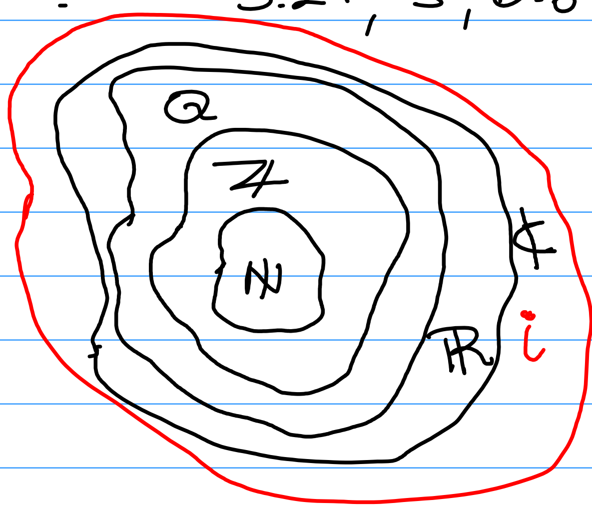
Introduction Complex Numbers

$\mathbb{N}$ Natural : $1, 2, 3, 4, \ldots$

$\mathbb{Z}$ Integer : Natural + 0 + Negative Integers

$\mathbb{Q}$ Rational : $-3/2, -2/2, -1/2, 5/3, \ldots$

$\mathbb{R}$ Real : $3.24, 5, 6.62, 1.000.000$



$$i^2 = -1 \Rightarrow \sqrt{-1} = i$$

Ex] $\underbrace{\sqrt{-25} = \sqrt{i^2 5^2} = \sqrt{(i5)^2} = i \cdot 5}$

Complex Numbers:

$$z = a + ib$$

$$a \rightarrow Re(z) \qquad b \rightarrow Im(z)$$

Ex] $z = 1 + 2i \rightarrow Re(z) = 1, \quad Im(z) = 2$

$z = 4i \rightarrow Re(z) = 0, \quad Im(z) = 4$

Arithmetics

1) Addition: $z_1 = a+ib$
$z_2 = c+id$

$$z_1+z_2 = (a+c)+i(b+d)$$

Ex] $z_1 = 1+i$, $z_2 = 3+2i$ $\Big\}$ $z_1+z_2 = (1+3)+i(2+1) =$

$= 4+i\cdot 3$

2) Substruction:

Ex] $z_1 = 2+i$, $z_2 = 1+2i$

$$z_1 - z_2 = (2-1)+(1-2)i = 1-i$$

3) Multiplication: $z_1 = a+ib$
$z_2 = c+id$

$(i^2) = -1$

$$z_1 \cdot z_2 = (a+ib)(c+id) = ac + iad + ibc + i^2 bd =$$

$$= ac - bd + i(ad + b\cdot c)$$

Ex] $\left.\begin{array}{l} z_1 = 1 + 2i \\ z_2 = 5 - i \end{array}\right\}$ $z_1 \cdot z_2 = (1+2i)(5-i) =$

$$= 5 - i + 10i + 2 = 7 - 9i$$

4) Division : $z_1 = a + ib$
$z_2 = c + id$

$$\frac{z_1}{z_2} = \frac{(a+ib)}{c+id}\,\frac{(c-id)}{(c-id)} =$$

$$\boxed{\bar{z}_2 = c - id}$$

$$= \frac{ac - iad + ibc + bd}{c^2 - \cancel{icd} + \cancel{idc} + d^2} =$$

$$= \frac{(ac+bd) + i(-ad+bc)}{c^2+d^2}$$

Ex] $\left.\begin{array}{l} z_1 = 3 + 4i \\ z_2 = 1 + 3i \end{array}\right\}$

$$\frac{z_1}{z_2} = \frac{3+4i}{(1+3i)}\,\frac{(1-3i)}{(1-3i)} = \frac{3 - 9i + 4i + 12}{1^2 + 3^2} =$$

$$= \underbrace{\frac{15}{4}} - \underbrace{\frac{5i}{4}} =$$

Complex Conjugate

Ex) $z = a + ib$

$\bar{z} = a - ib$

1) $z + \bar{z} = (a+a) + i\overset{0}{(\cancel{b}-b)} = 2a = 2\,\mathrm{Re}(z) \Rightarrow$

$\Rightarrow \boxed{\mathrm{Re}(z) = \frac{z+\bar{z}}{2}}$

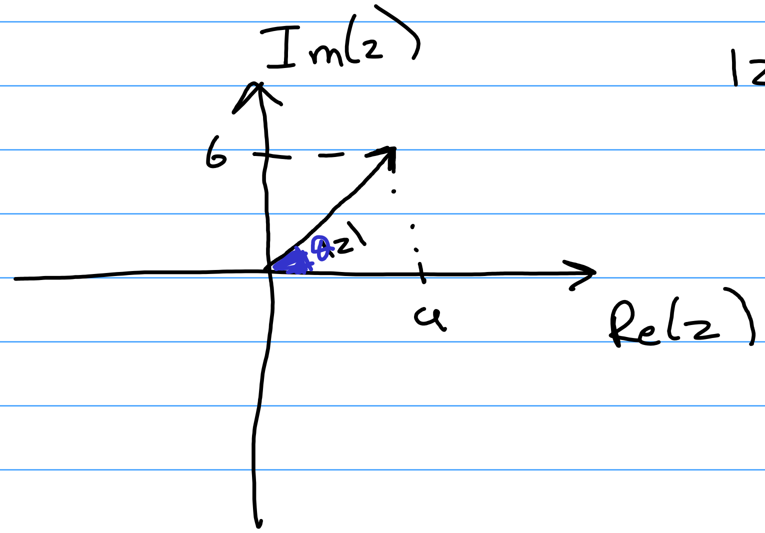
2) $z - \bar{z} = (a-a) + i(b-(-b)) = 2\cdot ib = 2i\,\mathrm{Im}(z) \Rightarrow$

$\Rightarrow \boxed{\mathrm{Im}(z) = \frac{z-\bar{z}}{2i}}$

3) $z\cdot\bar{z} = (a+ib)(a-ib) = a^2 - iab + iab + b^2 =$

$= \boxed{a^2 + b^2 = |z|^2}$

$z = a + ib$



$|z| = \sqrt{a^2 + b^2}$

$\sin\theta = \dfrac{b}{|z|}$ $\qquad\qquad$ $\cos\theta = \dfrac{a}{|z|}$

$\Rightarrow \quad b = |z|\sin\theta \qquad\qquad a = |z|\cos\theta$

$z = a + ib = |z|\cos\theta + i|z|\sin\theta =$

$= |z|(\cos\theta + i\sin\theta)$

Euler's Formula : $\quad e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$\Rightarrow \quad \boxed{z = |z|\, e^{i\theta}}$