Design of FIR Digital Filters using Window method

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1 Objectives

- Familiarization with design of FIR digital filters using window method.
- Comparison of response for designed FIR filters with different window functions.

2 Background Theory

In previous experiment we discussed the most commonly used techniques for design of IIR filters based on transformations of continuous-time IIR systems into discrete time IIR systems. The major difficulty lies in the implementation of the non-iterative direct design method for IIR filters. However FIR filters are almost entirely restricted to discrete-time implementations. Thus the design techniques for FIR filters are based on directly approximating the desired frequency response of the discrete time system. Furthermore, most techniques for approximating the magnitude response of the FIR system assume a linear phase constraint; thereby avoiding the problem of spectrum factorization that complicates the direct design of IIR filters.

The simplest method of FIR design is called the window method. This method generally begins with an ideal desired frequency response $H_d(\omega)$. The impulse response $h_d(n)$ of the filter exhibiting this desired frequency response can be obtained from inverse Fourier transform of $H_d(\omega)$. However this impulse response exists for $n = -\infty$ to $+\infty$ and hence truncation is needed to make the finite duration impulse response. The truncation is similar to the multiplication of the $h_d(n)$ with the window function $w(n)$. The multiplication in discrete time domain is equivalent to the convolution of the two in frequency domain, which actually gives the frequency response of the truncated FIR filter.

But depending on the tapering of the window to zero at each end, the nature of the window differs. For example, the rectangular window exhibits the most abrupt changes while approaching to zero at each end. For the specific value of length of the window, the rectangular window exhibits main lobe with the greatest width and the lowest side lobe attenuation, than

the other windows like Bartlett, Hanning, Hamming, Blackman etc. There are also adjustable windows like Kaiser windows whose windowing function $w(n)$ can be adjusted by changing the value of parameter β according to the stop band attenuation A as given below,

$$\beta = \begin{cases} 0.1102(A - 8.7) & \text{if } A > 50 \\ 0.5842(A - 0.21)^{0.4} + 0.07886(A - 21) & \text{if } 21 \leq A \leq 50 \\ 0.0 & \text{if } A < 21 \end{cases}$$

where $A = -20 \log_{10} \delta$. The length of the Kaiser window is given by,

$$M = \frac{\delta - 7.95}{14.36 \Delta\omega} + 1, \quad \text{where } \Delta\omega = \frac{\omega_s - \omega_p}{2}$$

To determine and plot the different types of windowing functions and their frequency responses, built-in functions like `Bartlett`, `Hamming`, `Hanning`, `Blackman`, `Kaiser` etc. are available. There is a function `fir1` that can be used to obtain the frequency response of the designed FIR filter. Use MATLAB `help` for further information of the functions.

3 Lab Exercises

Problem 1

Design an FIR linear phase digital filter approximating the ideal frequency response

$$H(\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \frac{\pi}{6} \\ 0, & \text{for } \frac{\pi}{6} < |\omega| \leq \pi \end{cases}$$

a. Plot the window function for Hamming and its frequency response for length of $M = 31$.

b. Using the Hamming window plot the frequency response of the truncated FIR filter.

c. Repeat parts (a) and (b) for the Hanning, Blackman and Bartlett windows.

d. Repeat parts (a), (b) and (c) for filter length of $M = 61$.

```
function [h] = window_selector(M, selected_filter)
    switch selected_filter
        case 'hamming'
            h = hamming(M);
        case 'hanning'
            h = hanning(M);
        case 'blackman'
            h = blackman(M);
        case 'bartlett'
            h = bartlett(M);
        otherwise
            error('The window you want is not found.')
    end
end
```

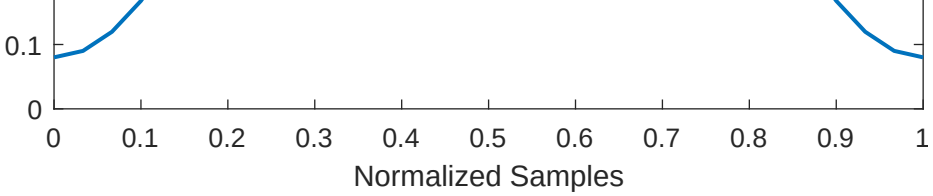
Listing 1: Matlab function to select particular window function based on user input

```
function [fig,mlW,slA] = freqRespPlot(H,w,str)
    mag = abs(H);
    magdB = 20 * log10(mag);
    mlW=2*w(find(magdB<max(magdB)-3,1,'first'))/pi;
    pks=findpeaks(magdB,'Threshold',0);
    pks = pks(find(pks < 0, 1, 'first'):end);
    slA=max(pks)-max(magdB);
    fig=figure();
    l = tiledlayout(2, 1);
    nexttile
    plot(w/pi, magdB, 'LineWidth', 1.5)
    grid on
    ylabel('Magnitude (dB)')
    title('Magnitude plot', 'FontWeight', 'normal')
    nexttile
    plot(w/pi, mag, 'LineWidth', 1.5)
    grid on
    ylabel('Gain')
    title('Linearized form', 'FontWeight', 'normal')
    xlabel(l, 'Normalized Frequency (\times \pi rad/sample)')
    title(l, sprintf('Frequency response of %s\n(PUL074BEX007)', str));
end
```

Listing 2: Matlab function to plot the frequency response for given `H` and `w` and return the main lobe width and relative side lobe attenuation

```
f = [0 pi/6 pi/6 pi];
H = [1 1 0 0];
selected_window = input('Enter the type of window: ', 's');
M = input('Enter the value of M: ');
N = M-1;
w = 0:pi/N:pi;
h = window_selector(M, selected_window);
f1=figure();
l=tiledlayout(1,1);
nexttile
plot(w/pi,h,'LineWidth', 1.5);
ylabel('Amplitude')
xlabel('Normalized Samples')
title(l,sprintf('%d-point symmetric %s window',M, selected_window));
[hWindow,wWindow] = freqz(h,1,1024);
f2=freqRespPlot(hWindow,wWindow, sprintf('%s window function', selected_window));
B = fir1(N,1/6,h);
[hFir,wFir] = freqz(B,1,1024);
[f3,mlWF,slAF]=freqRespPlot(hFir, wFir,sprintf('FIR filter using %s window',selected_window));
fprintf('Parameters for %s:\n', sprintf('FIR filter using %s window',selected_window));
fprintf('Main lobe width = %.3f\n',mlWF)
fprintf('Relative side lobe attenuation = %.3f\n',slAF)
print(f1,sprintf('../Figures/%d-%s',M, selected_window), '-depsc')
print(f2,sprintf('../Figures/%d-%s-resp',M, selected_window), '-depsc')
print(f3,sprintf('../Figures/%d-%s-fir',M, selected_window), '-depsc')
```

Listing 3: Matlab script for visualizing window function and frequency responses of the window function and the designed truncated FIR filter




```
>> lab_6_1
Enter the type of window: hamming
Enter the value of M: 31
Parameters for FIR filter using hamming window:
Main lobe width = 0.281
Relative side lobe attenuation = -52.126
```

Figure 1: Plot for 31-point symmetric hamming window

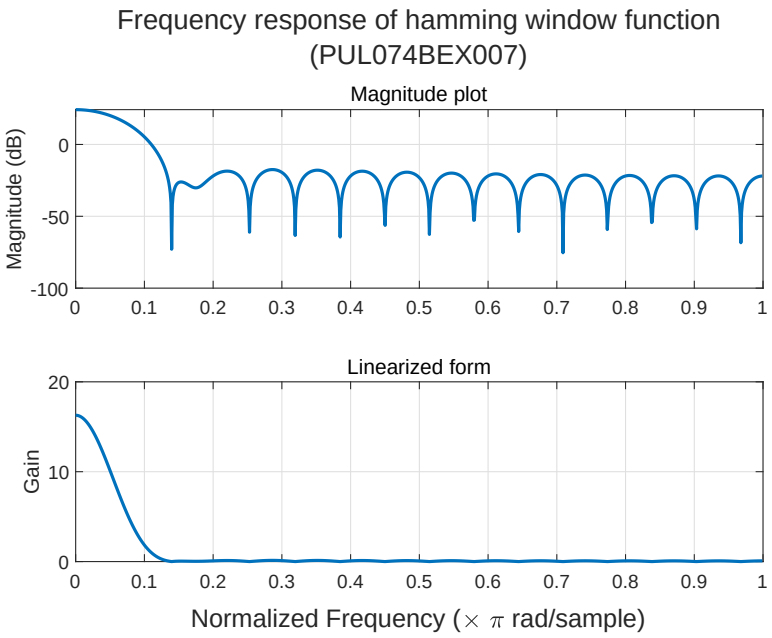


Figure 2: Plot for frequency response for hamming window function with M=31

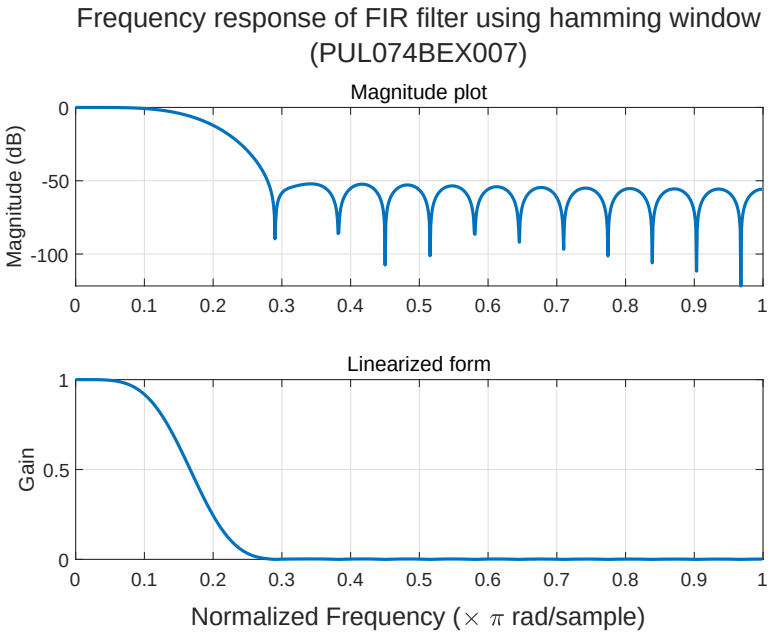
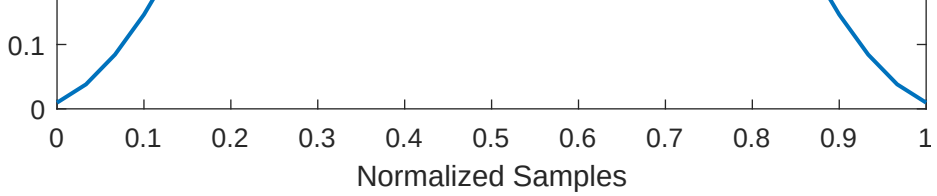


Figure 3: Plot for frequency response for FIR filter using hamming window for M=31




```
>> lab_6_1
Enter the type of window: hanning
Enter the value of M: 31
Parameters for FIR filter using hanning window:
Main lobe width = 0.279
Relative side lobe attenuation = -43.887
```

Figure 4: Plot for 31-point symmetric hanning window

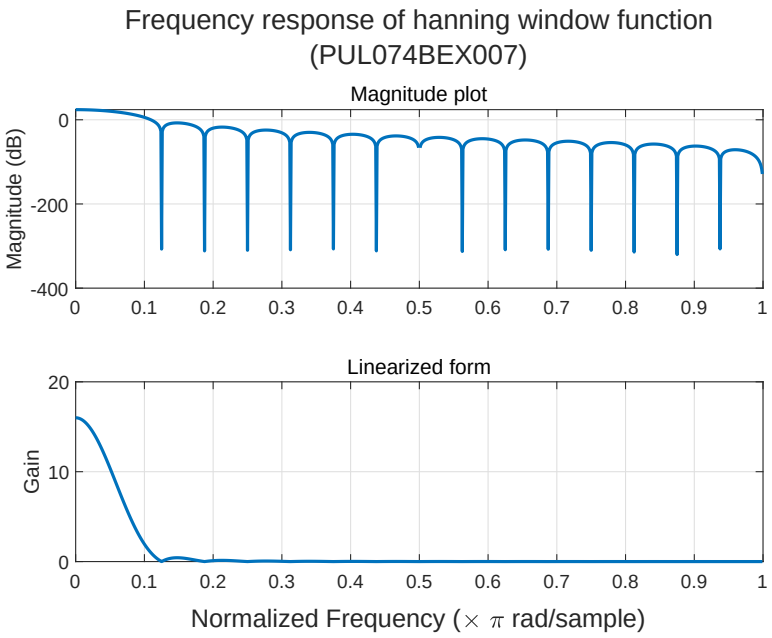


Figure 5: Plot for frequency response for hanning window function with M=31

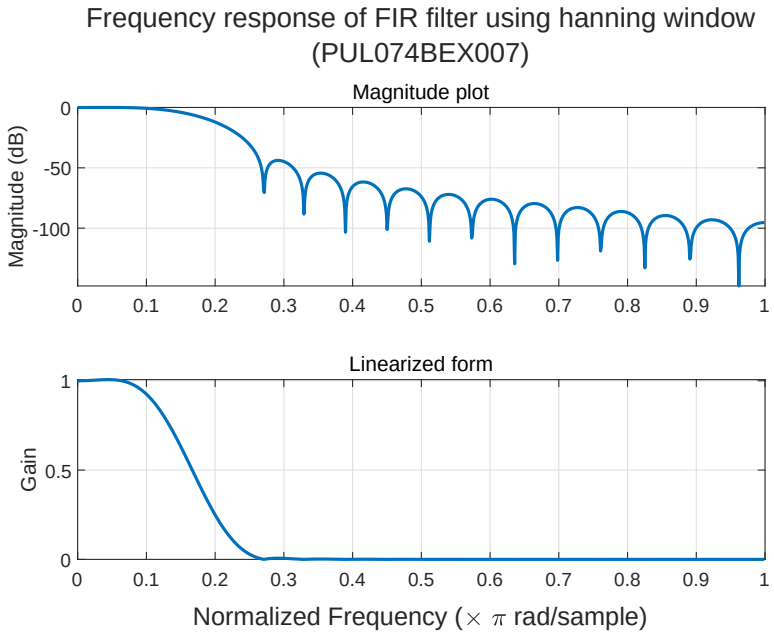
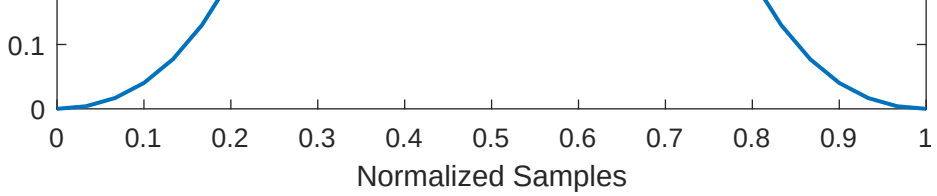


Figure 6: Plot for frequency response for FIR filter using hanning window for M=31




```
>> lab_6_1
Enter the type of window: blackman
Enter the value of M: 31
Parameters for FIR filter using blackman window:
Main lobe width = 0.266
Relative side lobe attenuation = -74.275
```

Figure 7: Plot for 31-point symmetric blackman window

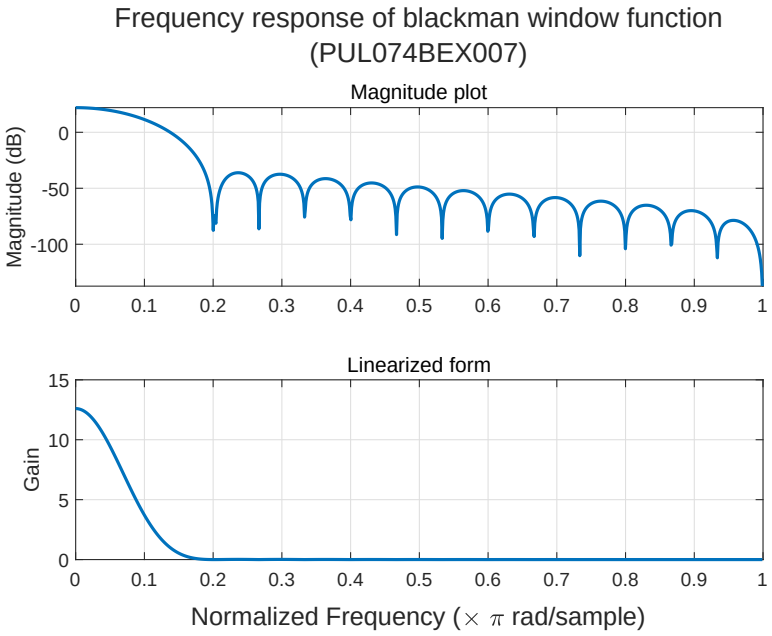


Figure 8: Plot for frequency response for blackman window function with M=31

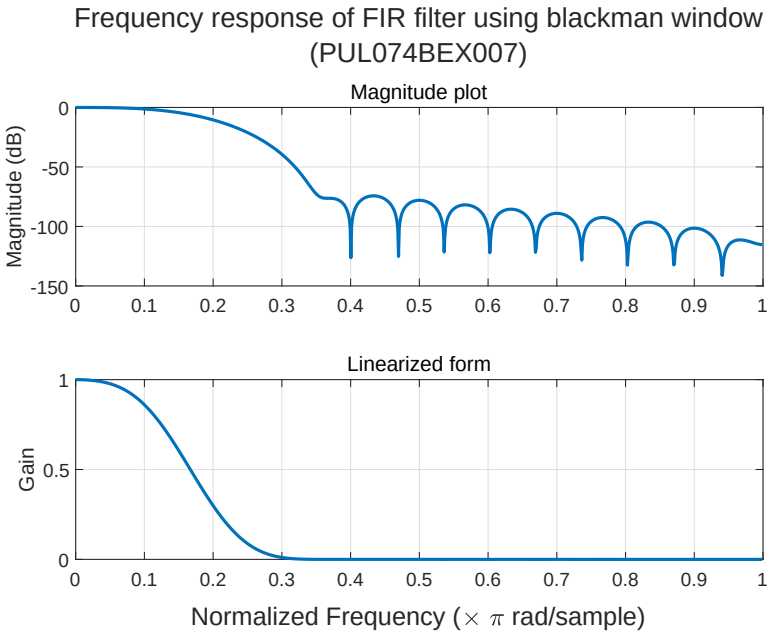
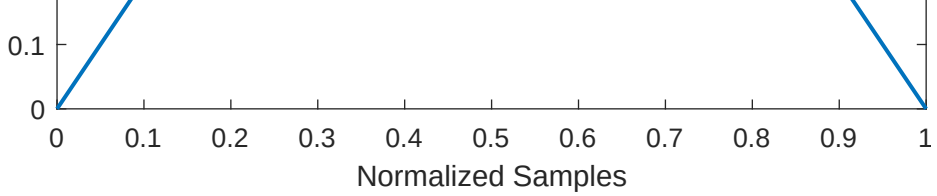


Figure 9: Plot for frequency response for FIR filter using blackman window for M=31




```
>> lab_6_1
Enter the type of window: bartlett
Enter the value of M: 31
Parameters for FIR filter using bartlett window:
Main lobe width = 0.285
Relative side lobe attenuation = -28.025
```

Figure 10: Plot for 31-point symmetric bartlett window

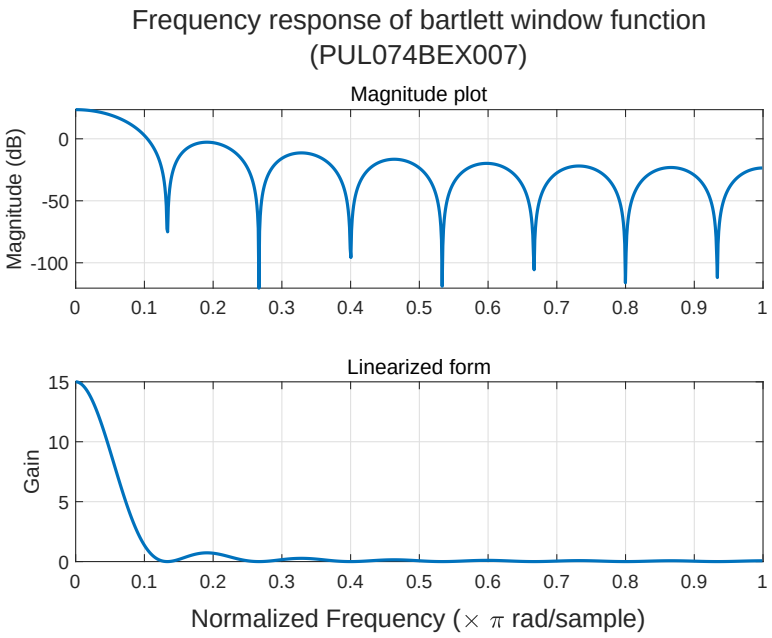


Figure 11: Plot for frequency response for bartlett window function with M=31

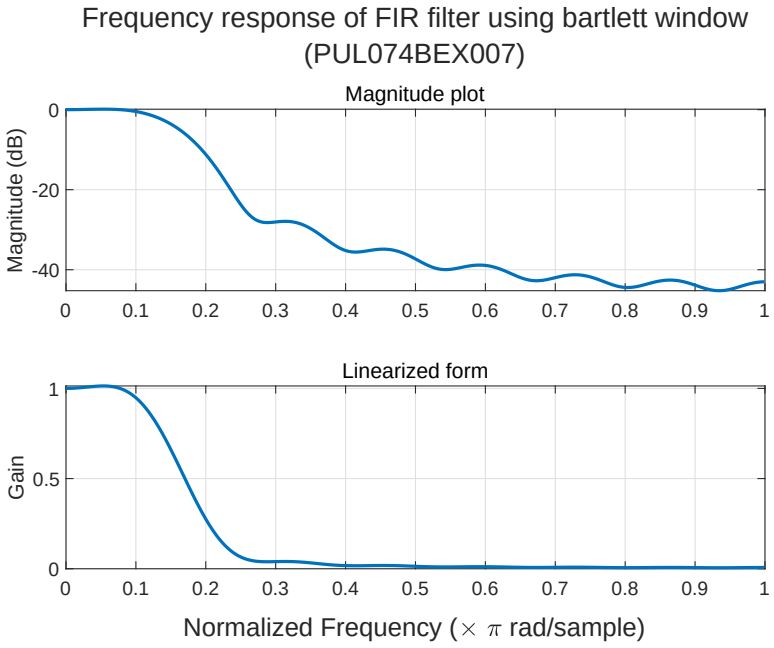
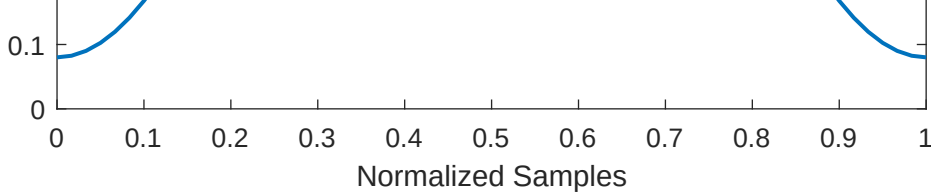


Figure 12: Plot for frequency response for FIR filter using bartlett window for M=31




```
>> lab_6_1
Enter the type of window: hamming
Enter the value of M: 61
Parameters for FIR filter using hamming window:
Main lobe width = 0.307
Relative side lobe attenuation = -51.966
```

Figure 13: Plot for 61-point symmetric hamming window

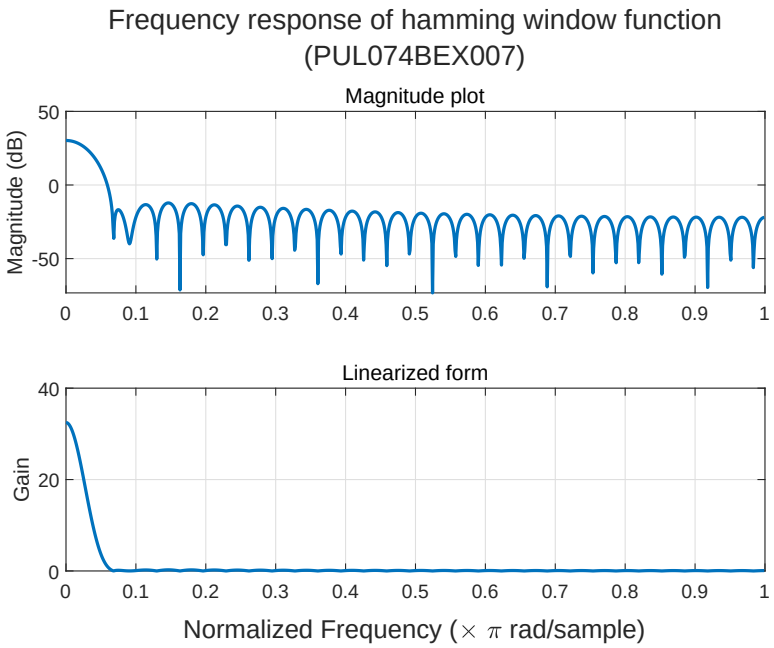


Figure 14: Plot for frequency response for hamming window function with M=61

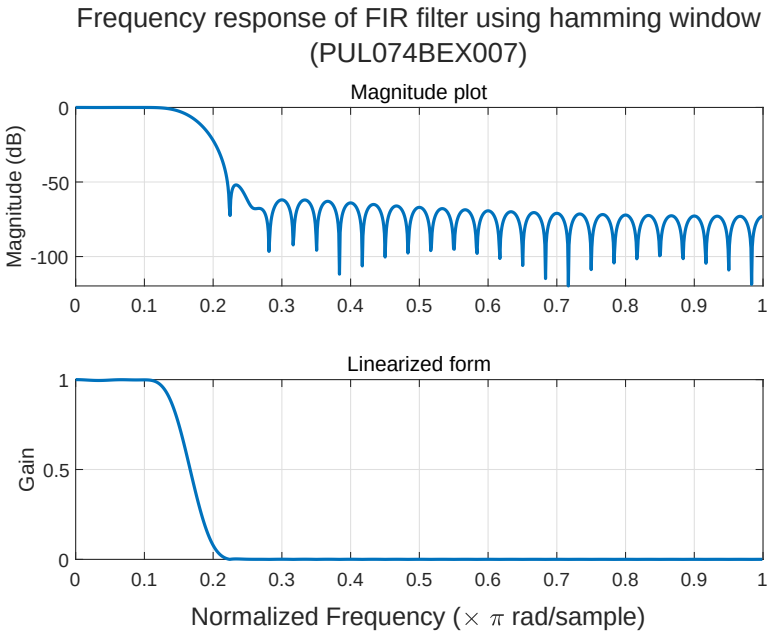
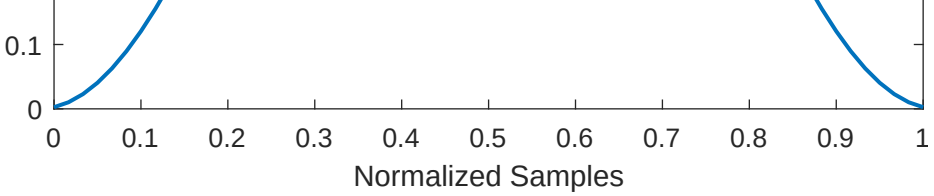


Figure 15: Plot for frequency response for FIR filter using hamming window for M=61




```
>> lab_6_1
Enter the type of window: hanning
Enter the value of M: 61
Parameters for FIR filter using hanning window:
Main lobe width = 0.307
Relative side lobe attenuation = -44.021
```

Figure 16: Plot for 61-point symmetric hanning window

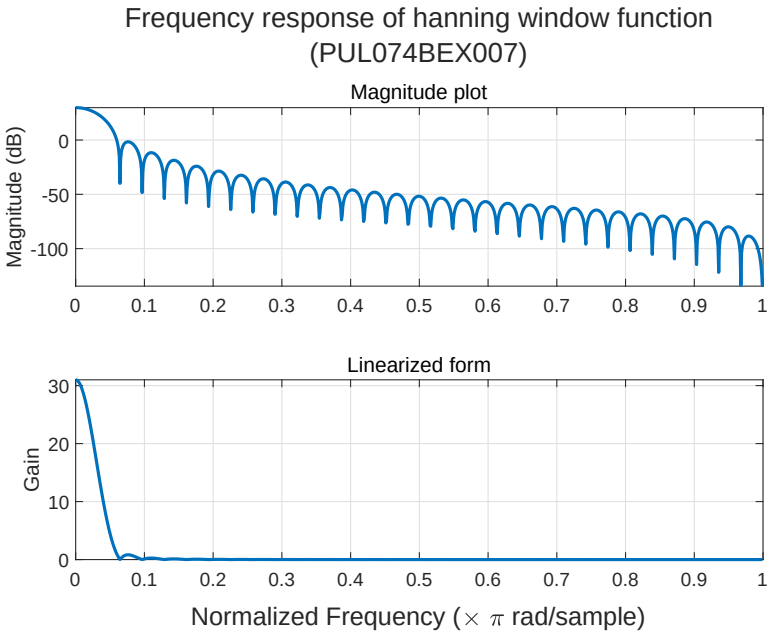


Figure 17: Plot for frequency response for hanning window function with M=61

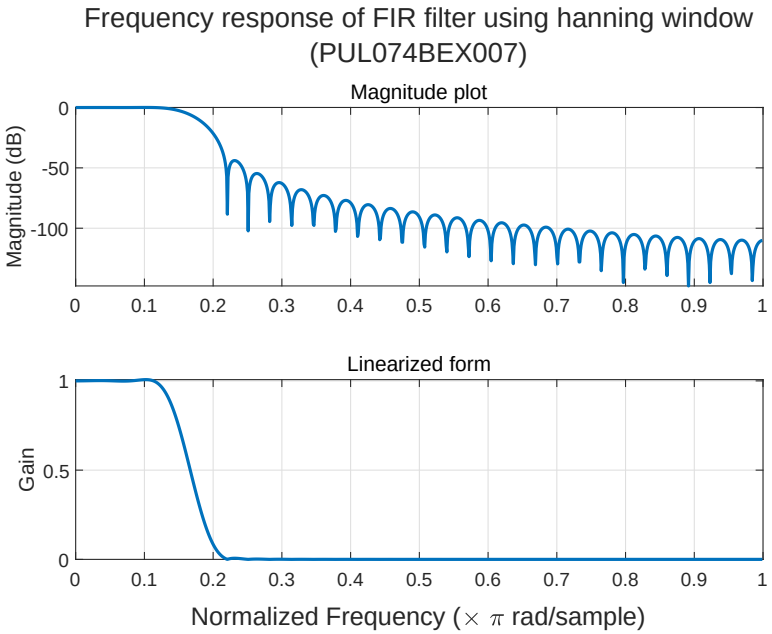
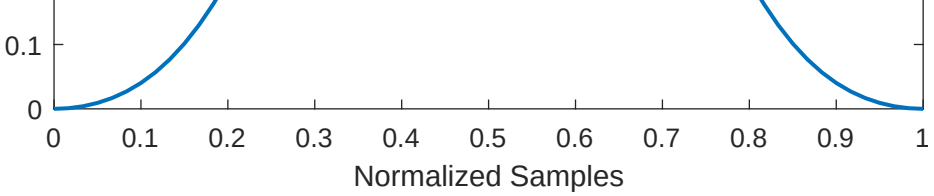


Figure 18: Plot for frequency response for FIR filter using hanning window for M=61




```
>> lab_6_1
Enter the type of window: blackman
Enter the value of M: 61
Parameters for FIR filter using blackman window:
Main lobe width = 0.299
Relative side lobe attenuation = -75.617
```

Figure 19: Plot for 61-point symmetric blackman window

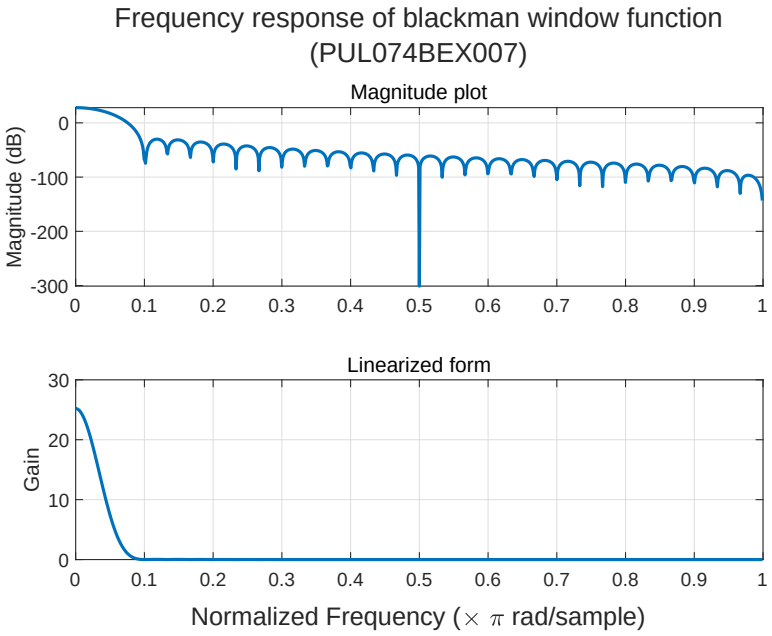


Figure 20: Plot for frequency response for blackman window function with M=61

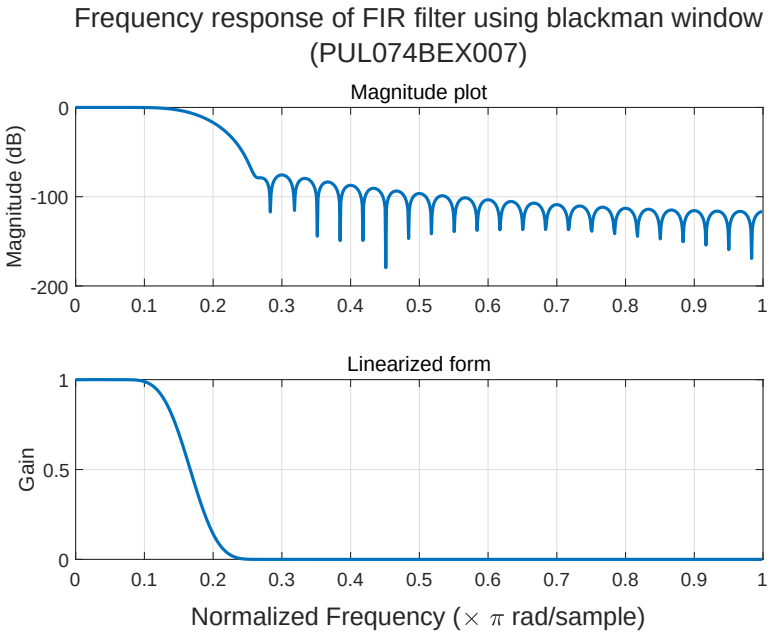
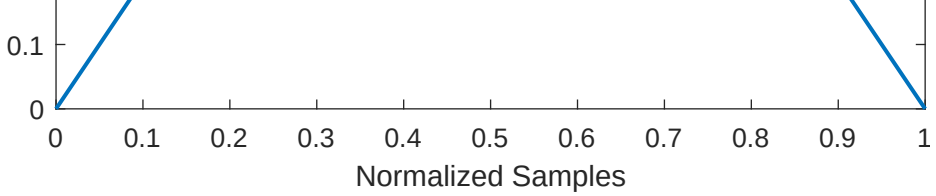


Figure 21: Plot for frequency response for FIR filter using blackman window for M=61




```
>> lab_6_1
Enter the type of window: bartlett
Enter the value of M: 61
Parameters for FIR filter using bartlett window:
Main lobe width = 0.309
Relative side lobe attenuation =
```

Figure 22: Plot for 61-point symmetric bartlett window

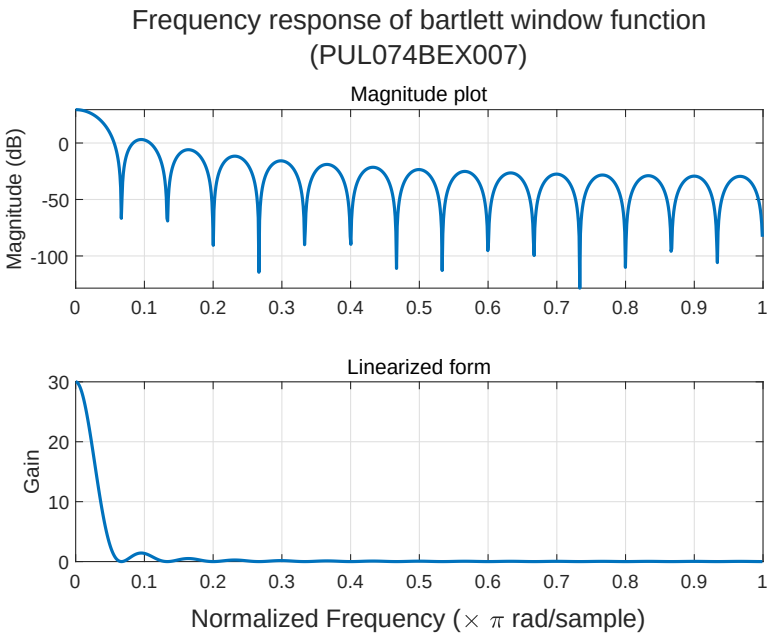


Figure 23: Plot for frequency response for bartlett window function with M=61

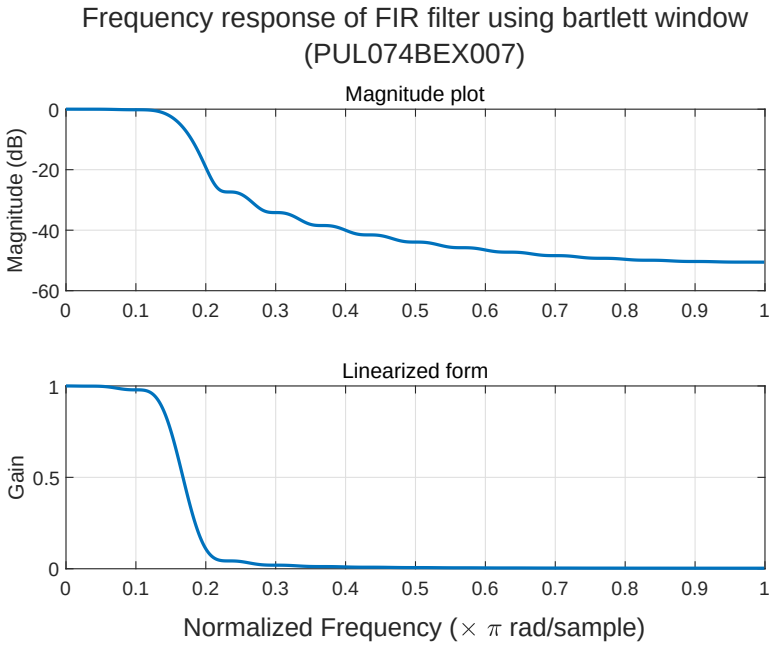


Figure 24: Plot for frequency response for FIR filter using bartlett window for M=61

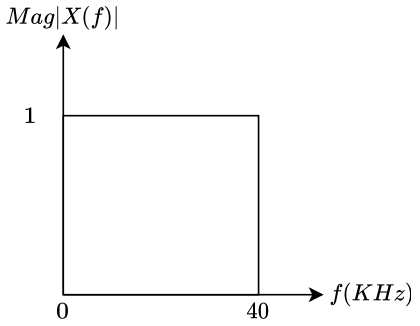
Problem 2

Discuss the effects of different types of the windowing functions on the frequency response of the FIR filter. Carry out the comparative study on the basis of the peak side lobe level, the approximate transition width of the main lobe etc. Also discuss on the effects of increasing value of M.

The frequency response of each designed FIR filter with different window functions, namely, hamming, hanning, blackman, bartlett are observed in the Figures above. The parameters such as the main lobe width and the relative side lobe attenuation are also determined and printed in the command console in MATLAB. It can be observed that the fourier transforms of the windows are generally concentrated at $\omega = 0$, which in fact is the desired property. In comparison, the different window responses have varying main lobe width and relative side lobe attenuation, among which there is a trade off. This tradeoff is distinctly visible in blackman window that has the main lobe width of approximately $0.266(\times\pi$ rad/samples$)$ and the relative side lobe attenuation is -74.275 dB. Likewise, for bartlett window, the side lobe level is increased with narrower main lobe width. By increasing the value of M, the main lobe width increases along with increased number of side lobes.

Problem 3

An analog signal $x(t)$ consists of the sum of two components $x_1(t)$ and $x_2(t)$. The spectral characteristics of $x(t)$ is shown in Figure. The signal $x(t)$ is band limited to 40 kHz and is sampled at the rate of 100 kHz to yield the sequence $x(n)$



It is desired to suppress the signal $x_2(t)$ by passing the sequence $x(n)$ through a digital low pass filter. The allowable distortion on $|X_1(f)|$ is $\pm 2\%$ $(\delta_1 = 0.02)$ over the range $0 \leq |F| \leq 15$ kHz. Above 20 kHz, the filter must have an attenuation of at least 40 dB$(\delta_2 = 0.01)$.

a. For obtaining the filter with above specifications, use the Kaiser window and determine the length of the required window. Plot the frequency response of the filter and its impulse response also.

b. If the same filter is to be designed using Hamming window what would be the length of the required window. Plot the frequency and impulse response of the filter.

```
function [beta] = betaValue(A)
    switch true
        case A>50
            beta=0.1102.*(A-8.7);
        case (21<=A)&&(A<=50)
            beta=0.5842.*(A-21)
    ^0.4+0.07886.*(A-21);
        case A<21
            beta=0;
        otherwise
            beta=0;
    end
end
```

Listing 4: Matlab function to return the value of β based on the attenuation A

```
selected_window = input('Enter the
    type of window: ', 's');
str=sprintf('FIR filter using %s
    window',selected_window);
wsample = 100;
wp = 15;
ws = 20;
wpn = 2*wp/wsample;
wsn = 2*ws/wsample;
wn = (wpn + wsn)/2;
A = 40;
switch selected_window
    case 'kaiser'
        beta = betaValue(A);
        dw = (wsn - wpn)/2;
        M = ((A-7.95)./(14.36*dw))
    + 1;
        M = round(M);
        if(rem(M,2)==0)
            M=M+1;
            N=M-1;
        else
```

```
            N=M-1;
        end
        win = kaiser(M,beta);
    case 'hamming'
        M=61;
        N=M-1;
        win=hamming(M);
    otherwise
        error('The window you want is not found.')
end
B = fir1(N,wn,win);
[hFir,wFir] = freqz(B,1,1024);
[f1,mlW,slA]=freqRespPlot(hFir, wFir,str);
fprintf('Parameters for %s:\n',str);
fprintf('Window length: %d\n',M);
fprintf('Main lobe width = %.3f\n',mlW)
fprintf('Relative side lobe attenuation = %.3f\n',slA)
f2=figure();
l=tiledlayout(1,1);
y=impz(B,1);
nexttile
stem(y, 'LineWidth', 1.5)
xlabel('n')
ylabel('Amplitude')
title(l,sprintf('Impulse response of %s\n(PUL074BEX007)',str));
print(f1,sprintf('../Figures/%s-fir', selected_window), '-depsc')
print(f2,sprintf('../Figures/%s-imp', selected_window), '-depsc')
```

Listing 5: Matlab script for visualizing the frequency response of designed FIR filter from either kaiser or hamming window based on user input

```
>> lab_6_3
Enter the type of window: kaiser
Parameters for FIR filter using kaiser window:
Window length: 47
Main lobe width = 0.672
Relative side lobe attenuation = -40.228
```

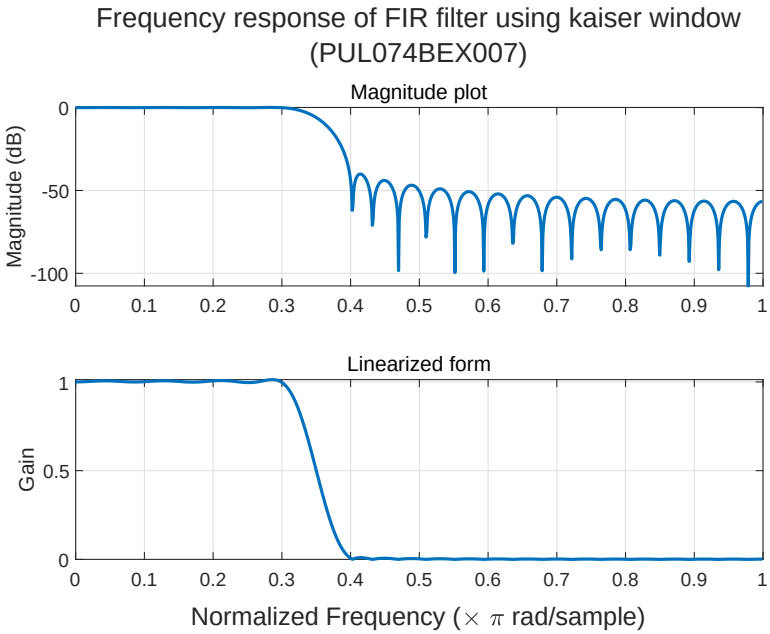


Figure 25: Plot for frequency response for FIR filter using kaiser window for M=47

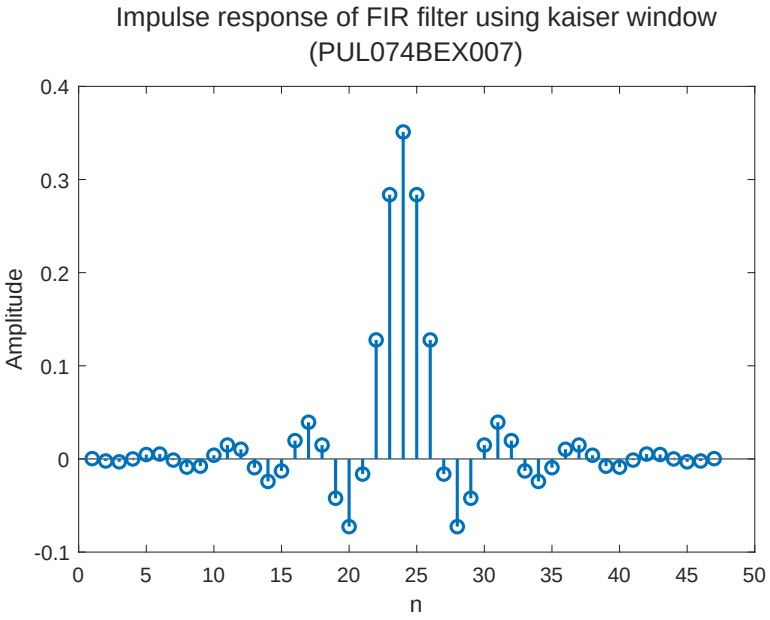


Figure 26: Plot for impulse response for FIR filter using kaiser window for M=47

```
>> lab_6_3
Enter the type of window: hamming
Parameters for FIR filter using hamming window:
Window length: 61
Main lobe width = 0.674
Relative side lobe attenuation = -54.289
```

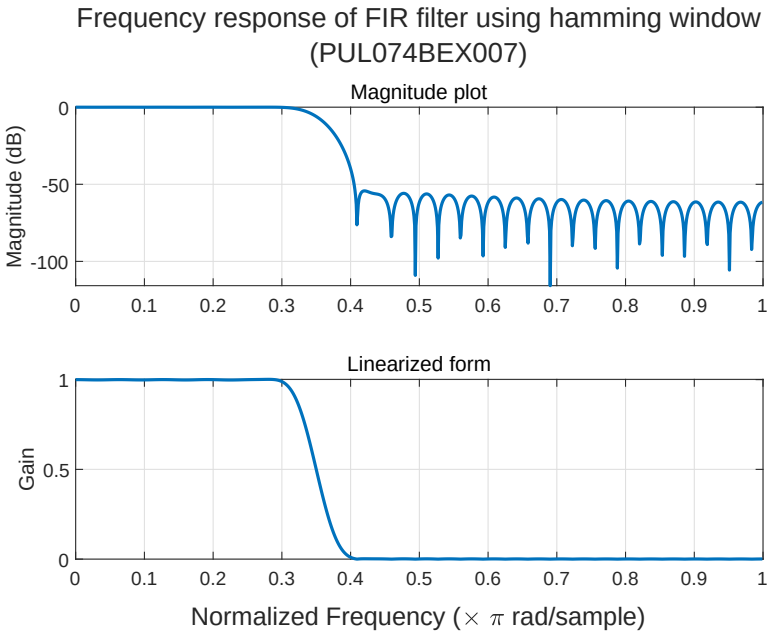


Figure 27: Plot for frequency response for FIR filter using hamming window for M=61

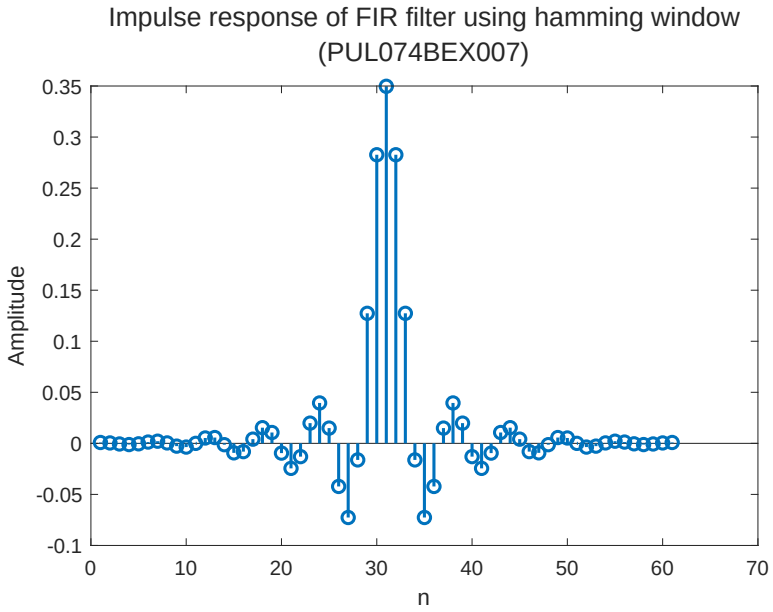


Figure 28: Plot for impulse response for FIR filter using hamming window for M=61

4 Discussion and Conclusion

In this lab experiment we dealt with the design and analysis of FIR filters using various windowing functions. Firstly, a window function such as Hamming, Hanning, Blackman, Bartlett or Kaiser is used to get the desired window function. The window functions like `Hamming`, `Hanning`, `Blackman`, `Bartlett`, `Kaiser` etc. are used to return the window function. Then the `fir1` function is used to obtain the frequency response of the designed FIR filter. The frequency responses of the window function, and designed FIR filter are plotted and analysis of parameters such as the main lobe width and relative side lobe attenuation is performed. The effect of increasing the window length was also noted on these parameters and the responses were plotted. Lastly, a kaiser window was used to design a FIR filter at given requirements. The value of β, which in fact is a function of attenuation A was used to return the window function to be used. The length of kaiser window was calculated to be M=47. An equivalent design using the hamming window for M=61 was attempted.

Hence, the objectives of the lab experiment were fulfilled.