MA1014 Probability and Queueing Theory

Unit I

(Probability and Random Variables)

1. The Random variable which can take infinite number of values in an interval is called ……..
   (a) Random Variable (b) Continuous R.V (c) Discrete R.V (d) Range space
   Ans: (b)

2. The conditions satisfied by the pdf are ……
   (a) $p(x) \geq 0 \,\&\, \sum p(x) = 1$ (b) $f(x) \geq 0 \,\&\, \int_{-\infty}^{\infty} f(x)dx = 1$
   (c) $p(x) \leq 0 \,\&\, \sum p(x) = 0$ (d) $f(x) \leq 0 \,\&\, \int_{-\infty}^{\infty} f(x)dx = 1$
   Ans: (b)

3. The cumulative distribution function F(x) is a ……… function of X.
   (a) Increasing (b) non- increasing (c) non- decreasing (d) decreasing
   Ans: (c)

4. If X is a continuous R.V, then $\frac{d}{dx}F(x) = f(x)$ at all points here F(x) is—
   (a) integrable (b) Constant (c) 1 (d) Differentiable
   Ans: (d)

5. The value of 'k' from the following table is------

x	-2	-1	0	1	2	3
p(x)	0.1	k	0.2	2k	0.3	3k

   (a) 1 (b) $\frac{1}{10}$ (c) $\frac{1}{15}$ (d) $\frac{2}{3}$ **Ans: (c)**

6. From the above table the value of P(X<2) is
   (a) 1 (b) $\frac{1}{15}$ (c) $\frac{1}{2}$ (d) $\frac{1}{30}$ **Ans: (c)**

7. The value of $P(1/2\, X < 2/3)$ from the above table is ----
   (a) 0.5 (b) ∞ (c) undefined (d) 1 **Ans: (c)**

8. The Relation between Variance and Standard deviation is ------
   (a) $\text{var} = S.D^2$ (b) $\text{var} = \sqrt{S.D}$
   (c) $\text{var} - S.D = 0$ (d) $\text{var} = \sqrt[2]{S.D}$ **Ans: (a)**

9. The Relation between Covariance and Mean is ------
   (a) $\text{cov}(X,Y) = E(XY) - E(X)E(Y)$
   (b) $\text{cov}(X,Y) = E(XY) + E(X)E(Y)$
   (c) $\text{cov}(X,Y) = E(XY) - (E(X)E(Y))^2$
   (d) $\text{cov}(X,Y) = E(XY)^2 - (E(X)E(Y))^2$ **Ans: (a)**

10. The value of k if the pdf $f(x) = kx^2e^{-x}$, $x \geq 0$ is --------
    (a) 0.5 (b) ∞ (c) 0 (d) 1 **Ans: (a)**

11. The Relation between Variance and Mean is ------

(a) $VarX = E(x) - (E(x))^2$ (b) $VarX = E(x^2) - (E(x))^2$

(c) $VarX = (E(x))^2 - E(x^2)$ (d) $VarX = (E(x))^2 - E(x)$ **Ans: (b)**

12. The functions of R.V $Y = g(x)$ is given by

(a) $h(y) = f(x)\left|\frac{dx}{dy}\right|$ (b) $h(x) = f(y)\left|\frac{dx}{dy}\right|$

(c) $h(y) = f(x)\left|\frac{dy}{dx}\right|$ (d) $h(x) = f(y)\left|\frac{dy}{dx}\right|$ **Ans: (a)**

13. The generalized form of Tchebycheff's inequality is ------------

(a) $P\left[|X-\mu| < k\sigma\right] = 1 - \frac{1}{k^2}$ (b) $P\left[|X-\mu| > k\sigma\right] = 1 - \frac{1}{k^2}$

(c) $P\left[|X-\mu| < k\sigma\right] = \frac{1}{k^2}$ (d) $P\left[|X-\mu| > k\sigma\right] = \frac{1}{k^2}$ **Ans: (a)**

14. The conditions satisfied by the pmf is ……

(a) $p(x) \geq 0 \,\&\, \sum p(x) = 1$ (b) $f(x) \geq 0 \,\&\, \int_{-\infty}^{\infty} f(x)dx = 1$

(c) $p(x) \leq 0 \,\&\, \sum p(x) = 0$ (d) $f(x) \leq 0 \,\&\, \int_{-\infty}^{\infty} f(x)dx = 1$ **Ans: (a)**

15. If Var(X) = 4, then Var(4X+5) is

(a)89 (b) 69 (c) 64 (d) 9 **Ans(c)**

16. If X and Y are independent random variables with Var 2 and 3 respectively, Then Var(3X+4Y) is

(a) 66 (b) 7 (c) 25 (d) 18 **Ans: (a)**

17. If X and Y are independent random variables with Var 2 and 3 respectively, Then Var(3X - 4Y) is

(a) 66 (b) 7 (c) 25 (d) 18 **Ans: (a)**

18. If E(X) =3 ,then E(3X+4) is

(a) 15 (b) 13 (c) 9 (d) 10 **Ans: (b)**

19. Var(aX) is

(a) aVar(X) (b) a^2 Var(X) (c) Var(X) (d) 0 **Ans: (b)**

20. Var (aX+b) =

(a) aVar(X)+b (b) a^2 Var(X) (c)aVar(X) (d)Var(X) **Ans: (b)**

Unit II

(Theoretical Distributions)

1. A discrete R.V X has moment generating function $M_x(t) = \left(\frac{1}{4} + \frac{3}{4}e^t\right)^5$. Then E(X) and Var(X) is

a) $\frac{15}{4}, \frac{15}{4}$ b) $\frac{15}{4}, \frac{15}{16}$ c) $\frac{1}{4}, \frac{5}{4}$ d) $\frac{1}{4}, \frac{3}{4}$ **Ans: (b)**

2. Mean and Variance of Binomial Distribution is

a) np, npq b) nq, n/q c) pq , p+q =1, d) p+q,p-q **Ans: (a)**

3. If on an average, 9 ships out of 10 arrive safely to a port then the variance of the number of ships returning safely out of 150 ships is

a) 135 b) 13.5 c) 1.35 d) 12 **Ans: (b)**

4. If X and Y are independent Poisson variates with parameters λ_1 and λ_2, then X+Y is also a Poisson variate with parameter

a) $\lambda_1 + \lambda_2$ b) $\lambda_1 - \lambda_2$ c) λ_1 / λ_2 d) $\lambda_1 . \lambda_2$ **Ans: (a)**

5. Let X be a random variable following Poisson distribution such that P(X=2) = 9P(X=4) + 90P(X=6), then the mean of X is

a)1 b) 2 c)0 d)5 **Ans: (a)**

6. If X is a random variable with geometric distribution, then P[X > s+t / X > s] =

a) P[X > s] b) P[X > t] c) P[X < t] d) P[X < s] **Ans: (b)**

7. If the probability of success on each trial is 1/3 , then the expected number of trials required for the first success is

a) 2/3 b) 3 c) 2 d)1/3 **Ans: (b)**

8. A typist types 2 letters errorneously for every 100 letters. Then the probability that the tenth letter typed is the first letter with error is

a) 0.0167 b) 2.335 c) .0001 d) 0.1 **Ans: (a)**

9. Four coins are tossed simultaneously the probability of getting 2 heads is

a) 3/4 b)11/16 c)3/8 d)3 **Ans: (c)**

10. Poisson distribution is a limiting case of

a)Binomial distribution b) uniform distribution

c) Geometric distribution d) Normal distribution. **Ans: (a)**

11. The mean and variance of poisson distribution is

a) λ b) λ^2 c) λ^3 d) pq **Ans: (a)**

12. If the moment generating function of the random variable is $e^{4(e^t-1)}$, Find $P(X = \mu + \sigma)$

where μ and σ^2 are the mean and variance of poisson

a) $\frac{e^4 4^6}{6!}$ b) $\frac{e^{-4} 4^6}{6!}$ c) $\frac{e^{-6} 6^4}{4!}$ d) $\frac{e^6 6^4}{4!}$ **Ans: (b)**

13. Variance of Exponential distribution is

a) $\frac{1}{\lambda}$ b) $\frac{1}{\lambda^2}$ c) $\frac{1}{\sqrt{\lambda}}$ d) λ **Ans: (b)**

14. Memory less property is satisfied by

a) Exponential distribution b) Uniform distribution C) Normal distribution

d) Binomial distribution **Ans: (a)**

15. Moment generating function of exponential disturibution is ____________

16. All odd order moments of a Normal distribution about its mean are

a) Zero b) one c) infinity d) uniform **Ans: (a)**

17. Total area under the standard normal curve is equal to

a)0 b) 1 c)2 d) ∞ **Ans: (b)**

18. If for a poisson variate, $E(X^2) = 6$, what is E(X)

a)1 b) 2 c) 6 d)3 **Ans: (b)**

19. If X has uniform distribution in (-3,3) Then $P(|x-2| < 2)$ IS

a) 0 b)1 c)1/2 d)2 **Ans: (c)**

20. Which of the following distribution satisfies Memoryless Property?

a) Binomial distribution b) Poisson distribution c) Geometric distribution

d) Normal distribution. **Ans: (c)**

UNIT III

TESTING OF HYPOTHESES

1. If θ_0 is a population parameter and θ is the corresponding sample statistic and if we set up the null hypotheses $H_0: \theta = \theta_0$ then the right-tailed alternative hypotheses is

(a) $H_1: \theta = \theta_0$ (b) $H_1: \theta > \theta_0$ (c) $H_1: \theta < \theta_0$ (d) $H_1: \theta \neq \theta_0$ **Ans: (b)**

2. The size of large sample is :

(a)Exact (b) Less than 30 (c) Greater than 30 (d) Equal to 30 **Ans: (c)**

3. The statistic to test the significance difference between sample proportion and population proportion is

(a) $\dfrac{p-P}{\sqrt{\dfrac{p}{n}}}$ (b) $\dfrac{p+P}{\sqrt{\dfrac{pQ}{n}}}$ (c) $\dfrac{p-P}{\sqrt{\dfrac{pQ}{n}}}$ (d) $\dfrac{p-P}{\sqrt{\dfrac{Q}{n}}}$

Ans: (c)

4. The statistic to test the significance difference between the sample mean and population mean is

(a) $Z = \dfrac{\bar{X}-\mu}{\dfrac{\sigma}{\sqrt{n}}}$ (b) $Z = \dfrac{\bar{X}+\mu}{\dfrac{\sigma}{\sqrt{n}}}$ (c) $Z = \dfrac{\bar{X}}{\dfrac{\sigma}{\sqrt{n}}}$ (d) $Z = \dfrac{\bar{X}-\mu}{\dfrac{\sigma}{n}}$

Ans: (a)

5. If σ_1 and σ_2 are equal and not known then the test statistic is

(a) $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{s_1^2}{n_2} + \dfrac{s_2^2}{n_1}}}$ (b) $Z = \dfrac{\bar{X}_1 + \bar{X}_2}{\sqrt{\dfrac{s_1^2}{n_2} + \dfrac{s_2^2}{n_1}}}$ (c) $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{s_1^2}{n_2} - \dfrac{s_2^2}{n_1}}}$ (d) $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

Ans: (a)

6. The sample is said to be small if

(a) $n > 30$ (b) $n > 100$ (c) $n < 60$ (d) $n < 30$ **Ans: (d)**

7. The t – distribution is used to test the significance of the difference between

(a)Mean of two small samples (b) Variance of two small samples

(c) Mean of two large samples (d) Variance of two large samples **Ans: (a)**

8. If $n_1 = n_2 = n$, then the degrees of freedom to test mean of the two small samples is

(a) $n_1 + n_2 - 2$ (b) $n_1 + n_2 + 2$ (c) $2n - 2$ (d) $2n + 2$ **Ans: (c)**

9. The use of F-distribution is to test the

(a) Mean of two small samples (b) Variance of two small samples

(c) Mean of two large samples (d) Variance of two large samples **Ans: (b)**

10. The value of test statistic F is

(a) $F > 1$ (b) $F < 1$ (c) $F = 1$ (d) $F = 0$ **Ans: (a)**

11. The statistics to test the significance difference between means of two samples is

(a) $\dfrac{\overline{x_1}-\overline{x_2}}{\sqrt{\left(\dfrac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2-2}\right)\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$ (b) $\dfrac{\overline{x_1}+\overline{x_2}}{\sqrt{\left(\dfrac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2-2}\right)\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$

(c) $\dfrac{\overline{x_1}-\overline{x_2}}{\sqrt{\left(\dfrac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2-2}\right)\left(\dfrac{1}{n_1}-\dfrac{1}{n_2}\right)}}$ (d) $\dfrac{\overline{x_1}-\overline{x_2}}{\sqrt{\left(\dfrac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2-2}\right)}}$ **Ans:**

(a)

12. Chi square distribution is used to

(a) To test the mean of two small samples (b) To test the mean of two large samples

(c) To test the goodness of fit (d) To test the variance of two populations

Ans: (c)

13. In Chi square test , the number of observations in the sample is

(a) ≥ 50 (b) ≤ 50 (c) 10 (d) 100 **Ans: (a)**

14. In Chi square test , the condition to choose small n is

(a) $4 \leq n$ (b) $4 \leq n \leq 16$ (c) $n \geq 16$ (d) $n \leq 4$ **Ans: (b)**

15. The statistic of chi square test is

(a) $\chi^2 = \sum \dfrac{(O_i - E_i)^2}{E_i}$ (b) $\chi^2 = \sum \dfrac{(O_i - E_i)^2}{E_i^2}$

(c) $\chi^2 = \sum (O_i - E_i)$ (d) $\chi^2 = \sum \dfrac{(O_i - E_i)^2}{E_i}$ **Ans: (a)**

16. The number of degrees of freedom of Chi square test is

(a) n-2 (b) n-3 (c)n-4 (d) n-1 **Ans: (d)**

17. The value of χ^2 for 2 x 2 contingency table is

(a) $\chi^2 = \dfrac{N(ad-bc)}{(a+b)(c+d)(a+c)(b+d)}$ (b) $\chi^2 = \dfrac{N(ad+bc)^2}{(a+b)(c+d)(a+c)(b+d)}$

(c) $\chi^2 = \dfrac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$ (d) $\chi^2 = \dfrac{N(ad-bc)^2}{(a-b)(c+d)(a+c)(b+d)}$ **Ans: (c)**

Unit-IV

(Principles of Queueing theory)

1. In Queueing system,the Number of arrivals per unit time always follows _____ distribution.

(a) poisson (b) exponential (c) Binomial d)Normal **Ans: (a)**

2. In the model M/M/1 , the first M represents ___________

a) server b) arrival c) no. of servers d)departure **Ans: (b)**

3. In the model M/M/1 , then 1 represents ___________

a)single server b) multiple server c) single arrival d) multiple arrival **Ans: (a)**

4. The average waiting time of a customer in the (M/M/1):($\infty/FIFO$) system is

a) $\dfrac{1}{\mu-\lambda}$ b) $\dfrac{\lambda}{\mu-\lambda}$ c) $\dfrac{\mu}{\mu-\lambda}$ d) $\dfrac{\mu}{\mu+\lambda}$ **Ans: (a)**

5 . Mean arrival rate is denoted by

a) $\dfrac{1}{\lambda}$ b) λ c) μ d) $\dfrac{1}{\mu}$ **Ans: (a)**

6. The number of arrivals per unit time has a poisson distribution with mean _____

a) $\dfrac{1}{\lambda}$ b)λ c)μ d) $\dfrac{1}{\mu}$ **Ans: (b)**

7. The number of customers in the system in M/M/1 model is

a) $\dfrac{1}{\mu-\lambda}$ b) $\dfrac{\lambda}{\mu-\lambda}$ c) $\dfrac{\mu}{\mu-\lambda}$ d) $\dfrac{\mu}{\mu+\lambda}$ **Ans: (b)**

8. The probability that the arrival enter the service without wait is

a)1 + P(arrival has to wait) b)P(arrival has to wait) – 1

c)1 - P(arrival has to wait) d)zero **Ans: (c)**

9. Average number of customer in the system when ρ=1 in (M/M/1) :(K/FIFO) is ________

a) K/2 b)2K c) K d)0 **Ans: (a)**

10. The number of customer in the system are always ________

a) mutually exclusive b) mutually exhaustive

c) mutually exclusive and exhaustive d) unique **Ans: (c)**

11. The relation between $E(N_s)$ and $E(N_q)$ is

a) $E(N_s) = E(N_q) + \frac{\lambda}{\mu}$ b) $E(N_s) = E(N_q) - \frac{\lambda}{\mu}$

c) $E(N_s) = E(N_q) + \frac{1}{\mu}$ d) $E(N_s) = E(N_q) + \lambda\mu$ **Ans: (a)**

UNIT –V

(MARKOV CHAIN)

1. A discrete parameter markov process is called a

(a)Markov process (b) stationary process (c) random process (d) Markov chain

Ans: (d)

2. A square matrix , in which the sum of all the elements of each row is one is called a

(a)unitary matrix (b) diagonal matrix (c) stochastic matrix (d) skew matrix **Ans: (c)**

3. A stochastic matrix P is said to be regular if all the entries of P^m are

(a)negative (b) positive (c) semi positive (d) either positive or negative **Ans: (b)**

4. If $\pi = (\pi_1, \pi_{2,...,}\pi_n)$ is the steady state distribution of the chain whose tpm is the n^{th} order square matrix P, then

(a) $\pi P = \pi$ (b) $\pi\mu = \pi$ (c) $\pi A = n$ (d) $\pi P = P$. **Ans: (a)**

5.The conditional probability $P[X_n = a_j/X_{n-1} = a_i]$ is called

(a) second tpm (b)one-step transition probability (c) homogeneous (d) n-step tpm **Ans:(b)**

6. If the one-step tpm does not depend on the step ie. $p_{ij}(n-1,n) = p_{ij}(m-1,m)$ the markov chain is called

(a) stationary chain (b) discrete chain (c) homogeneous markov chain

(d) regular markov chain **Ans: (c)**

7. The conditional probability $P[X_n = a_j/X_0 = a_i]$ is called

(a) second tpm (b)one-step tpm (c) homogeneous (d) n-step transition probability **Ans:(d)**

8. If P is the tpm of a homogeneous Markov chain , then the n-step tpm $P^{(n)} = P^n$ is known as

(a) probability theorem (b) Chapman- Kolmogorov Theorem

(c) Markov theorem (d) Chapman theorem **Ans: (b)**

9. State i of a Markov chain is said to be ----- with period d_i if $d_i > 1$

(a) periodic (b) not periodic (c) aperiodic (d) biperiodic **Ans: (a)**

10. State i of a Markov chain is said to be ----- with period d_i if $d_i = 1$

(a) periodic (b) not periodic (c) aperiodic (d) biperiodic **Ans: (c)**

11. Every state can be reached from every other state , the Markov chain is said to be

(a) homogeneous (b) reducible (c) irreducible (d) recurrent **Ans: (c)**

12. A non null persistent and aperiodic state is called

(a) markov (b) irreducible (c) recurrence (d) ergodic **Ans: (d)**

13. A state i is said to be ------ if the return to state i is certain.

(a) persistent (b) non persistent (c) ergodic (d) periodic **Ans:(a)**

14. A state i is said to be ------ if the return to state i is uncertain.

(a) persistent (b) non persistent (c) transient (d) periodic **Ans: (c)**

15. A state i is said to be ------ if the mean recurrence time μ_{ii} is finite.

(a) persistent (b) non persistent (c) transient (d) non null persistent **Ans:(d)**

16. A state i is said to be ------ if the mean recurrence time $\mu_{ii} = \infty$.

(a) persistent (b) non persistent (c) null persistent (d) non null persistent **Ans:(c)**

17. If a markov chain is finite irreducible , all its states are

(a) persistent (b) null persistent (c) non null persistent (d) recurrent **Ans: (c)**

18. A Markov chain is completely specified when

(a) intial probability distribution (b) tpm (c) absorbing state (d) both a & b are given **Ans:(d)**

19.If $\pi P = \pi$, where P = $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$ then values of (π_1, π_2) is

(a) (1/3,2/3) (b) (1/2,1/2) (c) (2/3,1/3) (d) (0,1) **Ans:(a)**

20.If the tpm of a markov chain is P = $\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ find $P[X_1 = 3/X_0 = 2]$.

(a) 0.1 (b) 0.2 (c) 0.4 (d) 0.6 **Ans:(b)**

====================