Understanding Analysis Attempt/Solution

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0.1 Some Preliminaries

Exercise 0.1.1

(a) Prove that $\sqrt{3}$ is irrational. Does a similar argument work to show $\sqrt{6}$ is rational?

(b) Where does the proof break down if we try to prove $\sqrt{4}$ is irrational?

SOLUTION

(a) PROOF AFSOC that $\sqrt{3}$ is rational, so $\exists m, n \in \mathbb{Z}$, such that

$$\sqrt{3} = \frac{m}{n},$$

where $\frac{m}{n}$ is in the lowest reduced terms. By squaring both sides, we obtain $3 = (\frac{m}{n})^2 \implies 3n^2 = m^2$. Now, we know that m^2 is a multiple of 3 and thus m must also be a multiple of 3. We can then write $m = 3k$, deriving

$$\begin{aligned}(\sqrt{3})^2 &= \left(\frac{3k}{n}\right)^2 \\ 3n^2 &= 9k^2 \\ n^2 &= 3k^2\end{aligned}$$

Similar to above, we can conclude that n is a multiple of 3. However this is a contradiction since m, n are both multiples of 3 but we assumed that $\frac{m}{n}$ was in its lowest reduced term. Thus we conclude that $\sqrt{3}$ is irrational.

The same proof for $\sqrt{3}$ works for $\sqrt{6}$ as well.

(b) We cannot conclude that $\sqrt{4} = \frac{m}{n}$ imply that m is a multiple of 4, as we have

$$4n^2 = m^2 \implies 2n = m,$$

preventing us from reaching our contradiction that m/n is not in its lowest terms.

Exercise 0.1.2

Show that there is no rational number r satisfying $2^r = 3$.

SOLUTION

PROOF If $r = 0$, then $2^r = 1 \neq 3$. Suppose $r = p/q$ to get $2^p = 3^q$, which is not possible as 2 and 3 share no common factors. Hence r is not rational.