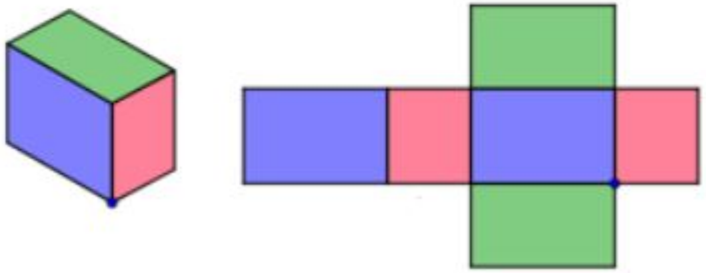
What is a Net?

Anti-prisms are n-sided polyhedrons



Differ in symmetry

Geometric Non-congruence

Figure 1. The antiprism of order $n = 5$.

(t_n)

Figure 2. A net of the antiprism.

(s_n)

Figure 3. A symmetric net.

A Closer Look

pairs of adjacent and oppositely oriented triangular faces, *neck size (h)*

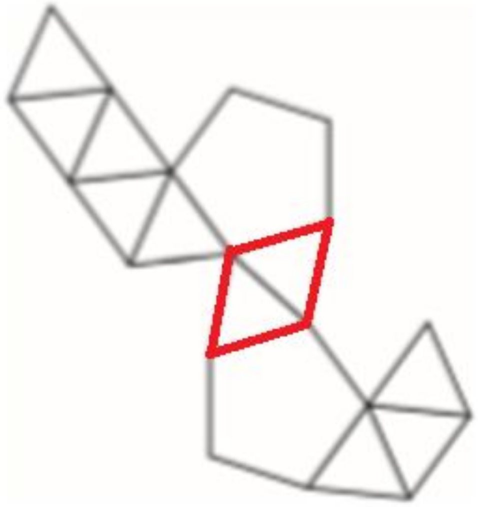


Figure 2. A net of the antiprism.

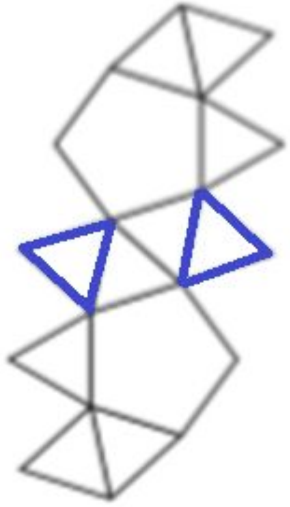
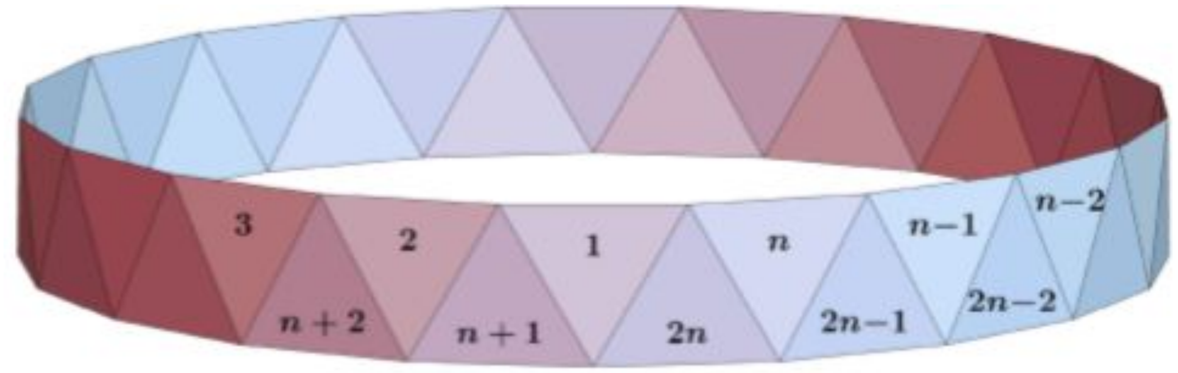


Figure 3. A symmetric net.

triangular faces attached to the neck that form the *collar* (*lapels (k)*)

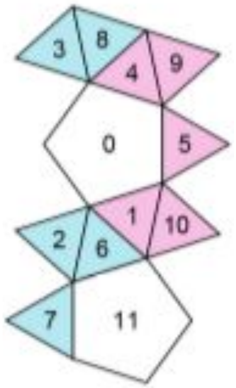
A Step Back

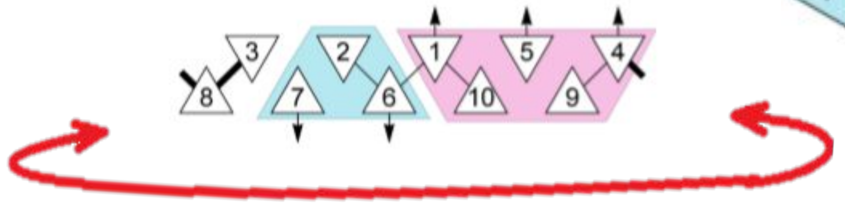
The labeling method of triangular faces in *n*-gons



Example of a 5-sided polyhedron:

- Arrows indicate connections to face
- Lines indicate a connections by an edge on the net
- Gaps indicate connections not noticeable on the net





A wrap-around edge

THE CONNECTION BETWEEN SYMMETRIC NETS & FIBONACCI

Main Results

The Relationship

(1) $$s_n = F_{2n}$$

(2) $$t_n = \frac{1}{2} s_n (s_n + 1)$$

(t_n)

Figure 2. A net of the antiprism.

(s_n)

Figure 3. A symmetric net.

- F_n is the usual j^{th} Fibonacci number (F_1= 1, F_2= 1, F_{j+2}=F_{j+1}+F_j for all j ≤ 1)
- The focus of these claims is to identify the symmetric nets of group (Tn)
- Size of S_n turns out to be the evenly indexed Fibonacci numbers

Properties of Symmetric Nets

In symmetric nets $k_1 = k_2$ (k)

$1 \le h \le n,\ 0 \le k \le n-1,\ 1 \le h + k \le n \qquad (3)$

Each pair (h,k) must satisfy (3) in T_n

Lemma 1. For each n ≥1, the number $s_n(h,k)$ of symmetric nets of the n-antiprism having neck size h and lapel size k is given by

$$s_n(h,k) = \begin{cases} 1 & \textit{if} \quad h+k=n \\ F_{2(n-h-k)} & \textit{if} \quad 1 \le h+k < n. \end{cases} \qquad (4)$$

Theorem 1.

There are F_{2n} symmetric nets of the n-antiprism. That is, $s_n = F_{2n}$.

$$s_n(h, *) := \sum_{k=0}^{n-h} s_n(h, k)$$

Proof. By **Lemma 1**, the number of symmetric nets with neck size h is

$$= 1 + \sum_{k=0}^{n-h-1} F_{2(n-h-k)}$$

$$= 1 + F_2 + F_4 + \cdots + F_{2(n-h)}$$

$$= F_{1+2(n-h)},$$

Note:

The last equality follows from telescoping the preceding sum after expressing the even-indexed Fibonacci numbers like so: $F_{2i} = F_{2i+1} - F_{2i-1}$ $(i \geq 1)$

$$s_n = \sum_{h=1}^{n} s_n(h, *)$$

$$= \sum_{h=1}^{n} F_{1+2(n-h)}$$

$$= F_1 + F_3 + \cdots + F_{2n-1}$$

$$s_n = F_{2n}.$$

Theorem 2

There are precisely $S_n \frac{S_{\mathrm{n}}-1}{2}$ nets of the n-antiprism. That is, T_n = $F_{2n} \frac{F_{2\mathrm{n}}+1}{2}$ [2]

The purpose of this formula allows for the chance to look at pairs of symmetric nets. These two nets are used construct a unique net in T_n

PROOF of Theorem 2.

Let $N_1, N_2 \in S_n$ such that N_1 and N_2 are not necessarily distinct and can be used to construct $\exists$ $N_3 \in T_n$ Let V be an injective function such that, V :$S_n^{(2)} \rightarrow T_n$ and V^{-1} : $T_n \rightarrow S_n^{(2)}$.

Case 1: Let $N_1, N_2 \in S_n$ such that $h_1 = h_2$.

Case 2: Let $N_1, N_2 \in S_n$ such that $h_1 \neq h_2$. In this case, let $\Delta h = h_2 - h_1$ (Assume $h_2 > h_1$)

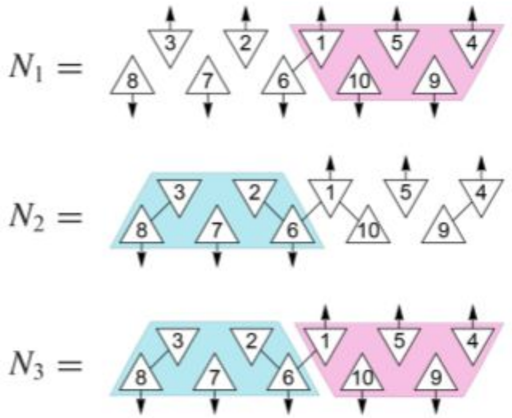
i) Δh is odd

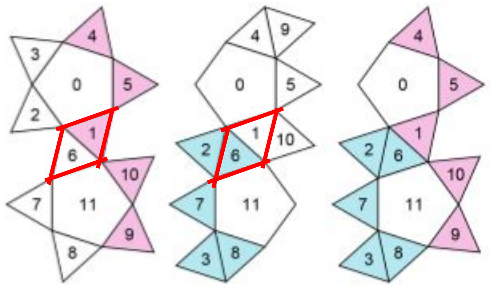
ii) Δh is even

Case 1

Case 1: Let $N_1, N_2 \in S_n$ such that $h_1 = h_2$.

Easiest to evaluate, the two opposing sides join to create unique $N_3 \in T_n$





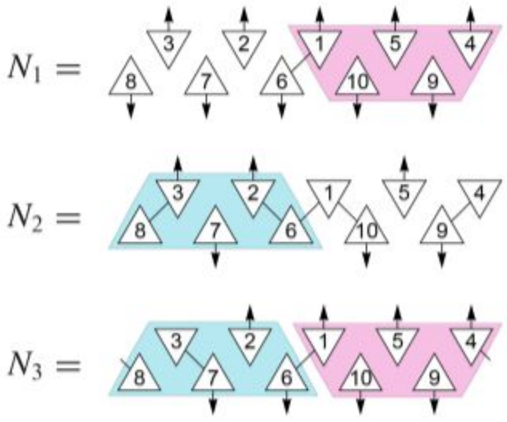
$h_1 = 1$ $h_2 = 1$

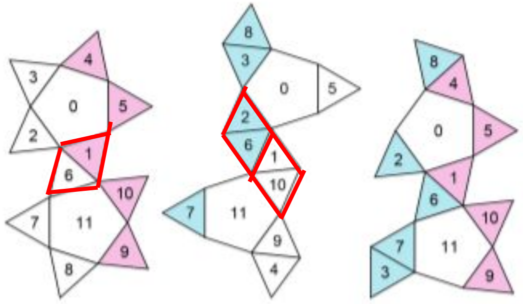
All the arrow-gram criteria for a net are met

Clearly, the resulting net N_3 will be symmetric if and only if $N_1 = N_2$.

Case 2

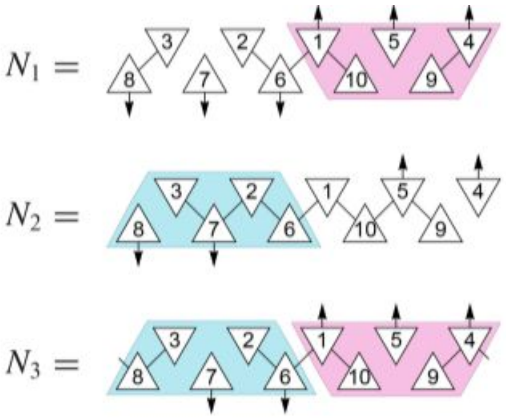
Case 2: Let $N_1, N_2 \in S_n$ such that $h_1 \neq h_2$.

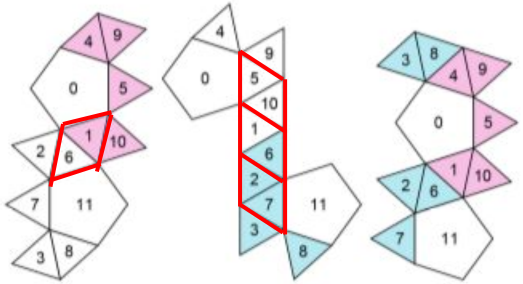




$h_1 = 1$ $h_2 = 2$

Δh is odd





$h_1 = 1$ $h_2 = 3$

Δh is even

This is an attempt at reproducing the results from Case 1.

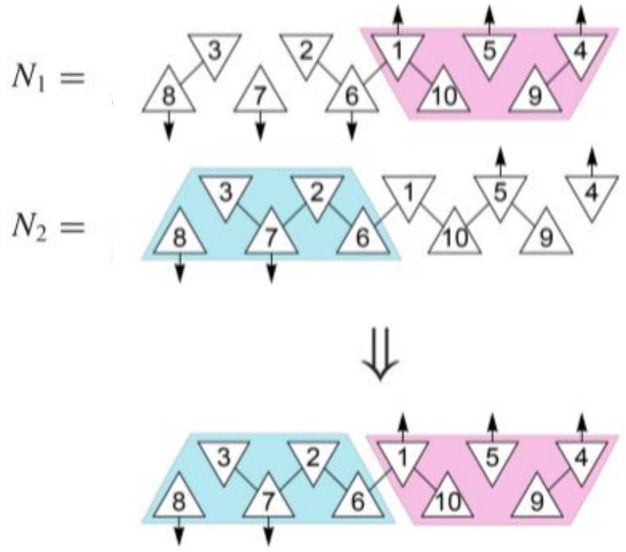
- Δh is odd, an illegal arrow-gram is the result of this step
- Δh is even, a legal arrow-gram is obtained for the same reasons as given in Case 1, but it turns out that the results are then not unique.

Case 2 cont.

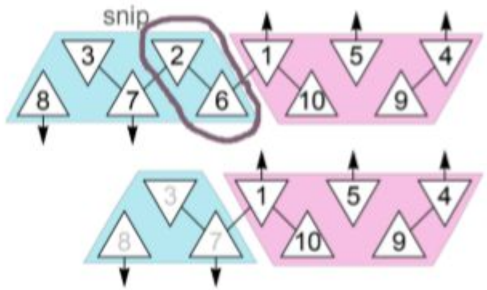
Case 2: Let $N_1, N_2 \in S_n$ such that $h_1 \neq h_2$.

For Example: Δh is even

Let's connect the two sides as in Case 1, using the right side of N_1, the left side of N_2



Shift and compress the left side of this new arrow-gram toward the center by removing Δh vertices(and edges), starting at the center resulting in a neck size of h_1.



The remainder of the left side will be shifted right (and flipped if Δ h is odd) by Δh places

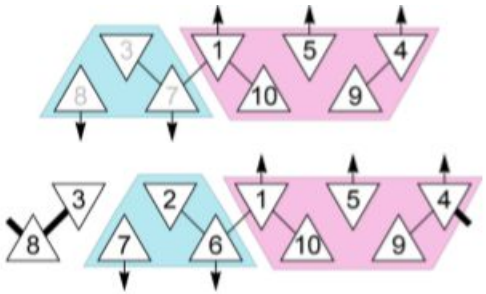
Case 2 cont.

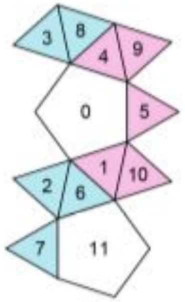
Case 2: Let $N_1, N_2 \in S_n$ such that $h_1 \neq h_2$.

For Example: Δh is even

Add Δh isolated band vertices at the far to refill the band.

Add a wraparound edge and starting there, add Δh edges from left to right.





In each case, the symmetric nets $N_1, N_2 \in S_n$ combine to form a net N_3 with neck size of $\min(h_1, h_2)$ and lapel sizes k_1 and k_2

Because we can look at this relationship as an injective function such that;

$V : S_n^{(2)} \rightarrow T_n$, that is, an injective function V from the set of all pairs from Sn into Tn

By completing both cases as $V^{-1} : T_n \rightarrow S_n^{(2)}$ that is, starting with any $N \in T_n$ we can split it in to $N_1, N_2 \in S_n$

Thus, the theorem follows.

Conclusion

The purpose of the paper was to give insight in another property of the Famous Fibonacci Sequence

Being able to wrap our heads around these unconventional 3D-shapes

QUESTIONS?