Analysis of Filter Networks

Lab Exercises on June 2, 2021

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1   Objective

- Analyze the given filter networks.

2   Required Tools

2.1   Proteus Design Suite

Proteus Design Suite, which is a professional PCB layout, circuit design and simulation tool, will be used to simulate the circuit for the filters to be analyzed. Proteus provides access to simulated circuit elements and different analyzer components to visualize the response plot for the given circuits.

2.2   MATLAB

MATLAB is a computing platform that allows users to analyze data, develop algorithms and visualize plots, that would otherwise be a complex task.

3   Filter Networks

(a)   (b)
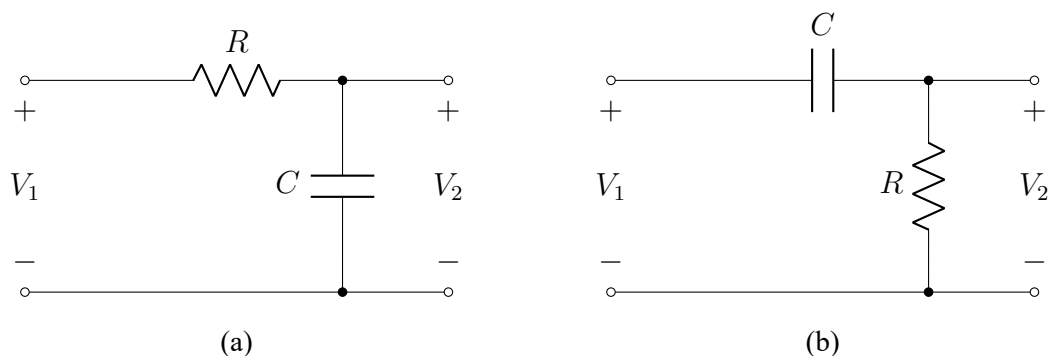
Figure 1: Filter networks to be analyzed

4 Exercises

Problem 1

Derive the transfer function of each of the network and determine the nature of filter network; (i.e. whether it is lowpass, highpass, bandpass or bandstop).

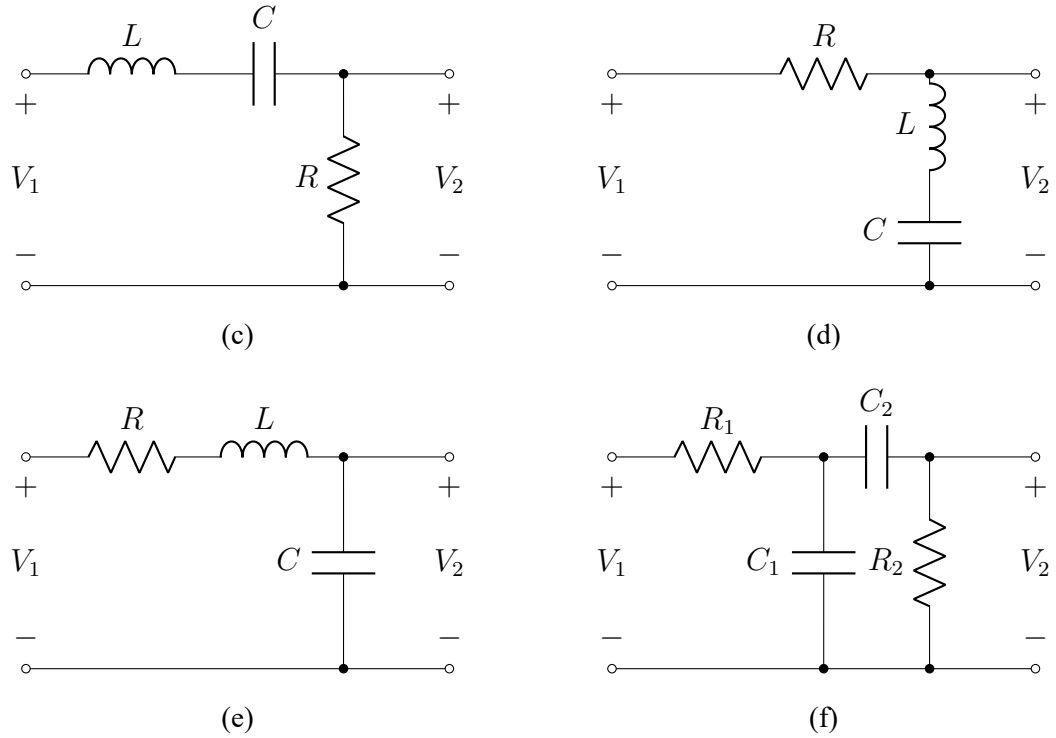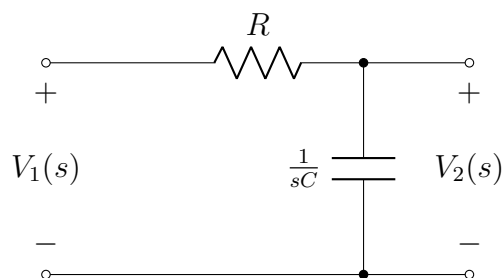**Figure 1a**

Using voltage divider rule,

$$V_2(s) = \frac{V_1(s).\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$\therefore H(s) = \frac{1}{sRC + 1} \tag{1}$$

From the transfer function in Equation 1, the pole is located at $-\frac{1}{RC}$ and zero is at ∞. Such type of s-plane singularity position represents a **low pass** filter.

Figure 1b
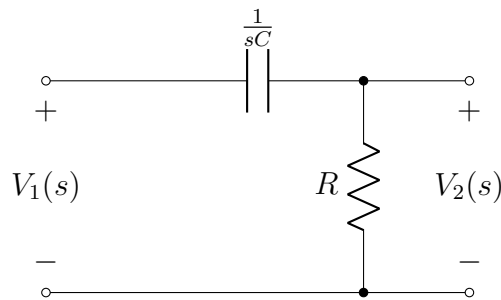
Using voltage divider rule,

$$V_2(s) = \frac{V_1(s).R}{R + \frac{1}{sC}}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{sRC}{sRC + 1}$$

$$\therefore H(s) = \frac{sRC}{sRC + 1} \tag{2}$$

From the transfer function in Equation 2, the pole is located at $-\frac{1}{RC}$ and zero is at 0. Such type of s-plane singularity position represents a **high pass** filter.

Figure 1c
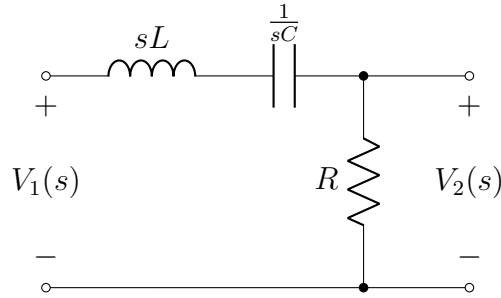
Using voltage divider rule,

$$V_2(s) = \frac{V_1(s).R}{R + \frac{1}{sC} + sL}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{sRC}{s^2LC + sRC + 1}$$

$$\therefore H(s) = \frac{sRC}{s^2LC + sRC + 1} \tag{3}$$

The transfer function in Equation 3 is of a **band pass** filter. It can also be identified as the combination of L and C in the given path provides 0 impedance at resonant frequency, which means the path is short-circuited and voltage across R is present at output. However, at other frequencies either the capacitor or inductor causes an open-circuit resulting in a band pass nature.

Figure 1d
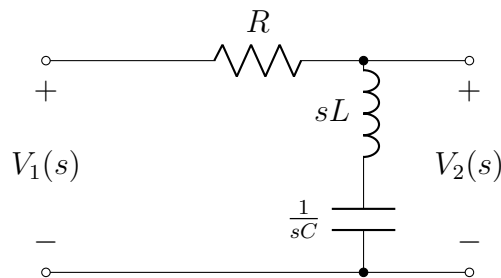
Using voltage divider rule,

$$V_2(s) = \frac{V_1(s)\left(sL + \frac{1}{sC}\right)}{R + \frac{1}{sC} + sL}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{s^2LC + 1}{sC\left(R + \frac{1}{sC} + sL\right)}$$

$$\therefore H(s) = \frac{s^2LC + 1}{s^2LC + sRC + 1} \tag{4}$$

The transfer function in Equation 4 is of a **band stop** filter. It can also be identified as the combination of L and C across which the output is measured provides 0 impedance at resonant frequency, which means the path is short-circuited and nearly 0 voltage is present at output. However, at other frequencies either the capacitor or inductor causes an open-circuit resulting in a band stop nature.
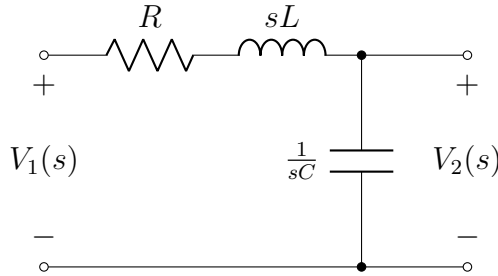
Figure 1e

Using voltage divider rule,

$$V_2(s) = \frac{V_1(s)\left(\frac{1}{sC}\right)}{R + \frac{1}{sC} + sL}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{1}{sC\left(R + \frac{1}{sC} + sL\right)}$$

$$\therefore H(s) = \frac{1}{s^2LC + sRC + 1} \tag{5}$$

The transfer function in Equation 5 is of a **low pass** filter. It can also be identified as the combination of L and R in the series path provides low impedance at low frequency during

which the capacitor provides large impedance, which means the series path is short-circuited and parallel path is open-circuited. However, at higher frequencies the inductor provides larger impedance and capacitor provides lower impedance resulting in a low pass nature.

Figure 1f
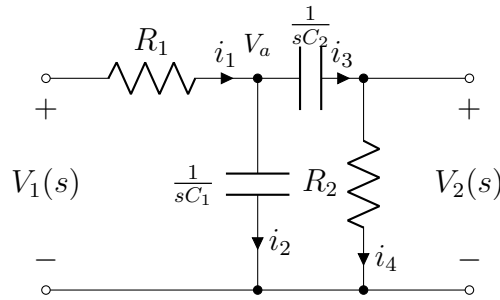
To find the transfer function, using nodal analysis as,

$$\because i_3 = i_4 \Rightarrow \frac{V_a(s) - V_2(s)}{\frac{1}{sC_2}} = \frac{V_2(s)}{R_2}$$

$$sR_2C_2V_a(s) = V_2(s)(sR_2C_2 + 1)$$

$$V_a(s) = V_2(s)\frac{sR_2C_2 + 1}{sR_2C_2} \tag{6}$$

$$\because i_1 = i_2 + i_3 \Rightarrow \frac{V_1(s) - V_a(s)}{R_1} = \frac{V_a(s)}{\frac{1}{sC_1}} + \frac{V_a(s) - V_2(s)}{\frac{1}{sC_2}}$$

$$V_1(s) - V_a(s) = sR_1C_1V_a(s) + sR_1C_2V_a(s) - sR_1C_2V_2(s)$$

$$V_1(s) = V_a(s)(sR_1C_1 + sR_1C_2 + 1) - sR_1C_2V_2(s) \tag{7}$$

Substituting value of $V_a(s)$ from Equation 6 into Equation 7, we get,

$$V_1(s) = V_2(s)\frac{sR_2C_2 + 1}{sR_2C_2}(sR_1C_1 + sR_1C_2 + 1) - sR_1C_2V_2(s)$$

$$sR_2C_2V_1(s) = V_2(s)(s^2R_1R_2C_1C_2 + s^2R_1R_2C_2{}^2 + sR_2C_2 + sR_1C_1 + sR_1C_2 + 1)$$

$$- s^2R_1R_2C_2{}^2V_2(s)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{sR_2C_2}{s^2R_1R_2C_1C_2 + sR_1C_1 + sR_1C_2 + sR_2C_2 + 1}$$

$$\therefore H(s) = \frac{sR_2C_2}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1} \tag{8}$$

The transfer function in Equation 8 is of a **band pass** filter. It can also be identified as the circuit is a cascaded form of a RC low pass and high pass filters such that a band between their overlapping pass frequencies is created.

Problem 2

Plot the magnitude and phase response of each network from the derived transfer function (using MATLAB or any tool) and note down the followings:

a. Gain in passband   b. Half power frequency/frequencies   c. Bandwidth

```
1  function  Ashlesh_bodePlot(num,den)    5  bode(h,options);
2  h=tf(num,den);                         6  grid on;
3  options = bodeoptions;                 7  end
4  options.FreqUnits = 'Hz';
```

Listing 1: Matlab function for entering transfer function and plotting bode diagram

```
1  function [R, L, C, R1, C1, R2, C2]    4  C1 = 0.01e-6; R2 = 10e3;
       = Ashlesh_componentValues()       5  C2 = 0.1e-6;
2  R = 1e3; L = 1e-3;                     6  end
3  C = 0.1e-6; R1 = 1e3;
```

Listing 2: Matlab function that returns the component values

```
1  [R, L, C, R1, C1, R2, C2] =           8   figure(4);
       Ashlesh_componentValues();        9   Ashlesh_bodePlot([L*C 0 1],[L*C R*C
2  figure(1);                                    1]);
3  Ashlesh_bodePlot([1],[R*C 1]);        10  figure(5);
4  figure(2);                            11  Ashlesh_bodePlot([1],[L*C R*C 1]);
5  Ashlesh_bodePlot([R*C 0], [R*C 1]);   12  figure(6);
6  figure(3);                            13  Ashlesh_bodePlot([R2*C2 0],[R1*R2*
7  Ashlesh_bodePlot([R*C 0],[L*C R*C            C1*C2 R1*C1+R1*C2+R2*C2 1]);
       1]);
```

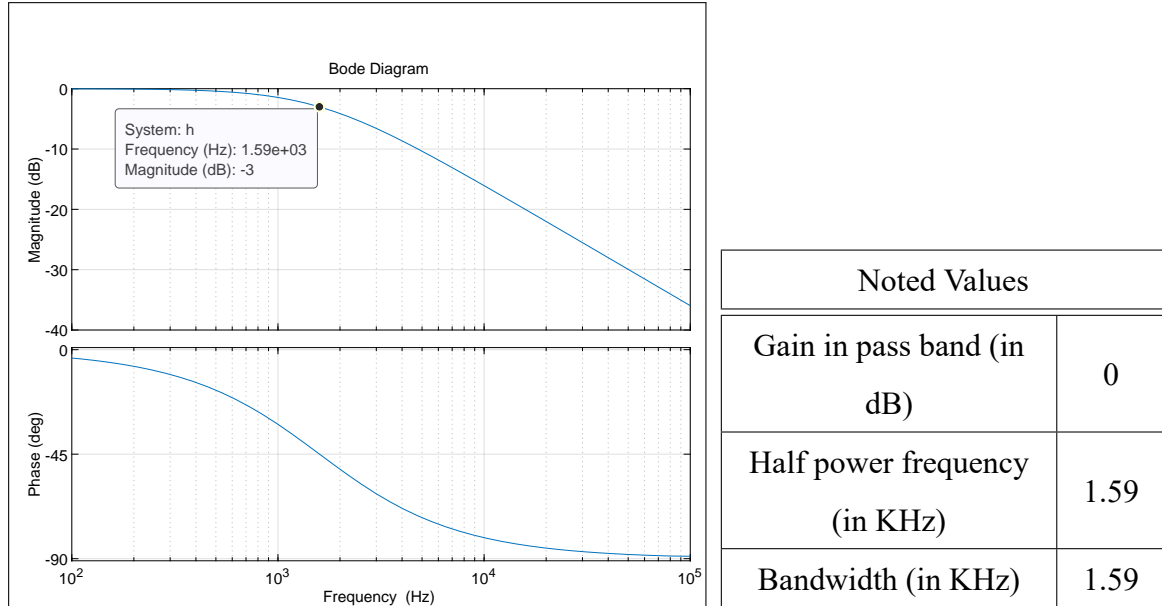Listing 3: Matlab script for different filter networks

Figure 2: Observation for Equation 1

Noted Values	
Gain in pass band (in dB)	0
Half power frequency (in KHz)	1.59
Bandwidth (in KHz)	1.59
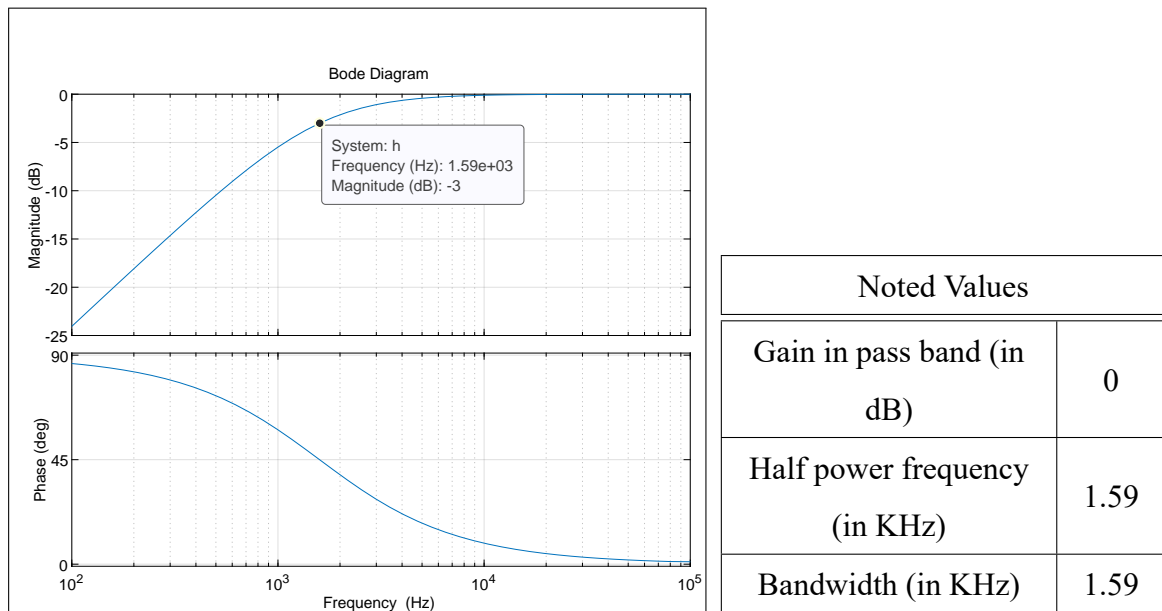
Figure 3: Observation for Equation 2

Noted Values	
Gain in pass band (in dB)	0
Half power frequency (in KHz)	1.59
Bandwidth (in KHz)	1.59

Figure 4: Observation for Equation 3
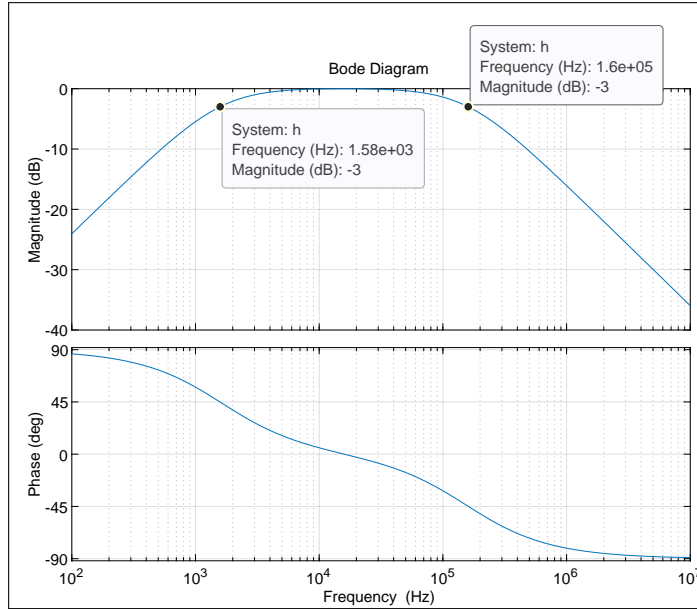
Noted Values	
Gain in pass band (in dB)	0
Half power frequencies (in KHz)	1.58, 160
Bandwidth (in KHz)	158.42

Figure 5: Observation for Equation 4
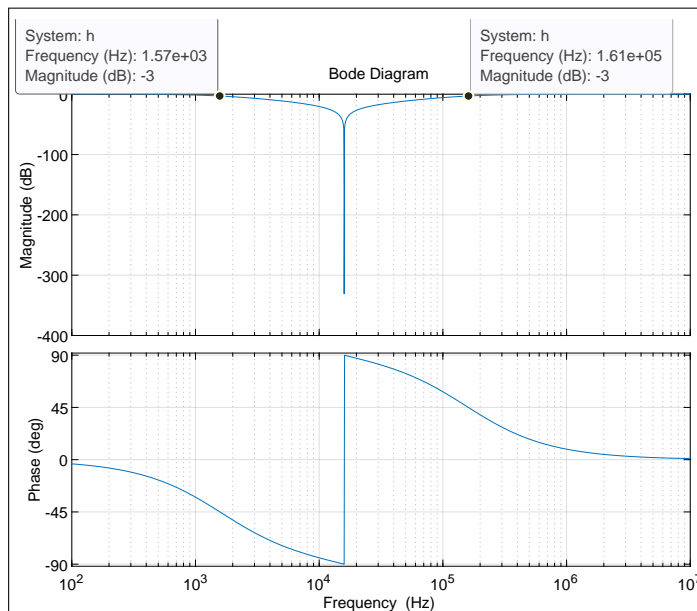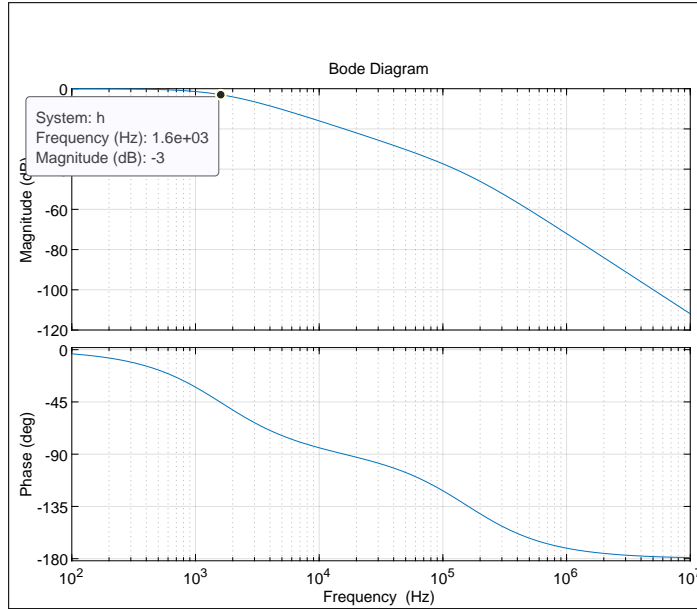
Noted Values	
Gain in pass band (in dB)	0
Half power frequencies (in KHz)	1.57, 161
Bandwidth (in KHz)	159.43

Figure 6: Observation for Equation 5

Noted Values	
Gain in pass band (in dB)	0
Half power frequency (in KHz)	1.6
Bandwidth (in KHz)	1.6

Figure 7: Observation for Equation 8

Noted Values	
Gain in pass band (in dB)	-0.9
Half power frequency (in KHz)	0.180, 14.1
Bandwidth (in KHz)	13.92

Problem 3

Observe the magnitude response of given filter networks by realizing the circuit using circuit simulation tools and note down the followings:

a. Gain in passband   b. Half power frequency/frequencies   c. Bandwidth

Figure 8: Simulated filter networks
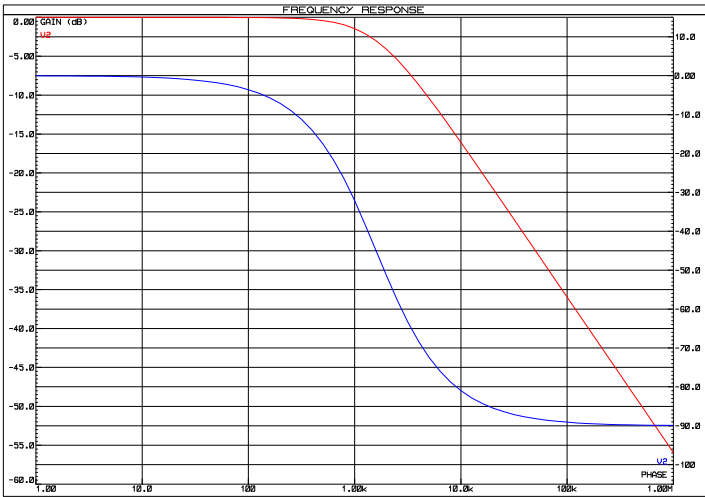
Noted Values	
Gain in pass band (in dB)	0
Half power frequency (in KHz)	1.59
Bandwidth (in KHz)	1.59

Figure 9: Observation for Figure 8a

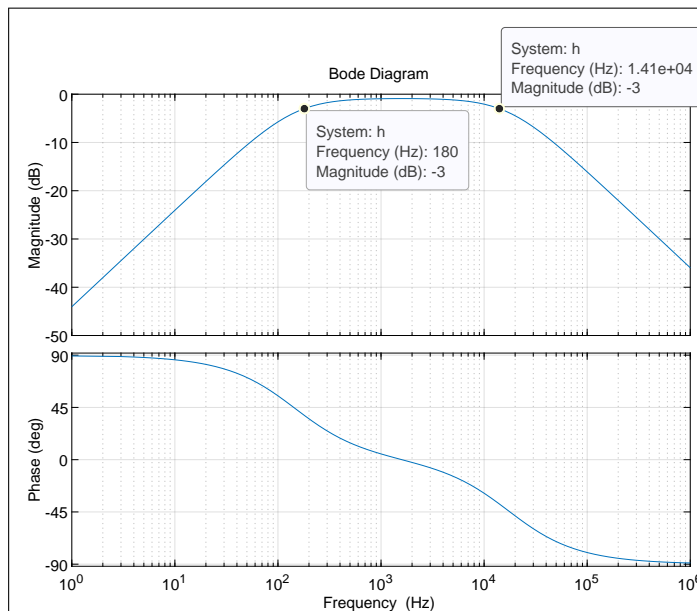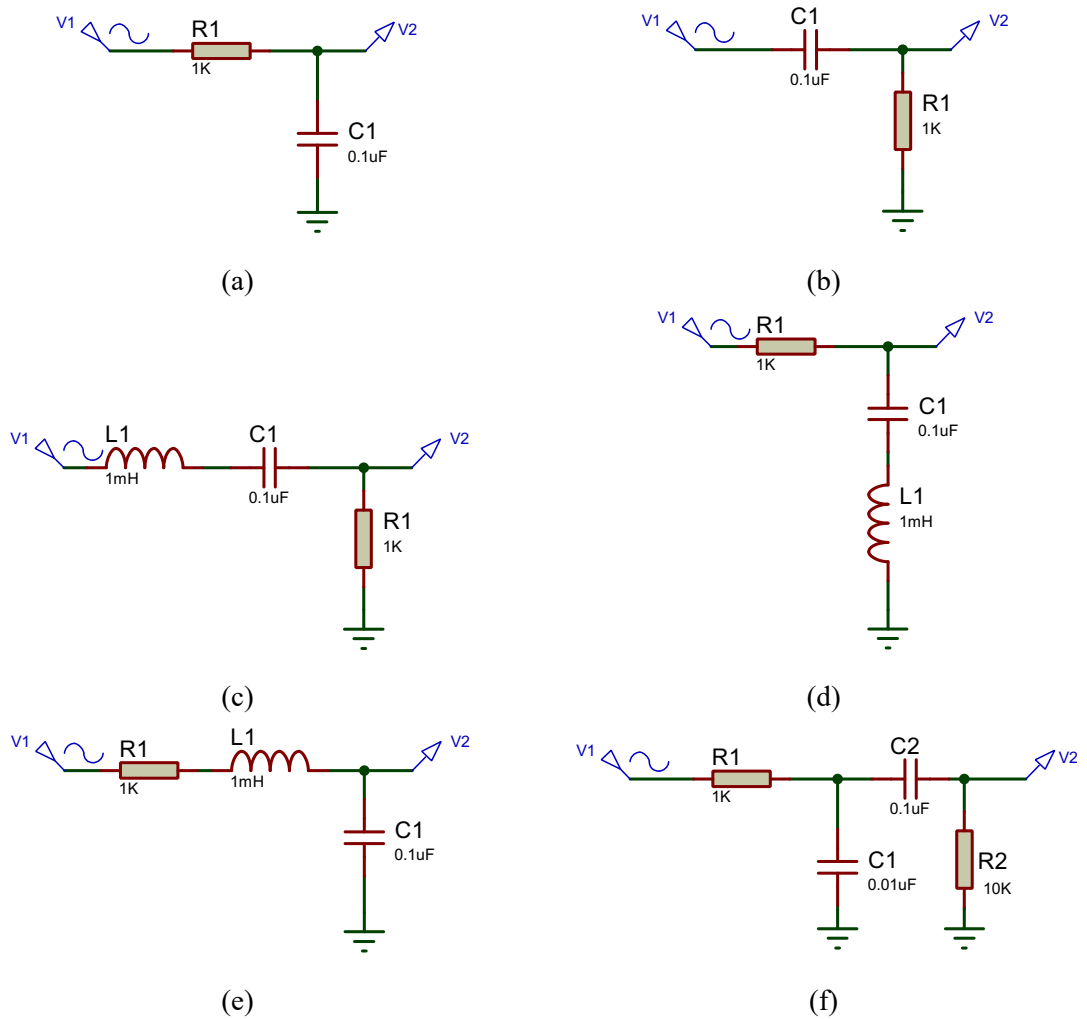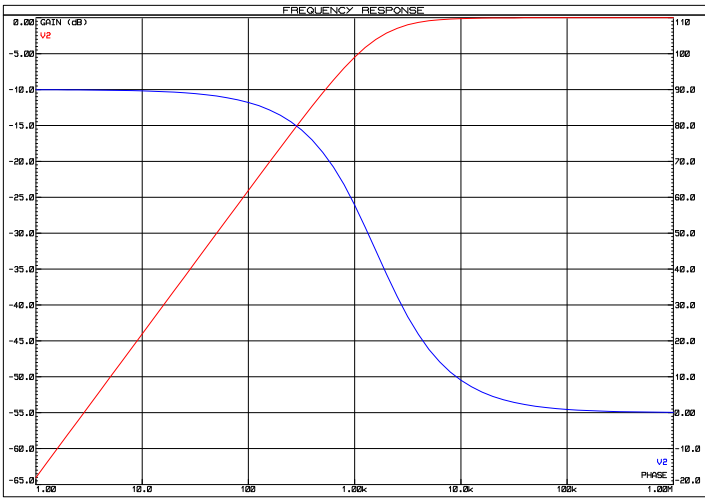| Noted Values	
Gain in pass band (in dB)	0
Half power frequency (in KHz)	1.6
Bandwidth (in KHz)	1.6

Figure 10: Observation for Figure 8b

Figure 11: Observation for Figure 8c

Noted Values	
Gain in pass band (in dB)	0
Half power frequencies (in KHz)	1.59, 160
Bandwidth (in KHz)	158.41

Figure 12: Observation for Figure 8d

Noted Values	
Gain in pass band (in dB)	0
Half power frequencies (in KHz)	1.57, 162
Bandwidth (in KHz)	160.43
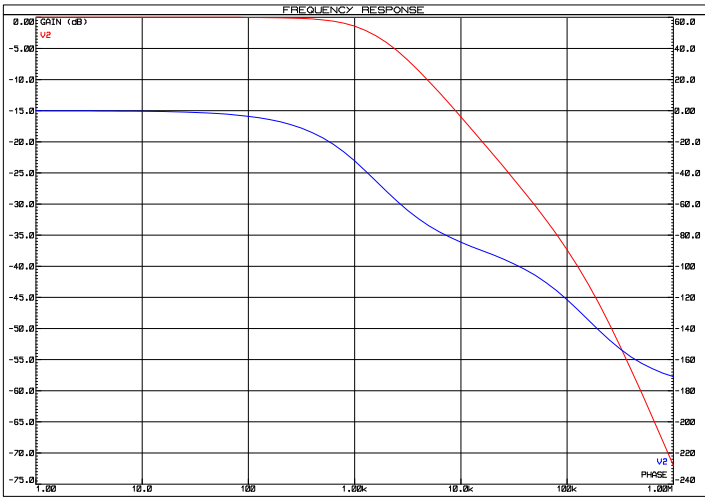
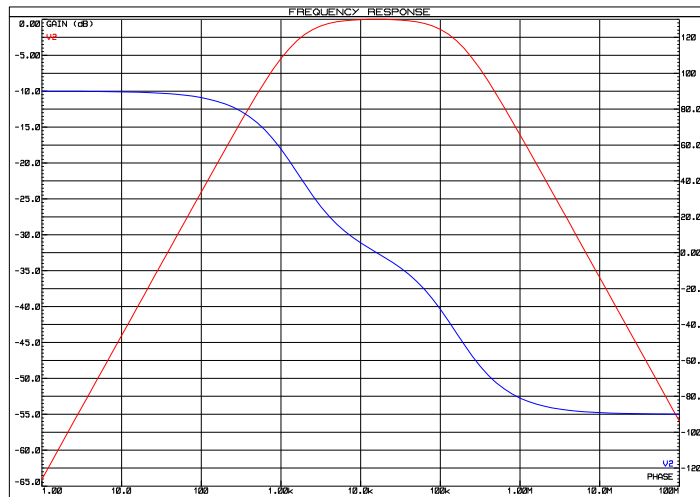| Noted Values	
Gain in pass band (in dB)	0 dB
Half power frequency (in KHz)	1.6
Bandwidth (in KHz)	1.6

Figure 13: Observation for Figure 8e
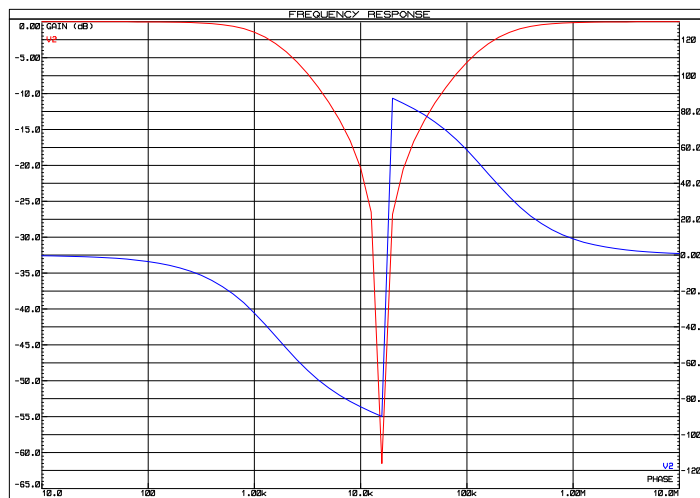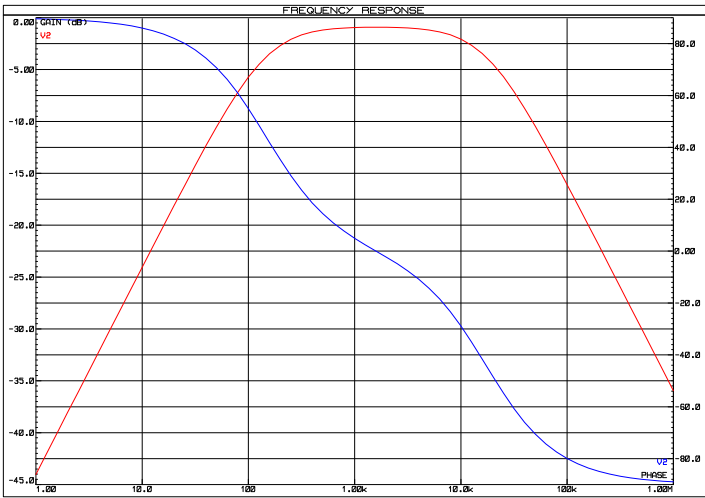

Noted Values	
Gain in pass band (in dB)	-0.9 dB
Half power frequency (in KHz)	0.183, 14.1
Bandwidth (in KHz)	13.917

Figure 14: Observation for Figure 8f

Compare the result obtained in Problem 2 and Problem 3.

The observations from Figure 2 to Figure 7 are for the various network's response plotted using MATLAB's control system toolbox. Similarly, the observations from Figure 9 to Figure 14 are for the same networks simulated in Proteus. It is clear from the two sets that the variation in observation is negligible. Derived transfer functions were used to observe the response in MATLAB, which gives the theoretical values. Similarly, since Proteus is a simulated environment, practical errors aren't visible in the observed values.

Component Values the result obtained in Problem 2 and Problem 3.

Component	Value
R	1 KΩ
L	1 mH
C	0.1 μF
R_1	1 KΩ
C_1	0.01 μF
R_2	10 KΩ
C_2	0.1 μF

Table 1: Original component values

Problem 4

Observe the effect in magniture response of above networks by increasing or decreasing the element values. Comment on your results for each of the given filter network.

The values of individual components were varied by -30%, -15%, +15% and +30% as variables in MATLAB since it was easier than changing them manually in Proteus. The change in parameters such as Gain in pass band, Half power frequency/frequencies and Bandwidth due to the variation in component values were observed.

Figure	Component		−30			−15			0			+15			+30	
		G	f	ω	G	f	ω	G	f	ω	G	f	ω	G	f	ω
1a	R	0	2.27	2.27	0	1.87	1.87	0	1.59	1.59	0	1.38	1.38	0	1.22	1.22
	C	0	2.27	2.27	0	1.87	1.87				0	1.38	1.38	0	1.22	1.22
1b	R	0	2.28	2.28	0	1.88	1.88	0	1.59	1.59	0	1.39	1.39	0	1.23	1.23
	C	0	2.28	2.28	0	1.88	1.88				0	1.39	1.39	0	1.23	1.23
1c	R	0	2.24, 113	110.76	0	1.86, 137	135.14	0	1.58, 160	158.42	0	1.38, 184	182.62	0	1.22, 207	205.78
	L	0	1.59, 228	226.43	0	1.59, 188	186.41				0	1.58, 140	138.42	0	1.58, 124	122.42
	C	0	2.25, 160	157.75	0	1.86, 160	158.14				0	1.38, 160	158.62	0	1.22, 160	158.78
1d	R	0	2.22, 114	111.78	0	1.84, 138	136.16	0	1.57, 161	159.43	0	1.37, 185	183.63	0	1.21, 209	207.79
	L	0	1.57, 230	228.43	0	1.57, 189	187.43				0	1.57, 140	138.43	0	1.56, 125	123.44
	C	0	2.23, 161	158.77	0	1.85, 161	159.15				0	1.37, 161	159.63	0	1.21, 161	159.79
1e	R	0	2.31	2.31	0	1.89	1.89	0	1.6	1.6	0	1.39	1.39	0	1.23	1.23
	L	0	1.6	1.6	0	1.6	1.6				0	1.6	1.6	0	1.6	1.6
	C	0	2.3	2.3	0	1.89	1.89				0	1.39	1.39	0	1.23	1.23
1f	R_1	−0.6	0.173, 20.9	20.727	−0.7	0.177, 16.9	16.723	−0.9	0.18, 14.1	13.92	−1.05	0.184, 12	11.816	−1.2	0.189, 10.3	10.111
	C_1	−0.9	0.18, 20.1	19.92	−0.9	0.18, 16.5	16.32				−0.9	0.18, 12.2	12.02	−0.9	0.18, 10.8	10.62
	R_2	−1.3	0.276, 13.1	12.824	−1.06	0.217, 13.7	13.483				−0.8	0.154, 14.3	14.146	−0.7	0.134, 14.5	14.366
	C_2	−0.9	0.258, 14	13.742	−0.9	0.212, 14	13.788				−0.9	0.157, 14	13.843	−0.9	0.139, 14	13.861

[1] G : Gain in pass band (in dB)
[2] f : Half power frequency (in KHz)
[3] ω : Bandwidth (in KHz)

Table 2: Observation for increase and decrease in component values

Table 2 shows a detailed observation for the parameters variation with change in component values. For low pass filter and high pass filter given in Figure 1a and Figure 1b respectively, the variation in either R or C showed similar behavior variation. For decrease in the value, the half power frequency and bandwidth increased, whereas with increase in the value, the half power frequency and bandwidth decreased.

For band pass filter and band stop filter given in Figure 1c and Figure 1d respectively, the decrease in value of R increased the lower half power frequency whereas decreased the upper half power frequency, leading to a decrease in bandwidth. On the contrary, increase in the value of R decreased the lower limit of the band whereas increased the upper limit, leading to an increase in bandwidth. For variation in L, negligible change was seen in the lower limit, whereas inverse change was observed for the upper limit, leading to an over all inverse relation for the bandwidth. For variation in C, negligible variation was seen in the upper limit, whereas inverse relation with the lower limit was observed. This lead to an overall direct relationship with the bandwidth change for variation in C.

For low pass filter given in Figure 1e, variation in R or C had an identical inverse effect on the half power frequency and bandwidth. However, for the variation in L, no change was observed for any parameters.

No change in gain in pass band was observed yet.

For band pass filter given in Figure 1f, the decrease in R_1 decreased the lower limit of the band whereas the upper frequency increased leading to an increase in bandwidth. Similarly the gain in bass band also increased. On the other hand, increase in the value of R_1 increased the lower frequency and decreased the upper limit of the band which led to a decrease in bandwidth. The gain in pass band decreased during this variation. For variation in C_1, the lower frequency of the band remained unchanged, whereas the upper limit showed an inverse relation leading to an overall inverse relation with bandwidth. The gain in pass band remained unchanged. For decrease in R_2, the lower limit of the band increased and the upper frequency decreased which meant the bandwidth decreased. The gain in pass band decreased. Likewise, for the increase in the value of R_2, the lower frequency decreased and upper frequency increased leading to an

increase in bandwidth. The gain in pass band increased during this variation. For variation in C_2, the upper frequency of the band remained somewhat unchanged, whereas the lower limit showed an inverse relation. The relation with bandwidth was somewhat uneven as indicated by the tabulated value. The gain in pass band remained unchanged.

5   Conclusion

The lab experiment for Analysis of Filter Networks concluded following the realization of the objectives. The analysis of filter networks based on their network elements, transfer functions, simulated plot allowed the visualization of the responses and determination of the different parameters.