Tutorial Sheet: MA 5101 - 02-November-18

Indian Institute of Technology Tirupati

Department of Mathematics

MA5101 – Mathematics for Engineers – Tutorial Sheet

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Problem 1: Find the particular solution of the following ODE using **Green's function**

a) $y'' - y = f(x)$ (d) $y'' - y = \frac{1}{x}, y(1) = 0, y'(1) = 0$

b) $y'' - y = e^{2x}, y(0) = 0, y'(0) = 0$ (e) $y'' + 4y = x, y(0) = 0, y'(0) = 0$

c) $y'' + 4y = \sin 2x, y(0) = 1, y'(0) = -2$ (f) $y'' + 4y = 3, y'(0) = 0, y\left(\frac{\pi}{2}\right) = 0$

$$(g) y'' + 4y = f(x), y(0) = 1, y'(0) = -2, f(x) = \begin{cases} 0 & x < 0 \\ \sin 2x & 0 \le x \le 2\pi \\ 0 & x > 2\pi \end{cases}$$

Problem 2: Find the radius and interval of convergence of the following **power series**

$$(a) \sum_{m=0}^{\infty} (m+1)mx^m \quad (b) \sum_{m=0}^{\infty} x^m \quad (c) \sum_{m=0}^{\infty} \frac{(-1)^{n+1}}{n} x^n \quad (d) \sum_{m=0}^{\infty} \frac{5^n}{n!} x^n$$

Problem 3: Find the power series solution for the following ODEs

a) $y' + y = 0$ (b) $y' - 5y = 0$ (c) $(1 + x)y' = y$ (d) $y' = -2xy$

Problem 4: Identify ordinary, singular, regular singular and irregular singular points for the following ODEs.

a) $(x^2 - 4)^2 y'' + 3(x - 2)y' + 5y = 0$ (c) $3xy'' + y' - y = 0$

b) $x^3 y'' + 4x^2 y' + 3y = 0$ (d) $(x^2 + x - 6)y'' + (x + 3)y' + (x - 2)y = 0$

Problem 5: Use **Frobenius method** to find two independent solutions for the following ODE

a) $x(x - 1)y'' + (3x - 1)y' + y = 0$ (c) $(x^2 - x)y'' - xy' + y = 0$

b) $(x + 2)^2 y'' + (x + 2)y' - y = 0$ (d) $x^2 y'' + 6xy' + (4x^2 + 6)y = 0$

Problem 6: Legendre's equation

(a) Using $n = 0$, prove that $P_0(x) = 1$ and $y_2(x) = x + \frac{1}{3!}x^3 + \frac{1}{5}x^5 + \cdots = \frac{1}{2}\ln\frac{1+x}{1-x}$
Verify this by solving, the Legendre equation with $n = 0$.

(b) Using $n = 1$, prove that $y_2(x) = P_1(x) = x$ and

$$y_1 = 1 - x^2 - \frac{1}{3}x^4 - \frac{1}{5}x^6 - \cdots = 1 - \frac{1}{2}x \ln\frac{1+x}{1-x}$$

d) Applying the binomial theorem to $(x^2 - 1)^n$, differentiating it n times term by term, and comparing the result with $P_n(x)$, show that

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n[(x^2 - 1)^n]}{dx^n}$$

e) Using the following recurrence relation, list the first six Legendre polynomials

$$(k + 1)P_{k+1}(x) - (2k + 1)xP_k(x) + kP_{k-1}(x) = 0, k = 1,2,3$$

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f) Show that the following DE can be transformed to Legendre's equation

$$sin\theta \frac{d^2y}{d\theta^2} + cos\theta \frac{dy}{d\theta} + n(n+1)sin\theta y = 0$$

Problem 7: Bessel's equation: Prove the following

$$(a) J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x, \qquad (b)\, J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x, (c)\, J_{-n}(x) = (-1)^n J_n(x),$$

$$(d)\, J_0'(x) = -J_1(x), J_1'(x) = J_0(x) - J_1(x), J_2'(x) = \frac{1}{2}[J_1(x) - J_3(x)]$$

e) Derive $xJ_v'(x) = vJ_v(x) - xJ_{v+1}(x)$

Problem 8: Find general solution in terms of J_v, Y_v

a) $x^2y'' + xy' + (x^2 - 16)y = 0$ (c) $xy'' + 5y' + xy = 0, use\ y = \frac{u}{x^2}$

b) $xy'' - 5y' + xy = 0, y = x^3u$

Fourier Series and Fourier Integral

Problem 9: Find the Fourier coefficients of the periodic function $f(x)$

$$a)\, f(x) = \begin{cases} -k & if \quad -\pi < x < 0 \\ k & if \quad 0 < x < \pi \end{cases} \ and\ f(x + 2\pi) = f(x)$$

$$b)\, f(x) = \begin{cases} x & if \quad -\pi < x < 0 \\ \pi - x & if \quad 0 < x < \pi \end{cases} \ and\ f(x + 2\pi) = f(x)$$

$$c)\, f(x) = \begin{cases} -\cos^2 x & if \quad -\pi < x < 0 \\ \cos^2 x & if \quad 0 < x < \pi \end{cases} \ and\ f(x + 2\pi) = f(x)$$

$$d)\, f(x) = |x|, -\pi < x < \pi \ e)\, f(x) = x^2, -\pi < x < \pi$$

Problem 10: Find the Fourier sine integral representation of the function

$$a)\, f(x) = \begin{cases} \frac{\pi}{2} & if \quad 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases} \text{ and } b)\, f(x) = \begin{cases} \frac{\pi}{2} \cos x & if \quad 0 < |x| < \frac{\pi}{2} \\ 0 & if \quad |x| \geq \frac{\pi}{2} \end{cases}$$

Problem 11: Find the Fourier cosine integral representation of the function

$$a)\, f(x) = \begin{cases} x^2 & if \quad 0 < x < 1 \\ 0 & if \quad x > 1 \end{cases} \ and\ b)\, f(x) = \begin{cases} \sin x & if \quad 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases}$$

Problem 12: Find the Fourier sine integral representation of the function

$$a)\, f(x) = \begin{cases} x & if \quad 0 < x < 1 \\ 0 & if \quad x > 1 \end{cases} \text{ and } b)\, f(x) = \begin{cases} \cos x & if \quad 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases}$$

Problem 13: Prove that $y_m(x) = \sin mx, m = 1,2, \ldots$ form an orthogonal set on $[-\pi, \pi]$.