Signal Analysis Assignment #1

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PUL074BEX007

Problem 1

Prove the periodicity condition of continuous exponential signal $x(t) = e^{jw_0t}$.

Solution:

Here, $x(t) = e^{jw_0t}$, we know that for a continuous time signal $x(t)$ to be periodic with a period T, it must comply with $x(t) = x(t+T)$, i.e. the signal should be unaffected by the time shift of period T.

i.e. $e^{jw_0t} = e^{jw_0(t+T)} = e^{jw_0t}.e^{jw_0T}$,

so for $x(t)$ to be follow periodicity, the following equation must be true.

$$e^{jw_0T} = 1 \tag{1}$$

when $w_0 = 0$, equation (1) holds for all values of T,
when $w_0 \neq 0$, the fundamental period T_0 of $x(t)$, i.e. the smallest positive value of T for which equation (1) holds is given by, $T_0 = \frac{2\pi}{|w_0|}$.

This is the condition that a continuous exponential signal $x(t) = e^{jw_0t}$ must hold in order to be periodic.

Problem 2

Prove that discrete time complex exponential are periodic only if its frequency is rational.

Solution:

Let us take a discrete time complex exponential signal as $x[n] = e^{jw_0n}$. Considering the discrete time complex exponential with frequency of $w_o + 2\pi$,

$$e^{j(w_0+2\pi)n} = e^{jw_0n}.e^{j2\pi n} = e^{jw_0n} \tag{2}$$

The analysis of the equation (2) leads us to a conclusion that the signal $x[n] = e^{jw_0n}$ is not distinct like its continuous counterpart, rather the signal with frequency w_0 is completely identical to that with frequency $w_0 + (2n)\pi$ where n is 1,2,3,.... Furthermore, equation (2) also implies the periodicity of a discrete time complex exponential signal in with a period of say, N, such that $N > 0$.

For this to be true the signal must follow, $x[n] = x[n + N]$,
or, $e^{jw_0n} = e^{jw_0(n+N)} = (e^{jw_0n}).(e^{jw_0N})$,
so equivalently, for the signal $x[n] = e^{jw_0n}$ to be periodic, $e^{jw_0N} = 1$ must be true, which consequently means,
$w_0N = 2\pi m$, such that m is an integer.

or, $\boxed{\frac{w_0}{2\pi} = \frac{m}{N}}$. Since m and N both are integers, the ratio $\frac{m}{N}$ is rational.

This means the signal $x[n] = e^{jw_0n}$ is **periodic only if $\frac{w_0}{2\pi}$, i.e. the frequency of the signal is rational**.

Problem 3

Let $x_1(t) = cos6\pi t$ and $x_2(t) = sin30\pi t$. Determine if the function $y = x_1 + x_2$ is periodic, and if it is, find its fundamental period.

Solution:

Here, $x_1(t) = cos6\pi t$, $x_2(t) = sin30\pi t$

or, $T_1 = \frac{2\pi}{6\pi} = \frac{1}{3}$ and $T_2 = \frac{2\pi}{30\pi} = \frac{1}{15}$,

or, $\frac{T_1}{T_2} = \frac{\frac{1}{3}}{\frac{1}{15}} = 5$, which is a rational number.

Since the ratio $\frac{T_1}{T_2}$ is a rational number such that the two integers (numerator and denominator) are co-prime, **the sum of the two original periodic signals,** $x_1(t)$ **and** $x_2(t)$ **is a period function**.

Likewise, the fundamental period T can be calculated as,
$\boxed{T = T_1 = 5T_2 = \frac{1}{3}}$

Problem 4

Determine whether the signal is periodic or aperiodic signal?

1. $x(t) = sin(\frac{2\pi}{3}t)$

2. $x(t) = cos(\frac{\pi}{3}t) + sin(\frac{\pi}{4}t)$

Solution:

1. Here, $x(t) = sin(\frac{2\pi}{3}t)$,

   or, $T = \frac{2\pi}{\frac{2\pi}{3}}$=3

   A continuous time signal $x(t)$ is said to be periodic if there is a positive a time shift T such that $x(t) = x(t + T)$ for all values of t.

   or, $x(t + 3) = sin(\frac{2\pi}{3}(t + 3)) = sin(\frac{2\pi}{3}t + 2\pi) = sin(\frac{2\pi}{3}t)$

   Since the given signal complies to the condition for a signal to be periodic, $x(t) = sin(\frac{2\pi}{3}t)$ **is periodic**.

2. Here, $x(t) = cos(\frac{\pi}{3}t) + sin(\frac{\pi}{4}t)$,

   Let us represent the signal $x(t)$ as a sum of two signals $x_1(t)$ and $x_2(t)$ such that $x_1(t) = cos(\frac{\pi}{3}t)$ and $x_2(t) = sin(\frac{\pi}{4}t)$.

   From this, if T_1 is the time period of $x_1(t)$ and T_2 is that of $x_2(t)$,
   $T_1 = \frac{2\pi}{\frac{\pi}{3}} = 6$ and $T_2 = \frac{2\pi}{\frac{\pi}{4}} = 8$,

   so, $\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4}$, which is a rational number.

Since the ratio $\frac{T_1}{T_2}$ is a rational number such that the two integers (numerator and denominator) are co-prime, **the sum of the two original periodic signals, $x_1(t)$ and $x_2(t)$, is a period function**.

The fundamental period of $x(t)$ can be calculated as $T = 4T_1 = 3T_2 = 24$, so we can conclude that $x(t) = cos(\frac{\pi}{3}t) + sin(\frac{\pi}{4}t)$ is periodic function with a fundamental period of 24.