Signal Analysis Assignment #2

Due on September 7th, 2020

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PUL074BEX007

Problem 1

Find the even and odd components of $x(t) = e^{jt}$.

Solution:

For any signal $x(t)$ in the continuous time domain, if $x_e(t)$ and $x_o(t)$ represent the even and odd components of that signal, they can be calculated as follows,

$$x_e(t) = \frac{1}{2}\left\{x(t) + x(-t)\right\}$$
$$x_o(t) = \frac{1}{2}\left\{x(t) - x(-t)\right\}$$

The signal we have is, $x(t) = e^{jt}$. So, the time-reversed signal can be written as, $x(-t) = e^{-jt}$. So, from these statements we can figure out the even and odd components of the signal $x(t)$ as,

$$\begin{aligned} x_e(t) &= \frac{1}{2}\left\{e^{jt} + e^{-jt}\right\} \\ &= cos(t) \\ x_o(t) &= \frac{1}{2}\left\{e^{jt} - e^{-jt}\right\} \\ &= j\frac{1}{2j}\left\{e^{jt} - e^{-jt}\right\} \\ &= jsin(t) \end{aligned}$$

Thus, the even and odd components of the signal $x(t) = e^{jt}$ are $cos(t)$ and $jsin(t)$ respectively.

Problem 2

Find the even and odd components of the signal shown in Figure 1.

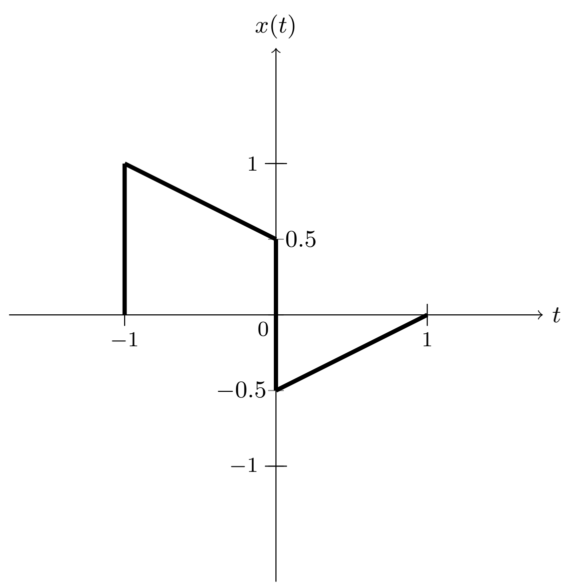
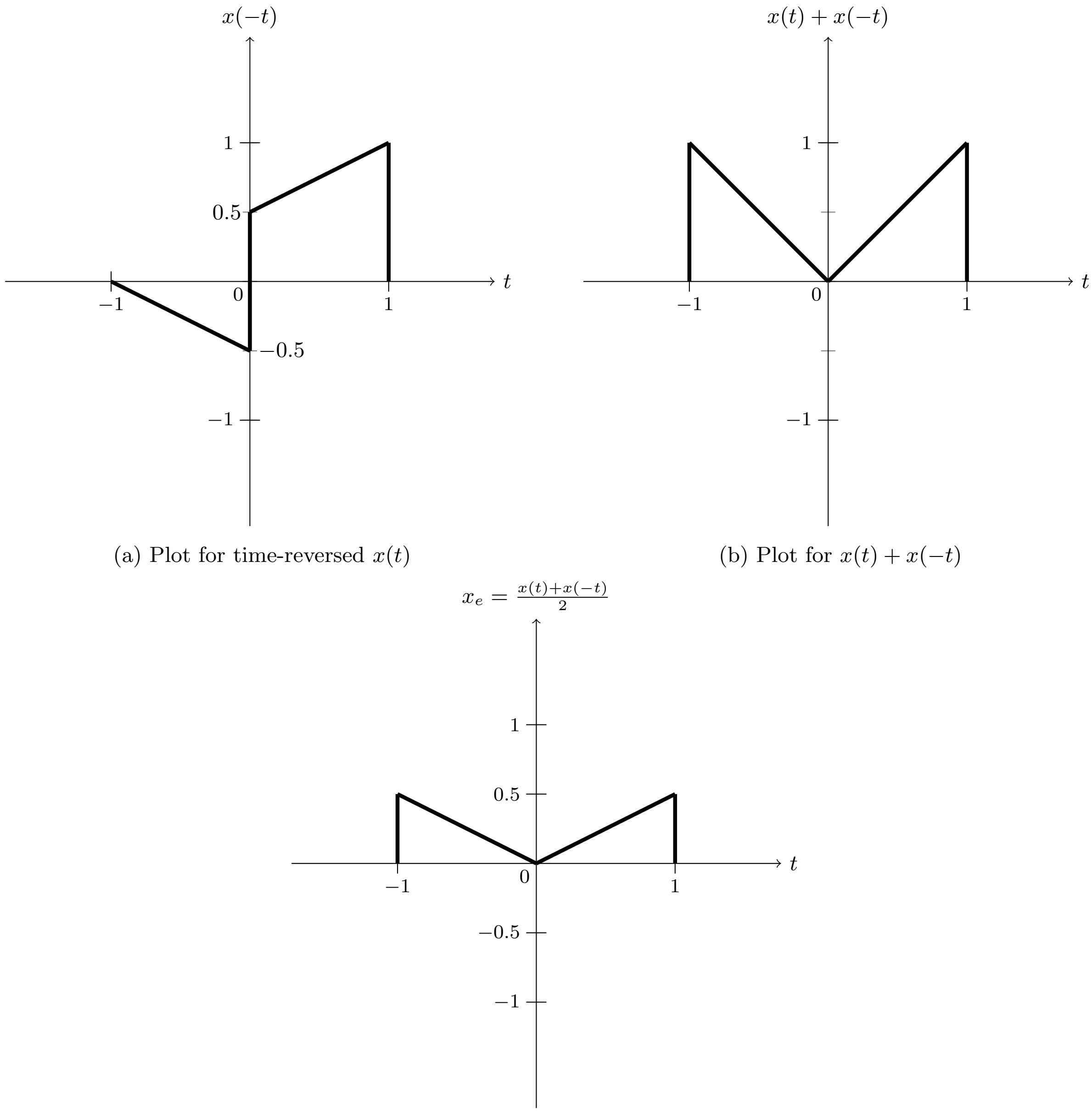


Figure 1: Plot for $x(t)$

Solution:

For any signal $x(t)$ in the continuous time domain, if $x_e(t)$ and $x_o(t)$ represent the even and odd components of that signal, they can be calculated as follows,

$$x_e(t) = \frac{1}{2}\left\{x(t) + x(-t)\right\}$$
$$x_o(t) = \frac{1}{2}\left\{x(t) - x(-t)\right\}$$

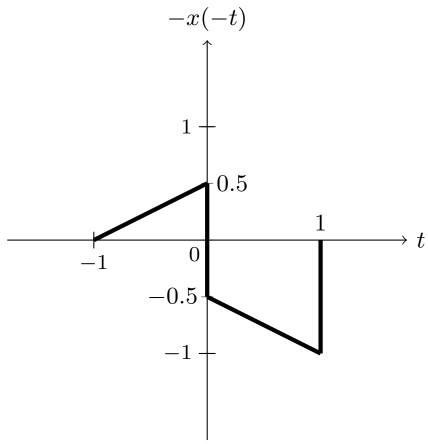


(a) Plot for time-reversed $x(t)$

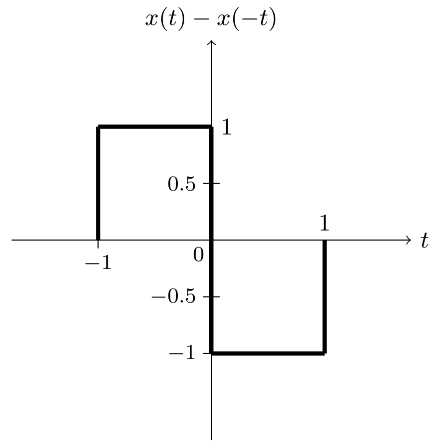
(b) Plot for $x(t) + x(-t)$

(c) Plot for even component of $x(t)$ i.e. $x_e(t)$

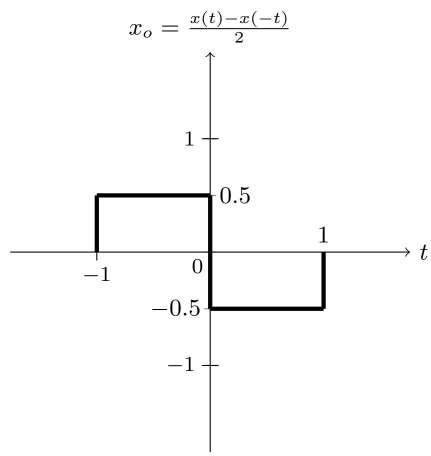
Figure 2: Plots for determining the even component of $x(t)$



(a) Plot for time-reversed and amplitude-reversed $x(t)$



(b) Plot for $x(t) - x(-t)$



(c) Plot for odd component of $x(t)$ i.e. $x_o(t)$

Figure 3: Plots for determining the odd component of $x(t)$

Figure 2c and Figure 3c represent the even and odd components of the original signal $x(t)$.

Problem 3

An exponential function $x(t) = e^{-2t}$ shown in Figure 4 is delayed by 1 second. Sketch and matematically describe the delayed function. Repeat the problem with $x(t)$ advanced by 1 second.

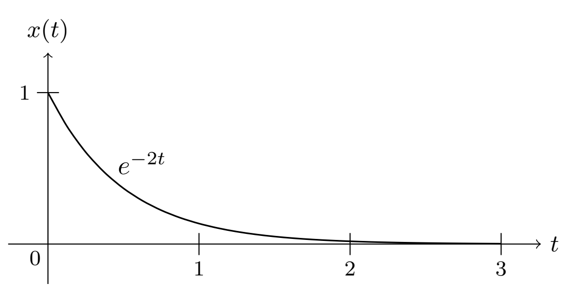


Figure 4: Plot for $x(t) = e^{-2t}$

Solution:

Time-delay and time-advancements are quite necessary transformations of a signal with great significance in communications. Advancing or reflecting a signal is only possible in real-time, whereas for a recorded signal, delay is also possible.
A signal $x(t-\tau)$ is said to be delayed in time by a positive value τ seconds for the original signal $x(t)$. It is to be noted that delaying a signal by τ seconds means that the original signal has been shifted to the right and the corresponding value of $x(0)$ now exists at $t-\tau=0$ for $x(t-\tau)$. This means the signal $x(t-\tau)$ starts τ seconds later than $x(t)$ hence beign delayed. Similarly, a signal $x(t+\tau)$ is said to be advanced in time by a positive value τ seconds for the original signal $x(t)$. The signal is shifted to the left and hence starts before the original signal $x(t)$. The corresponding value of $x(0)$ exists at $t+\tau=0$ for $x(t+\tau)$. This means the signal $x(t+\tau)$ starts τ seconds earlier than $x(t)$ hence beign advanced.

It is given that, $x(t) = e^{-2t}$,
so for the signal to be time-delayed by 1 second, we have,

$$x(t-1) = e^{-2(t-1)}$$

This means the signal $x(t)$ is shifted to the right by 1 second. Graphically, this is represented by Figure 5.

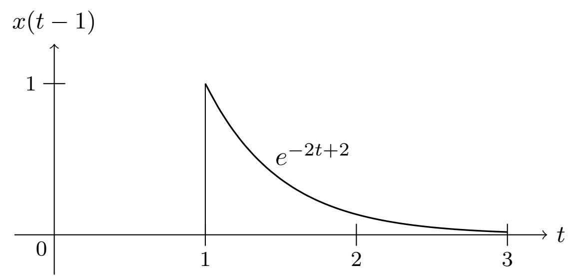


Figure 5: Plot for $x(t)$ delayed in time by 1 second, i.e. $x(t-1)$

Similarly, for the signal to be time-advanced by 1 second, we have,

$$x(t+1) = e^{-2(t+1)}$$

This means the signal $x(t)$ is shifted to the left by 1 second. Graphically, this is represented by Figure 6.

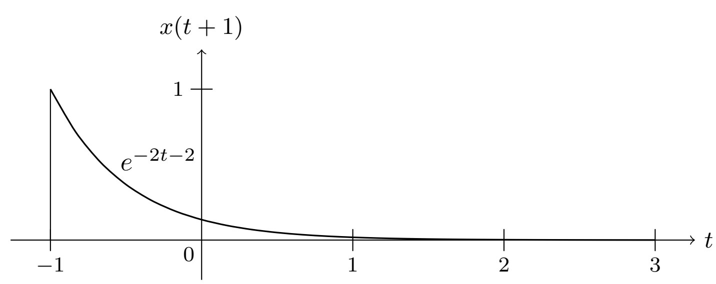


Figure 6: Plot for $x(t)$ advanced in time by 1 second, i.e. $x(t+1)$

Problem 4

Figure 7 shows a signal $x(t)$. Sketch and describe mathematically this signal time-compressed by factor 3. Repeat the problem for the same signal time-expanded by factor 2.

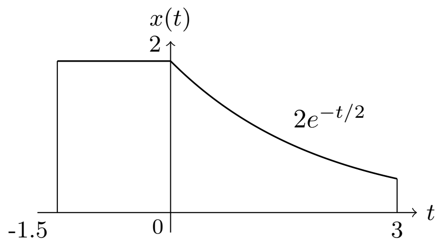


Figure 7: Plot for $x(t)$

Solution:

The signal $x(t)$ can be described as,

$$x(t) = \begin{cases} 2 & -1.5 \leq t < 0 \\ 2e^{-t/2} & 0 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}$$

The time-compressed result of a signal $x(t)$ by a factor of k is calculated as $x(kt)$. Since the horizontal axis is scaled by a factor of k, the resulting signal is time-compressed.
Mathematically,

$$x(3t) = \begin{cases} 2 & -1.5 \leq 3t < 0 \\ 2e^{-3t/2} & 0 \leq 3t < 3 \\ 0 & \text{otherwise} \end{cases}$$

This can be further reduced to get,

$$x(3t) = \begin{cases} 2 & -0.5 \leq t < 0 \\ 2e^{-1.5t} & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

A plot for the time-compressed signal is shown in Figure 8.

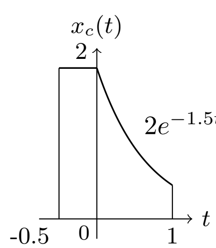


Figure 8: Plot for $x(t)$ time-compressed by factor 3

Similarly, the time-expanded result of a signal $x(t)$ by a factor of k is calculated as $x(\frac{t}{k})$. Since the horizontal axis is being reduced by a factor of k, the resulting signal is time-expanded by the same

factor.
Mathematically,

$$x\left(\frac{t}{2}\right) = \begin{cases} 2 & -1.5 \leq \frac{t}{2} < 0 \\ 2e^{-t/4} & 0 \leq \frac{t}{2} < 3 \\ 0 & \text{otherwise} \end{cases}$$

This can be further reduced to get,

$$x\left(\frac{t}{2}\right) = \begin{cases} 2 & -3 \leq t < 0 \\ 2e^{-t/4} & 0 \leq t < 6 \\ 0 & \text{otherwise} \end{cases}$$

A plot for the time-expanded signal is shown in Figure 9.

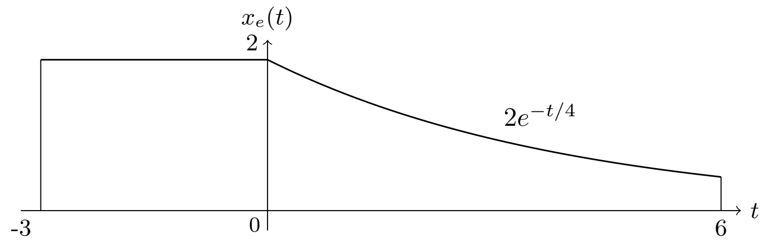


Figure 9: Plot for $x(t)$ time-expanded by factor 2

Problem 5

For the signal $x(t)$, sketch $x(-t)$, which is time-reversed $x(t)$.

$$x(t) = \begin{cases} e^{t/2} & -1 \geq t > -5 \\ 0 & \textbf{otherwise} \end{cases}$$

Solution:

A signal $x(-t)$ is said to be time reversed or reflected for the original signal $x(t)$. This is visualized by flipping the sigal $x(t)$ about the origin.

Mathematically,

$$x(-t) = \begin{cases} e^{-t/2} & -1 \geq -t > -5 \\ 0 & \text{otherwise} \end{cases}$$

$$x(-t) = \begin{cases} e^{-t/2} & 1 \leq t < 5 \\ 0 & \text{otherwise} \end{cases}$$

This can be simply realized by taking the mirror image of the signal $x(t)$ about the y-axis. Figure 10 represents the original signal $x(t)$ and Figure 11 represents the time-reversed signal $x(-t)$ for the given ranges.

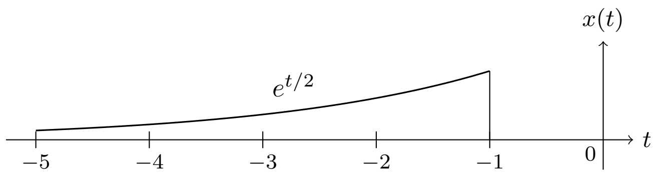


Figure 10: Plot for $x(t)$

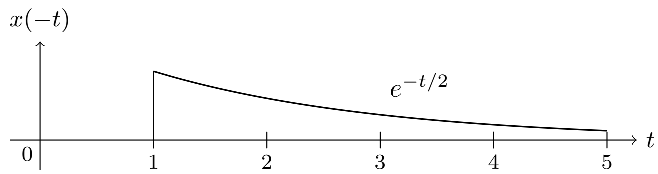


Figure 11: Plot for $x(-t)$

Problem 6

Define rectangular pulse, signum function and ramp function both in continuous and discrete time.

Solution:

Rectangular Pulse

A rectangular pulse function otherwise known as rectangle function or gate function is simply a function that has an amplitude of A inside a certain internal and 0 otherwise.

A rectangular pulse in **continuous time** with a period of T is defined as,

$$p_\tau(t) \triangleq \begin{cases} A & \frac{-LT}{2} \leq t \leq \frac{LT}{2} \\ 0 & \text{otherwise} \end{cases}$$

Likewise, a unit rectangular pulse with period of τ centered at (0,0) is defined as,

$$p_\tau(t) \triangleq \begin{cases} 1 & \frac{-\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

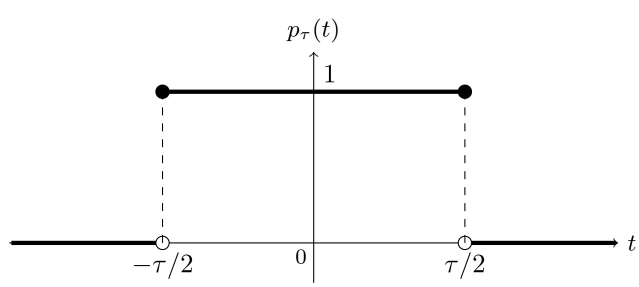


Figure 12: Plot for unit rectangular pulse in continuous time

Similarly, the discrete version of the rectangular pulse can be obtained by sampling. For $m = 2\left(\frac{\tau}{2T}\right)$ where T is the sampling period, a rectangular pulse in **discrete-time** is defined as,

$$p_m[n] \triangleq \begin{cases} A & \frac{-m}{2} \leq n \leq \frac{m}{2} \\ 0 & \text{otherwise} \end{cases}$$

A unit rectangular pulse in discrete-time centered at (0,0) is defined as,

$$p_m[n] \triangleq \begin{cases} 1 & \frac{-m}{2} \leq n \leq \frac{m}{2} \\ 0 & \text{otherwise} \end{cases}$$

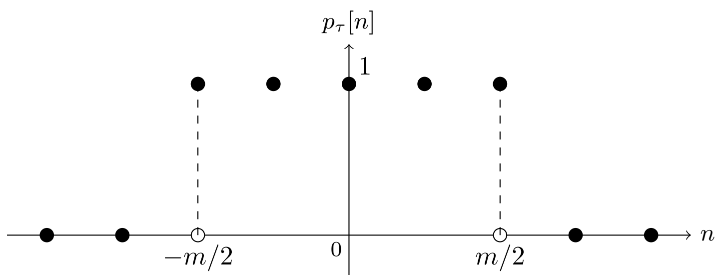


Figure 13: Plot for unit rectangular pulse in discrete time

Signum Function
A signum function is one that has amplitude +1 for the positive x-axis, −1 for the negative x-axis and 0 at origin, which is why it is also known as a sign function.

Mathematically, in **continuous time domain**, a signum function is defined as,

$$sgn[t] \triangleq \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

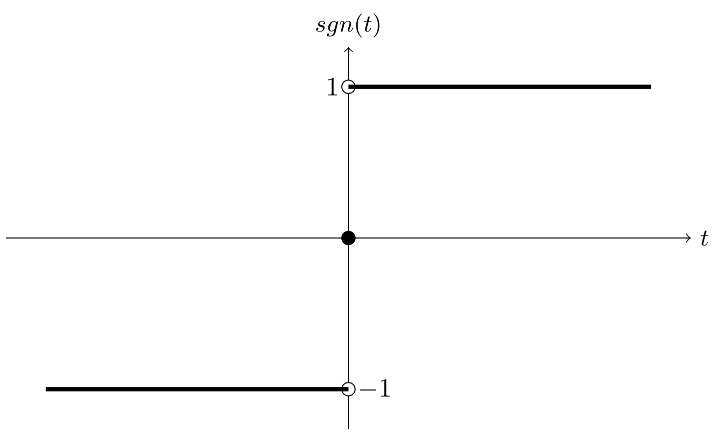


Figure 14: Plot for signum function in continuous time

Similarly, the **discrete time** version of the signum function is defined as,

$$sgn[n] \triangleq \begin{cases} 1 & n > 0 \\ 0 & n = 0 \\ -1 & n < 0 \end{cases}$$

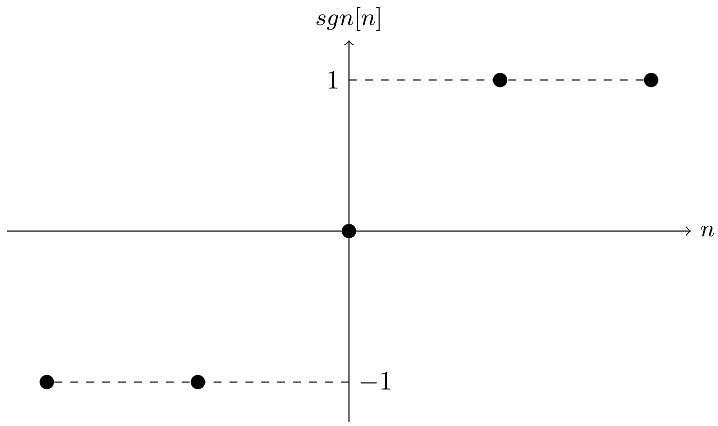


Figure 15: Plot for signum function in discrete time

Ramp Function

A unit ramp signal is one that has slope equal to one for the positive x-axis and 0 for the negative x-axis. The signal in **continuous time domain** can be defined as,

$$r(t) \triangleq \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

A regular ramp function with an arbitrary slope α for $t > 0$ can be defined as $r_\alpha = \alpha r(t)$.

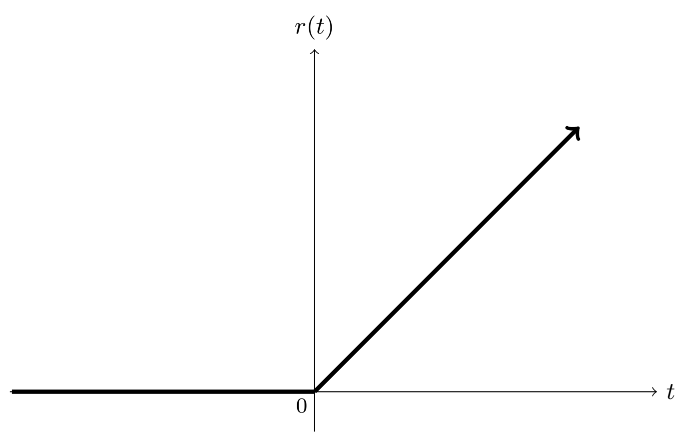


Figure 16: Plot for unit ramp function in continuous time

Likewise, the sampling of the unit-ramp signal $r(t)$gives the **discrete time** version of the ramp signal as,

$$r[nT] \triangleq r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

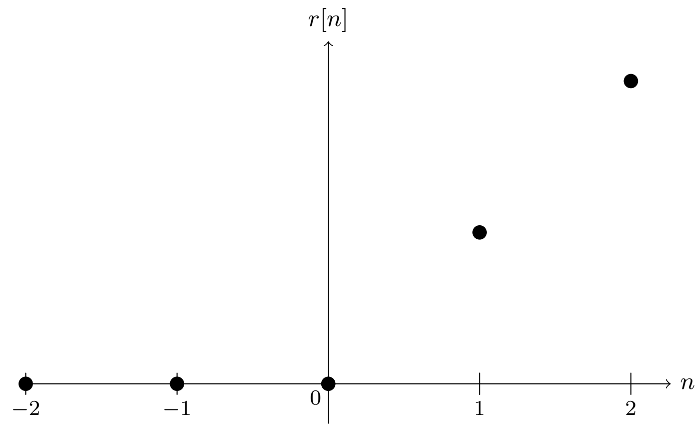


Figure 17: Plot for unit ramp function in discrete time