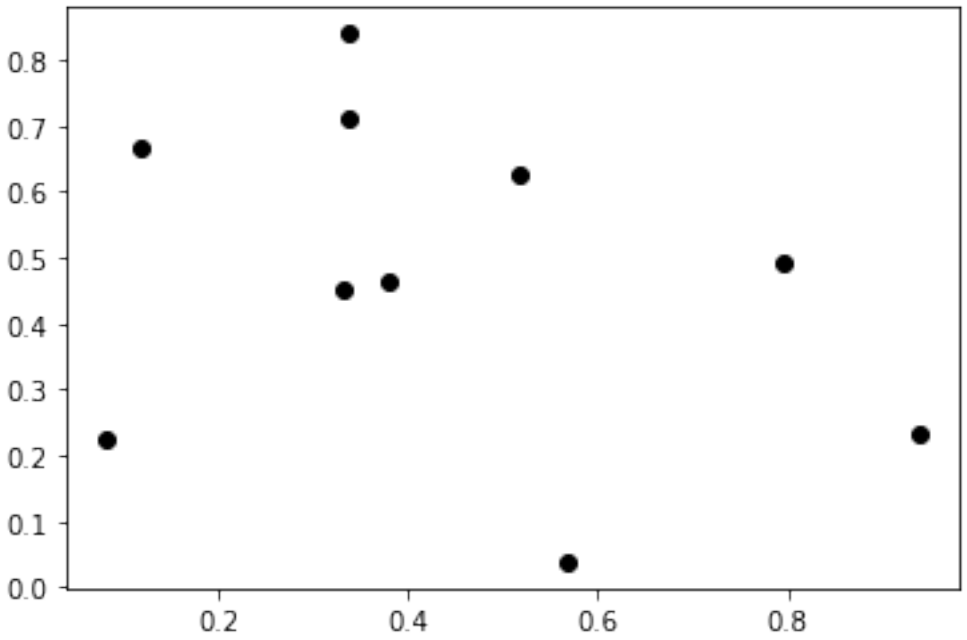
Ejemplos Alpha y VR

October 14, 2021

1 Ejemplo 1

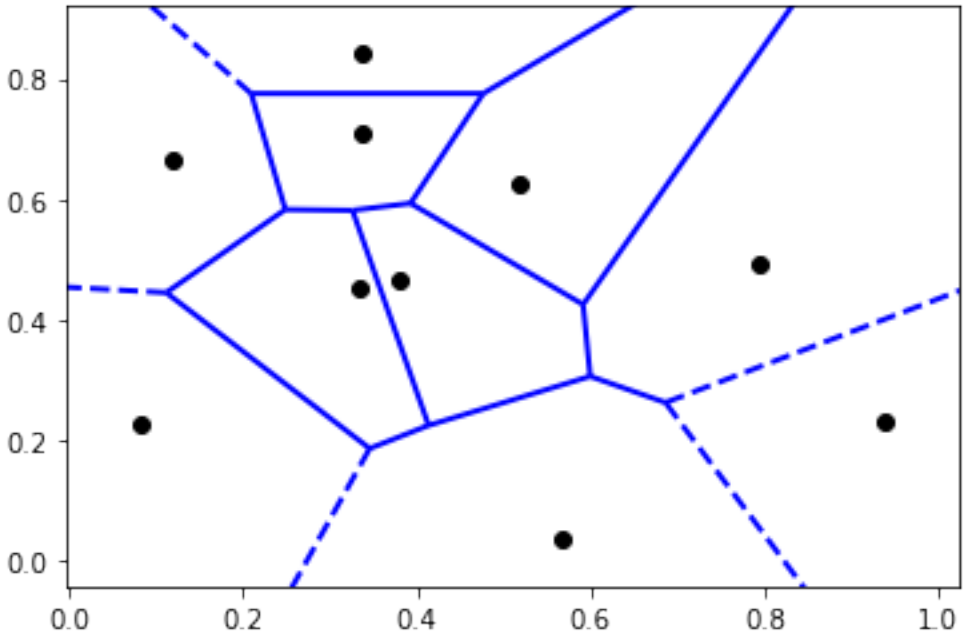
Consideramos diez puntos en el plano

[94]:
```
points=np.array([(0.38021546727456423, 0.46419202339598786), (0.
↪7951628297672293, 0.49263630135869474), (0.566623772375203, 0.
↪038325621649018426), (0.3369306814864865, 0.7103735061134965), (0.
↪08272837815822842, 0.2263273314352896), (0.5180166301873989, 0.
↪6271769943824689), (0.33691411899985035, 0.8402045183219995), (0.
↪33244488399729255, 0.4524636520475205), (0.11778991601260325, 0.
↪6657734204021165), (0.9384303415747769, 0.2313873874340855)])
plt.plot(points[:,0],points[:,1],'ko')
plt.show()
```



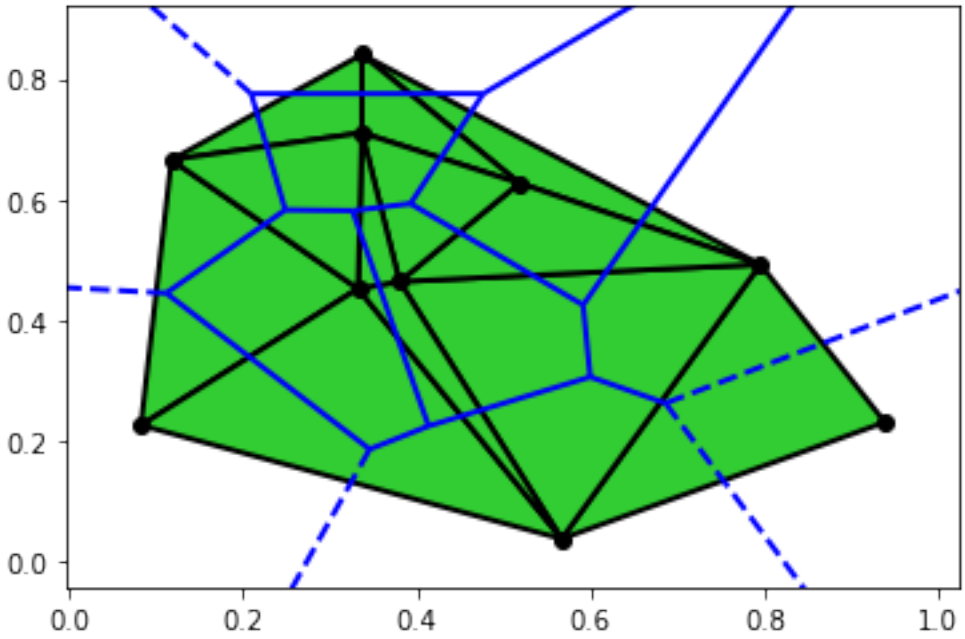
Calculamos y representamos su diagrama de Voronoi

[22]:
```
vor=Voronoi(points)
fig = voronoi_plot_2d(vor,show_vertices=False,line_width=2, line_colors='blue')
plt.plot(points[:,0],points[:,1],'ko')
plt.show()
```



Calculamos y representamos su triangulación de Delaunay sobre el diagrama de Voronoi

[23]:
```
tri = Delaunay(points)
fig = voronoi_plot_2d(vor,show_vertices=False,line_width=2, line_colors='blue' )
c=np.ones(len(points))
cmap = matplotlib.colors.ListedColormap("limegreen")
plt.tripcolor(points[:,0],points[:,1],tri.simplices, c, edgecolor="k", lw=2,␣
 ↪cmap=cmap)
plt.plot(points[:,0], points[:,1], 'ko')
plt.show()
```



Calculamos la filtración de alfa complejos asociada

[24]:
```
alpha=AlphaComplex(points)
```

Mostramos por pantalla la lista de símplices del complejo ordenadas según su umbral de aparación.

[25]:
```
alpha.filtrationorder()
```

[25]:
```
[(0,),
 (1,),
 (2,),
 (3,),
 (4,),
 (5,),
 (6,),
 (7,),
 (8,),
 (9,),
 (0, 7),
 (3, 6),
 (3, 5),
 (0, 5),
 (3, 8),
 (0, 3),
 (3, 7),
 (0, 3, 7),
```

```
 (0, 3, 5),
 (6, 8),
 (3, 6, 8),
 (1, 9),
 (7, 8),
 (5, 6),
 (3, 5, 6),
 (1, 5),
 (3, 7, 8),
 (4, 7),
 (2, 9),
 (0, 1),
 (0, 1, 5),
 (4, 8),
 (4, 7, 8),
 (0, 2),
 (2, 7),
 (0, 2, 7),
 (1, 2),
 (1, 2, 9),
 (2, 4),
 (2, 4, 7),
 (0, 1, 2),
 (1, 6),
 (1, 5, 6)]
```

Mostramos la lista de umbrales para los que aparecen nuevos símplices

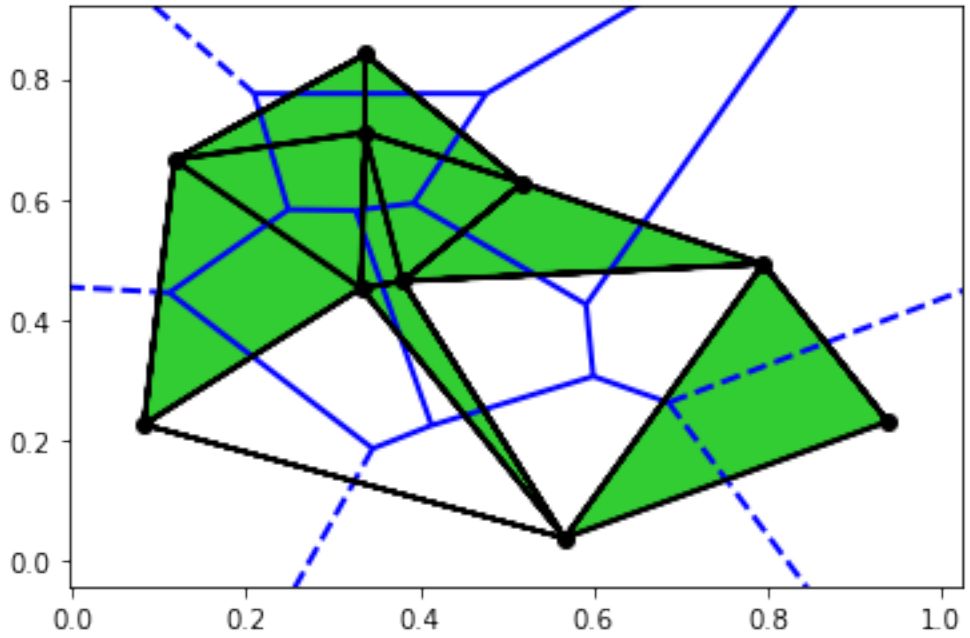
[26]:
```
alpha.thresholdvalues()
```

[26]:
```
[0.0,
 0.024594630110749274,
 0.06491550663246884,
 0.09964158316325628,
 0.1067160499498478,
 0.11181663867459748,
 0.12497889333174499,
 0.12926191918239144,
 0.12972005087987803,
 0.14290429288721868,
 0.14897699064479192,
 0.151309131049649,
 0.15386007896068243,
 0.15403831830169779,
 0.15498302512711043,
 0.16844581383081997,
 0.20947134070559964,
 0.2146289409772832,
```

```
0.22042128784724613,
0.22072722035185588,
0.23243829737556373,
0.24143718415372598,
0.25427755231828075,
0.25434295608514496,
0.259566655903143,
0.26605192026672236,
0.27011640994552,
0.7144105889522738]
```

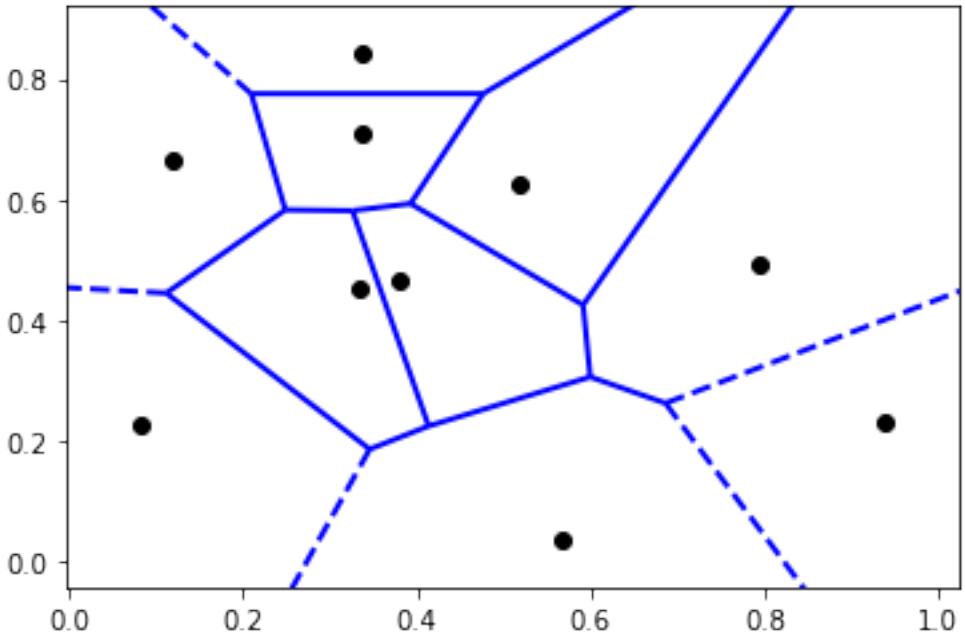
Calculamos y representamos gráficamente el alfa complejo para r=0.26

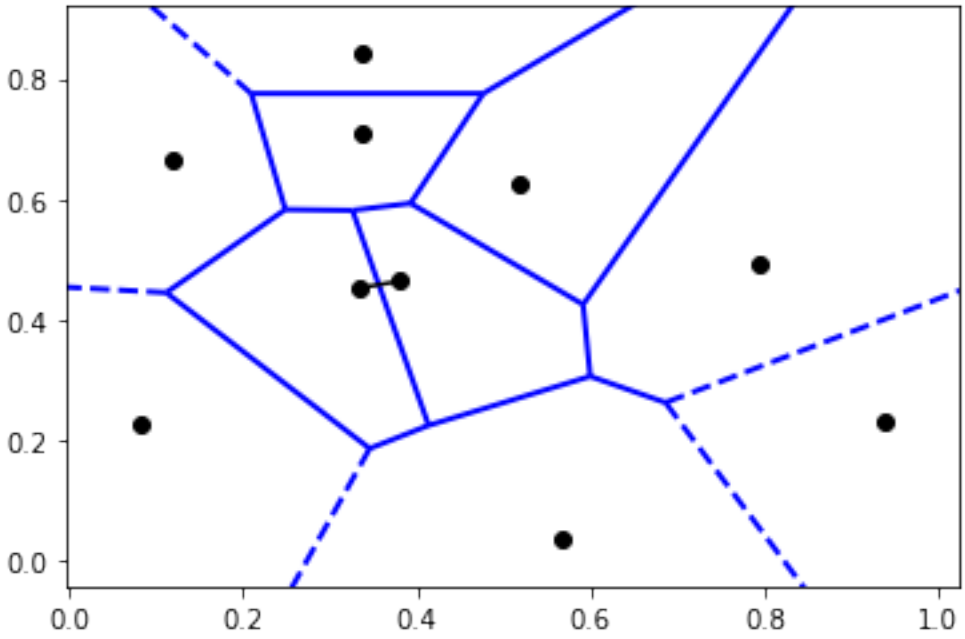
[53]:
```
K=alpha.sublevel(0.26)
plotalpha(points,K)
```

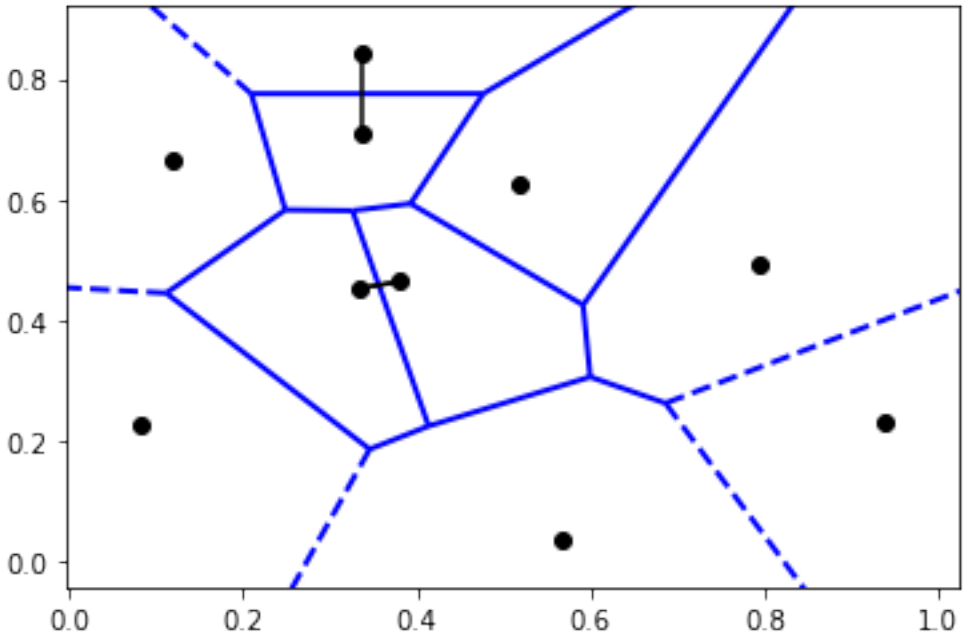


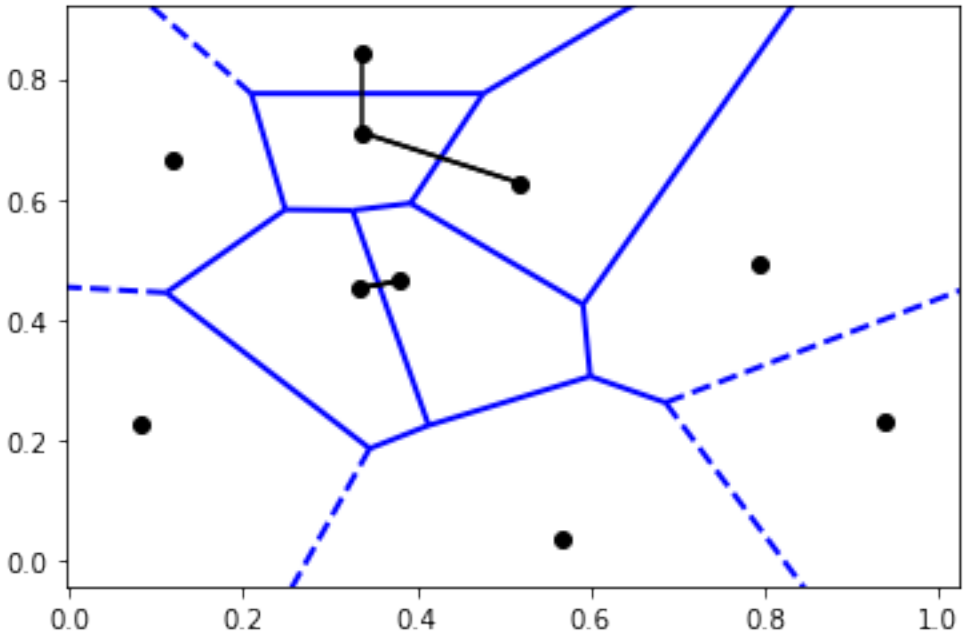
Representamos gráficamente toda la filtración

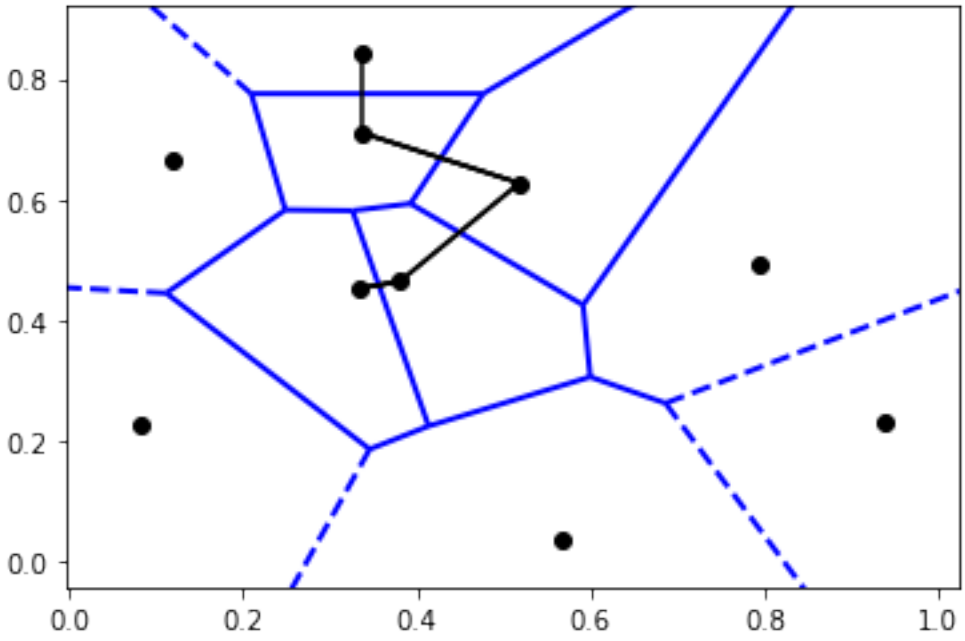
[55]:
```
for value in alpha.thresholdvalues():
    K=alpha.sublevel(value)
    plotalpha(points,K)
    plt.show()
```

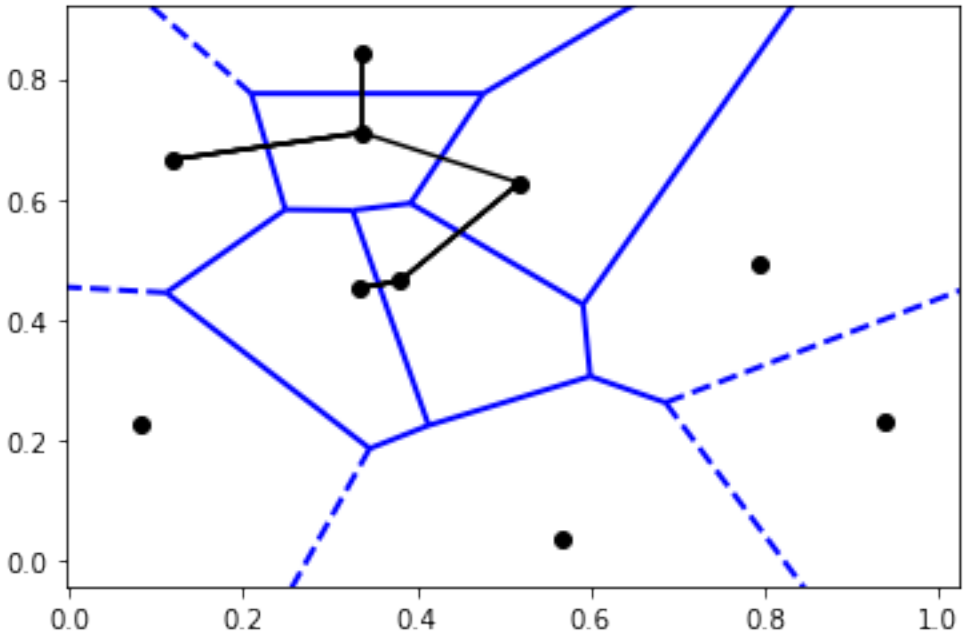

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

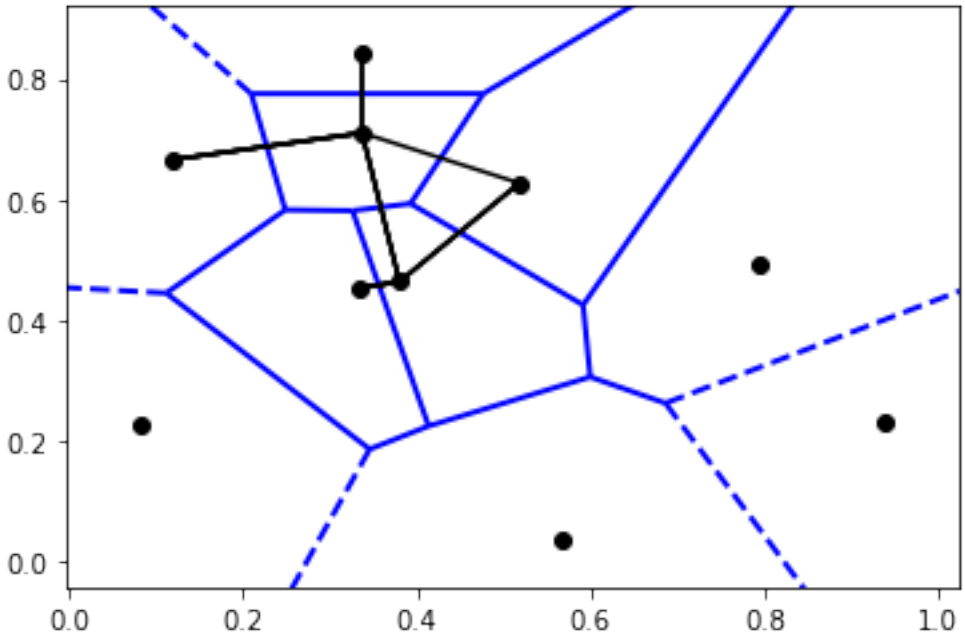

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

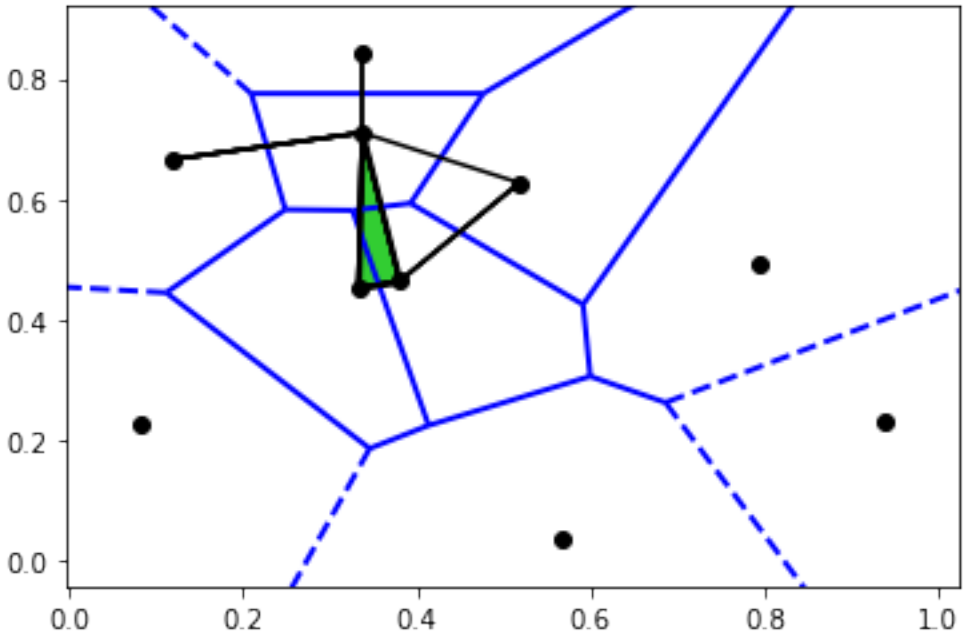

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

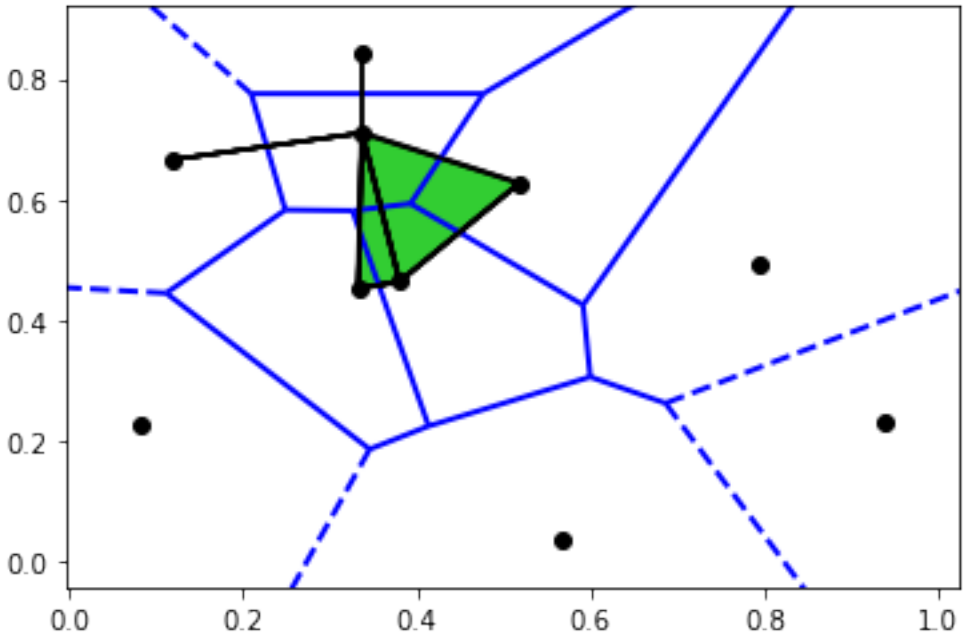

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

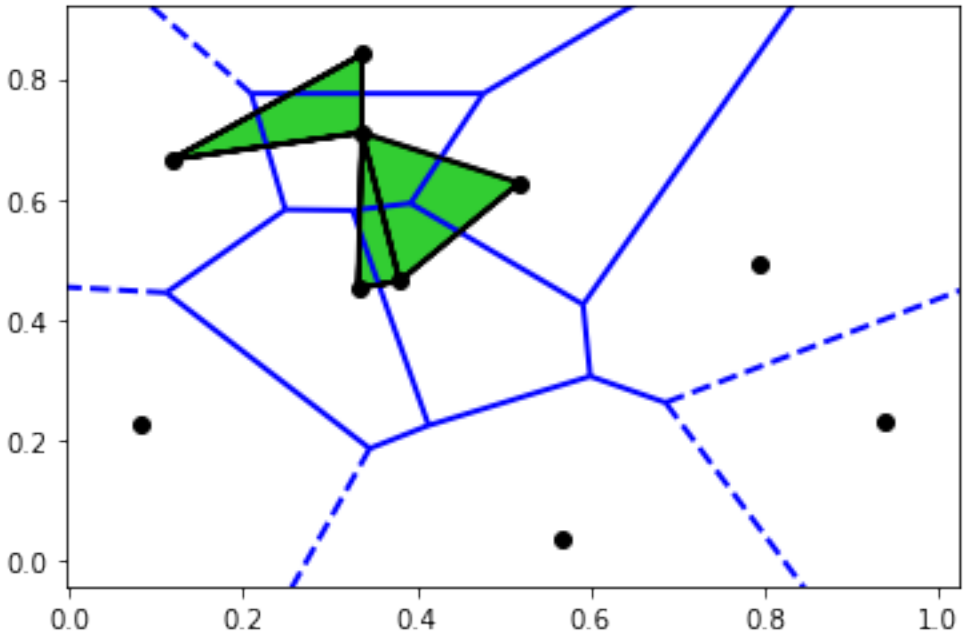

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

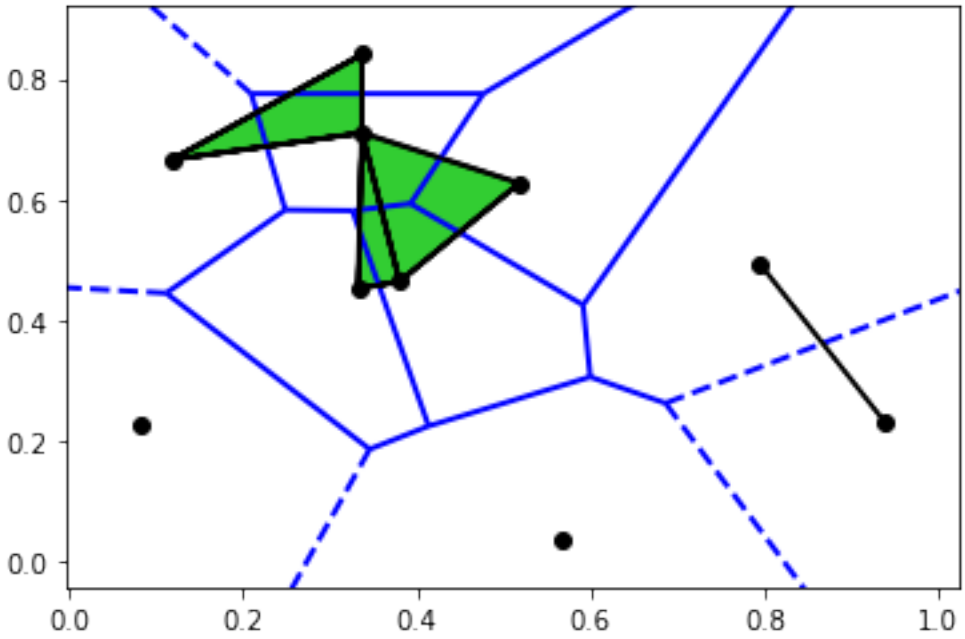

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

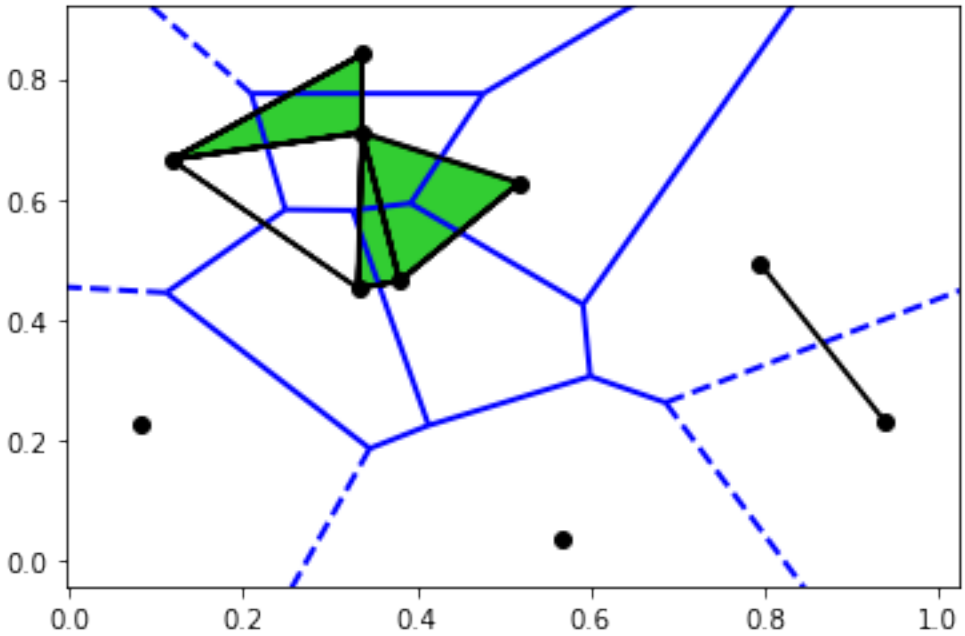

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

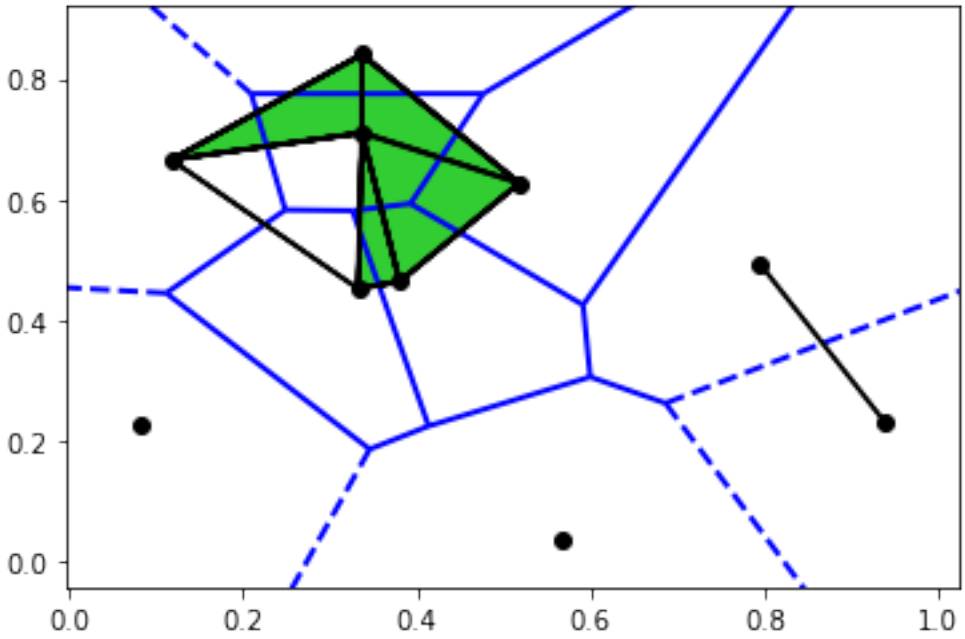

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

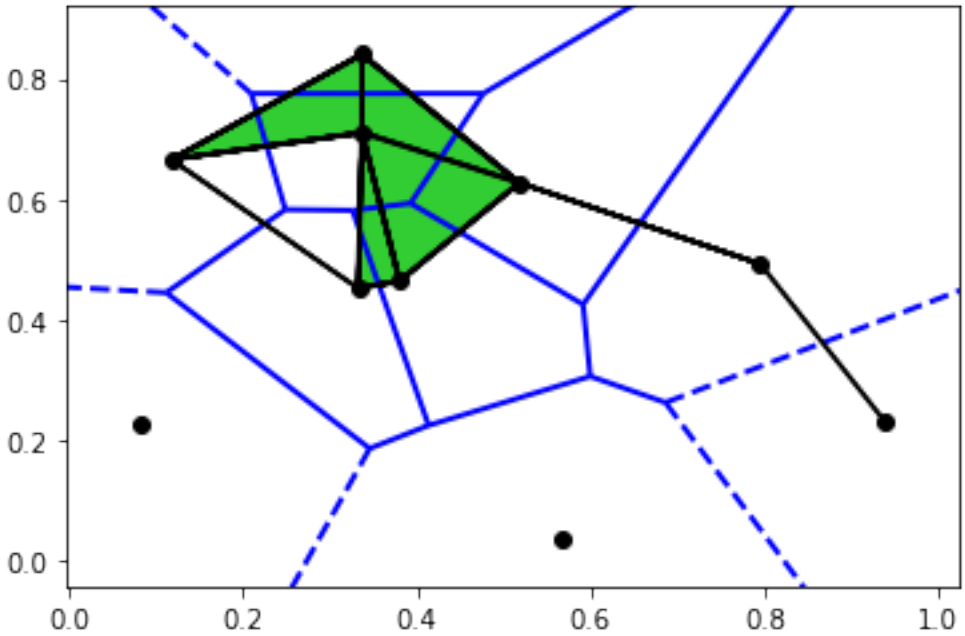

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

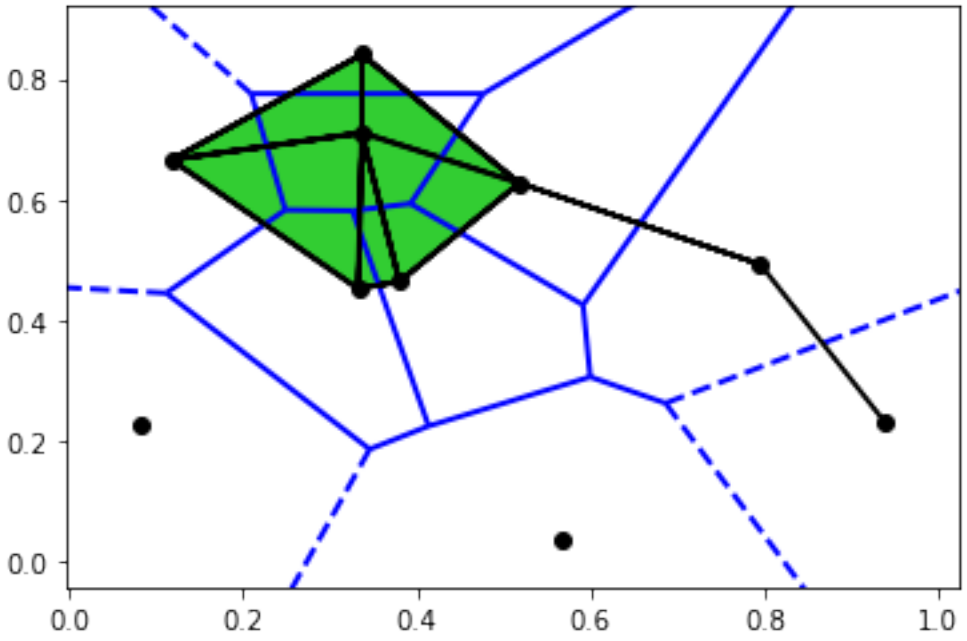

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

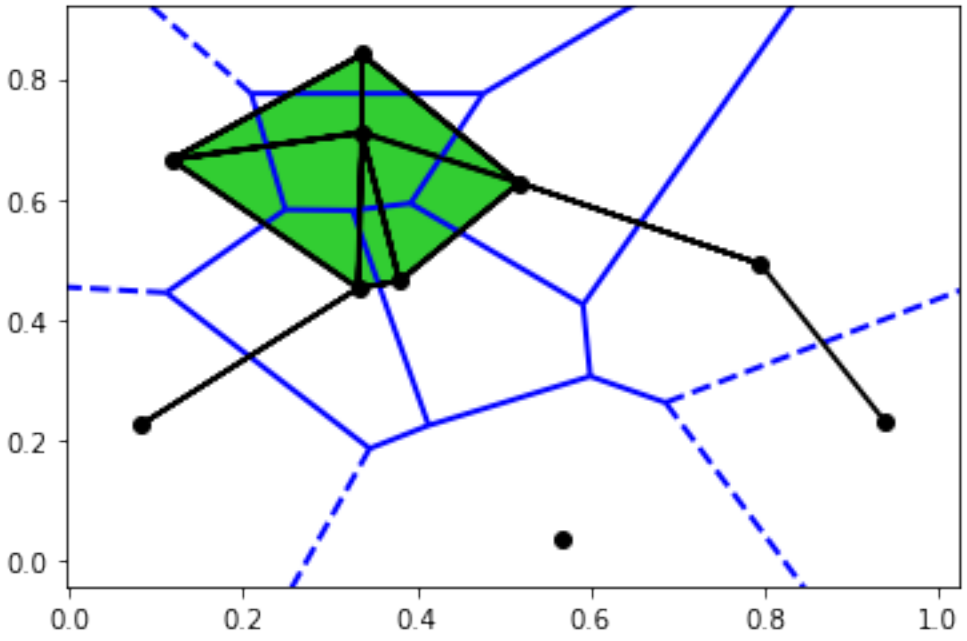

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

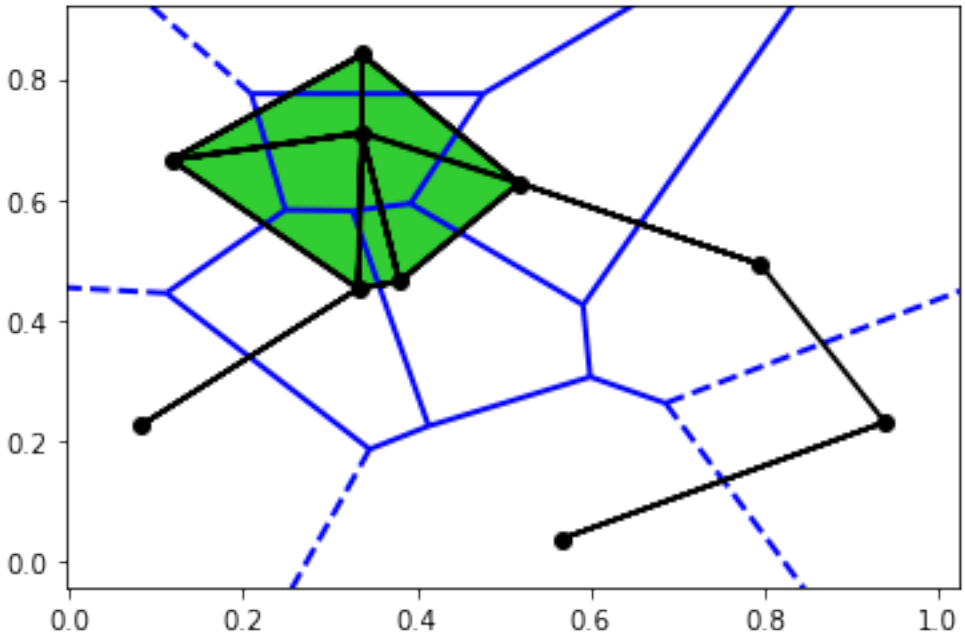

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

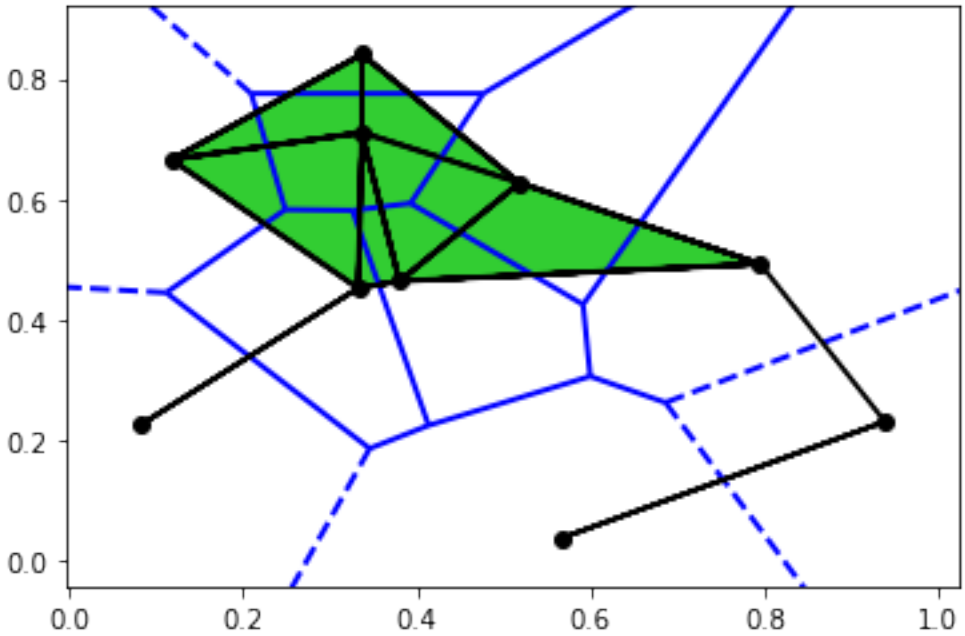

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

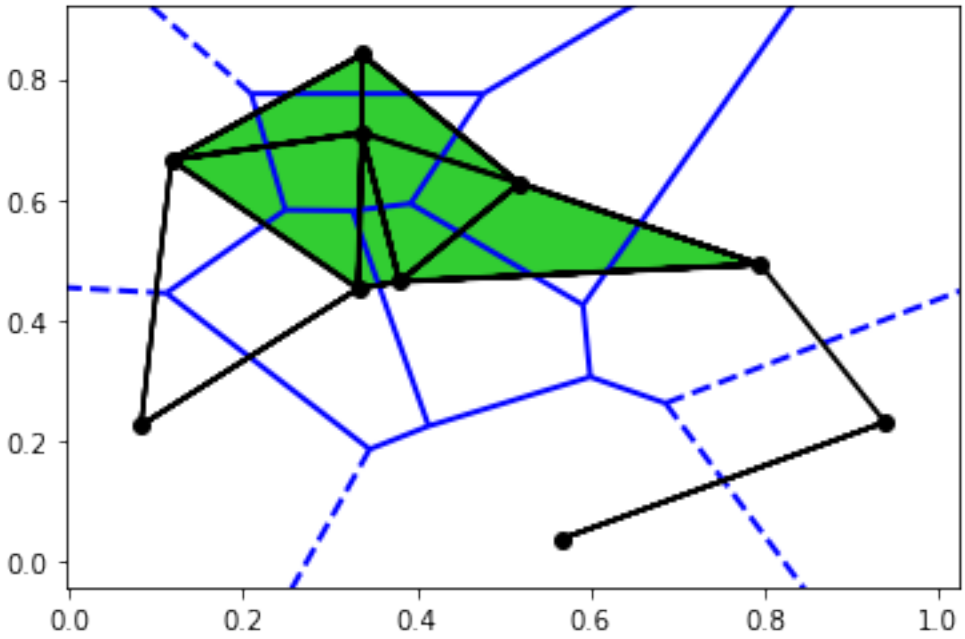

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

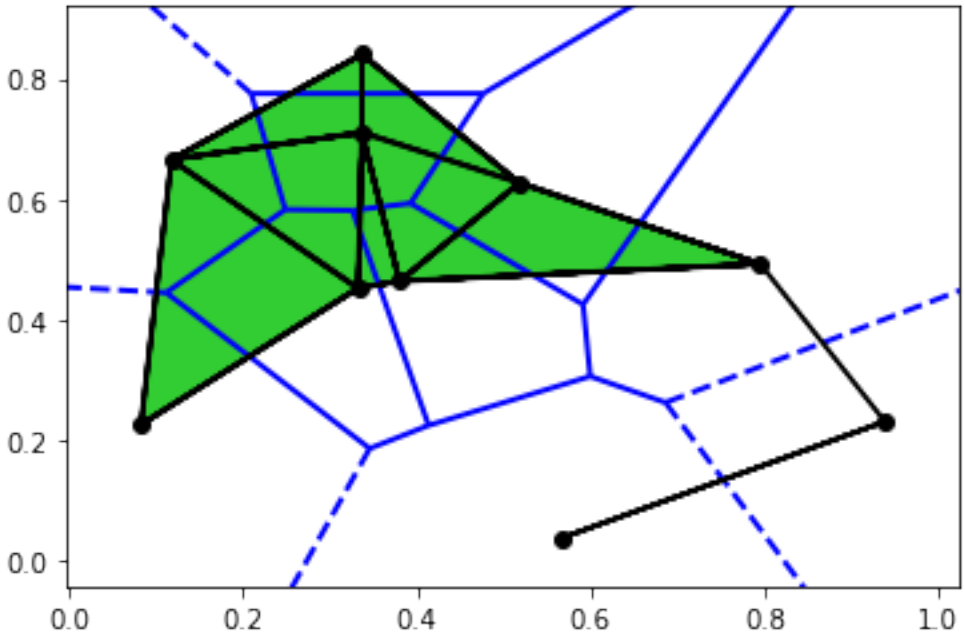

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

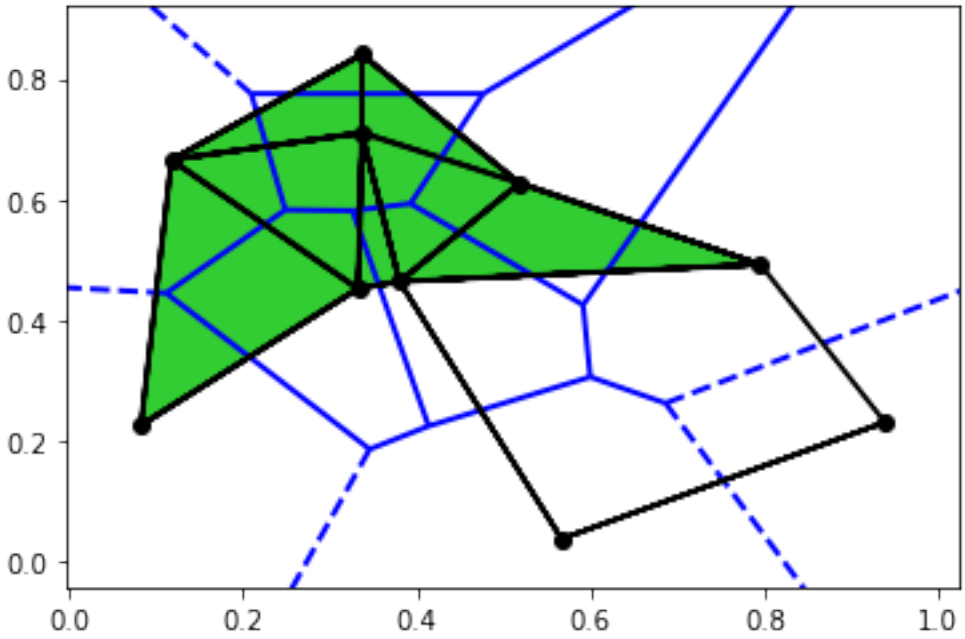

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

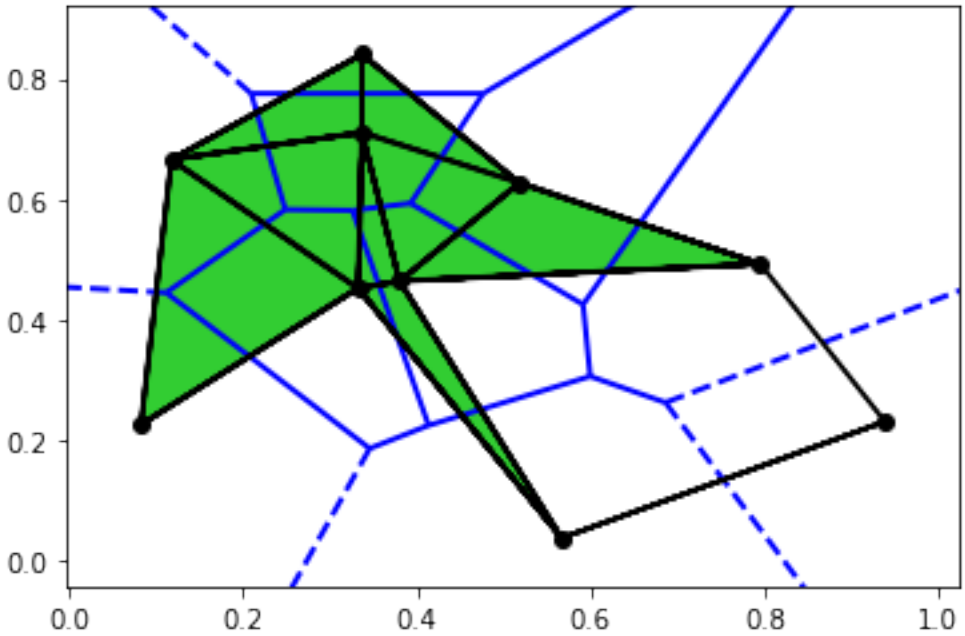

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

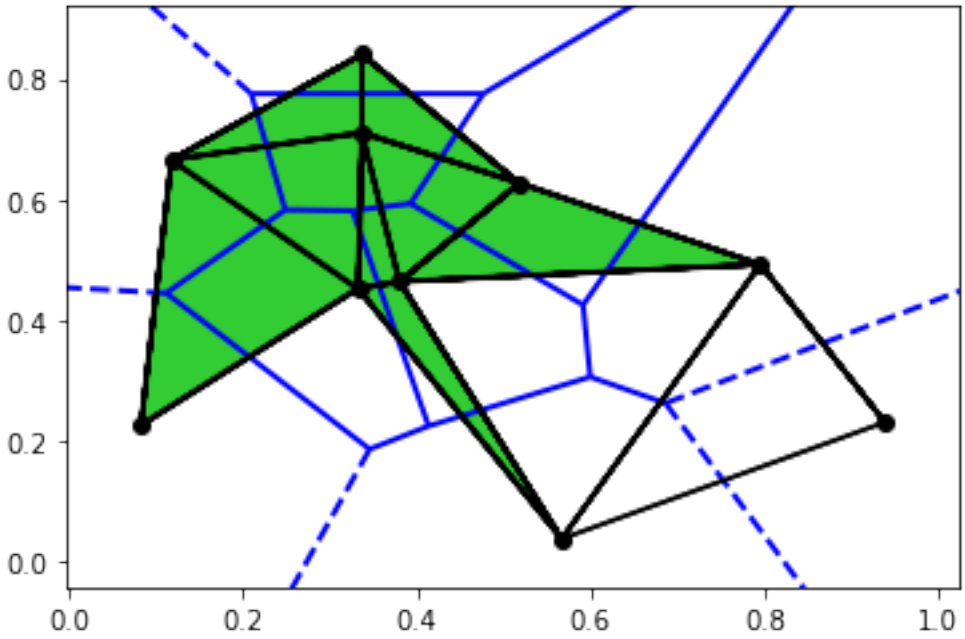

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

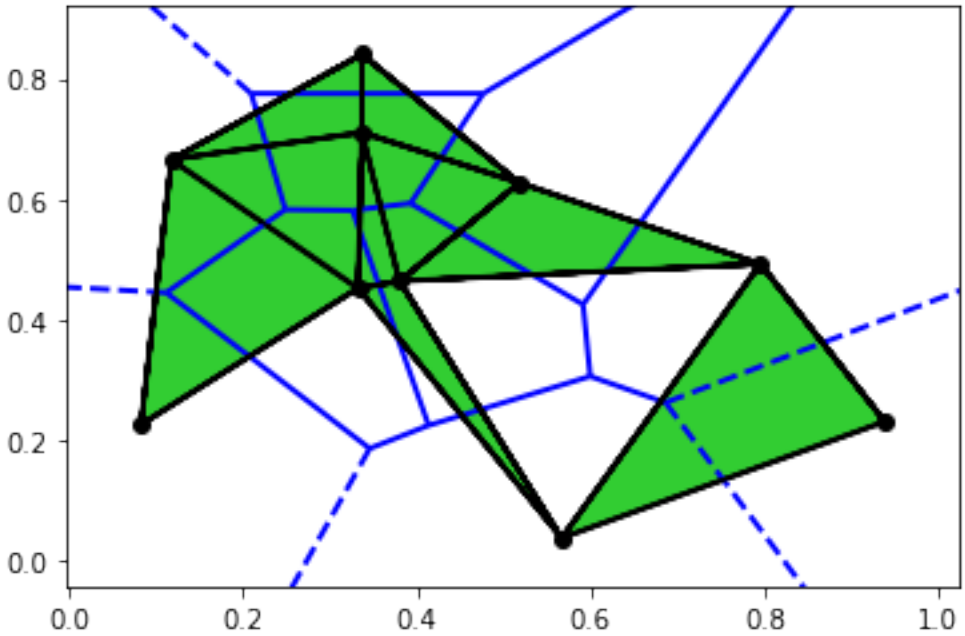

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

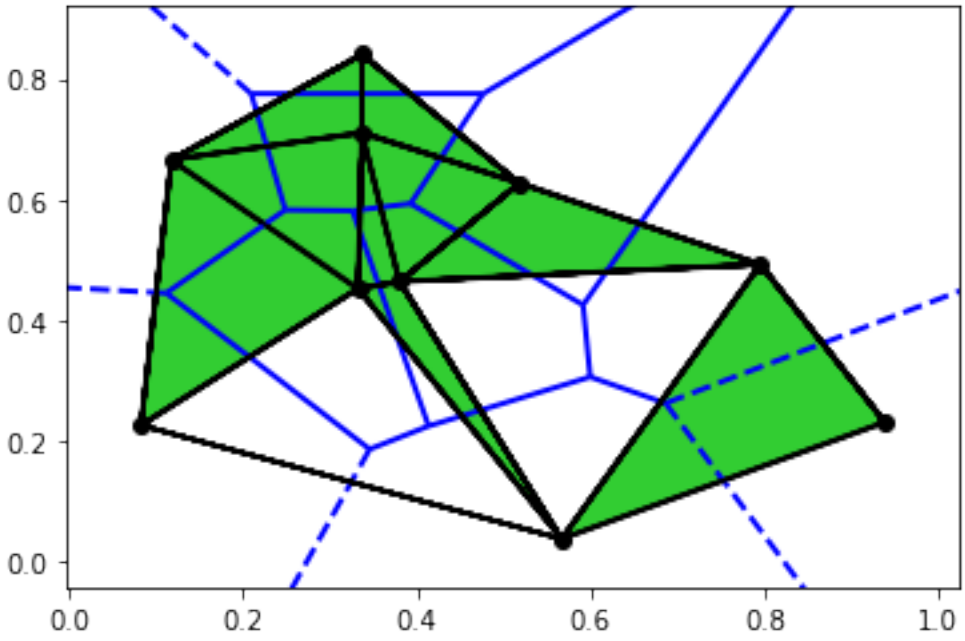

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

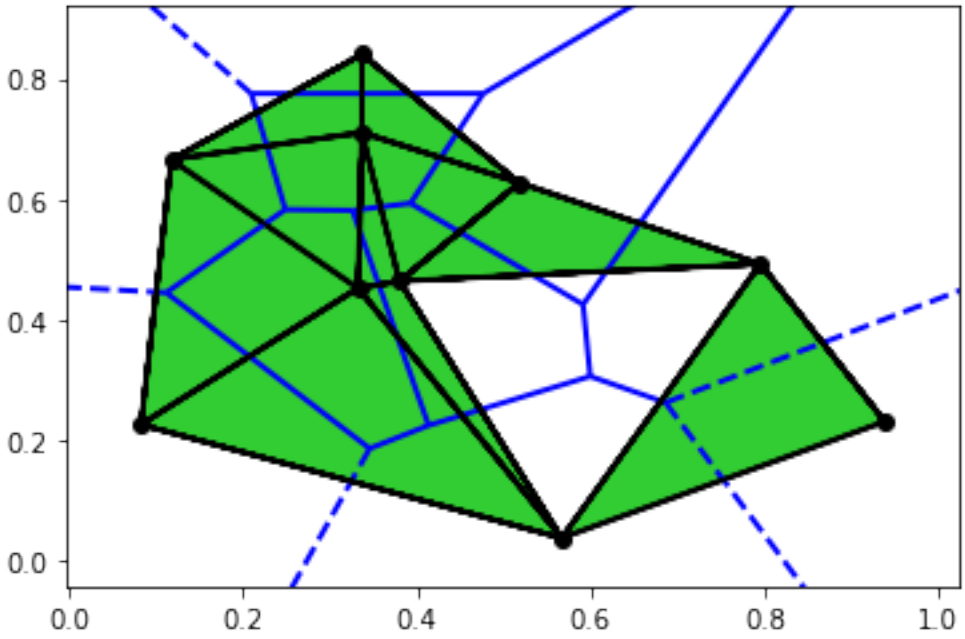

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

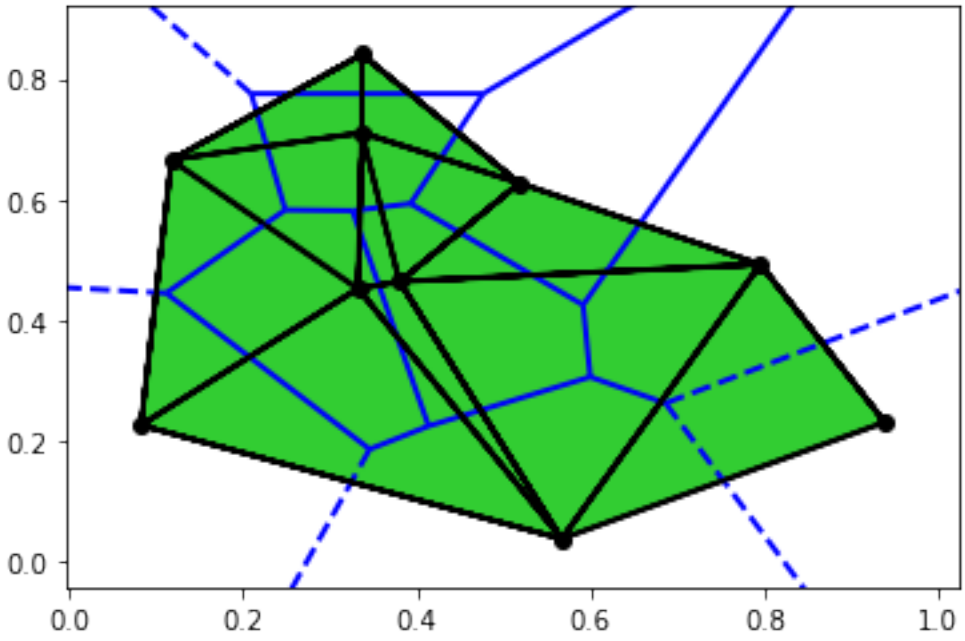

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

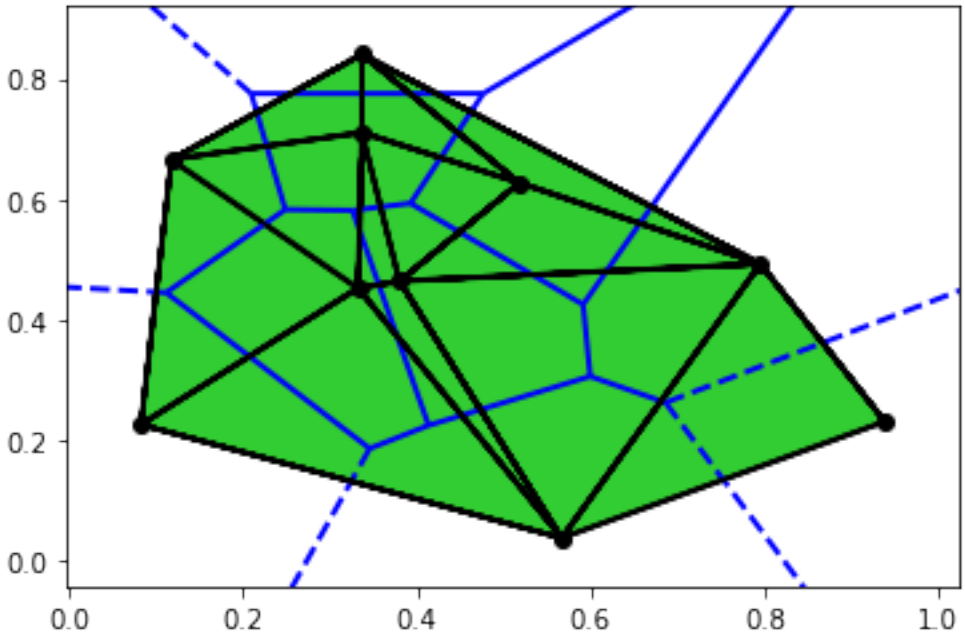

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

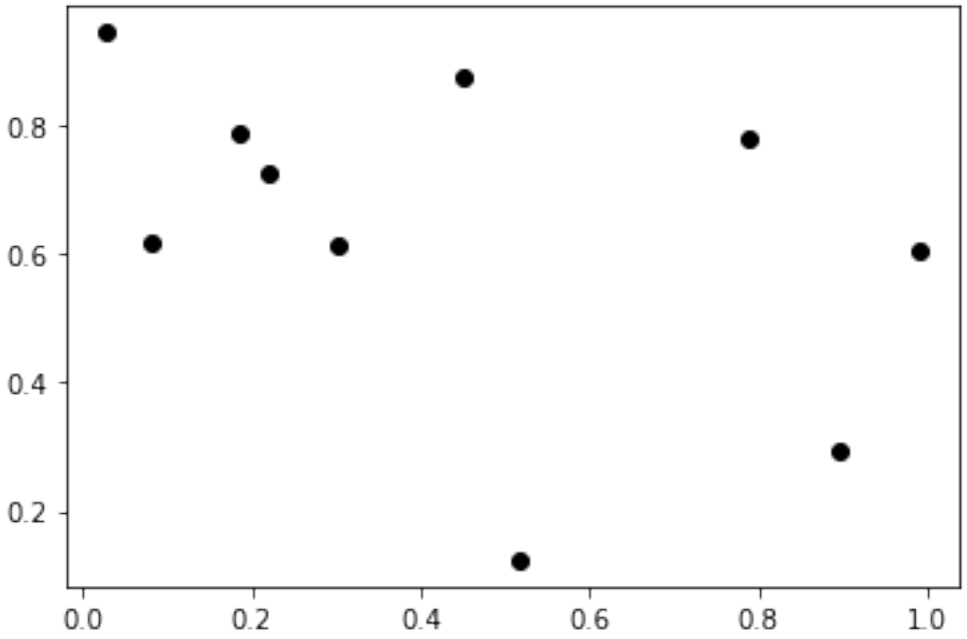

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

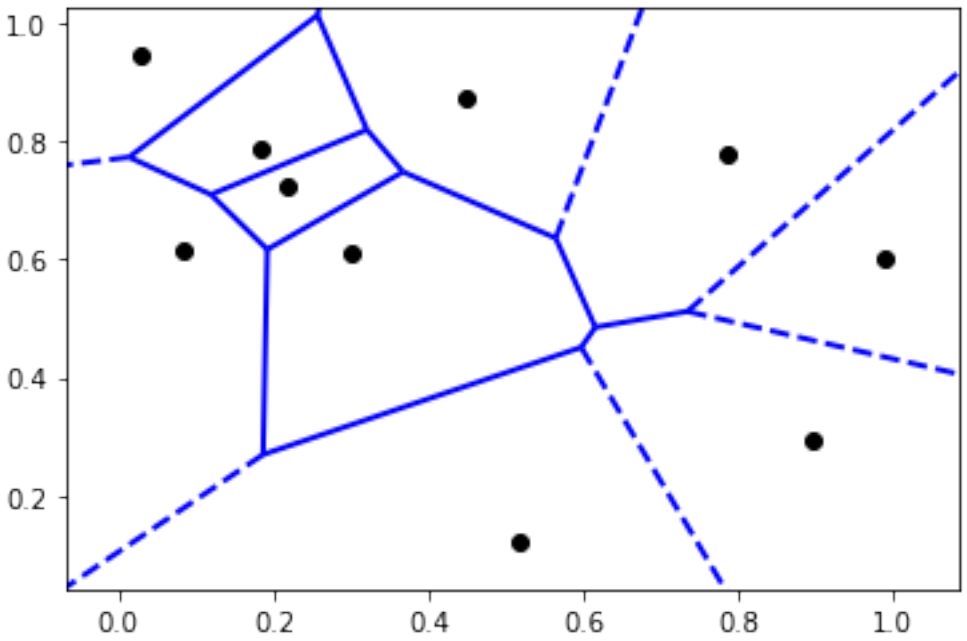

0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

2 Ejemplo 2

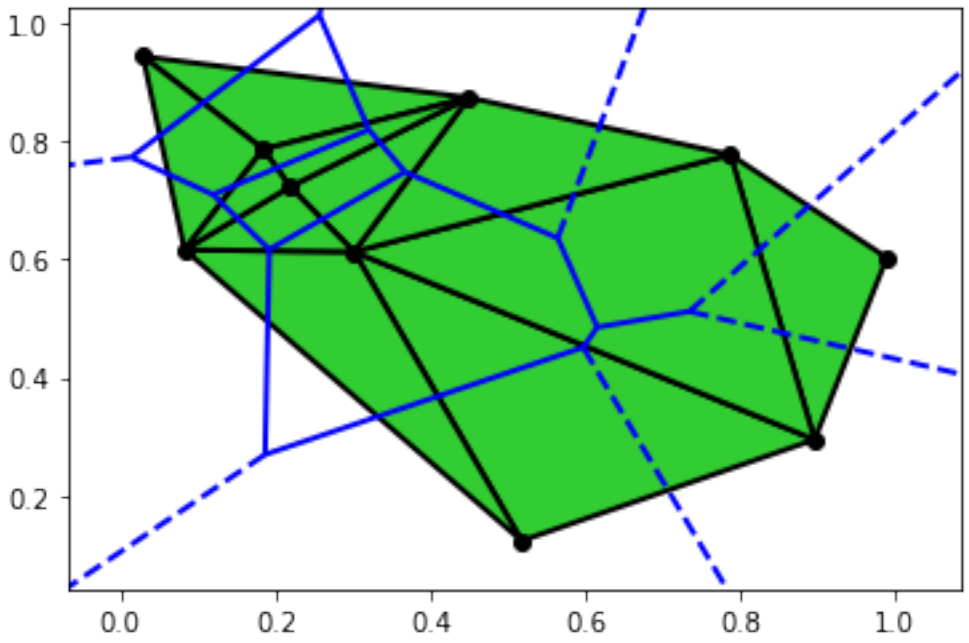
[59]:
```
points2=np.array([[0.8957641450573793, 0.2950833519989374],
 [0.028621391963087994, 0.9440875759025237],
 [0.517621505875702, 0.1236620161847416],
 [0.7871047164191424, 0.7777474116014623],
 [0.21869796914805273, 0.7233589914276723],
 [0.9891035292480995, 0.6032186214942837],
 [0.30113764052453484, 0.613321425324272],
 [0.18407448222466916, 0.7868606964403773],
 [0.4496777667376678, 0.874366215574117],
 [0.08225571534539433, 0.616710205071694]])
plt.plot(points[:,0],points[:,1],'ko')
plt.show()
```



[60]:
```
vor=Voronoi(points2)
fig = voronoi_plot_2d(vor,show_vertices=False,line_width=2,line_colors='blue')
plt.plot(points2[:,0],points2[:,1],'ko')
plt.show()
```



[61]:
```
fig = voronoi_plot_2d(vor,show_vertices=False,line_width=2,line_colors='blue')
tri = Delaunay(points2)
c=np.ones(len(points2))
cmap = matplotlib.colors.ListedColormap("limegreen")
plt.tripcolor(points2[:,0],points2[:,1],tri.simplices, c, edgecolor="k", lw=2,␣
 ↪cmap=cmap)
plt.plot(points2[:,0], points2[:,1], 'ko')
plt.show()
```



[62]:
```
alpha2=AlphaComplex(points2)
```

[63]:
```
alpha2.filtrationorder()
```

[63]:
```
[(0,),
 (1,),
 (2,),
 (3,),
 (4,),
 (5,),
 (6,),
 (7,),
 (8,),
 (9,),
 (4, 7),
 (4, 6),
 (4, 9),
 (7, 9),
 (4, 7, 9),
 (6, 9),
 (4, 6, 9),
 (1, 7),
 (3, 5),
 (4, 8),
 (7, 8),
 (4, 7, 8),
```

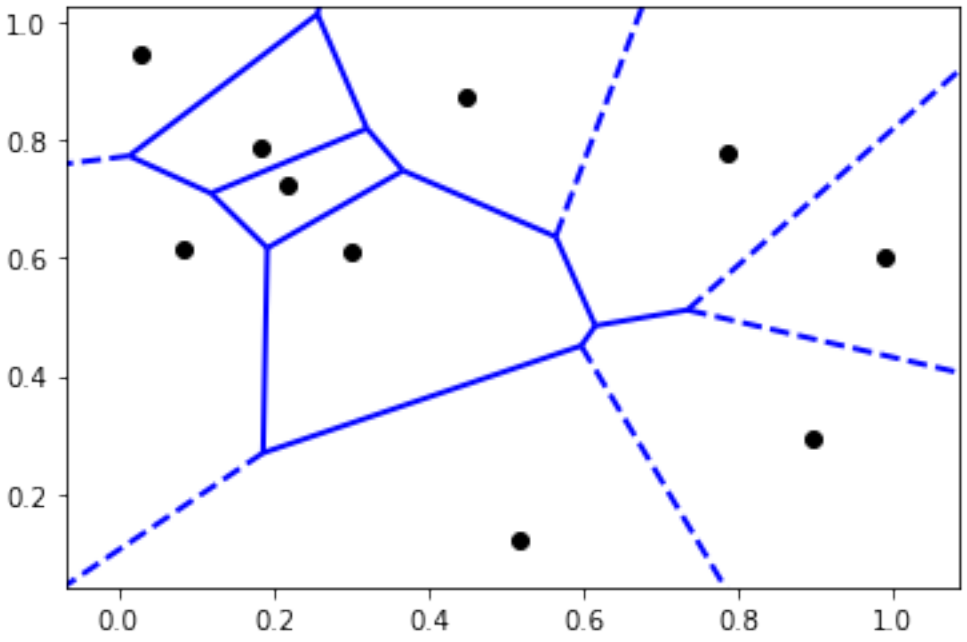
```
 (6, 8),
 (4, 6, 8),
 (0, 5),
 (1, 9),
 (1, 7, 9),
 (3, 8),
 (0, 2),
 (1, 8),
 (1, 7, 8),
 (3, 6),
 (3, 6, 8),
 (2, 6),
 (0, 3),
 (0, 3, 5),
 (0, 6),
 (0, 2, 6),
 (0, 3, 6),
 (2, 9),
 (2, 6, 9)]
```

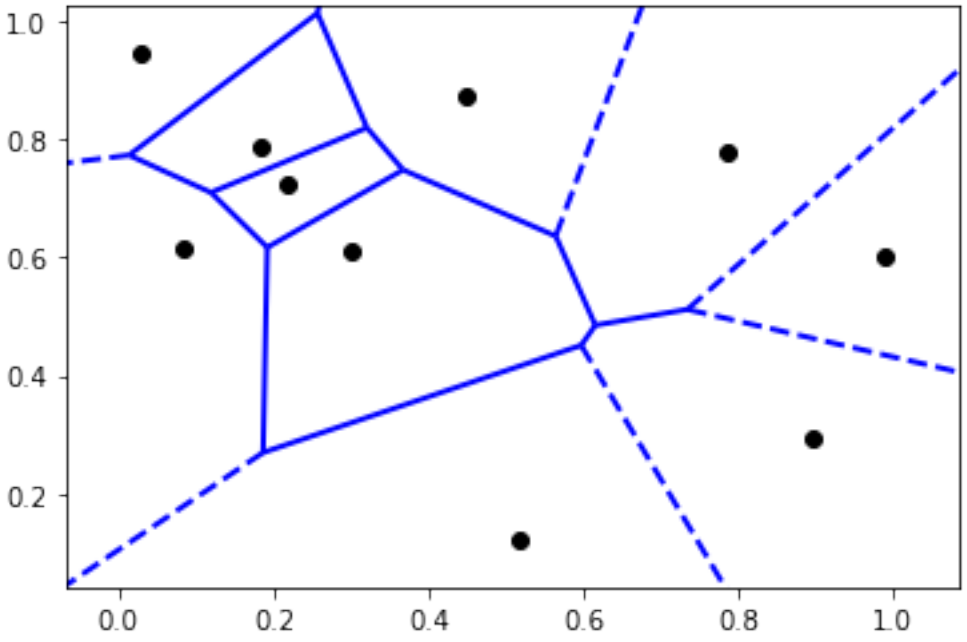
[65]:
```
alpha2.thresholdvalues()
```

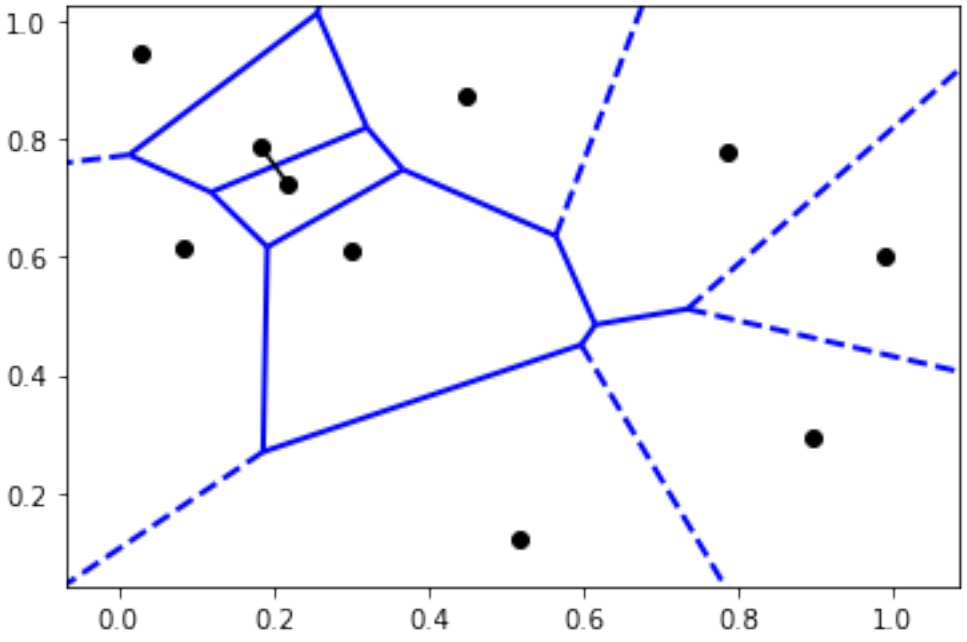
[65]:
```
[0.0,
 0.036163698601824126,
 0.0687469369692617,
 0.08658875829741432,
 0.10049698089739419,
 0.10945407826835805,
 0.10947698997829011,
 0.11055084225965854,
 0.1334764201653083,
 0.13798083982835635,
 0.13982338915839346,
 0.14027022785794788,
 0.150173692446446,
 0.15048145159235235,
 0.1609810430983044,
 0.171272970051592,
 0.17549368909767854,
 0.2075916243129811,
 0.23832327791636163,
 0.2639845483335834,
 0.2676899329991027,
 0.27072941952967894,
 0.3372151209564786,
 0.33722983315924704,
 0.3390763184810709,
```

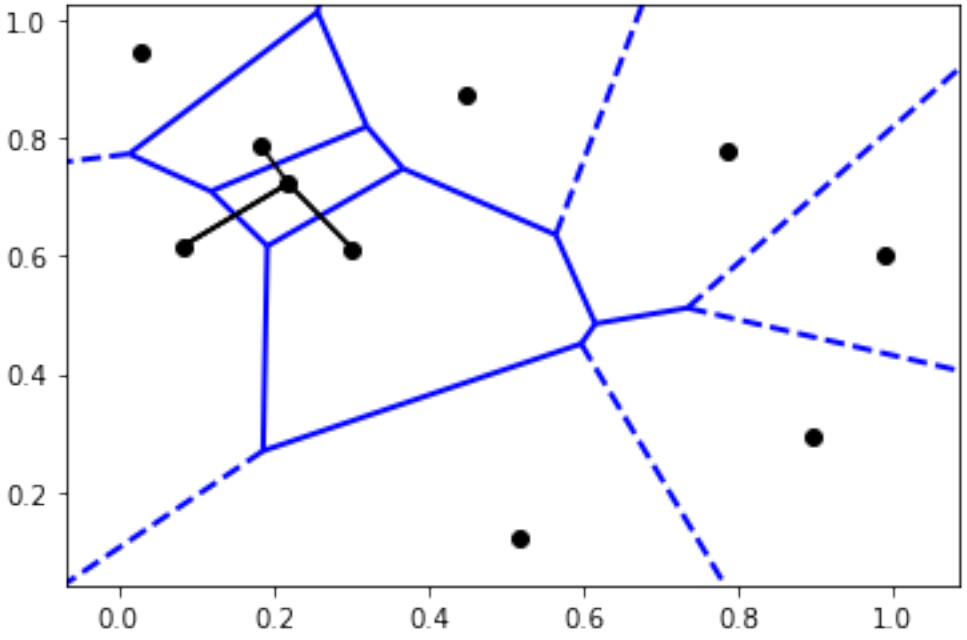
0.36210637012732]

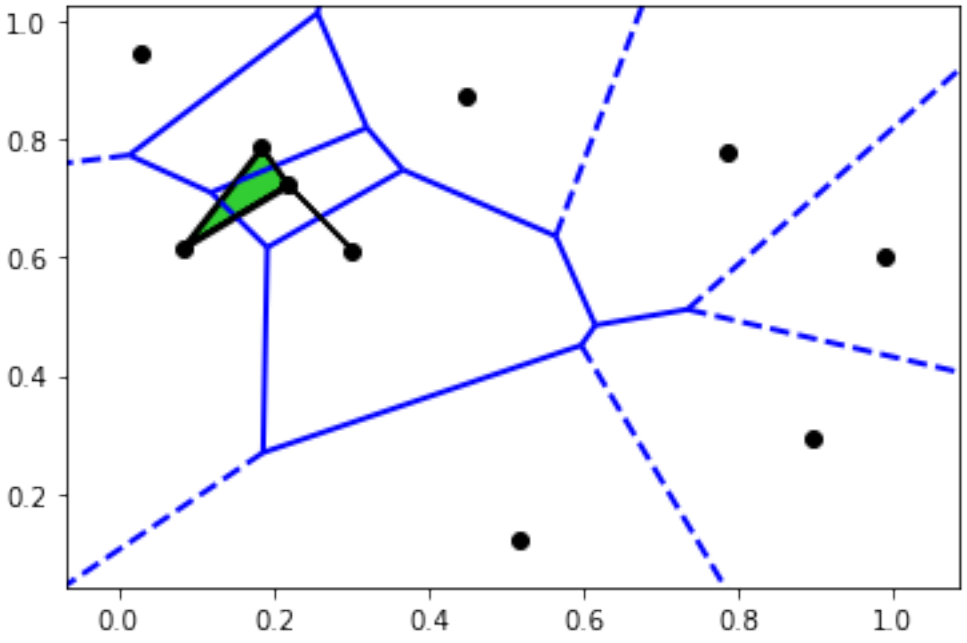
[66]:
```
for value in alpha.thresholdvalues():
    K2=alpha2.sublevel(value)
    figure=[]
    plotalpha(points,K2)
```

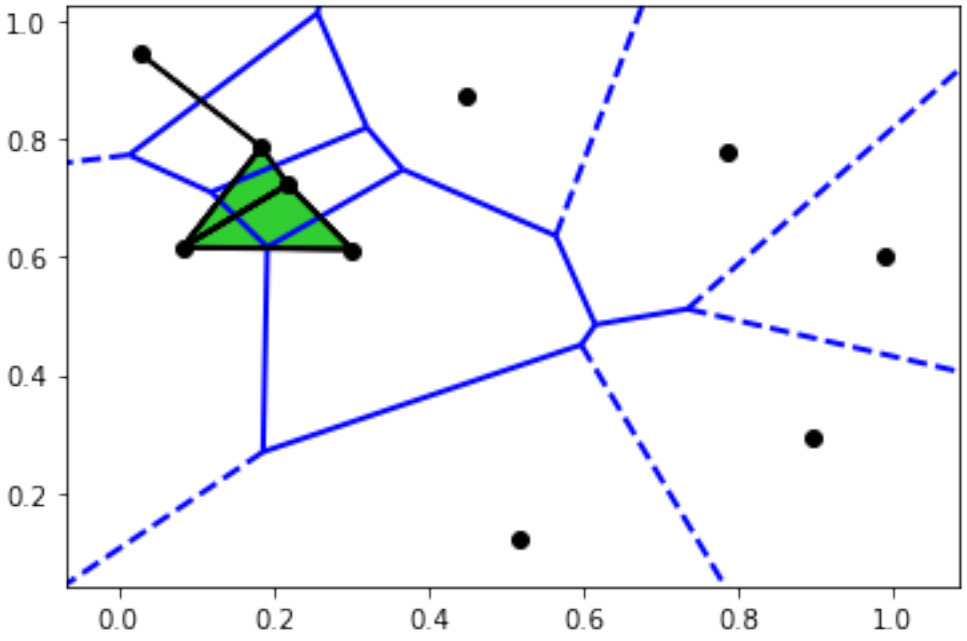


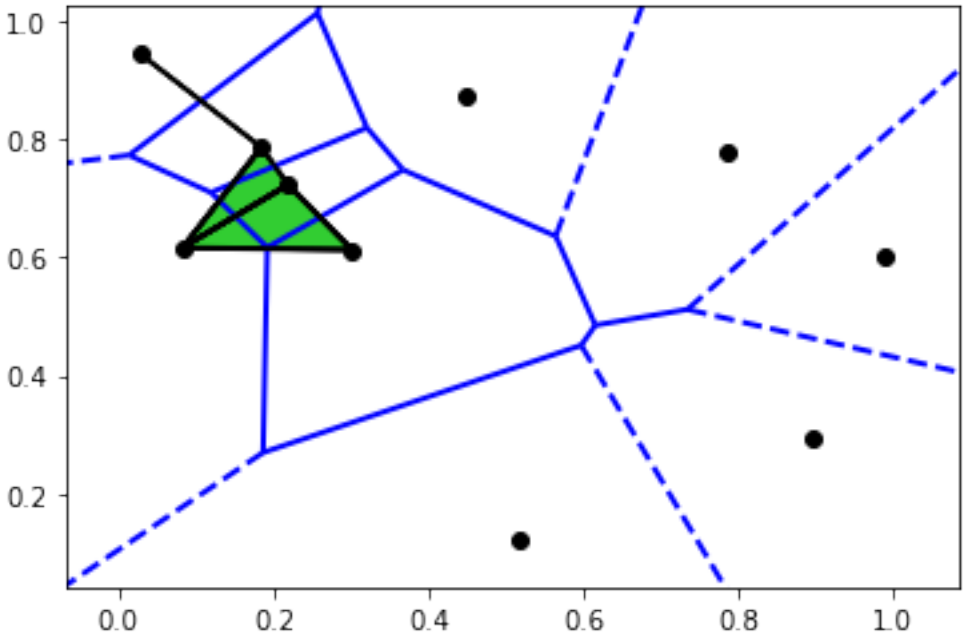


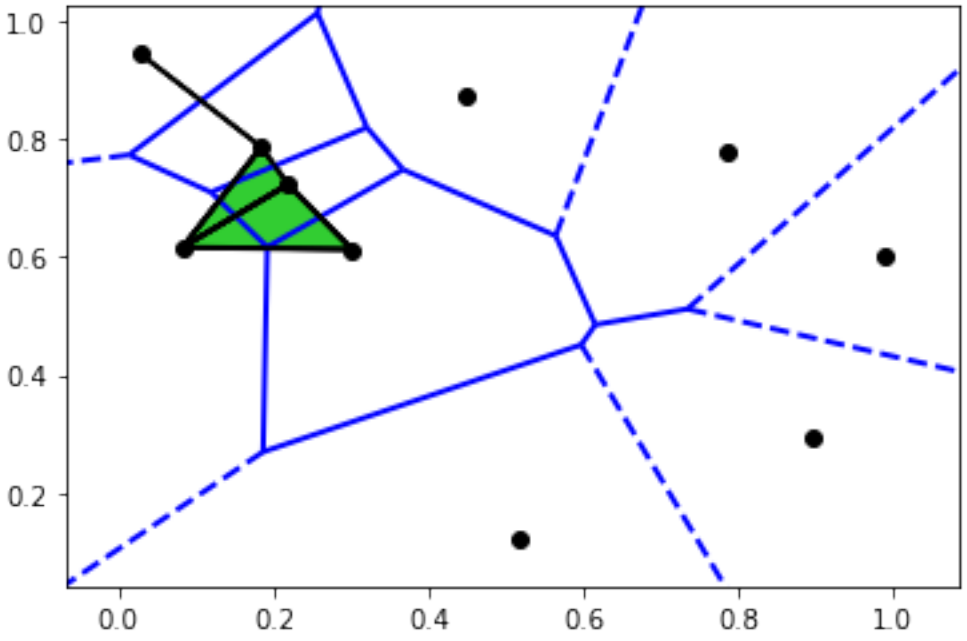

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

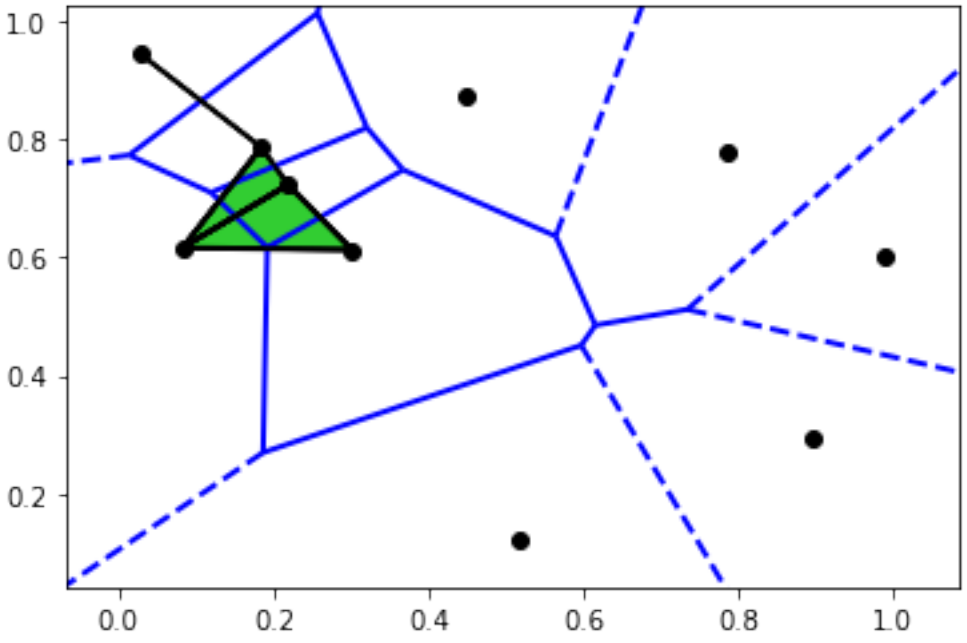

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

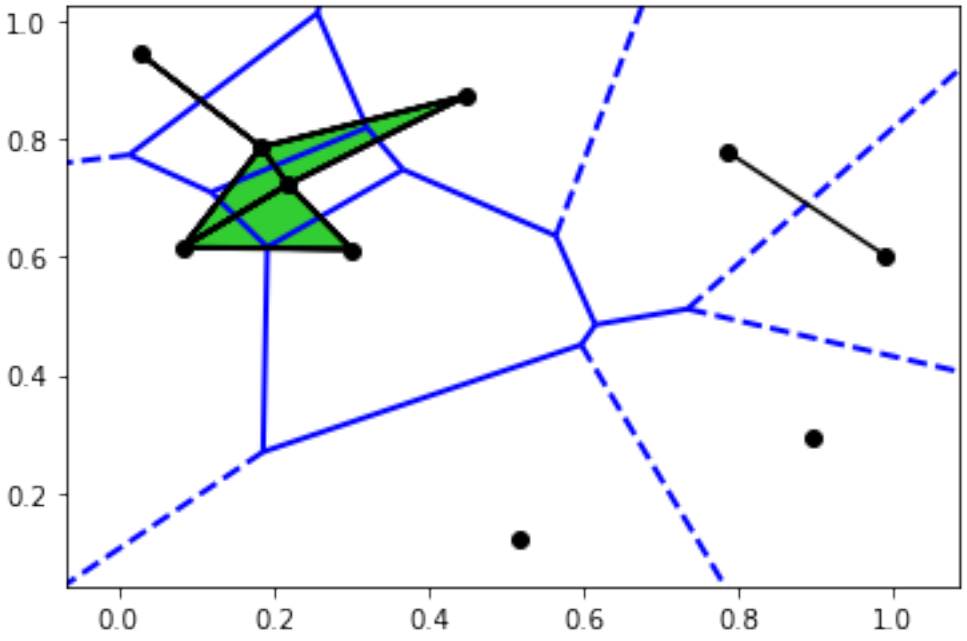

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

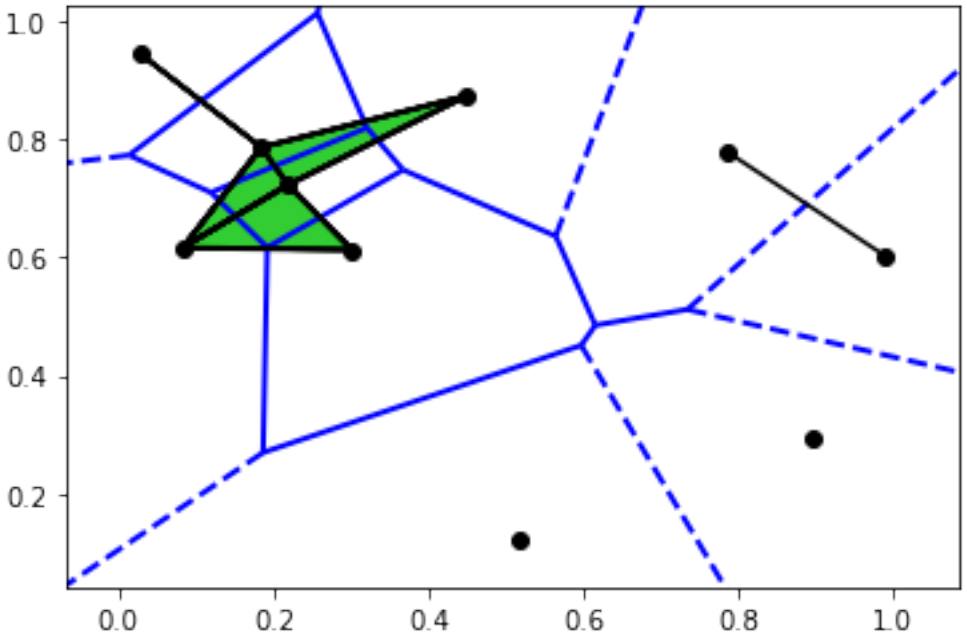

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

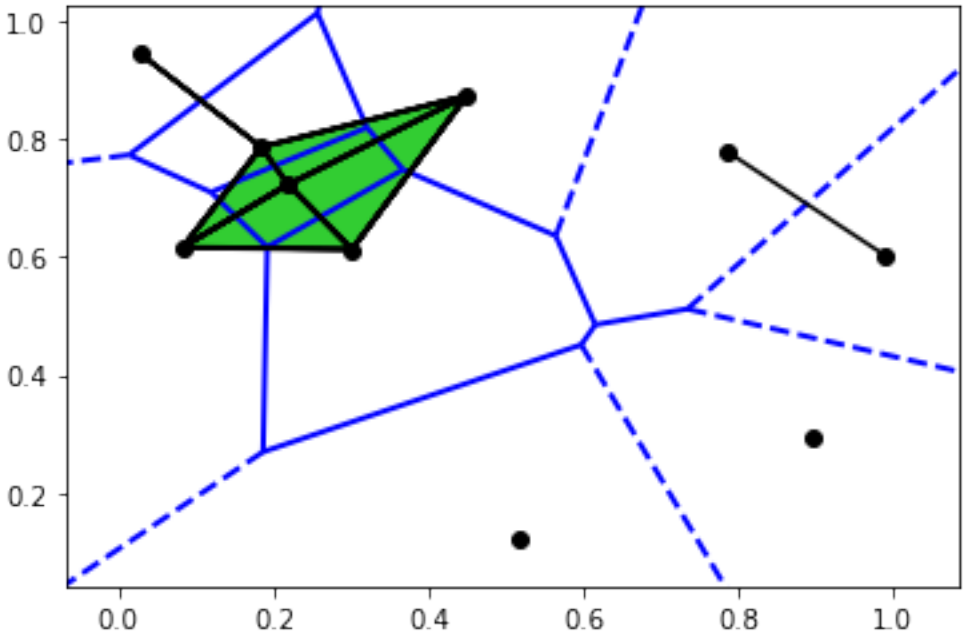

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

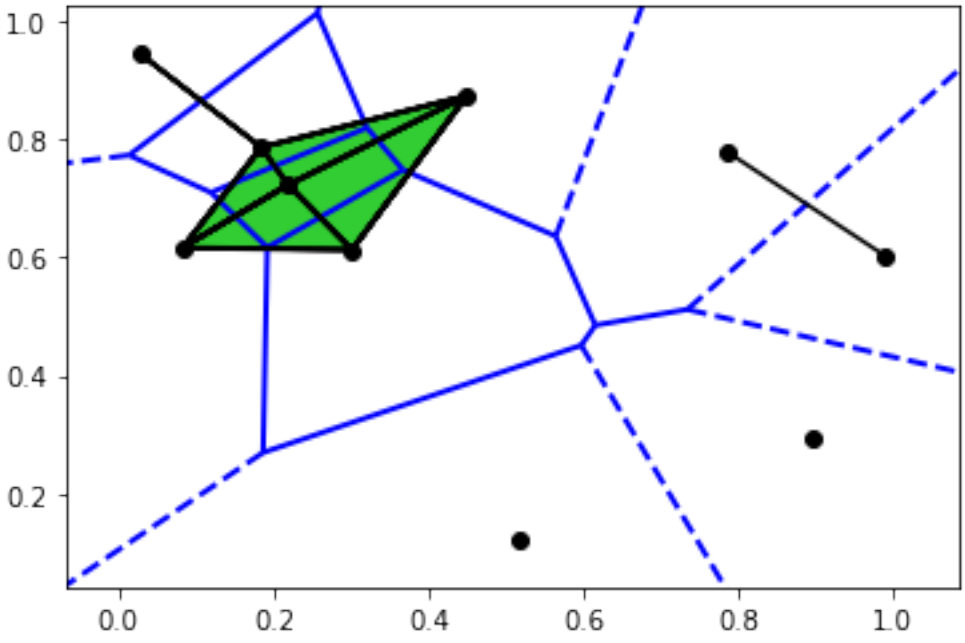

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

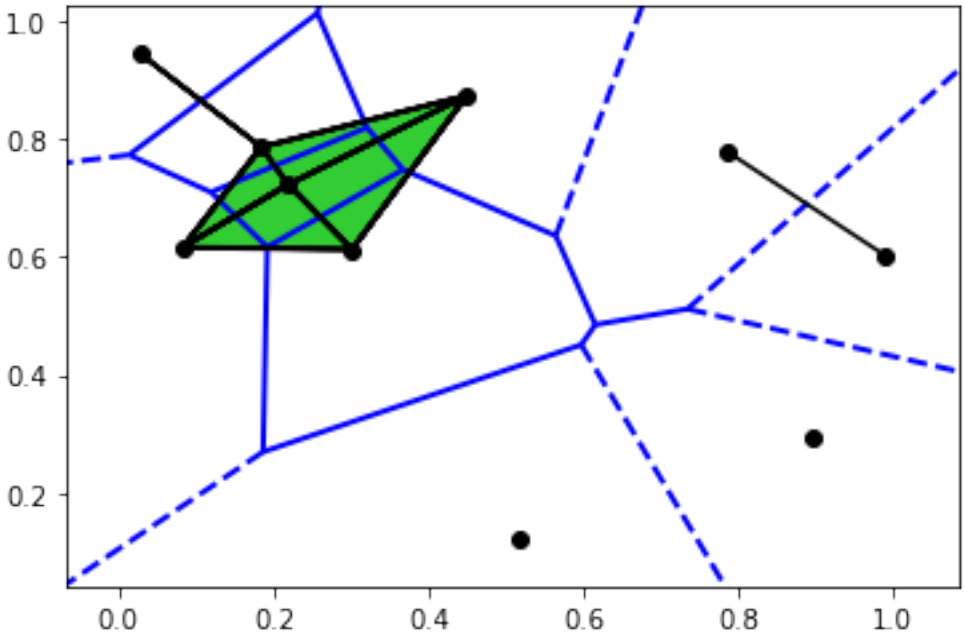

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

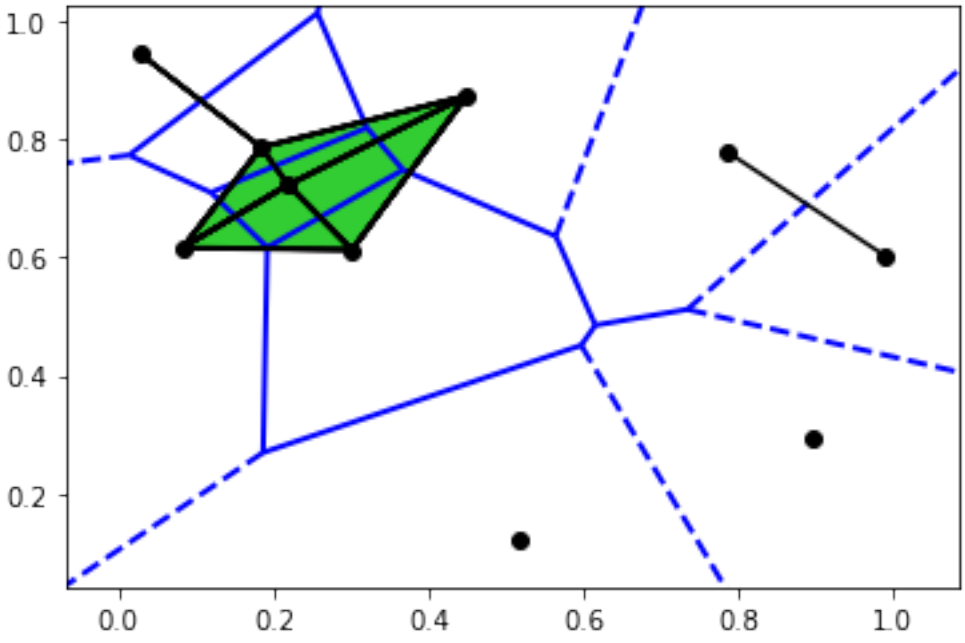

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

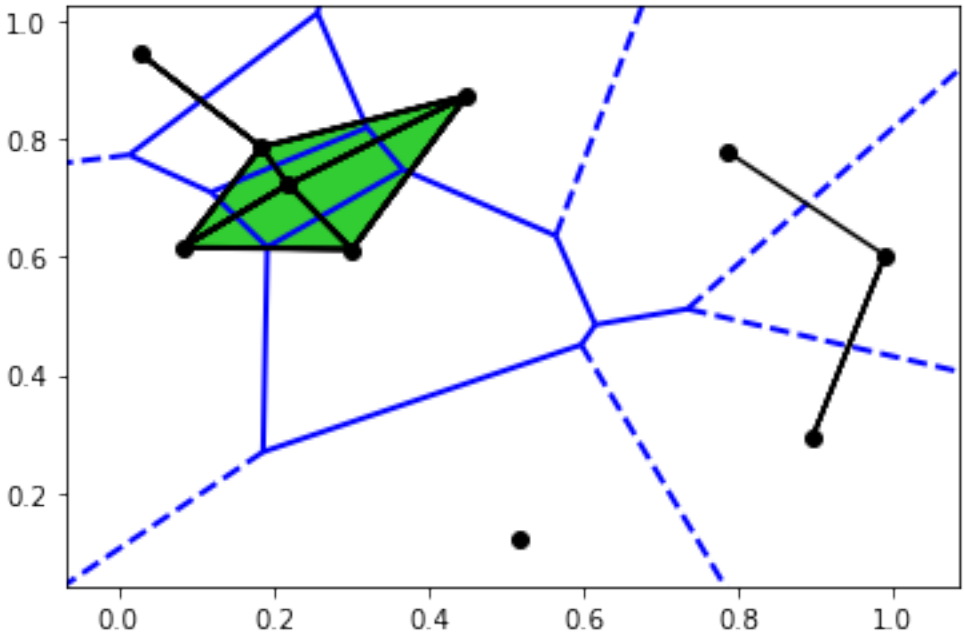

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

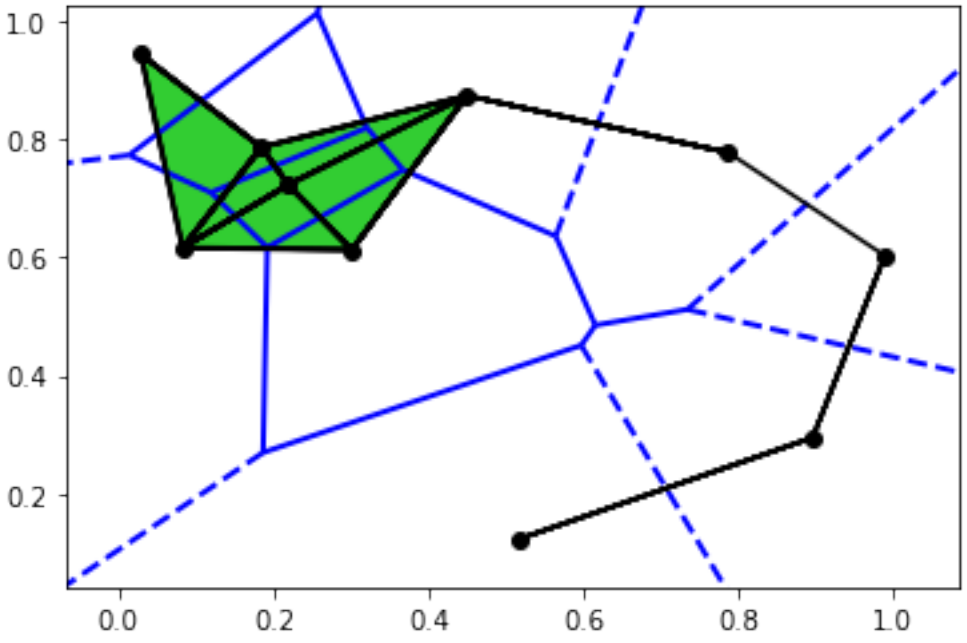

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

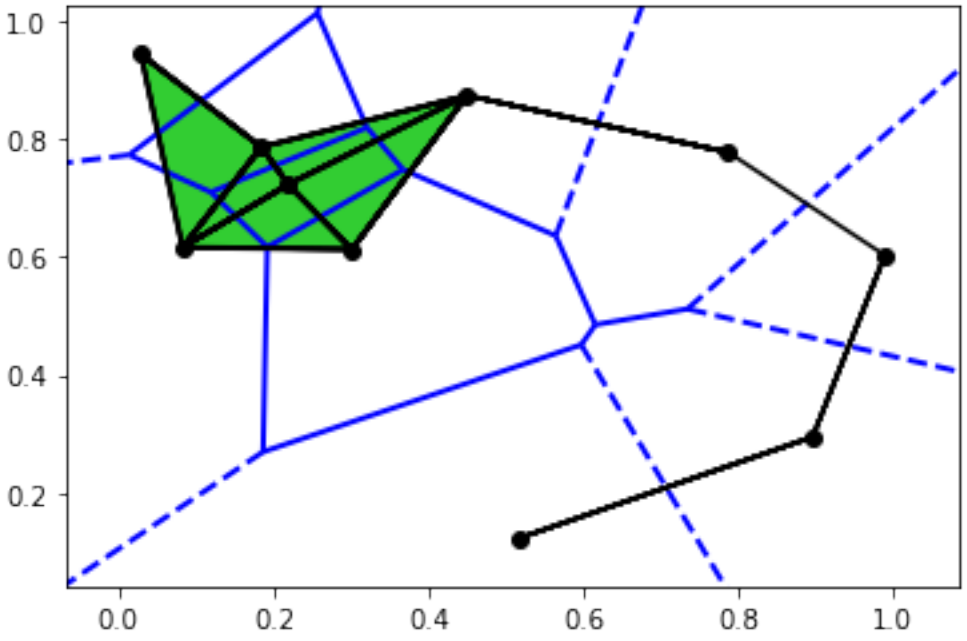

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

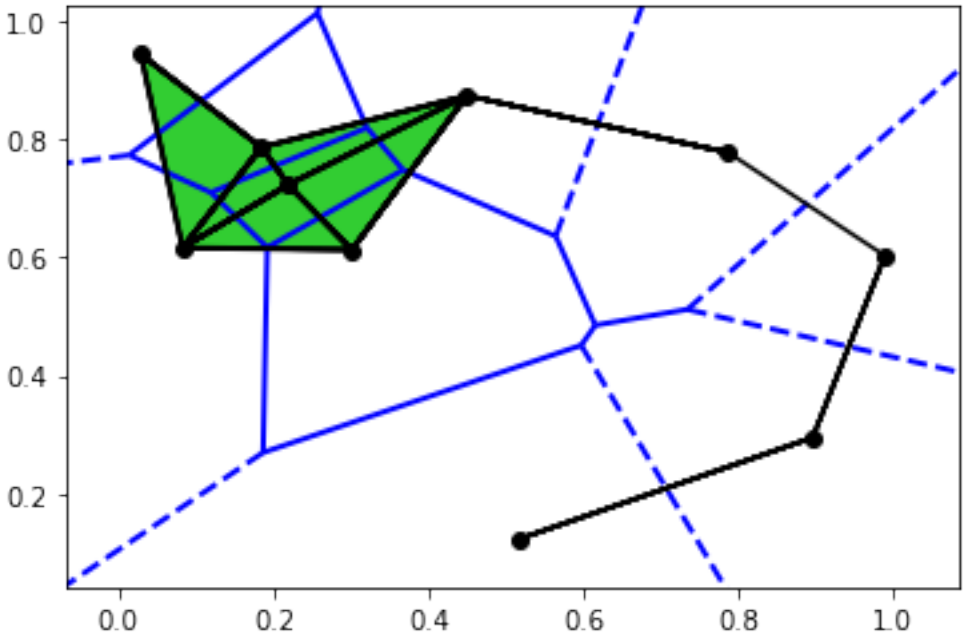

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

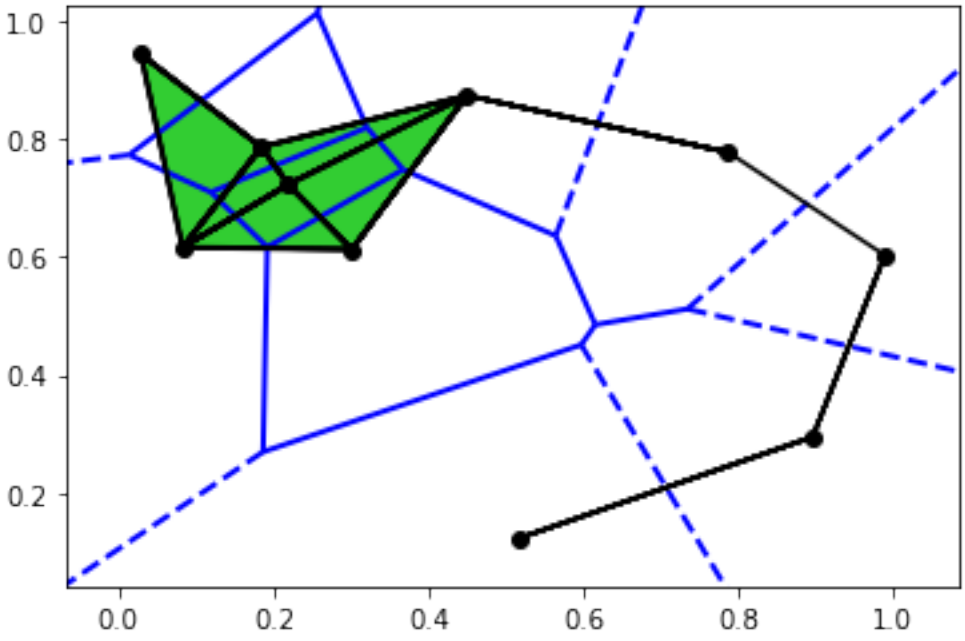

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

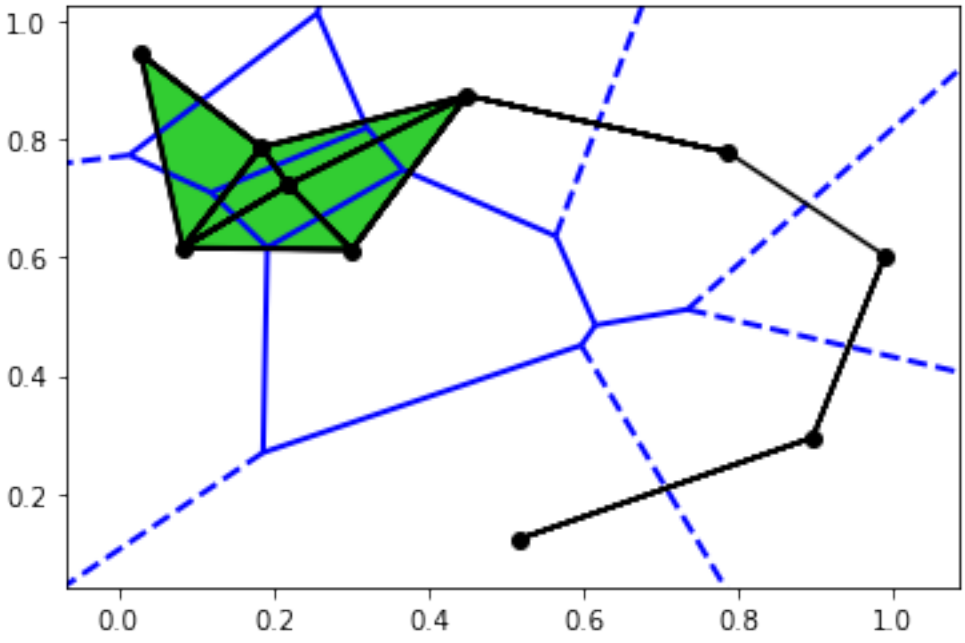

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

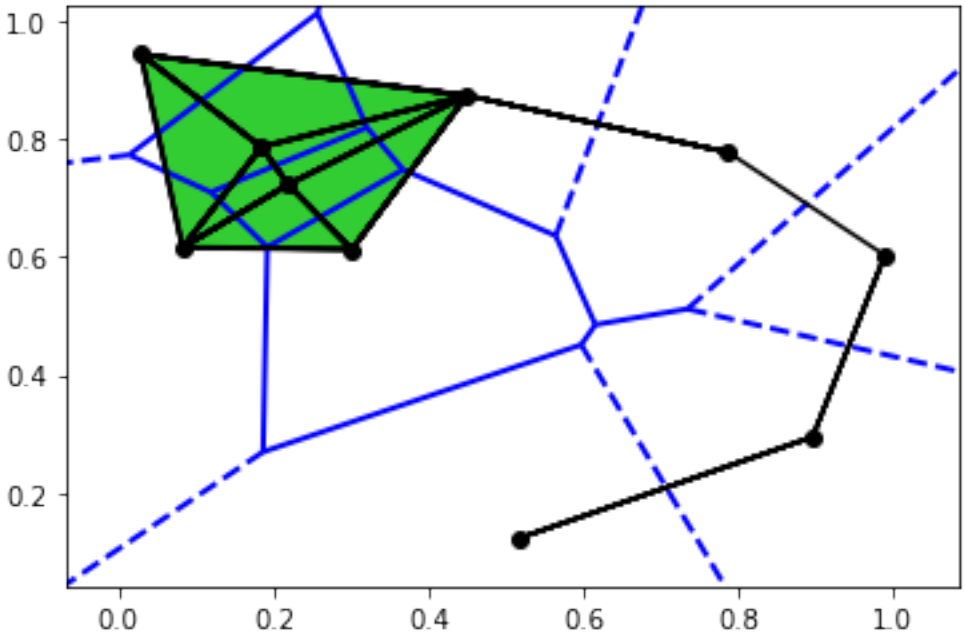

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

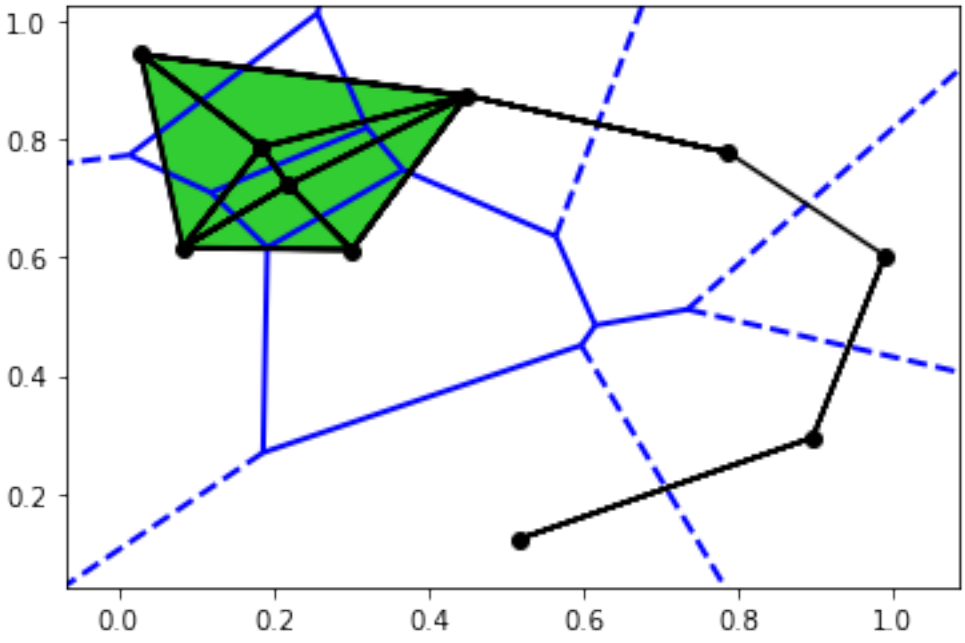

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

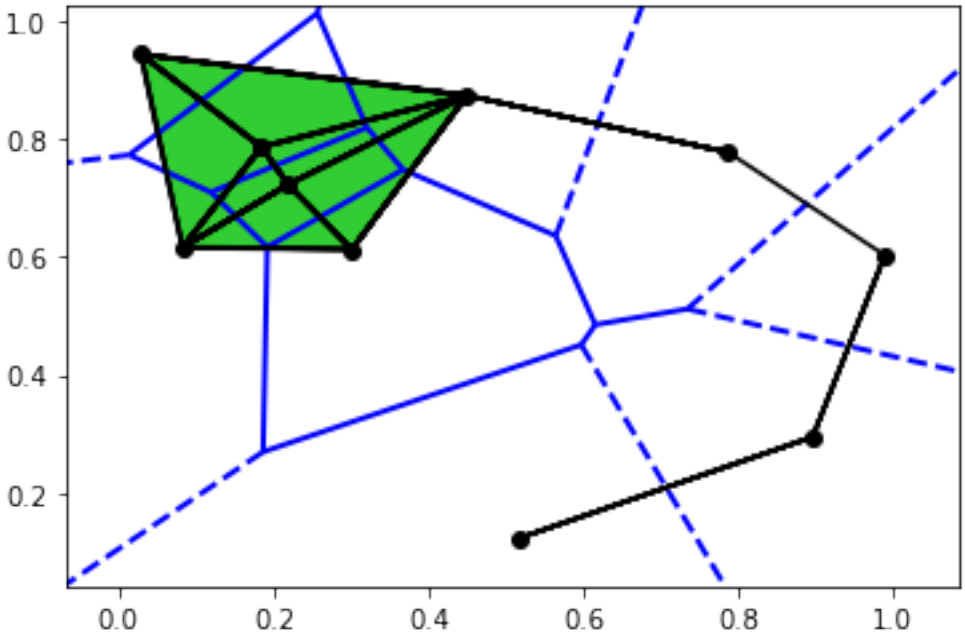

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

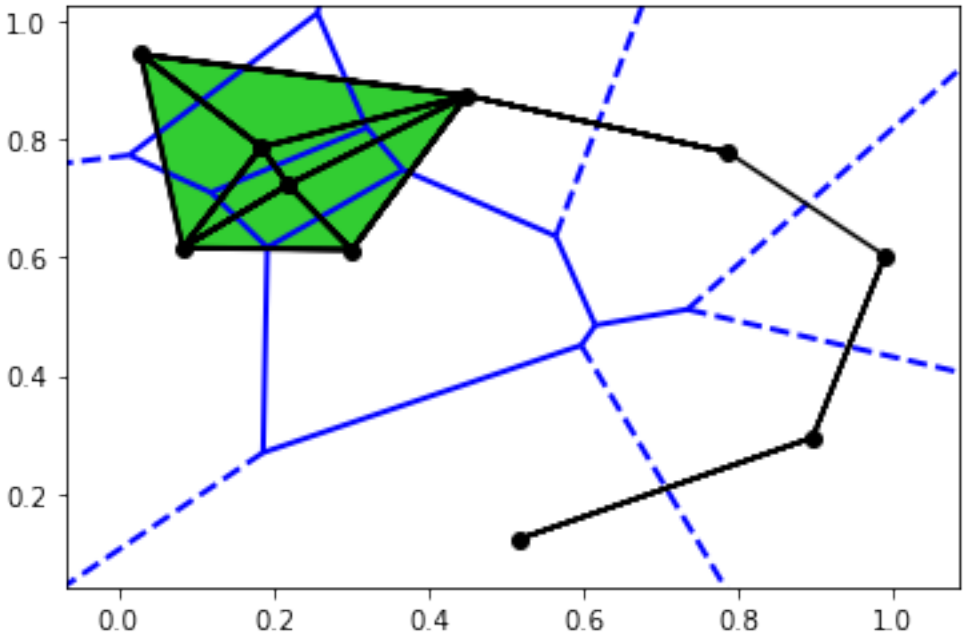

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

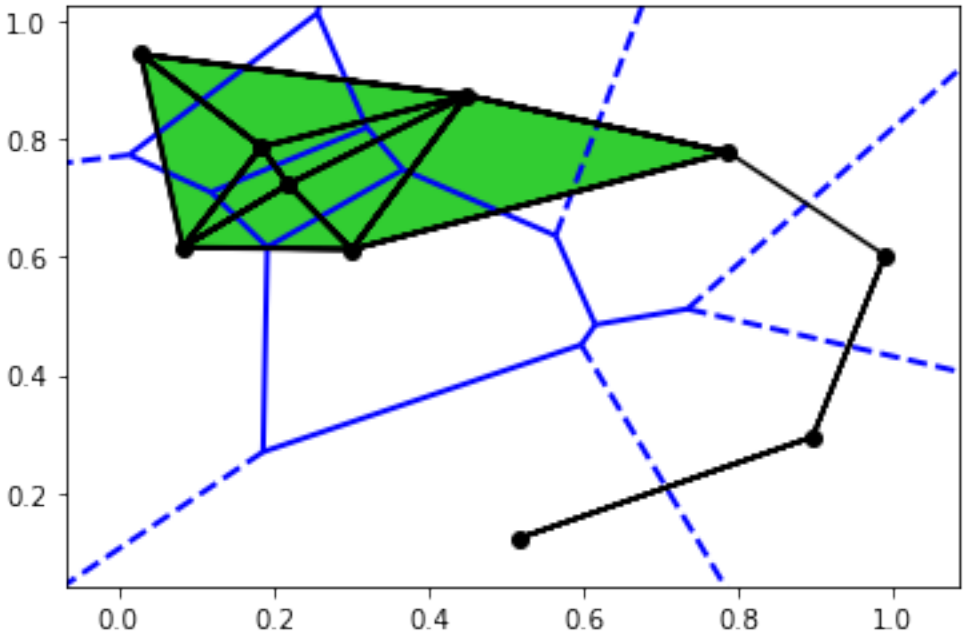

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

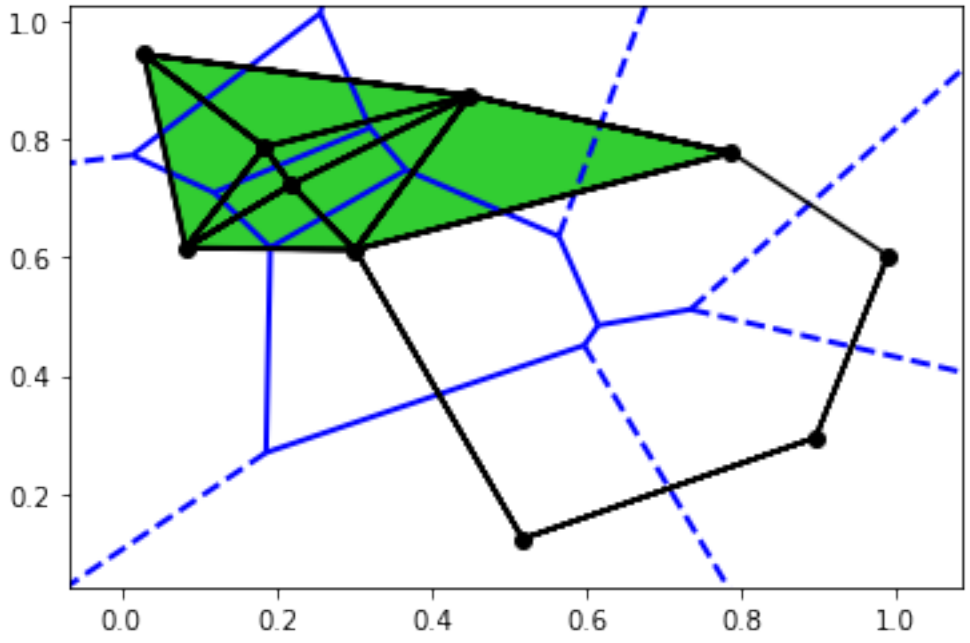

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

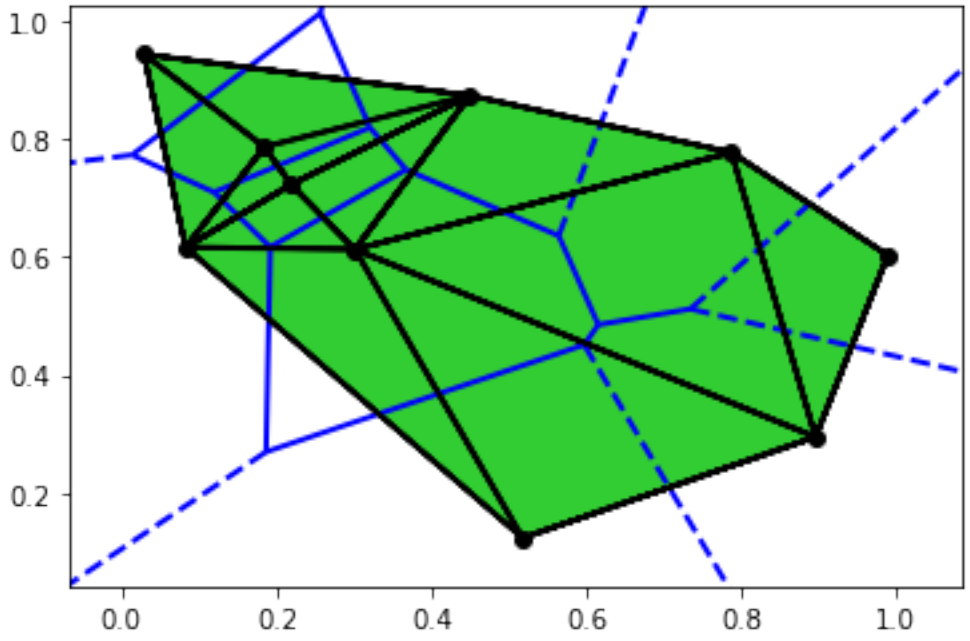

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

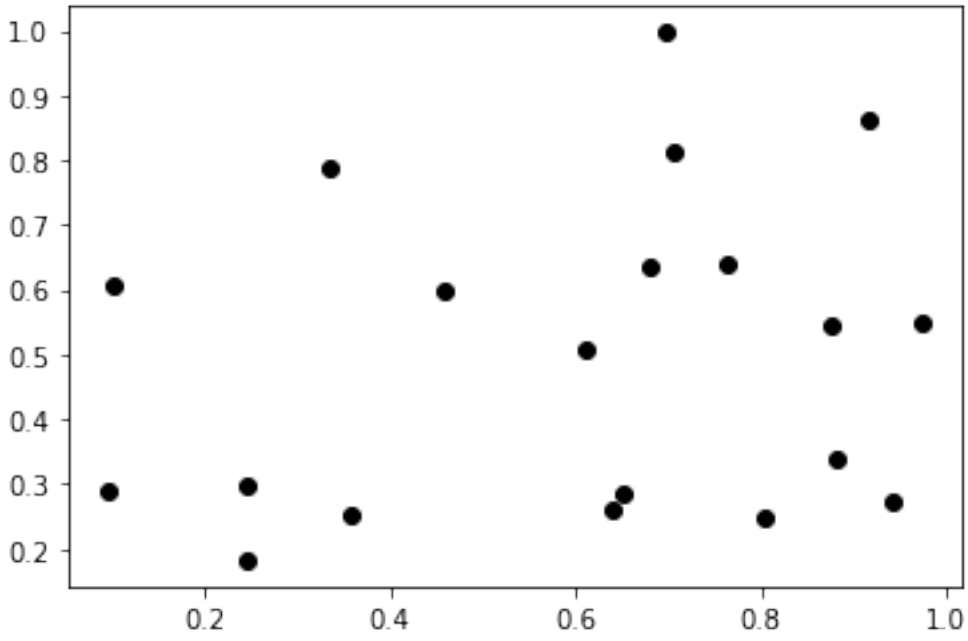

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

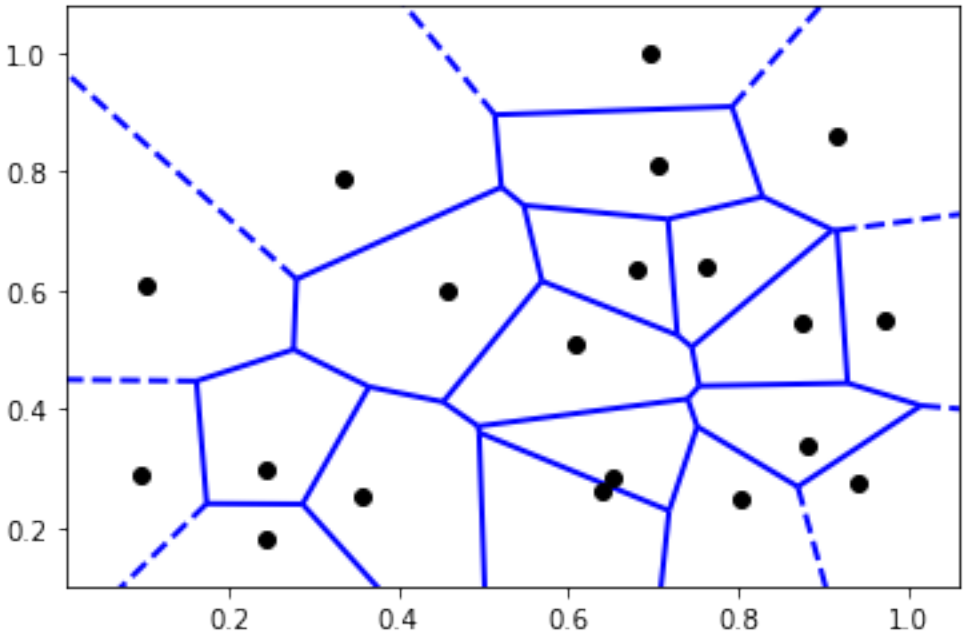

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

3 Ejemplo 3

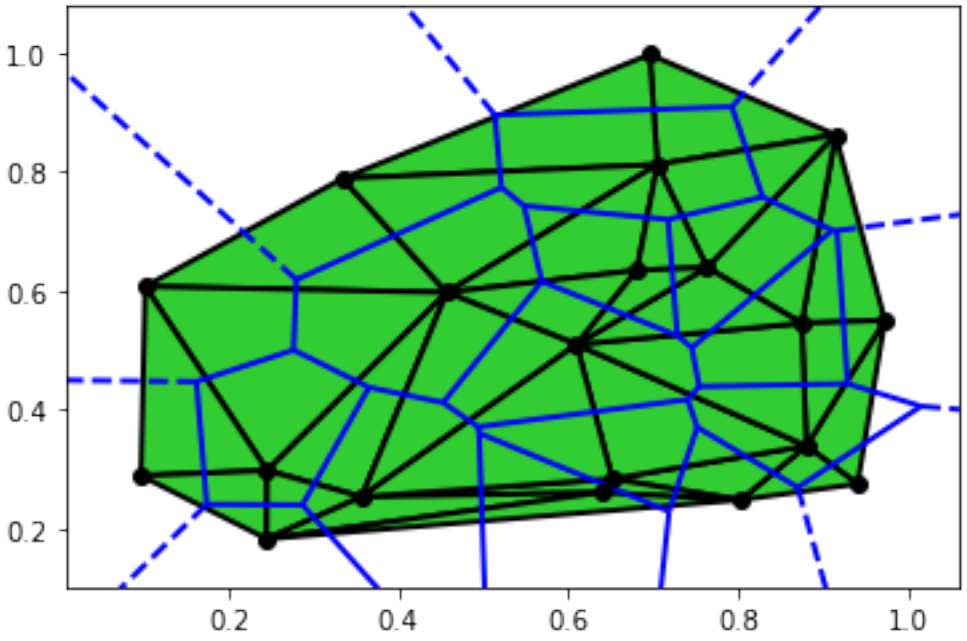
[67]:
```
points3=np.array([(0.8753299639906736, 0.5452963206013219),
 (0.915335120559448, 0.8622822047328554),
 (0.9411759819184322, 0.2748278885761678),
 (0.7052034033196758, 0.8122389578499669),
 (0.9734431558329487, 0.5500672178217452),
 (0.101349658961157, 0.6072126518098413),
 (0.6099428935549683, 0.5095146187792166),
 (0.6810379648990679, 0.6343196355745316),
 (0.763747595111296, 0.6389758508715849),
 (0.6521290891236327, 0.28340359060768416),
 (0.4569706839687516, 0.5970966728571825),
 (0.3339042514617916, 0.7888181435443109),
 (0.24447615661103717, 0.18247811626397858),
 (0.6961254832425103, 0.9974914431850389),
 (0.2452860638322797, 0.2974794924024807),
 (0.09631846692736679, 0.2887656085651358),
 (0.638575556222527, 0.26034722595932536),
 (0.803241921795395, 0.24803894619975986),
 (0.8809182300057703, 0.3389661339754195),
 (0.3565859265456749, 0.25327819736066515)])
plt.plot(points3[:,0],points3[:,1],'ko')
plt.show()
```



[68]:
```
vor=Voronoi(points3)
fig = voronoi_plot_2d(vor,show_vertices=False,line_width=2,line_colors='blue')
plt.plot(points3[:,0],points3[:,1],'ko')
plt.show()
```



[70]:
```
Del=Delaunay(points3)
fig = voronoi_plot_2d(vor,show_vertices=False,line_width=2,line_colors='blue')
c=np.ones(len(points3))
cmap = matplotlib.colors.ListedColormap("limegreen")
plt.tripcolor(points3[:,0],points3[:,1],Del.simplices, c, edgecolor="k", lw=2,␣
 ↪cmap=cmap)
plt.plot(points3[:,0], points3[:,1], 'ko')
plt.show()
```



[71]:
```
alpha3=AlphaComplex(points3)
```

[72]:
```
alpha3.filtrationorder()
```

[72]:
```
[(0,),
 (1,),
 (2,),
 (3,),
 (4,),
 (5,),
 (6,),
 (7,),
 (8,),
 (9,),
 (10,),
 (11,),
 (12,),
 (13,),
 (14,),
 (15,),
 (16,),
 (17,),
 (18,),
 (19,),
 (9, 16),
 (7, 8),
```

```
(2, 18),
(0, 4),
(12, 14),
(17, 18),
(14, 19),
(12, 19),
(2, 17),
(2, 17, 18),
(12, 14, 19),
(6, 7),
(0, 8),
(14, 15),
(9, 17),
(16, 17),
(9, 16, 17),
(6, 10),
(3, 7),
(12, 15),
(12, 14, 15),
(3, 8),
(3, 13),
(3, 7, 8),
(0, 18),
(1, 3),
(7, 10),
(6, 7, 10),
(10, 11),
(6, 9),
(4, 18),
(0, 4, 18),
(6, 8),
(6, 7, 8),
(1, 13),
(1, 3, 13),
(9, 18),
(9, 17, 18),
(1, 8),
(1, 3, 8),
(0, 6),
(0, 6, 8),
(16, 19),
(5, 11),
(2, 4),
(2, 4, 18),
(1, 4),
(5, 15),
(6, 18),
```

```
 (6, 9, 18),
 (0, 1),
 (0, 1, 8),
 (0, 6, 18),
 (0, 1, 4),
 (5, 14),
 (5, 14, 15),
 (3, 10),
 (3, 7, 10),
 (9, 19),
 (9, 16, 19),
 (5, 10),
 (5, 10, 11),
 (10, 19),
 (6, 19),
 (6, 9, 19),
 (10, 14),
 (10, 14, 19),
 (6, 10, 19),
 (3, 11),
 (3, 10, 11),
 (5, 10, 14),
 (11, 13),
 (3, 11, 13),
 (12, 16),
 (12, 16, 19),
 (12, 17),
 (12, 16, 17)]
```

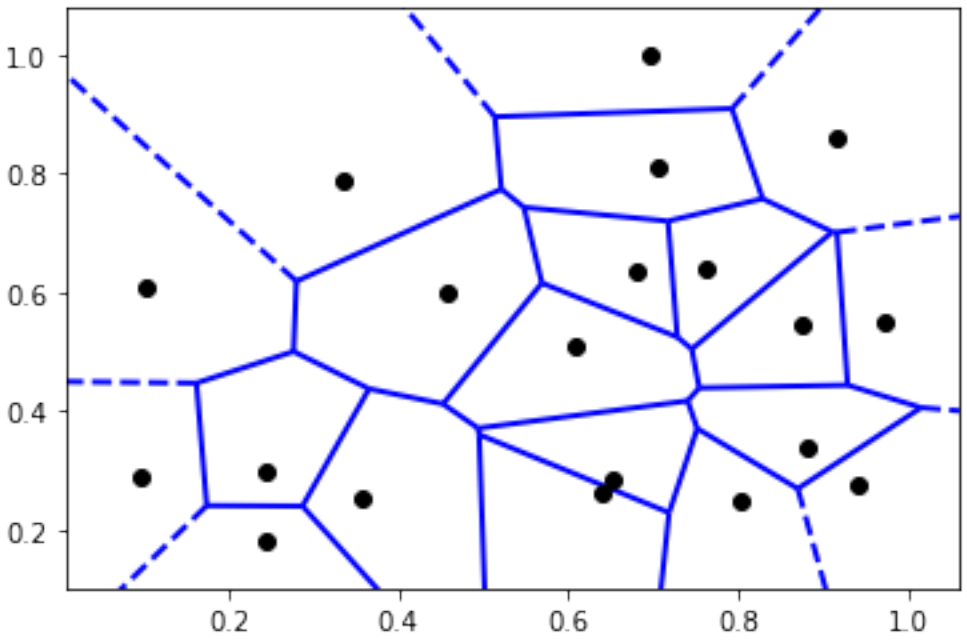
[74]:
```
alpha3.thresholdvalues()
```

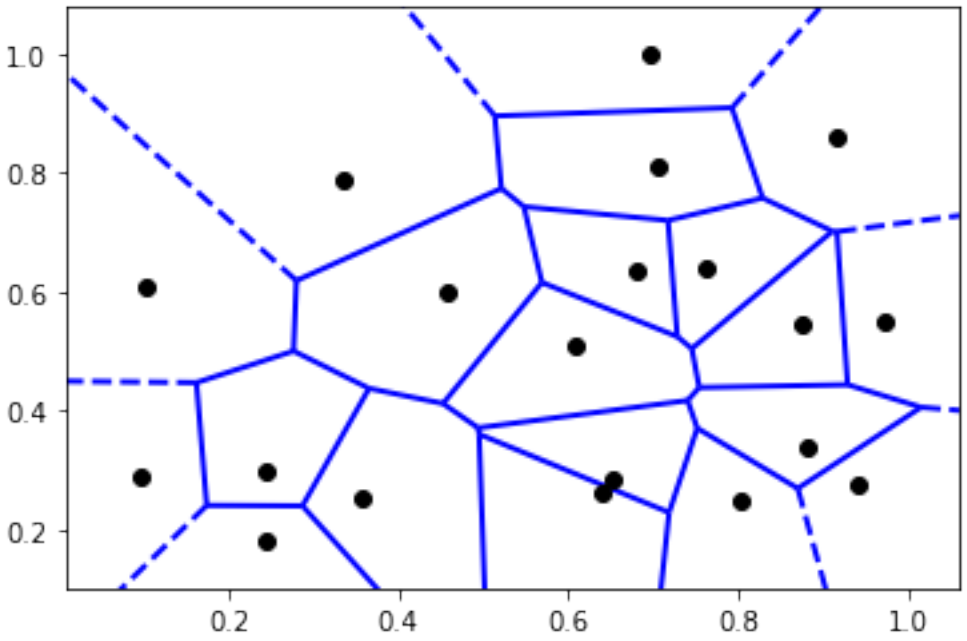
[74]:
```
[0.0,
 0.013372492334073723,
 0.04142029475612175,
 0.04400202037538318,
 0.04911455963807197,
 0.05750211401213061,
 0.05979415174994811,
 0.059877821276734536,
 0.06629715679818932,
 0.07025569794627363,
 0.07067990072251536,
 0.07113675651754964,
 0.0718171312896538,
 0.07284655008370443,
 0.07461111964572599,
 0.07759791600323905,
```

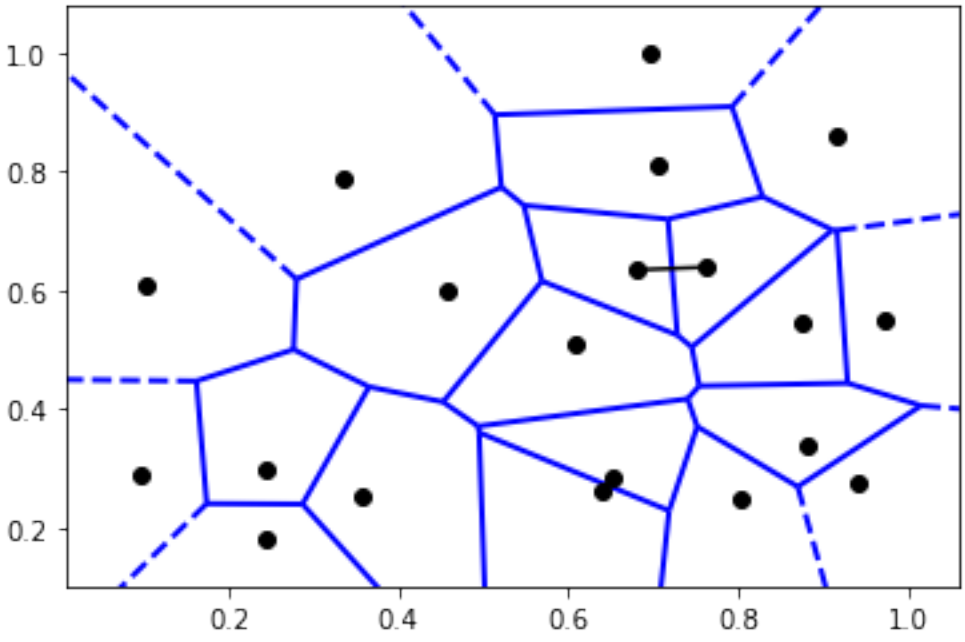
0.08646416102371164,
0.08813500028678328,
0.08977646358179073,
0.09116980315881915,
0.09136555055470971,
0.09144332484158732,
0.09273738722293785,
0.09314123008885496,
0.10320292465591177,
0.10800424198696701,
0.11356902673810587,
0.11356929577552029,
0.11391056703882392,
0.11500638260260286,
0.11527046280465017,
0.11970598498904293,
0.1287773303569409,
0.13100880710597215,
0.13251628383735528,
0.13494860646400642,
0.13552236642253898,
0.13591668163570714,
0.14103911018811138,
0.14753151622725796,
0.1506616276807287,
0.15878817794096886,
0.15924339251957695,
0.16008936460106923,
0.16032124553800148,
0.16058515378127633,
0.16100919728848836,
0.1612262808166077,
0.17077201079486737,
0.17124353257525474,
0.17190336980359205,
0.174932674429453,
0.17788243762217146,
0.17864341126113065,
0.1790867412845234,
0.1801713464965598,
0.18079179079051558,
0.18342630211201155,
0.1842390140019963,
0.18530372758297425,
0.1860185439096894,
0.18794891774059627,
0.20505743493329942,

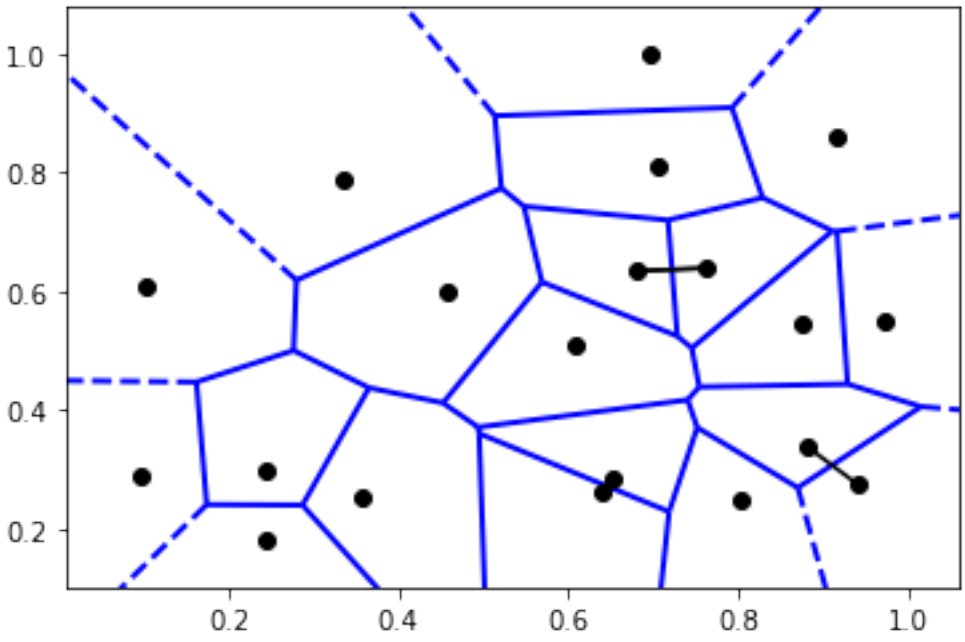
```
 0.20903538737809918,
 0.3918415394717987,
 1.0558267620678936]
```

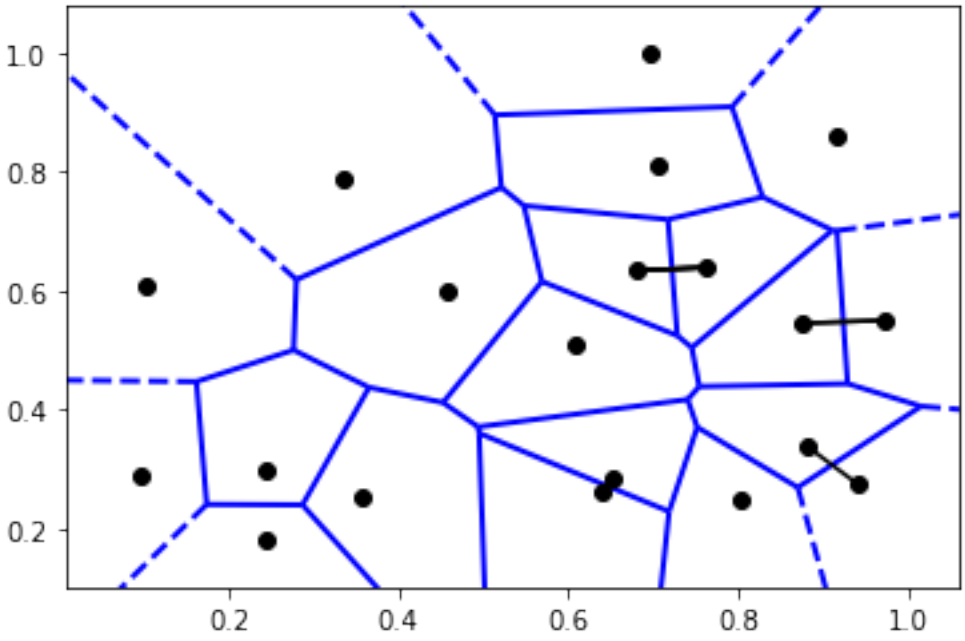
[77]:
```
for value in alpha3.thresholdvalues():
    K3=alpha3.sublevel(value)
    plotalpha(points3,K3)
```

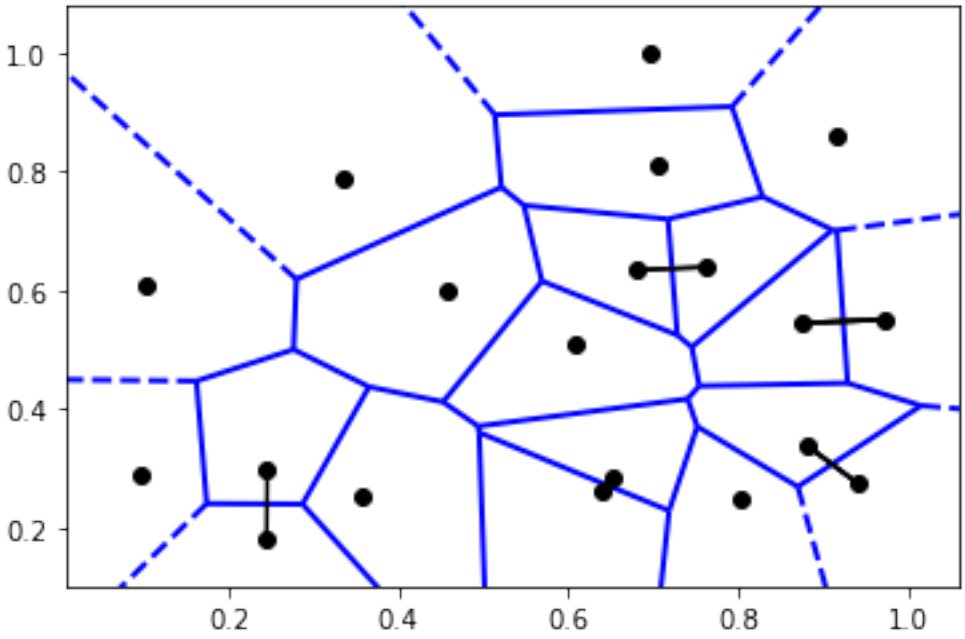


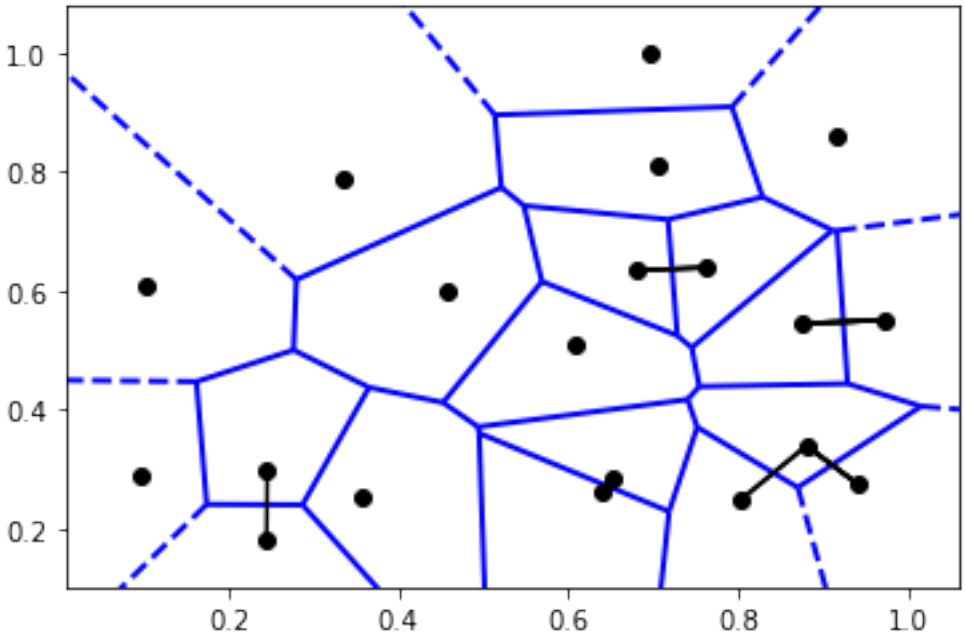


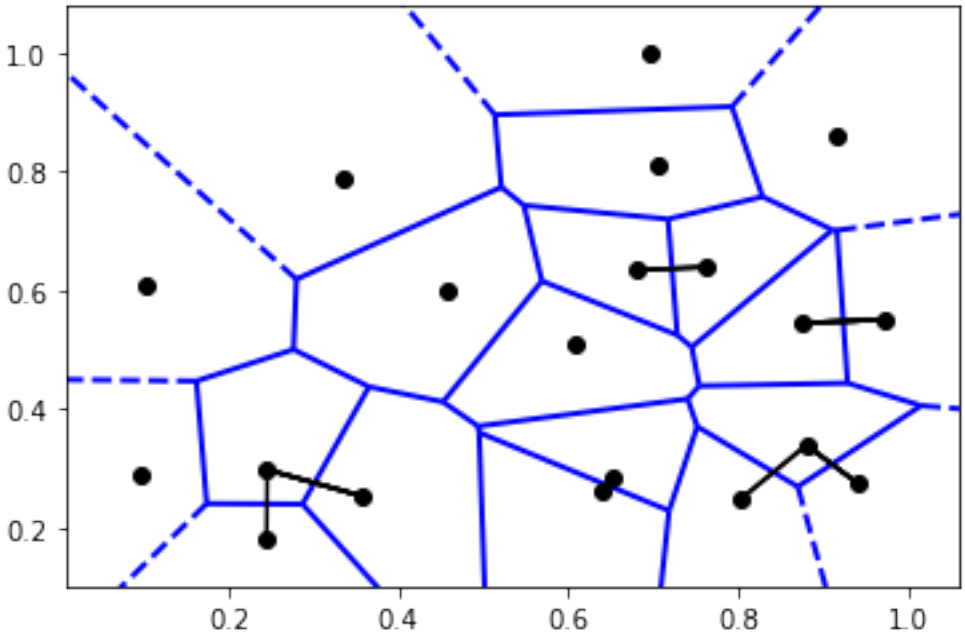

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

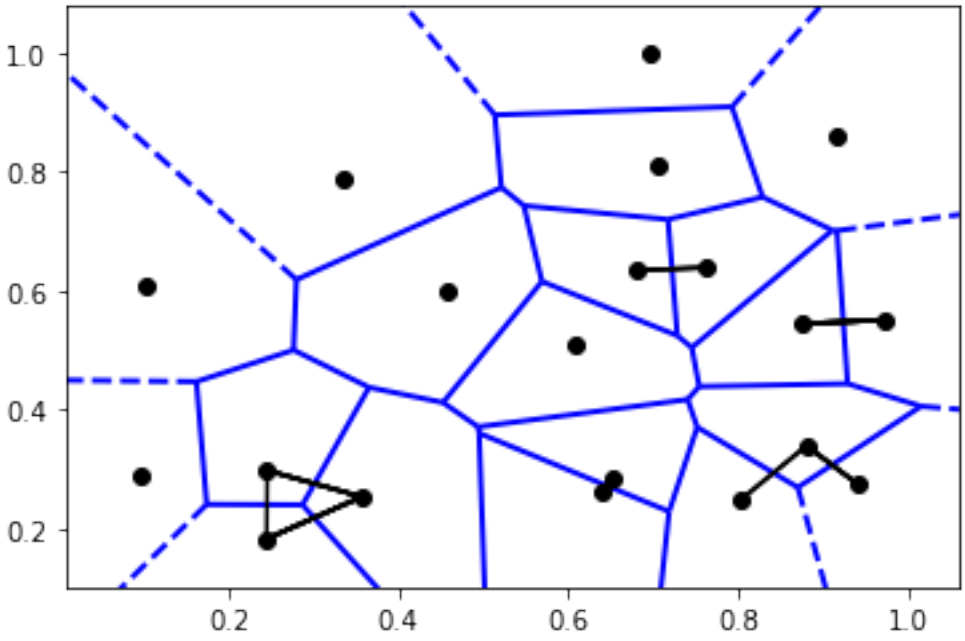

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

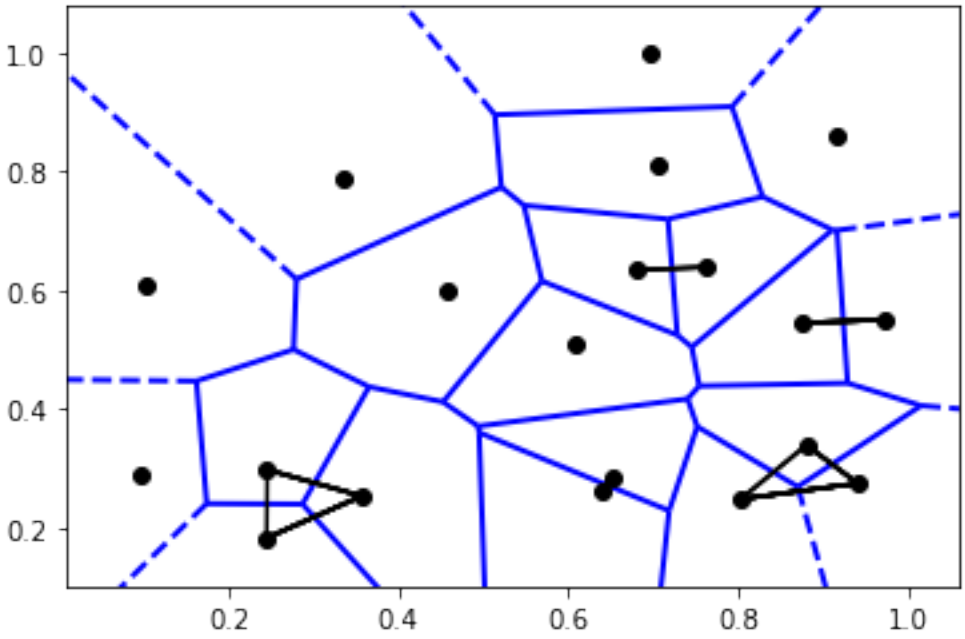

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

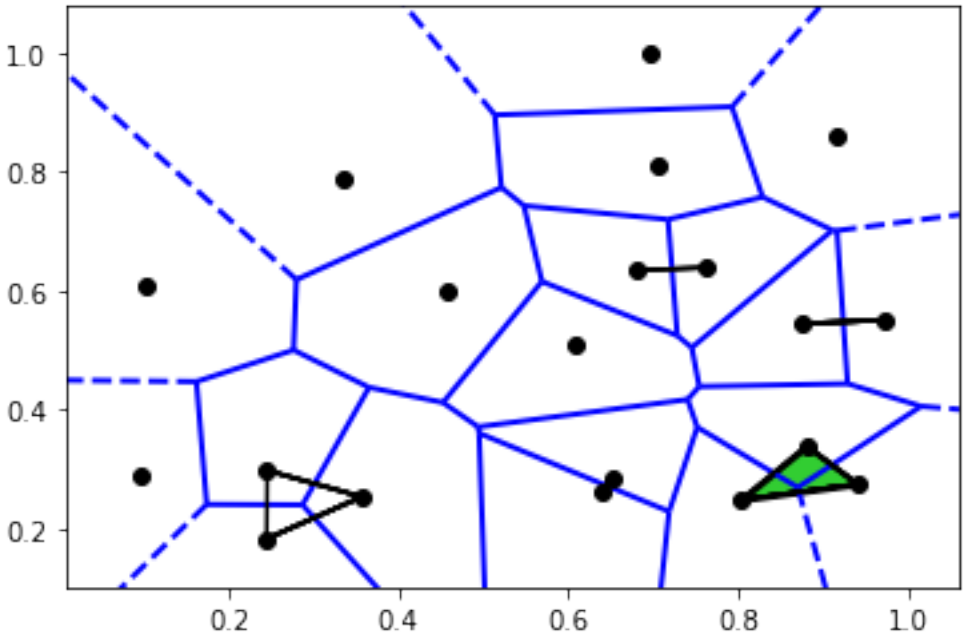

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

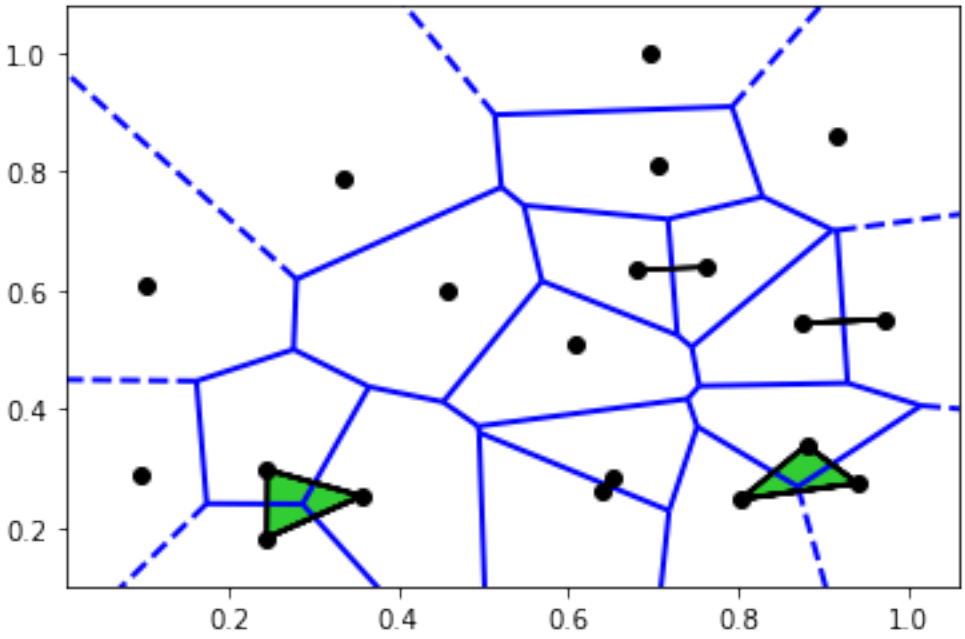

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

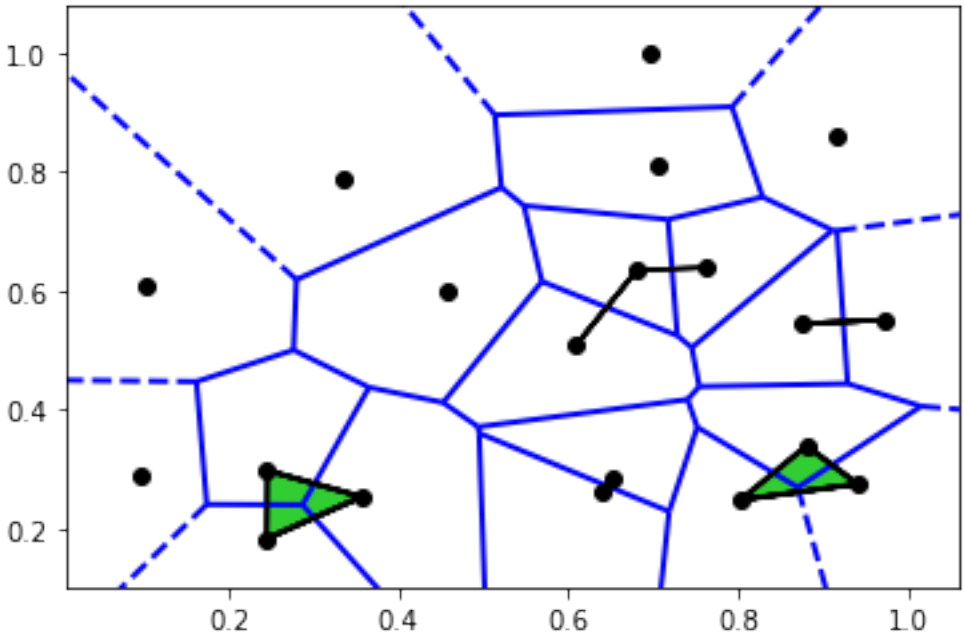

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

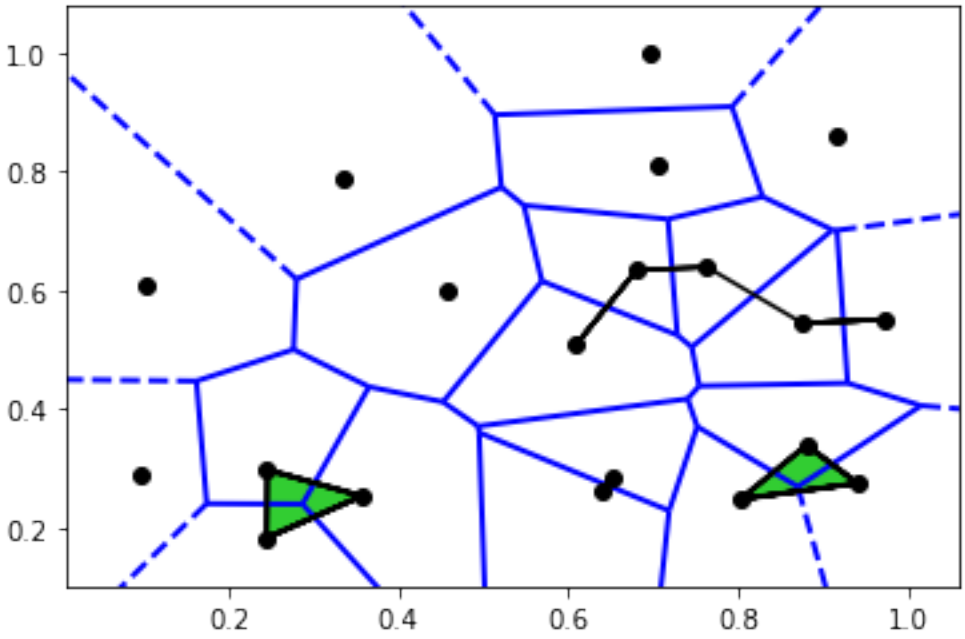

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

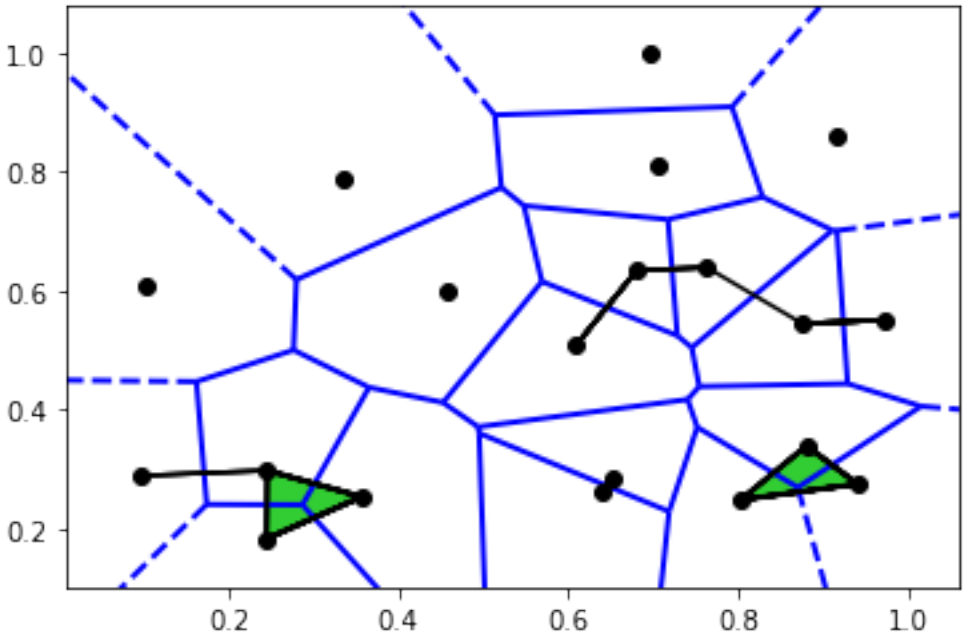

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

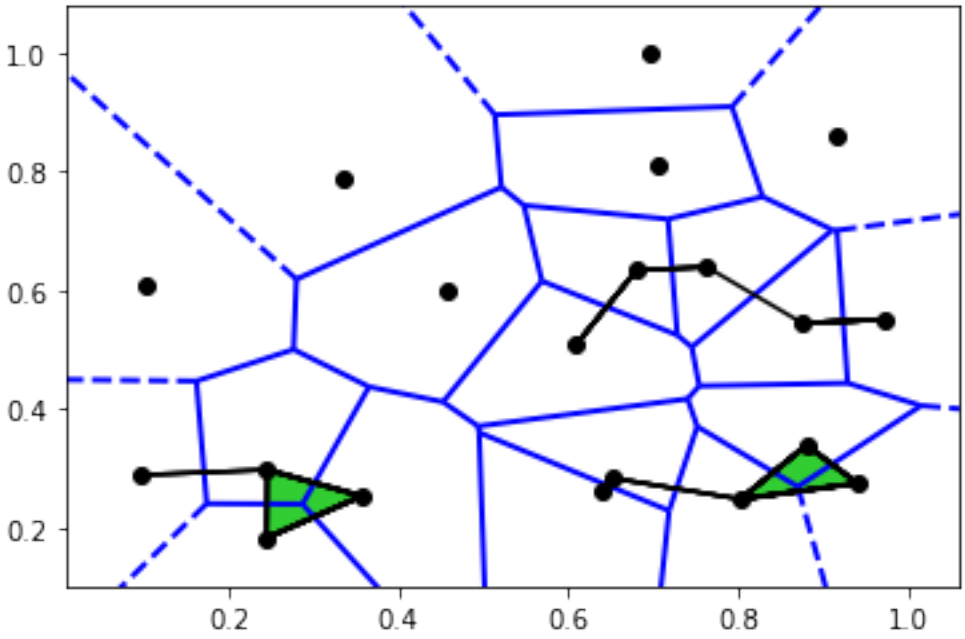

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

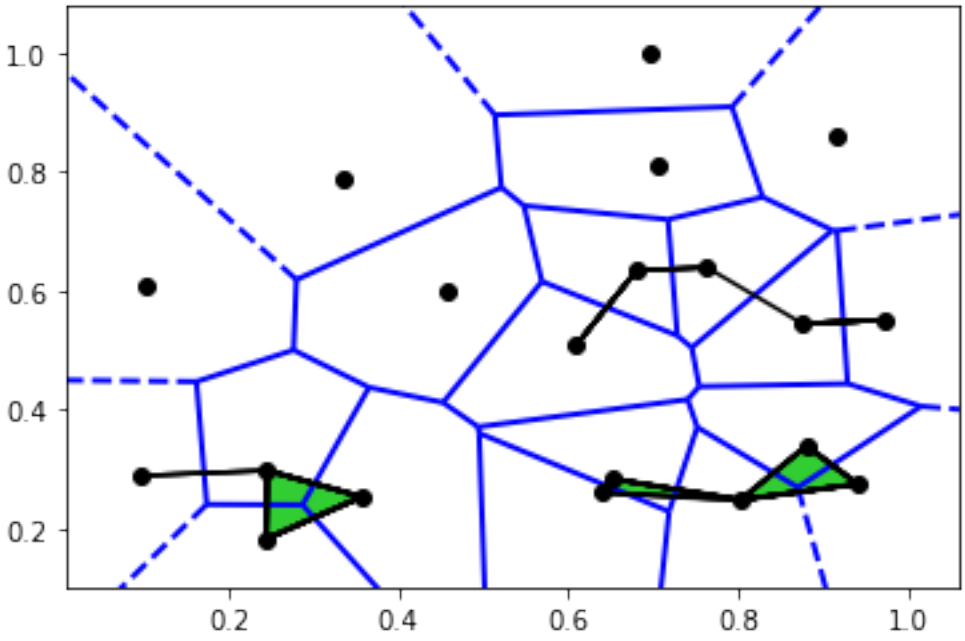

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

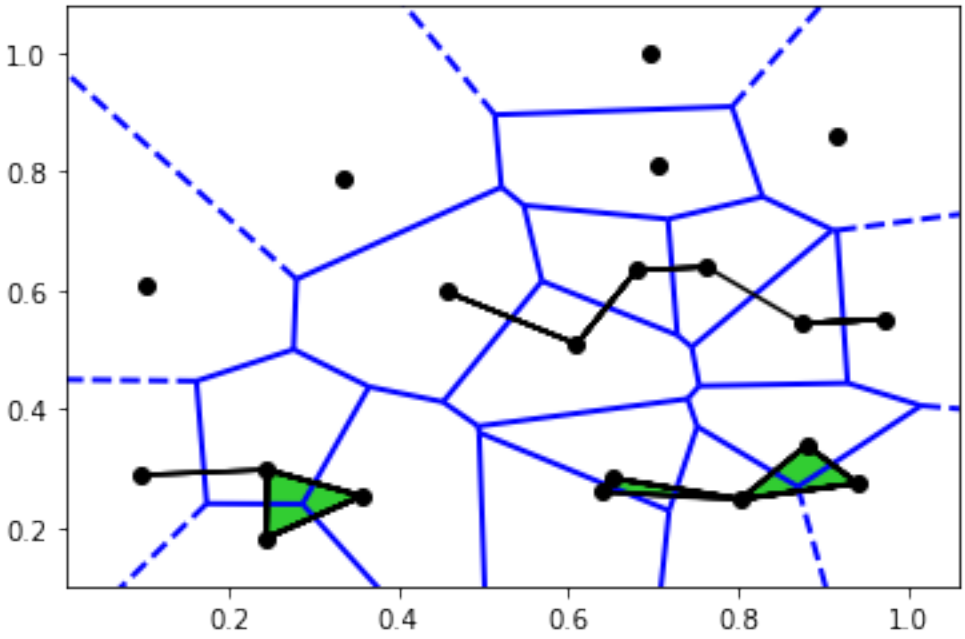

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

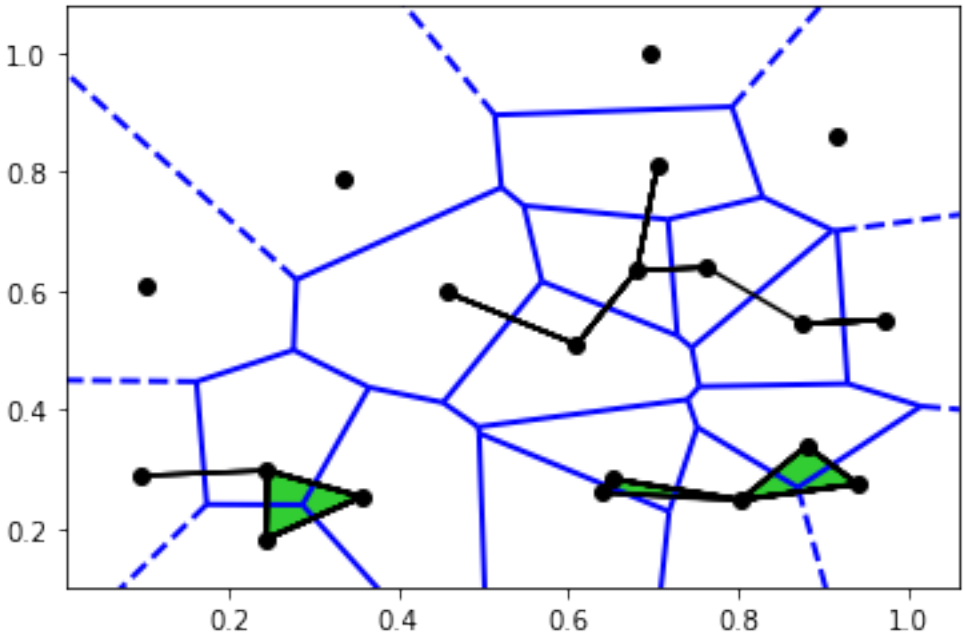

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

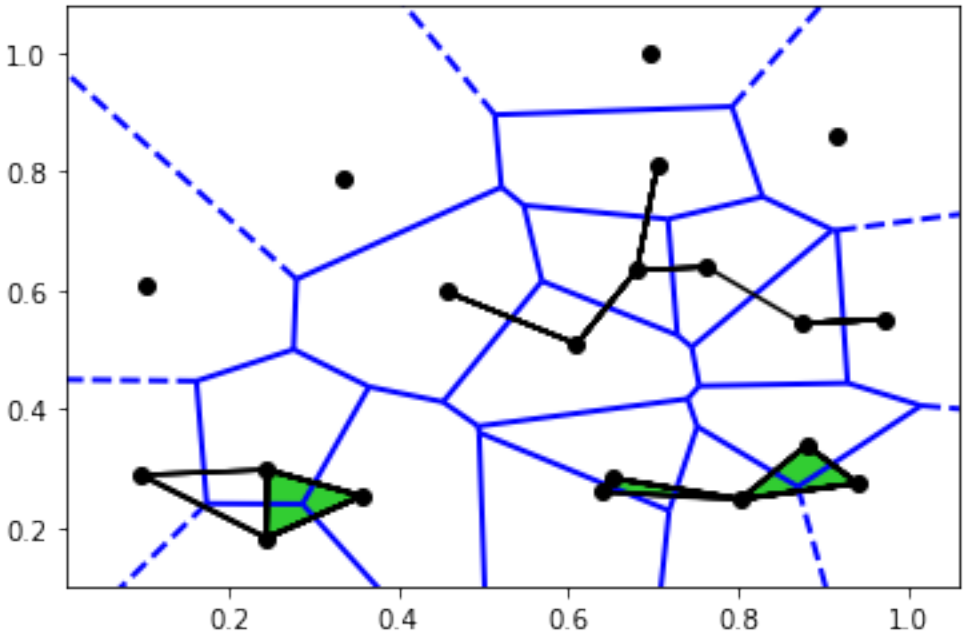

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

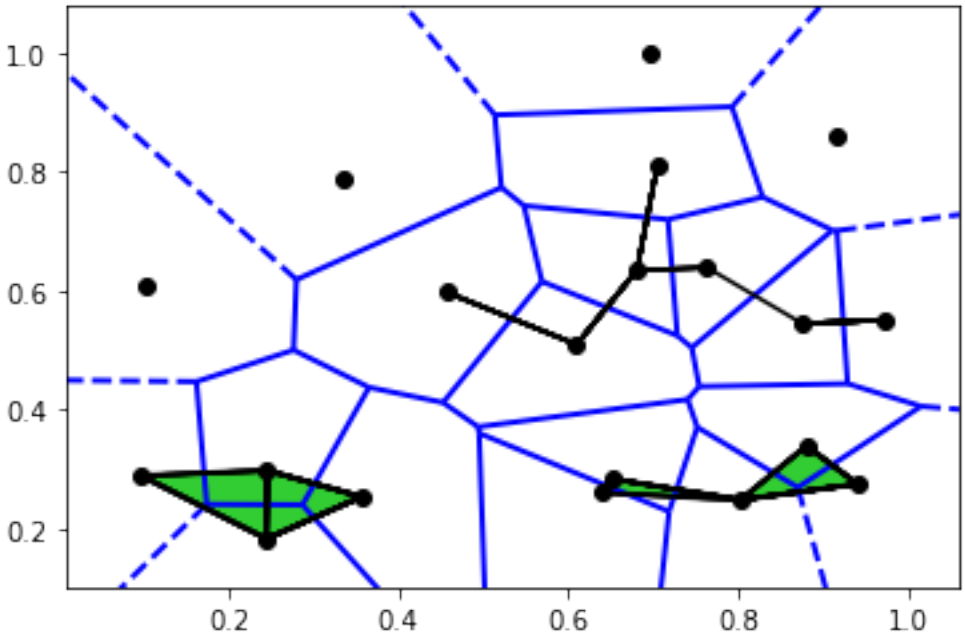

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

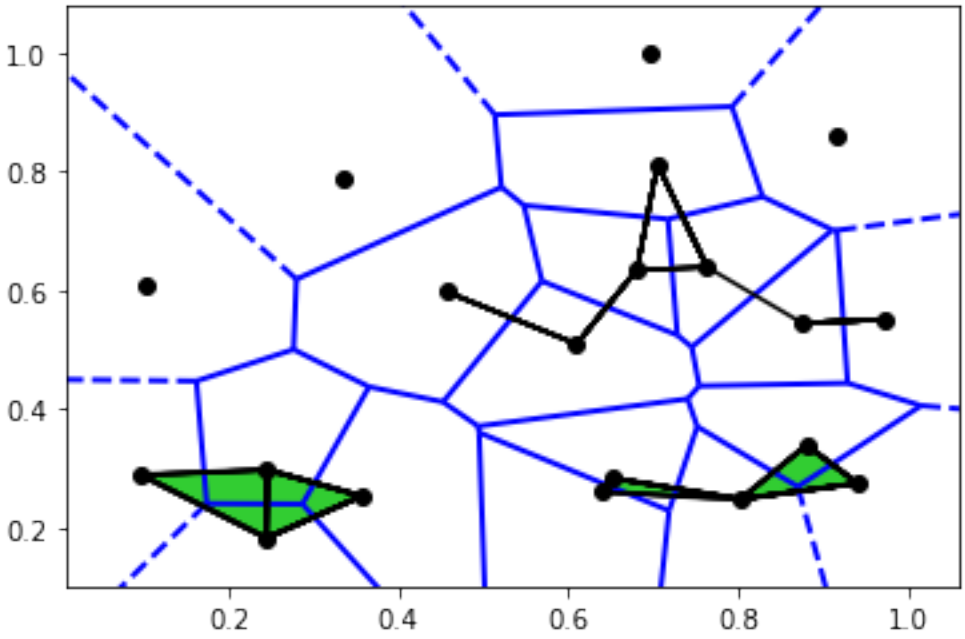

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

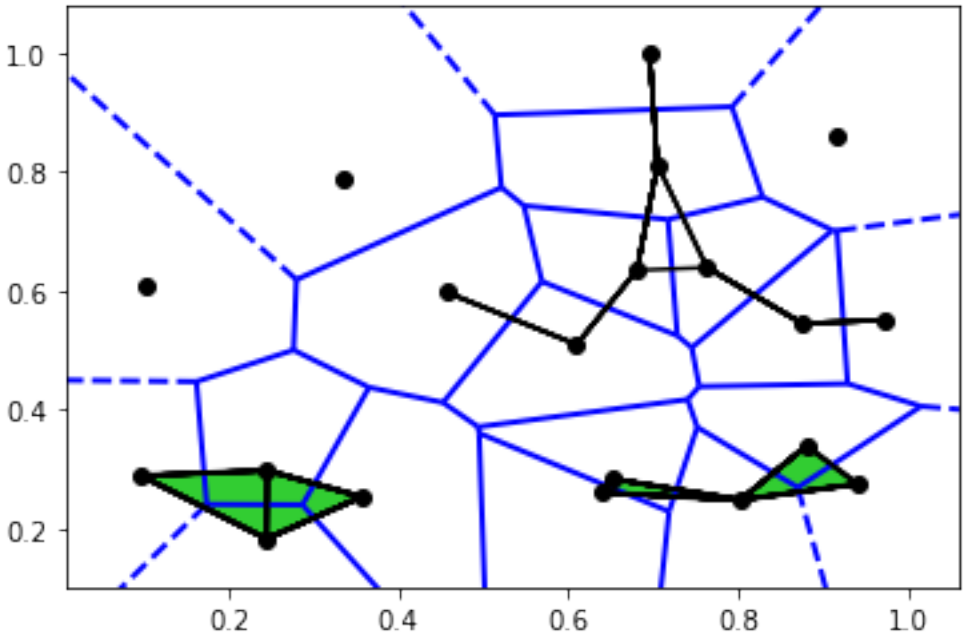

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

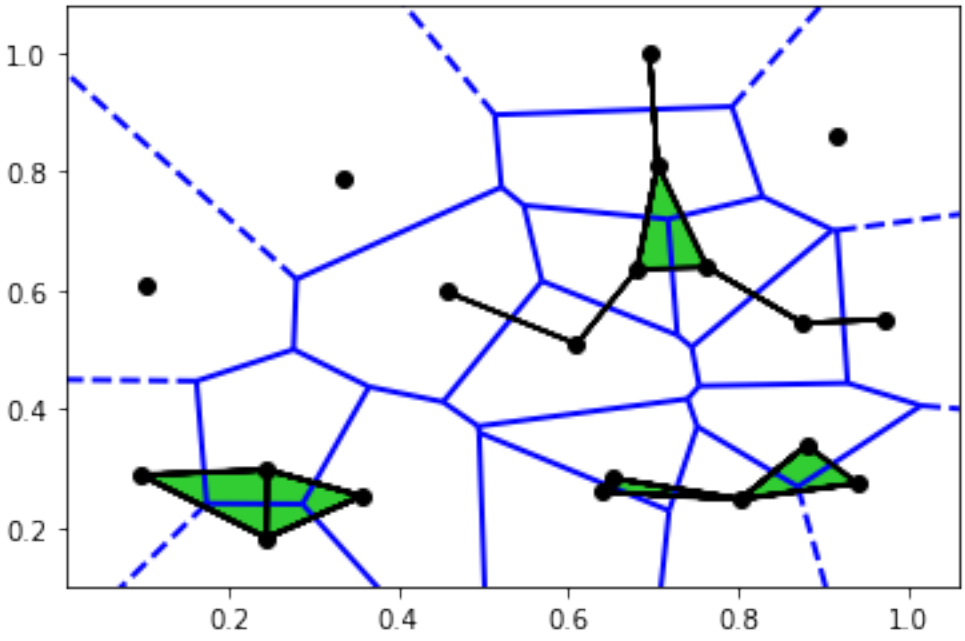

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

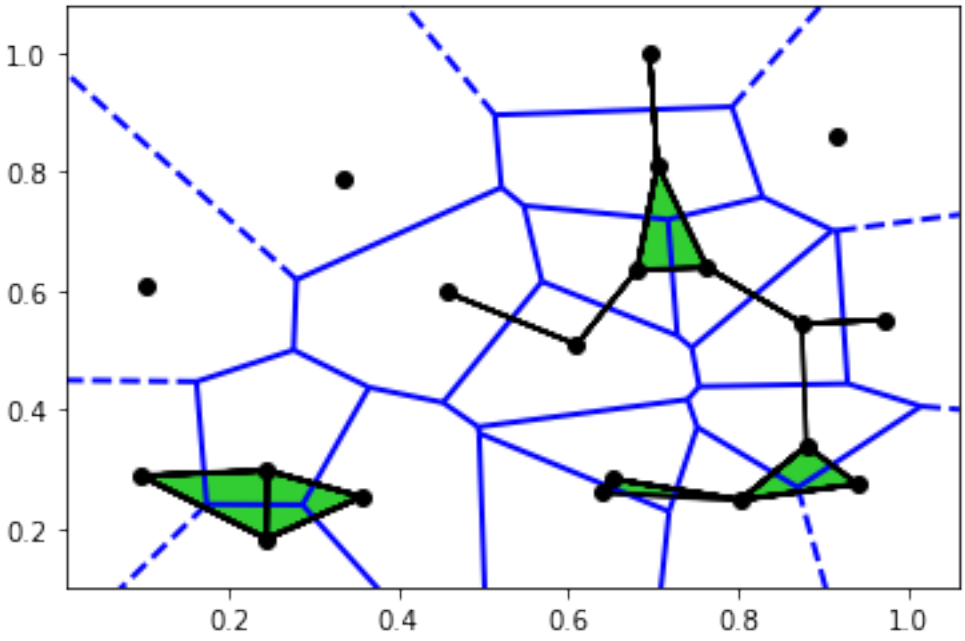

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

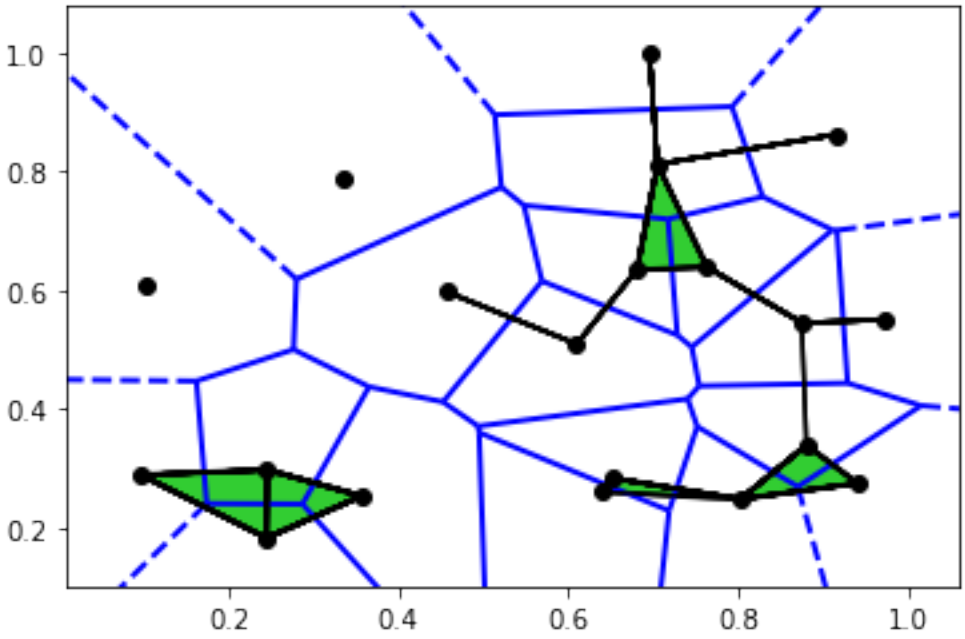

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

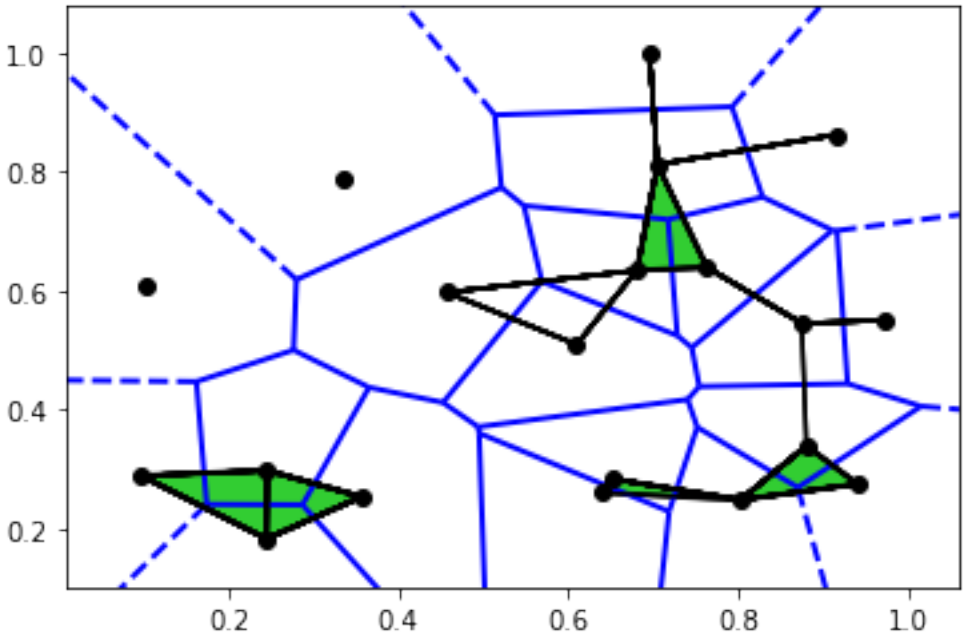

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

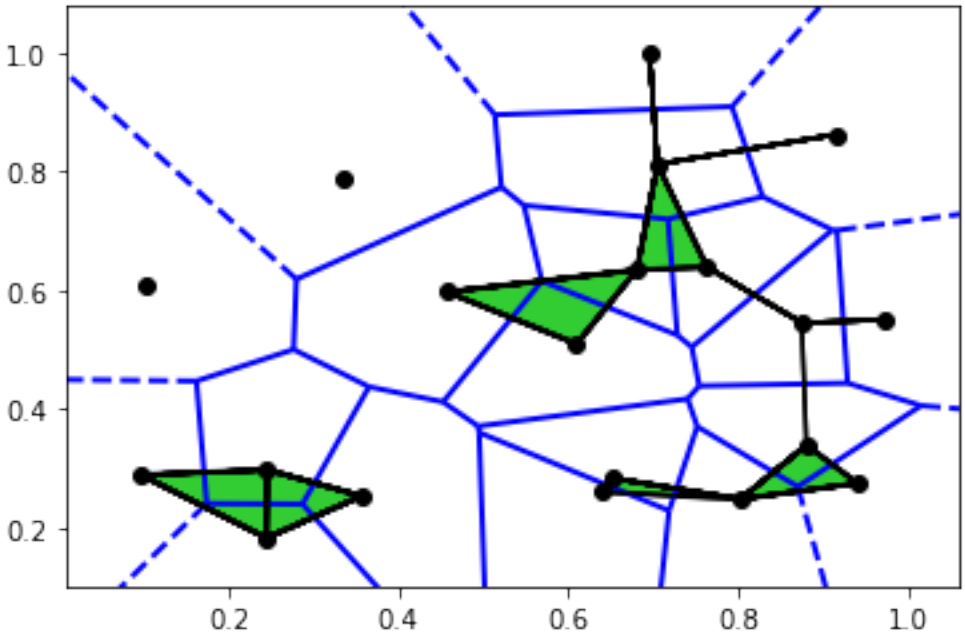

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

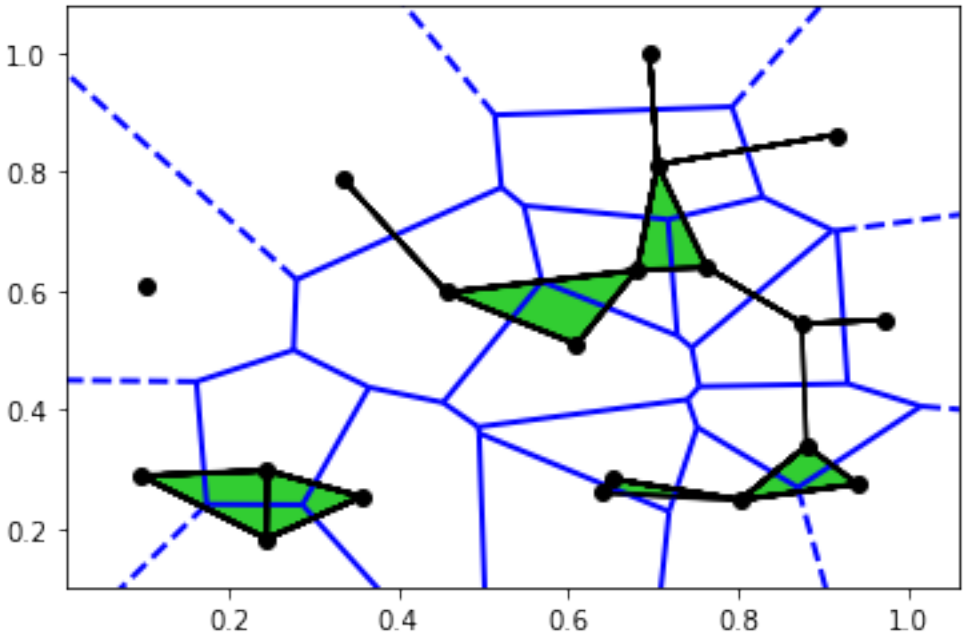

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

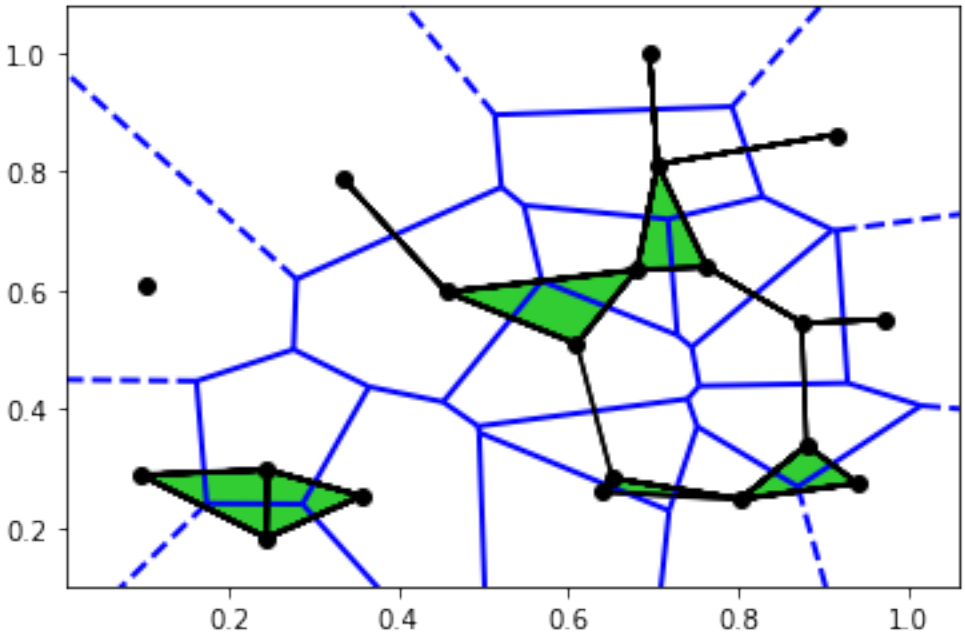

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

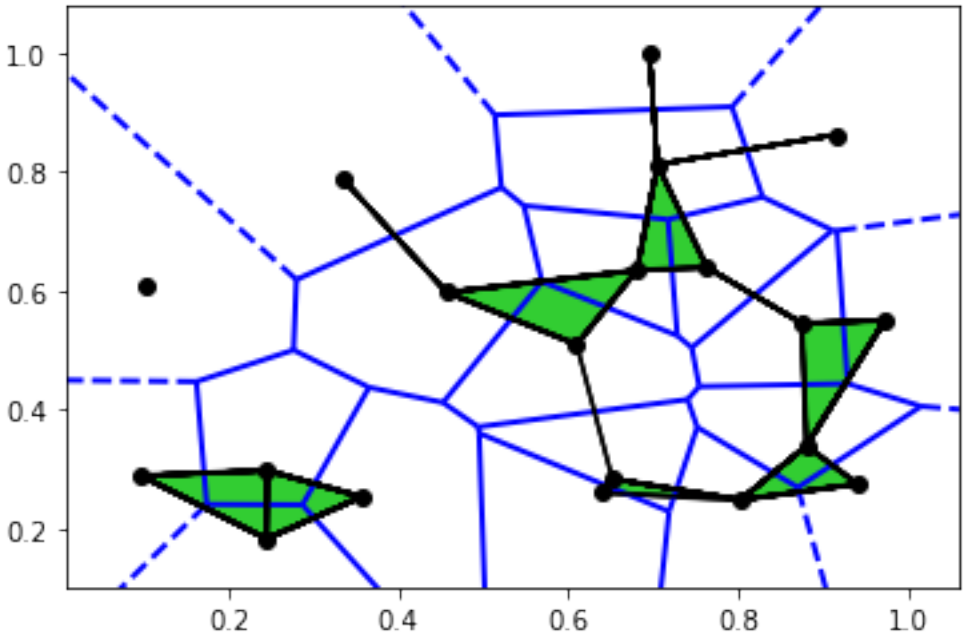

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

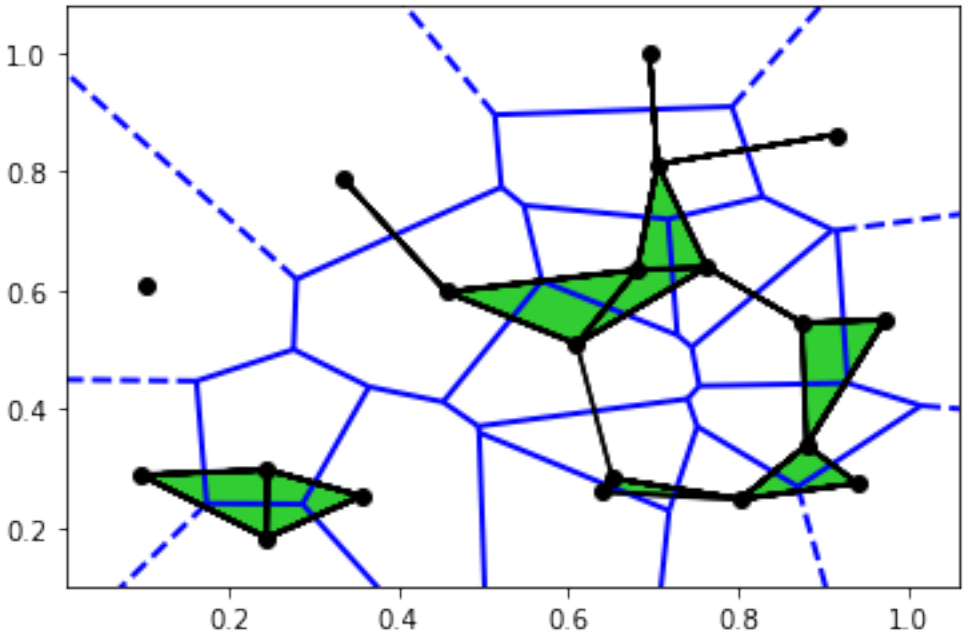

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

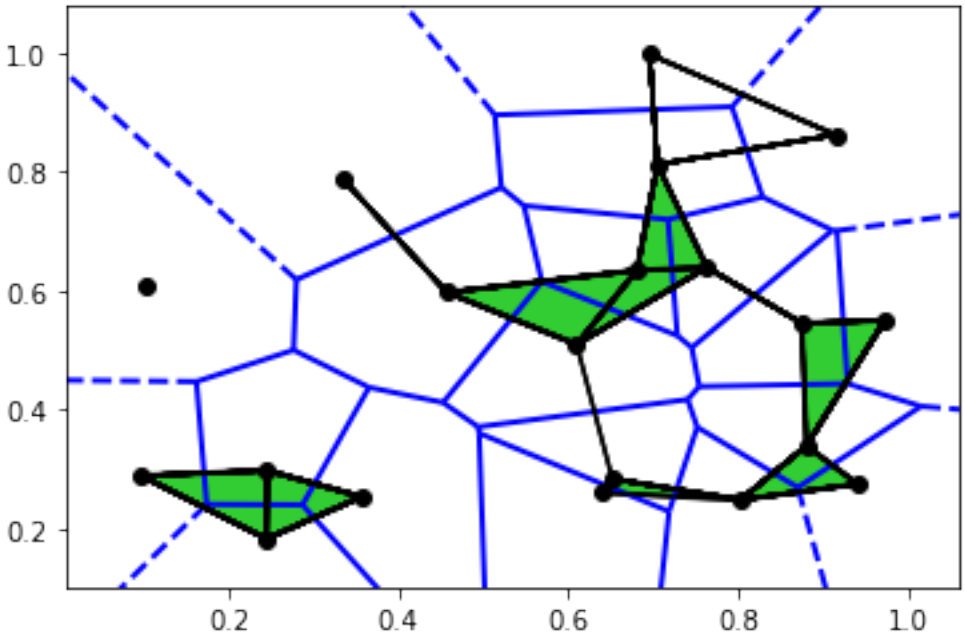

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

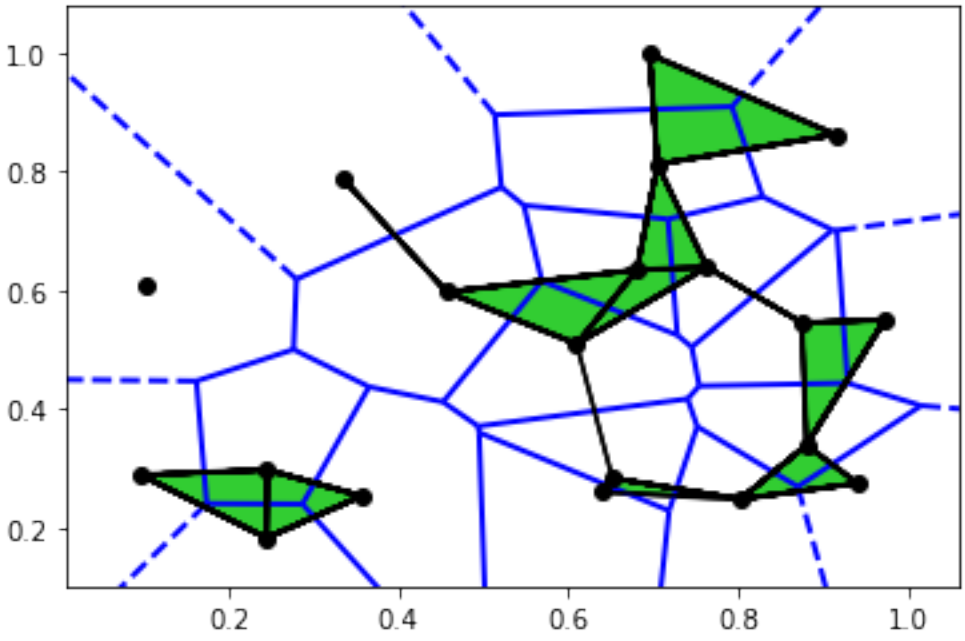

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

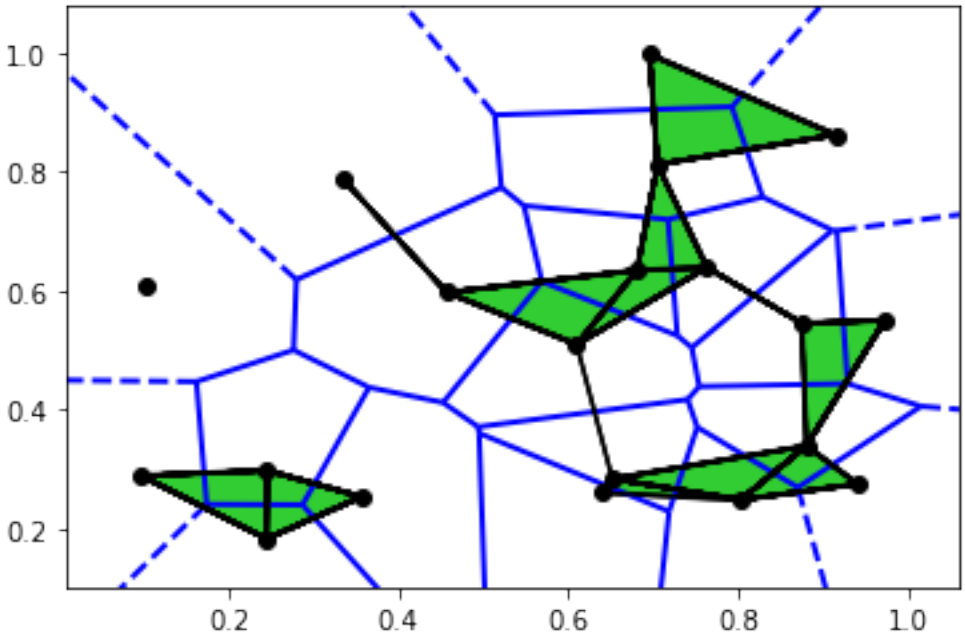

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

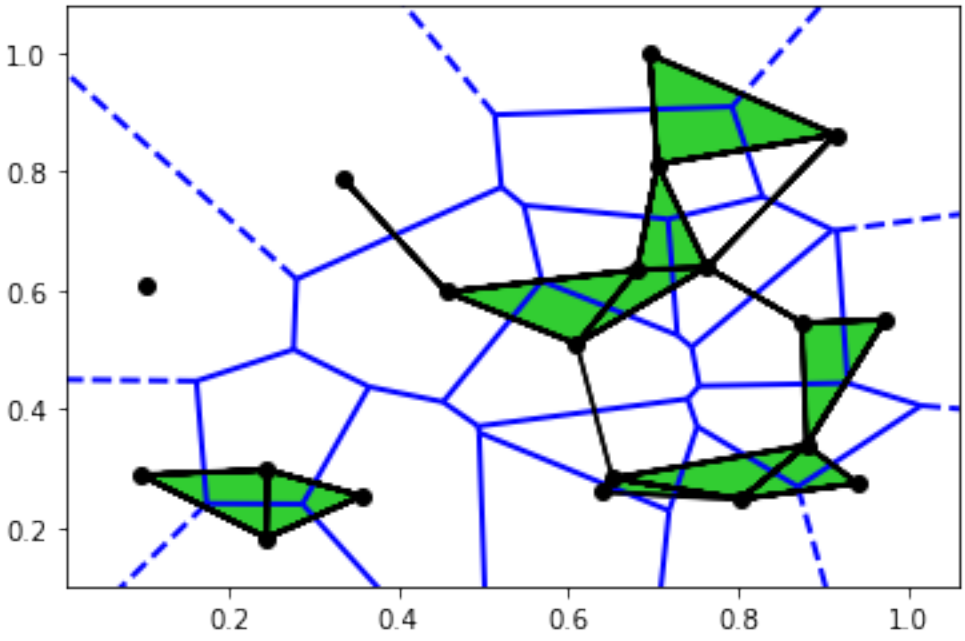

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

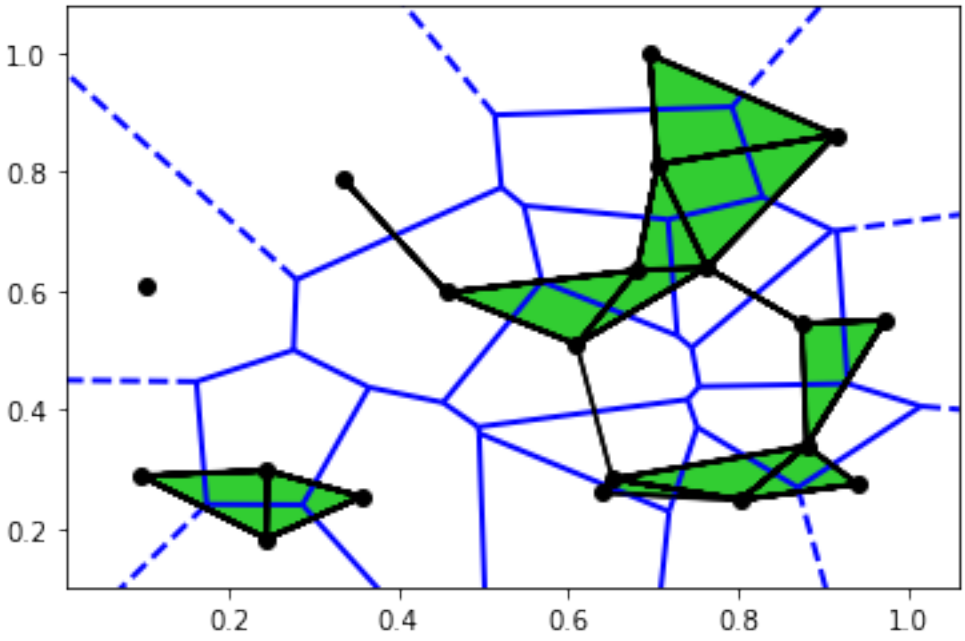

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

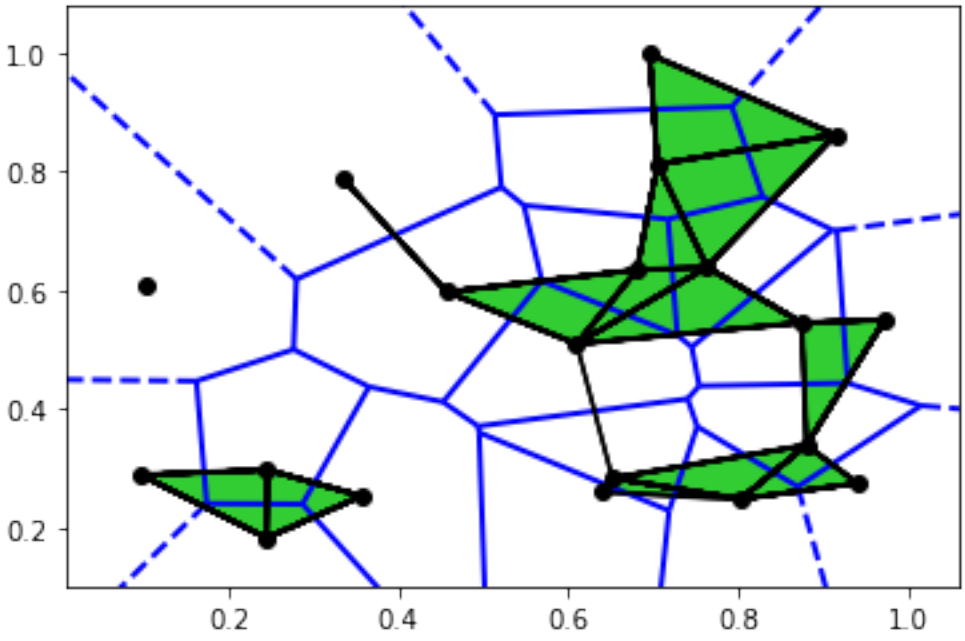

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

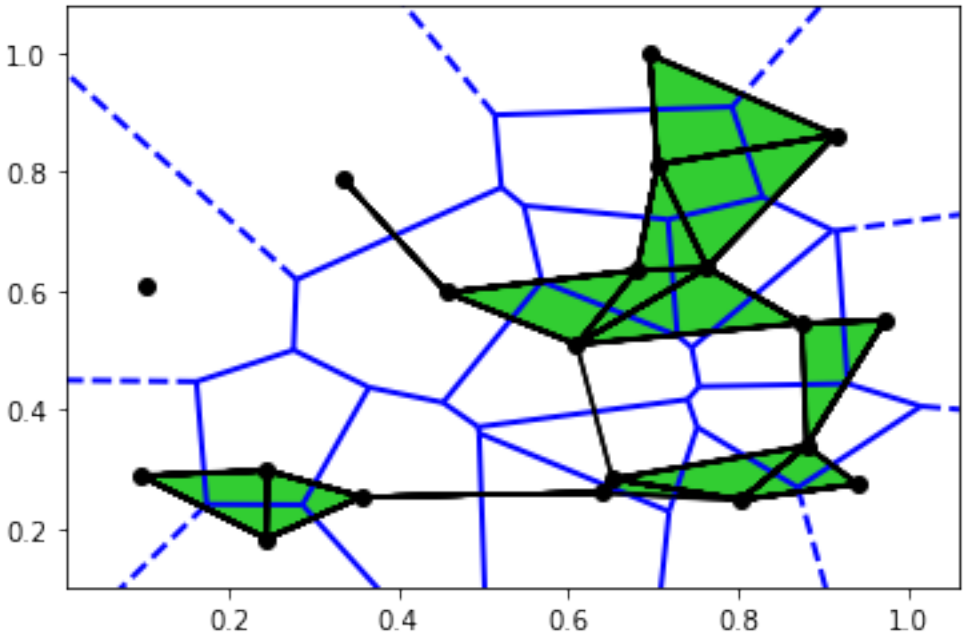

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

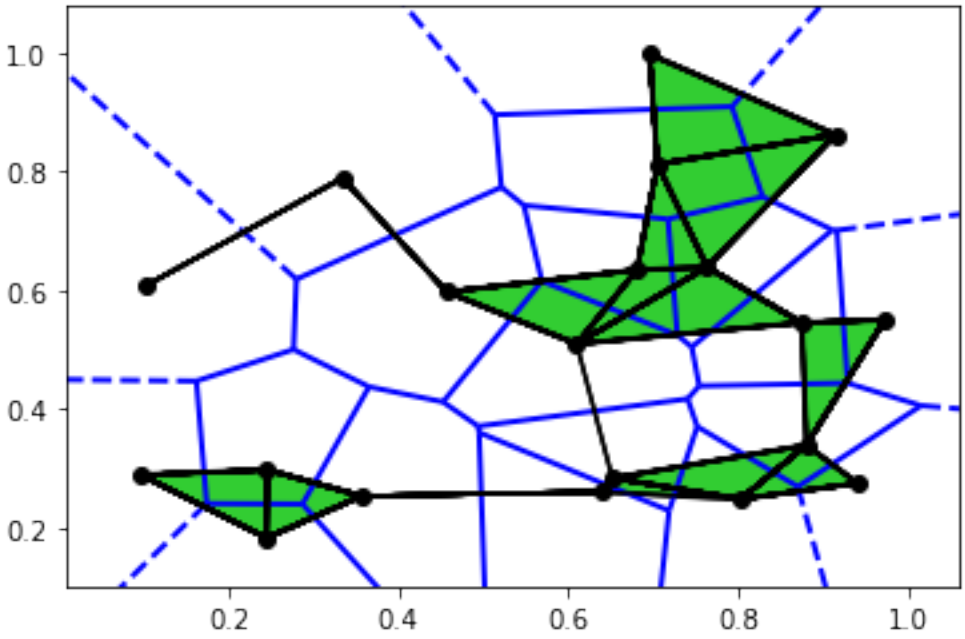

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

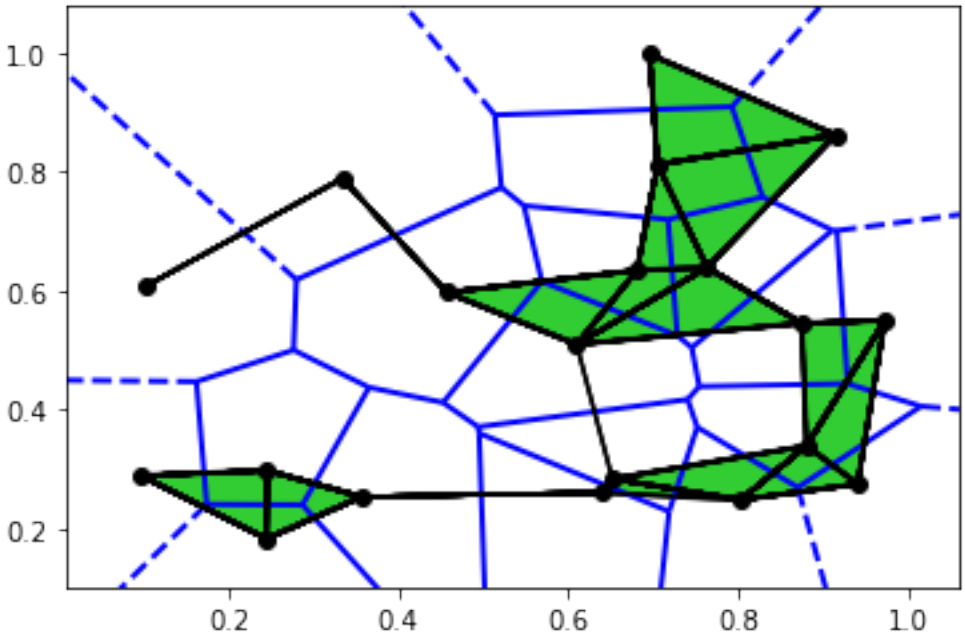

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

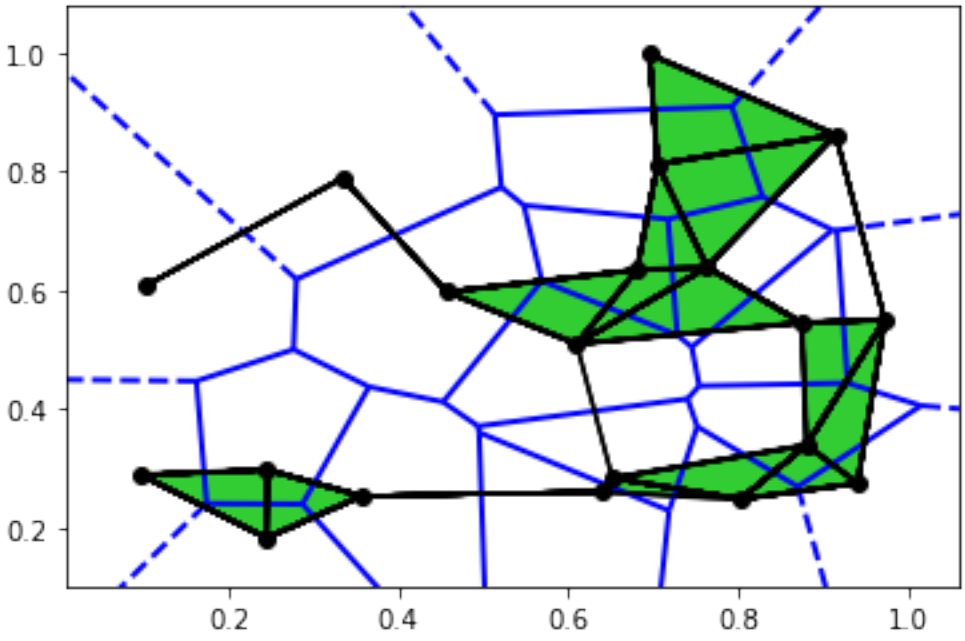

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

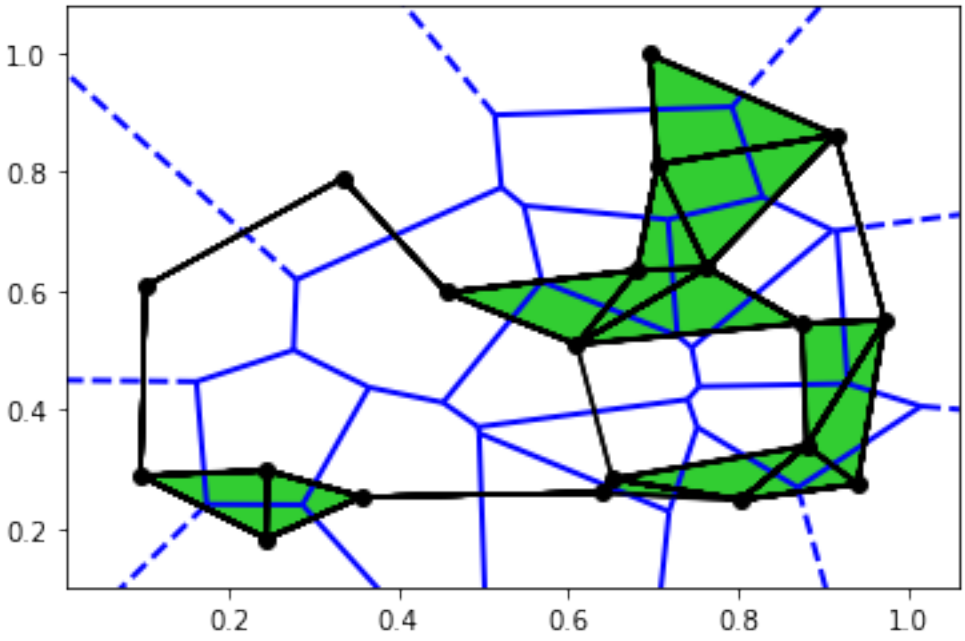

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

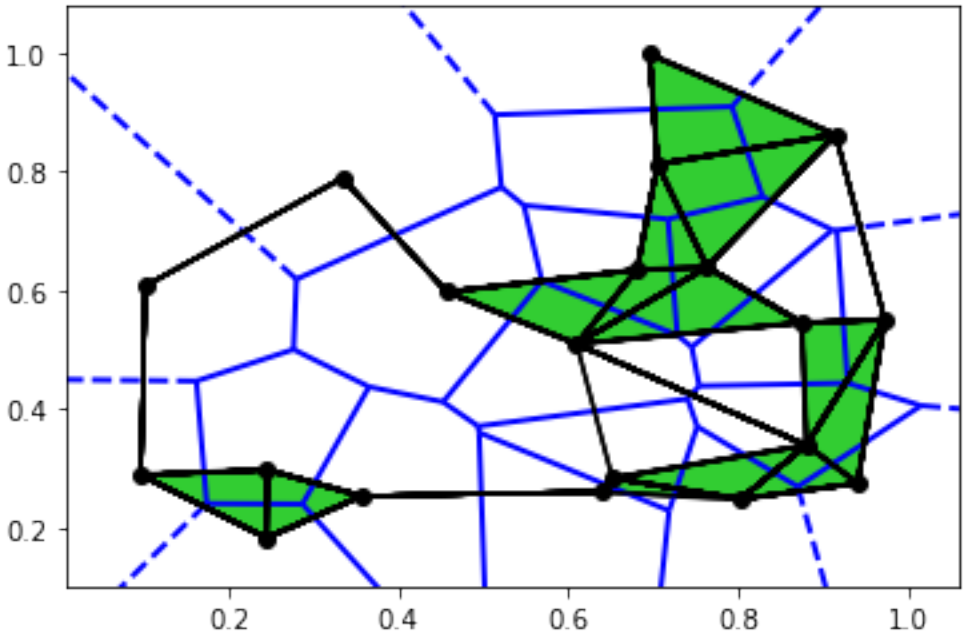

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

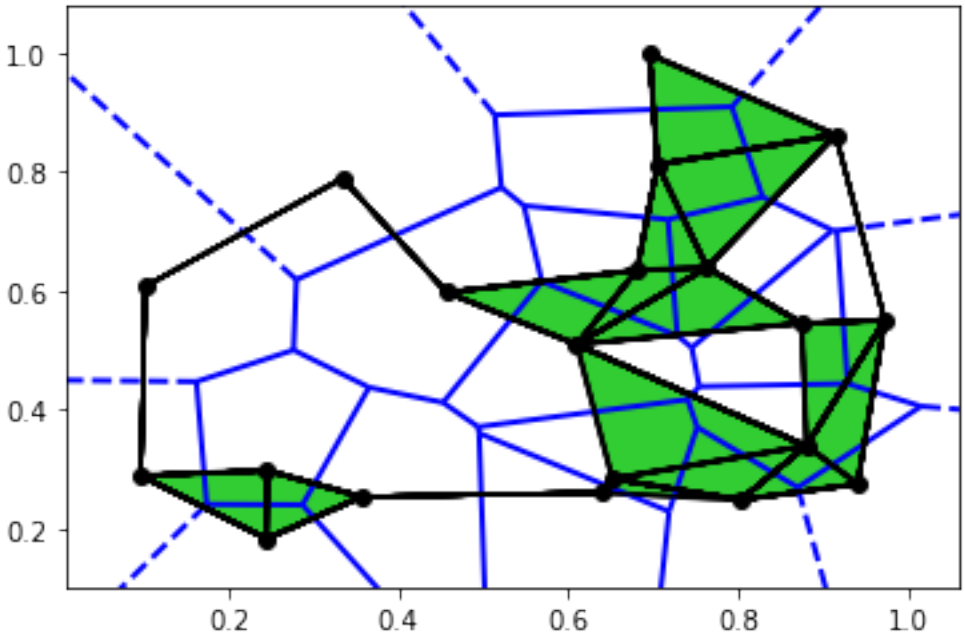

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

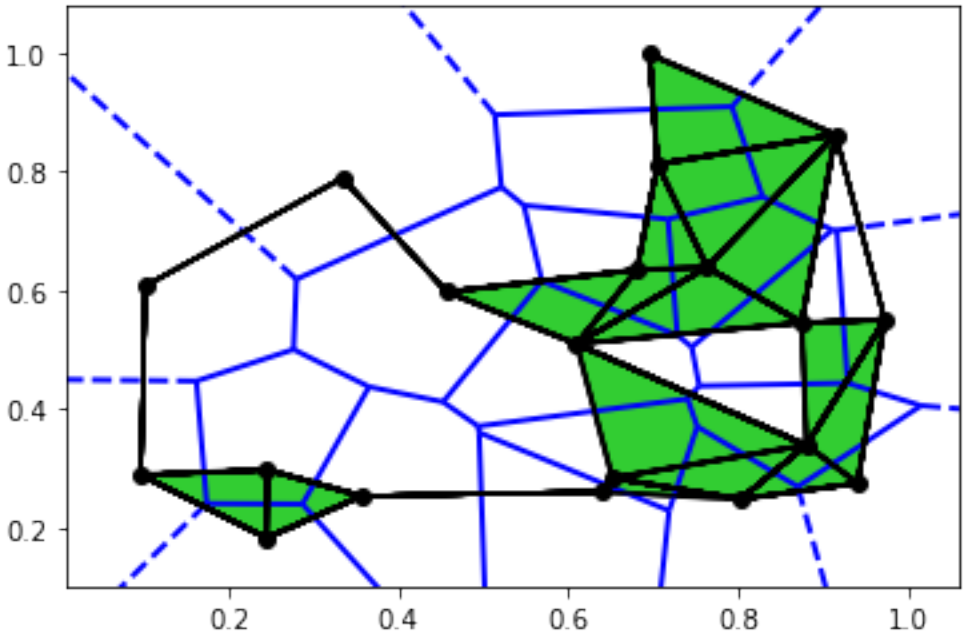

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

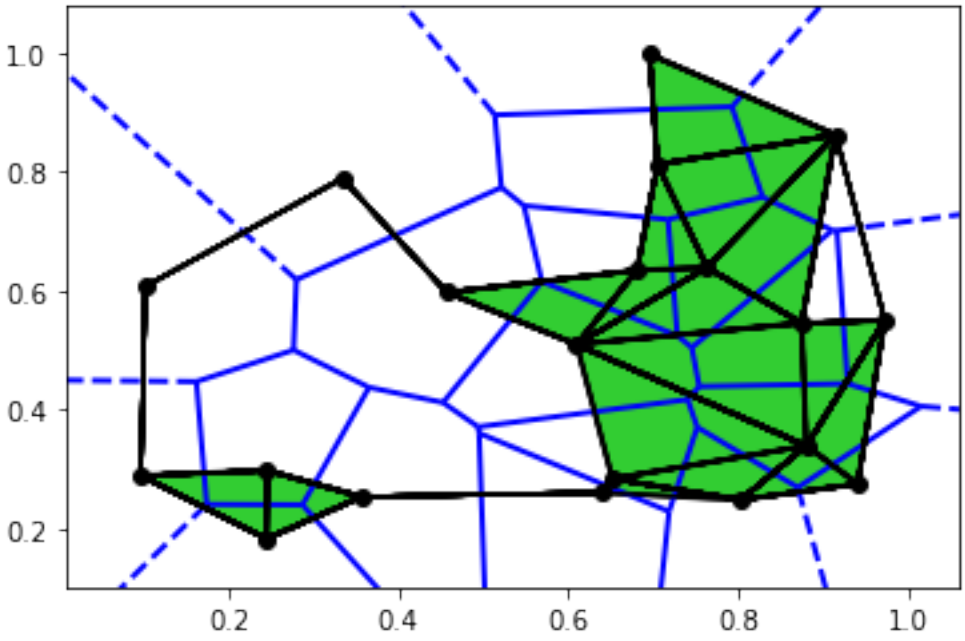

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

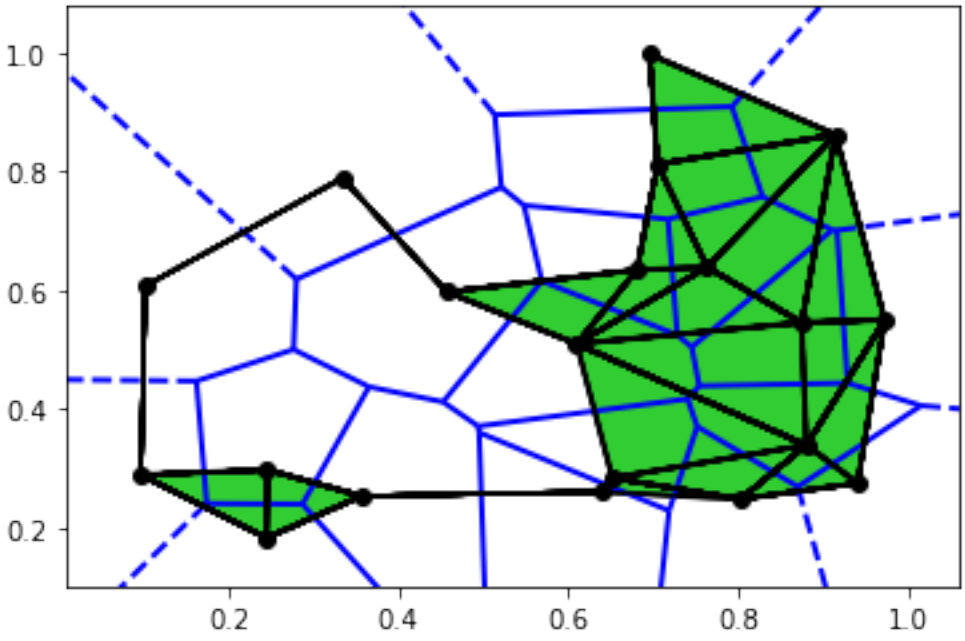

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

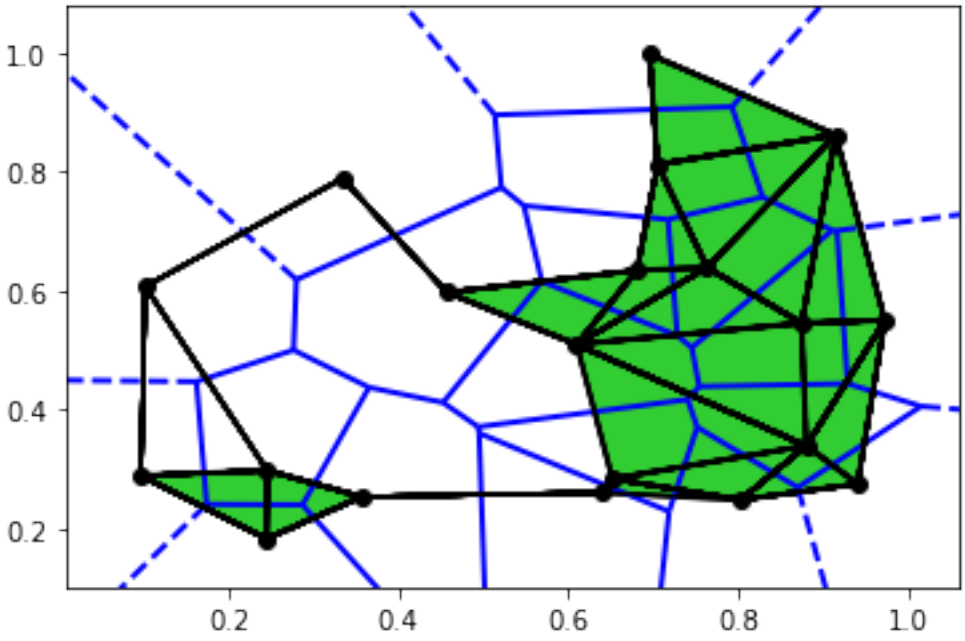

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

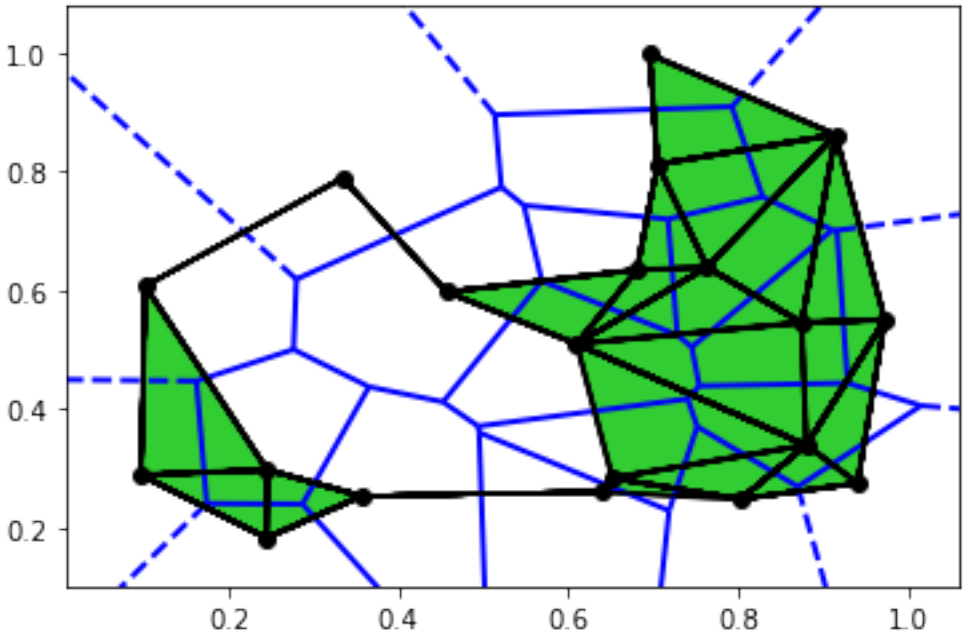

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

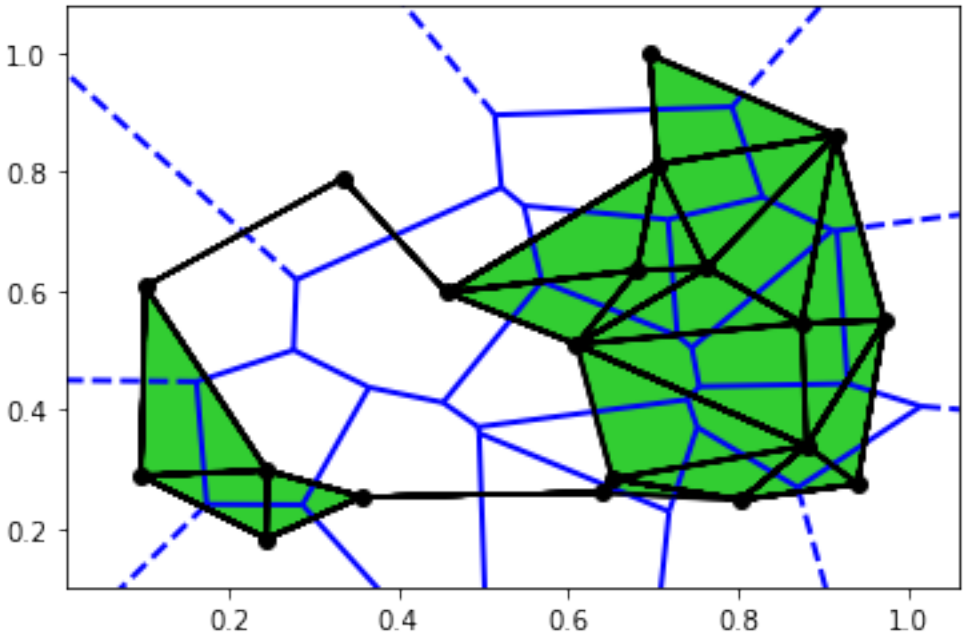

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

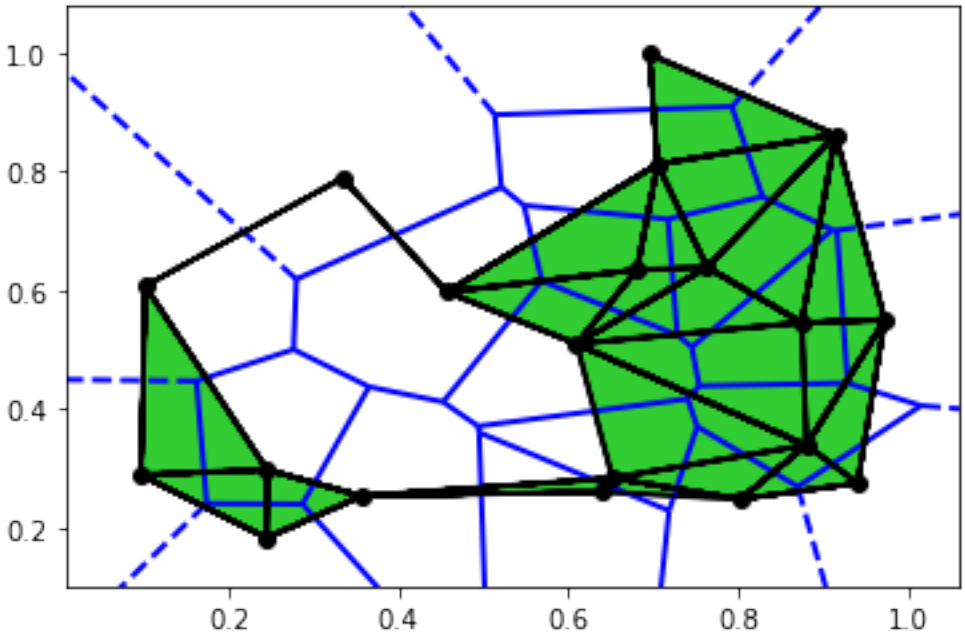

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

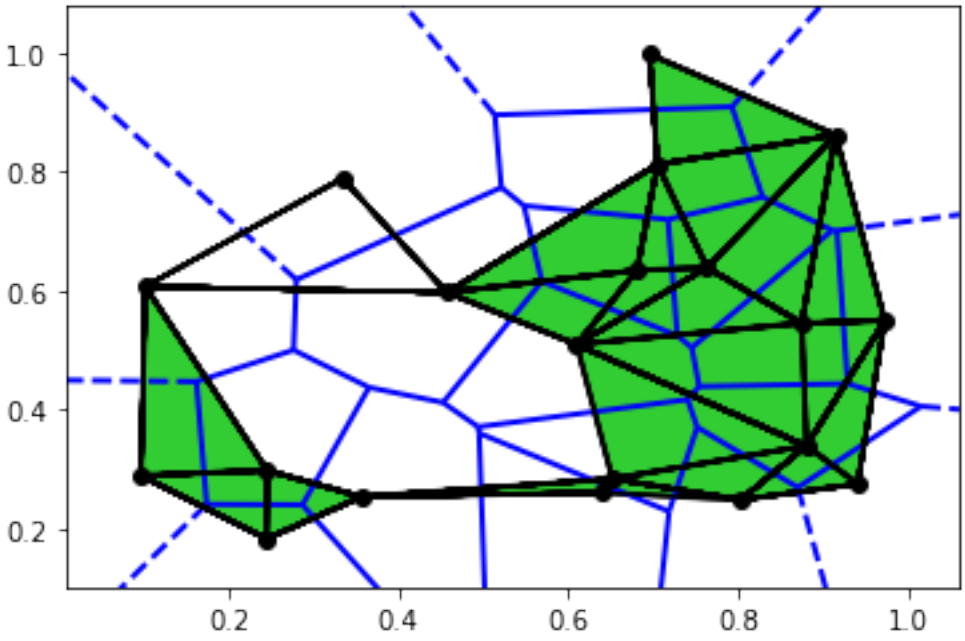

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

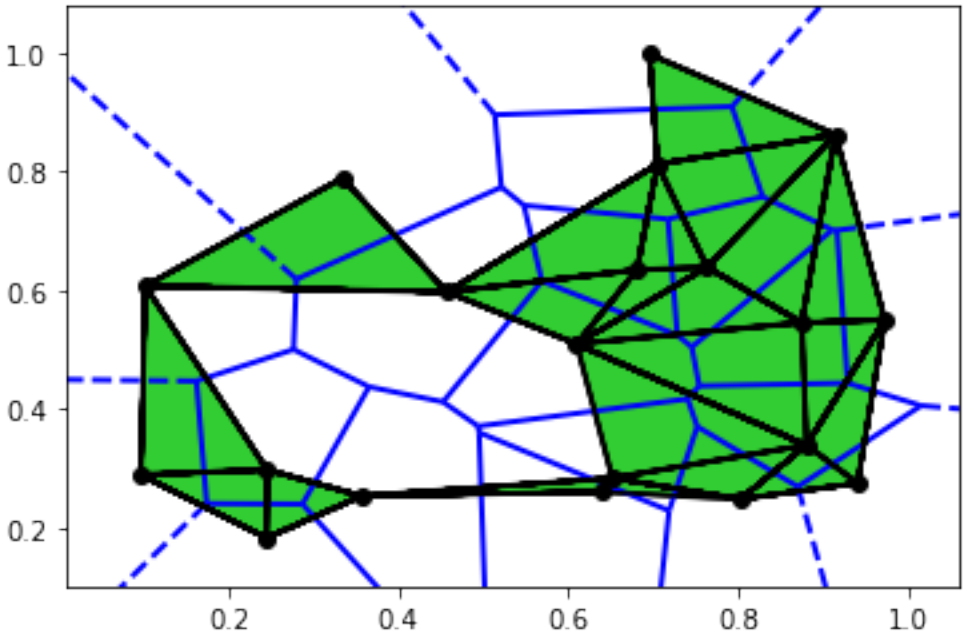

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

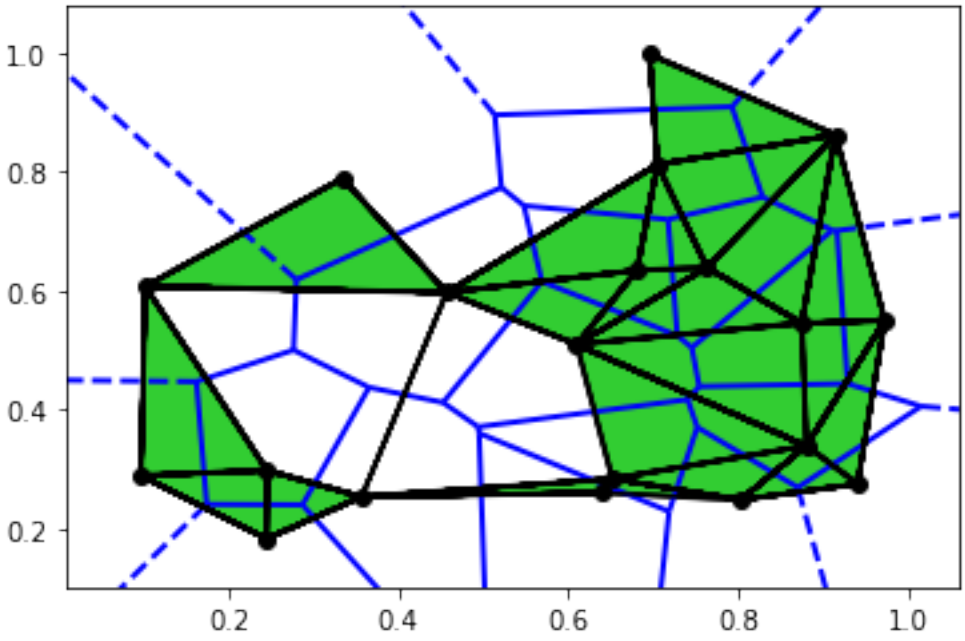

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

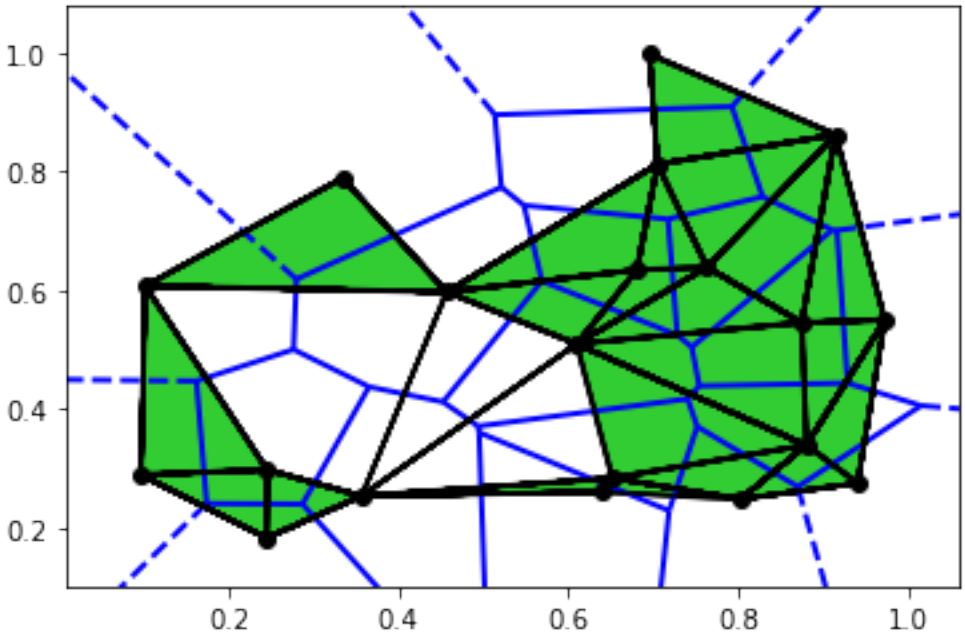

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

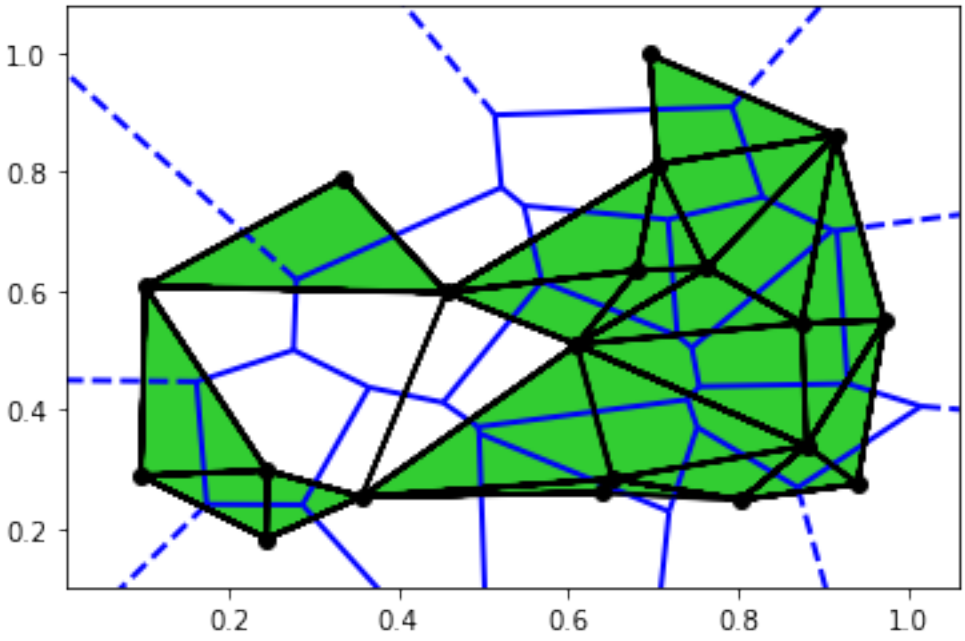

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

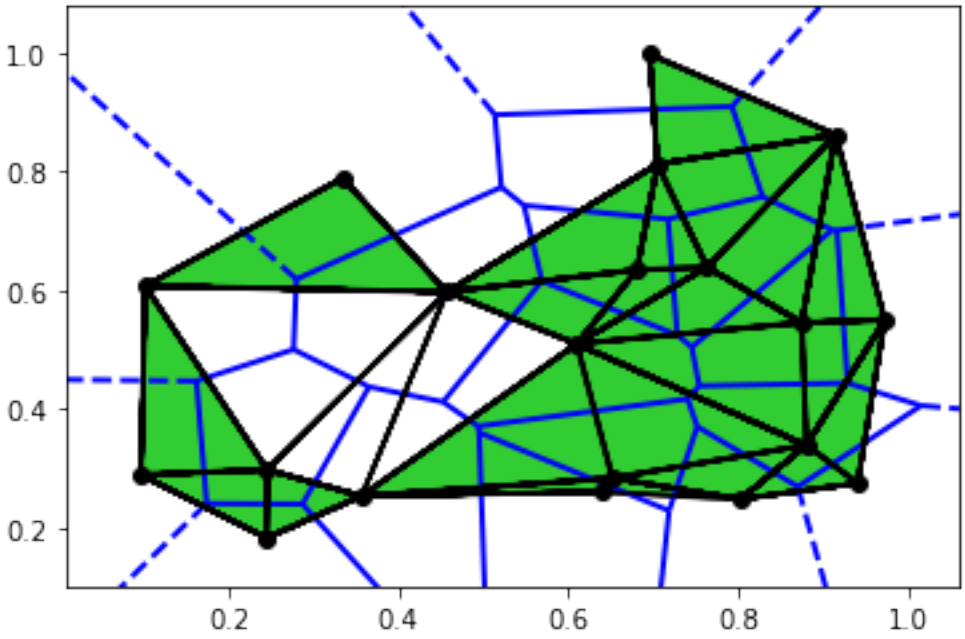

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

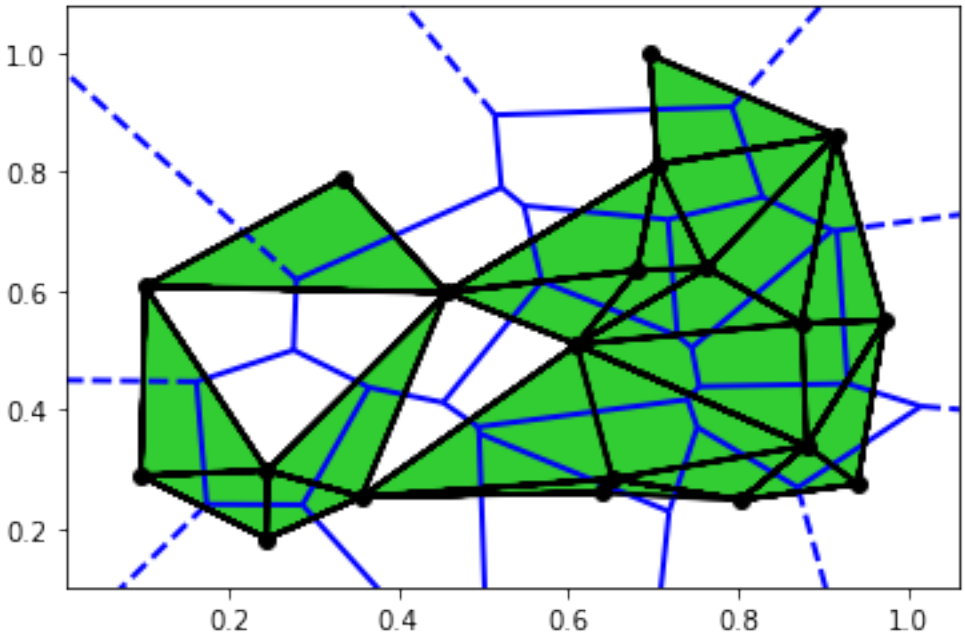

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

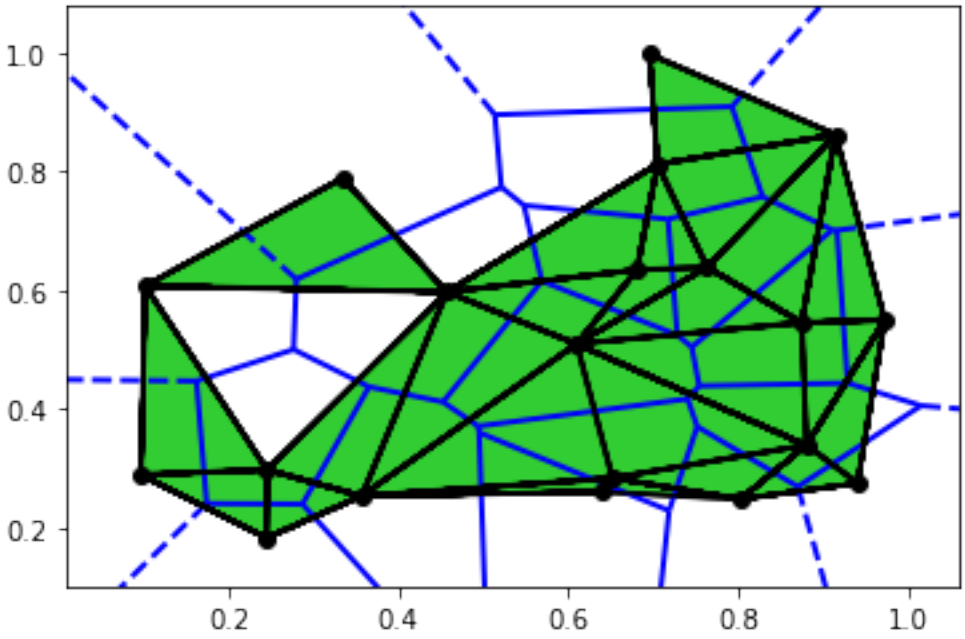

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

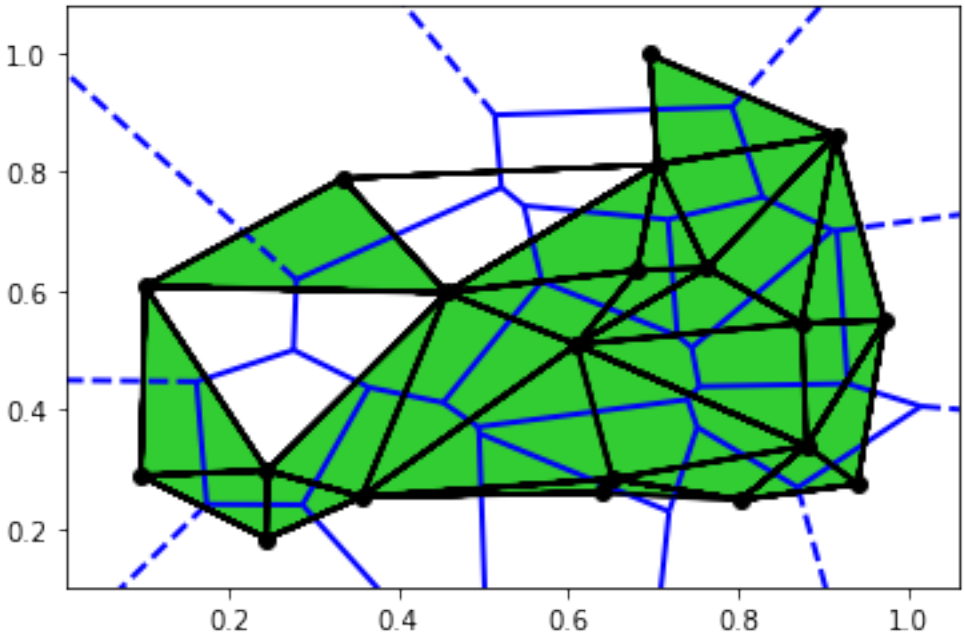

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

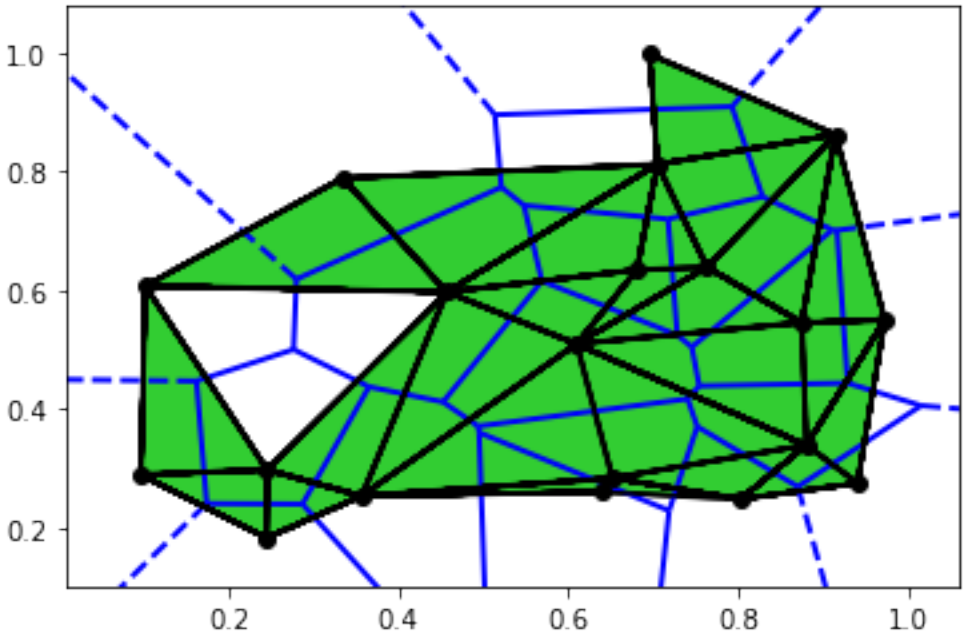

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

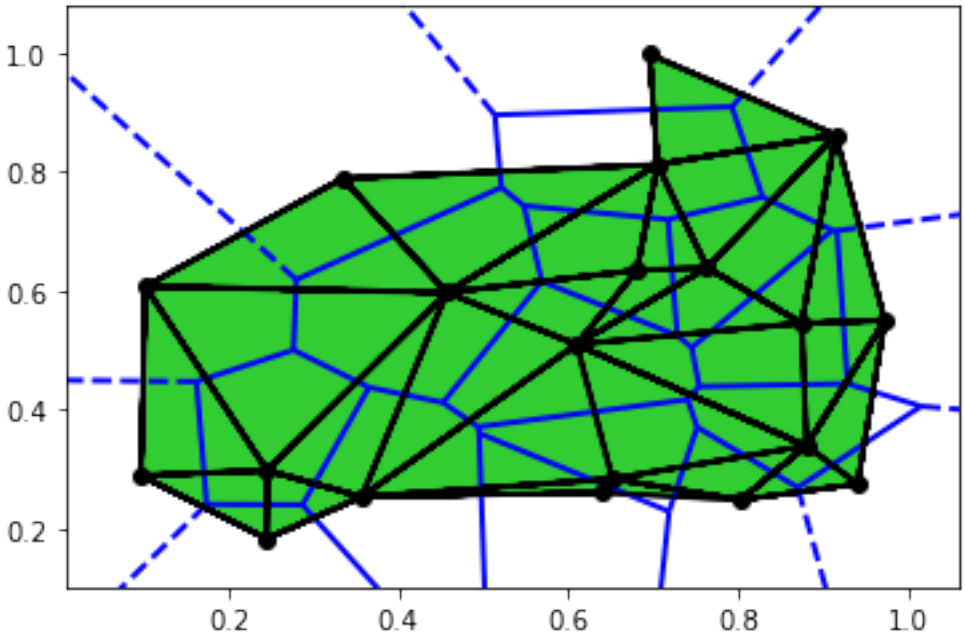

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

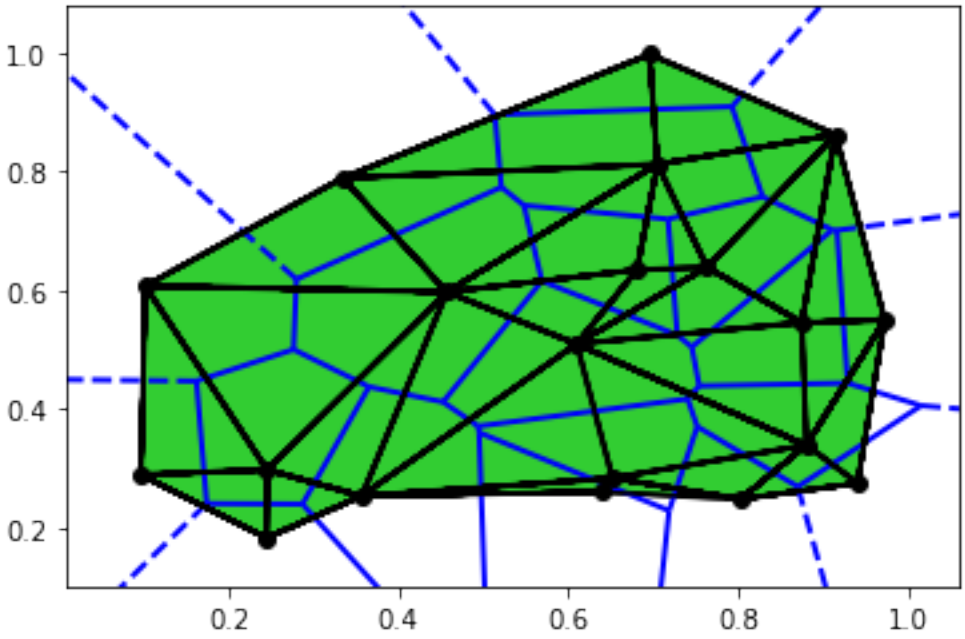

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

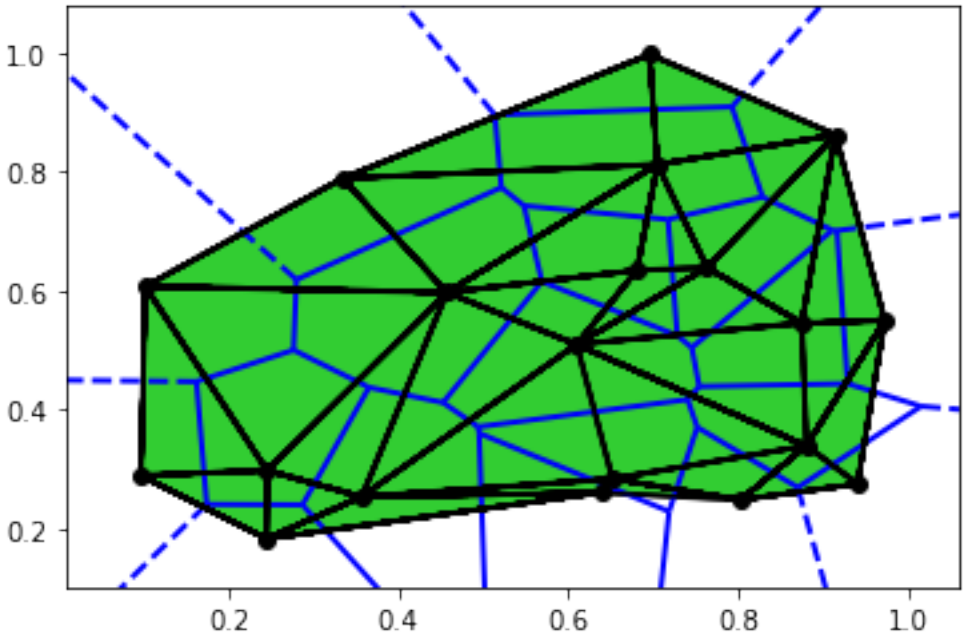

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

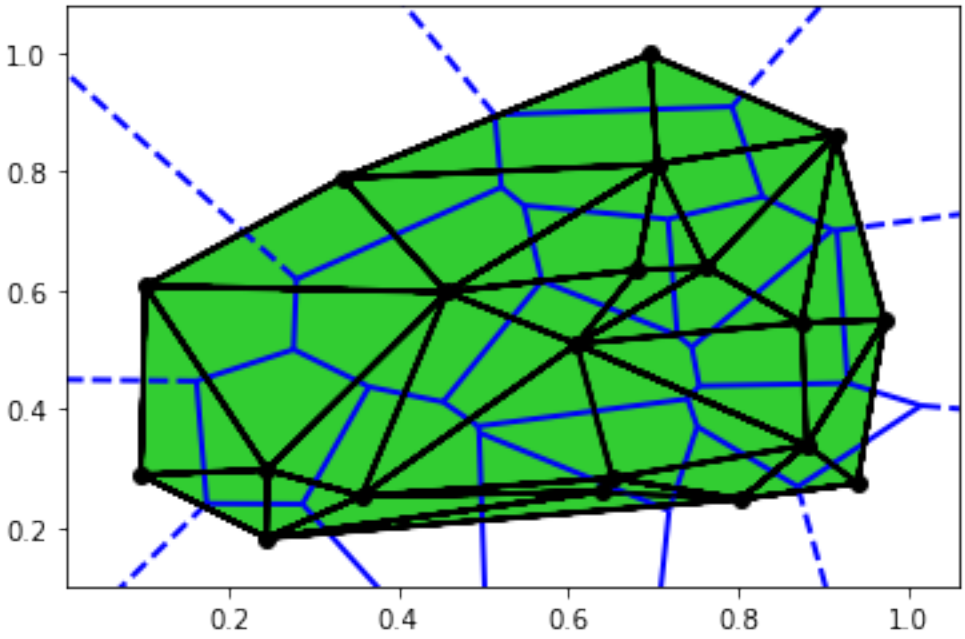

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

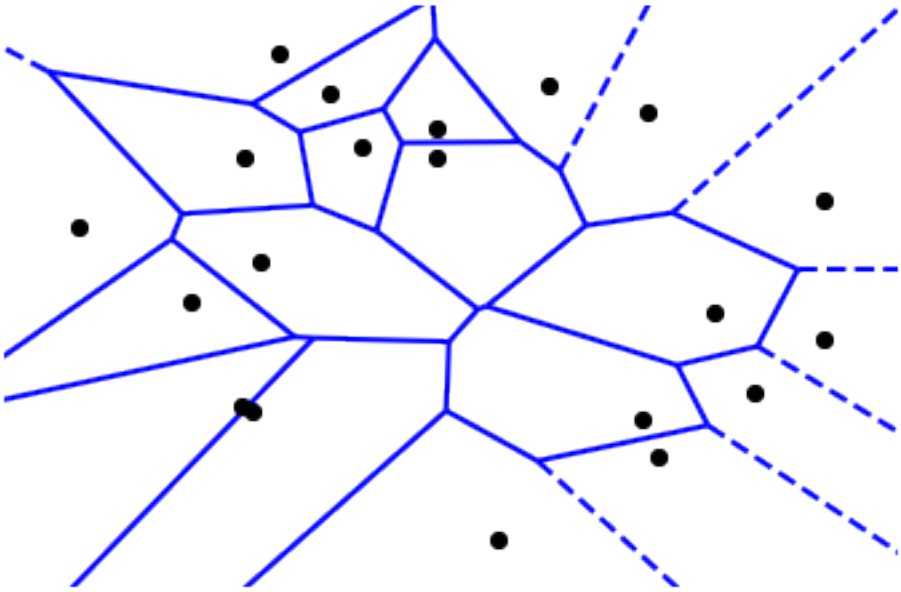

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0


1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

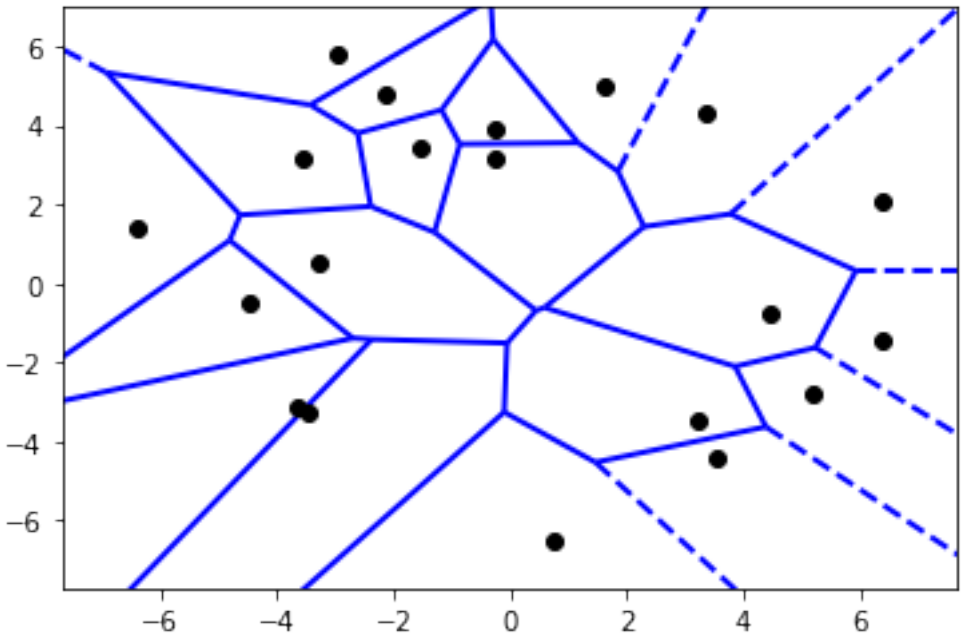
4 Ejemplo 4

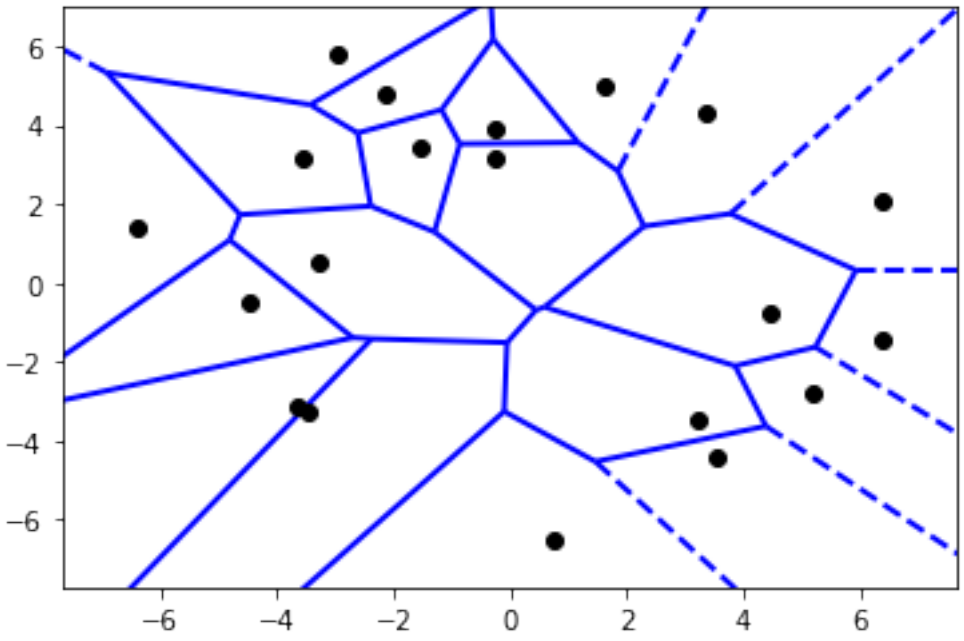
[93]:
```
circle = np.array([[ 0.7649936 , -6.49105706],
       [-0.26047978,  3.17414802],
       [ 5.16486466, -2.77709227],
       [ 6.3630621 ,  2.08442442],
       [ 1.63681198,  4.95671697],
       [-4.47103343, -0.4944843 ],
       [-2.12035066,  4.80887876],
       [-3.44986384, -3.25828704],
       [-3.55428879,  3.18415674],
       [-0.27202076,  3.89295058],
       [-2.9512385 ,  5.76602403],
       [ 6.36747098, -1.44195299],
       [ 3.36400365,  4.33230353],
       [ 3.23972602, -3.44494391],
       [-3.29551494,  0.50827386],
       [-1.55188576,  3.42645393],
       [-3.62672644, -3.14264111],
       [ 3.54177077, -4.42486894],
       [-6.39734363,  1.37489294],
       [ 4.46578318, -0.76225718]])
vor=Voronoi(circle)
fig = voronoi_plot_2d(vor,show_vertices=False,line_width=2,line_colors='blue')
plt.plot(circle[:,0], circle[:,1], 'ko')
plt.axis('off')
plt.show()
```

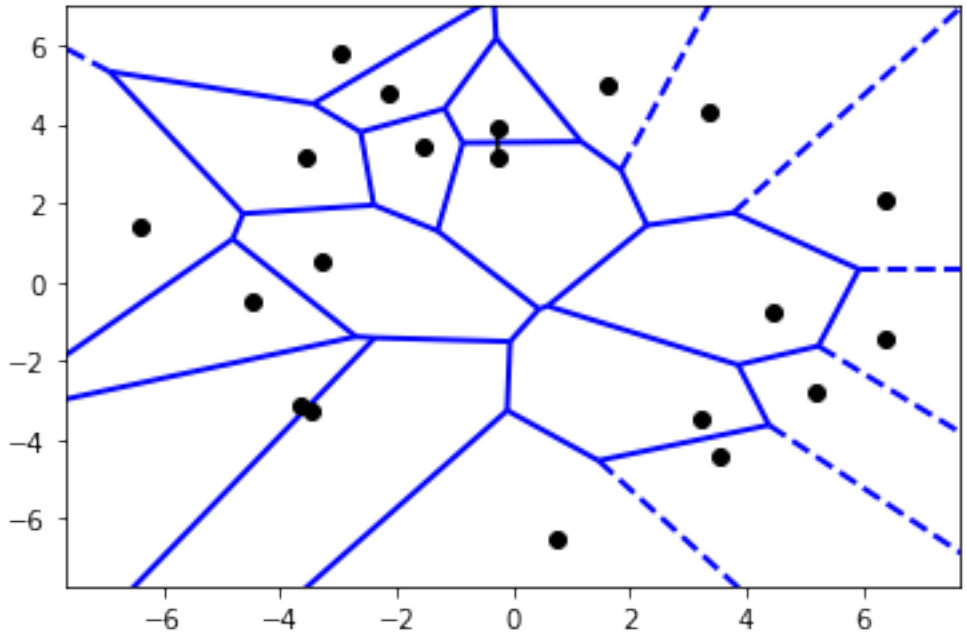


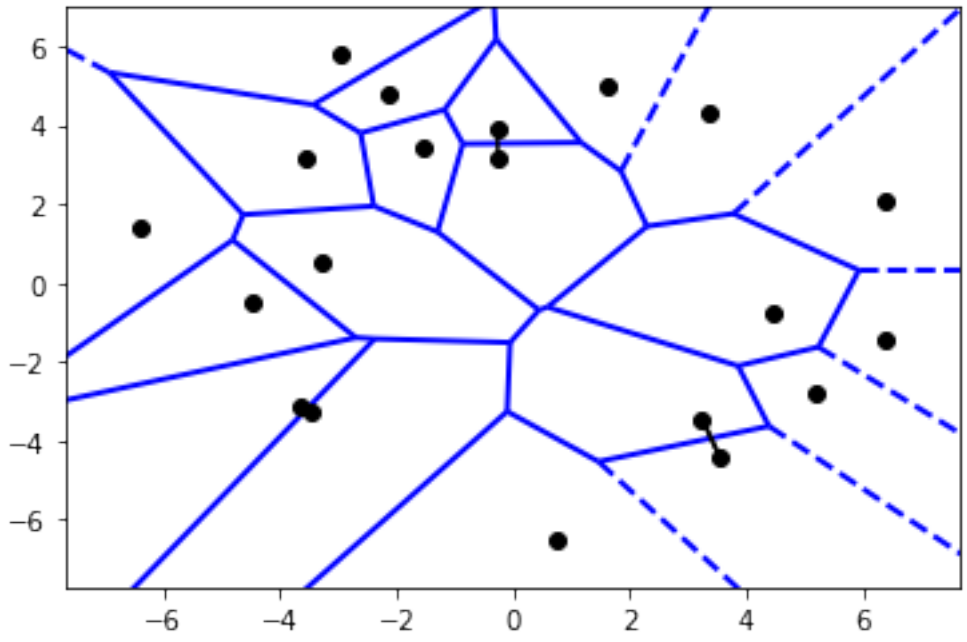
[86]:
```
alphacircle=AlphaComplex(circle)
```

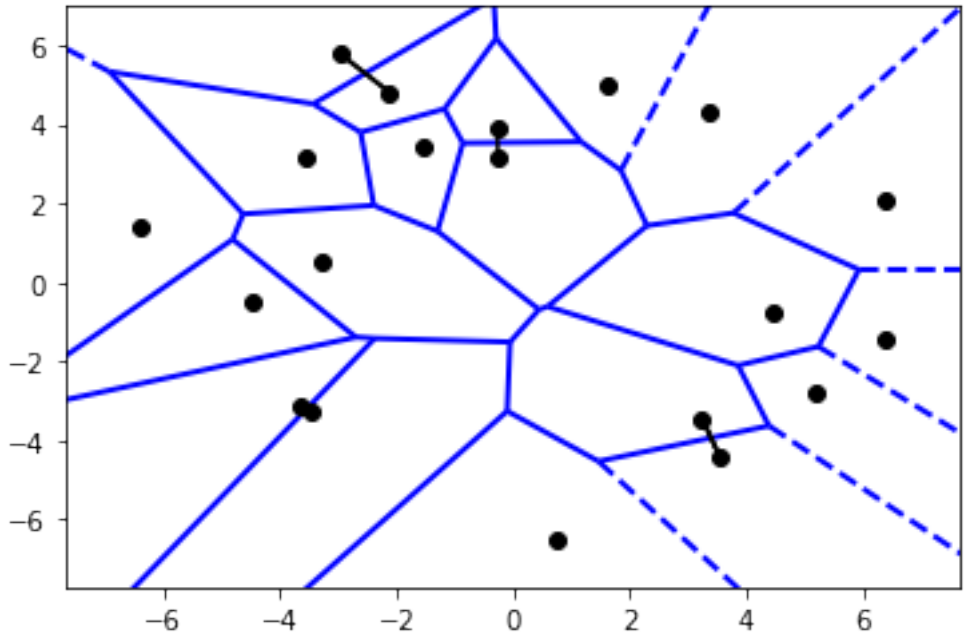
[90]:
```
thresholds=alphacircle.thresholdvalues()
for value in thresholds:
    K=alphacircle.sublevel(value)
    plotalpha(circle,K)
    plt.show()
```

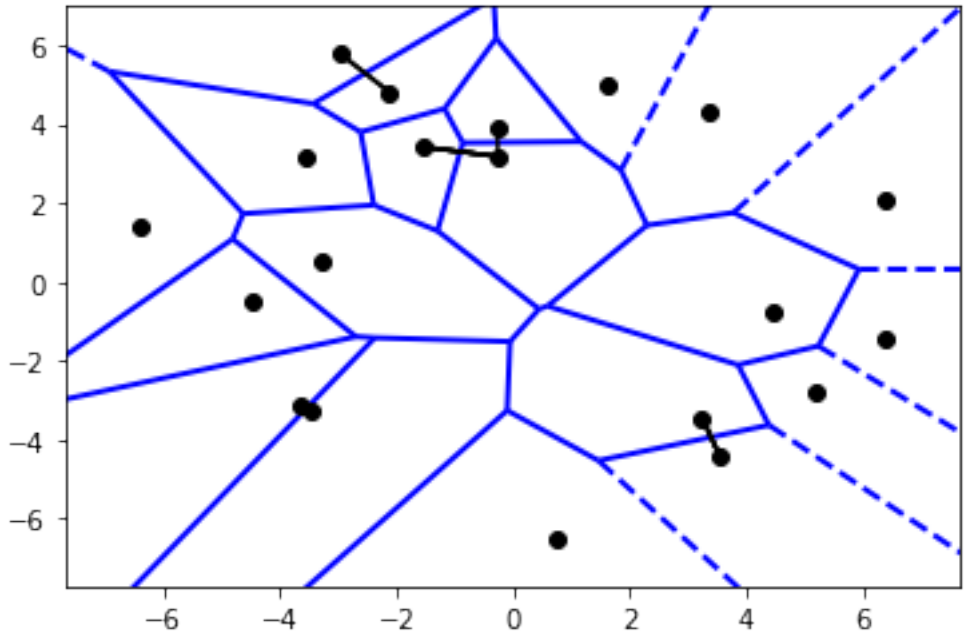


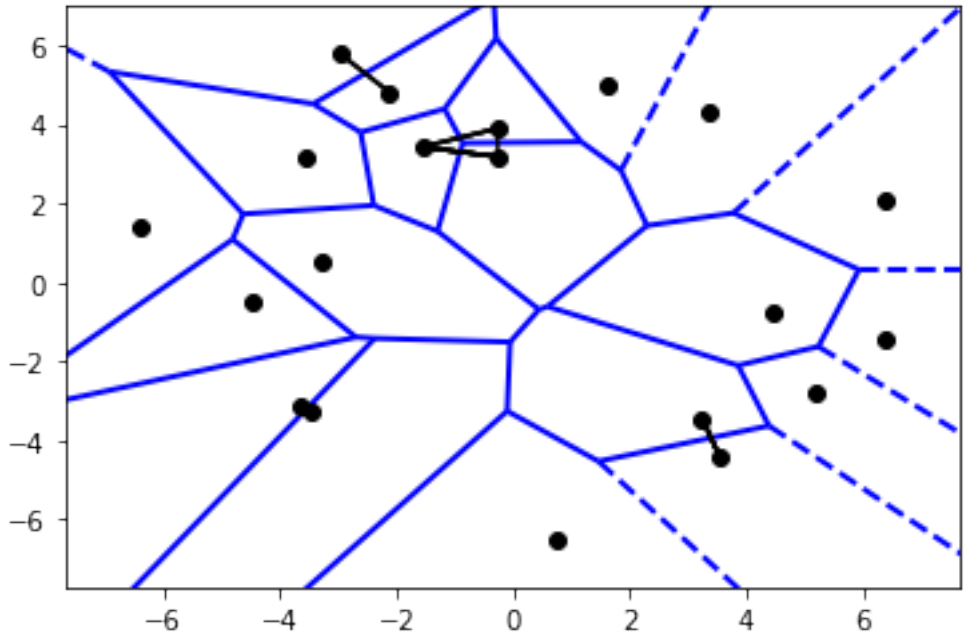


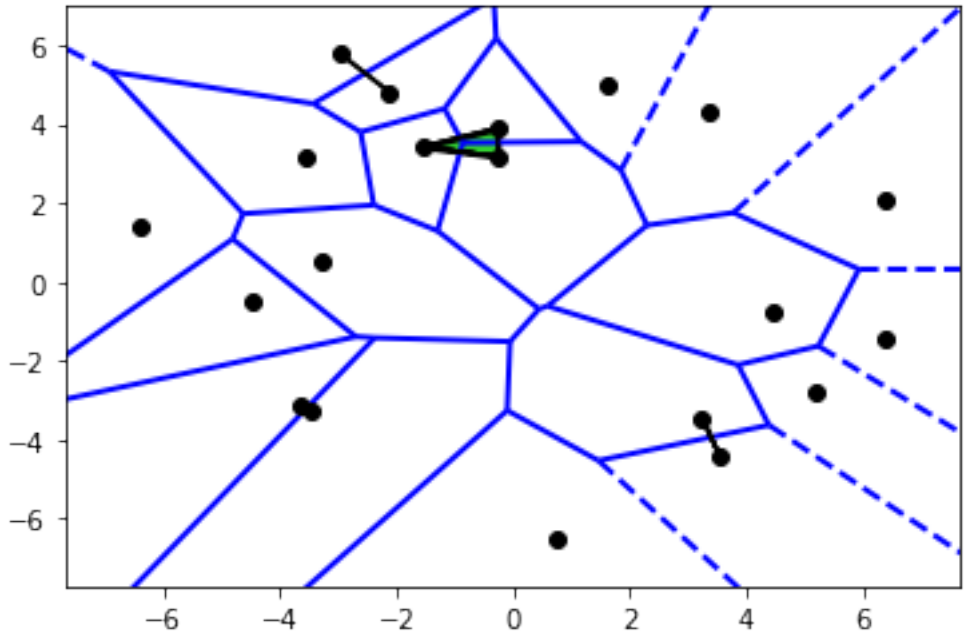

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

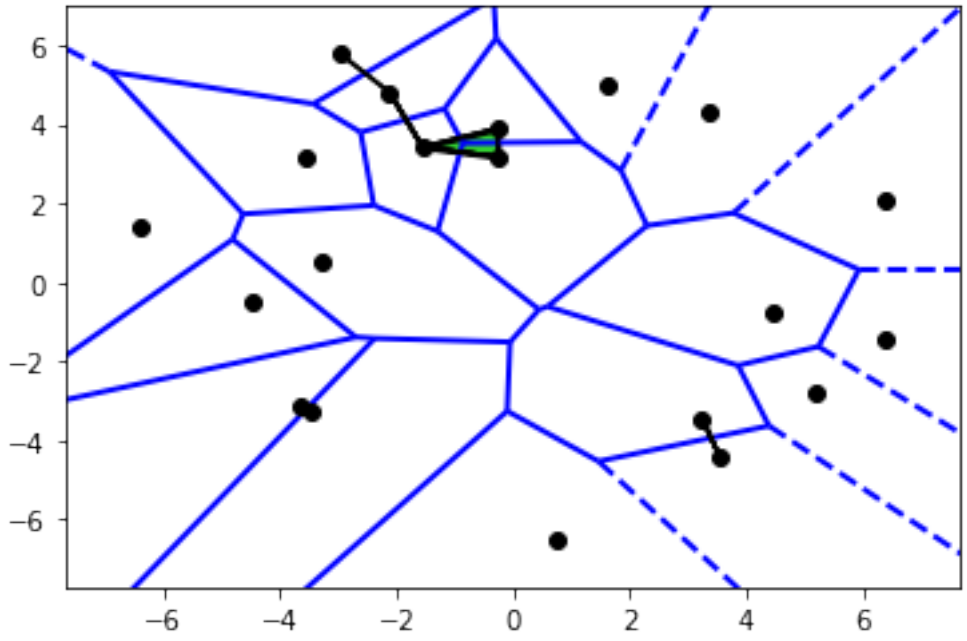

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

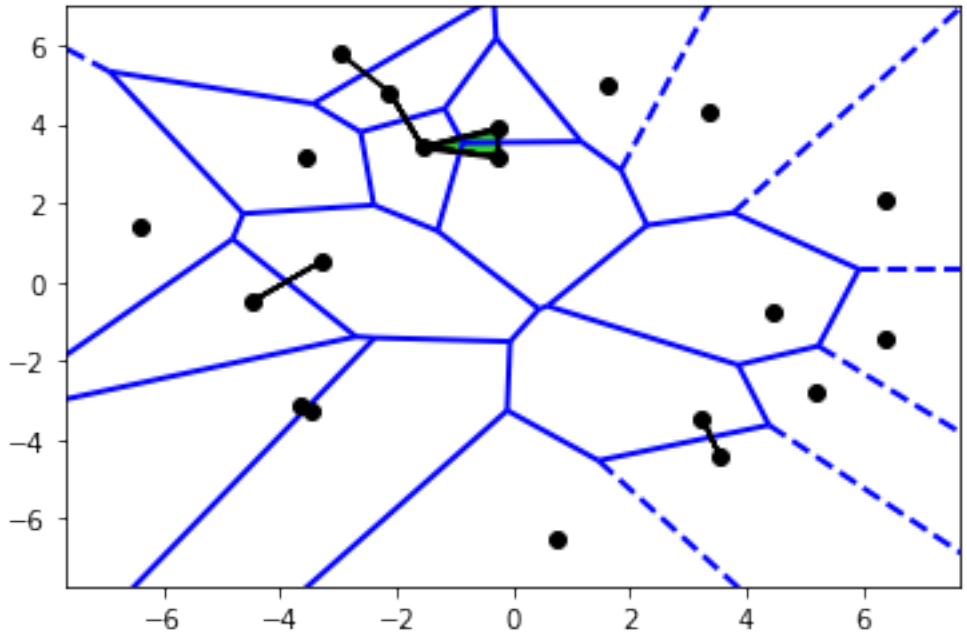

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

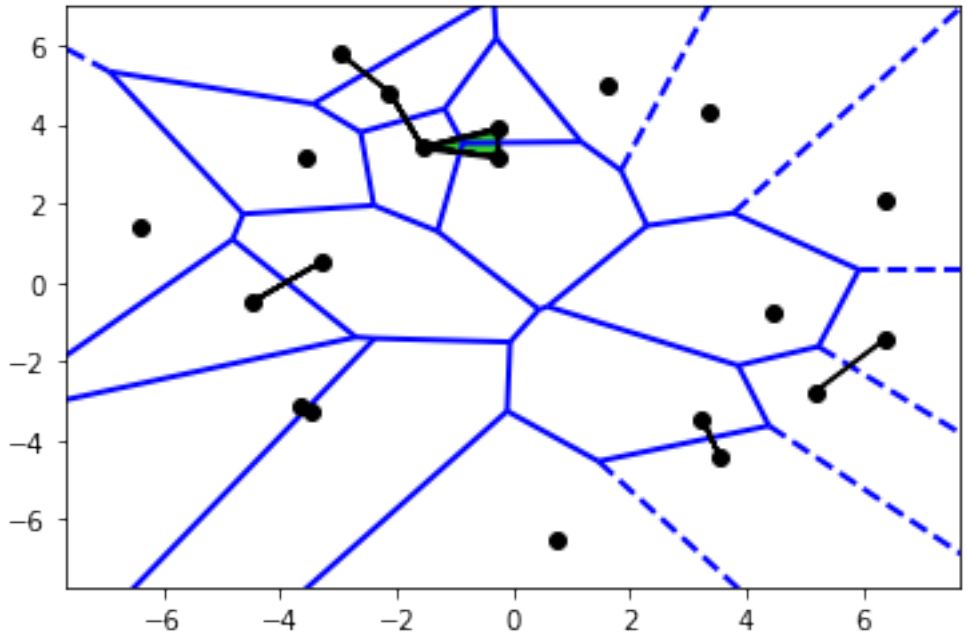

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

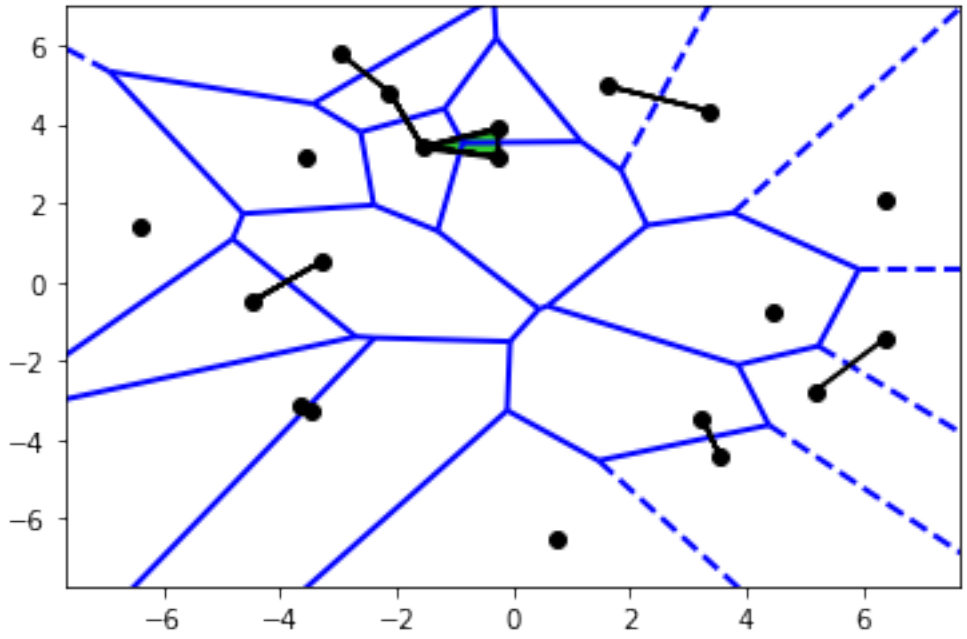

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

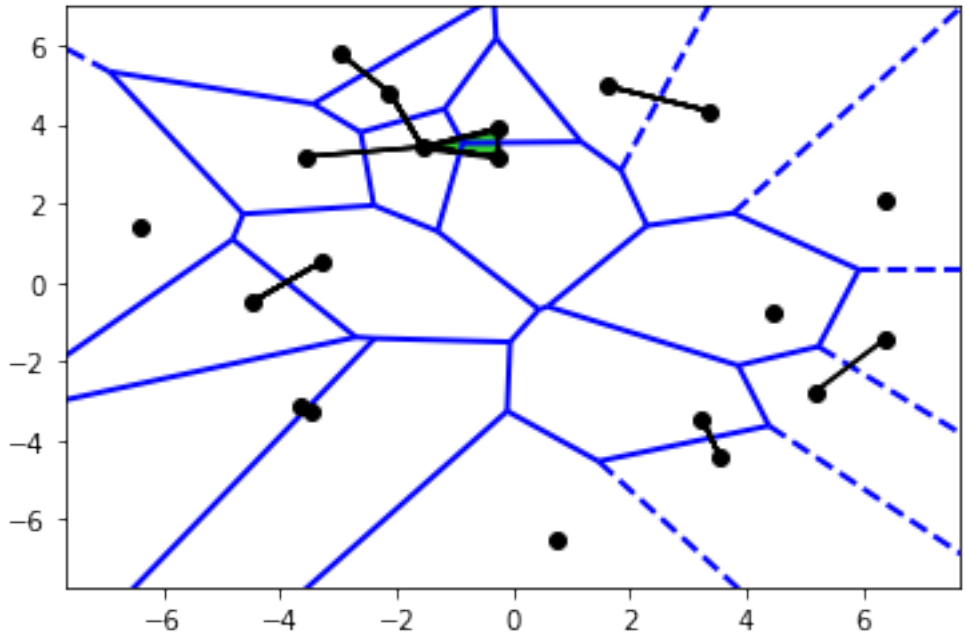

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

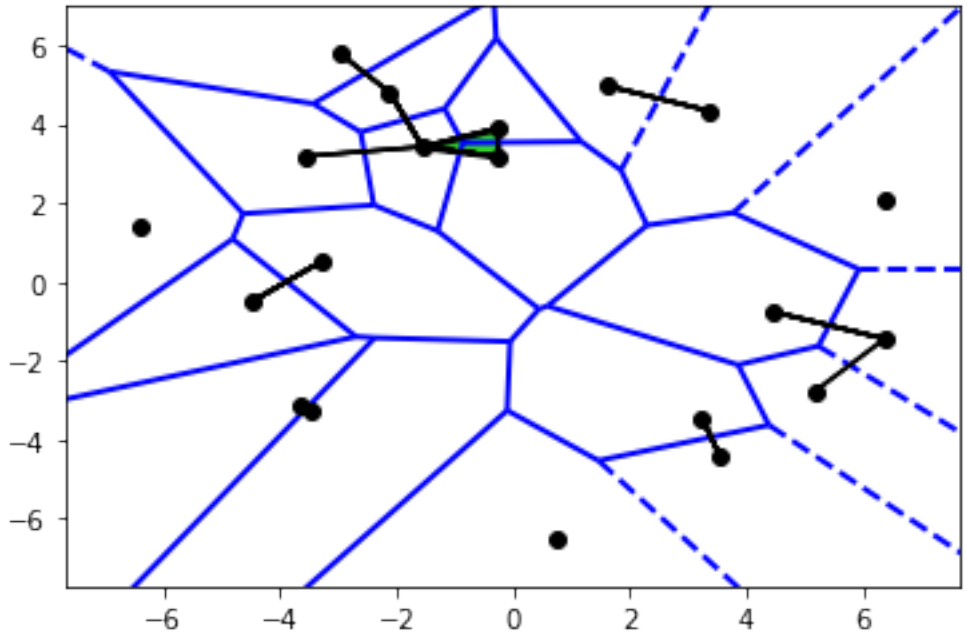

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

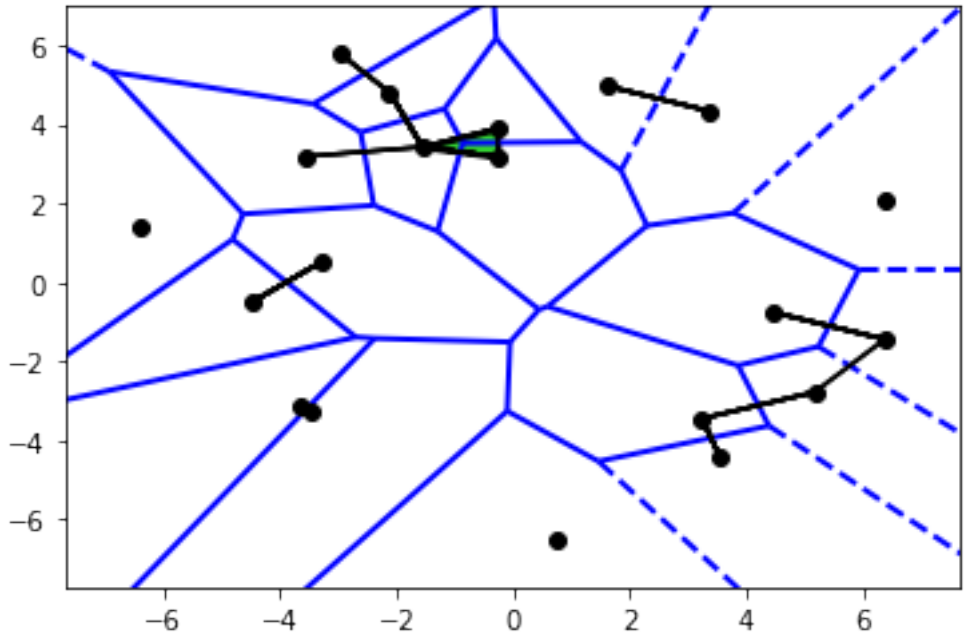

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

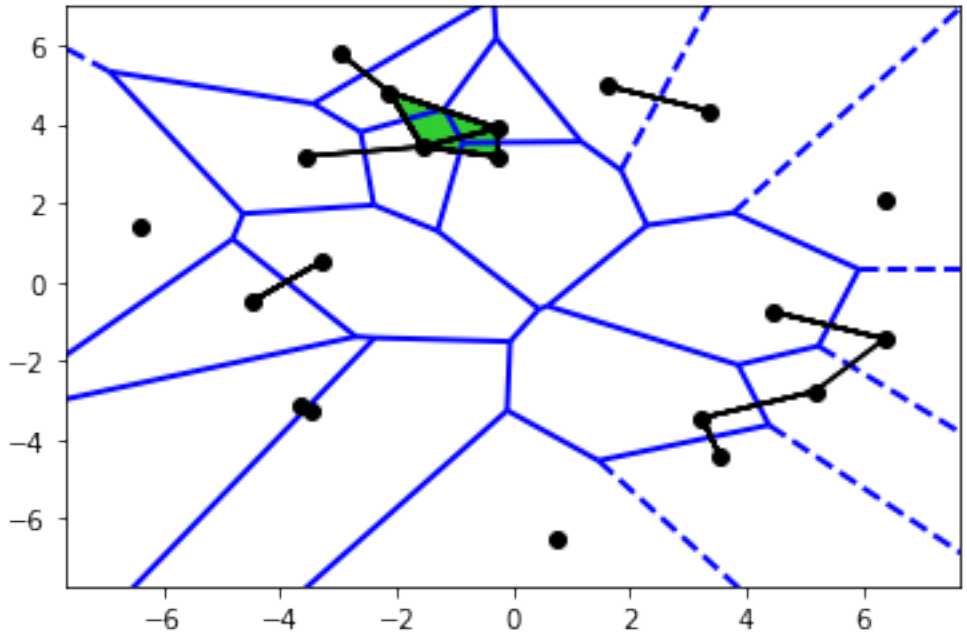

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

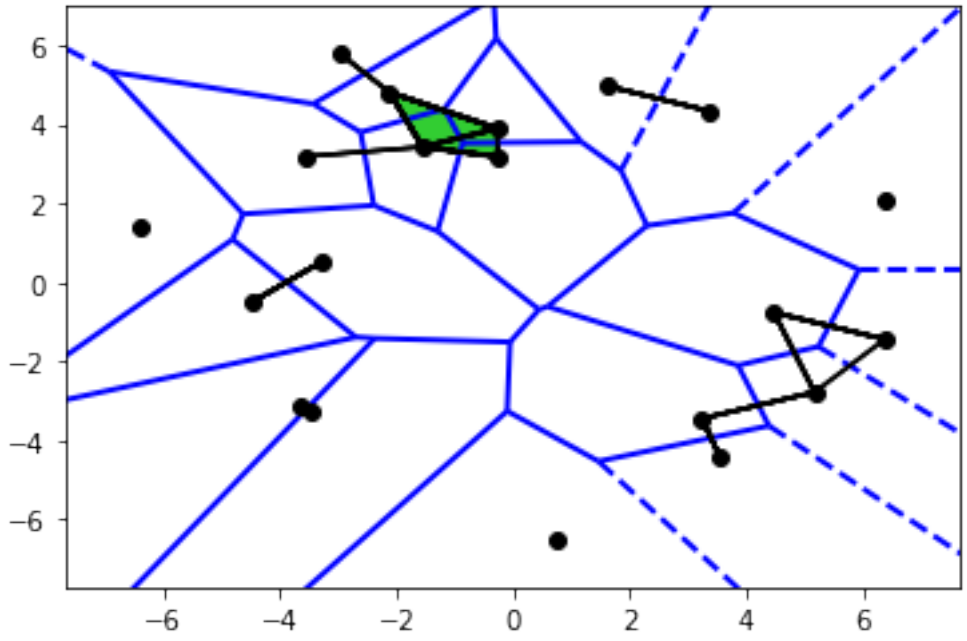

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

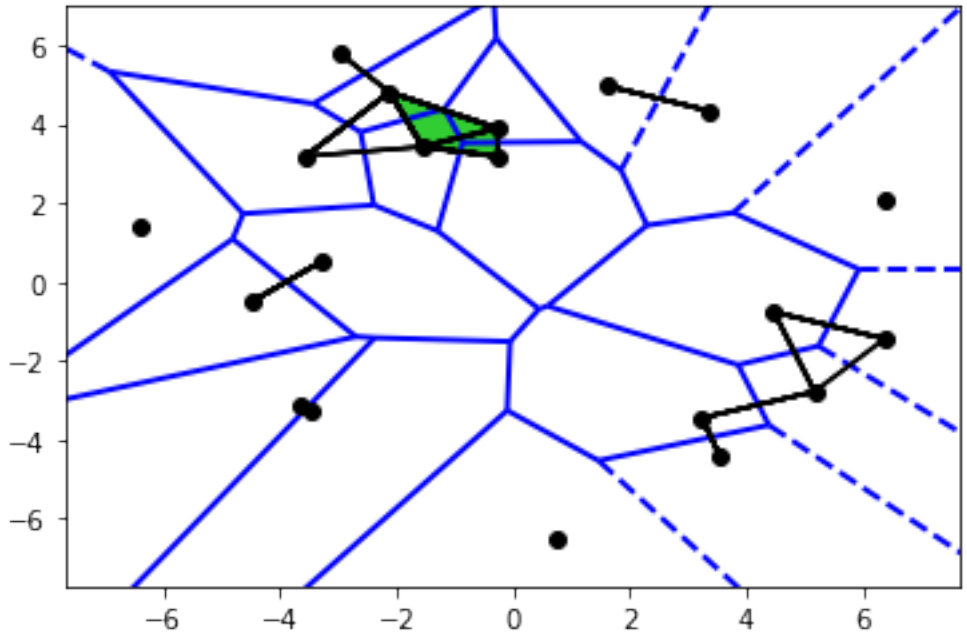

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

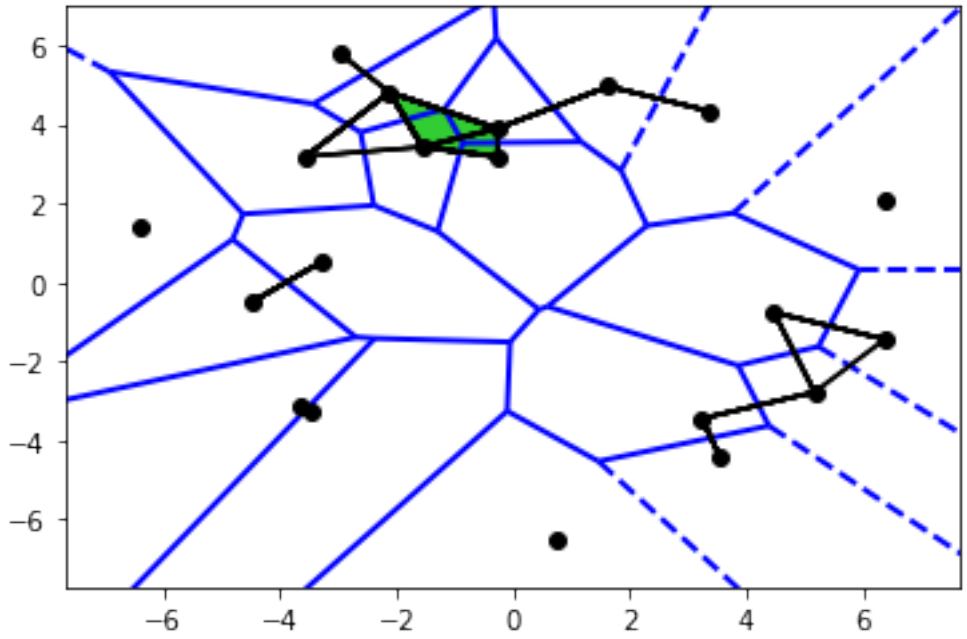

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

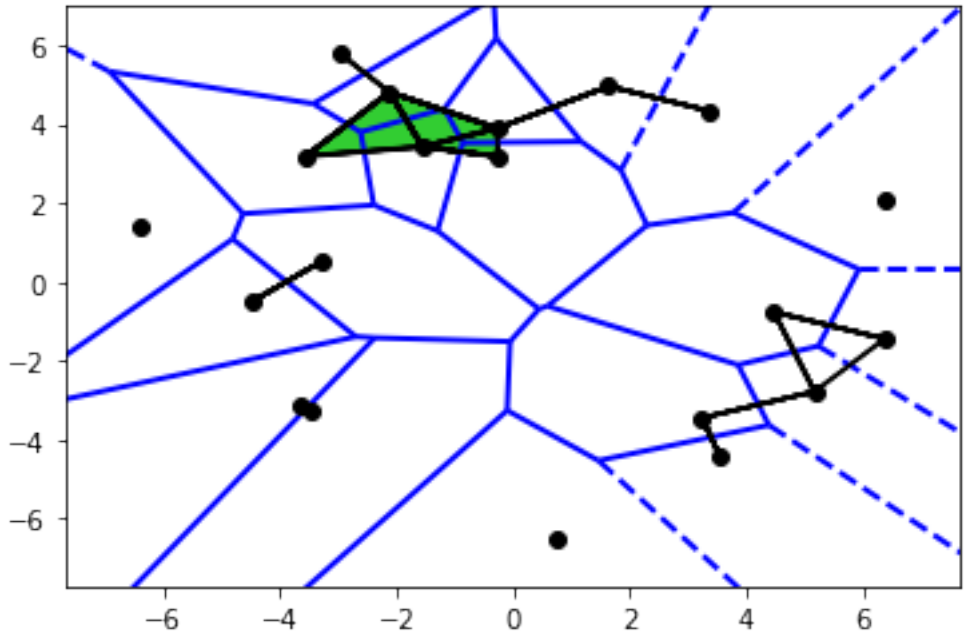

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

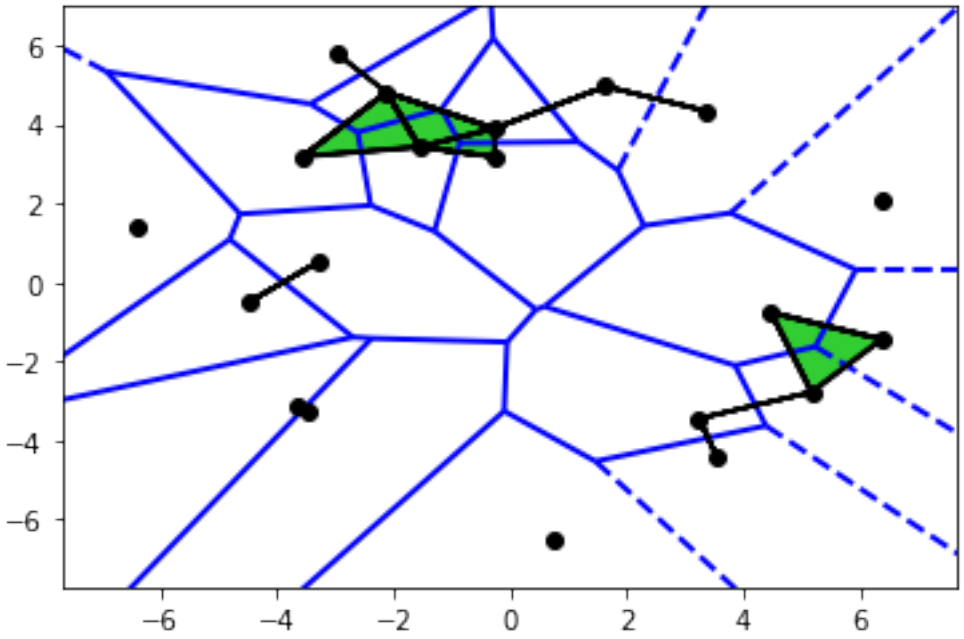

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

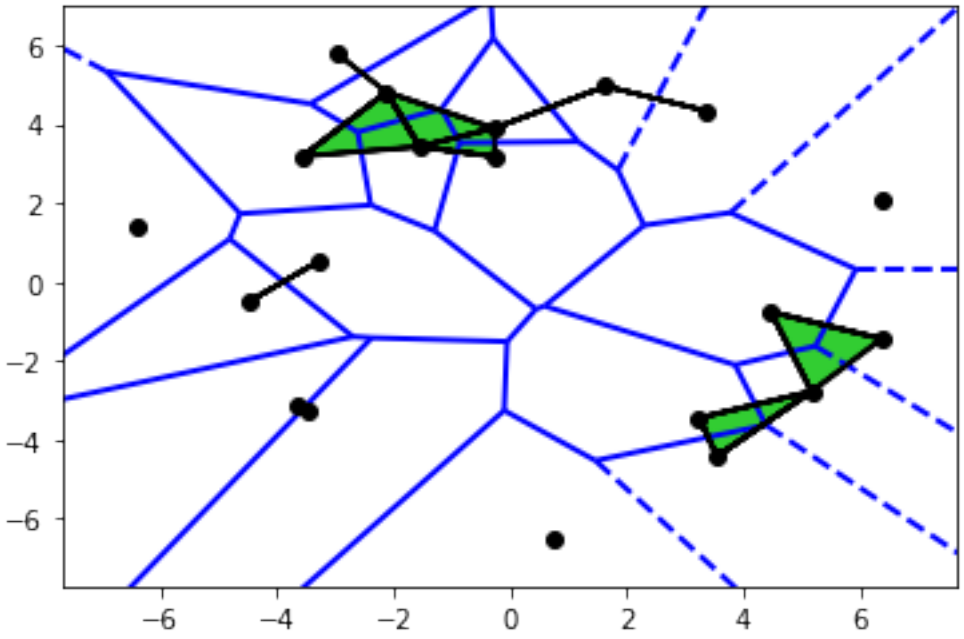

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

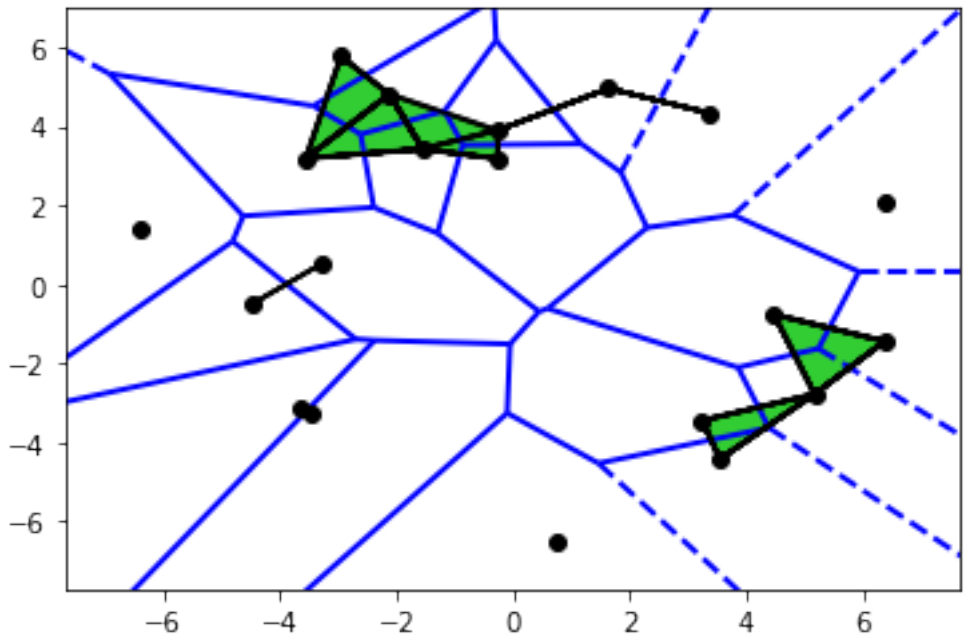

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

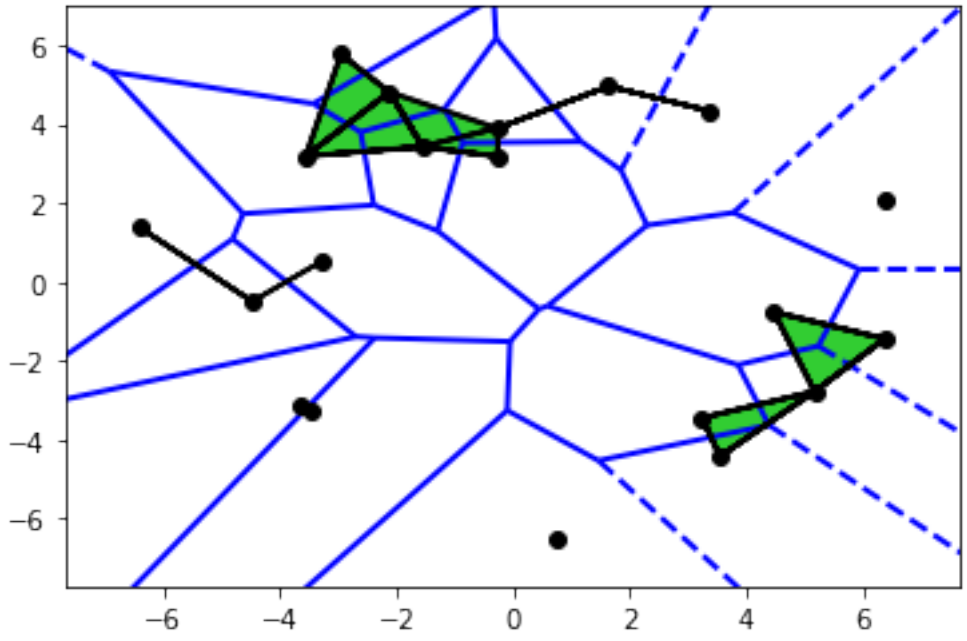

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

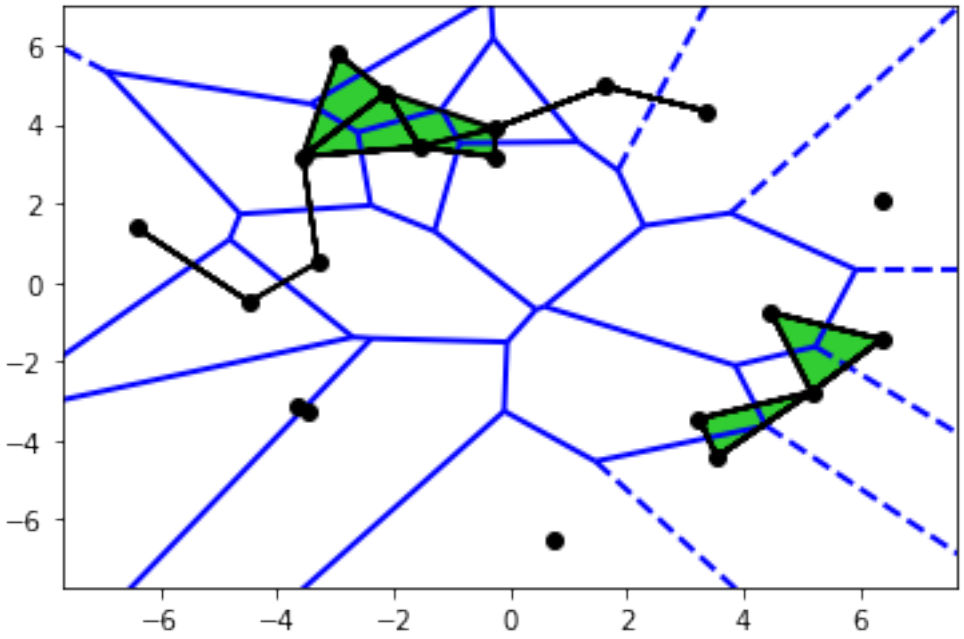

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

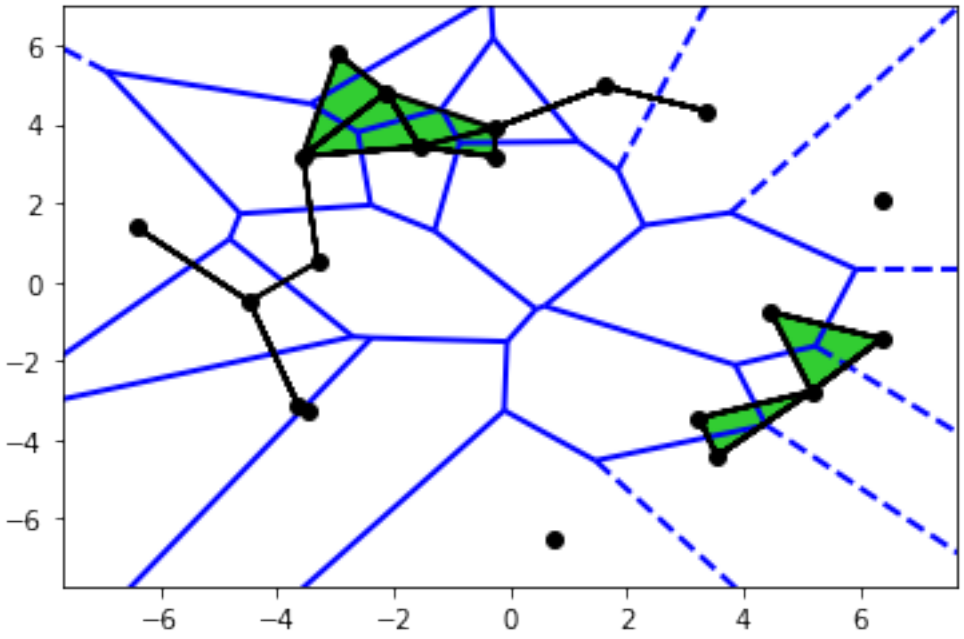

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

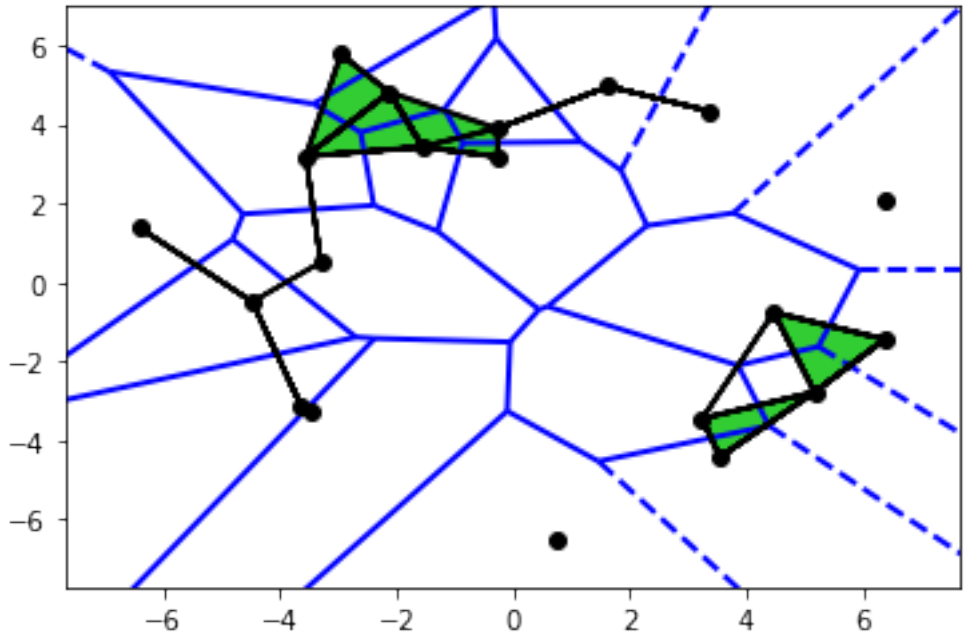

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

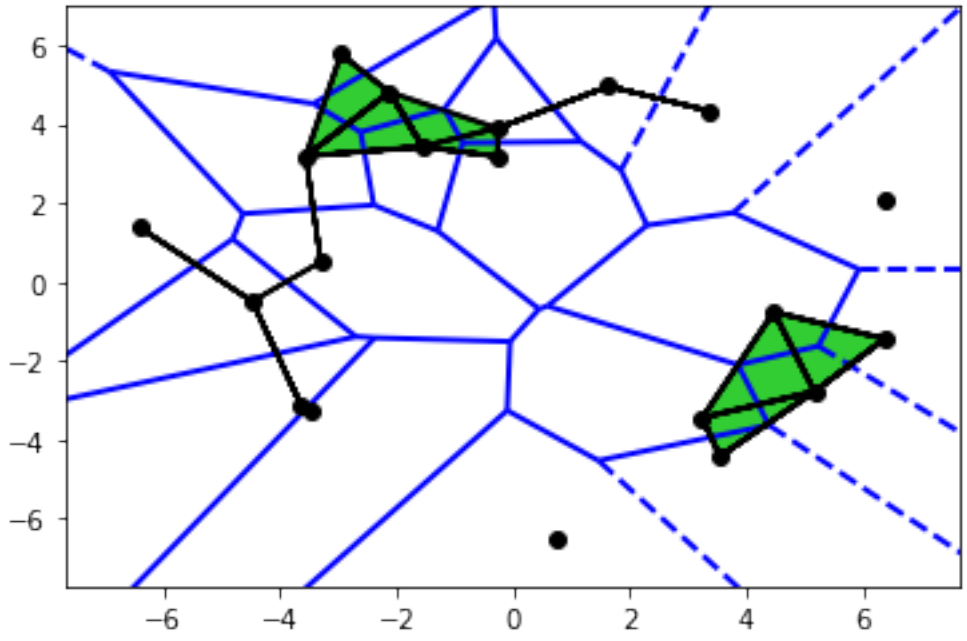

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

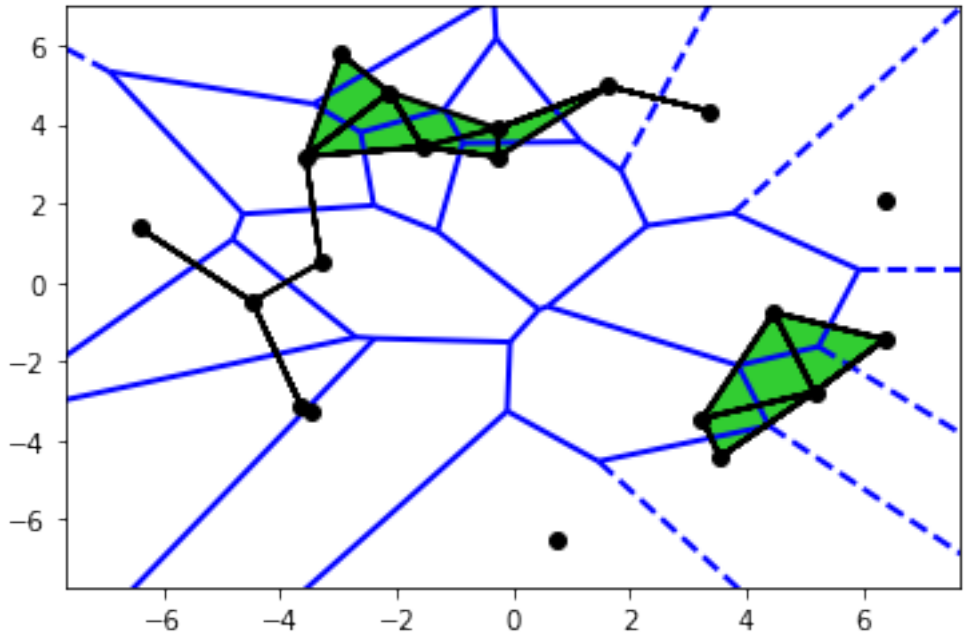

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

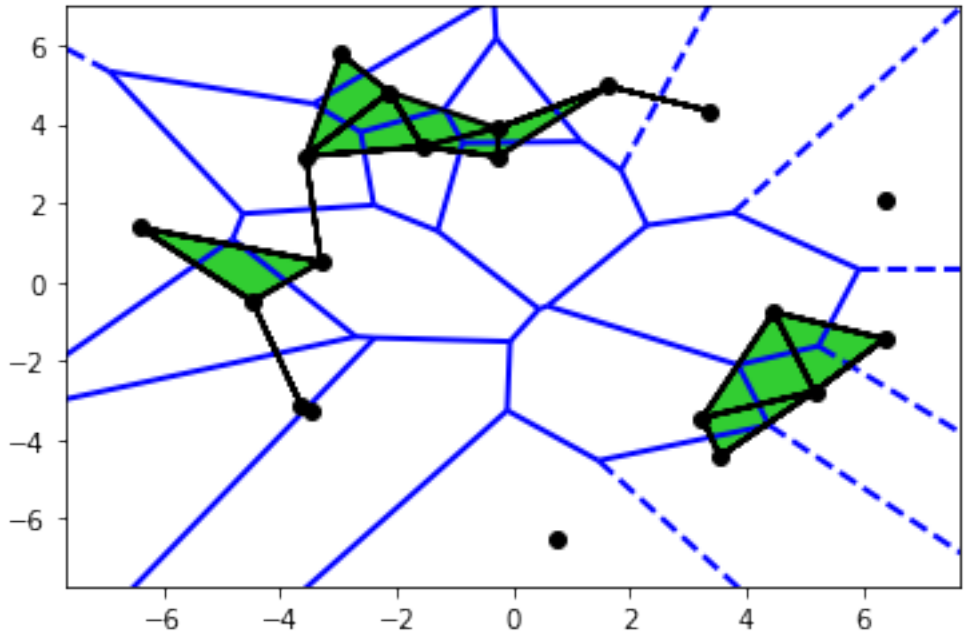

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

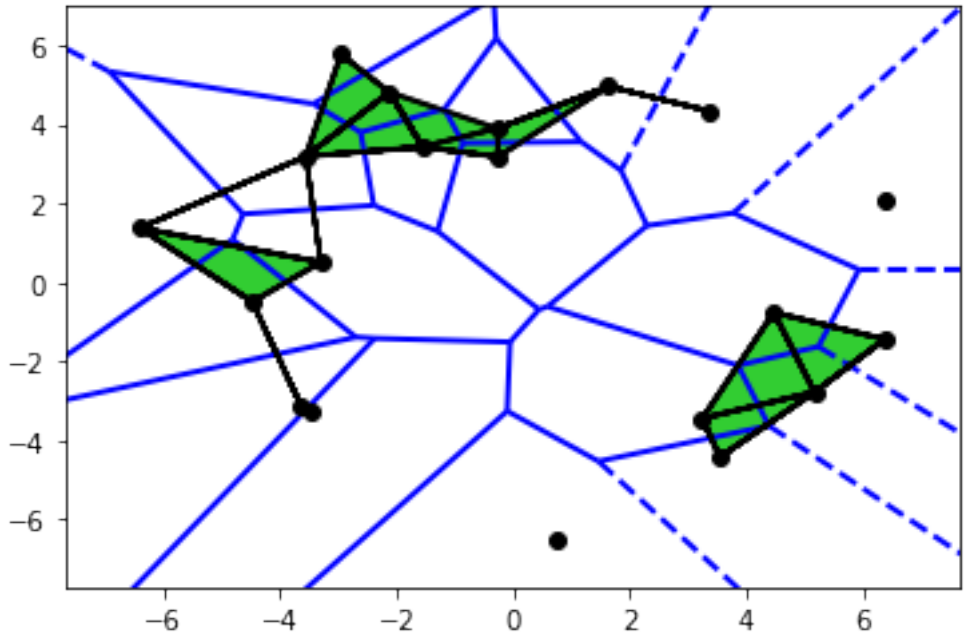

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

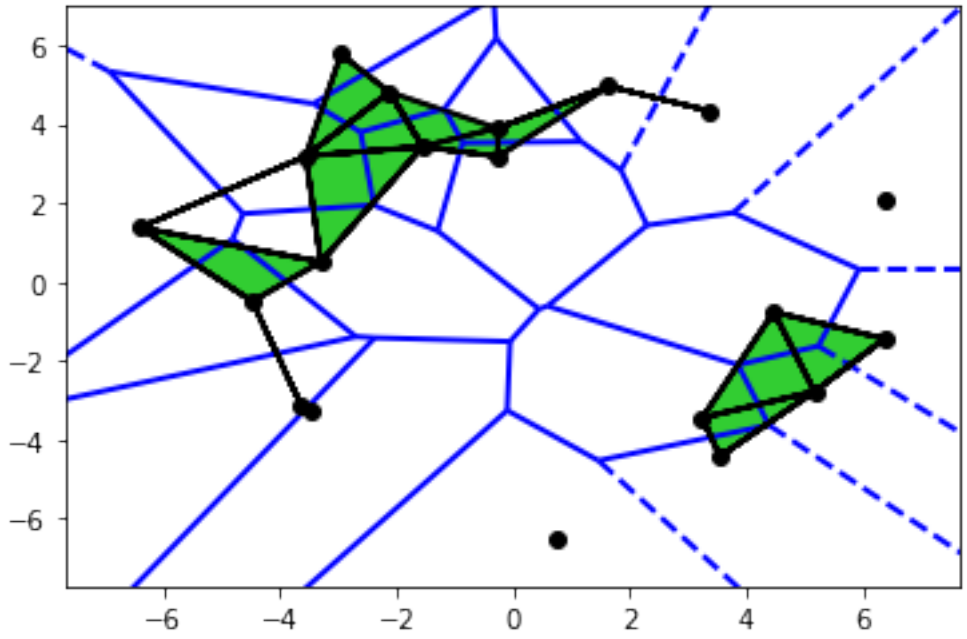

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

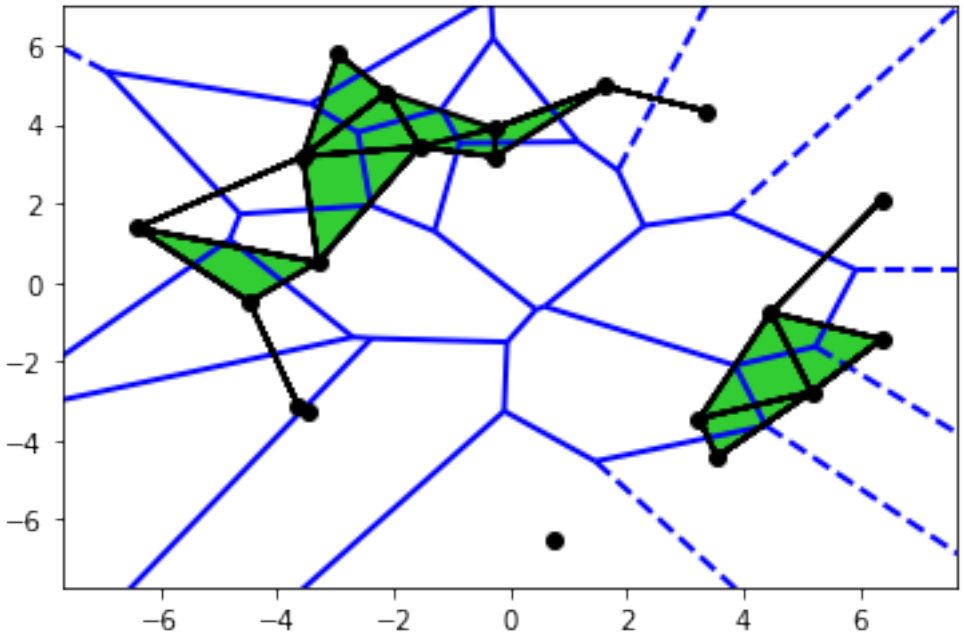

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

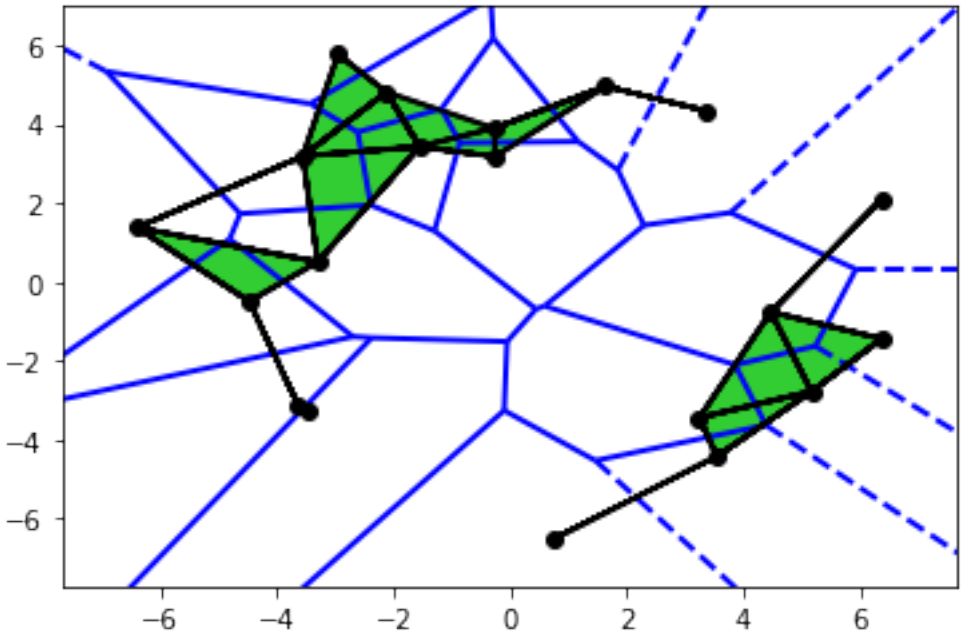

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

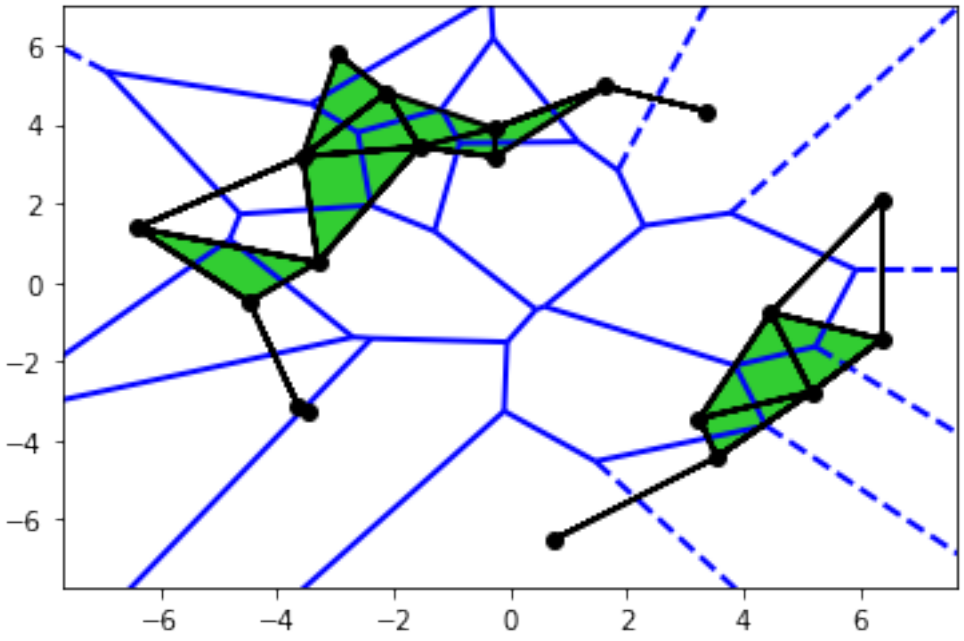

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

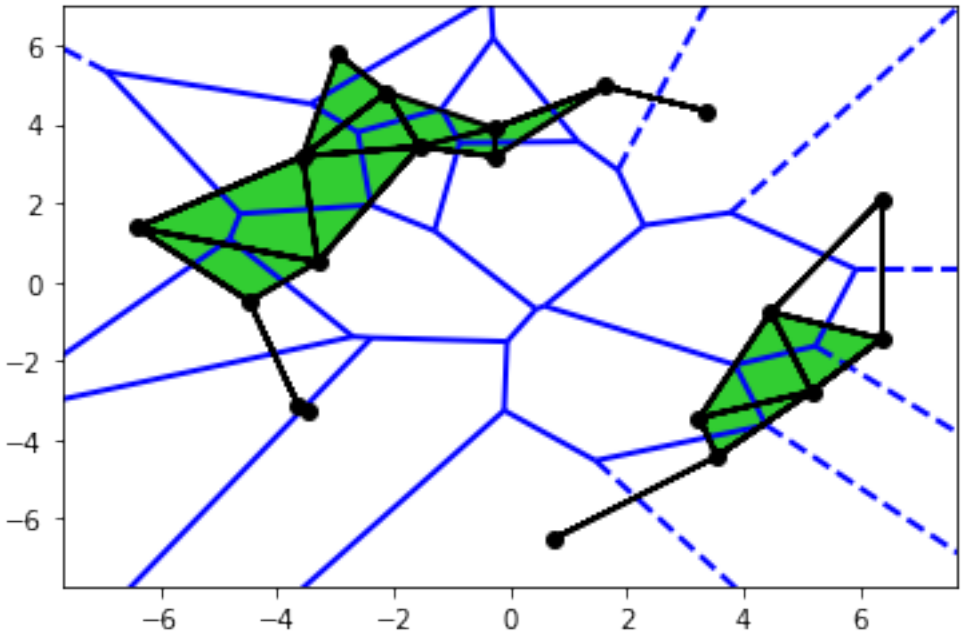

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

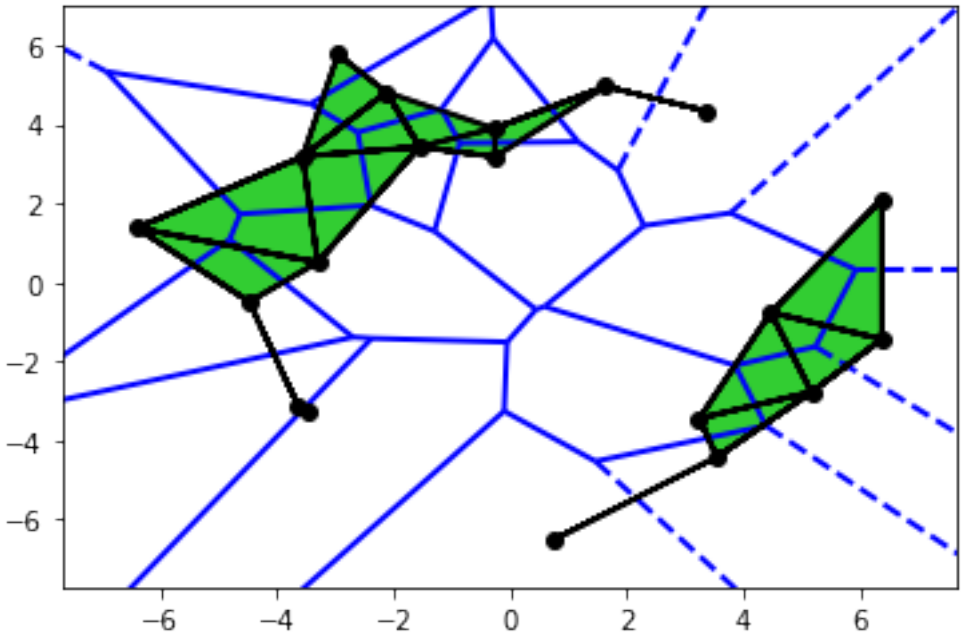

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

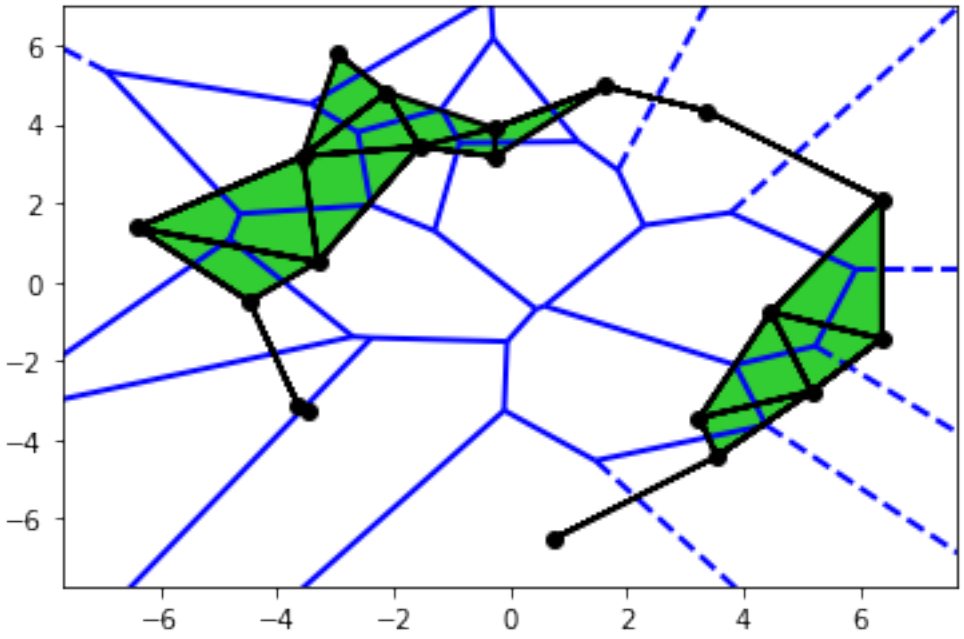

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

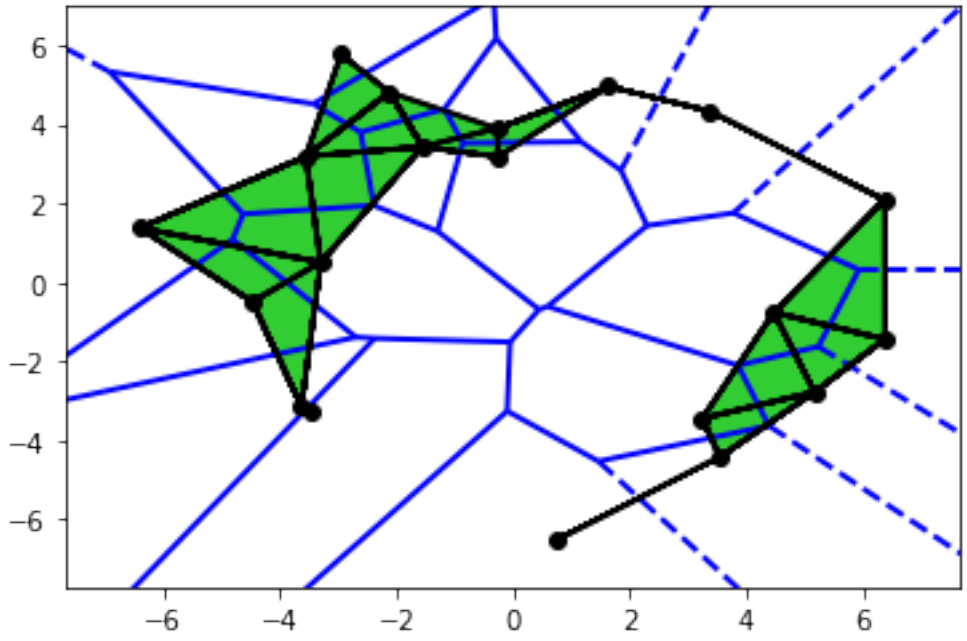

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

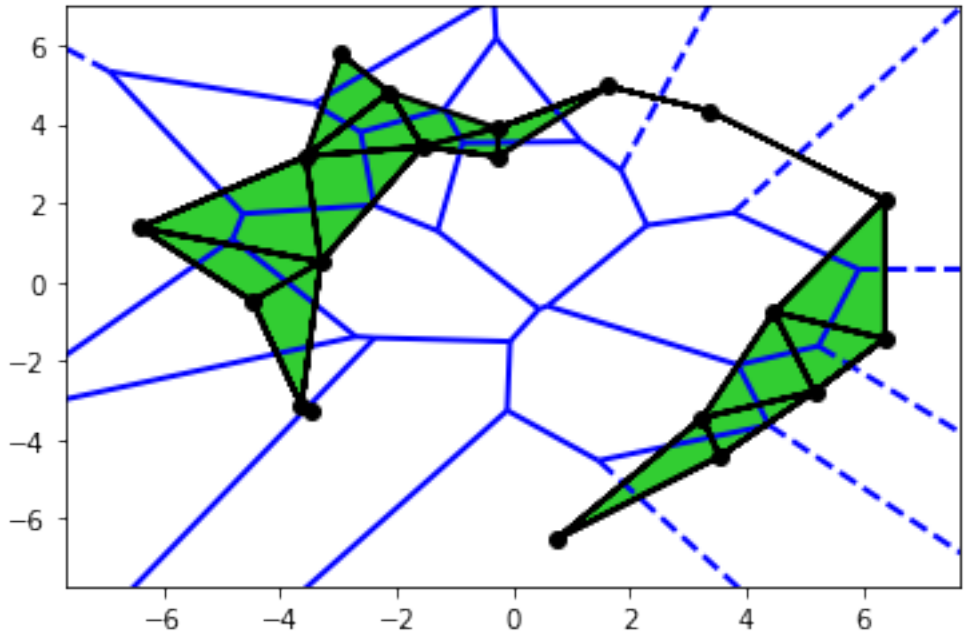

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

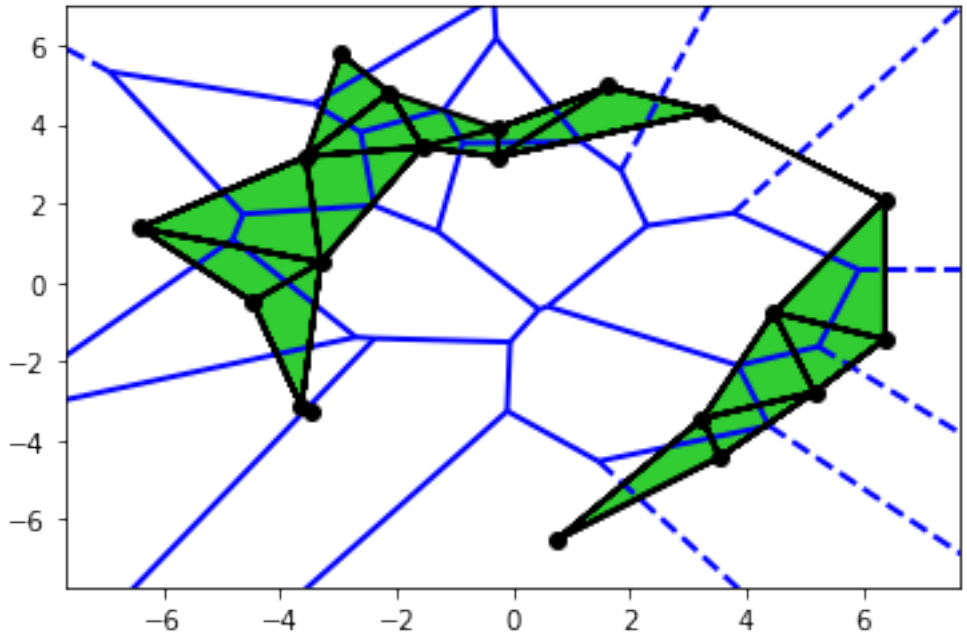

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

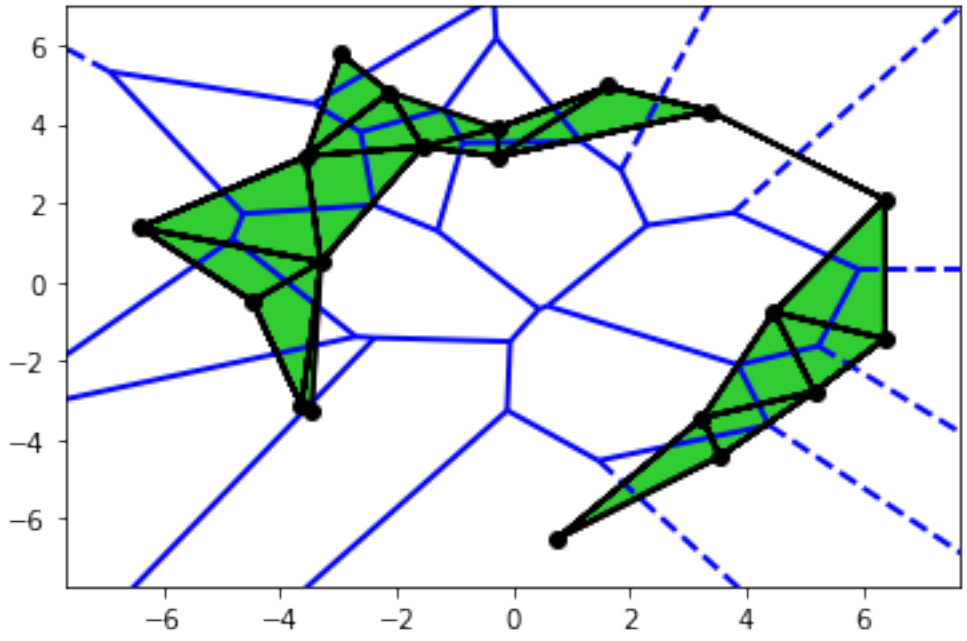

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

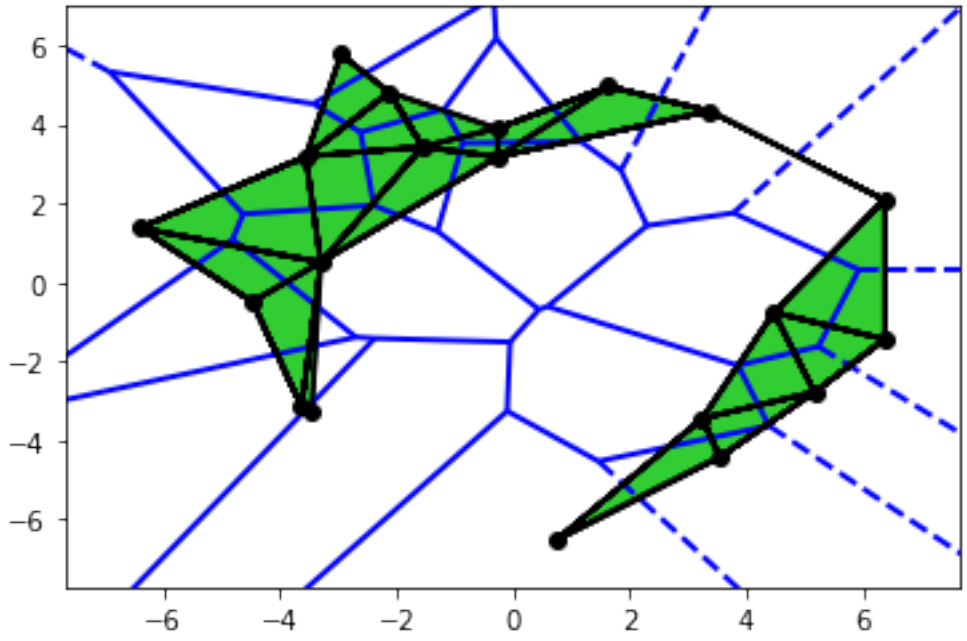

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

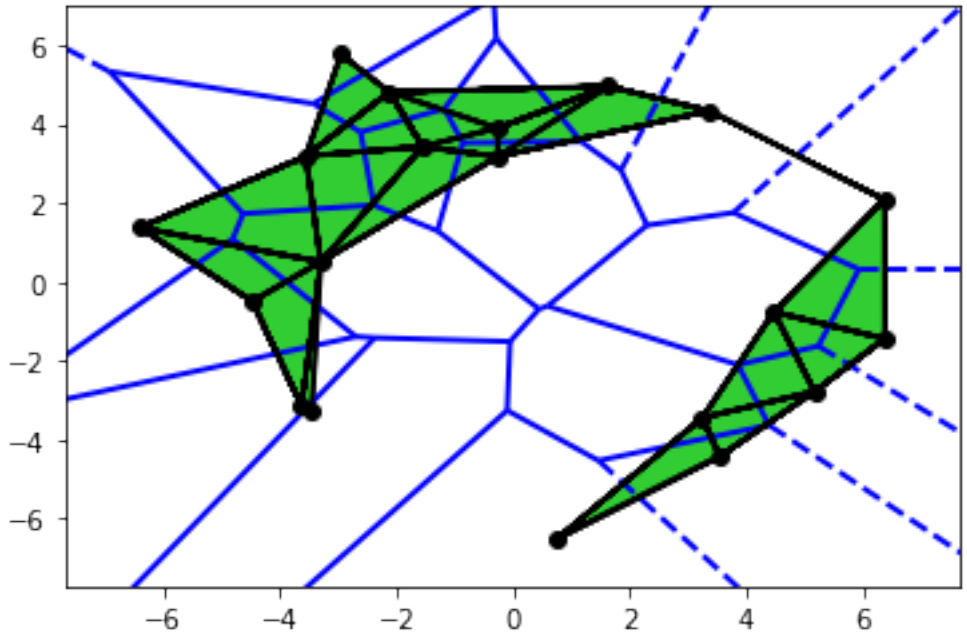

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

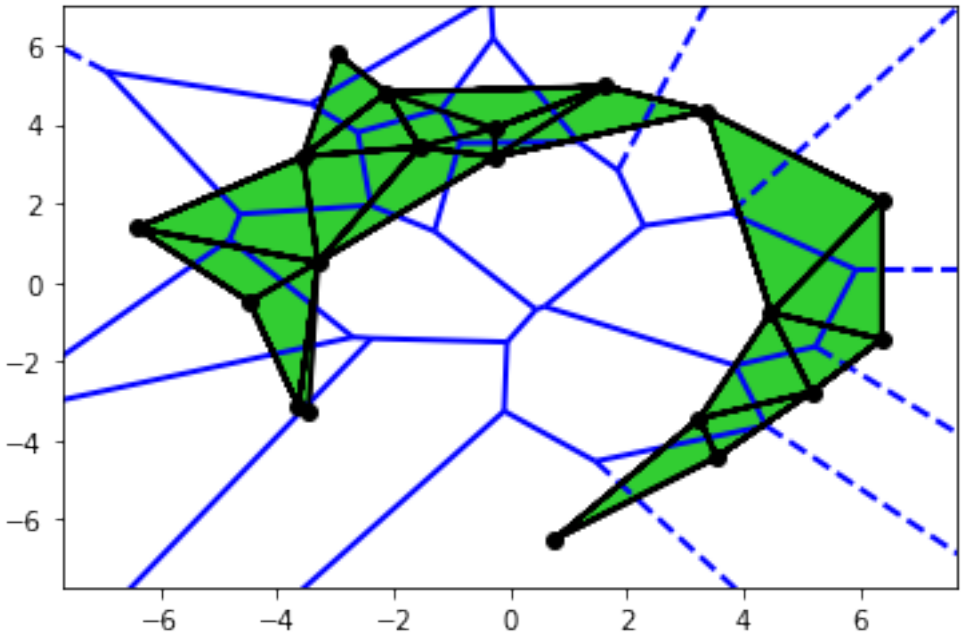

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

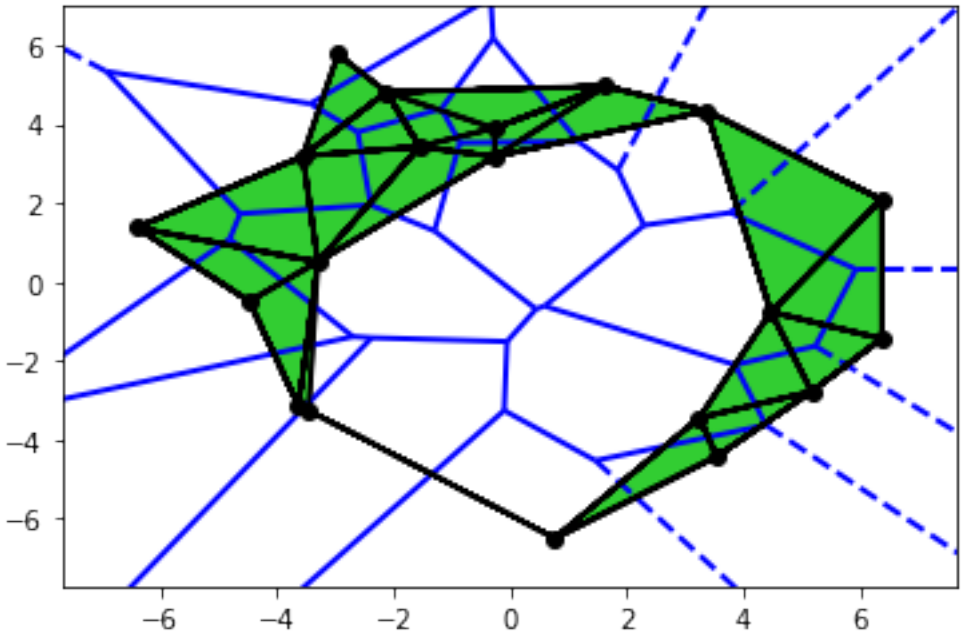

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

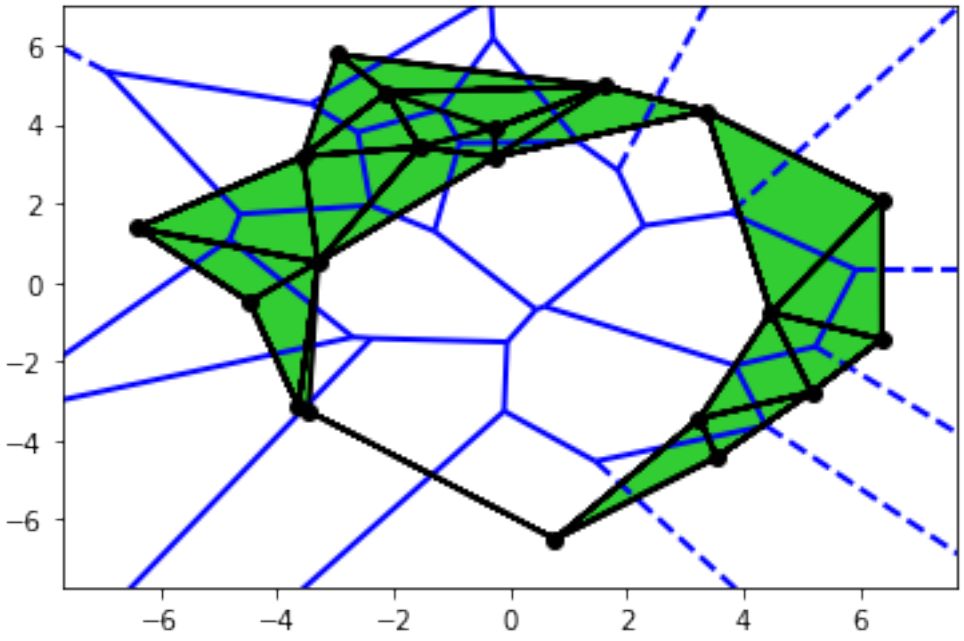

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

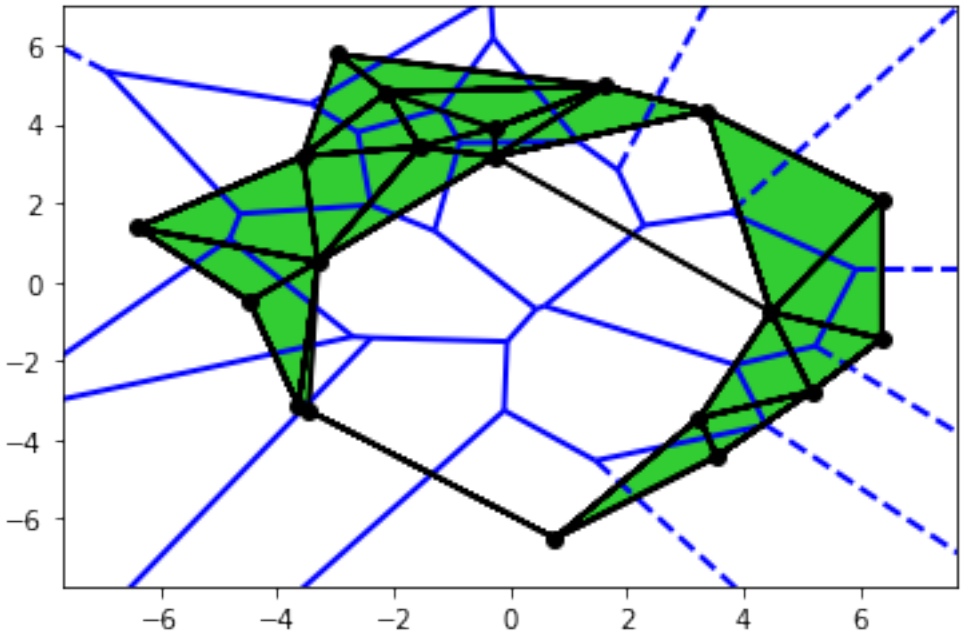

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

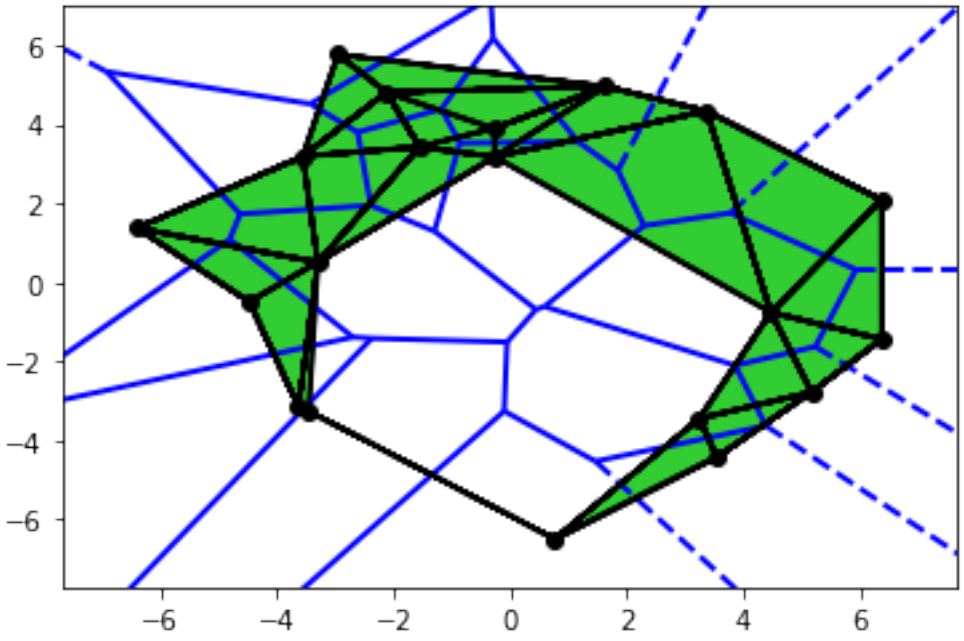

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

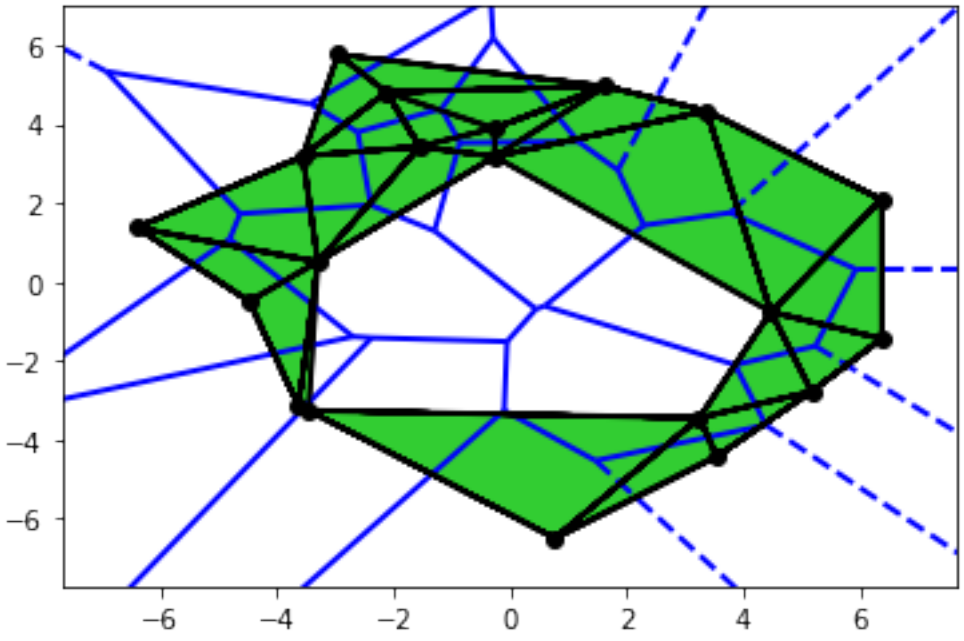

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

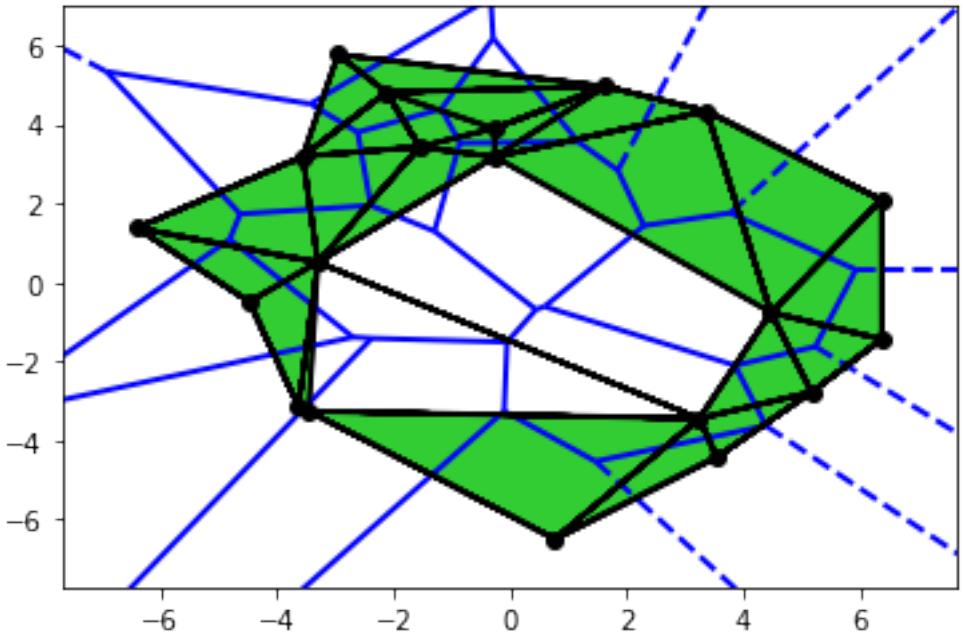

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

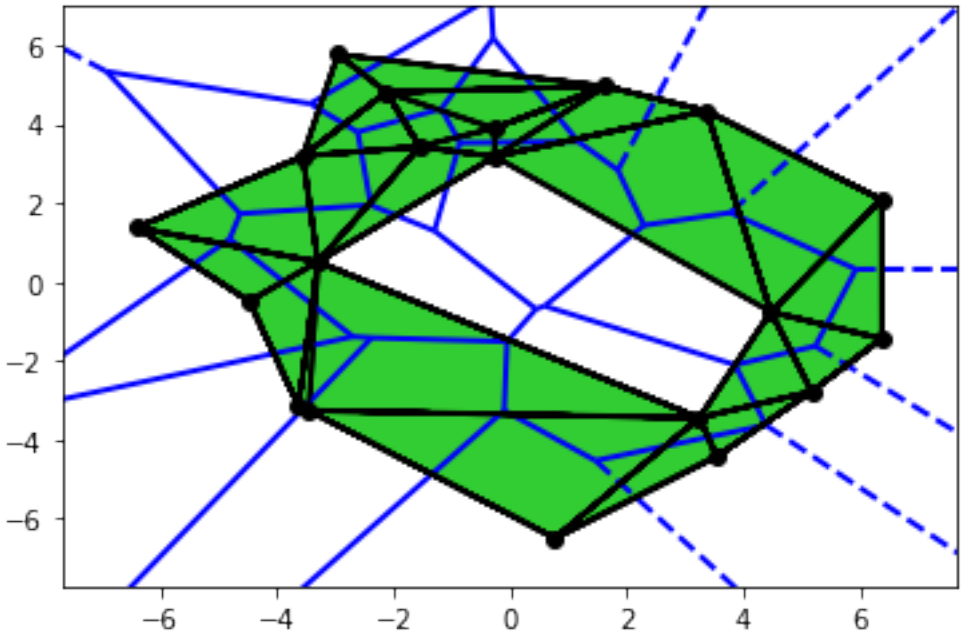

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

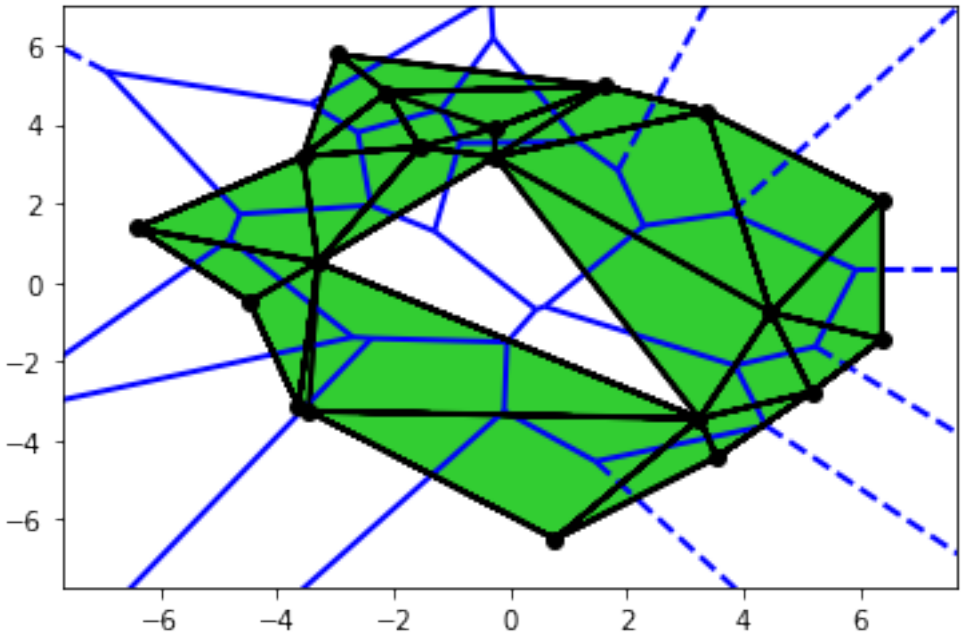

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

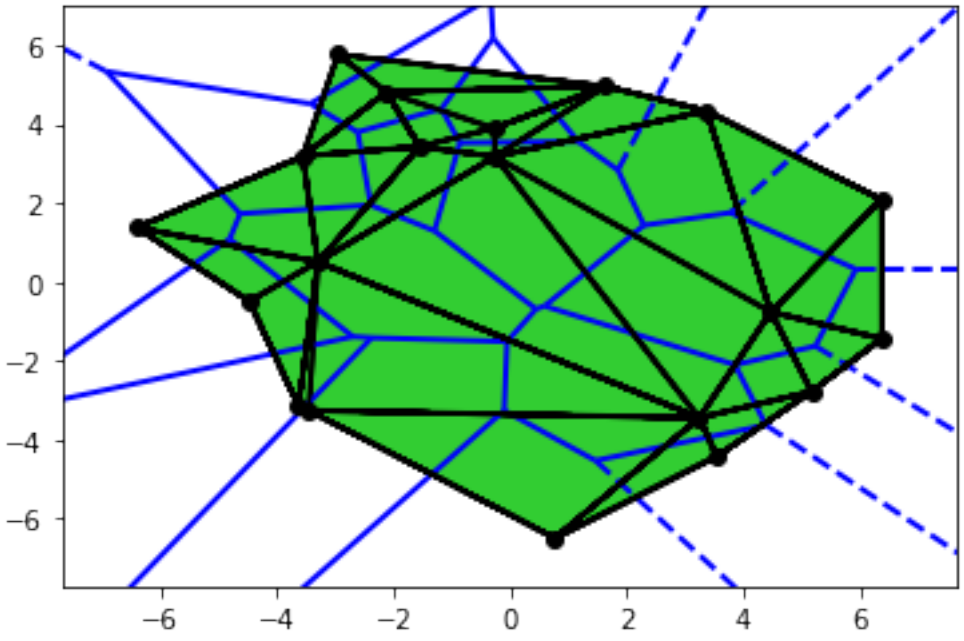

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

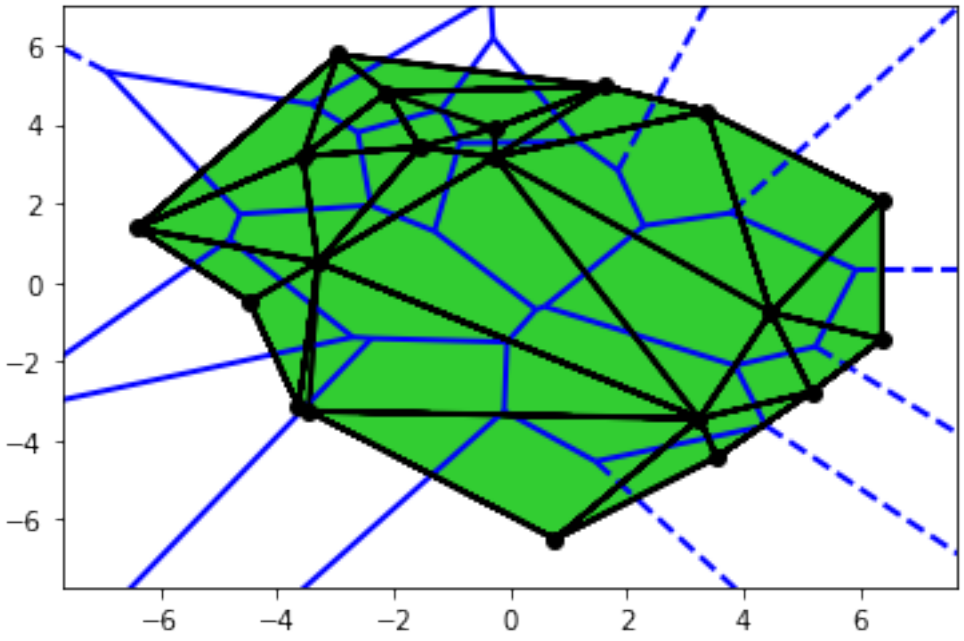

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

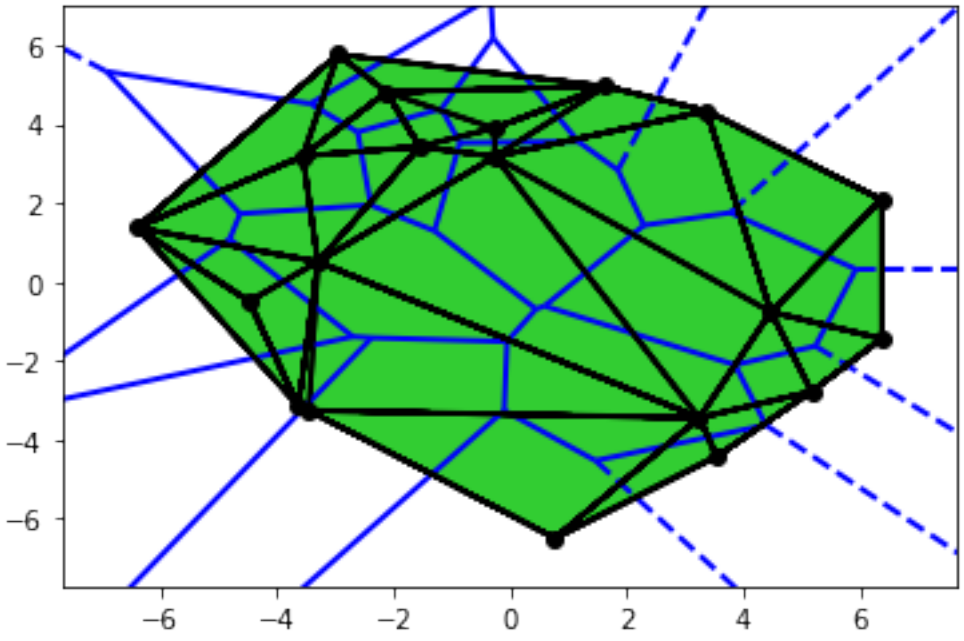

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

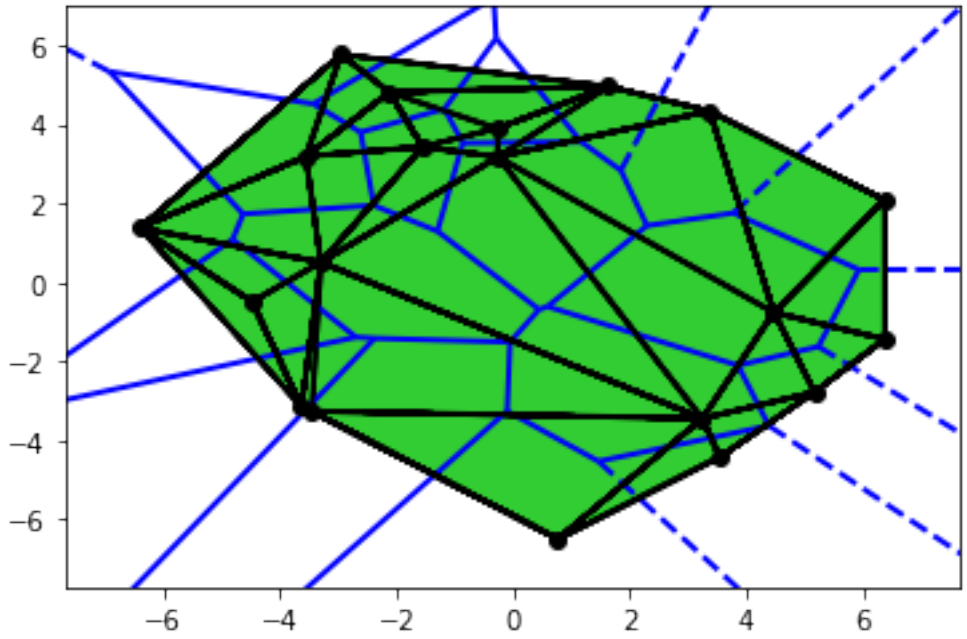

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

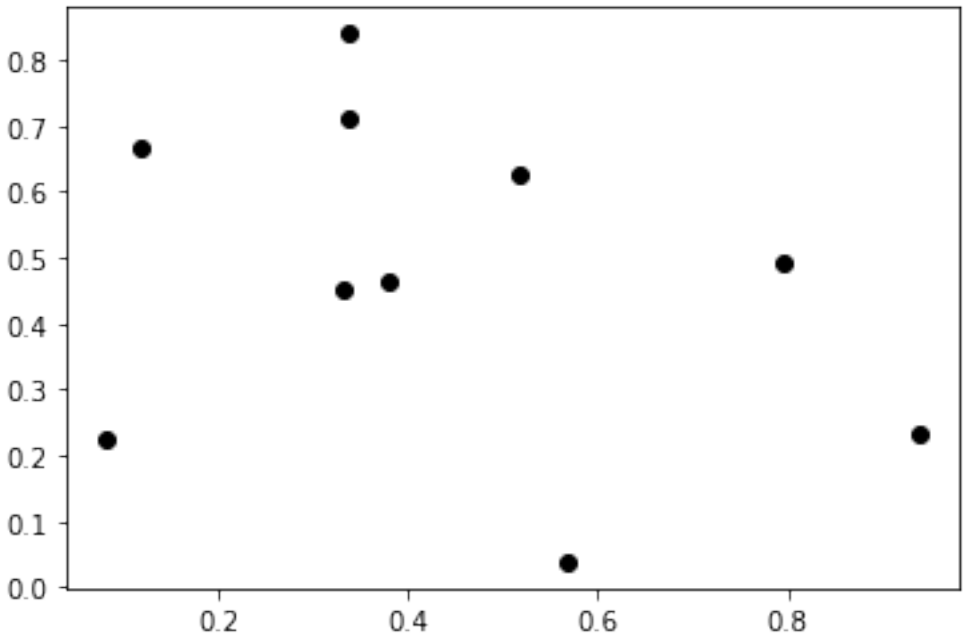

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6



5 Complejo de Vietoris-Rips

Calculamos la filtración de complejos de Vietoris-Rips asociada a la nube de puntos del Ejemplo 1

[96]:
```
plt.plot(points[:,0],points[:,1],'ko')
plt.show()
```



[98]:
```
VR=Vietoris_RipsComplex(points)
```

[99]:
```
VR.filtrationorder()
```

[99]:
```
[(0,),
 (1,),
 (2,),
 (3,),
 (4,),
 (5,),
 (6,),
 (7,),
 (8,),
 (9,),
 (0, 7),
 (3, 6),
 (3, 5),
 (0, 5),
 (3, 8),
 (0, 3),
 (0, 3, 5),
 (5, 7),
 (0, 5, 7),
 (3, 7),
 (0, 3, 7),
 (3, 5, 7),
 (0, 3, 5, 7),
 (5, 6),
 (3, 5, 6),
 (6, 8),
 (3, 6, 8),
 (1, 9),
 (7, 8),
 (3, 7, 8),
 (1, 5),
 (0, 8),
 (0, 3, 8),
 (0, 7, 8),
 (0, 3, 7, 8),
 (4, 7),
 (0, 6),
 (0, 3, 6),
 (0, 5, 6),
 (0, 6, 8),
```

```
(0, 3, 5, 6),
(0, 3, 6, 8),
(0, 4),
(0, 4, 7),
(6, 7),
(0, 6, 7),
(3, 6, 7),
(5, 6, 7),
(6, 7, 8),
(0, 3, 6, 7),
(0, 5, 6, 7),
(0, 6, 7, 8),
(3, 5, 6, 7),
(3, 6, 7, 8),
(0, 3, 5, 6, 7),
(0, 3, 6, 7, 8),
(5, 8),
(0, 5, 8),
(3, 5, 8),
(5, 6, 8),
(5, 7, 8),
(0, 3, 5, 8),
(0, 5, 6, 8),
(0, 5, 7, 8),
(3, 5, 6, 8),
(3, 5, 7, 8),
(5, 6, 7, 8),
(0, 3, 5, 6, 8),
(0, 3, 5, 7, 8),
(0, 5, 6, 7, 8),
(3, 5, 6, 7, 8),
(0, 3, 5, 6, 7, 8),
(0, 1),
(0, 1, 5),
(2, 9),
(4, 8),
(0, 4, 8),
(4, 7, 8),
(0, 4, 7, 8),
(1, 7),
(0, 1, 7),
(1, 5, 7),
(0, 1, 5, 7),
(0, 2),
(2, 7),
(0, 2, 7),
(1, 3),
```

```
(0, 1, 3),
(1, 3, 5),
(1, 3, 7),
(0, 1, 3, 5),
(0, 1, 3, 7),
(1, 3, 5, 7),
(0, 1, 3, 5, 7),
(1, 2),
(0, 1, 2),
(1, 2, 7),
(1, 2, 9),
(0, 1, 2, 7),
(2, 4),
(0, 2, 4),
(2, 4, 7),
(0, 2, 4, 7),
(3, 4),
(0, 3, 4),
(3, 4, 7),
(3, 4, 8),
(0, 3, 4, 7),
(0, 3, 4, 8),
(3, 4, 7, 8),
(0, 3, 4, 7, 8),
(1, 6),
(0, 1, 6),
(1, 3, 6),
(1, 5, 6),
(1, 6, 7),
(0, 1, 3, 6),
(0, 1, 5, 6),
(0, 1, 6, 7),
(1, 3, 5, 6),
(1, 3, 6, 7),
(1, 5, 6, 7),
(0, 1, 3, 5, 6),
(0, 1, 3, 6, 7),
(0, 1, 5, 6, 7),
(1, 3, 5, 6, 7),
(0, 1, 3, 5, 6, 7),
(5, 9),
(1, 5, 9),
(2, 5),
(0, 2, 5),
(1, 2, 5),
(2, 5, 7),
(2, 5, 9),
```

```
(0, 1, 2, 5),
(0, 2, 5, 7),
(1, 2, 5, 7),
(1, 2, 5, 9),
(0, 1, 2, 5, 7),
(4, 5),
(0, 4, 5),
(2, 4, 5),
(3, 4, 5),
(4, 5, 7),
(4, 5, 8),
(0, 2, 4, 5),
(0, 3, 4, 5),
(0, 4, 5, 7),
(0, 4, 5, 8),
(2, 4, 5, 7),
(3, 4, 5, 7),
(3, 4, 5, 8),
(4, 5, 7, 8),
(0, 2, 4, 5, 7),
(0, 3, 4, 5, 7),
(0, 3, 4, 5, 8),
(0, 4, 5, 7, 8),
(3, 4, 5, 7, 8),
(0, 3, 4, 5, 7, 8),
(0, 9),
(0, 1, 9),
(0, 2, 9),
(0, 5, 9),
(0, 1, 2, 9),
(0, 1, 5, 9),
(0, 2, 5, 9),
(0, 1, 2, 5, 9),
(7, 9),
(0, 7, 9),
(1, 7, 9),
(2, 7, 9),
(5, 7, 9),
(0, 1, 7, 9),
(0, 2, 7, 9),
(0, 5, 7, 9),
(1, 2, 7, 9),
(1, 5, 7, 9),
(2, 5, 7, 9),
(0, 1, 2, 7, 9),
(0, 1, 5, 7, 9),
(0, 2, 5, 7, 9),
```

```
(1, 2, 5, 7, 9),
(0, 1, 2, 5, 7, 9),
(4, 6),
(0, 4, 6),
(3, 4, 6),
(4, 5, 6),
(4, 6, 7),
(4, 6, 8),
(0, 3, 4, 6),
(0, 4, 5, 6),
(0, 4, 6, 7),
(0, 4, 6, 8),
(3, 4, 5, 6),
(3, 4, 6, 7),
(3, 4, 6, 8),
(4, 5, 6, 7),
(4, 5, 6, 8),
(4, 6, 7, 8),
(0, 3, 4, 5, 6),
(0, 3, 4, 6, 7),
(0, 3, 4, 6, 8),
(0, 4, 5, 6, 7),
(0, 4, 5, 6, 8),
(0, 4, 6, 7, 8),
(3, 4, 5, 6, 7),
(3, 4, 5, 6, 8),
(3, 4, 6, 7, 8),
(4, 5, 6, 7, 8),
(0, 3, 4, 5, 6, 7),
(0, 3, 4, 5, 6, 8),
(0, 3, 4, 6, 7, 8),
(0, 4, 5, 6, 7, 8),
(3, 4, 5, 6, 7, 8),
(0, 3, 4, 5, 6, 7, 8),
(1, 8),
(0, 1, 8),
(1, 3, 8),
(1, 5, 8),
(1, 6, 8),
(1, 7, 8),
(0, 1, 3, 8),
(0, 1, 5, 8),
(0, 1, 6, 8),
(0, 1, 7, 8),
(1, 3, 5, 8),
(1, 3, 6, 8),
(1, 3, 7, 8),
```

```
(1, 5, 6, 8),
(1, 5, 7, 8),
(1, 6, 7, 8),
(0, 1, 3, 5, 8),
(0, 1, 3, 6, 8),
(0, 1, 3, 7, 8),
(0, 1, 5, 6, 8),
(0, 1, 5, 7, 8),
(0, 1, 6, 7, 8),
(1, 3, 5, 6, 8),
(1, 3, 5, 7, 8),
(1, 3, 6, 7, 8),
(1, 5, 6, 7, 8),
(0, 1, 3, 5, 6, 8),
(0, 1, 3, 5, 7, 8),
(0, 1, 3, 6, 7, 8),
(0, 1, 5, 6, 7, 8),
(1, 3, 5, 6, 7, 8),
(0, 1, 3, 5, 6, 7, 8),
(2, 3),
(0, 2, 3),
(1, 2, 3),
(2, 3, 4),
(2, 3, 5),
(2, 3, 7),
(0, 1, 2, 3),
(0, 2, 3, 4),
(0, 2, 3, 5),
(0, 2, 3, 7),
(1, 2, 3, 5),
(1, 2, 3, 7),
(2, 3, 4, 5),
(2, 3, 4, 7),
(2, 3, 5, 7),
(0, 1, 2, 3, 5),
(0, 1, 2, 3, 7),
(0, 2, 3, 4, 5),
(0, 2, 3, 4, 7),
(0, 2, 3, 5, 7),
(1, 2, 3, 5, 7),
(2, 3, 4, 5, 7),
(0, 1, 2, 3, 5, 7),
(0, 2, 3, 4, 5, 7),
(1, 4),
(0, 1, 4),
(1, 2, 4),
(1, 3, 4),
```

```
(1, 4, 5),
(1, 4, 6),
(1, 4, 7),
(1, 4, 8),
(0, 1, 2, 4),
(0, 1, 3, 4),
(0, 1, 4, 5),
(0, 1, 4, 6),
(0, 1, 4, 7),
(0, 1, 4, 8),
(1, 2, 3, 4),
(1, 2, 4, 5),
(1, 2, 4, 7),
(1, 3, 4, 5),
(1, 3, 4, 6),
(1, 3, 4, 7),
(1, 3, 4, 8),
(1, 4, 5, 6),
(1, 4, 5, 7),
(1, 4, 5, 8),
(1, 4, 6, 7),
(1, 4, 6, 8),
(1, 4, 7, 8),
(0, 1, 2, 3, 4),
(0, 1, 2, 4, 5),
(0, 1, 2, 4, 7),
(0, 1, 3, 4, 5),
(0, 1, 3, 4, 6),
(0, 1, 3, 4, 7),
(0, 1, 3, 4, 8),
(0, 1, 4, 5, 6),
(0, 1, 4, 5, 7),
(0, 1, 4, 5, 8),
(0, 1, 4, 6, 7),
(0, 1, 4, 6, 8),
(0, 1, 4, 7, 8),
(1, 2, 3, 4, 5),
(1, 2, 3, 4, 7),
(1, 2, 4, 5, 7),
(1, 3, 4, 5, 6),
(1, 3, 4, 5, 7),
(1, 3, 4, 5, 8),
(1, 3, 4, 6, 7),
(1, 3, 4, 6, 8),
(1, 3, 4, 7, 8),
(1, 4, 5, 6, 7),
(1, 4, 5, 6, 8),
```

```
(1, 4, 5, 7, 8),
(1, 4, 6, 7, 8),
(0, 1, 2, 3, 4, 5),
(0, 1, 2, 3, 4, 7),
(0, 1, 2, 4, 5, 7),
(0, 1, 3, 4, 5, 6),
(0, 1, 3, 4, 5, 7),
(0, 1, 3, 4, 5, 8),
(0, 1, 3, 4, 6, 7),
(0, 1, 3, 4, 6, 8),
(0, 1, 3, 4, 7, 8),
(0, 1, 4, 5, 6, 7),
(0, 1, 4, 5, 6, 8),
(0, 1, 4, 5, 7, 8),
(0, 1, 4, 6, 7, 8),
(1, 2, 3, 4, 5, 7),
(1, 3, 4, 5, 6, 7),
(1, 3, 4, 5, 6, 8),
(1, 3, 4, 5, 7, 8),
(1, 3, 4, 6, 7, 8),
(1, 4, 5, 6, 7, 8),
(0, 1, 2, 3, 4, 5, 7),
(0, 1, 3, 4, 5, 6, 7),
(0, 1, 3, 4, 5, 6, 8),
(0, 1, 3, 4, 5, 7, 8),
(0, 1, 3, 4, 6, 7, 8),
(0, 1, 4, 5, 6, 7, 8),
(1, 3, 4, 5, 6, 7, 8),
(0, 1, 3, 4, 5, 6, 7, 8),
(3, 9),
(0, 3, 9),
(1, 3, 9),
(2, 3, 9),
(3, 5, 9),
(3, 7, 9),
(0, 1, 3, 9),
(0, 2, 3, 9),
(0, 3, 5, 9),
(0, 3, 7, 9),
(1, 2, 3, 9),
(1, 3, 5, 9),
(1, 3, 7, 9),
(2, 3, 5, 9),
(2, 3, 7, 9),
(3, 5, 7, 9),
(0, 1, 2, 3, 9),
(0, 1, 3, 5, 9),
```

```
(0, 1, 3, 7, 9),
(0, 2, 3, 5, 9),
(0, 2, 3, 7, 9),
(0, 3, 5, 7, 9),
(1, 2, 3, 5, 9),
(1, 2, 3, 7, 9),
(1, 3, 5, 7, 9),
(2, 3, 5, 7, 9),
(0, 1, 2, 3, 5, 9),
(0, 1, 2, 3, 7, 9),
(0, 1, 3, 5, 7, 9),
(0, 2, 3, 5, 7, 9),
(1, 2, 3, 5, 7, 9),
(0, 1, 2, 3, 5, 7, 9),
(2, 8),
(0, 2, 8),
(1, 2, 8),
(2, 3, 8),
(2, 4, 8),
(2, 5, 8),
(2, 7, 8),
(0, 1, 2, 8),
(0, 2, 3, 8),
(0, 2, 4, 8),
(0, 2, 5, 8),
(0, 2, 7, 8),
(1, 2, 3, 8),
(1, 2, 4, 8),
(1, 2, 5, 8),
(1, 2, 7, 8),
(2, 3, 4, 8),
(2, 3, 5, 8),
(2, 3, 7, 8),
(2, 4, 5, 8),
(2, 4, 7, 8),
(2, 5, 7, 8),
(0, 1, 2, 3, 8),
(0, 1, 2, 4, 8),
(0, 1, 2, 5, 8),
(0, 1, 2, 7, 8),
(0, 2, 3, 4, 8),
(0, 2, 3, 5, 8),
(0, 2, 3, 7, 8),
(0, 2, 4, 5, 8),
(0, 2, 4, 7, 8),
(0, 2, 5, 7, 8),
(1, 2, 3, 4, 8),
```

```
(1, 2, 3, 5, 8),
(1, 2, 3, 7, 8),
(1, 2, 4, 5, 8),
(1, 2, 4, 7, 8),
(1, 2, 5, 7, 8),
(2, 3, 4, 5, 8),
(2, 3, 4, 7, 8),
(2, 3, 5, 7, 8),
(2, 4, 5, 7, 8),
(0, 1, 2, 3, 4, 8),
(0, 1, 2, 3, 5, 8),
(0, 1, 2, 3, 7, 8),
(0, 1, 2, 4, 5, 8),
(0, 1, 2, 4, 7, 8),
(0, 1, 2, 5, 7, 8),
(0, 2, 3, 4, 5, 8),
(0, 2, 3, 4, 7, 8),
(0, 2, 3, 5, 7, 8),
(0, 2, 4, 5, 7, 8),
(1, 2, 3, 4, 5, 8),
(1, 2, 3, 4, 7, 8),
(1, 2, 3, 5, 7, 8),
(1, 2, 4, 5, 7, 8),
(2, 3, 4, 5, 7, 8),
(0, 1, 2, 3, 4, 5, 8),
(0, 1, 2, 3, 4, 7, 8),
(0, 1, 2, 3, 5, 7, 8),
(0, 1, 2, 4, 5, 7, 8),
(0, 2, 3, 4, 5, 7, 8),
(1, 2, 3, 4, 5, 7, 8),
(0, 1, 2, 3, 4, 5, 7, 8),
(2, 6),
(0, 2, 6),
(1, 2, 6),
(2, 3, 6),
(2, 4, 6),
(2, 5, 6),
(2, 6, 7),
(2, 6, 8),
(0, 1, 2, 6),
(0, 2, 3, 6),
(0, 2, 4, 6),
(0, 2, 5, 6),
(0, 2, 6, 7),
(0, 2, 6, 8),
(1, 2, 3, 6),
(1, 2, 4, 6),
```

```
(1, 2, 5, 6),
(1, 2, 6, 7),
(1, 2, 6, 8),
(2, 3, 4, 6),
(2, 3, 5, 6),
(2, 3, 6, 7),
(2, 3, 6, 8),
(2, 4, 5, 6),
(2, 4, 6, 7),
(2, 4, 6, 8),
(2, 5, 6, 7),
(2, 5, 6, 8),
(2, 6, 7, 8),
(0, 1, 2, 3, 6),
(0, 1, 2, 4, 6),
(0, 1, 2, 5, 6),
(0, 1, 2, 6, 7),
(0, 1, 2, 6, 8),
(0, 2, 3, 4, 6),
(0, 2, 3, 5, 6),
(0, 2, 3, 6, 7),
(0, 2, 3, 6, 8),
(0, 2, 4, 5, 6),
(0, 2, 4, 6, 7),
(0, 2, 4, 6, 8),
(0, 2, 5, 6, 7),
(0, 2, 5, 6, 8),
(0, 2, 6, 7, 8),
(1, 2, 3, 4, 6),
(1, 2, 3, 5, 6),
(1, 2, 3, 6, 7),
(1, 2, 3, 6, 8),
(1, 2, 4, 5, 6),
(1, 2, 4, 6, 7),
(1, 2, 4, 6, 8),
(1, 2, 5, 6, 7),
(1, 2, 5, 6, 8),
(1, 2, 6, 7, 8),
(2, 3, 4, 5, 6),
(2, 3, 4, 6, 7),
(2, 3, 4, 6, 8),
(2, 3, 5, 6, 7),
(2, 3, 5, 6, 8),
(2, 3, 6, 7, 8),
(2, 4, 5, 6, 7),
(2, 4, 5, 6, 8),
(2, 4, 6, 7, 8),
```

```
(2, 5, 6, 7, 8),
(0, 1, 2, 3, 4, 6),
(0, 1, 2, 3, 5, 6),
(0, 1, 2, 3, 6, 7),
(0, 1, 2, 3, 6, 8),
(0, 1, 2, 4, 5, 6),
(0, 1, 2, 4, 6, 7),
(0, 1, 2, 4, 6, 8),
(0, 1, 2, 5, 6, 7),
(0, 1, 2, 5, 6, 8),
(0, 1, 2, 6, 7, 8),
(0, 2, 3, 4, 5, 6),
(0, 2, 3, 4, 6, 7),
(0, 2, 3, 4, 6, 8),
(0, 2, 3, 5, 6, 7),
(0, 2, 3, 5, 6, 8),
(0, 2, 3, 6, 7, 8),
(0, 2, 4, 5, 6, 7),
(0, 2, 4, 5, 6, 8),
(0, 2, 4, 6, 7, 8),
(0, 2, 5, 6, 7, 8),
(1, 2, 3, 4, 5, 6),
(1, 2, 3, 4, 6, 7),
(1, 2, 3, 4, 6, 8),
(1, 2, 3, 5, 6, 7),
(1, 2, 3, 5, 6, 8),
(1, 2, 3, 6, 7, 8),
(1, 2, 4, 5, 6, 7),
(1, 2, 4, 5, 6, 8),
(1, 2, 4, 6, 7, 8),
(1, 2, 5, 6, 7, 8),
(2, 3, 4, 5, 6, 7),
(2, 3, 4, 5, 6, 8),
(2, 3, 4, 6, 7, 8),
(2, 3, 5, 6, 7, 8),
(2, 4, 5, 6, 7, 8),
(0, 1, 2, 3, 4, 5, 6),
(0, 1, 2, 3, 4, 6, 7),
(0, 1, 2, 3, 4, 6, 8),
(0, 1, 2, 3, 5, 6, 7),
(0, 1, 2, 3, 5, 6, 8),
(0, 1, 2, 3, 6, 7, 8),
(0, 1, 2, 4, 5, 6, 7),
(0, 1, 2, 4, 5, 6, 8),
(0, 1, 2, 4, 6, 7, 8),
(0, 1, 2, 5, 6, 7, 8),
(0, 2, 3, 4, 5, 6, 7),
```

```
(0, 2, 3, 4, 5, 6, 8),
(0, 2, 3, 4, 6, 7, 8),
(0, 2, 3, 5, 6, 7, 8),
(0, 2, 4, 5, 6, 7, 8),
(1, 2, 3, 4, 5, 6, 7),
(1, 2, 3, 4, 5, 6, 8),
(1, 2, 3, 4, 6, 7, 8),
(1, 2, 3, 5, 6, 7, 8),
(1, 2, 4, 5, 6, 7, 8),
(2, 3, 4, 5, 6, 7, 8),
(0, 1, 2, 3, 4, 5, 6, 7),
(0, 1, 2, 3, 4, 5, 6, 8),
(0, 1, 2, 3, 4, 6, 7, 8),
(0, 1, 2, 3, 5, 6, 7, 8),
(0, 1, 2, 4, 5, 6, 7, 8),
(0, 2, 3, 4, 5, 6, 7, 8),
(1, 2, 3, 4, 5, 6, 7, 8),
(0, 1, 2, 3, 4, 5, 6, 7, 8),
(4, 9),
(0, 4, 9),
(1, 4, 9),
(2, 4, 9),
(3, 4, 9),
(4, 5, 9),
(4, 7, 9),
(0, 1, 4, 9),
(0, 2, 4, 9),
(0, 3, 4, 9),
(0, 4, 5, 9),
(0, 4, 7, 9),
(1, 2, 4, 9),
(1, 3, 4, 9),
(1, 4, 5, 9),
(1, 4, 7, 9),
(2, 3, 4, 9),
(2, 4, 5, 9),
(2, 4, 7, 9),
(3, 4, 5, 9),
(3, 4, 7, 9),
(4, 5, 7, 9),
(0, 1, 2, 4, 9),
(0, 1, 3, 4, 9),
(0, 1, 4, 5, 9),
(0, 1, 4, 7, 9),
(0, 2, 3, 4, 9),
(0, 2, 4, 5, 9),
(0, 2, 4, 7, 9),
```

```
(0, 3, 4, 5, 9),
(0, 3, 4, 7, 9),
(0, 4, 5, 7, 9),
(1, 2, 3, 4, 9),
(1, 2, 4, 5, 9),
(1, 2, 4, 7, 9),
(1, 3, 4, 5, 9),
(1, 3, 4, 7, 9),
(1, 4, 5, 7, 9),
(2, 3, 4, 5, 9),
(2, 3, 4, 7, 9),
(2, 4, 5, 7, 9),
(3, 4, 5, 7, 9),
(0, 1, 2, 3, 4, 9),
(0, 1, 2, 4, 5, 9),
(0, 1, 2, 4, 7, 9),
(0, 1, 3, 4, 5, 9),
(0, 1, 3, 4, 7, 9),
(0, 1, 4, 5, 7, 9),
(0, 2, 3, 4, 5, 9),
(0, 2, 3, 4, 7, 9),
(0, 2, 4, 5, 7, 9),
(0, 3, 4, 5, 7, 9),
(1, 2, 3, 4, 5, 9),
(1, 2, 3, 4, 7, 9),
(1, 2, 4, 5, 7, 9),
(1, 3, 4, 5, 7, 9),
(2, 3, 4, 5, 7, 9),
(0, 1, 2, 3, 4, 5, 9),
(0, 1, 2, 3, 4, 7, 9),
(0, 1, 2, 4, 5, 7, 9),
(0, 1, 3, 4, 5, 7, 9),
(0, 2, 3, 4, 5, 7, 9),
(1, 2, 3, 4, 5, 7, 9),
(0, 1, 2, 3, 4, 5, 7, 9),
(6, 9),
(0, 6, 9),
(1, 6, 9),
(2, 6, 9),
(3, 6, 9),
(4, 6, 9),
(5, 6, 9),
(6, 7, 9),
(0, 1, 6, 9),
(0, 2, 6, 9),
(0, 3, 6, 9),
(0, 4, 6, 9),
```

```
(0, 5, 6, 9),
(0, 6, 7, 9),
(1, 2, 6, 9),
(1, 3, 6, 9),
(1, 4, 6, 9),
(1, 5, 6, 9),
(1, 6, 7, 9),
(2, 3, 6, 9),
(2, 4, 6, 9),
(2, 5, 6, 9),
(2, 6, 7, 9),
(3, 4, 6, 9),
(3, 5, 6, 9),
(3, 6, 7, 9),
(4, 5, 6, 9),
(4, 6, 7, 9),
(5, 6, 7, 9),
(0, 1, 2, 6, 9),
(0, 1, 3, 6, 9),
(0, 1, 4, 6, 9),
(0, 1, 5, 6, 9),
(0, 1, 6, 7, 9),
(0, 2, 3, 6, 9),
(0, 2, 4, 6, 9),
(0, 2, 5, 6, 9),
(0, 2, 6, 7, 9),
(0, 3, 4, 6, 9),
(0, 3, 5, 6, 9),
(0, 3, 6, 7, 9),
(0, 4, 5, 6, 9),
(0, 4, 6, 7, 9),
(0, 5, 6, 7, 9),
(1, 2, 3, 6, 9),
(1, 2, 4, 6, 9),
(1, 2, 5, 6, 9),
(1, 2, 6, 7, 9),
(1, 3, 4, 6, 9),
(1, 3, 5, 6, 9),
(1, 3, 6, 7, 9),
(1, 4, 5, 6, 9),
(1, 4, 6, 7, 9),
(1, 5, 6, 7, 9),
(2, 3, 4, 6, 9),
(2, 3, 5, 6, 9),
(2, 3, 6, 7, 9),
(2, 4, 5, 6, 9),
(2, 4, 6, 7, 9),
```

```
(2, 5, 6, 7, 9),
(3, 4, 5, 6, 9),
(3, 4, 6, 7, 9),
(3, 5, 6, 7, 9),
(4, 5, 6, 7, 9),
(0, 1, 2, 3, 6, 9),
(0, 1, 2, 4, 6, 9),
(0, 1, 2, 5, 6, 9),
(0, 1, 2, 6, 7, 9),
(0, 1, 3, 4, 6, 9),
(0, 1, 3, 5, 6, 9),
(0, 1, 3, 6, 7, 9),
(0, 1, 4, 5, 6, 9),
(0, 1, 4, 6, 7, 9),
(0, 1, 5, 6, 7, 9),
(0, 2, 3, 4, 6, 9),
(0, 2, 3, 5, 6, 9),
(0, 2, 3, 6, 7, 9),
(0, 2, 4, 5, 6, 9),
(0, 2, 4, 6, 7, 9),
(0, 2, 5, 6, 7, 9),
(0, 3, 4, 5, 6, 9),
(0, 3, 4, 6, 7, 9),
(0, 3, 5, 6, 7, 9),
(0, 4, 5, 6, 7, 9),
(1, 2, 3, 4, 6, 9),
(1, 2, 3, 5, 6, 9),
(1, 2, 3, 6, 7, 9),
(1, 2, 4, 5, 6, 9),
(1, 2, 4, 6, 7, 9),
(1, 2, 5, 6, 7, 9),
(1, 3, 4, 5, 6, 9),
(1, 3, 4, 6, 7, 9),
(1, 3, 5, 6, 7, 9),
(1, 4, 5, 6, 7, 9),
(2, 3, 4, 5, 6, 9),
(2, 3, 4, 6, 7, 9),
(2, 3, 5, 6, 7, 9),
(2, 4, 5, 6, 7, 9),
(3, 4, 5, 6, 7, 9),
(0, 1, 2, 3, 4, 6, 9),
(0, 1, 2, 3, 5, 6, 9),
(0, 1, 2, 3, 6, 7, 9),
(0, 1, 2, 4, 5, 6, 9),
(0, 1, 2, 4, 6, 7, 9),
(0, 1, 2, 5, 6, 7, 9),
(0, 1, 3, 4, 5, 6, 9),
```

```
(0, 1, 3, 4, 6, 7, 9),
(0, 1, 3, 5, 6, 7, 9),
(0, 1, 4, 5, 6, 7, 9),
(0, 2, 3, 4, 5, 6, 9),
(0, 2, 3, 4, 6, 7, 9),
(0, 2, 3, 5, 6, 7, 9),
(0, 2, 4, 5, 6, 7, 9),
(0, 3, 4, 5, 6, 7, 9),
(1, 2, 3, 4, 5, 6, 9),
(1, 2, 3, 4, 6, 7, 9),
(1, 2, 3, 5, 6, 7, 9),
(1, 2, 4, 5, 6, 7, 9),
(1, 3, 4, 5, 6, 7, 9),
(2, 3, 4, 5, 6, 7, 9),
(0, 1, 2, 3, 4, 5, 6, 9),
(0, 1, 2, 3, 4, 6, 7, 9),
(0, 1, 2, 3, 5, 6, 7, 9),
(0, 1, 2, 4, 5, 6, 7, 9),
(0, 1, 3, 4, 5, 6, 7, 9),
(0, 2, 3, 4, 5, 6, 7, 9),
(1, 2, 3, 4, 5, 6, 7, 9),
(0, 1, 2, 3, 4, 5, 6, 7, 9),
(8, 9),
(0, 8, 9),
(1, 8, 9),
(2, 8, 9),
(3, 8, 9),
(4, 8, 9),
(5, 8, 9),
(6, 8, 9),
(7, 8, 9),
(0, 1, 8, 9),
(0, 2, 8, 9),
(0, 3, 8, 9),
(0, 4, 8, 9),
(0, 5, 8, 9),
(0, 6, 8, 9),
(0, 7, 8, 9),
(1, 2, 8, 9),
(1, 3, 8, 9),
(1, 4, 8, 9),
(1, 5, 8, 9),
(1, 6, 8, 9),
(1, 7, 8, 9),
(2, 3, 8, 9),
(2, 4, 8, 9),
(2, 5, 8, 9),
```

```
(2, 6, 8, 9),
(2, 7, 8, 9),
(3, 4, 8, 9),
(3, 5, 8, 9),
(3, 6, 8, 9),
(3, 7, 8, 9),
(4, 5, 8, 9),
(4, 6, 8, 9),
(4, 7, 8, 9),
(5, 6, 8, 9),
(5, 7, 8, 9),
(6, 7, 8, 9),
(0, 1, 2, 8, 9),
(0, 1, 3, 8, 9),
(0, 1, 4, 8, 9),
(0, 1, 5, 8, 9),
(0, 1, 6, 8, 9),
(0, 1, 7, 8, 9),
(0, 2, 3, 8, 9),
(0, 2, 4, 8, 9),
(0, 2, 5, 8, 9),
(0, 2, 6, 8, 9),
(0, 2, 7, 8, 9),
(0, 3, 4, 8, 9),
(0, 3, 5, 8, 9),
(0, 3, 6, 8, 9),
(0, 3, 7, 8, 9),
(0, 4, 5, 8, 9),
(0, 4, 6, 8, 9),
(0, 4, 7, 8, 9),
(0, 5, 6, 8, 9),
(0, 5, 7, 8, 9),
(0, 6, 7, 8, 9),
(1, 2, 3, 8, 9),
(1, 2, 4, 8, 9),
(1, 2, 5, 8, 9),
(1, 2, 6, 8, 9),
(1, 2, 7, 8, 9),
(1, 3, 4, 8, 9),
(1, 3, 5, 8, 9),
(1, 3, 6, 8, 9),
(1, 3, 7, 8, 9),
(1, 4, 5, 8, 9),
(1, 4, 6, 8, 9),
(1, 4, 7, 8, 9),
(1, 5, 6, 8, 9),
(1, 5, 7, 8, 9),
```

```
(1, 6, 7, 8, 9),
(2, 3, 4, 8, 9),
(2, 3, 5, 8, 9),
(2, 3, 6, 8, 9),
(2, 3, 7, 8, 9),
(2, 4, 5, 8, 9),
(2, 4, 6, 8, 9),
(2, 4, 7, 8, 9),
(2, 5, 6, 8, 9),
(2, 5, 7, 8, 9),
(2, 6, 7, 8, 9),
(3, 4, 5, 8, 9),
(3, 4, 6, 8, 9),
(3, 4, 7, 8, 9),
(3, 5, 6, 8, 9),
(3, 5, 7, 8, 9),
(3, 6, 7, 8, 9),
(4, 5, 6, 8, 9),
(4, 5, 7, 8, 9),
(4, 6, 7, 8, 9),
(5, 6, 7, 8, 9),
(0, 1, 2, 3, 8, 9),
(0, 1, 2, 4, 8, 9),
(0, 1, 2, 5, 8, 9),
(0, 1, 2, 6, 8, 9),
(0, 1, 2, 7, 8, 9),
(0, 1, 3, 4, 8, 9),
(0, 1, 3, 5, 8, 9),
(0, 1, 3, 6, 8, 9),
(0, 1, 3, 7, 8, 9),
(0, 1, 4, 5, 8, 9),
(0, 1, 4, 6, 8, 9),
(0, 1, 4, 7, 8, 9),
(0, 1, 5, 6, 8, 9),
(0, 1, 5, 7, 8, 9),
(0, 1, 6, 7, 8, 9),
(0, 2, 3, 4, 8, 9),
(0, 2, 3, 5, 8, 9),
(0, 2, 3, 6, 8, 9),
(0, 2, 3, 7, 8, 9),
(0, 2, 4, 5, 8, 9),
(0, 2, 4, 6, 8, 9),
(0, 2, 4, 7, 8, 9),
(0, 2, 5, 6, 8, 9),
(0, 2, 5, 7, 8, 9),
(0, 2, 6, 7, 8, 9),
(0, 3, 4, 5, 8, 9),
```

```
(0, 3, 4, 6, 8, 9),
(0, 3, 4, 7, 8, 9),
(0, 3, 5, 6, 8, 9),
(0, 3, 5, 7, 8, 9),
(0, 3, 6, 7, 8, 9),
(0, 4, 5, 6, 8, 9),
(0, 4, 5, 7, 8, 9),
(0, 4, 6, 7, 8, 9),
(0, 5, 6, 7, 8, 9),
(1, 2, 3, 4, 8, 9),
(1, 2, 3, 5, 8, 9),
(1, 2, 3, 6, 8, 9),
(1, 2, 3, 7, 8, 9),
(1, 2, 4, 5, 8, 9),
(1, 2, 4, 6, 8, 9),
(1, 2, 4, 7, 8, 9),
(1, 2, 5, 6, 8, 9),
(1, 2, 5, 7, 8, 9),
(1, 2, 6, 7, 8, 9),
(1, 3, 4, 5, 8, 9),
(1, 3, 4, 6, 8, 9),
(1, 3, 4, 7, 8, 9),
(1, 3, 5, 6, 8, 9),
(1, 3, 5, 7, 8, 9),
(1, 3, 6, 7, 8, 9),
(1, 4, 5, 6, 8, 9),
(1, 4, 5, 7, 8, 9),
(1, 4, 6, 7, 8, 9),
(1, 5, 6, 7, 8, 9),
(2, 3, 4, 5, 8, 9),
(2, 3, 4, 6, 8, 9),
(2, 3, 4, 7, 8, 9),
(2, 3, 5, 6, 8, 9),
(2, 3, 5, 7, 8, 9),
(2, 3, 6, 7, 8, 9),
(2, 4, 5, 6, 8, 9),
(2, 4, 5, 7, 8, 9),
(2, 4, 6, 7, 8, 9),
(2, 5, 6, 7, 8, 9),
(3, 4, 5, 6, 8, 9),
(3, 4, 5, 7, 8, 9),
(3, 4, 6, 7, 8, 9),
(3, 5, 6, 7, 8, 9),
(4, 5, 6, 7, 8, 9),
(0, 1, 2, 3, 4, 8, 9),
(0, 1, 2, 3, 5, 8, 9),
(0, 1, 2, 3, 6, 8, 9),
```

```
(0, 1, 2, 3, 7, 8, 9),
(0, 1, 2, 4, 5, 8, 9),
(0, 1, 2, 4, 6, 8, 9),
(0, 1, 2, 4, 7, 8, 9),
(0, 1, 2, 5, 6, 8, 9),
(0, 1, 2, 5, 7, 8, 9),
(0, 1, 2, 6, 7, 8, 9),
(0, 1, 3, 4, 5, 8, 9),
(0, 1, 3, 4, 6, 8, 9),
(0, 1, 3, 4, 7, 8, 9),
(0, 1, 3, 5, 6, 8, 9),
(0, 1, 3, 5, 7, 8, 9),
(0, 1, 3, 6, 7, 8, 9),
(0, 1, 4, 5, 6, 8, 9),
(0, 1, 4, 5, 7, 8, 9),
(0, 1, 4, 6, 7, 8, 9),
(0, 1, 5, 6, 7, 8, 9),
(0, 2, 3, 4, 5, 8, 9),
(0, 2, 3, 4, 6, 8, 9),
(0, 2, 3, 4, 7, 8, 9),
(0, 2, 3, 5, 6, 8, 9),
(0, 2, 3, 5, 7, 8, 9),
(0, 2, 3, 6, 7, 8, 9),
(0, 2, 4, 5, 6, 8, 9),
(0, 2, 4, 5, 7, 8, 9),
(0, 2, 4, 6, 7, 8, 9),
(0, 2, 5, 6, 7, 8, 9),
(0, 3, 4, 5, 6, 8, 9),
(0, 3, 4, 5, 7, 8, 9),
(0, 3, 4, 6, 7, 8, 9),
(0, 3, 5, 6, 7, 8, 9),
(0, 4, 5, 6, 7, 8, 9),
(1, 2, 3, 4, 5, 8, 9),
(1, 2, 3, 4, 6, 8, 9),
(1, 2, 3, 4, 7, 8, 9),
(1, 2, 3, 5, 6, 8, 9),
(1, 2, 3, 5, 7, 8, 9),
(1, 2, 3, 6, 7, 8, 9),
(1, 2, 4, 5, 6, 8, 9),
(1, 2, 4, 5, 7, 8, 9),
(1, 2, 4, 6, 7, 8, 9),
(1, 2, 5, 6, 7, 8, 9),
(1, 3, 4, 5, 6, 8, 9),
(1, 3, 4, 5, 7, 8, 9),
(1, 3, 4, 6, 7, 8, 9),
(1, 3, 5, 6, 7, 8, 9),
(1, 4, 5, 6, 7, 8, 9),
```

```
 (2, 3, 4, 5, 6, 8, 9),
 (2, 3, 4, 5, 7, 8, 9),
 (2, 3, 4, 6, 7, 8, 9),
 (2, 3, 5, 6, 7, 8, 9),
 (2, 4, 5, 6, 7, 8, 9),
 (3, 4, 5, 6, 7, 8, 9),
 (0, 1, 2, 3, 4, 5, 8, 9),
 (0, 1, 2, 3, 4, 6, 8, 9),
 (0, 1, 2, 3, 4, 7, 8, 9),
 (0, 1, 2, 3, 5, 6, 8, 9),
 (0, 1, 2, 3, 5, 7, 8, 9),
 (0, 1, 2, 3, 6, 7, 8, 9),
 (0, 1, 2, 4, 5, 6, 8, 9),
 (0, 1, 2, 4, 5, 7, 8, 9),
 (0, 1, 2, 4, 6, 7, 8, 9),
 (0, 1, 2, 5, 6, 7, 8, 9),
 (0, 1, 3, 4, 5, 6, 8, 9),
 (0, 1, 3, 4, 5, 7, 8, 9),
 (0, 1, 3, 4, 6, 7, 8, 9),
 (0, 1, 3, 5, 6, 7, 8, 9),
 ...]
```

[100]:
```
VR.thresholdvalues()
```

[100]:
```
[0.0,
 0.024594630110749274,
 0.06491550663246884,
 0.09964158316325628,
 0.1067160499498478,
 0.11181663867459748,
 0.12497889333174499,
 0.12743785247513206,
 0.12897443080262758,
 0.13980240119025444,
 0.14003715958303004,
 0.14897699064479192,
 0.151309131049649,
 0.15403831830169779,
 0.16545560550649724,
 0.16844581383081997,
 0.1892487801168689,
 0.19044564833074332,
 0.19388331119567223,
 0.20104172877828352,
 0.20796056752930286,
 0.20947134070559964,
 0.22042128784724613,
```

```
0.23222927199764715,
0.23243829737556373,
0.23788130442736796,
0.2536662137792591,
0.25427755231828075,
0.259566655903143,
0.2733676600468666,
0.2875741410328021,
0.2887027457831261,
0.2954270609819514,
0.29587003677268414,
0.3024077728892547,
0.32252639022676216,
0.3322106227332085,
0.34957492983589655,
0.35510817894523095,
0.3802904532397188,
0.38445726127031843,
0.3857274202002728,
0.41706602888011096,
0.4278584620998067,
0.4279252460662376,
0.4642579923126173]
```