Economics 3020

Intermediate Macroeconomics, Spring 2018

Problem Set 1

The following questions are meant to demonstrate your basic understanding of the topics covered. Answer the following questions correctly to receive full credit.
Note: items marked with ** are not required, but recommended

1 Math Review Problems

Suppose we want to maximise a function f given by:

$$f(x) = -x^2 + 2x$$

(a) x can be any real number between $-\infty$ and $+\infty$. Can you find a value of x that maximises the function f? What is the maximum value of f? (Hint: one way to solve this is finding a value of x at which the slope of f equals zero)

(b) Suppose you have found a value of x (say) that makes the slope of f zero. But, this is not a sufficient condition in gneral. In other words, may be one candidate that potentially maximises f, but it is uncertain that f is actually maximised at . One way to establish "sufficency" is to show the function f is concave on the domain of x (in this case the domain is $-\infty < x < +\infty$). How do you show f is on a concave function?

(c) Now find a value of z at which the slope of the function h is zero:
$$h(z) = z^2 - 2z$$

Does the value you just found maximises the function h? Explain.

2

Consider the foloowing function:

$$f(x_1, x_2) = ax_1^{0.7} x_2^{0.3} - bx_1 - cx_2, \text{ where } a, b, \text{ and } c \text{ are constant.}$$

(a) Find the optimal value of x_1 that maximises f given x_2 (that is, treat x_2 as if it is fixed at a particular value or treat x_2 as if it is a constant). Also, find the optimal value of x_2 given x_1. (Don't worry about concavity)

(b) For a fixed x_2, is the slope of f (or the first derivative with respect to x_1) increasing or decreasing in x_1?

(c) For a fixed x_1, is the slope of f (or the first derivative with respect to x_2) increasing or decreasing in x_2

3   Firms' Optimisation Problem

Suppose the US economy has a Cobb-Douglas production function:

$$F(K, L) = AK^\alpha L^{1-\alpha}$$

where $0 < \alpha < 1$ is assumed to be a constant

(a) Derive the marginal product of capital (MPK). You can obtain MPK by taking the first derivative of F with respect to K, treating the other variables as constants (differentiate $F(K, L)$ with respect to K). That is, MPK is given by the following equation:

$$MPK = \frac{\partial F(K,L)}{\partial K}$$

(b) Find the marginal product of labour:

$$MPL = \frac{\partial F(K,L)}{\partial L}$$

(c) Derive firms' optimal level of capital (demand for capital). You need to solve for K (express K as function of other variables (A, L, and R/P and parameter α.

(d) How does the demand for K depend on A, L, and R/P? Are the results intuitive?

(e) Similarly, derive firms' optimal level of labour (demand for labour).

(f) How does the demand for L depend on A, K, W/P?

(g) Show that firms' real profit should be zero under optimal K and L, that is:

$$\frac{\Pi}{P} = F(K, L) - \frac{W}{P}L - \frac{R}{P}K = 0$$