Design of Active Filter using Tow Thomas Biquad Circuit

Lab Exercises on July 14, 2021

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1 Objectives

- To design a lowpass filter using Tow Thomas biquad circuit from given specifications.
- To obtain bandpass, highpass, bandstop and allpass filter using Tow Thomas biquad circuit.

2 Exercises

Problem 1

From the circuit shown in Figure 1 perform the following:

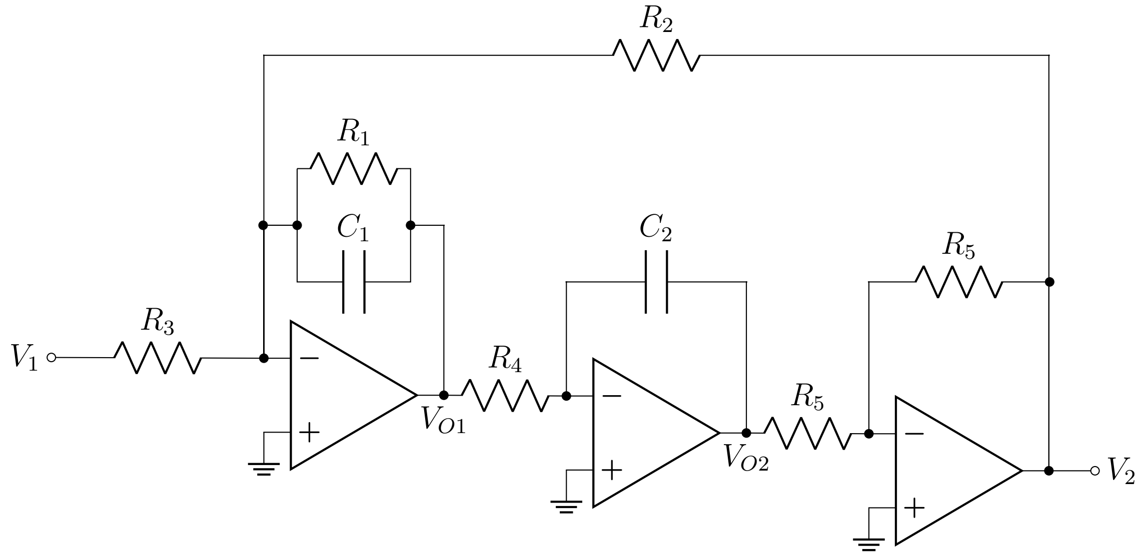


Figure 1: Tow Thomas biquad circuit

a. Derive the transfer function V_2/V_1 and determine the nature of the filter while taking output at V_2.

The derivation for transfer function using the circuit shown in Figure 1 is challenging if performed as a whole. So, we separate the circuit into three sections and determine the transfer function as:

Section I

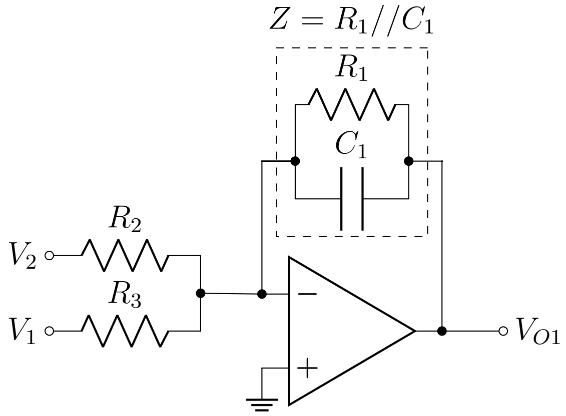


Figure 2: Lossy integrator (summer)

If we assume Z is the total impedance obtained due to the parallel combination of R_1 and C_1, then,

$$\frac{1}{Z} = \frac{1}{R_1} + sC_1 = \frac{1 + sR_1C_1}{R_1}$$
$$\Rightarrow Z = \frac{R_1}{1 + sR_1C_1}$$

From Figure 2, we can write,

$$V_{O1} = -\left(\frac{Z}{R_3}\right)V_1 - \left(\frac{Z}{R_2}\right)V_2 = -Z\left(\frac{V_1}{R_3} + \frac{V_2}{R_2}\right)$$
$$\therefore V_{O1} = -\left(\frac{R_1}{1 + sR_1C_1}\right)\left(\frac{V_1}{R_3} + \frac{V_2}{R_2}\right) \tag{1}$$

Section II

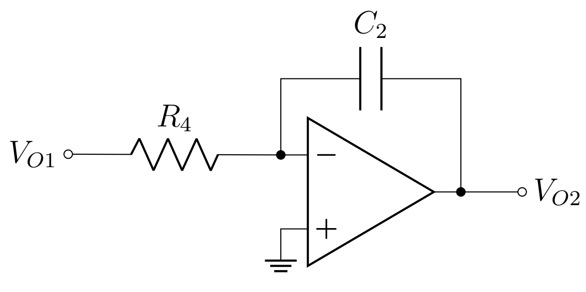


Figure 3: Inverting integration

From Figure 3, we can write,

$$V_{O2} = -\frac{V_{O1}}{sR_4C_2}$$

Substituting value of V_{O1} from Equation 1, we get,

$$\therefore V_{O2} = \left(\frac{R_1}{1 + sR_1C_1}\right)\left(\frac{1}{sR_4C_2}\right)\left(\frac{V_1}{R_3} + \frac{V_2}{R_2}\right) \tag{2}$$

Section III

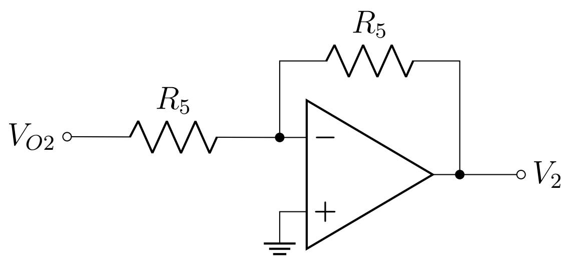


Figure 4: Unit gain inverter

From Figure 4, we can write,

$$V_2 = -\left(\frac{R_5}{R_5}\right)V_{O2} = -V_{O2}$$

Substituting value of V_{O2} from Equation 2, we get,

$$\text{or, } V_2 = -\left(\frac{R_1}{1 + sR_1C_1}\right)\left(\frac{1}{sR_4C_2}\right)\left(\frac{V_1}{R_3} + \frac{V_2}{R_2}\right)$$
$$\text{or, } V_2 = \frac{-V_1R_1}{sR_3R_4C_2(1 + sR_1C_1)} - \frac{V_2R_1}{sR_2R_4C_2(1 + sR_1C_1)}$$
$$\text{or, } V_2\left(1 + \frac{R_1}{sR_2R_4C_2(1 + sR_1C_1)}\right) = \frac{-V_1R_1}{sR_3R_4C_2(1 + sR_1C_1)}$$
$$\text{or, } \frac{V_2}{V_1} = \frac{-R_1R_2}{(sR_2R_4C_2(1 + sR_1C_1) + R_1)R_3}$$
$$\text{or, } \frac{V_2}{V_1} = \frac{-R_1R_2}{sR_2R_3R_4C_2(1 + sR_1C_1) + R_1R_3}$$
$$\text{or, } \frac{V_2}{V_1} = \frac{-R_1R_2}{s^2R_1R_2R_3R_4C_1C_2 + sR_2R_3R_4C_2 + R_1R_3}$$

$$\text{or, } \frac{V_2}{V_1} = \frac{-\left(\frac{R_1R_2}{R_1R_2R_3R_4C_1C_2}\right)}{s^2\left(\frac{R_1R_2R_3R_4C_1C_2}{R_1R_2R_3R_4C_1C_2}\right) + s\left(\frac{R_2R_3R_4C_2}{R_1R_2R_3R_4C_1C_2}\right) + \left(\frac{R_1R_3}{R_1R_2R_3R_4C_1C_2}\right)}$$

$$\therefore \frac{V_2}{V_1} = \frac{-\left(\frac{1}{R_3R_4C_1C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)} \tag{3}$$

We have the standard equation for the transfer function of a lowpass filter as,

$$T_{LP}(s) = \frac{-H\omega_o^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2} \tag{4}$$

Since Equation 3 is of the form shown in Equation 4, the nature of the filter while taking output at V_2 and input at V_1 is a lowpass.

b. Also obtain the transfer function while observing output at V_{O1} and input at V_1.

To obtain the transfer function while observing output at V_{O1} and input at V_1, we proceed as,

$$\frac{V_{O1}}{V_1} = \left(\frac{V_{O1}}{V_{O2}}\right)\left(\frac{V_{O2}}{V_2}\right)\left(\frac{V_2}{V_1}\right)$$

Substituting values for the terms on the RHS, we get,

$$\text{or, } \frac{V_{O1}}{V_1} = (-sR_4C_2)(-1)\left(\frac{-\left(\frac{1}{R_3R_4C_1C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\text{or, } \frac{V_{O1}}{V_1} = \frac{-s\left(\frac{R_4C_2}{R_3R_4C_1C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}$$

$$\therefore \frac{V_{O1}}{V_1} = \frac{-\left(\frac{1}{R_3C_1}\right)s}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)} \tag{5}$$

We have the standard equation for the transfer function of a bandpass filter as,

$$T_{BP}(s) = \frac{H\left(\dfrac{\omega_o}{Q}\right)s}{s^2 + s\left(\dfrac{\omega_o}{Q}\right) + \omega_o^2} \tag{6}$$

Since Equation 5 is of the form shown in Equation 6, the nature of the filter while taking output at V_{O1} and input at V_1 is a bandpass.

c. How can you obtain a bandstop, allpass and highpass filter using the Tow Thomas biquad circuit? Derive the transfer functions with necessary circuit diagrams.

To design bandstop, allpass and highpass filters using a Tow Thomas biquad circuit, we will require an additional op-amp to modify the original Tow Thomas biquad circuit shown in Figure 1, and create four op-amp biquad circuits.

Bandstop filter

Let us consider the voltages V_1 and V_{O1} are applied as input to the additional op-amp with inverting configuration as shown in Figure 5 for both bandpass and allpass filter design. We can then, for the output voltage V_2', write the relation,

$$V_2' = -(V_1 + V_{O1})$$
$$\Rightarrow \frac{V_2'}{V_1} = -\left(1 + \frac{V_{O1}}{V_1}\right)$$

Substituting value of $\frac{V_{O1}}{V_1}$ from Equation 5, we get,

$$\text{or, } \frac{V_2'}{V_1} = -\left(1 + \frac{-\left(\dfrac{1}{R_3C_1}\right)s}{s^2 + s\left(\dfrac{1}{R_1C_1}\right) + \left(\dfrac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\text{or, } \frac{V_2'}{V_1} = -\left(\frac{s^2 + s\left(\dfrac{1}{R_1C_1}\right) + \dfrac{1}{R_2R_4C_1C_2} - \left(\dfrac{1}{R_3C_1}\right)s}{s^2 + s\left(\dfrac{1}{R_1C_1}\right) + \left(\dfrac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\therefore \frac{V_2'}{V_1} = -\left(\frac{s^2 + s\left(\frac{1}{R_1C_1} - \frac{1}{R_3C_1}\right) + \frac{1}{R_2R_4C_1C_2}}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right) \tag{7}$$

We have the standard equation for the transfer function of a bandstop filter as,

$$T_{BS}(s) = \frac{-H(s^2 + \omega_o^2)}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2} \tag{8}$$

For Equation 7 to be of the form shown in Equation 8, $R_1 = R_3$ must be satisfied so that the nature of the filter while taking output at V_2' is a bandstop, whose final transfer function is given as,

$$\therefore T_{BS}(s) = \frac{V_2'}{V_1} = -\left(\frac{s^2 + \frac{1}{R_2R_4C_1C_2}}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right) \tag{9}$$

Allpass filter

We have the standard equation for the transfer function of an allpass filter as,

$$T_{AP}(s) = \frac{H\left(s^2 - \left(\frac{\omega_o}{Q}\right)s + \omega_o^2\right)}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2} \tag{10}$$

Comparing Equation 7 with Equation 10, we get two values of $\frac{\omega_o}{Q}$, one from numerator and one from denominator. On equating those values, we get,

$$-\left(\frac{1}{R_1C_1} - \frac{1}{R_3C_1}\right) = \frac{1}{R_1C_1} \Rightarrow \frac{2}{R_1C_1} = \frac{1}{R_3C_1} \Rightarrow R_3 = \frac{R_1}{2}$$

For Equation 7 to be of the form shown in Equation 10, $R_3 = \frac{R_1}{2}$ must be satisfied so that the nature of the filter while taking output at V_2' is an allpass, whose final transfer function is given as,

$$\therefore T_{AP}(s) = \frac{V_2'}{V_1} = -\left(\frac{s^2 - \left(\frac{1}{R_1C_1}\right)s + \frac{1}{R_2R_4C_1C_2}}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right) \tag{11}$$

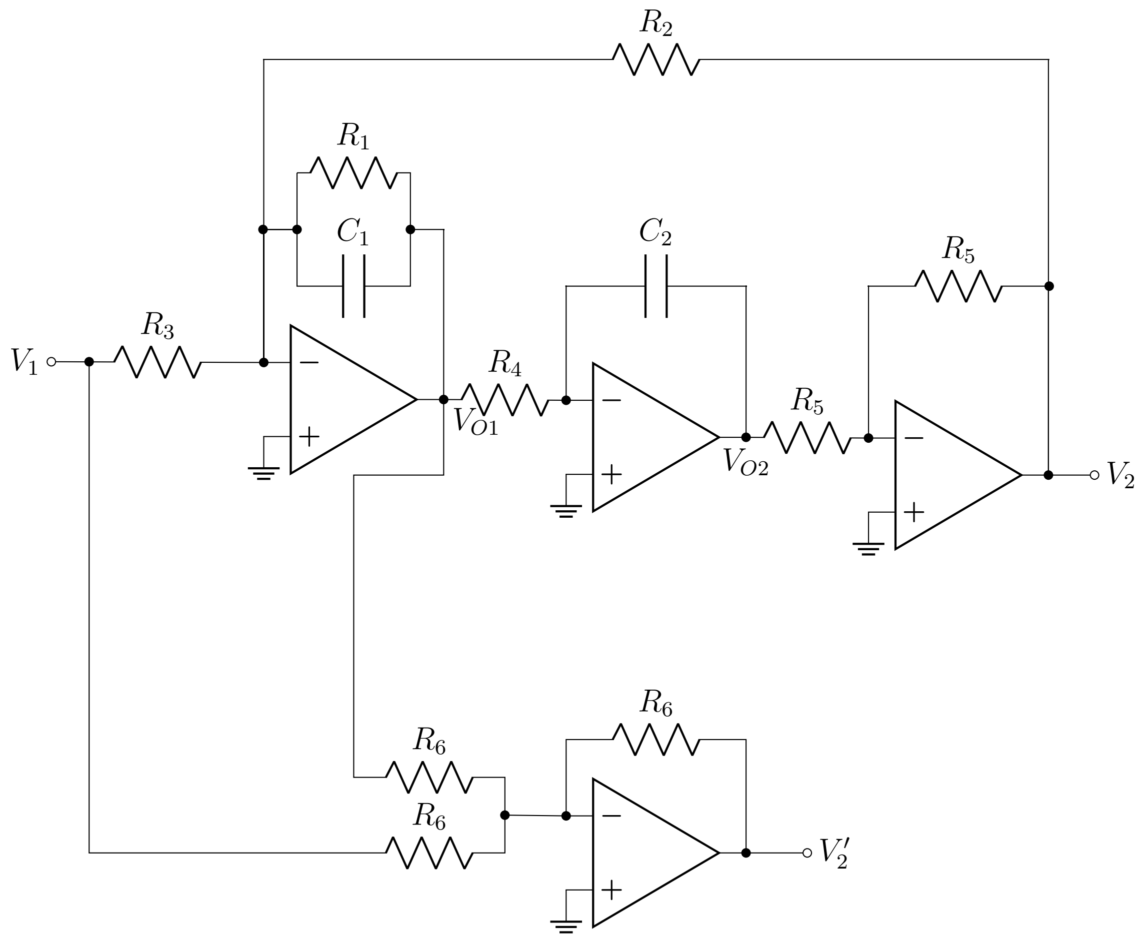


Figure 5: Four op-amp biquad circuit for bandstop and allpass filters

Highpass filter

Let us consider the voltages V_1, V_2 and V_{O1} are applied as input to the additional op-amp with inverting configuration as shown in Figure 6 for both bandpass and allpass filter design. We can then, for the output voltage V_2', write the relation,

$$
\begin{aligned}
V_2' &= -(V_1 + V_2 + V_{O1}) \\
\Rightarrow \frac{V_2'}{V_1} &= -\left(1 + \frac{V_2}{V_1} + \frac{V_{O1}}{V_1}\right)
\end{aligned}
$$

Substituting values of $\frac{V_2}{V_1}$ and $\frac{V_{O1}}{V_1}$ from Equation 3 and Equation 5 respectively, we get,

$$\text{or, } \frac{V_2'}{V_1} = -\left(1 - \frac{\left(\frac{1}{R_3R_4C_1C_2}\right) + \left(\frac{1}{R_3C_1}\right)s}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\text{or, } \frac{V_2'}{V_1} = -\left(\frac{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right) - \left(\frac{1}{R_3R_4C_1C_2}\right) - \left(\frac{1}{R_3C_1}\right)s}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\therefore \frac{V_2'}{V_1} = -\left(\frac{s^2 + \left(\frac{1}{R_1C_1} - \frac{1}{R_3C_1}\right)s + \left(\frac{1}{R_2R_4C_1C_2} - \frac{1}{R_3R_4C_1C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right) \tag{12}$$

We have the standard equation for the transfer function of a highpass filter as,

$$T_{HP}(s) = \frac{-Hs^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2} \tag{13}$$

For Equation 12 to be of the form shown in Equation 13, $R_1 = R_2 = R_3$ must be satisfied so that the nature of the filter while taking output at V_2' is a highpass, whose final transfer function is given as,

$$\therefore T_{HP}(s) = \frac{V_2'}{V_1} = -\left(\frac{s^2}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right) \tag{14}$$

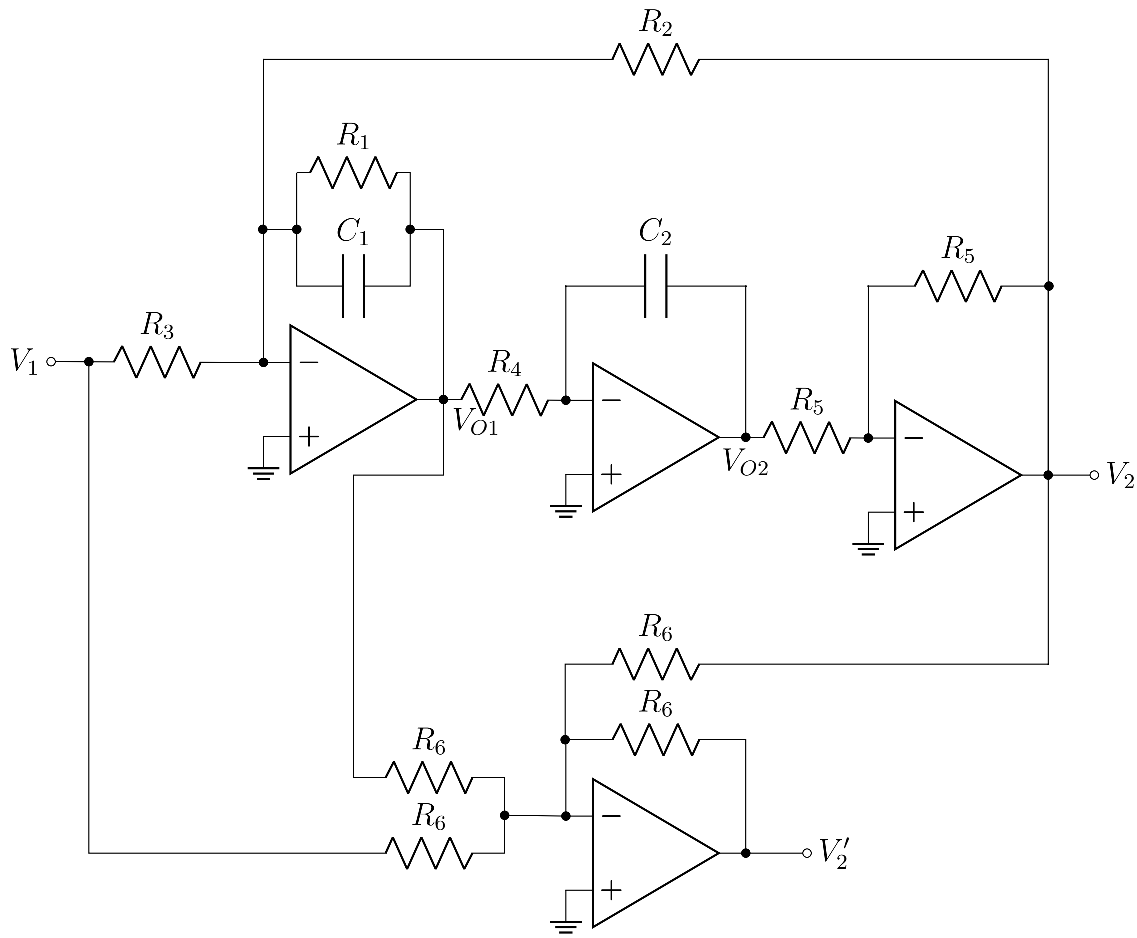


Figure 6: Four op-amp biquad circuit for highpass filter

Problem 2

Design a low-pass filter having poles at – 5000 ± j8660.25404 and a DC gain of 2 using Tow-Thomas biquad circuit. Your final circuit should consist of practically realizable elements. Realize the circuit and observe the magnitude response. And determine the characteristic features such as 3 dB frequency and DC gain.

For a lowpass filter with poles at $s = -\alpha \pm j\beta$, the denominator polynomial is calculated as,

$$f_{den}(s) = (s + \alpha + j\beta)(s + \alpha - j\beta) = (s + \alpha)^2 - j^2\beta^2$$
$$= (s + \alpha)^2 + \beta^2 = s^2 + 2\alpha s + \alpha^2 + \beta^2$$

For given pole location at $s = -5000 \pm j8660.25404$, i.e. $\alpha = 5000$ and $\beta = 8660.25404$, the denominator function is,

$$\begin{aligned} f_{den}(s) &= s^2 + 2 \times (5000)s + (5000)^2 + (8660.25404)^2 \\ &= s^2 + 10000s + 10^8 \end{aligned}$$

From Equation 4, the denominator function for a standard lowpass filter can be written as,

$$f'_{den}(s) = s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2$$

Comparing these equations, we get,

$$\begin{aligned} \omega_o^2 &= 10^8 \Rightarrow \omega_o = \sqrt{10^8} = 10^4 \\ \frac{\omega_o}{Q} &= 10^4 \Rightarrow Q = \frac{\omega_o}{10^4} = \frac{10^4}{10^4} = 1 \end{aligned}$$

Hence the filter parameters for the required filter are, $\omega_o = 10^4$, $Q = 1$ and $H = 2$. Comparing Equation 3 and Equation 4, we get the following relations,

$$\begin{aligned} \omega_o^2 &= \frac{1}{R_2R_4C_1C_2} \\ Q &= \sqrt{\frac{R_1^2C_1}{R_2R_4C_2}} \\ H &= \frac{R_2}{R_3} \end{aligned} \tag{15}$$

We normalize the design at $\omega_o = 1$ rad/s, $Q = 1$ and $H = 2$. Let us consider, $R_4 = 1\Omega$, $C_1 = C_2 = 1$ F. Using these values and the set of relations from Equation 15 to calculate the remaining elemental values as,

$$\begin{aligned} \omega_o^2 &= \frac{1}{R_2R_4C_1C_2} \Rightarrow R_2 = 1\Omega \\ Q &= \sqrt{\frac{R_1^2C_1}{R_2R_4C_2}} \Rightarrow R_1 = 1\Omega \\ H &= \frac{R_2}{R_3} \Rightarrow R_3 = 0.5\Omega \end{aligned}$$

Since the problem is to design the filter at $\omega_o = 10^4$ rad/s, $K_f = 10^4$. Let us choose $K_m = 10^4$. Also, as the value of R_5 doesn't affect in the actual response of the filter, we choose it as $R_5 = 1\Omega$. The scaled elemental values are,

$$R_1 = 1 \times 10^4 = 10\ \text{K}\Omega \qquad R_2 = 1 \times 10^4 = 10\ \text{K}\Omega$$

$$R_3 = 0.5 \times 10^4 = 5\ \text{K}\Omega \qquad R_4 = 1 \times 10^4 = 10\ \text{K}\Omega$$

$$C_1 = \frac{1}{10^4 \times 10^4} = 10\ \text{nF} \qquad C_2 = \frac{1}{10^4 \times 10^4} = 10\ \text{nF}$$

$$R_5 = 1 \times 10^4 = 10\ \text{K}\Omega$$

Note: The elemental notations are according to the Figure 1.

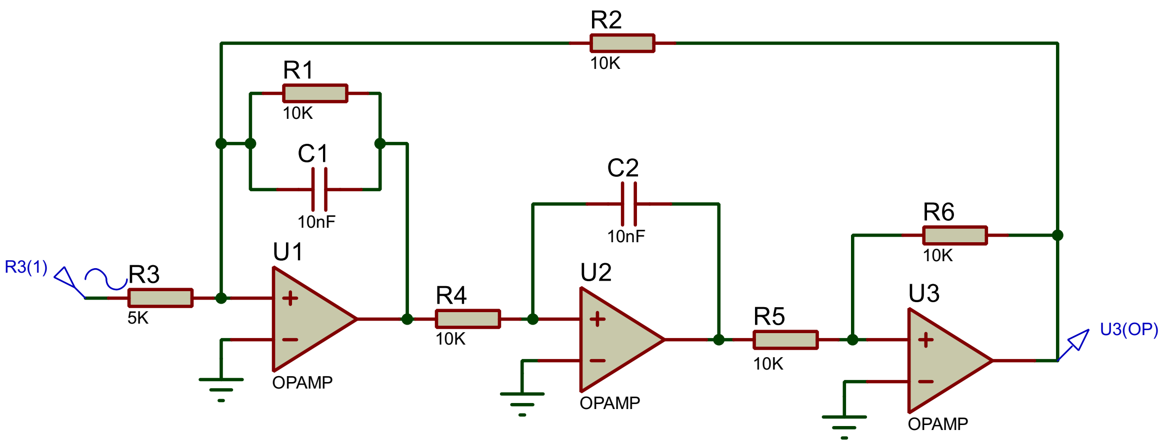
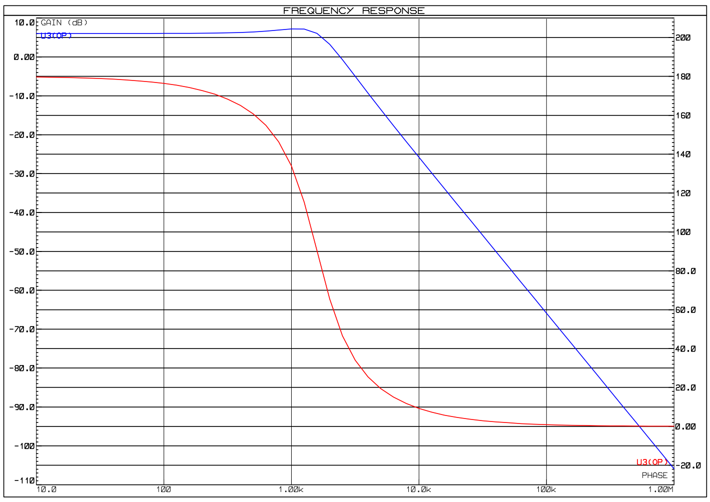


Figure 7: Proteus circuit for Tow Thomas biquad circuit with output taken at V_2



Noted Values	
DC gain (in dB)	6.02
Half power frequency (in KHz)	2.02

Figure 8: Observation for lowpass filter designed in Problem 2

a. Determine the nature of the response by observing output at V_{O1} with input V_1.

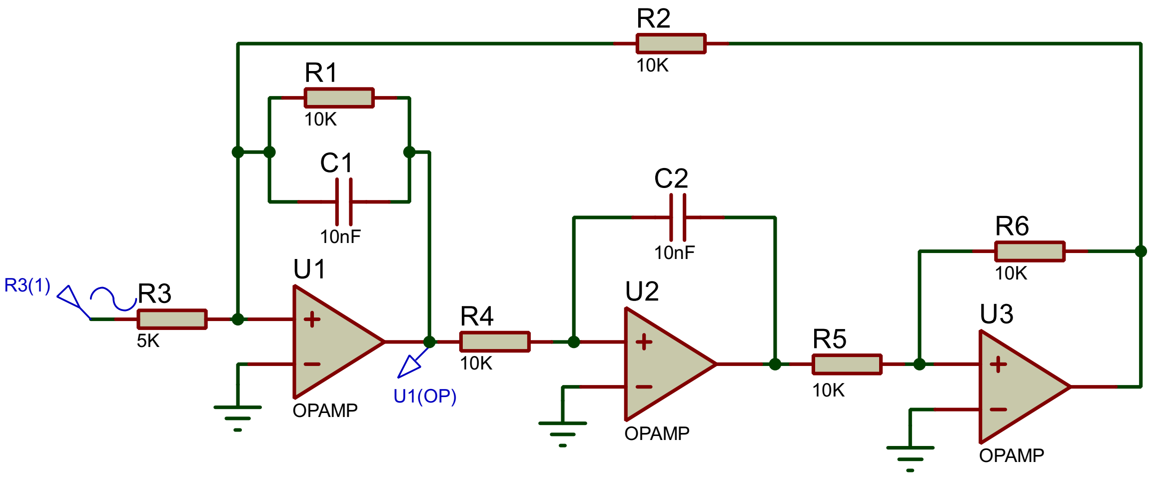
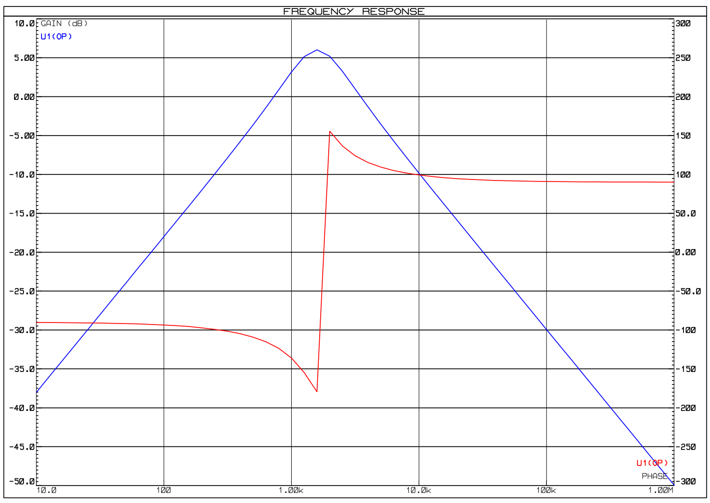


Figure 9: Proteus circuit for Tow Thomas biquad circuit with output taken at V_{O1}



Noted Values	
Gain in pass band (in dB)	6.02
Center frequency (in KHz)	1.58
Half power frequencies (in KHz)	0.987, 2.58
Bandwidth (in KHz)	1.593

Figure 10: Observation for bandpass filter designed in Problem 2

b. Observe the magnitude response by obtaining each of the following filter from your design and note down passband gain and half power frequencies:

i. Highpass filter

To design a highpass filter, we need a 4th op-amp in addition to the 3 op-amp Tow Thomas biquad circuit. We first design a general circuit using the required filter parameters, and then choose values that constraint a highpass response. As derived earlier, the required condition for a highpass response at V_2' is $R_1 = R_2 = R_3$.

The filter parameters for the required filter are, $\omega_o = 10^4$, $Q = 1$ and $H = 1$ (for a highpass filter, $H = 1$, as shown by comparing Equation 13 and Equation 14).

We normalize the design at $\omega_o = 1$ rad/s, $Q = 1$ and $H = 1$. Let us consider, $R_1 = R_2 = R_3 = 1\Omega$ and $C_2 = 1$ F. Using these values and the set of relations from Equation 15 to calculate the remaining elemental values as,

$$\omega_o^2 = \frac{1}{R_2 R_4 C_1 C_2} \Rightarrow R_4 C_1 = 1$$

$$Q = \sqrt{\frac{R_1^2 C_1}{R_2 R_4 C_2}} \Rightarrow R_4 = C_1$$

Solving for C_1 and R_4, we get, $C_1 = 1$ F and $R_4 = 1\Omega$.

Since the problem is to design the filter at $\omega_o = 10^4$ rad/s, $K_f = 10^4$. Let us choose $K_m = 10^4$. Also, as the values of R_5 and R_6 don't affect in the actual response of the filter, we choose it as $R_5 = R_6 = 1\Omega$. The scaled elemental values are,

$$R_1 = 1 \times 10^4 = 10 \text{ K}\Omega \qquad R_2 = 1 \times 10^4 = 10 \text{ K}\Omega$$

$$R_3 = 0.5 \times 10^4 = 5 \text{ K}\Omega \qquad R_4 = 1 \times 10^4 = 10 \text{ K}\Omega$$

$$R_5 = 1 \times 10^4 = 10 \text{ K}\Omega \qquad R_6 = 1 \times 10^4 = 10 \text{ K}\Omega$$

$$C_1 = \frac{1}{10^4 \times 10^4} = 10 \text{ nF} \qquad C_2 = \frac{1}{10^4 \times 10^4} = 10 \text{ nF}$$

Note: The elemental notations are according to the Figure 6.

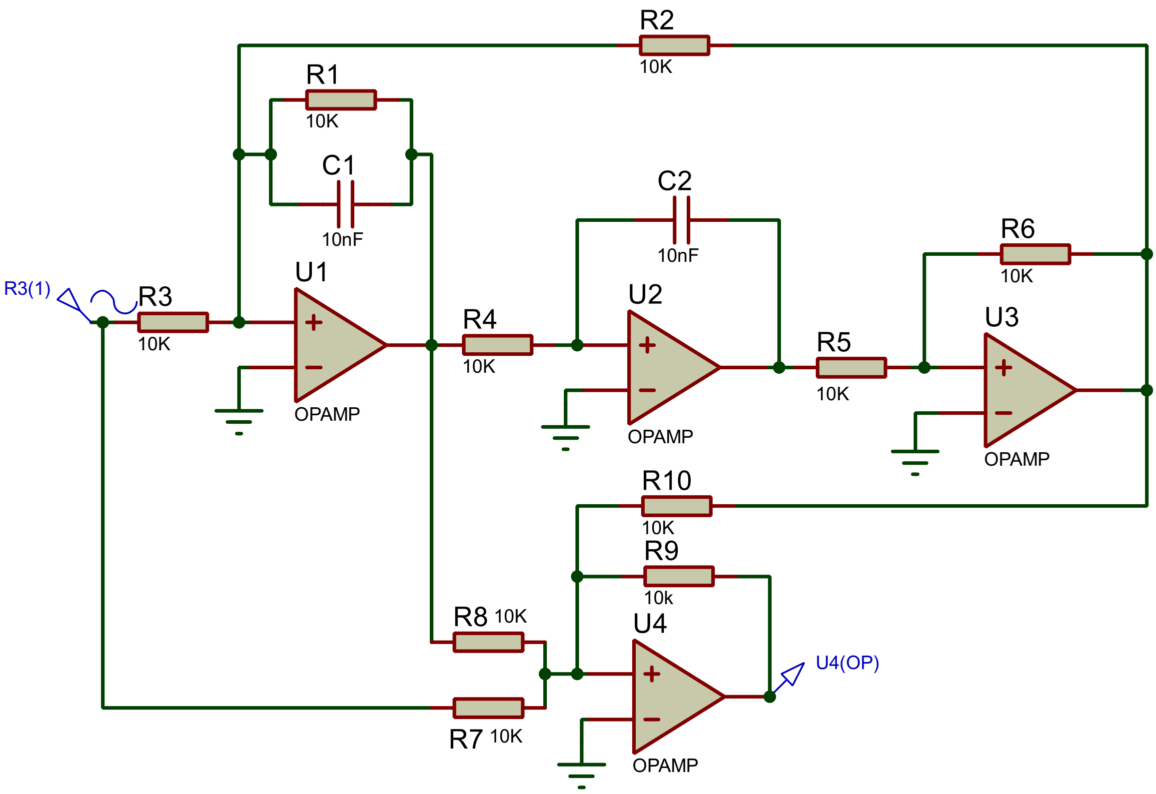
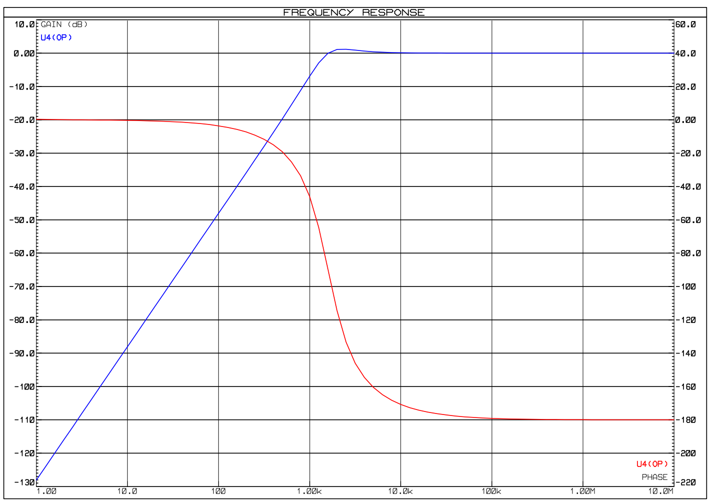


Figure 11: Proteus circuit for highpass filter using 4-opamp biquad circuit



Noted Values	
Gain in pass band (in dB)	0
Half power frequency (in KHz)	1.25

Figure 12: Observation for highpass filter designed in Problem 2

ii. Bandstop filter

To design a bandstop filter, we need a 4th op-amp in addition to the 3 op-amp Tow Thomas biquad circuit. We first design a general circuit using the required filter parameters, and then choose values that constraint a bandstop response. As derived earlier, the required condition for a bandstop response at V_2' is $R_1 = R_3$.

The filter parameters for the required filter are, $\omega_o = 10^4$, $Q = 1$ and $H = 2$.

We normalize the design at $\omega_o = 1$ rad/s, $Q = 1$ and $H = 2$. Let us consider, $R_1 = R_3 = 1\Omega$ and $C_2 = 1$ F. Using these values and the set of relations from Equation 15 to calculate the remaining elemental values as,

$$H = \frac{R_2}{R_3} \Rightarrow R_2 = 2\Omega$$
$$\omega_o^2 = \frac{1}{R_2 R_4 C_1 C_2} \Rightarrow R_4 C_1 = \frac{1}{2}$$
$$Q = \sqrt{\frac{R_1^2 C_1}{R_2 R_4 C_2}} \Rightarrow 2R_4 = C_1$$

Solving for C_1 and R_4, we get, $C_1 = 1$ F and $R_4 = 0.5\Omega$.

Since the problem is to design the filter at $\omega_o = 10^4$ rad/s, $K_f = 10^4$. Let us choose $K_m = 10^4$. Also, as the values of R_5 and R_6 don't affect in the actual response of the filter, we choose it as $R_5 = R_6 = 1\Omega$. The scaled elemental values are,

$$R_1 = 1 \times 10^4 = 10 \text{ K}\Omega \qquad R_2 = 2 \times 10^4 = 20 \text{ K}\Omega$$
$$R_3 = 1 \times 10^4 = 10 \text{ K}\Omega \qquad R_4 = 0.5 \times 10^4 = 5 \text{ K}\Omega$$
$$R_5 = 1 \times 10^4 = 10 \text{ K}\Omega \qquad R_6 = 1 \times 10^4 = 10 \text{ K}\Omega$$
$$C_1 = \frac{1}{10^4 \times 10^4} = 10 \text{ nF} \qquad C_2 = \frac{1}{10^4 \times 10^4} = 10 \text{ nF}$$

Note: The elemental notations are according to the Figure 5.

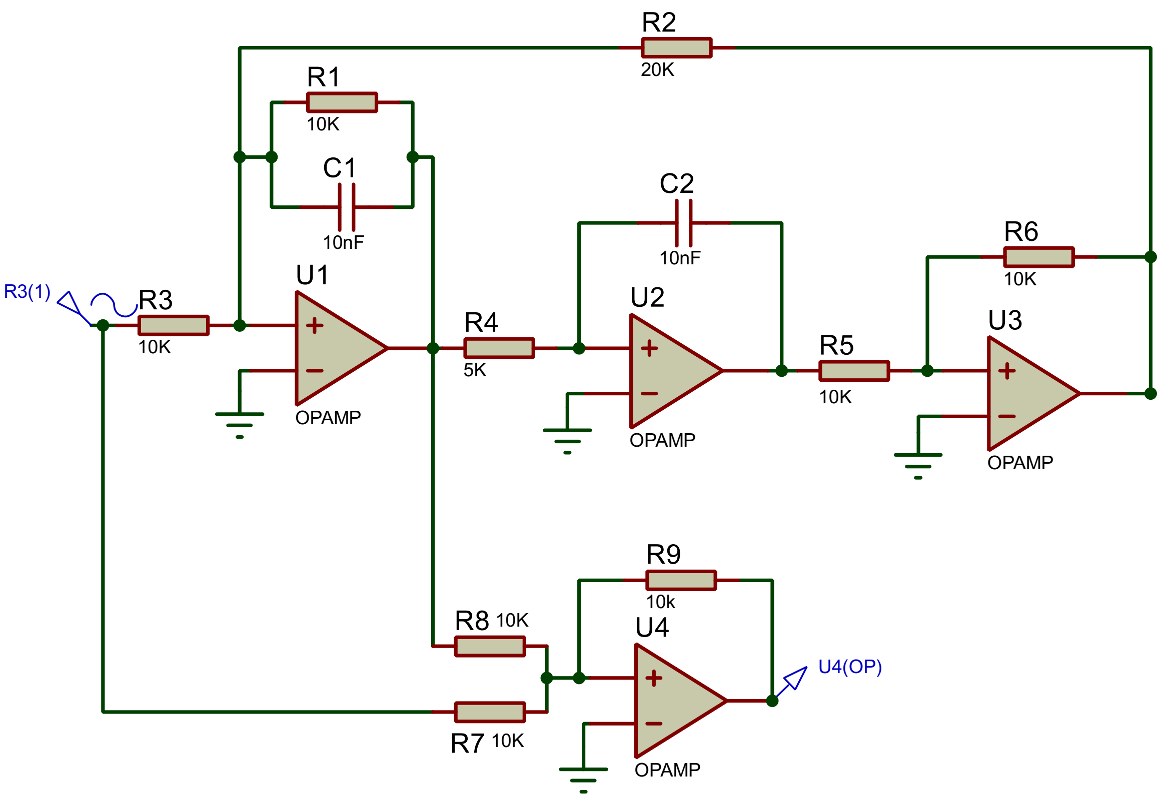
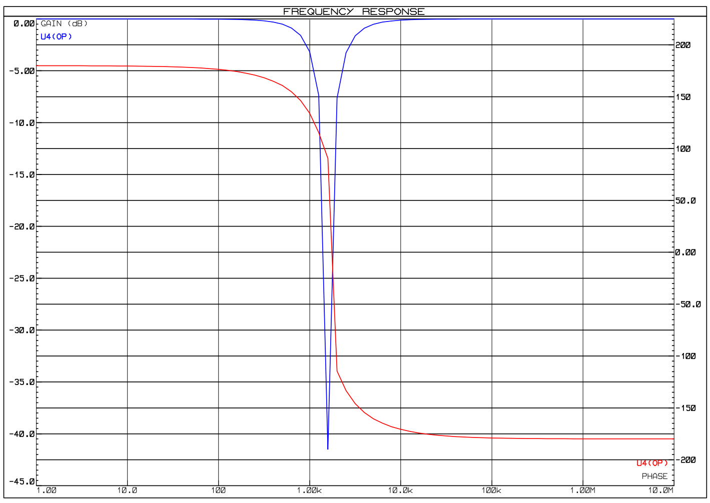


Figure 13: Proteus circuit for bandstop filter using 4-opamp biquad circuit



Noted Values	
Gain in pass band (in dB)	0
Center frequency (in KHz)	1.59
Half power frequencies (in KHz)	0.978, 2.52
Bandwidth (in KHz)	1.642

Figure 14: Observation for bandstop filter designed in Problem 2

ii. Allpass filter

To design an allpass filter, we need a 4th op-amp in addition to the 3 op-amp Tow Thomas biquad circuit. We first design a general circuit using the required filter parameters, and then choose values that constraint an allpass response. As derived earlier, the required condition for an allpass response at V_2' is $R_3 = \frac{R_1}{2}$.

The filter parameters for the required filter are, $\omega_o = 10^4$, $Q = 1$ and $H = 2$.

We normalize the design at $\omega_o = 1$ rad/s, $Q = 1$ and $H = 2$. Let us consider, $R_1 = 1\Omega$, $R_3 = 0.5\Omega$ and $C_2 = 1$ F. Using these values and the set of relations from Equation 15 to calculate the remaining elemental values as,

$$H = \frac{R_2}{R_3} \Rightarrow R_2 = 1\Omega$$

$$\omega_o^2 = \frac{1}{R_2 R_4 C_1 C_2} \Rightarrow R_4 C_1 = 1$$

$$Q = \sqrt{\frac{R_1^2 C_1}{R_2 R_4 C_2}} \Rightarrow R_4 = C_1$$

Solving for C_1 and R_4, we get, $C_1 = 1$ F and $R_4 = 1\Omega$.

Since the problem is to design the filter at $\omega_o = 10^4$ rad/s, $K_f = 10^4$. Let us choose $K_m = 10^4$. Also, as the values of R_5 and R_6 don't affect in the actual response of the filter, we choose it as $R_5 = R_6 = 1\Omega$. The scaled elemental values are,

$$R_1 = 1 \times 10^4 = 10 \text{ K}\Omega \qquad R_2 = 1 \times 10^4 = 10 \text{ K}\Omega$$

$$R_3 = 0.5 \times 10^4 = 5 \text{ K}\Omega \qquad R_4 = 1 \times 10^4 = 10 \text{ K}\Omega$$

$$R_5 = 1 \times 10^4 = 10 \text{ K}\Omega \qquad R_6 = 1 \times 10^4 = 10 \text{ K}\Omega$$

$$C_1 = \frac{1}{10^4 \times 10^4} = 10 \text{ nF} \qquad C_2 = \frac{1}{10^4 \times 10^4} = 10 \text{ nF}$$

Note: The elemental notations are according to the Figure 5.

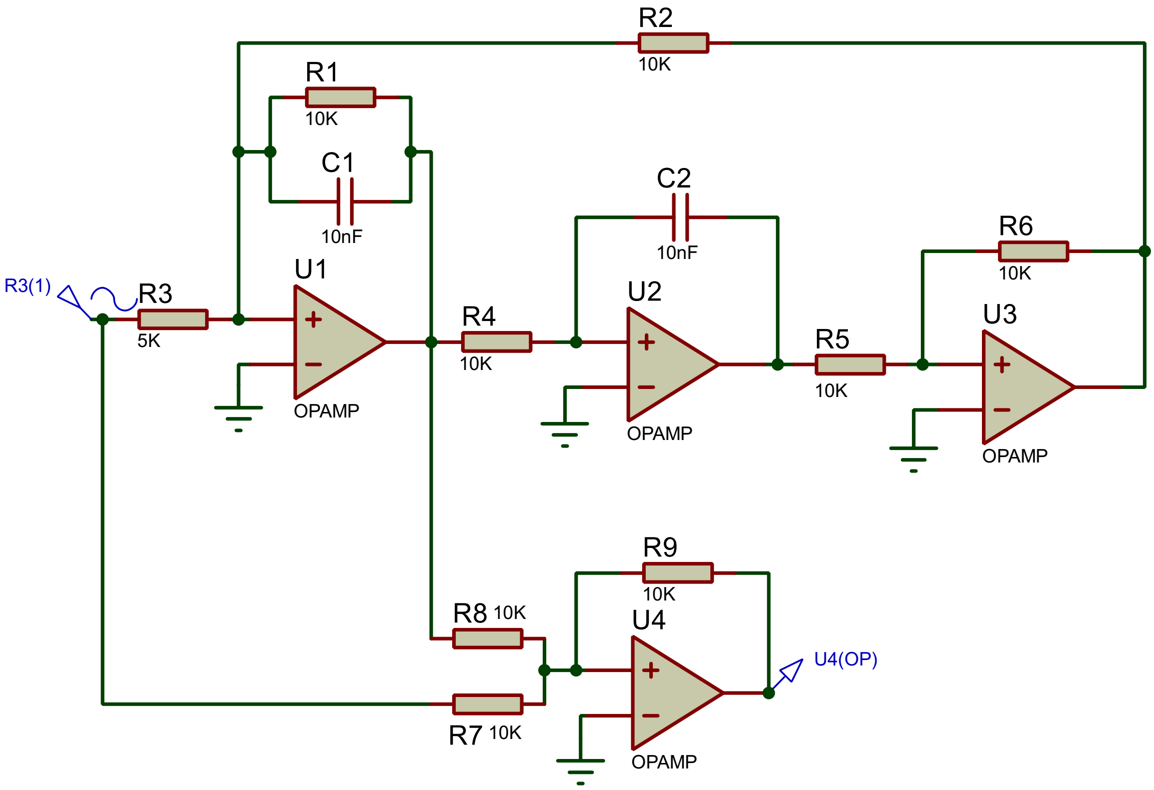
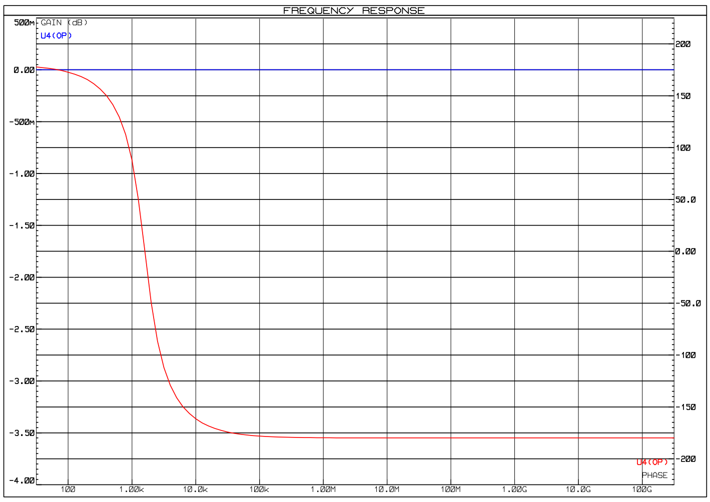


Figure 15: Proteus circuit for allpass filter using 4-opamp biquad circuit



Noted Values	
Gain in pass band (in μdB)	26.14

Figure 16: Observation for allpass filter designed in Problem 2

Problem 3

Realize the second order Butterworth lowpass filter having half power frequency of 3.5 KHz using the Tow Thomas biquad circuit and observe the response. By plotting the response show the half power frequency and DC gain.

For second order butterworth filter with half power frequency $\omega_o = 1$ rad/s, the denominator function is,

$$f_{den}(s) = s^2 + \sqrt{2}s + 1$$

From Equation 4, the denominator function for a standard lowpass filter can be written as,

$$f'_{den}(s) = s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2$$

Comparing these equations, we get,

$$\omega_o^2 = 1 \Rightarrow \omega_o = \sqrt{1} = 1$$
$$\frac{\omega_o}{Q} = \sqrt{2} \Rightarrow Q = \frac{\omega_o}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.7071$$

The filter parameters for the required filter are, $\omega_o = 3.5$ KHz, $Q = 0.7071$ and $H = 1$ (for a butterworth response, DC gain is unity).
We normalize the design at $\omega_o = 1$ rad/s, $Q = 0.7071$ and $H = 1$. Let us consider, $R_4 = 1\Omega$, $C_1 = C_2 = 1$ F. Using these values and the set of relations from Equation 15 to calculate the remaining elemental values as,

$$\omega_o^2 = \frac{1}{R_2R_4C_1C_2} \Rightarrow R_2 = 1\Omega$$
$$Q = \sqrt{\frac{R_1^2C_1}{R_2R_4C_2}} \Rightarrow R_1 = 0.7071\Omega$$
$$H = \frac{R_2}{R_3} \Rightarrow R_3 = 1\Omega$$

Since the problem is to design the filter at $\omega_o = 3.5$ KHz, $K_f = \frac{2\pi \times 3.5 \times 10^3}{1} \approx 22000$. Let us choose $K_m = 10^4$. Also, as the value of R_5 doesn't affect in the actual response of the filter, we choose it as $R_5 = 1\Omega$. The scaled elemental values are,

$$R_1 = 0.7071 \times 10^4 = 7.071\ \text{K}\Omega \qquad R_2 = 1 \times 10^4 = 10\ \text{K}\Omega$$

$$R_3 = 1 \times 10^4 = 10\ \text{K}\Omega \qquad R_4 = 1 \times 10^4 = 10\ \text{K}\Omega$$

$$C_1 = \frac{1}{22000 \times 10^4} = 4.54\ \text{nF} \qquad C_2 = \frac{1}{22000 \times 10^4} = 4.54\ \text{nF}$$

$$R_5 = 1 \times 10^4 = 10\ \text{K}\Omega$$

Note: The elemental notations are according to the Figure 1.

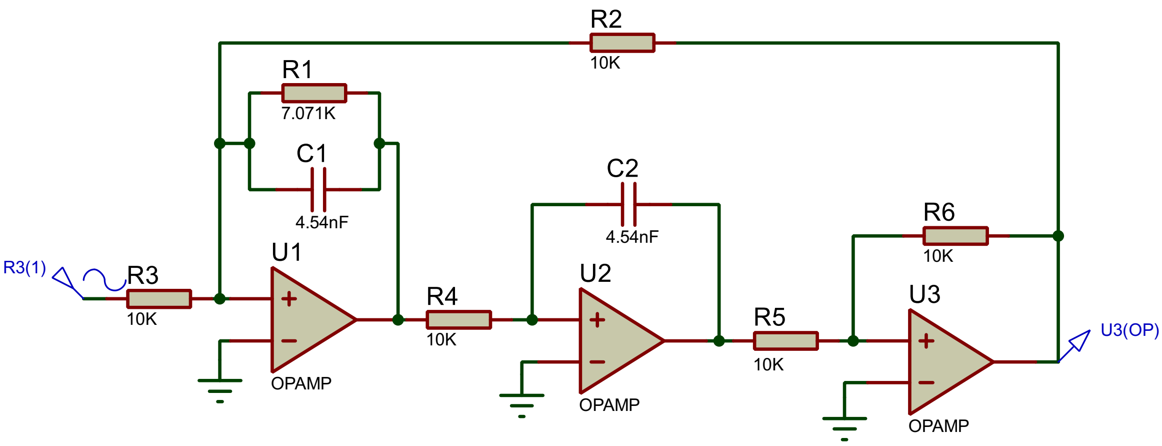
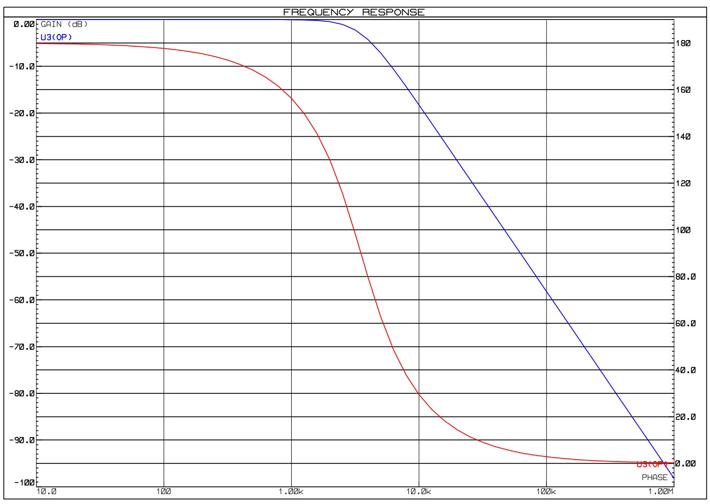


Figure 17: Proteus circuit for lowpass filter using Tow Thomas biquad circuit



Noted Values	
DC gain (in dB)	0
Half power frequency (in KHz)	3.49

Figure 18: Observation for lowpass filter designed in Problem 3

3 Discussion and Conclusion

In this lab experiment, we designed different types of filter using the Tow Thomas biquad circuit given in Figure 1. The derivation for the different responses was understood as a part of the experiment. An important thing to note is that the single Tow Thomas biquad circuit gives lowpass response at V_2 and bandpass response at V_{O1}, which was also verified and visualized from the actual design problem. Similarly, to design highpass, bandstop and allpass filters, an additional op-amp was used along with the original Tow Thomas biquad circuit as shown in Figure 5 and Figure 6, with some constraints in the values of resistance such that response taken at V_2' would be as we require. The design problems for these filters were also performed, and the observations are included above. Lastly, a second order butterworth filter was designed at half power frequency of $\omega_o = 3.5$ KHz using the Tow Thomas biquad circuit. This was performed by comparing the transfer functions and then applying tuning algorithm to constraint the elemental values for the required filter parameters.
Hence, the objectives of the lab were fulfilled with the understanding of the mentioned topics.