Communication System Assignment#1

Due on August 30, 2020

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PUL074BEX007

Problem 1

Determine $v(t) * w_1(t)$ and $v(t) * w_2(t)$. Show your work graphically and mathematically. Given that $v(t) = e^{-t}u(t)$, $w_1(t) = \delta(t)$, $w_2(t) = \delta(t-10)$.

Solution

Mathematical approach

The convolution of two functions, say, $y(t) = x(t) * h(t)$ is given by:

$$y(t) = x(t) * h(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \tag{1}$$

Also, the dirac(delta) function has a special integration property that we'll use to easily calculate the integration, the property being,

$$\int_{-d}^{d} f(t)\delta(t-a)dt = f(a) \tag{2}$$

such that $-\infty \leq -d < a < d \leq \infty$.
Likewise, since $d(t)$ is an even function the following equation holds true.

$$\delta(t) = \delta(-t) \tag{3}$$

Mathematically,

$$v(t) * w_1(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\delta(t-\tau)d\tau \tag{4}$$

and

$$v(t) * w_2(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\delta((t-10)-\tau)d\tau \tag{5}$$

Using equation (3) to rearrange equation (4) and (5) we get,

$$v(t) * w_1(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\delta(\tau-t)d\tau \tag{6}$$

and

$$v(t) * w_2(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\delta(\tau-(t-10))d\tau \tag{7}$$

From the property of delta function mentioned in equation(2), equation(6) and (7) can be reduced to get,

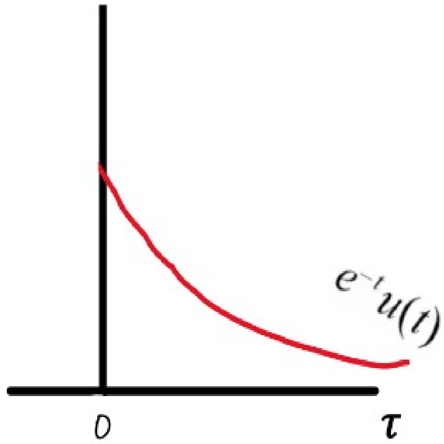
$$v(t) * w_1(t) = e^{-t}u(t) \tag{8}$$

and

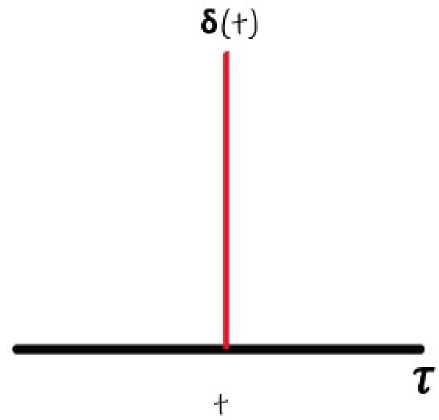
$$v(t) * w_2(t) = e^{-(t-10)}u(t-10) \tag{9}$$

These equations are the solutions to the convolution problem obtained matematically.

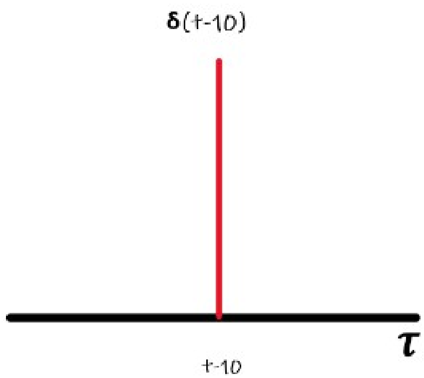
Graphical approach



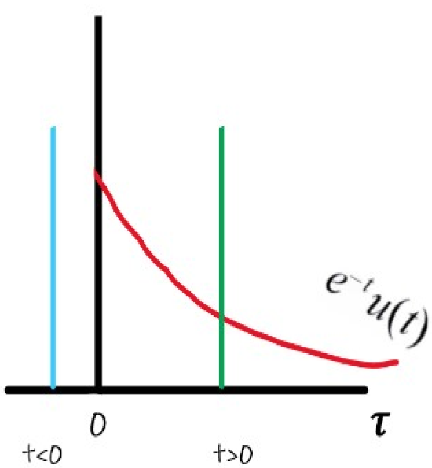
(a) Plot of $e^{-\tau}u(\tau)$ in τ axis



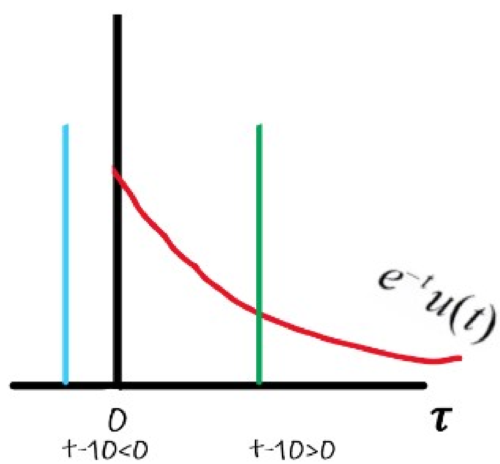
(b) Plot of $\delta(t)$ in τ axis



(c) Plot of $\delta(t-10)$ in τ axis



(d) Plot for $v(t) * w_1(t)$



(e) Plot for $v(t) * w_1(t)$s

Figure 1: Plot for calculating $v(t) * w_1(t)$ and $v(t) * w_2(t)$

Convolution of any two signals is simply the overlap of the signals represented all over the τ axis. So, figure(1 a), (1 b) and(1 c) show the three signals namely, $v(t)$,$w_1(t)$ and $w_2(t)$ in the τ axis.
When the values of t and $t-10$ in the respective figure (1 b) and (1 c) is either less than or greater than 0, the corresponding interaction with the $v(t)$ signal can be interpreted from the figure (1 d) and (1 e).

Both the signals $\delta(t)$ and $\delta(t-10)$ overlap with the signal $e^{-t}u(t)$ when they satisfy the condition, $t > 0$ and $t - 10 > 0$ respectively. This means the convolution of the signal $v(t)$ with $w_1(t)$ and $w_2(t)$ is nothing but the function $v(t)$ with $t = t$ and $t = t - 10$ respectively, i.e.

$$v(t) * w_1(t) = v(t) = e^{-t}u(t) \tag{10}$$

and

$$v(t) * w_2(t) = v(t-10) = e^{-(t-10)}u(t-10) \tag{11}$$

From this we can draw a conclusion that,

$$y(t) = x(t) * \delta(t \pm a) = x(t \pm a) \tag{12}$$