Exchange Economies

- Let's construct a market with only 2 players.
  - Assume there are 2 consumers, consumer A and consumer B.
  - Assume there are 2 products (goods), good x and good y.

  - Consumer A starts with an endowment of x_A^e and y_A^e
  - Consumer B starts with an endowment of x_B^e and y_B^e.

  ☆ This implies that in total there are $x_A^e + x_B^e = \bar{x}$ units of x in this market and $y_A^e + y_B^e = \bar{y}$ units of y.

Let (x_A^*, y_A^*) be the amount of x and y that consumer A consumes.

Let (x_B^*, y_B^*) be the same for consumer B.

- Consumers (A and B) know the prices in the market (P_x and P_y) and can sell their endowment to purchase more of either x or y. Consumers know nothing besides the prices.

Exchange Economies

- A market equilibrium in this model is defined by a set of prices (P_x, P_y) and consumption bundles (x_A^*, y_A^*) and (x_B^*, y_B^*) where:

  1) $x_A^* + x_B^* = \bar{x}$ (markets clear)
     $y_A^* + y_B^* = \bar{y}$ Supply equals demand

  2) (x_A^*, y_A^*) solves $\max\limits_{x,y} U_A(x,y) : P_x x + P_y y \leq P_x x_A^e + P_y y_A^e$

     (x_B^*, y_B^*) solves $\max\limits_{x,y} U_B(x,y) : P_x x + P_y y \leq P_x x_B^e + P_y y_B^e$

- Condition 1 is a simple concept, consumers cannot consume more than the total endowment of either good.
- Condition 1 is the idea that supply which is fixed (the endowments) equals demand (the total consumption of consumers).
- Condition 2 requires that consumers are behaving rationally by choosing their bundle by maximizing their utility subject to their budget.

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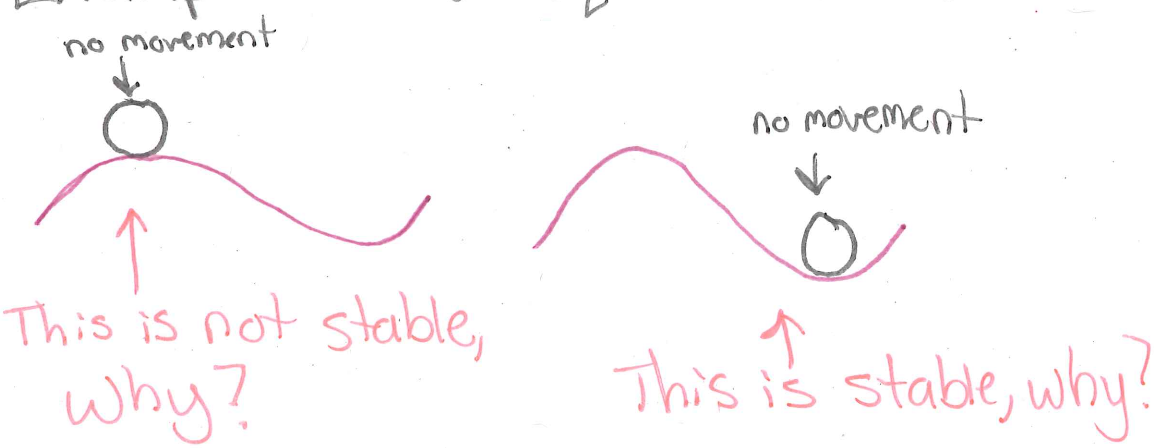
Important: You will not be asked to solve for this market equilibrium, but you need to be able to show that a given $(P_x, P_y, x_A^*, x_B^*, y_A^*, y_B^*)$ either <u>is</u> or <u>is not</u> an equilibrium.

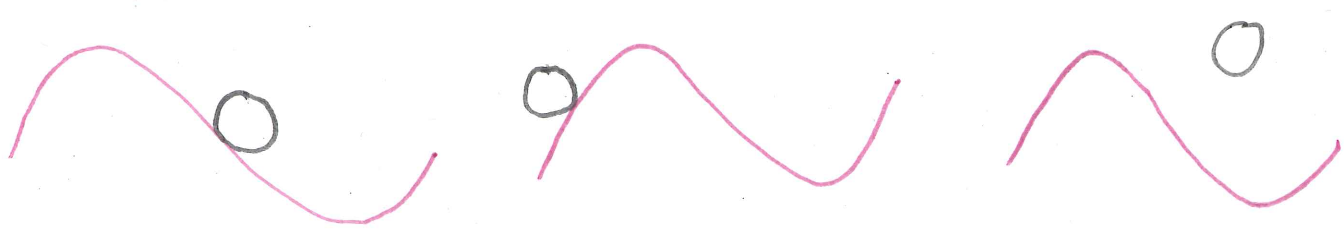
What is an equilibrium in general?

- imagine you put a marble on a track and let go, if the marble does not move we say that it is in equilibrium.

Examples of an equilibrium with a marble:



Examples that are not an equilibrium:



Exchange Economy

- Think of the ball being on the track as markets clearing.
  - A floating ball will fail even the most basic idea of what a valid answer is.
  - similarly, consumers having more than the total endowment is impossible.

- Think of the ball not moving along the track as consumers behaving optimally.
  - There is no reason for the ball to move
  - Similarly, there is no reason for a consumer to change their mind and want a different bundle.

Examples: Take as given, $x_A^e, x_B^e, y_A^e, y_B^e$ and let $U_A(x,y) = U_B(x,y) = x \cdot y$.

We know
$$x_A^* = \frac{I_A}{2P_x} = \frac{P_x x_A^e + P_y y_A^e}{2P_x}$$

$$y_A^* = \frac{I_A}{2P_x} = \frac{P_x x_A^e + P_y y_A^e}{2P_y}$$

$$x_B^* = \frac{I_B}{2P_x} = \frac{P_x x_B^e + P_y y_B^e}{2P_x}$$

$$y_B^* = \frac{I_B}{2P_x} = \frac{P_x x_B^e + P_y y_B^e}{2P_y}$$

Exchange Economy

- $$x_A^* + x_B^* = \frac{P_x x_A^e + P_y y_A^e + P_x x_B^e + P_y y_B^e}{2P_x}$$

$$= \frac{P_x \bar{x} + P_y \bar{y}}{2P_x} = \frac{\bar{x}}{2} + \frac{P_y}{2P_x}\bar{y}$$

Since $x_A^* + x_B^* = \bar{x}$ in equilibrium,

$$\Rightarrow \frac{\bar{x}}{2} + \frac{P_y}{2P_x}\bar{y} = \bar{x}$$

$$\Rightarrow \boxed{\frac{P_y}{P_x} = \frac{\bar{x}}{\bar{y}}} \Rightarrow P_x \bar{x} = P_y \bar{y}$$

(this should look familiar)

The Equilibrium

$$\Rightarrow \boxed{\begin{array}{ll} x_A^* = \frac{x_A^e}{2} + \frac{\bar{x}}{\bar{y}}\frac{y_A^e}{2}, & y_A^* = \frac{\bar{y}}{\bar{x}}\frac{x_A^e}{2} + \frac{y_A^e}{2} \\ x_B^* = \frac{x_B^e}{2} + \frac{\bar{x}}{\bar{y}}\frac{y_B^e}{2}, & y_B^* = \frac{\bar{y}}{\bar{x}}\frac{x_B^e}{2} + \frac{y_B^e}{2} \end{array}}$$

Double Check: $x_A^* + x_B^* = \frac{x_A^e}{2} + \frac{\bar{x}}{\bar{y}}\frac{y_A^e}{2} + \frac{x_B^e}{2} + \frac{\bar{x}}{\bar{y}}\frac{y_B^e}{2}$

$$= \frac{\bar{x}}{2} + \frac{\bar{x}}{\bar{y}} \cdot \frac{\bar{y}}{2} = \bar{x} \checkmark$$

$y_A^* + y_B^* = \bar{y}$ by similar math ✓

How do we know consumers are behaving optimally?

Exchange Economies

For practice proving that a market is in, see A3.

Key Properties of a market equilibrium:

① Only the price ratio ($\frac{P_y}{P_x}$ or $\frac{P_x}{P_y}$) matters for consumer choice and by extension, only $\frac{P_x}{P_y}$ is important in equilibrium.

(i.e. in a market is in equilibrium when $P_x=2$ and $P_y=3$, then it is in equilibrium when $P_x=10$ and $P_y=15$)

This is true because

$$P_x x^* + P_y y^* \leq P_x x^e + P_y y^e$$

$$\Longleftrightarrow$$

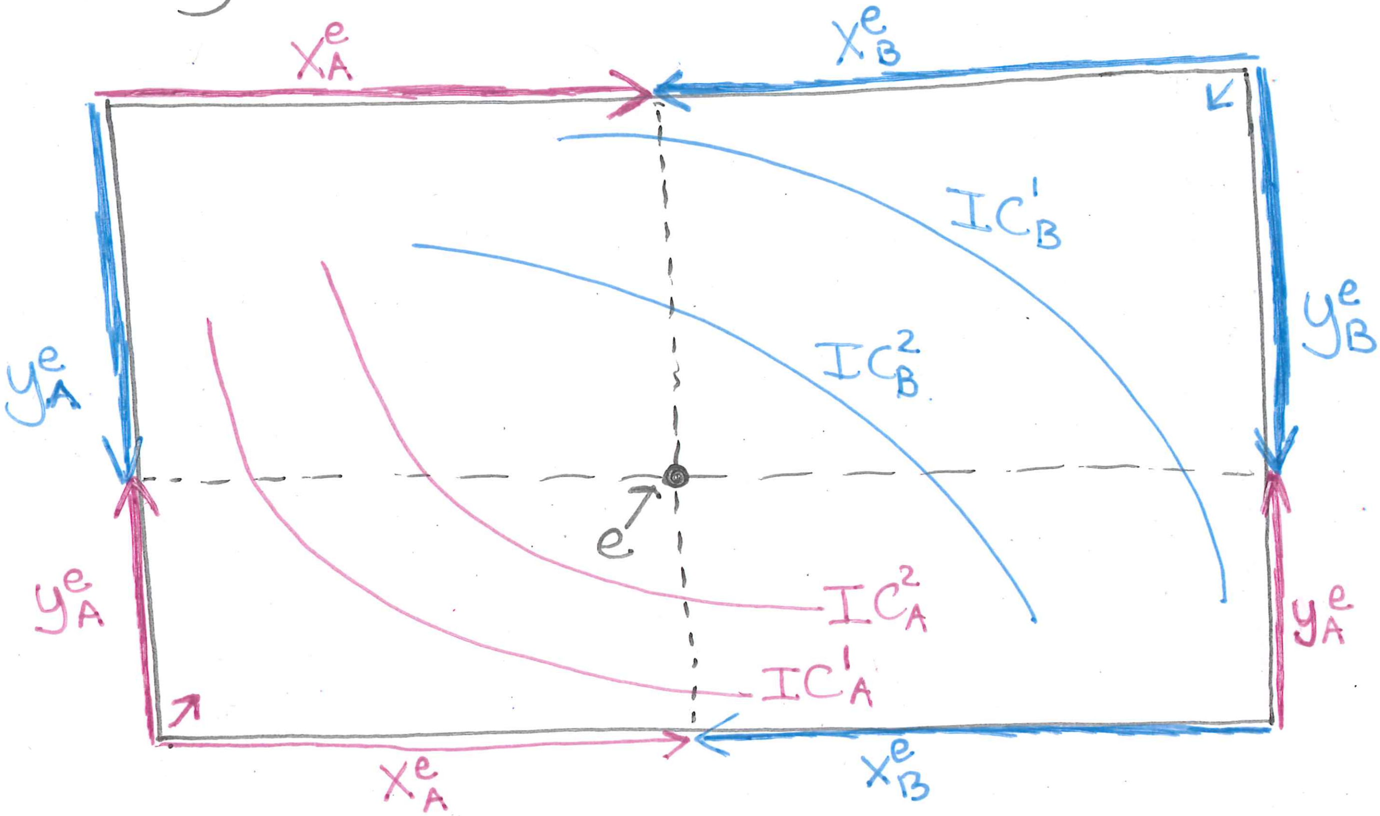
$$\frac{P_x}{P_y} x^* + y^* \leq \frac{P_x}{P_y} x^e + y^e$$ (why does this show only the ratio matters?)

② Any earlier solutions to utility max problems still apply where I is replaced with $P_x x^e + P_y y^e$

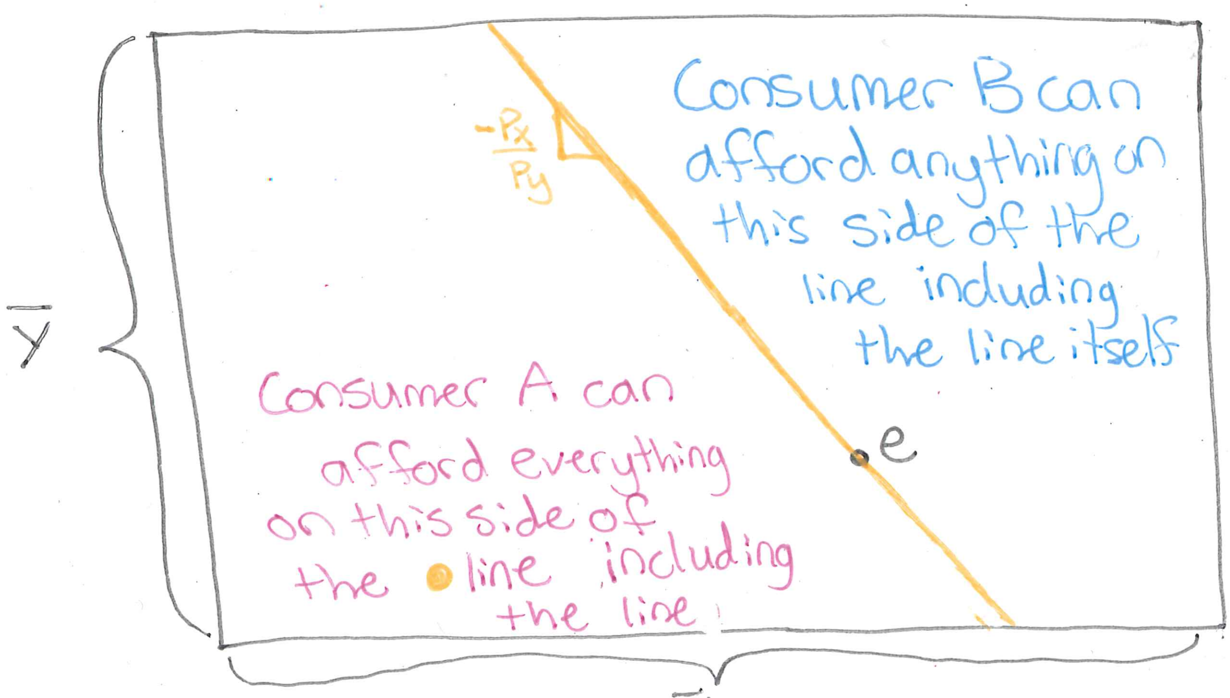
- By extension, price changes also effect income now

Exchange Economies

Graphing this market:

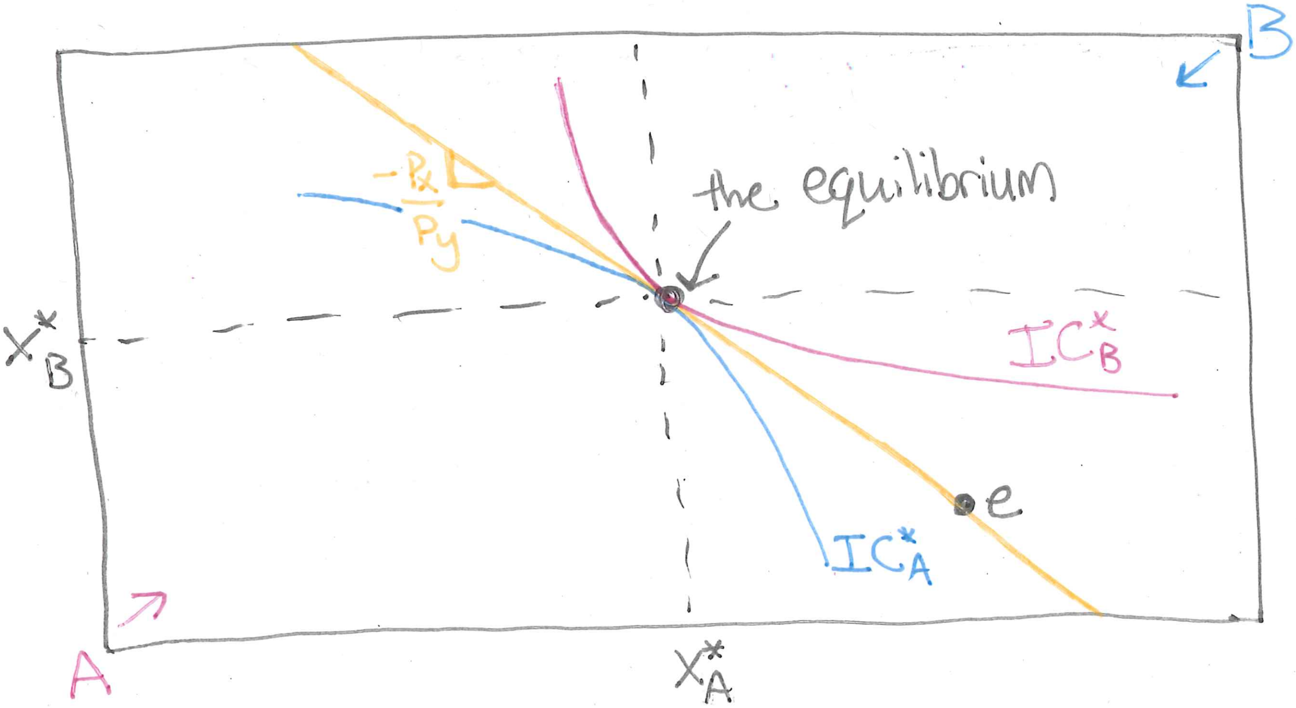


This graph (and the 2 consumer exchange problem) are called an <u>Edgeworth Box</u>!



Exchange Economy

What does an equilibrium look like?

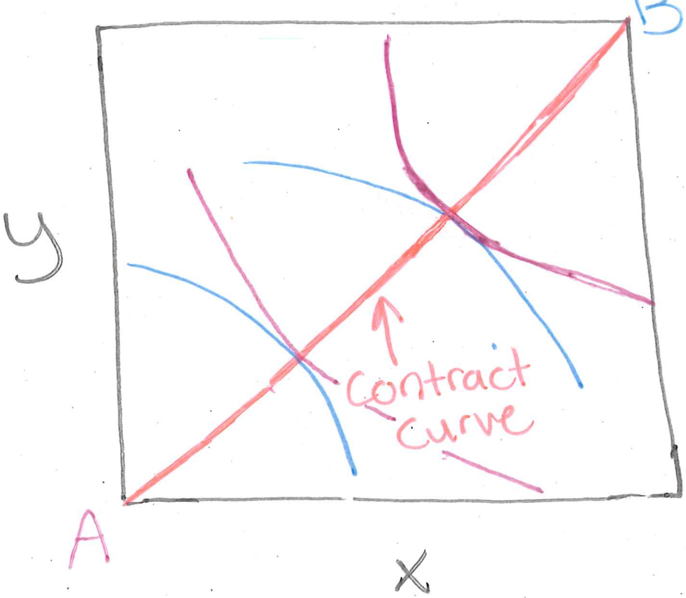


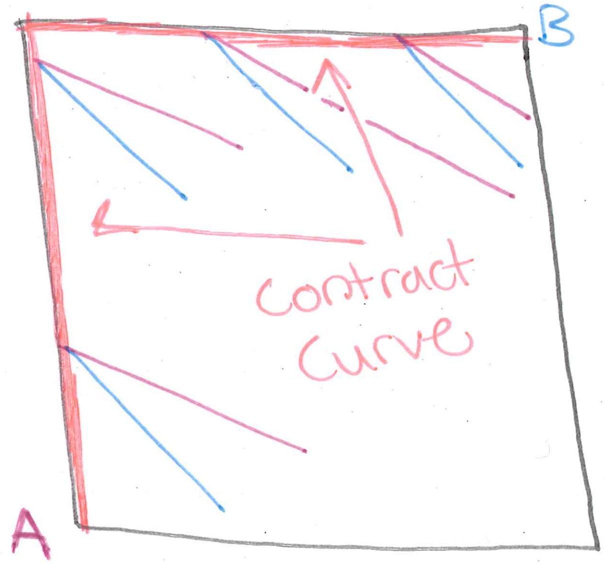
Notice that in equilibrium, IC_A and IC_B are tangent. Thus, there is no bundle where both players have a higher utility (and markets clear)

- In any market, we say an allocation is Pareto Efficient (or Pareto Optimal) if and only if there is no other allocations where every player (consumer) is better off. (weak vs strong discussed in lecture)

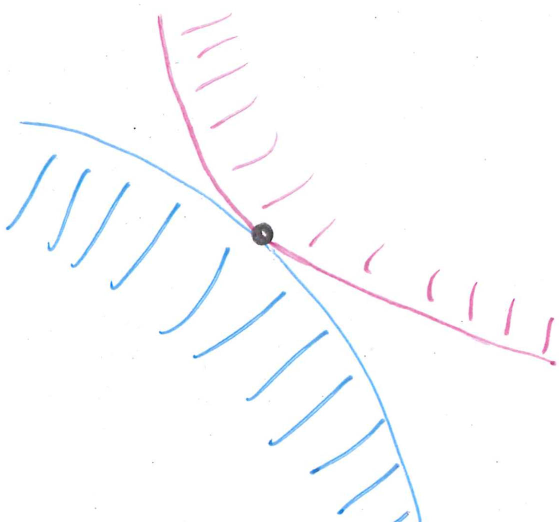
Exchange Economies

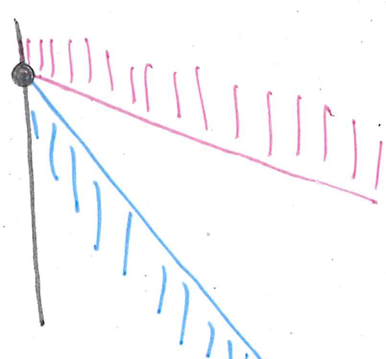
The contract curve is the set of all Pareto Efficient bundles.





Visually: For a point, Shade the region A is better off and shade the region B is better off, if there is no overlap then the point is pareto efficient.





For more, see A3 and A4.