

# Understanding Analysis Attempt/Solution

pand4monium

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# Contents

0.1 Some Preliminaries . . . . .	1
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## 0.1 Some Preliminaries

### Exercise 0.1.1

(a) Prove that  $\sqrt{3}$  is irrational. Does a similar argument work to show  $\sqrt{6}$  is rational?

(b) Where does the proof break down if we try to prove  $\sqrt{4}$  is irrational?

### SOLUTION

(a) PROOF AFSOC that  $\sqrt{3}$  is rational, so  $\exists m, n \in \mathbb{Z}$ , such that

$$\sqrt{3} = \frac{m}{n},$$

where  $\frac{m}{n}$  is in the lowest reduced terms. By squaring both sides, we obtain  $3 = (\frac{m}{n})^2 \implies 3n^2 = m^2$ . Now, we know that  $m^2$  is a multiple of 3 and thus  $m$  must also be a multiple of 3. We can then write  $m = 3k$ , deriving

$$\begin{aligned} (\sqrt{3})^2 &= \left(\frac{3k}{n}\right)^2 \\ 3n^2 &= 9k^2 \\ n^2 &= 3k^2 \end{aligned}$$

Similar to above, we can conclude that  $n$  is a multiple of 3. However this is a contradiction since  $m, n$  are both multiples of 3 but we assumed that  $\frac{m}{n}$  was in its lowest reduced term. Thus we conclude that  $\sqrt{3}$  is irrational.

The same proof for  $\sqrt{3}$  works for  $\sqrt{6}$  as well.

(b) We cannot conclude that  $\sqrt{4} = \frac{m}{n}$  imply that  $m$  is a multiple of 4, as we have

$$4n^2 = m^2 \implies 2n = m,$$

preventing us from reaching our contradiction that  $m/n$  is not in its lowest terms.

### Exercise 0.1.2

Show that there is no rational number  $r$  satisfying  $2^r = 3$ .

### SOLUTION

PROOF If  $r = 0$ , then  $2^r = 1 \neq 3$ . Suppose  $r = p/q$  to get  $2^p = 3^q$ , which is not possible as 2 and 3 share no common factors. Hence  $r$  is not rational.

### Exercise 0.1.3

Decide which of the following represent true statements about the nature of sets. For any that are false, provide a specific example where the statement in question does not hold.

- (a) If  $A_1 \supseteq A_2 \subseteq A_3 \subseteq A_4 \dots$  are all sets containing an infinite number of elements, then the intersections  $\bigcap_{n=1}^{\infty} A_n$  is infinite as well.
- (b) If  $A_1 \supseteq A_2 \subseteq A_3 \subseteq A_4 \dots$  are all finite, nonempty sets of real numbers, then the intersection  $\bigcap_{n=1}^{\infty} A_n$  is finite and non-empty.
- (c)  $A \cap (B \cup C) = (A \cap B) \cup C$ .
- (d)  $A \cap (B \cap C) = (A \cap B) \cap C$ .
- (e)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ .

### SOLUTION

(a) False. Consider  $A_n = \{n, n+1, n+2, \dots\}$ , then  $\bigcap_{n=1}^{\infty} A_n = \emptyset$ .

- (b) True. Since all  $A_n$  are nonempty,  $\exists n \in \mathbb{N}$  such that  $A_n = \{x\}$  for some real  $x$ . Hence  $\bigcap_{n=1}^{\infty} A_n \subseteq \{x\}$  which is empty. Since  $A_1$  is finite,  $\bigcap_{n=1}^{\infty} A_n \subseteq \{x\} \subset A_1$  is finite.
- (c) False. If  $A = \emptyset$ , then  $\emptyset = C$
- (d) True. Intersection is associative as evident that both LHS and RHS implies the  $x \in A, B, C$
- (e) True. Drawing a Venn Diagram illustrates this.