Increasing the Fisher Information Content in the Matter Power Spectrum through Reconstruction

(Dated: November 2, 2016)

We adapt a new reconstruction method called Adaptive particle mesh (APM) to 139 non-linear density fields given by independent N-body simulations, in order to recover some of the information lost in the non-linear regime of large-scale structure. Through analyzing the power spectra of both density fields from simulations and deformation potentials from reconstructions, we find that after reconstruction, the non-linear regime of correlation matrix shinks to some extent. We also find that the Fisher information has a increase by a factor up to 400 at the translinear plateau.

PACS numbers:

I. INTRODUCTION

Power spectrum of matter density field is essential in modern cosmology research since it is related to many cosmology parameters. Many dedicated experiments attemp to constrain some specific parameters.

Fisher information is widely used for making predictions for the errors and covariances of parameter estimates. Rimes and Hamilton[**?**] first studied the Fisher information as a function of scale contained in the matter power spectrum given by N-body simulation, and find that there's a plateau at translinear scales ($k \simeq 0.2 - 0.8h\text{Mpc}^{-1}$), which showed that in this region, power spectrum contains little information over and above that in the linear power spectrum. Aprt from numerical trend, Fisher information was also estimated in data from observation. For example, Lee and Pen [**?**] measured the information content in the falaxy angular power spectrum with the help of the Rimes-hamilton technique, and also found that the informtation saturation.

After that, many approaches were put forward to recover parts of the lost information in the power spectrum of matter density field, aiming at the information function closer to that of the power spectrum for linear density fields. Neyrinck, Szapudi and Rimes [**?**] argued that more information could be extracted on non-linear scales if the masses of the largest haloes in a survey are known. Neyrinck, Szapudi and Szalay [**?**] found that nonlinearities in the dark-matter power spectrum are dramatically smaller if the density field first undergoes a logarithmic mapping, yielding 10 times more cumulative signal-to-noise at $z = 0$. Zhang et al. [**?**] suggested that using Wavelet Wiener filter to seperate Gaussian and non-Gaussian structure in wavelet space is possible to increase the Fisher information by a factor of three before reaching the translinear plateau. Similar steps was also done in the angular power spectrum of weak lensing [**?**] and the result showed that there's three times more information compared to that of logarithmic mapping. Later, Neyrinck [**?**] applied Gaussianizing transformation method to cosmology and found that it can greatly multiply the Fisher information.

Pen [**?**][**?**] introduced new N-bosy algorithm call Adapted Particle-mesh (APM), which scaled linearly with the number of particles for the computational effort per time step, aiming at offering higher resolution. This method can be used to reconstruct the matter density field and is hopeful for tracing a non-linear density field back to it's linear part[**?**].

This paper is organized as follows. In Section II, we present the main steps of the N-body simulation code that was used to simulate the dark matter density fields. In Section III, we briefly describe the reconstruction algorithm. In Section IV, we calculate and compare the power spectra, correlation matrix and Fisher information given by simulation and reconstruction. Conclusion and discussion are presented in Section V

II. N-BODY SIMULATION OF DARK MATTER DENSITY FIELDS

We run 139 simulations with a box size of 300 h^{-1}Mpc, resolution of 1024^3 cells and 512^3 particles, using the cosmological simulation code CubeP3M(CITA Computing 2008?).The initial condition is given by reading CMBFAST transfer function and then evolving the power linearly to $z = 100$. Then Zel'dovich approximation is used to calculate the displacement field and velocity field, which are assigned to the particles. The cosmological parameters used are $\Omega_M = 0.32$, $\Omega_\Lambda = 0.679$, $h = 0.67$, $\sigma_8 = 0.83$, and $n_s = 0.96$. And we use different seeds to produce the initial conditions for different simulations so that those simulations are indipendent to each other. Then the initial densities are evolved up to $z = 0$. Projection of one of those density fields is plotted in Fig. II (a), in which the magnitude is the average of number of particles per cell over the dimension perpendicular to the paper.

III. RECONSTRUCTION ALGORITHM

We use the algorithm and numerical method called Adaptive Particle-mesh (APM), discribed in citebib:...,bib:... to reconstruct the density field. The basic idea is to build a PM scheme on a curvilinear coordinate system, in which the number of the particles per grid cell is set approximately constant. Consider a numerical grid of curvilinear coordinates $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$. In order to determine the physical position of each grid point, one needs to specify the Euclidean coordinate $\boldsymbol{x}(\boldsymbol{\xi}, t)$ as a function of grid position. In the Euclidean coordinate, the flat metric is Kronecker delta function δ_{ij}, while the

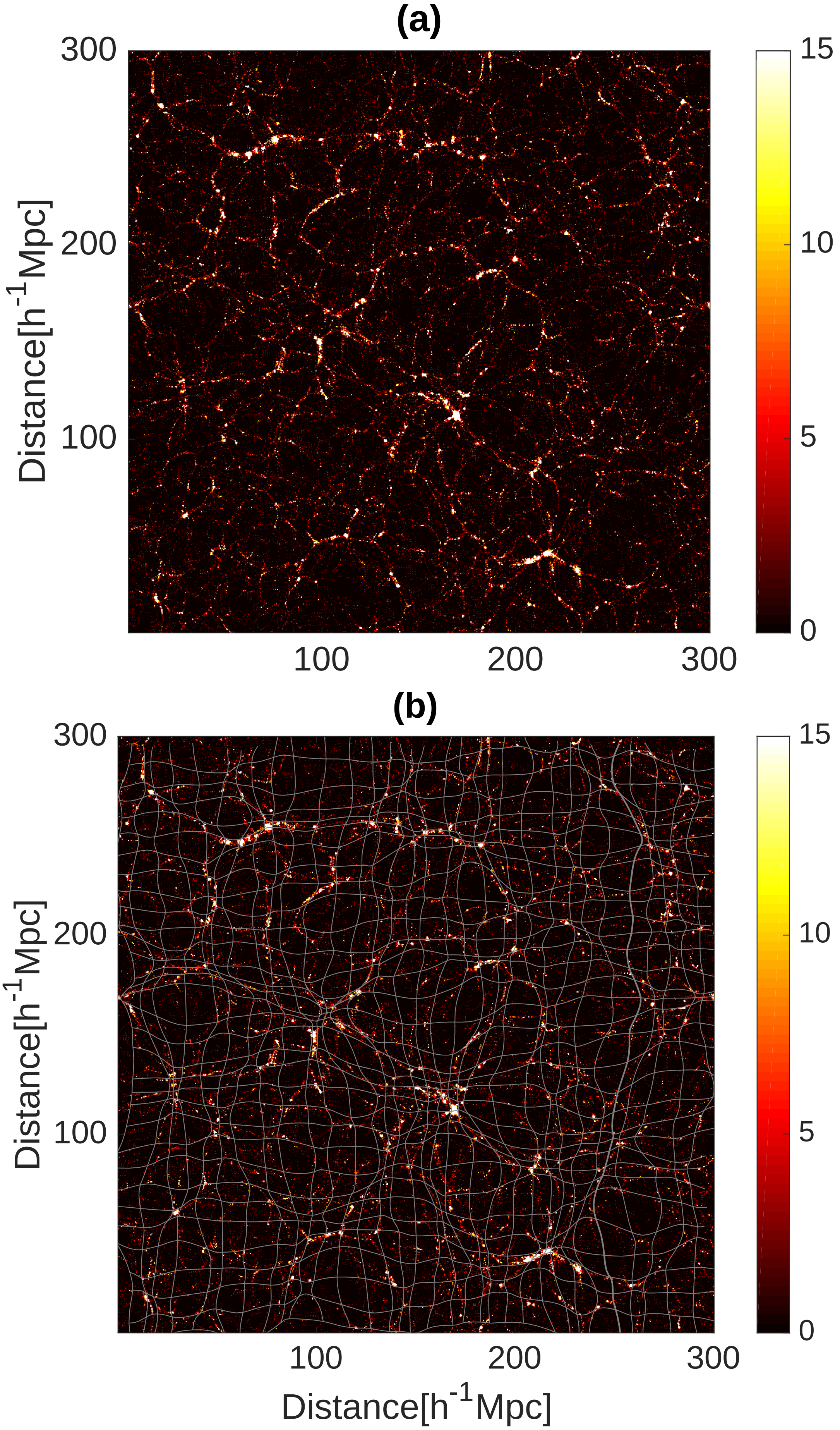


FIG. 1: (a) Map of a randomly selected density field from 139 N-body simulations, with a 300 h^{-1}Mpc width box and 1024^3 pixels. It's the projection along the axis perpendicular to the paper, and the magnitude is the average of number of particles per cell.(b) The density field and the deformed grids of a random selected layer of the selected density field in (a).

curvilinear metric is given by

$$g_{\mu\nu} = \frac{\partial x^i}{\partial \xi^\mu} \frac{\partial x^j}{\partial \xi^\nu} \delta_{ij}. \tag{1}$$

We use the convention that Latin indices denote Cartesian coordinate, while Greek indices denote the curvilinear grid coordinate. In principle, there are many different methods to connect the Cartesian coordiante and curvilinear coordinate of each grid cell. In APM method, the connetction is described by an irrotational deformation,

$$x^i = \xi^\mu \delta^i_\mu + \Delta x^i, \tag{2}$$

where

$$\Delta x^i = \frac{\partial \phi}{\partial \xi^\nu} \delta^i_\nu. \tag{3}$$

This choice of the deformation can minimize mesh distortion and twisting. ϕ is called the deformation potential, and Δx^i the lattice displacement. The deformation potential can be given in terms of the continuity equation in curvilinear coordinate,

$$\frac{\partial \sqrt{g}\rho}{\partial t} + \partial_\mu \left[\rho \sqrt{g} e^\mu_i \left(v^i - \Delta \dot{x}^i\right)\right] = 0 \tag{4}$$

where $\sqrt{g} \equiv (\partial x^i / \partial \xi^\alpha)$ is the volume element and $e^i_\mu = \partial \xi^\mu / \partial x^i$ is the triad. $\Delta \dot{x} = \delta^{i\nu} \partial_\nu dot\phi$ is chosen such that the first term in equation 4 is zero, resulting in a constant mass per volume element. And the velocity field divergence is replaced by the deviation density field $\Delta\rho = \bar{\rho} - \rho\sqrt{g}$, which ideally should be zero. Then the deformation potential is described in the elliptic equation,

$$\partial_\mu (\rho \sqrt{g} e^\mu_i \delta^{i\nu} \partial_\nu \Delta\phi) = \Delta\rho \tag{5}$$

The equation 5 can be solved using multigrid algorithm described in Ref. .... Then the displacement is given by the gradiant of the deformation potential as in 3. One layer of the deformed grids and the original density field of that layer is given in Fig. II (b). As expeted, there's no grid crossing after reconstruction.

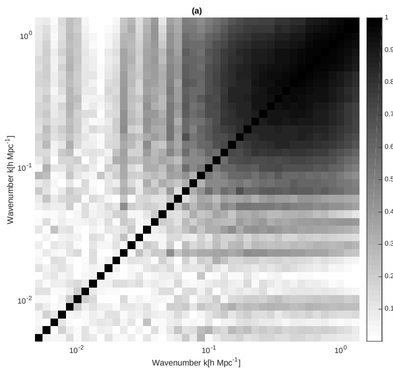
IV. POWER SPECTRA AND INFORMATION CONTENT

The power spectrum is the Fourier transform of the correlation function and measures the amoutn of clustering in the matter distribution in terms of the wavenumber k in unit of $h\mathrm{Mpc}^{-1}$,

$$\langle \delta(\boldsymbol{k})\, \delta(\boldsymbol{k'}) \rangle = (2\pi)^3 P(\boldsymbol{k})\, \hat{\delta}(\boldsymbol{k} - \boldsymbol{k'}), \tag{6}$$

where $\delta(\boldsymbol{k})$ is the density fluctuation in wave space, while $\hat{\delta}$ is the delta funciton. Of equal interest is Δ^2_k, the power spectrum in its dimensionless form, defined as

$$\Delta^2_k \equiv \frac{k^3 P(k)}{2\pi^2} \tag{7}$$



The power spectra of the mass distributions are calculated using the "Nearest Grid Point" (NGP) mass assignment scheme, which calculates the position of each particle based on which grid point it is nearest. In Fig.... we plot the mean power spectrum (and error bars) of 139 density fields and reconstruced deformation potentials. .....(needed?) To calculate the cumulative Fisher information of the density fields, the covariance matrix of the power spectra should be first given. Mathematically, the the covariance matrix is defined as

$$\mathrm{Cov}(k, k') \equiv \frac{1}{N-1} \sum_{i=1}^{N} \left[P_i(k) - \langle P(k) \rangle\right] \left[P_j(k') - \langle P(k') \rangle\right], \tag{8}$$

where angle brackets mean the expected values. The cross-correlation coefficient matrix, or for short, the correlation matrix, is a normalized version of the covariance matrix,

$$\mathrm{Corr}(k, k') = \frac{\mathrm{Cov}(k, k')}{\sqrt{\mathrm{Cov}(k, k)\, Cov(k', k')}}. \tag{9}$$

The corelation matrices for density fields from simulations and deformation potentials from reconstructions are shown in Fig. IVIV 2. For the original density fields, the linear regime, where $k < 0.1$, is diagonal, while in the non-linear regime, the power spectra of different k modes are strongly correlated by at least 60% . For the reconstructed deformation potential correlation matrix, however, the linear regime expand up to k 0.2. The correlation matrix is closer to that for the power spectra of linear density fields. The cumulative, or Fisher, information function of k_n is then defined as the sum of the elements of inverse of subsection of the normalized covariance matrix up to k_n scale

$$I(< k_n) = \sum_{i,j=1}^{n} \left[C^{-1}_{norm}(k_i, k_j)\right](i, j \le n), \tag{10}$$

where C^{-1}_{norm} is the normalized covariance matrix, defined as

$$C^{-1}_{norm}(k, k') = \frac{\mathrm{Cov}(k, k')}{\langle P(k) \rangle \langle P(k) \rangle}. \tag{11}$$

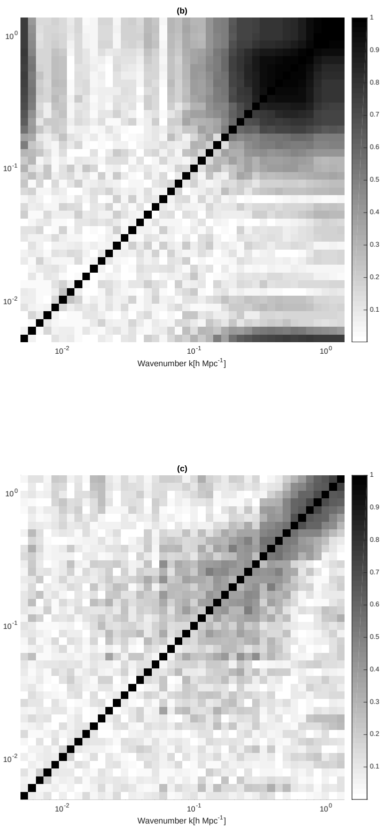


FIG. 2: Cross-correlation coefficient matrix as found from 139 power spectra of (**a**) the non-linear density field from simulation, (**b**) the deformation potential field from reconstruction, (**c**) the linear density field from simulation and (**d**) a normal random number field.

The signal-to-noise ratio, sometimes also called Fisher information, can be given by the inverse matrix of covariance. Since the signal-to-noise ratio was given in some work, we also present it for a better comparison.

$$\left(\frac{S}{N}\right)^2 (k_n) = \sum_{i,j=1}^{n} P_i \mathrm{Cov}^{-1}(i,j) P_j \tag{12}$$

As seen above, cumulative information is a measurement of the number of independent Fourier modes presented in a field up to a given k_n, which represents how linear a field is. We plot the cumulative information of the power specta of density fields from simulations and deformation potentials from reconstructions in Fig.3. In the translinear regime, where k 0.2, the cumulative information of the non-linear density field has a flat plateau. It indicates that there's nearly no independent information in the non-linear regime of the power spectrum. While the information curve of the reconstructed deformation potential keeps incresing at that point and reach it's plateau at k 0.5 up to a factor of 400, which is better than the result of any other existing reconstruction method??? One can also find that the reconstrcted Fisher information is higher even than that of linear density field in large at the scale from k 0.1 to k 1.(Unbelievable?)(Not knowing the situation in smaller scale so far) It indicates that APM method can strongly recover the lost information within this scale.

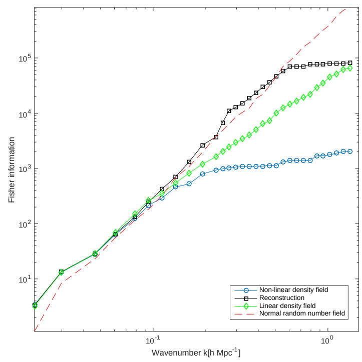


FIG. 3: (Color online)Cumulative information in the power spectra as a function of wavenumber. The black squares correspond to the non-linear density field by simulation; the blue cycles correspond to the the reconstructed deformation potential; the red dash line correspond to fields of normal random number.

V. CONCLUSION AND DISCUSSION

We use the code "CubeP3M" to generate 139 independent dark matter density fields, then give the reconstructed deformation potentials which are pure divergent using APM method. We analyze the power spectra of both the density fields and the deformation potentials, after which we give the cross correlation matrix. We find that the power spectra are highly correlated on small scales, since these scales are in non-linear regime. But after reconstruction, the strongly correlated regime shinks from (...) to (...). We also calculate the cumulative information, and find that the plateau of the reconstructed information curve in the tranlinear regime rises by a factor of 400.

[1] M. Viel, M. G. Haehnelt, and V. Springel, JCAP **1006**, 015 (2010), 1003.2422.
[2] Y. Ali-Haimoud and S. Bird, Mon. Not. Roy. Astron. Soc. **428**, 3375 (2012), 1209.0461.
[3] F. Führer and Y. Y. Y. Wong, JCAP **1503**, 046 (2015), 1412.2764.
[4] H. Dupuy and F. Bernardeau, JCAP **1401**, 030 (2014), 1311.5487.

[5] E. Massara, F. Villaescusa-Navarro, and M. Viel, JCAP **1412**, 053 (2014), 1410.6813.

[6] M. Shoji and E. Komatsu, Phys. Rev. **D81**, 123516 (2010), [Erratum: Phys. Rev.D82,089901(2010)], 1003.0942.

[7] D. Inman, J. D. Emberson, U.-L. Pen, A. Farchi, H.-R. Yu, and J. Harnois-Déraps, Phys. Rev. D **92**, 023502 (2015), 1503.07480.

[8] J. Harnois-Deraps, U.-L. Pen, I. T. Iliev, H. Merz, J. D. Emberson, and V. Desjacques, Mon. Not. Roy. Astron. Soc. **436**, 540 (2013), 1208.5098.

[9] H.-R. Yu, J. D. Emberson, D. Inman, T.-J. Zhang, U.-L. Pen, J. Harnois-Déraps, S. Yuan, H.-Y. Teng, H.-M. Zhu, X. Chen, et al., ArXiv e-prints (2016), 1609.08968.

[10] D. Blas, J. Lesgourgues, and T. Tram, JCAP **1107**, 034 (2011), 1104.2933.

[11] E. Bertschinger, arXiv preprint astro-ph/9503125 (1995).

[12] A. Ringwald and Y. Y. Y. Wong, JCAP **0412**, 005 (2004), hep-ph/0408241.

[13] J. Lesgourgues and T. Tram, JCAP **1109**, 032 (2011), 1104.2935.

[14] F. Villaescusa-Navarro, S. Bird, C. Pena-Garay, and M. Viel, JCAP **1303**, 019 (2013), 1212.4855.

[15] A. Banerjee and N. Dalal, ArXiv e-prints (2016), 1606.06167.

[16] C. Loken, D. Gruner, L. Groer, R. Peltier, N. Bunn, M. Craig, T. Henriques, J. Dempsey, C.-H. Yu, J. Chen, et al., in *Journal of Physics: Conference Series* (IOP Publishing, 2010), vol. 256, p. 012026.