

$$y' = (A - By)y$$

and fish are caught at a rate Hy proportional to y. Find the model

Problems

Problem 10: Schaefer Model

$$y' = (A - H - By)y \tag{11}$$

Problems

Problem 11: Fresh Air

In a room containing of air, 20 cubic m^3 of fresh air flows in per minute, 600 m^3 and the mixture (made practically uniform by circulating fans) is exhausted at a rate of 600 cubic metre per minute. What is the amount of fresh air at any time if there are no initial air? After what time will 90% of the air be fresh?

Problems

Problem 11

$$y' = \frac{600(20000 - y)}{20000} \tag{12}$$

Problems

Problem 12: Fresh Air

Suppose that in winter the daytime temperature in a European Space Agency office building is maintained at $21 \circ C$. The heating is shut off at 10 P M. and turned on again at 6 A M. On a certain day the temperature inside the building at 2 A M. was found to be $18 \circ C$. The outside temperature was $10 \circ C$ at 10 P M. and had dropped to $4 \circ C$ by 6 A M. What was the temperature inside the building when the heat was turned on at 6 A M.?

Problems

Problem 12: Newton's Law of cooling

$$y' = k(T - T_A) \tag{13}$$

Thank you for your attention