The advertising sales response curve

An empirical analysis in the fast moving consumer goods

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31-07-2015

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Introduction

Companies operate in a competitive environment where their limited resources have to be allocated optimally in order to survive. Managers constantly seek new opportunities to reduce their costs. While advertising is an important factor in the success or failure of a company's marketing strategy, it also is a cost. Therefore the knowledge of the return of advertising spending is of paramount importance for managers. The increased availability of data and propagation of statistical analysis techniques has led to an increase of quantifying the effect of advertising on sales.

Little (1979) proposed in his seminal paper that the shape of the advertising sales response curve is concave or S-shaped. Dubé et al (2005) show that in the case of an S-shaped response curve the optimal advertising schedule is pulsing: high amounts of advertising pressure in a short period of time. A complicating factor is the delayed effect of advertising; consumers remember past advertising and may act upon it in future periods. Broadbent (1979,1983) developed the elegant concept of adstock, which incorporates the carry-over effect in a simple manner. He posits that advertising exposure builds awareness, while in the absence of advertising adstock will decay at constant rate.

Dekimpe and Hanssens (2007) describe three characteristics which sales response functions might exhibit: diminishing returns, a ceiling effect and a threshold effect. Diminishing returns occur, because for example consumer cease to be responsive when they have learned the basic message contained in the advertising. A ceiling effect can occur because there is natural limit of consumers that can be reached. A threshold effect occurs when advertising below a certain level has no or little impact. The existing literature is inconclusive regarding the shape of the advertising sales response curve. The predominant response function is concave, but threshold effects may exist.

Three adstock models will be fitted to data from a confectionaries company. The first model, the logarithmic adstock model only incorporates diminishing returns to advertising by concavity of the adstock function. The second model, the negative exponential adstock model is bounded from above and thus incorporates a ceiling effect. The third model, the logistic adstock model, extends the negative exponential model with a threshold effect. The main focus of this thesis is which adstock model is most appropriate for the advertising sales response curve.

The thesis is structured as follows; the first chapter provides the theoretical framework of this thesis. The second chapter is a description of the data. In chapter 3 the estimation results of the adstock models are presented, assuming exogeneity of advertising in the single equation models in chapter 3. In chapter 4, this assumption is tested by estimating a vector autoregressive model and subsequently performing Granger causality tests. In chapter 5 the advertising sales response curves are estimated semiparametric. Chapter 6 summarizes the main results.

Chapter 1: Theory

Marketing Mix Modeling

Marketing mix modeling is the art of quantifying the effect of marketing on sales. The modeler has to choose and adapt the relevant variables using the appropriate statistical analytical techniques and select an outcome metric, such as revenue, volume sales or profits. The statistical techniques were developed by econometricians and were first applied to consumer packaged goods (CPGs), since there was good access to data on sales and marketing variables. Borden first used the term as follows (Culliton, 1948):

> *"An executive is a mixer of ingredients, who sometimes follows a recipe as he goes along, sometimes adapts a recipe to the ingredients immediately available, and sometimes experiments with or invents ingredients no one else has tried."*

Borden (1964) identifies twelve elements of the marketing mix: product, pricing, branding, distribution, personal selling, advertising, promotions, packaging, display, servicing, physical handling, and fact-finding. MacCarthy (1964) simplified Borden's work into the well-known four P's of marketing: price, promotion (advertising), product and place (distribution).

The effect of advertising on sales has been the subject of many econometric studies. Initially, sales was treated as a function of advertising expenditures in single-equation models (e.g. Lambin and Palda, 1969, Lambin, Naert and Bultez, 1975). These models treat advertising as exogenous, later this assumptions was relaxed in subsequent simultaneous equation models, starting with Bass (1969) and including work by Bass and Parsons (1969) and Hanssens (1980). More recently VAR models are used to estimate the effect of advertising on sales (Dekimpe and Hanssens, 1995; 1999). The latter allows for feedback effects (past own performance helps explain current spending), but also incorporating competitive interactions.

For the single equation models advertising is assumed to be exogenous, since the advertisement campaigns are determined on a yearly basis, where the data available for this study has a weekly interval. However the equation should satisfy diminishing returns to advertising for several reasons. First, consumers cease to be responsive once they have learned the basic message contained in the advertising (saturation effect), markets deplete as successful advertising causes purchasing which then removes the buyers from the market, at least temporarily (market-depletion effects), and, finally, there are natural ceilings to the number or percent of customers that can be targeted or reached (ceiling effect). Diminishing returns imply that, while sales still increase in advertising support, each additional unit of advertising brings less in incremental sales than the previous unit did.

As a consequence, the basic advertising-response function is concave, reflecting diminishing returns. A simple linear specification would not accommodate this, as each additional unit of advertising spending would have the same impact on sales. Instead we use the following multiplicative model in Equation 1.1:

$$S_t = e^c A_t^\beta X_t^\gamma Z_t^\delta e^{u_t} \qquad (1.1)$$

where S_t is sales or another performance metric at time *t*, *c* is the base level of sales; A_t stands for the advertising pressure at time *t*, X_t are other marketing variables such as price and distribution; Z_t corresponds to environmental factors; and u_t is the error term. We expect $0 < \beta < 1$ in estimation, a condition that results in concavity.

The model implies that with zero advertising comes zero sales, and with infinite advertising comes infinite sales. Zero-sales is usually unrealistic, and this is addressed by adding a small constant to the advertising term. In addition, there is always a limit or ceiling to sales, usually determined by the prevailing market conditions. While there are other ways to represent concavity (see e.g. Hanssens et al., 2001:100-2), the multiplicative function is particularly appealing as it recognizes that marketing-mix effects interact with each other. In addition, taking the natural logarithm linearizes the model:

$$\log(S_t) = c + \beta \log(A_t) + \gamma \log(X_t) + \delta \log(Z_t) + u_t, \qquad (1.2)$$

making the model easier to estimable and the parameters are easily interpreted as elasticities.

In some cases, the response is S-Shaped, i.e. there is a minimum or threshold-level of ad spend below which there is little or no impact, followed by a range of advertising spending with rapidly increasing sales response. At even higher spending levels, past a certain inflection point, the usual diminishing returns appear. For all practical purposes, concavity and S-shape are sufficient functional forms to capture the essence of advertising response. In some cases, the response may be even simpler, if all the advertising spending observations lie in a restricted range of the data, the response function may well be approximated by a linear function.

The Hierarchy of Effects Model

The predominant model of how advertising works is the Hierarchy of Effects Model (HEM) of Lavidge and Steiner (1961). In their model consumers have to go sequentially through a series of six stages. In the first step consumers become aware of the product. This could be because the consumer saw an advertisement, although a consumer does not necessarily becomes aware after seeing an advertisement. The consumer might forget the message contained in the advertisement. Second, consumers gain knowledge about the product. These two steps together form the cognitive part of the process.

The third phase is the affective phase, which consists of three steps. The first step of the affective phase is the liking of a particular brand. The consumer must have a positive attitude regarding the brand, otherwise it will not buy the product. The fourth step is the emergence of a preference of the consumer for a specific brand. The fifth step is the conviction of the consumer of desiring the product. And the final step is the actual buying of the product, which takes place in the conative stage or behavioral stage of the process.

Figure 1.1: Hierarchy of Effects Model

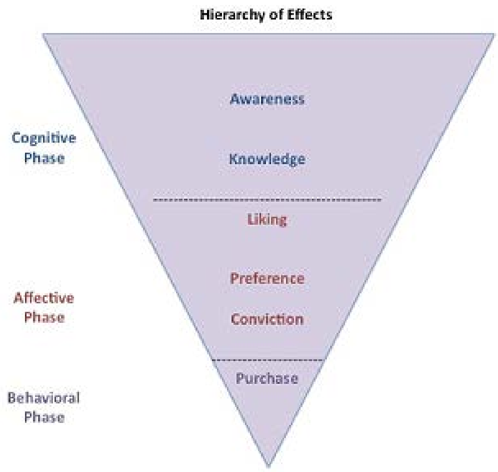


Figure 1.1 is a graphical illustration of the Hierarchy of Effects Model. It is shaped like a pyramid because along the process the brand will lose potential consumers. An advertisement can affect a consumer in all possible stages. For example a “Buy now!”

advertisement, this induces the consumer to go from the fifth step to the sixth step. Or creating a positive attitude regarding the brand, like the following slogan of a confectionary brand: “Kids and grown-ups love it so – the happy world of Haribo”. But most importantly, the basic premise of the Hierarchy of Effects Model is that advertising effects occur over a period of time.

Adstock

The cornerstone of the econometric modeling of this delayed – or carry-over effect is the Adstock concept developed by Broadbent (1979, 1984). Broadbent posits that each advertising exposure builds awareness, while in the absence of advertising Adstock will decay at a constant rate (see e.g. Dekimpe and Hanssens, 2007). It has been used in studies on e.g. advertising awareness (Brown, 1986), television advertising effectiveness (Tellis and Weiss, 1995), television scheduling (Broadbent et al., 1997; Ephron and MacDonald, 2002), trial of new products (Steenkamp and Gielens, 2003), product-harm crises (Cleeren et al., 2007), competitive advertising interference (Danaher et al., 2008) and timing and magnitude of advertising (Gijsenberg et al., 2009, 2011).

In line with the definition of Broadbent (1984), the aforementioned studies have the operationalization of the Adstock concept in common, namely the exponential model in Equation 1.3:

$$Adstock_t = (1-\lambda)A_t + \lambda Adstock_{t-1}, \quad (1.3)$$

where A_t is advertising at time t and λ a smoothing parameter bounded between 0 and 1. Hanssens et al. (2001, 142-52) exhibit several other methods for modeling advertising carryover. However, Danaher et al. (2008) argue that the exponential model is most compatible with the multiplicative model. It is easy to shat that Equation 1.3 implies that

$$Adstock_t = (1-\lambda)\sum_{i=1}^{t} \lambda^{t-i} A_{t-i}, \quad (1.4)$$

which shows that Adstock is a geometrically weighted average of current and past advertising.

More recently, various other operationalization of the Adstock concept are discussed and evaluated by respectively Joseph (2006) and Raj et al (2012). These operationalizations are not necessarily used within the multiplicative specification, but in a simple linear – or log-linear model. Starting with the most basic operationalization, the simple decay-effect model in Equation 1.5 (Broadbent, 1979):

$$Adstock_t = A_t + \lambda Adstock_{t-1}. \tag{1.5}$$

Its drawback when implemented in a simple linear model or log-linear model is that it only captures the dynamic effect of advertising. A simple model that would also accommodate the diminishing returns effect of advertising is the log decay model in Equation 1.6:

$$Adstock_t = \log A_t + \lambda Adstock_{t-1}. \tag{1.6}$$

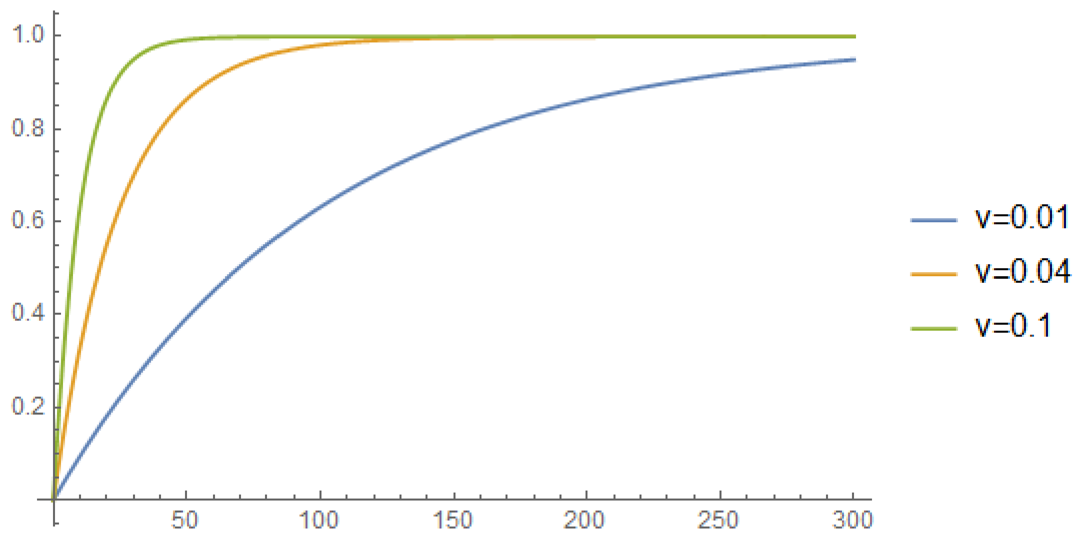
This model has two limitations; it does not specify any threshold effect of advertising on awareness and it is rather inflexible, as it does not allow for varying saturation levels (Joseph, 2006).

A model which allows for different saturation levels or learning rates is the negative exponential model in Equation 1.7:

$$Adstock_t = 1 - e^{-vA_t} + \lambda Adstock_{t-1}. \tag{1.7}$$

The parameter *v* accommodates different saturation levels. The ceiling effect is also accounted for, since it is a bounded function. For values of *v* close to zero it will represent constant returns or no learning at all. This is illustrated in Figure 1.2.

Figure 1.2: Negative exponential function

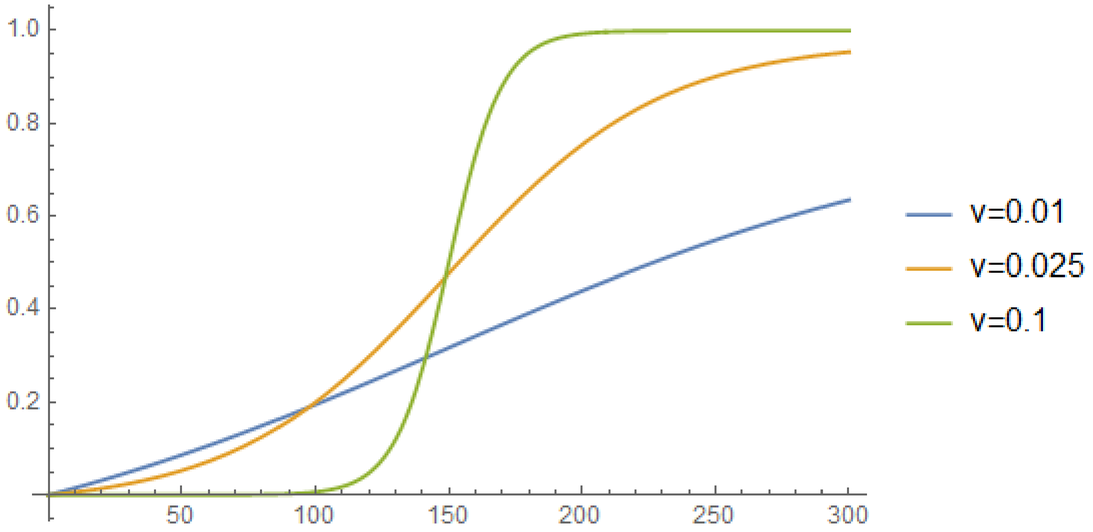


As stated before, the response of sales to advertising might also show threshold effect. This phenomena is accounted for by the logistic model in Equation 1.8:

$$Adstock_t = \frac{1}{1+e^{-v(A_t-I)}} - \frac{1}{1+e^{-vI}} + \lambda Adstock_{t-1}. \quad (1.8)$$

The parameter v performs the same function as in the negative exponential model, which is illustrated in Figure 1.3.

Figure 1.3: Logistic function



Chapter 2: Data

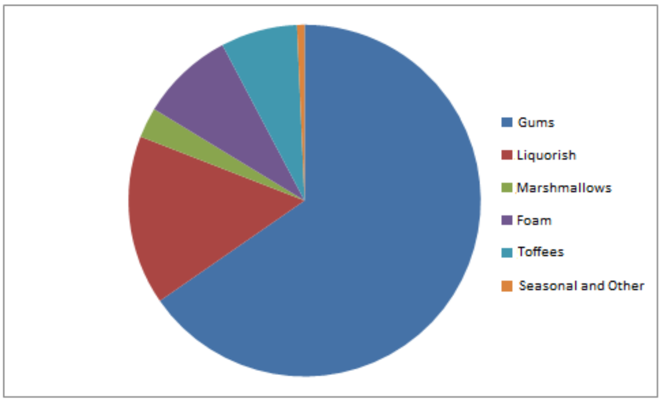
Dependent variable: sold volume

Weekly sales data is available from the first week of 2011 until week 40 of 2013 from a leading confectionary company in The Netherlands, which will be called The Confectionary Company. The sales data are only from supermarkets, which constitutes between 50% and 60% of the total sales during the period under study. Other important selling points are for example: gas stations, sports canteens and vending machines at schools. The revenues as well as the sales in volume are available. But revenues as dependent variable may create problems in a market response function (Farris, Parry & Ailwadi, 1992; Jacobson & Aaker, 1993).

$$R \equiv P \times Q = f(P) \tag{2.9}$$

As we can see from Equation 2.1, when price is included as an explanatory variable and revenues as dependent variable this causes spurious correlation, because price is on both sides of the equation. The same effect occurs to a lesser degree when there is any monetary variable, say advertising expenditures, on the right-hand side of the equation. Then inflation can impact both sided of the equation. Therefore the dependent variable will be the volume sold.

Figure 2.3: Distribution of the categories



The sales are divided into 6 categories: liquorish, gums, marshmallows, foam, toffees, and seasonal & other. In the period for which the data is available, gums is the largest category in volume, which accounts for 65% of sales. Liquorish is the second largest category with 15% of sales, foam accounts for 9% and toffees follow with 7% of sales. Marshmallows and the combined category of seasonal & other represent respectively 3% and 1% of volume sold. In Figure 2.3 the distribution of volume sold over the categories is presented graphically. The category toffees is sold under a different brand name than the other categories. The categories seasonal and other are not sold continuously throughout the year. Therefore these three categories will be discarded in further analysis. The categories foam and marshmallows are added together, otherwise there would be too much difference between the sizes of the categories. The volume sold of the categories gums, liquorish and foam & marshmallows are plotted in Figure 2.4 till 2.6, respectively.

Figure 2.4: Sold volume gums

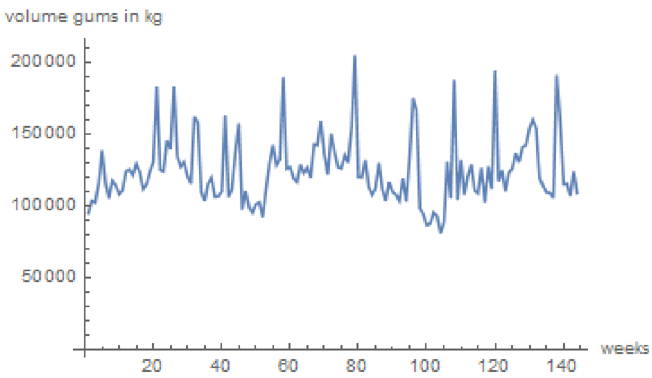


Figure 2.5: Sold volume liquorish

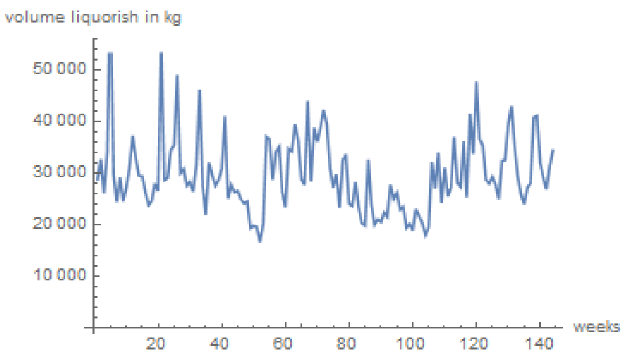
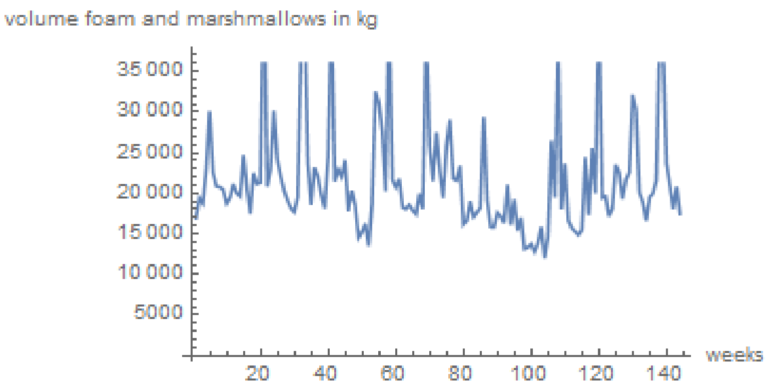


Figure 2.6: Sold volume foam & marshmallows



The Confectionary Company is the market leader in the category gums with 33% of market share, the second largest manufacturer has a market share of 21% and the third largest manufacturer has a market share of 5%. In the market for liquorish they are the third largest manufacturer with a market share of 7%. The market leader has a share of 19%. In the category foam and marshmallows they are the second largest manufacturer with 11% of market share, the market leader has 12% market share and the third largest manufacturer has 9% of market share. The house brands of the supermarkets also have significant share in the categories gums, liquorish and foam & marshmallows, with respectively 16%, 19% and 17% of market share.

Marketing-mix variables

Price

One of the most important determinants of sales is the price of the product. The price elasticity of price on own brand sales is negative and elastic (Hanssens, Parsons & Schultz, 2001). The Confectionary Company already sells 106 products in the category gums. They come in a variety of shapes, for example: bears, cherries, frogs and peaches. And there are nine different packagings to choose from, from little bags of 60 grams to large buckets of 2 kilograms. Therefore there is not one price which could be included as an explanatory variable. If price would be defined as revenues divided by volume, then volume would appear on both sides of the equation. As discussed earlier, this would lead to spurious correlation.

Some managers believe that the minimum price change to induce a change in the consumers purchasing behavior is approximately 15% (Della Bitta and Monroe, 1980). Gupta and Cooper (1992) indeed find in an experiment that consumers do not change their intentions unless the promotional discount exceeds a threshold level of 15%. Leeflang et al. (2001) also find threshold effects using a semiparametric approach for the effect of price on sales. During the period from week 1 of 2011 until week 40 of 2013 the average price of gums, weighted by volume sold, has increased by 5.95%. According to the CBS inflation during this period was 7.79%, thus the real price for gums has decreased with 1.84%. Since this change is small, much smaller than 15%, price will not be included as an explanatory variable. The figures are similar for liquorish and foam & marshmallows.

Trade promotions

Trade promotions are an effective instrument in order to increase sales substantially in the short term (Blattberg, Briesch & Fox, 1995). The main types of trade promotions are display – and feature advertising with or without temporary price reductions. Display advertising are in-store promotional fixtures such as point-of-sale displays, which are located near cash registers to encourage impulse buying. Another example of display advertising is feature displays, which can be located at the end of an aisle to draw attention to a product. Feature advertising is advertisements as in-store flyers, store magazines local and newspapers. Display – and feature advertising are almost always used together with temporary price reductions (Horváth et al., 2005).

Unfortunately there is no detailed data available of which products were on offer that week. There is also no data available on the size of the price reduction and if the price reduction was used together with display and/or feature advertising. However, we do know if there were gums on offer in the stores of a particular retailer. The effect of trade promotions will be modeled by a retailer specific dummy variable.

Distribution

The third marketing-mix variable which will be included as an explanatory variable is the distribution of gums. Increasing distribution (availability) increases sales. It is one of the strongest drivers of sales (Hanssens, Parsons & Schultz, 2001). This empirical generalization has been established for some time (Nutall, 1965; Farley & Leavitt, 1968; Parsons, 1974, Leone & Schultz, 1980; Reibstein & Farris, 1995). The distribution number is defined as the percentage of stores of the retailer carrying the product weighted by their market share. Like price, there are 106 different weekly distribution variables, therefore distribution will be a weighted average of the product specific distribution figures. The weights will be the yearly volume sold of the product.

Advertising

One of the first marketing generalizations is that the short-term elasticity of advertising on own brand sales of frequently purchased consumer goods is positive but low (Leone &

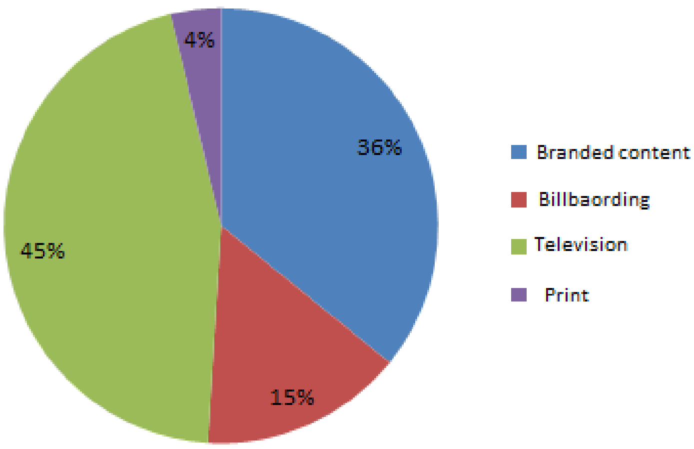
Schultz, 1980). This generalization is further supported by the literature survey of Aaker and Carman (1982) and by the meta-analysis of Assmus, Farley and Lehmann (1984). It is one of the weakest marketing-mix instruments, however that does not imply that advertising is the least profitable instrument (Jones, 1990). The average long-term effect of successful advertising spending is approximately double its initial spending. The 90% duration interval for advertising of mature, frequently purchased, low-prices consumer goods is brief, averaging between six and nine months (Clarke, 1976).

Advertising exposure is measured in Gross Rating Points (GRPs), a term used in marketing to measure the size of an advertisement. GRPs quantify impressions of the target audience as a percentage as follows:

$$GRPs(\%) = Reach(\%) \times Average\ frequency \qquad (2.2)$$

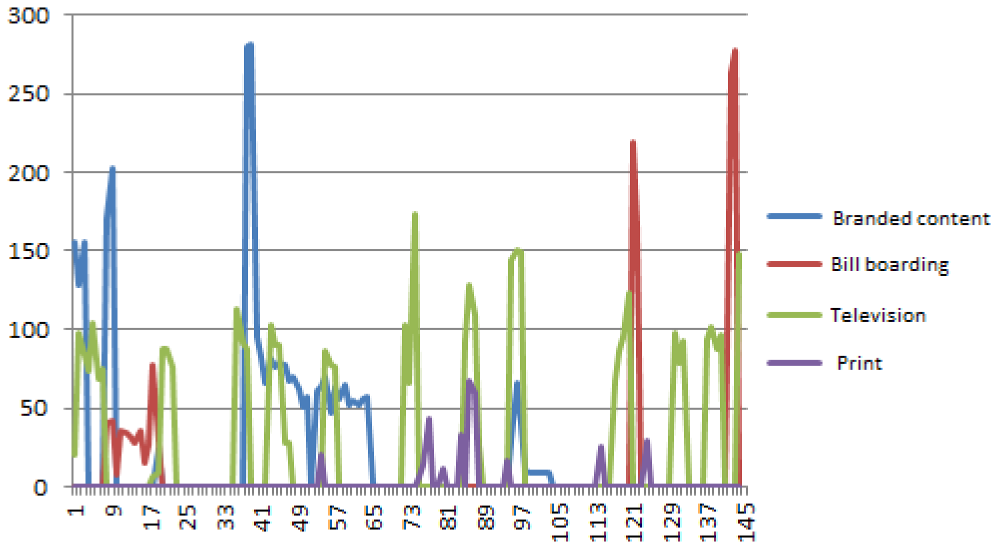
An advertisement exposure of 100 GRPs could mean that the 100% of the target audience is reached once or 50% of the target audience is reached twice or 25% of the audience is reached four times. Note that the probability of seeing the advertisement once is larger than seeing it twice and the probability of seeing the advertisement twice is larger than seeing the advertisement three times, and so on. A mixture is more likely, for intance a distribution where 45% of the target audience is reached once, 20% is reached twice and 5% of the target audience is reached three times. The media channels through which is advertised are television, print, branded content and billboards. The distribution of the total GRPs deployed over the media channels is presented in Figure 2.7

Figure 2.7: Distribution of advertising over the various channels



There are three basic patterns to allocate advertising budgets over time. First, burst or pulsing, where advertising only appears one, two or three times a year. Dubé, Hitsch and Manchanda (2005) show that the optimal schedule over time is pulsing if the sales response function to advertising is S-shaped. The second advertising schedule over time is flighting, where advertising is on for several consecutive weeks, followed by several weeks of no advertising. The third advertising schedule is continuous with advertising, even at low weight, nearly all the time (Hanssens, Parsons & Schultz, 2001). Figure 2.8 shows the GRPs deployed through the various media channels from week 1 in 2011 until week 40 in 2013.

Figure 2.8: Advertising



The advertising schedule of The Confectionary Company is in most congruence with a flighting schedule.

Macro-economic and climatological variables

Macro-economic indicator variables such as economic climate, consumer confidence and propensity to buy will be added to the model in order to determine whether they have a significant impact on the sales of confectionaries. Also climatological variables such average temperature, hours of sun and the amount of rainfall will be tested for a significant impact on sales.

Chapter 3

§3.1 Method: Single equation models

A general distributed lag model for market response is the Autoregressive Distributed Lag Model (ADL) (Tellis, Chandy and Thaivanich, 2000) presented in Equation 7. While advertising can influence a consumer's brand choice and may create the initial purchase, consumers will only buy a brand again only if they are satisfied with the product. In order to capture this purchase feedback effect, an autoregressive (AR) process for sales will be added to the linearized multiplicative model in Equation 2.2 with a moving average (MA) process for advertising. This results in Equation 3.1:

$$\log(S_t) = \alpha + \sum_{i=0}^{q} \beta_i \log(A_{t-i}) + \sum_{i=1}^{p} \gamma_i \log(S_{t-i}) + \delta \log(X_t) + u_t, \qquad (3.1)$$

where S_t is the volume sold in week t and A_t is the advertising level measured in Gross Rating Points in week t. Since we cannot take the log of zero, we have added 1 unit to the advertising variable in all weeks. X_t contains the exogenous variables; such as the marketing mix variables and environmental variables.

The above model will be compared with three adstock models, which results in Equation 3.2:

$$log(S_t) = f_t = f\big(\alpha, \beta, \gamma, \delta, Adstock_t, S_t, \ldots, S_{t-p}, X_t, \lambda, v, I\big) + u_t$$

$$= \alpha + \beta Adstock_t(A_t, \lambda, v, I) + \sum_{i=1}^{p} \gamma_i \, log(S_{t-i}) + \delta \, log(X_t) + u_t. \qquad (3.2)$$

The first adstock model is the logarithmic transformation in Equation 3.3, which only takes the diminishing effect of advertising on sales into account. The only difference with the benchmark model is the way the delayed effect is included in the model.

$$Adstock_t(A_t, \lambda) = \log(A_t) + \lambda Adstock_{t-1}(A_{t-1}, \lambda) \qquad (3.3)$$

The negative exponential model in Equation 3.4 not only accounts for diminishing returns, but also accounts for a ceiling effect.

$$Adstock_t(A_t, \lambda, v) = 1 - e^{-vA_t} + \lambda Adstock_{t-1}(A_{t-1}, \lambda, v) \qquad (3.4)$$

And Equation 3.5 incorporates a third effect, namely a threshold effect:

$$Adstock_t(A_t, \lambda, v, I) = \frac{1}{1+e^{-v(A_t - I)}} - \frac{1}{1+e^{vI}} + \lambda Adstock_{t-1}(A_{t-1}, \lambda, v, I). \quad (3.5)$$

The parameters of the model of the adstock model in Equation 3.2 is estimated by minimizing the residual sum of squares formulated in Equation 3.6, using the function fmincon in Matlab. Fmincon is a constrained optimization function in Matlab. The residual sum of squared residuals will be minimized, subjected to various constraints, depending on the adstock model:

$$SSR(\beta, \gamma, \lambda) = \sum_{t=1}^{T} (log(S_t) - \beta Adstock_t(A_t, \lambda, v, I) - \sum_{i=1}^{p} \gamma_i \, log(S_{t-i}) - \gamma \, log(X_t))^2$$
$$subject\ to: 0 \leq \lambda < 1 \quad (3.6)$$

The t- and F-tests are based on $s^2(X'X)^{-1}$ with

$$s^2 = \frac{1}{T-k} \sum_{t=1}^{T} \hat{u}_t{}^2, \quad (3.7)$$

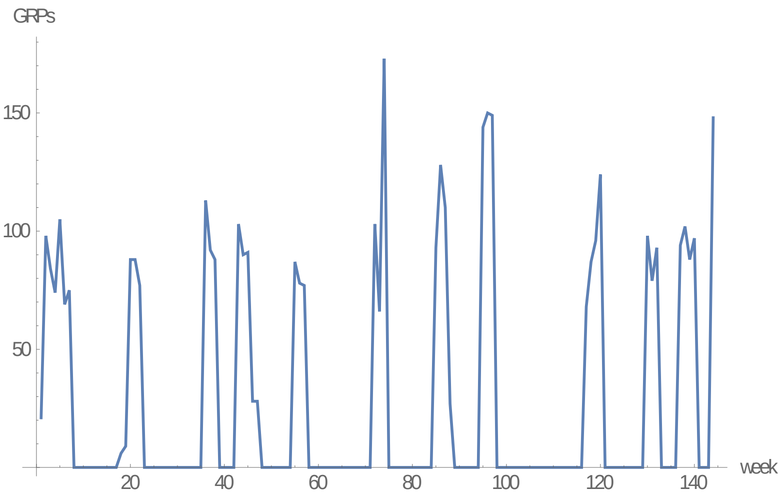
where $\hat{u}_t$ is the residual series of the nonlinear least squares estimation of Equation 3.2 and *k* is the number of coefficients estimated in Equation 3.2. Here *X* is defined as follows:

$$X = \begin{pmatrix} \frac{\partial f_1}{\partial \alpha} & \frac{\partial f_1}{\partial \beta} & \frac{\partial f_1}{\partial \gamma'} & \frac{\partial f_1}{\partial \delta'} & \frac{\partial f_1}{\partial \lambda} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial I} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_T}{\partial \alpha} & \frac{\partial f_T}{\partial \beta} & \frac{\partial f_T}{\partial \gamma'} & \frac{\partial f_T}{\partial \delta'} & \frac{\partial f_T}{\partial \lambda} & \frac{\partial f_T}{\partial v} & \frac{\partial f_T}{\partial I} \end{pmatrix}, \quad (3.8)$$

where the partial derivatives with respect to *v* applies for the negative exponential and logistic model and the partial derivative with respect to *I* only applies for the logistic adstock model (Heij et al., 2004).

There are three basic types of basic advertising schemes. A *burst* schedule, where only one, two or three times a year is advertised; a *flighting* schedule with advertising being on for several weeks, followed by several weeks off; and the third basic pattern is *continuous* with advertising being shown, even at a low weight nearly all the time. This pattern is also called ‘drip’ (Hanssens, Parsons and Schultz, 2001, 57). The television advertising schedule in Figure 3.1 is an example of a flighting scheme.

Figure 3.1:Television advertising



In Figure 3.2 the three different adstock functions are plotted. The three graphs differ most in shape for GRP levels below the inflection point *I* of the logistic function. The logarithmic function increases fast compared with the other two functions. The shapes of the negative exponential function and logistic function are quite different, this is due to the chosen parameters of the functions. If the saturation parameter of the logistic function was set to 0.08 and the inflection point was set at 25 GRPs, the negative exponential function and the logistic function would be more alike.

Figure 3.2:Adstock functions

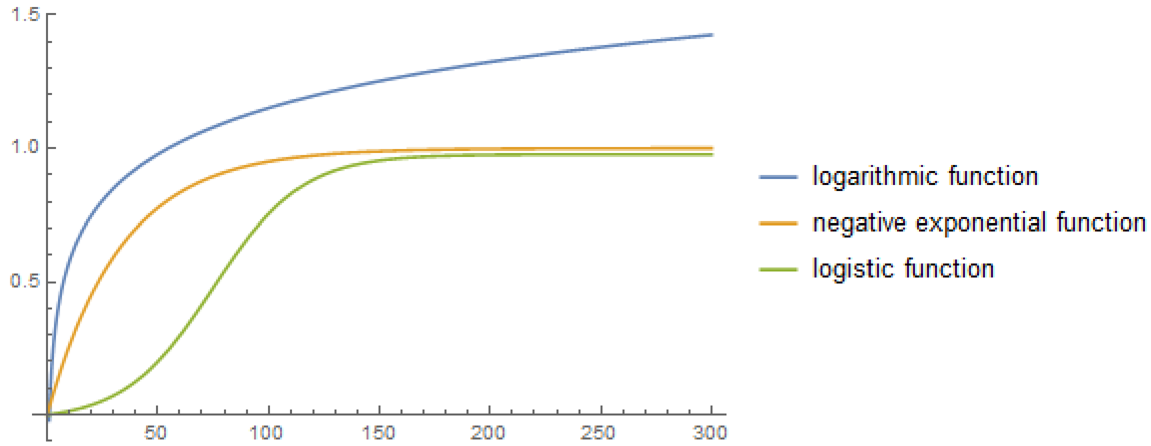
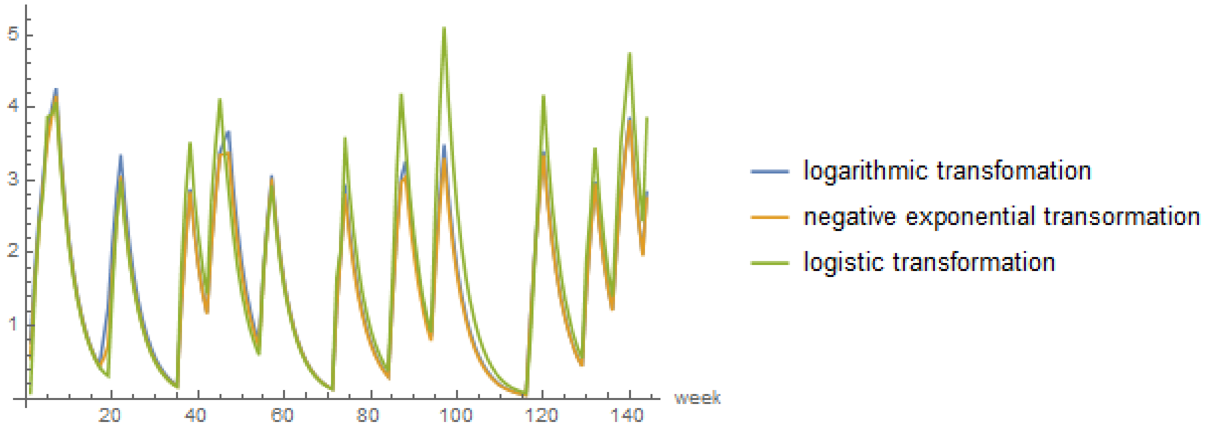
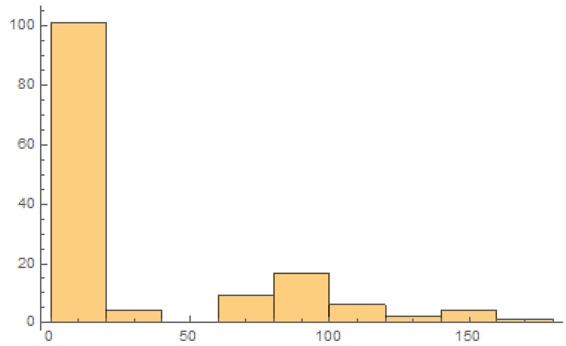


Figure 3.3: Adstock transformations



The three adstock transformations of the deployed amount of GRPs through television are illustrated in Figure 3.3. All three are plotted using a decay rate of 0.8. This value was found in a study of household-products group by (George, Mercer, and Wilson, 1996). The adstock transformations are scaled in order to compare the shape of the three adstock transformations. The logarithmic adstock transformation and negative exponential adstock transformation practically coincide. The logistic adstock transformation also does not differ much in shape, but creates higher peaks. This can be explained by the distribution of the deployed amount of GRPs, see Figure 3.4.

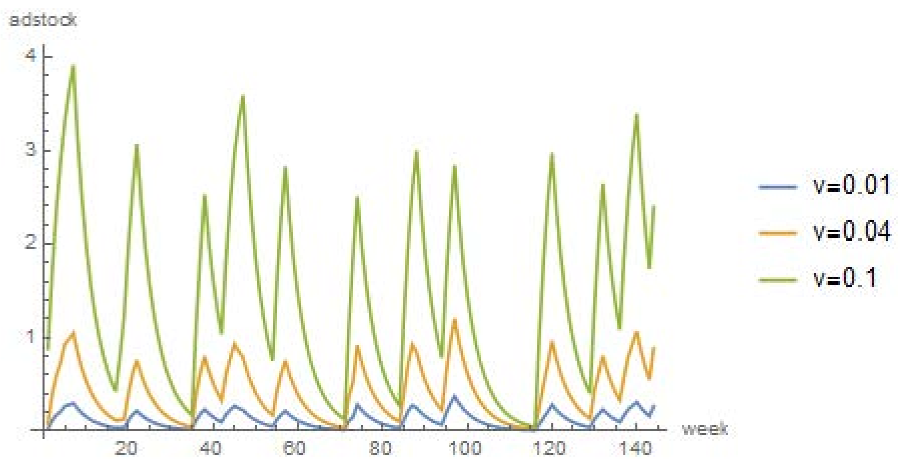
Figure 3.4: GRPs of television



In 99 of the 144 weeks there was no advertising on television. Of the remaining 45 weeks, in 38 of those weeks, advertising pressure was between 50 and 150 GRPs. The lower part of the graph of the three functions, where they differ most in shape, is hardly used.

To illustrate the effect of the saturation parameter of the negative exponential adstock model a few graphs are plotted in Figure 3.5. The saturation parameter v mostly affects the adstock level, for high values of v the impact of the deployed amount of GRPs is larger. For example, when the saturation parameter *v* equals 0.1, the impact of deploying 50 GRPs is already 0.99. The shape becomes spikier for higher levels of saturation. The overall shape of the adstock transformation is the same for all three saturation levels.

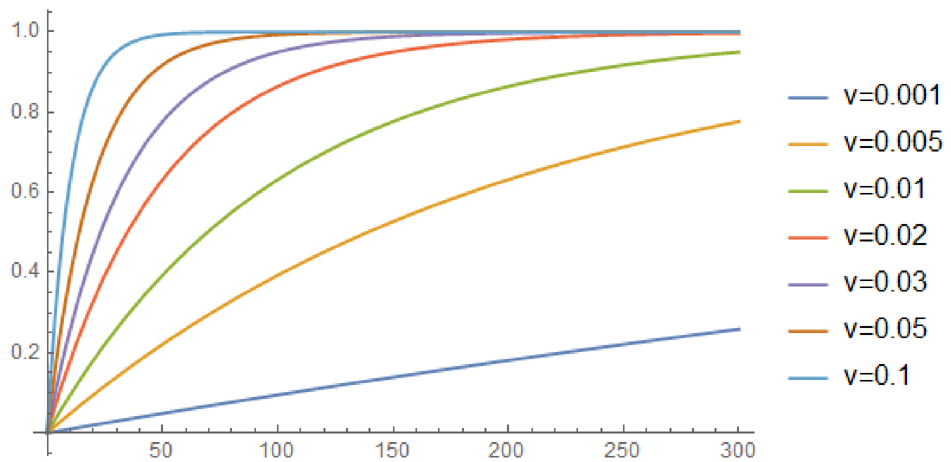
Figure 3.5: Negative exponential adstock transformations



This will also be the case for the logistic transformation, therefore an illustration of the logistic adstock model for different values of the saturation parameter *v* will be omitted.

A larger saturation parameter in the negative exponential model implies a more concave function. The smaller the saturation parameter is, the more linear the advertising-sales response function is. In Figure 3.6 the negative exponential function is plotted for different values of the saturation parameter *v*. For values higher than 0.03 there is little

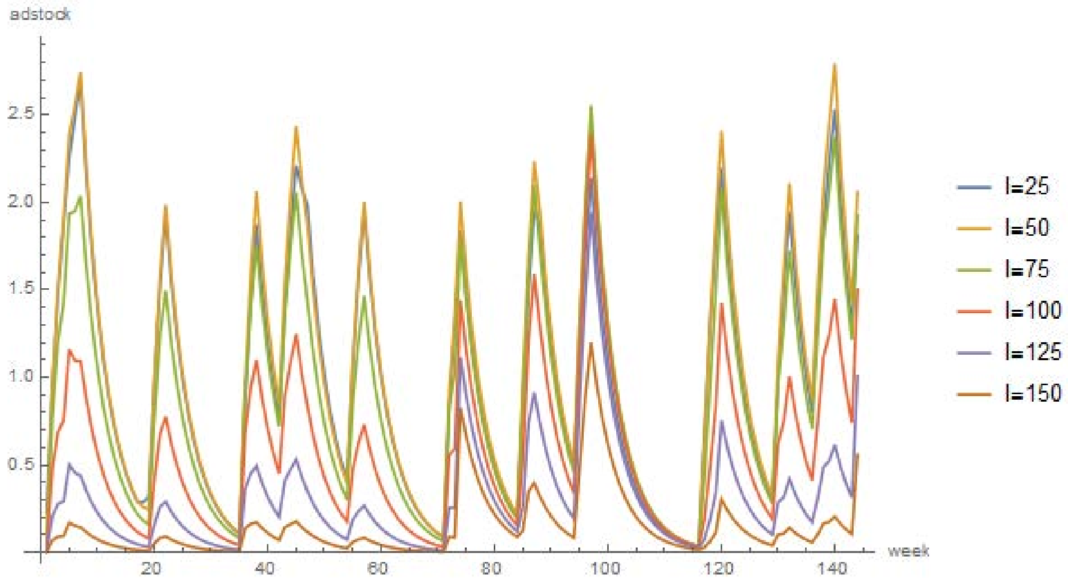
Figure 3.6: Negative exponential function



difference in the response of sales between deploying 100 GRPs and 200 GRPs, an impact of respectively 0.95 and 1.00. And for values below 0.001 there is too much difference in the response of sales to high levels of advertising. For v is 0.001 an increase from 500 GRPs to 600 GRPs would imply a 15% rise in sales. Considering the above, the initial feasible region for v is set from 0.005 to 0.03.

The effect of the inflection point I of the logistic adstock model is illustrated in Figure 3.7 below. The decay parameter λ still equals 0.8 and the saturation parameter v is 0.05. The logistic transformation with inflection points below 75 the graphs almost coincide.

Figure 3.7: Logistic adstock transformation



This is again because when advertised, the deployed amount of GRPs generally is larger than 60. When the inflection point I is below 60, the majority of the deployed amounts of GRPs lies on the concave part of the logistic function. When the inflection point lies somewhere between 60 and 120, the observations all lie around the inflection point. A large part of the deployed amount of GRPs will lie on the convex part of the curve, but there is also a substantial part, which will lie on the concave part of the function. This explains why the logistic adstock transformations almost coincide for inflection points below 75 and differ more in shape for inflection points between 60 and 120 GRPs.

§3.2 Results: Single equation models

Autoregressive Distributed Lag model

The two main categories of sales are gums and liquorish, which represent 65% and 15% of volume sold respectively. The categories marshmallows and foam are added together because otherwise there would be too much difference between the sizes of the categories, together they represent 12% of sold volume. The category toffees is sold under a different brand name than the other categories. The products in the category seasonal and other are not sold continuously during the year. Therefore these three categories will be discarded in further analysis.

First, stationarity of the three time-series have to be determined. In Figure 3.8 to 3.10 the sold volume of the three categories is plotted.

Figure 3.8: Volume gums

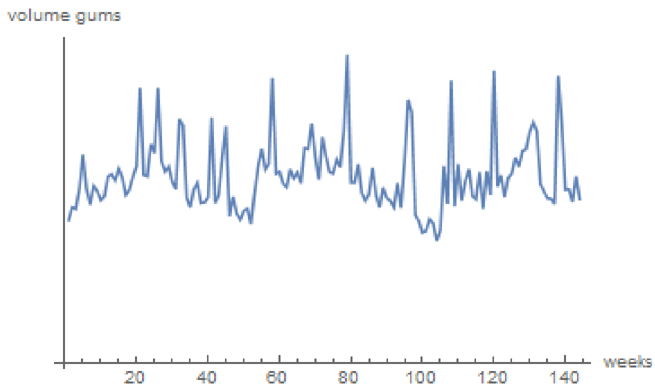


Figure 3.9: Volume liquorish

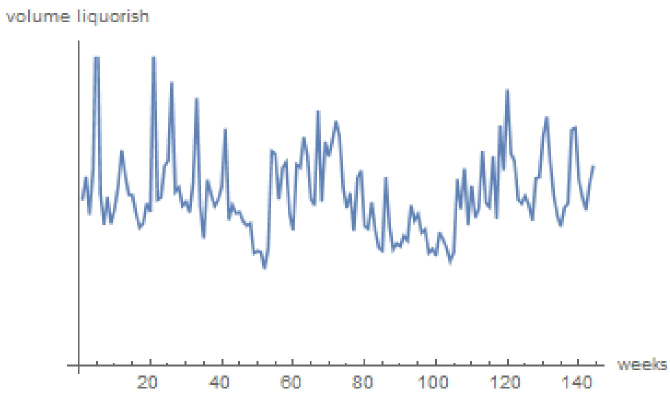
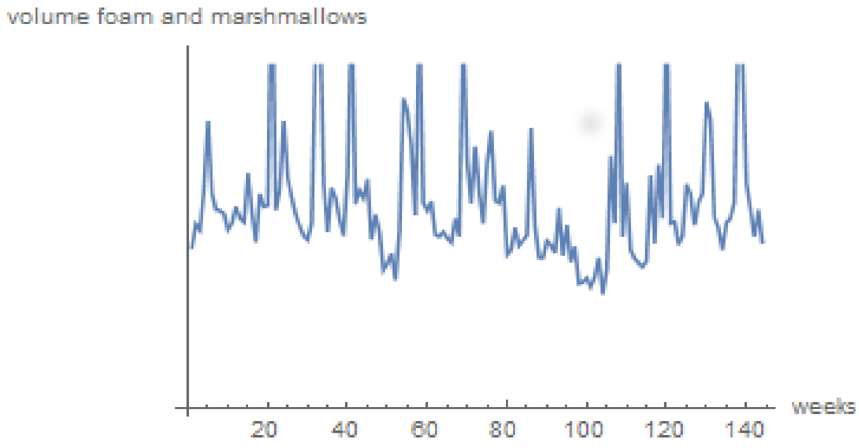


Figure 3.10: Volume foam and marshmallows



All three time-series does not seem to have a deterministic trend, therefore the Augmented Dickey-Fuller test is performed with only an intercept term. The number of augmented term is determined with Bayes Information Criterion. The test equation and hypothesis are (notation is taken from: Heij, et al., 2004):

$$\Delta y_t = \alpha + \rho y_{t-1} + \Delta y_{t-1} + \dots + \Delta y_{t-k} + \varepsilon_t$$

H_0: $\rho = 0$ *(non-stationary;unit root)*

H_1: $\rho < 0$ *(stationary around a constant)*

The null hypothesis is rejected for all three time-series at the 1% significance level. The three time-series will be modelled at their levels.

The estimation results of the log of sold volume of gums are presented in Table 3.1. The R^2 is 72,6%, which is reasonable. Without the lag of volume sold, R^2 would still be 69.9%. The promotions of the largest retailers have a significant effect on sales, their impact is accordance to the size of the market share. For example, if there is a promotion at Albert Heijn, the sold volume of gums rises with 43.1%. The holidays Halloween and Sint Maarten have a large positive impact on sold volume, 30.3% and 38.9% respectively. The beginning of the summer vacations of the elementary schools also causes a rise in the sales of gums of 15.4%. As for the response of sales on advertising, there is a small significant immediate impact of advertising through television on sales. A 1% increase in advertising pressure leads to a 0.011% immediate raise in sold volume of gums. Lodish et al. (1995, 129) found an average advertising elasticity for established products of 0.05. The long-run impact of television advertising is $\frac{\sum_{i=0}^{p} \beta_i}{1-\sum_{i=1}^{q} \gamma_i} = \frac{0.011}{1-0.156} = 0.013$. The purchase feedback effect is 0.156.

Table 3.1: Estimation results of the ADL model for gums

ADL model		
Dependent variable	log of volume sold gums (in kg)	
Variable	Coefficient	Standard Error
Constant	9.817**	0.592
Q2	0.075**	0.019
Q4	-0.134**	0.023
Promo AH	0.431**	0.041
Promo Dirk	0.156**	0.033
Promo Plus	0.117**	0.030
Halloween	0.303**	0.073
Sint Maarten	0.389**	0.074
Summer vacation	0.154**	0.033
TV advertising	0.011**	0.004
First lag of sales	0.156**	0.051
Sig.: 5%(*), 1%(**)		
R-squared	0.726	

The results of the diagnostic tests for the ADL model of the category gums can be found in Table 3.2. In order to determine whether the errors are normality distributed, the Jarque-Bera test is performed. The null hypothesis that the errors are normally is rejected, with a p-value of 0.020. The main cause of non-normality are the outliers. To test whether the errors are serially correlated, the Breusch-Godfrey LM test with one lag is performed. The null hypothesis of no serial correlation is not rejected, with a p-value of 0.45. And finally a general misspecification test, the Ramsey's RESET test with only the squared fitted values added to the model is conducted. Again the null hypothesis of a correct specification is not rejected, with a p-value of 0.46.

Table 3.2: Diagnostic tests of the ADL model for gums

	Value	p-value
Jarque-Bera test	7.789	0.020
Ramsay's RESET	0.736	0.463
LM-AR(1)	0.573	0.451

Although price is not included in the model, as discussed earlier, this would cause endogeneity. Also the price of the products of The Confectionary Company has not changed significantly during the period for which the data is available. Distribution is not significant when included in the model. Also various macro-economic indicators such as the consumers' confidence index and the propensity to buy are not significant when included in the model. Furthermore, climatological variables such as average temperature and hours of sun are also not significant. Also, sold volume or revenue of the main competitors in the category gums do not have a significant impact.

The estimation results of the ADL model for the category liquorish is presented in Table 3.3. The R^2 is 59.6%, which is rather low. The difference in explained variation between the categories gums and liquorish is mainly caused by the dummy variables Sint Maarten, Halloween and beginning of the summer vacation of the elementary schools. These are not significant in the model for liquorish, while they have relatively large impact on the sales of gums. Besides the significant effect of television advertising, there is also a significant effect of branded content in the category liquorish. The immediate impact of television advertising and branded content is 0.015 and 0.018, respectively. The long-run impact of television advertising and branded content is 0.022 and 0.025. The purchase feedback effect is 0.314.

Table 3.3: Estimation results of the ADL model for liquorish

ADL model		
Dependent variable	log of volume sold liquorish (in kg)	
Variable	Coefficient	Standard Error
Constant	6.961**	0.663
Q2	0.085**	0.032
Q4	-0.138**	0.040
Promo AH	0.219**	0.073
Promo Agrimarkt	0.216**	0.072
Promo C1000	0.228**	0.060
Promo Dirk	0.129*	0.052
Promo Plus	0.236**	0.048
TV advertising	0.015*	0.007
Branded content	0.018*	0.008
First lag of sales	0.314**	0.065
Sig.: 5%(*), 1%(**)		
R-squared	0.596	

The diagnostic tests of the ADL model for liquorish are presented in Table 3.4. The p-value of the Jarque-Bera test is 0.113. Unlike the residuals of the model for gums, we cannot reject the assumption of normally distributed residuals for the model of liquorish. The p-value of the Ramsay's Reset test is 0.785, thus the null hypothesis of no general misspecification cannot be rejected. The p-value of the Breusch Godfrey LM test for first order autocorrelation of the residuals is 0.419. So the null hypothesis of no serial correlation cannot be rejected.

Table 3.4: Diagnostic tests of the ADL model for liquorish

	Value	p-value
Jarque-Bera test	4.368	0.113
Ramsay's RESET test	0.274	0.785
LM-AR(1)	0.658	0.419

The results of the ADL model for the combined categories foam and marshmallows are presented in Table 3.5. The estimation results are similar to the estimation results for the category liquorish. The R^2 is rather low with 58.0%. Like in the model for liquorish, the dummy variables Halloween, Sint Maarten and the beginning of summer vacation of the elementary school are not significant. Besides television advertising, branded content also has a significant impact on sales. The immediate effect of television and branded content is 0.027 and 0.021, respectively. Furthermore, the long-run impact of television advertising and branded content is 0.036 and 0.029, respectively. The purchase feedback effect is 0.249.

Table 3.5: Estimation results of the ADL model for foam & marshmallows

ADL model		
Dependent variable	log of sold volume foam and marshmallows (in kg)	
Variable	Coefficient	Standard Error
Constant	7.355**	0.612
Q2	0.096**	0.036
Q4	-0.169**	0.045
Promo AH	0.550**	0.076
Promo C1000	0.186**	0.068
Promo Plus	0.265**	0.056
TV advertising	0.027**	0.008
Branded content	0.021*	0.010
First lag of sales	0.249**	0.062
Sig.: 5%(*), 1%(**)		
R-squared	0.580	

The results diagnostic tests of ADL model for foam and marshmallows are presented in Table 3.6. The null hypothesis of the Jarque-Bera test of normally distributed residuals is rejected with a p-value of 0.000. The Breusch-Godfrey LM test is performed for first order

Table 3.6: Diagnostic tests of the ADL model for foam and marshmallows

	Value	p-value
Jarque-Bera test	63.157	0.000
Ramsay's RESET test	0.357	0.722
LM-AR(1)	1.501	0.223

autocorrelation of the residuals. With a p-value of 0.223 the null hypothesis of no autocorrelation cannot be rejected. The null hypothesis of a general misspecification of Ramsay's RESET test can also not be rejected with a p-value of 0.722.

Logarithmic adstock model

An adstock transformation will be applied on the advertising channels, which are significant in the ADL models. The exogenous variables in the adstock models will be the same as in the corresponding ADL models. Starting with the logarithmic adstock model for gums, for convenience the logarithmic adstock transformation in Equation 3.3 is repeated in Equation 3.9.

$$Adstock_t(A_t, \lambda) = \log(A_t) + \lambda Adstock_{t-1} \tag{3.9}$$

The decay parameter λ of television advertising is estimated at zero, thus the logarithmic adstock model for gums reduces to the ADL model in Table 4.1. In the ADL models for liquorish and foam & marshmallows, there is a significant effect of advertising through television and branded content, however in the logarithmic adstock models only television advertising is significant. The estimation results of the logarithmic adstock models for the categories liquorish and foam & marshmallows can be found in Table 3.7 and 3.8.

Table 3.7: Results of the logarithmic adstock model for liquorish

Logarithmic adstock model		
Dependent variable:	log of sold volume liquorish (kg's)	
Variable	Coefficient	Standard Error
Constant	6.958*	0.683
Q2	0.068*	0.032
Q4	-0.138*	0.038
Promo AH	0.216*	0.074
Promo Agrimarkt	0.220*	0.074
Promo C1000	0.211*	0.061
Promo Dirk	0.133**	0.053
Promo Plus	0.226*	0.050
TV advertising	0.017**	0.008
First lag of sales	0.316*	0.067
Sig.: (*) 1%, (**) 5%		
R-squared	0.583	

Table 3.8: Results of the logarithmic adstock model for foam & marshmallows

Logarithmic adstock model		
Dependent variable:	log of sold volume foam & marshmallows (in kg)	
Variable	Coefficient	Standard Error
Constant	7.267*	0.632
Q2	0.076**	0.036
Q4	-0.129*	0.042
Promo AH	0.559*	0.078
Promo C1000	0.166**	0.069
Promo Plus	0.252*	0.057
TV advertising	0.029*	0.010
First lag of sales	0.260*	0.064
Sig.: (*) 1%, (**) 5%		
R-squared	0.565	

Negative exponential adstock model

For convenience the negative exponential adstock transformation in Equation 3.5 is formulated again in Equation 3.10.

$$Adstock_t(A_t, \lambda, v) = 1 - e^{-vA_t} + \lambda Adstock_{t-1},$$

$$with\ \lambda \in (0,1). \tag{3.10}$$

The estimation results of the negative exponential adstock transformation of television advertising can be found in Table 3.9. The linear effect β and the saturation parameter v have the wrong sign, but both are insignificant.

Table 3.9: *Results of the negative exponential adstock model for gums*

Negative exponential model for gums			
Coefficient	Estimate	S.E.	p-value
β_{tv}	-0.222	1.818	0.903
λ_{tv}	0.000	0.394	1.000
v_{tv}	-0.002	0.016	0.889

The decay parameter λ of television advertising is estimated at zero, which is in accordance with the ADL model.

In the estimation procedure for the category liquorish the function fmincon in Matlab gives a warning that the results might be inaccurate due to (near) singularity. When the standard errors of the coefficients are calculated, the $(X'X)^{-1}$ matrix as defined in Equation 3.9 is also close to singular and therefore cannot be determined. The results in Table 3.3 show that the coefficients of television advertising and branded content do not significantly differ from each other. In order to reduce the number of parameters the deployed GRPs of television and branded content are added together. The results can be found in Table 3.10.

Table 3.10: Results of the negative exponential adstock model for liquorish

Negative exponential model for liquorish			
Coefficient	Estimate	S.E.	p-value
β_{tvbc}	0.145	0.068	0.033
λ_{tvbc}	0.000	0.338	1.000
v_{tvbc}	0.015	0.991	0.332

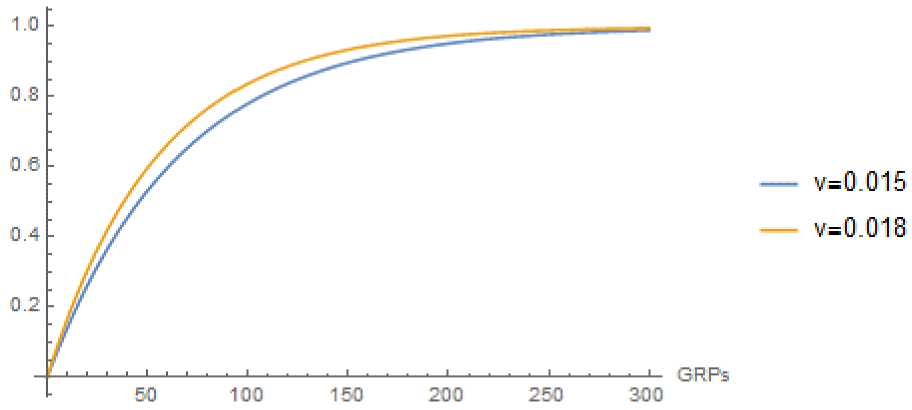
The same identification problem occurs in the category foam and marshmallows, therefore the GRPs of television and branded content are added together. The estimation results are presented in Table 3.11

Table 3.11: Results of the negative exponential adstock model of foam & marshmallows

Negative exponential model for foam & marshmallows			
Coefficient	Estimate	S.E.	p-value
β_{tvbc}	0.195	0.068	0.004
λ_{tvbc}	0.000	0.274	1.000
v_{tvbc}	0.018	0.015	0.221

The saturation parameter *v* is insignificant in the categories liquorish and foam & marshmallows is, but the linear coefficient *β* is significant for the estimated values of *v*. A plot of the negative exponential function for both values of *v* is presented in Figure 3.4

Figure 3.4: Plots of negative exponential model



The saturation parameters *v* is insignificant in the models for gums, liquorish and foam & marshmallows. This can be explained by use of the Taylor's series expansion of e^x in Equation 3.11:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots. \tag{3.11}$$

Rewriting the negative exponential function as an infinite sum using this Taylor's series expansion gives Equation 3.12.

$$1 - e^{-vx} = 1 - (1 - vx + \frac{v^2x^2}{2} - \frac{v^3x^3}{6} + \cdots) = vx - \frac{v^2x^2}{2} + \frac{v^3x^3}{6} - \cdots \tag{3.12}$$

The coefficient β is the linear effect of advertising on sales, whereas the parameter v models the nonlinear effect of advertising on sales. When there is no non-linearity in the effect of advertising on sales, the polynomials of the Taylor expansion from order two and higher are zero, and Equation 3.12 reduces to vx, thus the effect of advertising reduces to βv, which is unidentified.

Logistic adstock model

The logistic adstock transformation in Equation 3.5 is formulated again in Equation 3.13. It distinguishes itself from the other transformation by the incorporation of a threshold effect.

$$Adstock_t(A_t, v, I) = \frac{1}{1+e^{-v(A_t-I)}} - \frac{1}{1+e^{vI}} + \lambda Adstock_{t-1} \tag{3.13}$$

The estimation results of the category gums can be found in Table 3.12.

Table 3.12: Results of the logistic adstock model for gums

Logistic adstock model for gums			
Coefficient	Estimate	S.E.	p-value
β_{tv}	0.081	0.022	0.000
λ_{tv}	0.003	0.293	0.992
v_{tv}	4.103	16.802	0.807
I_{tv}	77.673	1.740	0.000

The parameters of the logistic S-curve imply that for GRP levels below the threshold of 78 GRPs there is no effect of television advertising and for GRPs levels above the threshold level the impact is an increase of 8.4% in sales with a negligible carry-over effect.

When the advertising effect of television and branded content are estimated simultaneously, the matrix $(X'X)^{-1}$ is again close to singular and the standard errors cannot be calculated. Since the coefficients of television advertising and branded content are not significantly different from each other, the GRPs deployed through television and branded content are added together. The results are present in Table 3.13. The estimated coefficients for imply that for GRP levels below 53, there is no impact on sales, whereas GRP levels above 53 increases sales with 13.3%.

Table 3.13: Results of the logistic adstock model for liquorish

Logistic adstock model for liquorish			
Coëfficiënt	Estimate	S.E.	p-value
β_{tvbc}	0.125	0.037	0.001
λ_{tvbc}	0.003	0.292	0.992
v_{tvbc}	1.929	15.291	0.900
I_{tvbc}	53.160	9.557	0.000

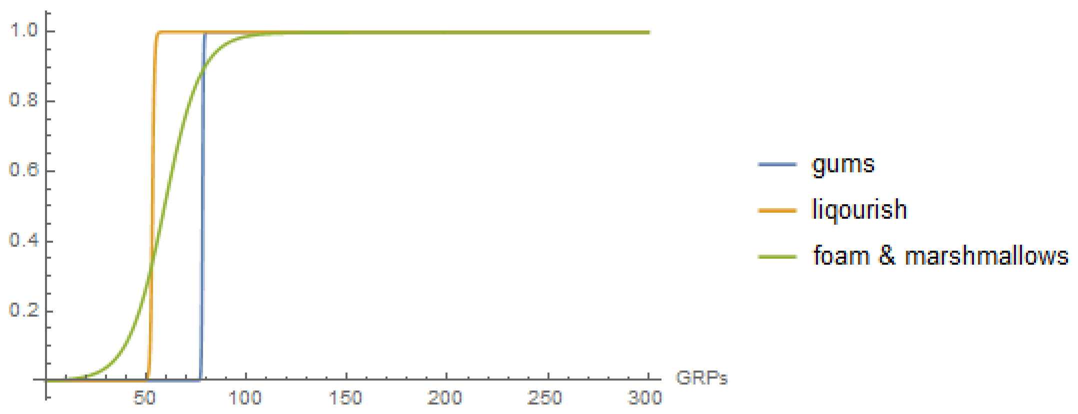
The standard errors of the coefficients for the category foam and marshmallows cannot be estimated simultaneously, because the matrix $(X'X)^{-1}$ is close to singular. When they are estimated separately, like in the category liquorish, both β_{tv} and β_{bc} are negative and not significant. Therefore a positive impact will be imposed on both advertising channels, which give the results in Table 3.14.

Table 3.14: Results of the logistic adstock model for foam and marshmallows

Logistic adstock model for foam & marshmallows			
Coëfficiënt	Estimate	S.E.	p-value
β_{tvbc}	0.185	0.050	0.002
λ_{tvbc}	0.005	0.244	0.854
v_{tvbc}	0.111	0.149	0.459
I_{tvbc}	59.436	10.080	0.000

The estimated coefficients imply a threshold effect for advertising, where advertising below 27 GRPs does not have effect on sales, whereas GRP levels above 27 imply an increase of 14.4%. Like in the model for liquorish the effect of branded content is not significant. The estimated logistic function for all three categories is plotted in Figure 3.5.

Figure 3.5: Estimated logistic functions



Chapter 4

§4.1 Method: Structural Vector Autoregressive model

The single-equation models imply a unidirectional relationship running from advertising to sales. Vector Autoregressive (VAR) models assume a bidirectional relationship between advertising and sales. Baghestani (1991) was one the first researchers that showed advertising and sales are cointegrated and therefore possess a long-run equilibrium condition. In general, a long-run equilibrium relationship between advertising and sales can be caused by firm specific decision rules, for example fixing advertising spending as a percentage of revenue. Furthermore, advertising may create a new costumer who will not only make an initial purchase but also repurchases in the future (Givon and Horsky, 1990).

Besides firm-specific decision rules (1), contemporaneous effects (2) and purchase reinforcement (3), Dekimpe and Hanssens (1995) identify three other channels through which advertising can influence a brand's performance:

4. Carry-over effects: consumer's remember past advertising and may act upon it in future periods.

5. Feedback effects: advertising may be influenced by current and past sales, and should not be treated exogenous.

6. Competitive reactions: in the long-run competitive reactions can negate the short-run positive effect of advertising.

Equation 4.1 is a bivariate VAR(p) model with as endogenous variables sales and advertising:

$$\begin{bmatrix} S_t \\ A_t \end{bmatrix} = \begin{bmatrix} \pi_{11}^1 & \pi_{12}^1 \\ \pi_{21}^1 & \pi_{22}^1 \end{bmatrix} \begin{bmatrix} S_{t-1} \\ A_{t-1} \end{bmatrix} + \cdots + \begin{bmatrix} \pi_{11}^p & \pi_{12}^p \\ \pi_{21}^p & \pi_{22}^p \end{bmatrix} \begin{bmatrix} S_{t-p} \\ A_{t-p} \end{bmatrix} + \begin{bmatrix} u_{S,t} \\ u_{A,t} \end{bmatrix}, \qquad (4.1)$$

where p is the order of the model and $\vec{\boldsymbol{u}}_{\boldsymbol{t}} = [u_{S,t},\ u_{A,t}]$ is a white noise vector of innovations. For ease of exposition, the intercepts and exogenous variables have been omitted. The delayed response is captured by π_{12}^j for $j = 1, \ldots, p$; purchase reinforcement by π_{11}^j; feedback effects by π_{21}^j and firm-specific decision rules by π_{22}^j. Competitive advertising is not included as endogenous variable (nor as exogenous variable), thus the effect of competitive advertising is not incorporated in the model.

Rather than interpreting the individual parameters of the VAR model, one focuses on the Impulse Response Functions (IRFs) and cumulative IRFs (CIRFs) derived from these parameters. Impulse Response Functions reflect the complex interplay of included variables and provide a concise description of the system's dynamic structure. Also, they lend itself well to a graphical representation and are easy to interpret. In order to understand IRFs the mathematically equivalent infinite-order Vector-Moving-Average representation in Equation 5.2 is presented:

$$\begin{bmatrix} S_t \\ A_t \end{bmatrix} = \begin{bmatrix} u_{S,t} \\ u_{A,t} \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{bmatrix} \begin{bmatrix} u_{S,t-1} \\ u_{A,t-1} \end{bmatrix} + \begin{bmatrix} a_{11}^2 & a_{12}^2 \\ a_{21}^2 & a_{22}^2 \end{bmatrix} \begin{bmatrix} u_{S,t-2} \\ u_{A,t-2} \end{bmatrix} + \cdots , \qquad (4.2)$$

a_{12}^k is the impact of a of a one-unit advertising shock $u_{A,t-k}$ on S_t. A sequence of successive a_{ij}^k is called an Impulse Response Function (IRF). However the interpretation of a_{ij}^k is not as straightforward as it seems. The innovations $u_{S,t}$ and $u_{A,t}$ are often contemporaneously correlated, which means that the off-diagonal elements of the covariance matrix $\boldsymbol{\Sigma}$ of $\vec{\boldsymbol{u}}_{\boldsymbol{t}}$ are positive. Thus advertising shocks $u_{S,t}$ not only have a direct impact through a_{ij}^k, but also indirectly through their correlation with $u_{S,t}$ (Dekimpe & Hanssens, 1995).

As we have seen in the previous chapter, there is a contemporaneous relation of advertising on sales. Equation 4.1 is called a reduced-form model because it does not show this effect explicitly. Although the concurrent effect is represented by the off diagonal-diagonal elements of the covariance matrix $\boldsymbol{\Sigma}$ of $\vec{\boldsymbol{u}}_{\boldsymbol{t}}$, the direction of the relationship cannot be established. An explicit expression can be obtained by a linear transformation of the reduced-form model, which fits into the framework of a Structural Vector Autoregressive (SVAR) model. A contemporaneous effect of sales on advertising is precluded on logical grounds. We will assume that the advertising schedule in week *t* cannot be adjusted in response to sales results in week *t*. Thus the contemporaneous relationship between adverting and sales will solely be attributed to advertising.

Because the covariance matrix $\boldsymbol{\Sigma}$ is positive definite, there exists a Cholesky decomposition of $\boldsymbol{\Sigma}$ with an upper triangular matrix $\boldsymbol{L}^{-1}$ with unit diagonal elements and a diagonal matrix $\boldsymbol{G}$ such that $\boldsymbol{\Sigma} = \boldsymbol{LGL}'$. Define $\vec{\boldsymbol{e}}_{\boldsymbol{t}} = \boldsymbol{L}^{-1}\vec{\boldsymbol{u}}_{\boldsymbol{t}}$, then $E(\vec{\boldsymbol{e}}_{\boldsymbol{t}}) = \boldsymbol{L}^{-1}E(\vec{\boldsymbol{u}}_{\boldsymbol{t}}) = \boldsymbol{0}$ and

$Cov(\vec{\boldsymbol{e}}_{\boldsymbol{t}}) = \boldsymbol{L}^{-1}\,\boldsymbol{\Sigma}(\boldsymbol{L}^{-1})' = \boldsymbol{L}^{-1}\,\boldsymbol{\Sigma}(\boldsymbol{L}')^{-1} = \boldsymbol{G}$. Since $\boldsymbol{G}$ is a diagonal matrix, the components of $\vec{\boldsymbol{e}}_{\boldsymbol{t}}$ are uncorrelated, which also aids the interpretation of the IRF. Multiplying $\boldsymbol{L}^{-1}$ from left to the reduced form model in Equation 4.1, we obtain (Tsay, 2002):

$$\boldsymbol{L}^{-1}\begin{bmatrix} S_t \\ A_t \end{bmatrix} = \boldsymbol{L}^{-1}\begin{bmatrix} \pi_{11}^{1} & \pi_{12}^{1} \\ \pi_{21}^{1} & \pi_{22}^{1} \end{bmatrix}\begin{bmatrix} S_{t-1} \\ A_{t-1} \end{bmatrix} + \cdots + \boldsymbol{L}^{-1}\begin{bmatrix} \pi_{11}^{p} & \pi_{12}^{p} \\ \pi_{21}^{p} & \pi_{22}^{p} \end{bmatrix}\begin{bmatrix} S_{t-p} \\ A_{t-p} \end{bmatrix} + \boldsymbol{L}^{-1}\begin{bmatrix} u_{S,t} \\ u_{A,t} \end{bmatrix}. \quad (4.3)$$

The individual equations for sales and advertising are formulated in Equation 4.4:

$$S_t = \sum_{i=0}^{p} {}^{*}\pi_{11}^{i} A_{t-i} + \sum_{i=1}^{p} {}^{*}\pi_{12}^{i} S_{t-i} + e_{S,t}$$

$$A_t = \sum_{i=1}^{p} \pi_{21}^{i} A_{t-i} + \sum_{i=1}^{p} \pi_{22}^{i} S_{t-i} + e_{A,t}. \quad (4.4)$$

The assumption that advertising and sales are endogenous will be tested by a Granger causality test. Granger (1980) states formally:

$$E[\boldsymbol{Y}_{\boldsymbol{t+1}} | J_t] = E[\boldsymbol{Y}_{\boldsymbol{t+1}} | J_t'], \quad (4.5)$$

where J_t' and J_t denote the information on time *t* with and without past and present values of $\boldsymbol{X}_{\boldsymbol{t}}$ respectively. This concept of causality will be tested by a Wald test on the joint significance of the lags of advertising in the equation for sales and vice versa.

§4.2 Results: Structural Vector Autoregressive Model

For each category of confectionaries a SVAR model will be estimated. In the model for gums only television advertising is included as endogenous variable, since only television advertising is significant in the ADL model for gums. In the categories liquorish and foam & marshmallows advertising through television and branded content are added together, because in the ADL models both television advertising and branded content are significant and the coefficient estimates of television advertising and branded are not significantly different from each other (Table 3.3 and 3.5). The exogenous variables in the SVAR models are the same explanatory variables as in the single equation models. Stationarity of the endogenous variables will be determined by performing the Augmented Dickey Fuller (ADF) test (notation is adapted from Heij, et al., 2004):

$$\Delta y_t = (\alpha) + (\beta t) + \rho y_{t-1} + \Delta y_{t-1} + \cdots + \Delta y_{t-2} + \varepsilon_t$$

$H_0: \rho = 0$ *(non-stationary; unit root)*

$H_1: \rho < 0$ *(level/trend stationary)*

The number of augmented terms is determined with the Bayes Information Criterion (BIC). Furthermore, the inclusion of a constant and/or trend term has to be decided.

We have already shown that the sold volume of gums, liquorish and foam & marshmallows are level stationary. Figure 4.1 and 4.2 contain the plots of the log of advertising through television and branded content & television, respectively. Both plots do not indicate the presence of a deterministic trend. Therefore the ADF test is performed on both series with only an intercept term. The null hypothesis of a unit root is rejected for both series for any conventional significance level.

Figure 4.1: Television advertising

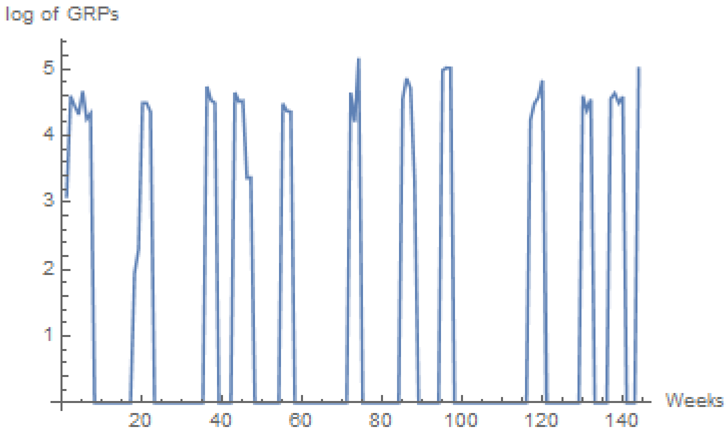
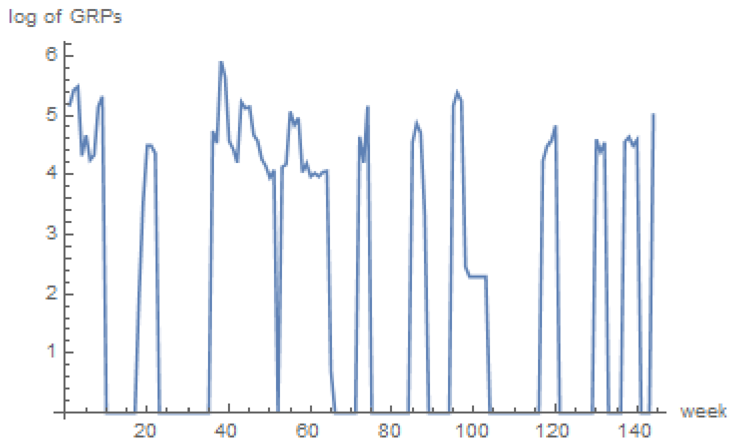


Figure 4.2: Branded content and television advertising



Gums

The lag length of the SVAR model is determined by comparing the Akaike information criterion (AIC), the Hannan-Quinn (HQ) criterion, the Bayesian information criterion (BIC) and the final prediction error (FPE), which are presented in Table 4.1.

Table 4.1: Selection criteria for optimal lag length

	Number of lags					
	1	2	3	4	5	6
AIC(p)	2.110	2.099	2.029*	2.060	2.106	2.100
HQ(p)	2.300	3.324	2.287*	2.353	2.434	2.462
BIC(p)	2.577*	2.651	2.665	2.781	2.912	2.991
FPE(p)	0.028	0.028	0.026*	0.027	0.028	0.028

The optimal lag length of the SVAR model according to the AIC, HQ criterion and FPE is three, whereas the BIC indicate an optimal lag length of one. Since we have not found a delayed effect of advertising in the single equation models and a parsimonious model is preferred, a SVAR model with one lag is estimated. In order to determine normality of the residuals the Jarque-Bera test for multivariate series is performed. Further, to test for residual autocorrelation the Lagrange Multiplier (LM) test for residual autocorrelation is performed. The results of the diagnostic tests can be found in Table 4.2.

Table 4.2: Diagnostic tests for SVAR(1)-X model of gums

	p-value
Jarque-Bera test	0.00
LM test for autocorrelation	
First order	0.06
Second order	0.81
Granger causality Wald test	
Advertising Granger causes sales	0.31
Sales Granger causes advertising	0.30

The null hypothesis of normally distributed residuals is rejected for any conventional significance level. The null hypothesis of no first order and second order serial autocorrelation of the LM test cannot be rejected with p-values of 0.06 and 0.81 respectively. We have found no statistical evidence that advertising Granger causes sales or sales Granger causes advertising, with p-values 0.31 and 0.30 respectively.

As the focus of this thesis is the effect of advertising on sales, only the IRF and CIRF of gums with advertising as impulse and sales as response are presented in Figure 4.3 and 4.4 respectively. A 1% percent increase in television advertising leads to an immediate increase of sales with 0.013% (π_{11}^{0}), which is significant with a p-value of 0.01. The IRF in Figure 4.3 shows that there is small significant effect until period two and the cumulative IRF (CIRF) in Figure 4.4 shows that there is a significant effect until period four. However the confidence interval of the IRF indicates there is no significant persistent effect of advertising on sales.

Figure 4.3: IRF of gums

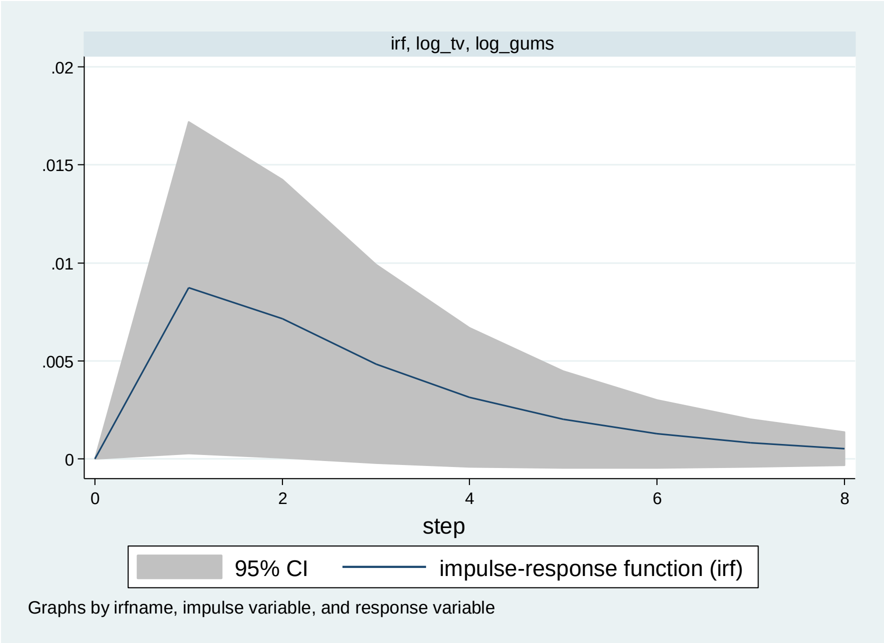
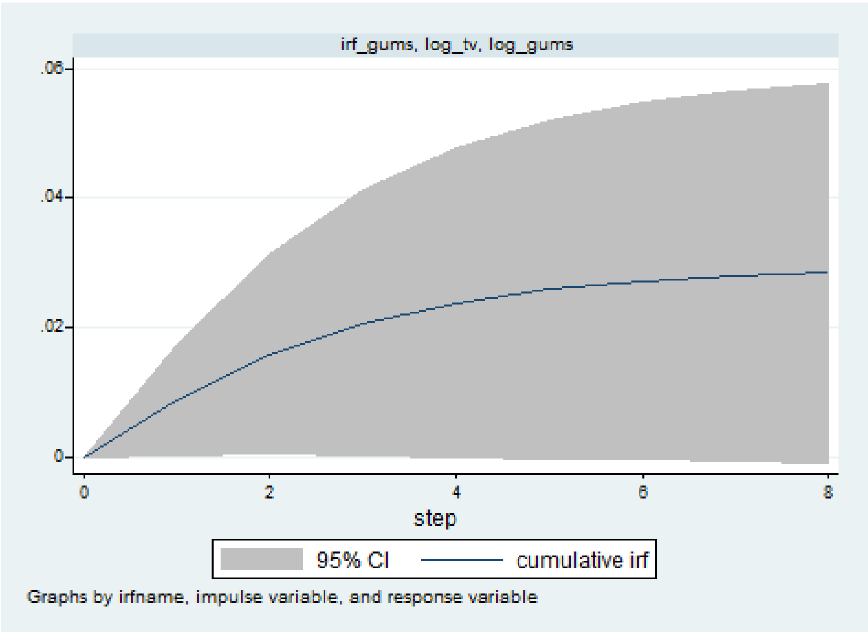


Figure 4.4: Cumulative IRF of gums



Liquorish

The values of the lag selection criteria of the model for liquorish are presented in Table 4.3.

Table 4.3: Selection criteria optimal lag length for liquorish

	Number of lags					
	1	2	3	4	5	6
AIC(p)	3.005	3.005	2.989*	3.008	3.047	3.074
HQ(p)	3.177*	3.212	3.230	3.284	3.357	3.419
BIC(p)	3.429*	3.514	3.583	3.687	3.810	3.922
FPE(p)	0.069	0.069	0.681*	0.070	0.072	0.074

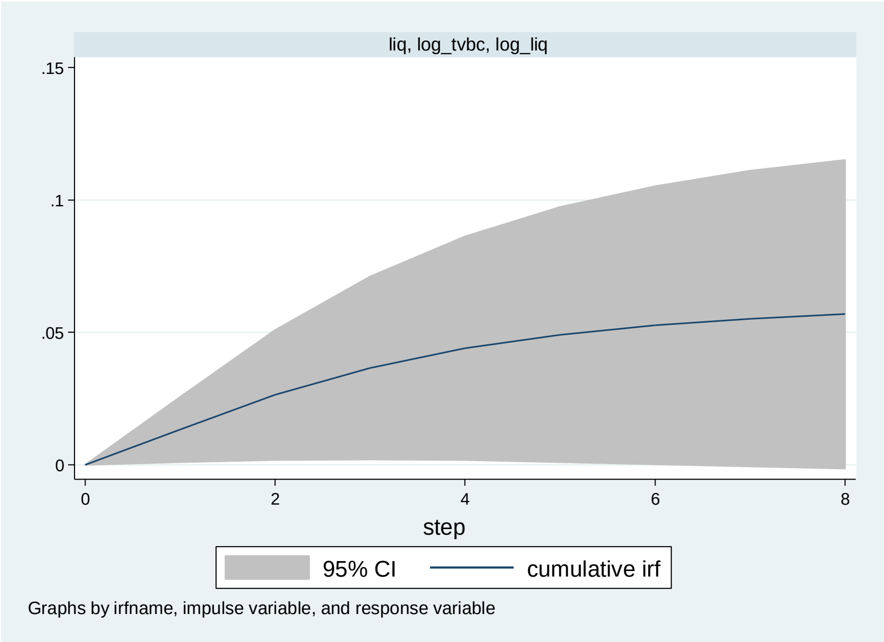
The optimal lag length of the SVAR(*p*)-X model according to the AIC and FPE is three, whereas the BIC and HQ criterion indicate an optimal lag length of one. Again a parsimonious is preferred and therefore a SVAR(1)-X model is estimated. The diagnostic tests are presented in Table 4.4. We must reject the null hypothesis of normally distributed residual for any

Table 4.4: Diagnostic tests for SVAR(1)-X model of liquorish

	p-value
Jarque-Bera test	0.00
LM test for autocorrelation	
First order	0.26
Second order	0.40
Granger causality Wald test	
Advertising Granger causes sales	0.04
Sales Granger causes advertising	0.43

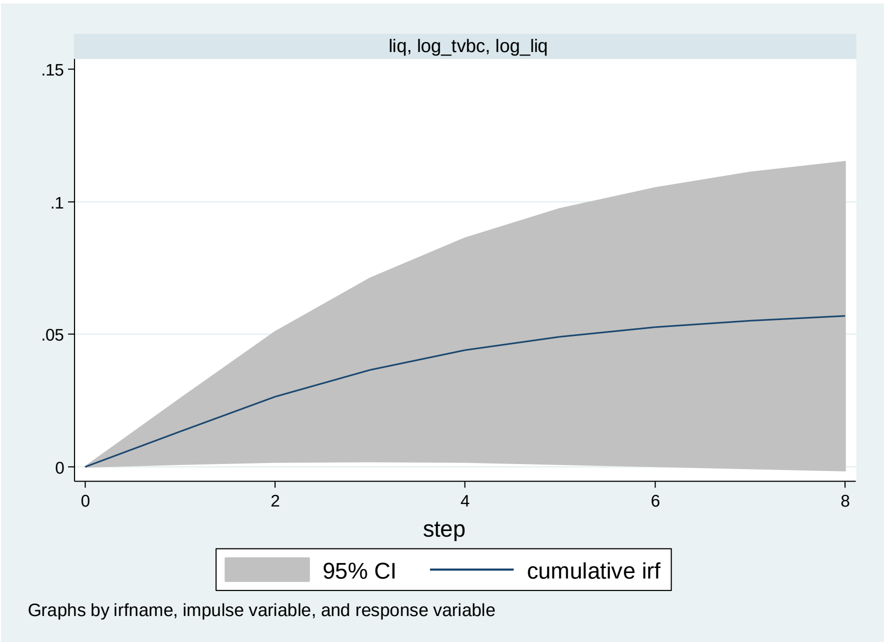
conventional significance level. The Lagrange multiplier test does not indicate the presence of first and second order autocorrelation in the residuals. The Granger causality Wald test does not indicate that sales Granger causes advertising, however with a p-value of 0.04 there is statistical evidence that advertising Granger causes sales. As the focus of this thesis is the effect of advertising on sales, only the IRF and CIRF of liquorish with as response sales and as impulse advertising are presented in Figure 4.5 and 4.6 respectively.

Figure 4.5: IRF of liquorish



The instantaneous effect of advertising on sales is 0.024 (π_{11}^{0}), with a standard error and p-value of 0.008 and 0.003 respectively. This means a 1% increase in television advertising or advertising through branded content increases sales in the same week with 0.024%. The 95% confidence interval of the IRF of liquorish shows, that there is a significant effect of advertising until period three. The 95% of the CIRF of the category liquorish shows there is a significant cumulative effect until period 6.

Figure 4.6: Cumulative IRF of liquorish



Foam and marshmallows

The results of the selection criteria for the optimal lag length can be found in Table 4.5.

Table 4.5: Selection criteria optimal lag length for foam and marshmallows

	Number of lags					
	1	2	3	4	5	6
AIC(p)	3.281	3.286	3.279*	3.317	3.337	3.371
HQ(p)	3.419*	3.458	3.486	3.559	3.612	3.681
BIC(p)	3.621*	3.710	3.788	3.911	4.015	4.134
FPE(p)	0.091	0.912	0.091*	0.095	0.097	0.100

As in the model for liquorish, the optimal lag length of the SVAR model according to the AIC and FPE is three, whereas the BIC and HQ criterion indicate an optimal lag length of one. Again a SVAR model with lag length one is estimated. The results of the diagnostic tests are presented in Table 4.6. The null hypothesis of normally distributed residual is rejected for

Table 4.6: Diagnostic tests for SVAR(1)-X model of foam and marshmallows

	p-value
Jarque-Bera test	0.00
LM test for autocorrelation	
First order	0.09
Second order	0.35
Granger causality Wald test	
Advertising Granger causes sales	0.01
Sales Granger causes advertising	0.26

any conventional significance level. The Lagrange multiplier test does not indicate there is first or second order autocorrelation in the residuals. As in the category liquorish, there is statistical evidence that advertising Granger causes sales, but not vice versa. Since our main interest is the effect of advertising on sales, only the IRF and CIRF with advertising as impulse and sales as response are presented in Figure 4.7 and 4.8 respectively.

The instantaneous effect of advertising through television or branded content is 0.035 (π_{11}^{0}), which implies an advertising elasticity if 0.035%. The standard error and p-value are 0.009 and 0.00 respectively. As in the model for liquorish there is a significant effect until period four. The CIRF for the category foam & marshmallows is presented in Figure 4.8. The 95% confidence interval indicates that after 8 periods there is significant cumulative effect of advertising of 0.077%. Whereas advertising in the categories gums and liquorish does not have a significant cumulative on sales, the cumulative effect in the category foam & marshmallows stabilizes after 15 periods at 0.082%, with a 95% confidence interval from 0.006% to 0.157%

Figure 4.7: IRF foam & marshmallows

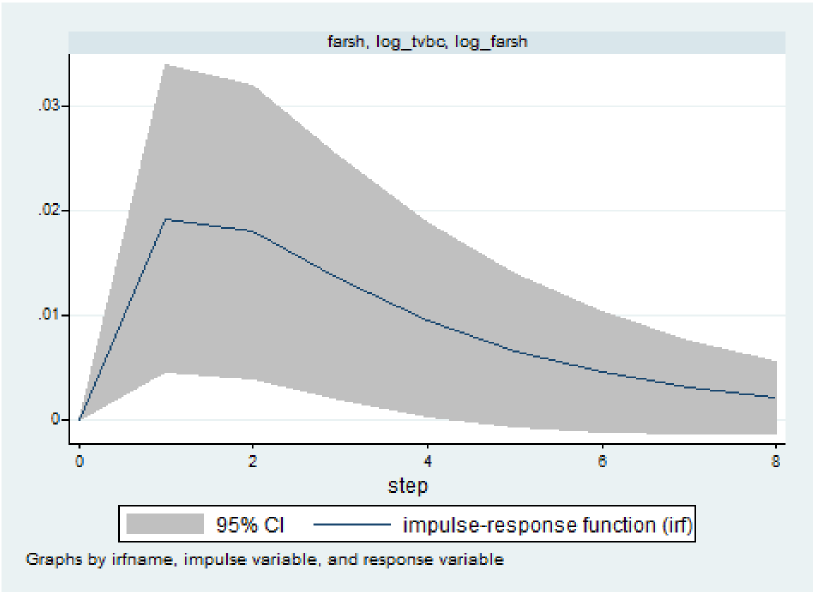
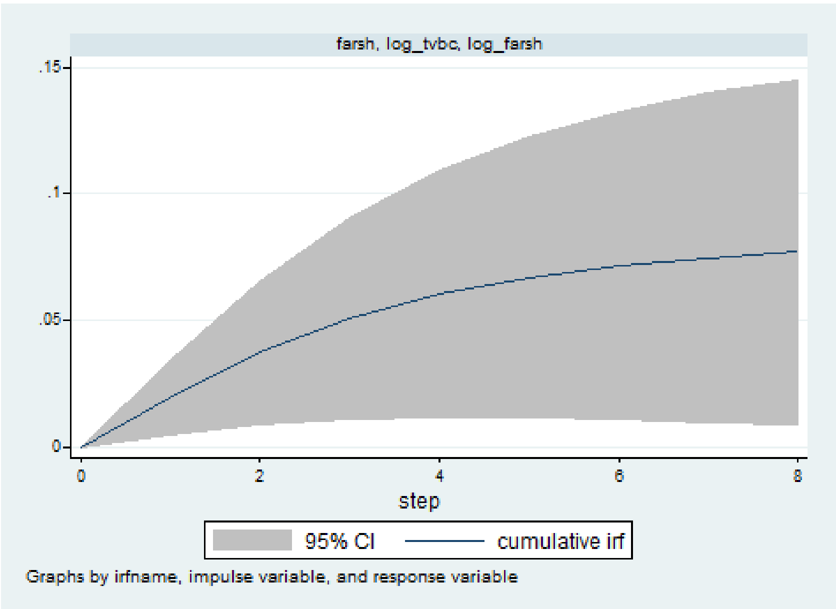


Figure 4.8: Cumulative IRF of foam % marshmallows



Chapter 5

§5.1 Method: Semiparametric estimation

Previously we assumed a particular functional form for the response curve of advertising on sales, now the advertising-sales response curve will be estimated semiparametrically by a cubic smoothing spline estimator. The cubic smoothing spline estimator minimizes the penalized residual sum of squares

$$PRSS(\lambda) = \sum_{i=1}^{N}(y_i - m(x_i))^2 + \lambda \int (m''(x))^2 dx, \qquad (5.1)$$

where λ is a smoothing parameter. The parametric terms are left out for ease exposition. The second term is introduced to penalize roughness. For $\lambda \to 0$ leads to a very rough fit since then $\widehat{m}(x_i) = y_i$. Large values of λ lead to a smoother curve (Cameron and Trivedi, 2005).

The estimation is conducted with the package SemiPar in R, where the parameter λ is estimated by restricted maximum likelihood using certain connections between penalized splines and linear mixed models. Details are given in Ruppert, Wand and Carroll (2000) and Wand (2003). The default basis function is used which is radial: $|x - \kappa_k|^p$, where κ_p is the location of the k^{th} knot, and p is the degree. The default degree for the radial basis is three, so the basis function is (Wand et al., 2005):

$$m(x) = \beta_0 + \beta_1 x + \sum_{k=1}^{K} u_k \, |x - \kappa_k|^3. \qquad (5.2)$$

with

$$\boldsymbol{u} \equiv [u_1, \dots, u_k]' \sim N\left(0, \sigma_u^2 \Omega^{-\frac{1}{2}} \left(\Omega^{-\frac{1}{2}}\right)'\right),$$

$$\Omega \equiv [|\kappa_k - \kappa_{k'}|^3], 1 \leq k, k' \leq K. \qquad (5.3)$$

§5.2 Results: Semiparametric estimation

Gums

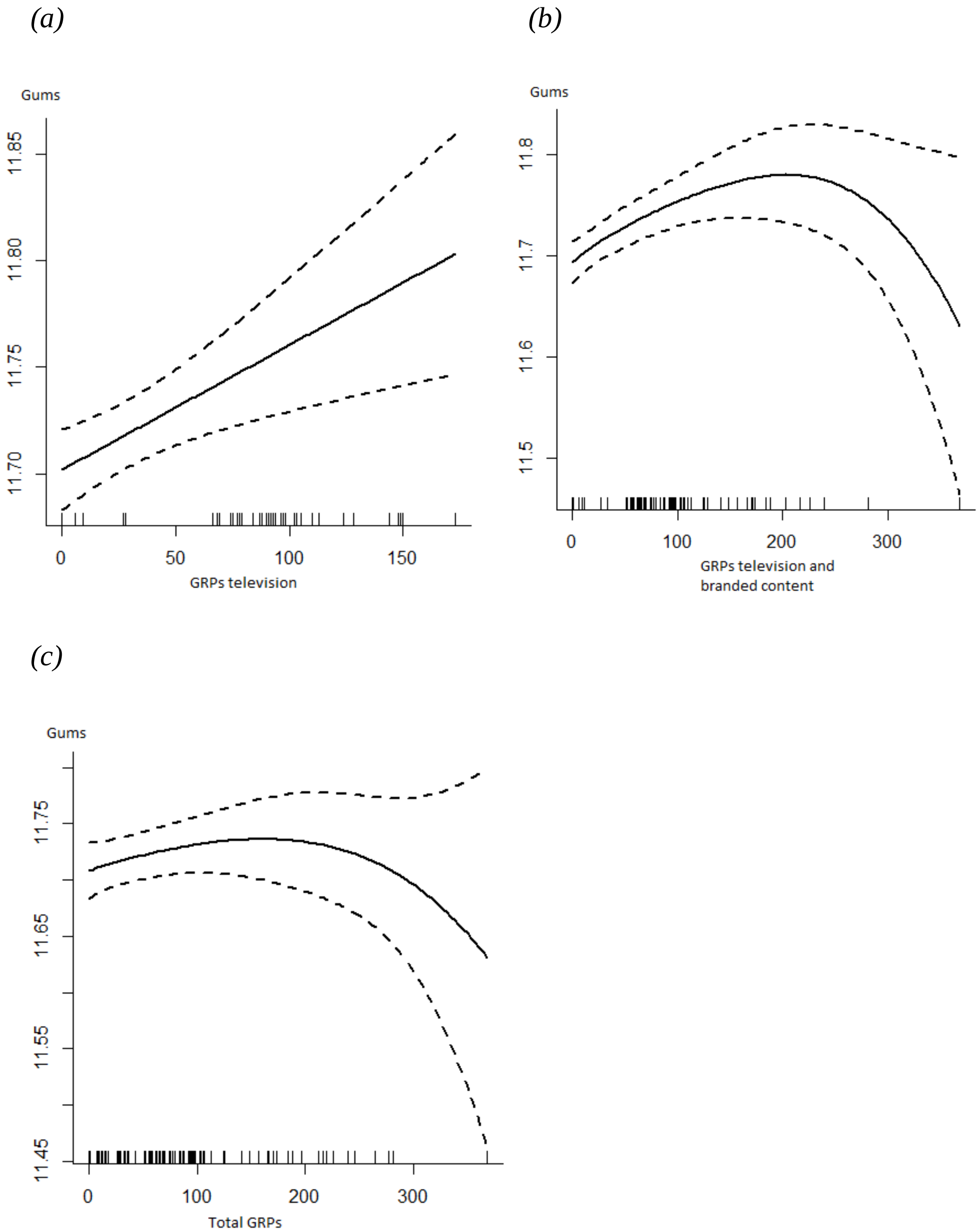
The linear components of the semiparametric model are the same variables as in the ADL model for gums. In Table 5.1 the estimation results of the linear components of the semiparametric model are presented, the nonparametric component is television advertising. The estimation results are not significantly different from the estimation results of the ADL model for gums in Table 3.1.

Table 5.1: Semiparametric estimation results of gums

Dependent variable	log of sold volume gums (in kg)	
Variable	Coefficient	Standard Error
Constant	9.822**	0.587
Q2	0.075**	0.019
Q4	-0.134**	0.023
Promo AH	0.428**	0.040
Promo Dirk	0.152**	0.032
Promo Plus	0.117**	0.029
Halloween	0.289**	0.073
Sint Maarten	0.373**	0.074
Summer vacation	0.154**	0.034
First lag of sales	0.156**	0.050
Sig.: 5%(*), 1%(**)		

Figure 5.1a shows that the response of the volume of gums to television advertising seems to be linear, however towards the end of the response curve the 95% confidence interval is quite wide. Therefore we should be careful drawing any conclusions about the shape of the response curve. When the GRPs deployed through television and branded content are added together, sales response curve is concave and there clearly is a ceiling effect around 200 GRPs, see Figure 5.1b. The sales response curve when the GRPs of all media channels are added together is shown in Figure 5.1c. Now it seems that advertising has little or no impact at all. If we assume there is a small effect of advertising, then the sales response curve is concave and again there is a ceiling effect at around 200 GRPs.

Figure 5.1: Response curves of gums



Liquorish

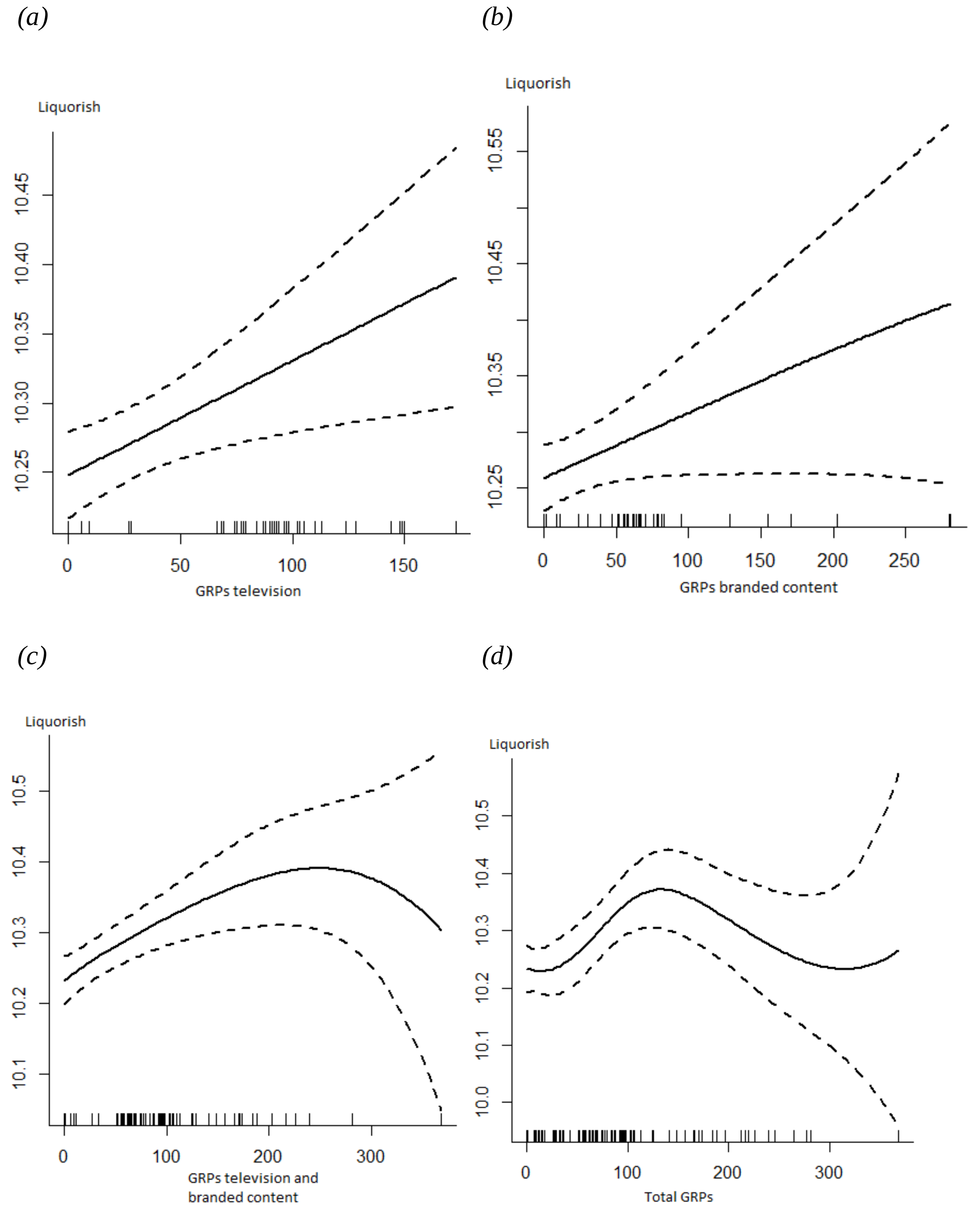
The coefficient estimates of the linear components of the semiparametric model are presented in Table 5.2. The linear components are the same variables as in the ADF model for liquorish. Although in the ADL model for liquorish the effect of advertising through television and branded content is estimated separately, the results in Table 5.1 are obtained by adding the GRPs deployed through television advertising and branded content together. Again the coefficient estimates are not significantly different from the estimates in the ADL model.

Table 5.2: Semiparametric estimation results of liquorish

Dependent variable	log of sold volume liquorish (in kg)	
Variable	Coefficient	Standard Error
Constant	7.100**	0.656
Q2	0.087**	0.032
Q4	-0.161**	0.038
Promo AH	0.206**	0.066
Promo Agrimarkt	0.231**	0.071
Promo C1000	0.132**	0.050
Promo Dirk	0.132**	0.052
Promo Plus	0.237**	0.048
First lag of sales	0.302**	0.064
Sig.: 5%(*), 1%(**)		

In the ADL models for liquorish both advertising through television and branded content are significant, therefore both advertising channels are estimated semiparametric. Figures 5.2a and 5.2b indicate that the response of sales to advertising is linear, however the 95% confidence intervals are quite wide. Therefore we should be careful drawing any conclusions based on the estimated response curve. When the GRPs from television advertising and branded content are added together the response curve is concave and shows a ceiling effect around 250 GRPs, see Figure 5.2c. For Figure 5.2d the GRPs from all media channels are added together, the response curve is now S-shaped. The inflection point is around 75 GRPs and the ceiling effect occurs at around 150 GRPs.

Figure 5.2: Response curves of liquorish



Foam and marshmallows

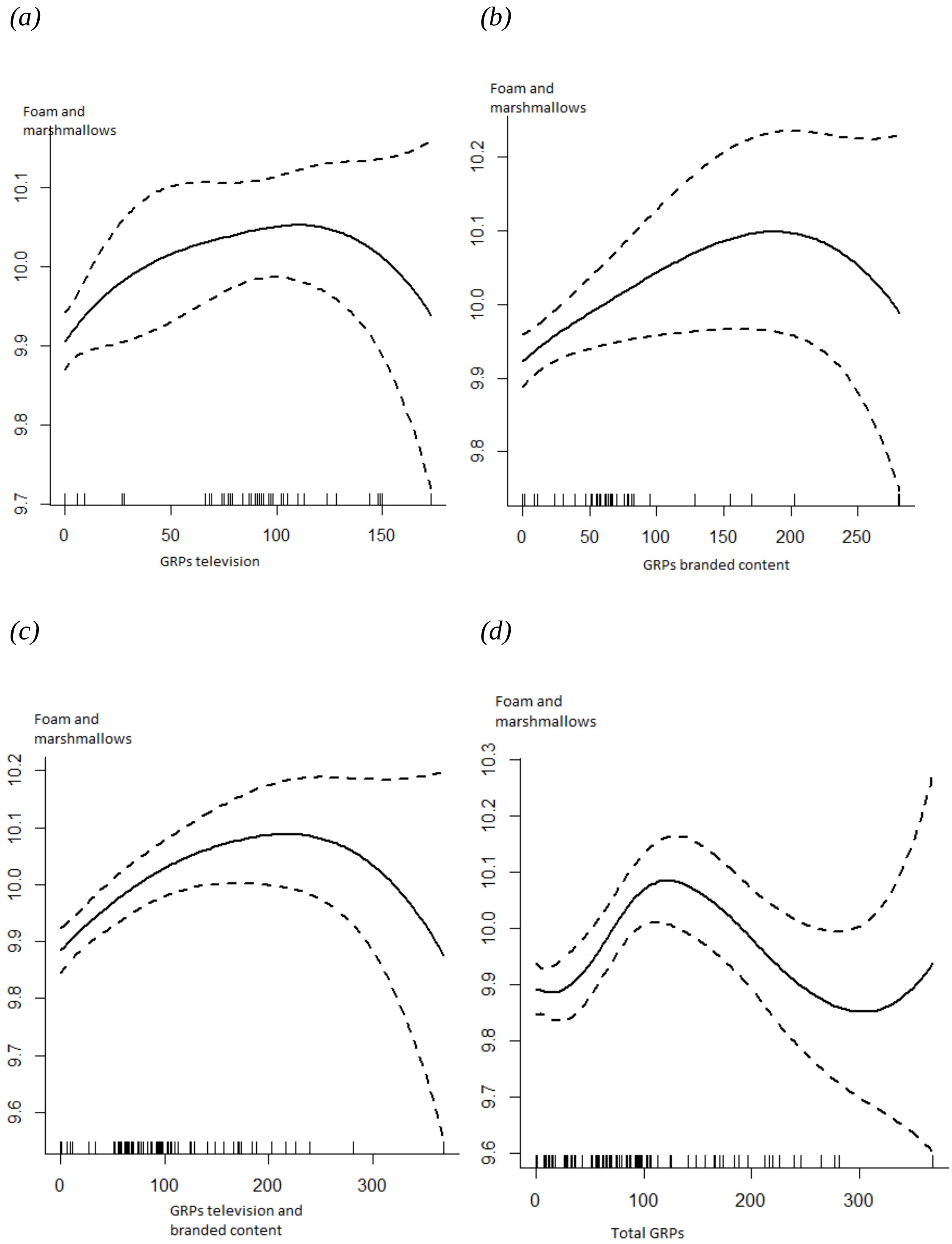
The coefficient estimates of the linear component of the semiparametric model for the category foam and marshmallows are presented in Table 5.3. The nonparametric part is television and branded content added together, since they were both significant in the ADL model. Again the coefficient estimates are not significantly different from the estimates in the ADL model.

Table 5.2: Semiparametric estimation results of foam and marshmallows

Dependent variable	log of sold volume foam and marshmallows (in kg)	
Variable	Coefficient	Standard Error
Constant	7.473**	0.613
Q2	0.096**	0.035
Q4	-0.165**	0.042
Promo AH	0.537**	0.075
Promo C1000	0.154**	0.056
Promo Plus	0.263**	0.055
First lag of sales	0.244**	0.060

The sales response curves for television advertising and branded content in Figures 5.3a and 5.3b respectively, show a concave and bounded response curve. The 95% confidence intervals are too wide to draw any conclusions about the shape of the response curve. When the GRPs of television and branded content are added together, the sales response curve is concave and bounded. Like the sales response curve of liquorish, when the GRPs from all the media channels are added together, the curve is S-shaped. The inflection point is again around 75 GRPs and the ceiling effect for foam & liquorish also occurs around 150 GRPs.

Figure 5.3: Response curves of foam & marshmallows



Conclusion

In this thesis the advertising sales response curve is estimated for confectionaries sold through supermarkets in the Netherlands. The data covers the period from week 1 of 2011 until week 40 of 2013. The sales are divided into three categories: gums, liqourish and foam & marshmallows. In the category gums the holidays Halloween and Sint Maarten have a significant positive impact on sales, whereas these holidays do not effect the sales of the other two categories. Also the start of the summer vacations of the elementary schools only cause an increase in the sales of the category gums. It seems that the category gums has a larger share of custumors younger than twelve compared to the other two categories. You could argue whether the results can be compared across the three categories.

However we argue that we can compare the results across the three categories, since there are still enough characteristics which they have in common: the sales of the three categories exhibit the same quarterly effects; the sold volume of all three categories are stationary; and the markets of all three categories are stable. Furthermore, there was no negative effect found of competitive promotions or advertising in all three categories. Also, there was no significant effect of economical or climatological variables.

The question of endogeneity of advertising is adressed by estimating SVAR-X models, where sales and advertising are included as endogenous variables. The Granger causality tests for all three categories show that sales does not lead advertising. Therefore the assumption that advertising is exogenous for weekly data cannot be rejected. In the categories liquorish and foam & marshmallows the Granger causality test do indicate that advertising lead sales.

The single equation models did not find a delayed effect of advertising on sales, probably because the brand is well established. Consumers are already familiar with the brand and do not have to go through the series of six steps of the Hierarchy of Effects Model. Therefore the logistic adstock models reduces to the conventional Autoregressive Distributed Lag models.

The semiparametric regressions clearly show that when the effect of television and branded are estimated separately, the sales response curve is linear. Therefore even a linear specification would suffice. This is caused because the majority of the observations of television and branded content lies between 50 and 100 GRPs, which lies on the linear part of the advertising sales response curve. However, when the GRPs from television advertising and branded content are added together, the range of observation increases and the advertising sales response curve clearly shows diminishing returns and a ceiling effect. The resemblance of the negative exponential response curves in Figure 3.4 and the semiparatmetric response curves in Figure 5.1a, 5.2b and 5.3c is striking, indicating a ceiling effect at around 200 GRPs.

The evidence for a threshold effect and thus an S-shaped response curve is inconclusive. Figures 5.2d and 5.3d clearly show an S-shaped functions. However the majority of the observations which lie on the lower part of the response curve are from the media channels print and bill boarding, for which both no significant effect was found. Thus the estimated threshold effect might be caused because advertising through the media channels print and bill boarding have no significant effect on sales.

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