Unit - 5

Z - Transform

* Let $\{x(n)\}$ be a sequence defined for $n = 0, \pm 1 \ldots$ then two sided z-transform of sequence $x(n)$ is defined by. $Z\{x(n)\} = X(z) = \sum_{n=\infty}^{\infty} x(n) z^{-n}$

$z \rightarrow$ complex variable.

If $x(n)$ is a causel sequence, i.e, $x(n) = 0$ for $n < 0$.

Then, $Z\{x(n)\} = X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$.

Inverse :- $Z^{-1}\{X(z)\} = \{x(n)\}$

Unit Sample Sequence :-

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

Unit Step Sequence :-

$$U(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

* If $f(t)$ is function defined for discret value of t where $t = nT$, T being sampling period, then.

$$Z\{f(t)\} = \sum_{n=0}^{\infty} f(t) z^{-n} = \sum_{n=0}^{\infty} f(nT) z^{-n}.$$

Properties of Z-transform :-

1. Z transform is linear.

(i) $Z[af(t) + bg(t)] = a\,Z[f(t)] + b\,Z[gt]$

$\boxed{Or}$

(ii) $Z[a\{x(n)\} + b\{y(n)\}] = aZ\{x(n)\} + b\,Z\{y(n)\}$

Proof :- (i) $Z\left[af(t) + bg(t)\right] = \sum_{n=0}^{\infty}[af(t) + bg(t)]\,z^{-n}$

$$= \sum_{n=0}^{\infty} a\cdot f(nT)\,z^{-n} + \sum_{n=0}^{\infty} b\cdot g(nT)\,z^{-n}$$

$$= a\,Z[f(t)] + b\,Z\,[g(t)]$$

(ii) $Z[a\{x(n)\} + b\{y(n)\}]$

$$= \sum_{n=0}^{\infty}[a\{x(n)\} + b\,\{y(n)\}]\,z^{-n}$$

$$= \sum_{n=0}^{\infty} a\cdot x(n)\,z^{-n} + \sum_{n=0}^{\infty} b\cdot y(n)\,z^{-n}$$

$$= aZ\{x(n)\} + b\,Z\,\{y(n)\}$$

2. $Z[a^n f(t)] = f\left(\frac{z}{a}\right)$.

$\boxed{Or}$ $\quad Z[\{a^n x(n)\}] = X\left(\frac{z}{a}\right)$

Proof :- Since $Z[f(t)] = \sum_{n=0}^{\infty} f(nT)\cdot z^{-n}$

$$\Rightarrow Z[a^n f(t)] = \sum_{n=0}^{\infty} a^n\cdot f(nT)\cdot z^{-n}$$

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$$= \sum_{n=0}^{\infty} f(nT) \cdot \left(\frac{z}{a}\right)^{-n}$$

$$= F\left(\frac{z}{a}\right)$$

$\boxed{\text{or}}$

$$\because Z[\{u(n)\}] = \sum_{n=0}^{\infty} u(n) \cdot z^{-n}$$

$$\Rightarrow Z[\{a^n \cdot u(n)\}] = \sum_{n=0}^{\infty} a^n \cdot u(n) \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} u(n) \cdot \left(\frac{z}{a}\right)^{-n}$$

$$= X\left(\frac{z}{a}\right)$$

3 Prove that $Z[\delta(n)] = 1$.

Proof :- $\because Z\{u(n)\} = \sum_{n=0}^{\infty} u(n) \cdot z^{-n}$

$$\Rightarrow Z[\delta(n)] = \sum_{n=0}^{\infty} \delta(n) \cdot z^{-n}$$

$$\Rightarrow \delta(0) \cdot z^0 + \delta(1) z^{-1} + \delta(2) z^{-2} + -\,.$$

$$\Rightarrow 1 \cdot 1 + 0 + 0 + \quad -$$

$$= 1$$

V. Imp

4. Prove that $Z[u(n)] = \frac{z}{z-1} \quad |z|>1.$

Proof :- $Z[u(n)] = \sum_{n=0}^{\infty} u(n) \cdot z^{-n}.$

$$= \sum_{n=0}^{\infty} z^{-n}. \quad \left[\because \text{ for } n>1 \quad u(n)=1\right]$$

$$= \sum_{n=0}^{\infty} \frac{1}{z^n}.$$

$$\Rightarrow 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots$$

$$\Rightarrow \frac{1}{1-\frac{1}{z}}, \quad \left|\frac{1}{z}\right| < 1$$

$$\Rightarrow \frac{z}{z-1}, \quad |z|>1$$

5. $Z[a^n u(n)] = \frac{z}{z-a}$ if $|z|>|a|.$

Proof :- $Z[a^n \{u(n)\}] = \sum_{n=0}^{\infty} a^n \cdot u(n) \cdot z^{-n}$

$$= \sum_{n=0}^{\infty} a^n \cdot z^{-n}.$$

$$= \sum_{n=0}^{\infty} \left(\frac{z}{a}\right)^{-n}$$

$$= 1 + \frac{1}{\left(\frac{z}{a}\right)} + \frac{1}{\left(\frac{z}{a}\right)^2} + \cdots$$

$$= \frac{1}{1 - 1/\left(\frac{z}{a}\right)}$$

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$$= \frac{1}{1-\frac{a}{z}}$$

$$= \frac{z}{z-a}$$

V. Imp

6 Prove that $Z[n \cdot f(t)] = -z\frac{d}{dz}F(z)$.

Proof :- We know,

$$F(z) = Z[f(t)] = \sum_{n=0}^{\infty} f(nT)\, z^{-n}.$$

On differentiating w.r.t. z.

$$\frac{d}{dz}F(z) = \sum_{n=0}^{\infty} -n \cdot f(nT) \cdot z^{-n-1}$$

$$= \frac{d\,F(z)}{dz} = \frac{-1}{z} \cdot \sum_{n=0}^{\infty} n \cdot f(nT) \cdot z^{-n}.$$

$$= -z\frac{d}{dz}f(z) = \sum_{n=0}^{\infty} n \cdot f(nT) \cdot z^{-n}$$

$$\Rightarrow -z\frac{d}{dz}F(z) = Z[n f(t)]$$

Q. Find z-transform of $u(n) = k$.

Sol →

$$Z[k] = \sum_{n=0}^{\infty} k \cdot z^{-n}.$$

$$= k \cdot \sum_{n=0}^{\infty} z^{-n}$$

$$= k \cdot \frac{1}{1-\frac{1}{z}} = \frac{kz}{z-1}$$ Ans

Q. Find Z-transform if $x(n) = a^n$.

Sol → $Z[a^n] = \sum_{n=0}^{\infty} a^n \cdot z^{-n}$

$= \sum_{n=0}^{\infty} \left(\frac{z}{a}\right)^{-n}$

$= \frac{1}{1-\frac{a}{z}}$

$= \frac{z}{z-a}$ Ans

Q Z-transform of $x(n) = (-1)^n$.

Sol → $Z[(-1)^n] = \sum_{n=0}^{\infty} (-1)^n z^{-n}$

$= \sum_{n=0}^{\infty} \left(\frac{-1}{z}\right)^n$

$= \frac{1}{1-\left(-\frac{1}{z}\right)}$

$= \frac{z}{z+1}$ Ans

Q Z-transform of $x(n) = n$.

Sol → $Z[n] = \sum_{n=0}^{\infty} n \cdot z^{-n} = -z\frac{d}{dz}[Z(1)]$

$Z[1] = \sum_{n=0}^{\infty} 1 \cdot z^{-n} = \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1}$

$\Rightarrow Z[n] = -z\frac{d}{dz}\left(\frac{z}{z-1}\right)$

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$$\Rightarrow Z[n] = -z \cdot \left[\frac{(z-1)\cdot 1 - z(1)}{(z-1)^2} \right]$$

$$\Rightarrow \frac{z}{(z-1)^2}$$

So, $Z[n] = \dfrac{z}{(z-1)^2}$ Ans

Q Z- transform of $x(n) = n \cdot a^n$.

Sol→ $$Z[n \cdot a^n] = -z \frac{d}{dz} \cdot Z(a^n).$$

Now, $$Z[a^n] = \sum_{n=0}^{\infty} a^n z^{-n}.$$

$$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

$$= \frac{1}{1 - \frac{a}{z}} = \frac{z}{z-a}$$

So, $$Z[n \cdot a^n] = -z \frac{d}{dz} \left(\frac{z}{z-a}\right)$$

$$= -z \left[\frac{(z-a)\cdot 1 - z(1).}{(z-a)^2} \right].$$

$$Z[n \cdot a^n] = \frac{+za}{(z-a)^2}$$ Ans

Imp

Q Find z transform of $z[\sin n\theta]$ and $z[\cos n\theta]$

Sol→ $\because Z[u(n)] = \sum_{n=0}^{\infty} u(n) \cdot z^{-n}$

$$Z[a^n] = \sum_{n=0}^{\infty} a^n \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

$$= \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}.$$

Let $a = e^{i\theta}$.

$$\Rightarrow a^n = e^{in\theta}$$

$$\therefore Z[e^{in\theta}] = \frac{z}{z-e^{i\theta}}$$

$$\Rightarrow Z[\cos n\theta + i\sin n\theta] = \frac{z}{z-\cos\theta - i\sin\theta}.$$

$$\Rightarrow Z[\cos n\theta + i\sin n\theta] = \frac{z(z-\cos\theta+i\sin\theta)}{(z-\cos\theta)^2+\sin^2\theta}$$

$$= \frac{z(z-\cos\theta)+iz\sin\theta}{z^2+\cos^2\theta-2z\cos\theta+\sin^2\theta}$$

$$= \frac{z(z-\cos\theta)+iz\sin\theta}{z^2-2z\cos\theta+1}$$

On Comparing.

$$Z[\cos n\theta] = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$$

$$Z[\sin n\theta] = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$$

Ans

Q Find z-transform of $Z[r^n \cos n\theta]$ & $Z[r^n \sin n\theta]$

Sol→ $Z[a^n] = \sum_{n=0}^{\infty} a^n z^{-n}$

$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$

$= \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}$.

Let $a = re^{i\theta}$

$a^n = r^n e^{in\theta}$

$\Rightarrow Z[r^n e^{in\theta}] = \frac{z}{z - re^{i\theta}}$

$\Rightarrow Z[r^n(\cos n\theta + i\sin n\theta)] = \frac{z}{z - r\cos\theta - ir\sin\theta}$

$\Rightarrow Z[r^n \cos n\theta + r^n i \sin n\theta] = \frac{z(z - r\cos\theta + ir\sin\theta)}{(z - r\cos\theta)^2 + r^2\sin^2\theta}$

$\Rightarrow Z[r^n \cos n\theta + ir^n \sin n\theta] = \frac{z(z - r\cos\theta) + irz\sin\theta}{z^2 - 2rz\cos\theta + r^2(\cos^2\theta + \sin^2\theta)}$

$\Rightarrow Z[r^n \cos n\theta + ir^n \sin n\theta] = \frac{z(z - r\cos\theta) + irz\sin\theta}{z^2 - 2rz\cos\theta + r^2}$

On Comparing.

$Z[r^n \cos n\theta] = \frac{z(z - r\cos\theta)}{z^2 - 2rz\cos\theta + r^2}$

$Z[r^n \sin n\theta] = \frac{rz\sin\theta}{z^2 - 2rz\cos\theta + r^2}$

Ans

Q Find Z-transform of $f(t) = t$

Sol→ $Z[f(t)] = \sum_{n=0}^{\infty} f(nT)\, z^{-n}$.

$$\Rightarrow Z[t] = \sum_{n=0}^{\infty} nT \cdot z^{-n}.$$

$$= T \cdot \sum_{n=0}^{\infty} n z^{-n}.$$

$$= T \cdot \left[-z \frac{d}{dz} F(1) \right]$$

$$= T \cdot \left[-z \frac{d}{dz} \left(\frac{z}{z-1} \right) \right]$$

$$= T \cdot \left[\frac{z}{(z-1)^2} \right]$$

$$= \frac{Tz}{(z-1)^2} \quad \text{Ans}$$

Q Z-Transform of $f(t) = e^{-at}$.

Sol→ $Z[e^{-at}] = \sum_{n=0}^{\infty} e^{-anT} z^{-n}$.

$$= \sum_{n=0}^{\infty} (e^{-aT})^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{e^{-aT}}{z} \right)^n$$

$$= \frac{1}{1 - \frac{e^{-aT}}{z}} = \frac{z}{z - e^{-aT}} \quad \text{Ans}$$

Q Z- transform of $x(n) = n^2$.

Sol→ $Z[n^2] = -z\frac{d}{dz} Z(n)$.

Now, $Z(n) = -z\frac{d}{dz} Z(1)$.

$$Z(1) = \sum_{n=0}^{\infty} 1 \cdot z^{-n} = \sum_{n=0}^{\infty} \frac{1}{z^n} = \frac{1}{1-\frac{1}{z}}$$

$$= \frac{z}{z-1}$$

$$\Rightarrow Z(n) = -z\frac{d}{dz}\left(\frac{z}{z-1}\right)$$

$$= -z\,\frac{(z-1)-z}{(z-1)^2}$$

$$Z(n) = \frac{z}{(z-1)^2}$$

$$\Rightarrow Z[n^2] = -z\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$= -z\left[\frac{(z-1)^2 \cdot 1 - z \cdot 2(z-1)}{(z-1)^4}\right]$$

$$= -z\left[\frac{(z-1)[z-1-2z]}{(z-1)^4}\right]$$

$$= \frac{z(z+1)}{(z-1)^3} \quad \text{Ans}$$

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Q Find z-transform of $x(n) = n(n-1)$

Sol → $Z[n(n-1)] = Z[n^2 - n]$

$= Z[n^2] - Z[n]$

$= \frac{z(z+1)}{(z-1)^3} - \frac{z}{(z-1)^2}$ ← Wondering how? ↓ see prev. Ques

$= \frac{z(z+1) - z(z-1)}{(z-1)^3}$

$\Rightarrow \frac{z(z+1-z+1)}{(z-1)^3}$

$\Rightarrow \frac{2z}{(z-1)^3}$ Ans

First Shifting Theorem :-

If $Z\{f(t)\} = F(z)$ then

$Z\{e^{-at} f(t)\} = F(z \cdot e^{aT})$

Proof :- As we know, $Z\{f(t)\} = F(t) = \sum_{n=0}^{\infty} f(nT) \cdot z^{-n}$.

$\Rightarrow Z[e^{-at} f(t)] = \sum_{n=0}^{\infty} e^{-anT} f(nT) z^{-n}$.

$= \sum_{n=0}^{\infty} f(nT) (e^{aT} z)^{-n}$.

$= F(e^{aT} z)$

Q. Find Z-transform of $Z[e^{-at}]$

Sol→ $Z[e^{-at}] = Z[e^{-at} \cdot 1]$

$$= [Z(1)]_{z \to ze^{aT}}$$

$$Z(1) = \sum_{n=0}^{\infty} 1 \cdot z^{-n}$$

$$= \frac{z}{z-1}$$

$$\Rightarrow [Z[1]]_{z \to ze^{aT}}$$

$$= \left[\frac{z}{z-1}\right]_{z \to ze^{aT}}$$

$$= \frac{ze^{aT}}{ze^{aT}-1} \Rightarrow \frac{z}{z-e^{-aT}} \text{ Ans}$$

Q. $Z[e^{-at} \cdot t]$

Sol→ $Z[e^{-at} \cdot t] = [Z(t)]_{z \to ze^{aT}}$

Now, $Z(t) = \sum_{n=0}^{\infty} nT \, z^{-n}$

$$= T \sum_{n=0}^{\infty} (n \cdot 1) z^{-n}$$

$$= T\left[-z \frac{d}{dz} Z(1)\right]$$

$$= T\left[\frac{z}{(z-1)^2}\right]$$

$$\Rightarrow \left[Z(t)\right]_{z \to ze^{aT}}$$

$$= \left[\frac{Tz}{(z-1)^2}\right]_{z \to ze^{aT}}$$

$$= \frac{Tze^{aT}}{(ze^{aT}-1)^2} \quad \text{Ans}$$

Q $Z\left[e^{-at}\cos bt\right]$

Sol → $Z\left[e^{-at} \cdot \cos bt\right] = \left[Z(\cos bt)\right]_{z \to ze^{aT}}$ --- ①

Now, $Z(\cos bt)$:-

$$Z(a^n) = \sum_{n=0}^{\infty} a^n z^{-n}.$$

$$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

$$= \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}$$

Let $a = e^{i\theta}$.

$$\Rightarrow a^n = e^{in\theta}$$

$$\Rightarrow Z\left[e^{in\theta}\right] = \frac{z}{z-e^{i\theta}}$$

$$\Rightarrow Z\left[\cos n\theta + i\sin n\theta\right] = \frac{z}{z-\cos\theta - i\sin\theta}$$

$$Z(\cos n\theta) + iZ(\sin n\theta) = \frac{z(z-\cos\theta - i\sin\theta)}{(z-\cos\theta)^2 + \sin^2\theta}$$

On Comparing.

$$Z(\cos n\theta) = \frac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1}$$

replacing n by b.

$$Z(\cos b\theta) = \frac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1}$$

Now, putting this in eq. (1)

$$\left[Z(\cos bt)\right]_{z \to ze^{aT}} = \left[\frac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1}\right]_{z \to ze^{aT}}$$

$$= \frac{ze^{aT}(ze^{aT} - \cos\theta)}{ze^{2aT} - 2ze^{aT}\cos\theta + 1} \quad \text{Ans}$$

Try Yourself :- (1) $Z(\sin bt \cdot e^{-at})$.

Q Find z-transform of $Z[e^{-t} t^2]$

Sol→ $Z[e^{-t} \cdot t^2] = \left[Z(t^2)\right]_{z \to ze^{T}}$

Now, $Z(t^2) = \sum_{n=0}^{\infty} (nT)^2 z^{-n}$.

$$= T^2 \sum_{n=0}^{\infty} n^2 z^{-n}$$

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$$= T^2 \sum_{n=0}^{\infty} n \cdot n \, z^{-n}$$

$$= T^2 \left[-z \frac{d}{dz} Z(n) \right] \quad \text{--- (2)}$$

Now, $Z(n) = -z \frac{d}{dz} Z(1)$ --- (3)

$$Z(1) = \sum_{n=0}^{\infty} 1 \cdot z^{-n} = \frac{z}{z-1}$$

Putting $Z(1)$ in (3), we get

$$Z(n) = -z \frac{d}{dz}\left(\frac{z}{z-1}\right)$$

$$= \frac{z}{(z-1)^2}$$

Putting $Z(n)$ in eq. (2), we get

$$Z(t^2) = T^2 \left[-z \frac{d}{dz}\left(\frac{z}{(z-1)^2}\right)\right]$$

$$= T^2 z \left[\frac{(z-1)^2 - z(2(z-1))}{(z-1)^4}\right]$$

$$= T^2 \frac{z(z+1)}{(z-1)^3}$$

Putting $Z(t^2)$ in eq. (1).

$$Z(e^{-t} t^2) = \left[T^2 \frac{z(z+1)}{(z-1)^3} \right]_{z \to z e^{T}}$$

$$= T^2 \frac{ze^T(ze^T+1)}{(ze^T-1)^3}$$ Ans

Try Yourself :-

Q ① $Z[e^{-2t}\, t^3]$

② $Z[e^{3t} \cdot \cos t]$.

Second Shifting Theorem :-

If $Z[f(t)] = F(z)$.

then, $Z\{f(t+T)\} = Z[f(z) - f(0)]$.

Proof :- $Z\{f(t)\} = \sum_{n=0}^{\infty} f(nT)\, z^{-n} = f(z)$

$$Z\{f(t+T)\} = \sum_{n=0}^{\infty} (nT+T)\, z^{-n}$$

$$= \sum_{n=0}^{\infty} (n+1)T\, z^{-n}.$$

$$= \sum_{n=0}^{\infty} (n+1)T\, z^{-n-1+1}$$

$$= z\sum_{n=0}^{\infty} (n+1)T\, z^{-(n+1)}.$$

Let $(n+1) = p$.

$$\Rightarrow z\sum_{n=0}^{\infty} (pT)\, z^{-p}.$$

$$\Rightarrow z\left[\sum_{p=1}^{\infty} (pt) z^{-p} + f(0) - f(0)\right]$$

$$\Rightarrow z\left[\sum_{p=0}^{\infty} (pt) z^{-p} - f(0)\right]$$

$$= z\left[f(z) - f(0)\right]$$

Q Find z-transform of $z\left[e^{2(t+T)}\right]$

Sol→ $z\left[e^{2(t+T)}\right]$

On Comparing with $z\left[f(t+T)\right]$

$$f(t) = e^{2t}$$

$$z\left[e^{2(t+T)}\right] = z\left[f(z) - f(0)\right]$$

Here, $f(z) = z(e^{2t})$.

$$\Rightarrow f(z) = \sum_{n=0}^{\infty} e^{2nT} z^{-n}$$

$$= \sum_{n=0}^{\infty} (e^{2T})^{n} z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{e^{2T}}{z}\right)^{n}$$

$$= \frac{1}{1 - \frac{e^{2T}}{z}} = \frac{z}{z - e^{2T}}$$

$$\Rightarrow Z[e^{2(t+T)}] = Z[f(z) - f(0)]$$

$$= Z\left[\frac{Z}{Z-e^{2T}} - 1\right]$$

$$= \frac{Ze^{2T}}{Z-e^{2T}}$$ Ans

Q. Find Z-transform of $Z[\sin(t+T)]$

Sol→ $Z[\sin(t+T)]$

On comparing with $Z[f(t+T)]$

$$f(t) = \sin t$$

$$Z[\sin(t+T)] = Z[f(z) - f(0)]$$

Here, $f(z) = Z(\sin t)$.

$$Z(\sin t) = \frac{Z\sin T}{Z^2 - 2Z\cos T + 1}$$

$$\Rightarrow Z[\sin(t+T)] = Z[f(z) - f(0)]$$

$$= Z\cdot\left[\frac{Z\sin T}{Z^2-2Z\cos T+1} - \sin 0\right]$$

$$= \frac{Z^2\sin T}{Z^2 - 2Z\cos T + 1}$$ Ans

Q Find Z-transform of $Z[(t+T)e^{-(t+T)}]$

Sol→ $Z[(t+T)e^{-(t+T)}]$

$= Z[(t+T)e^{-t} \cdot e^{-T}]$

$= e^{-T} Z[(t+T)e^{-t}]$

$= e^{-T} Z[t \cdot e^{-t} + e^{-t} \cdot T]$

$= e^{-T}[Z(e^{-t} \cdot t) + T Z(e^{-t})]$

$= e^{-T}[[Z(t)]_{z \to ze^{T}} + T Z(e^{-t})]$ ——(1)

Now, $[Z(t)]_{z \to ze^{T}}$

$= Z(t) = \sum_{n=0}^{\infty} nT z^{-n}$

$= T \sum_{n=0}^{\infty} n z^{-n}$

$= T \cdot \sum_{n=0}^{\infty} n \cdot 1 z^{-n}$

$= T\left[-z \frac{d}{dz} Z(1)\right]$

$= T\left(\frac{z}{(z-1)^2}\right)$

$Z(e^{-t}) = \sum_{n=0}^{\infty} e^{-nT} z^{-n}$

$= \frac{z}{z-e^{-T}}$

Putting these in eq. (1).

$$= e^{-T}\left[\left[\frac{TZ}{(Z-1)^2}\right]_{Z\to Ze^T} + \frac{TZ}{Z-e^{-T}}\right]$$

$$= e^{-T}\left[\frac{TZe^T}{(Ze^T-1)^2} + \frac{T\cancel{Z}}{(Z-e^{-T})}\right]$$

$$= e^{-T}\left[\frac{TZe^T}{(Ze^T-1)^2} + \frac{TZe^T}{(Ze^T-1)}\right]$$

$$\Rightarrow e^{-T}\times e^T\left[\frac{TZ}{(Ze^T-1)^2} + \frac{TZ}{Ze^T-1}\right]$$

$$\Rightarrow TZ\left[\frac{1+Ze^T-1}{(Ze^T-1)^2}\right]$$

$$= \frac{TZ^2e^T}{(Ze^T-1)^2} \quad \underline{\underline{\text{Ans}}}$$

$\underline{\underline{Q}}$ find Z-transform of $x(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$

Sol $\rightarrow$

$$Z(x(n)) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 0 \cdot\cdot + \sum_{n=0}^{\infty} n\cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} n\cdot z^{-n}$$

$$= -z\frac{d}{dz}z(1)$$

$$= -z\frac{d}{dz}\left(\frac{z}{z-1}\right) = \frac{z}{(z-1)^2} \quad \underline{\underline{\text{Ans}}}$$

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Q Find z-transform of $x(n) = \begin{cases} 1 & n = k \\ 0 & \text{otherwise} \end{cases}$

Sol → $$Z\{x(n)\} = \sum_{n=0}^{\infty} x(n) \cdot z^{-n}$$

$$= x(0) \cdot z^{0} + \dots x(k) z^{-k} + \dots$$

$$= 0 + \dots + 1 \cdot z^{-k} + 0 \dots$$

$$= z^{-k}.$$

$$= \frac{1}{z^{k}} \quad \text{Ans}$$

Q Find z-transform of ab^{n}, where $a \cdot b \neq 0$.

Sol → $$Z[ab^{n}] = \sum_{n=0}^{\infty} ab^{n} z^{-n}$$

$$= a \sum_{n=0}^{\infty} b^{n} z^{-n}$$

$$= a \sum_{n=0}^{\infty} \left(\frac{b}{z}\right)^{n}$$

$$= a \left[\frac{1}{1 - \frac{b}{z}}\right]$$

$$= \frac{az}{z-b} \quad \text{Ans}$$

Q Find the z-transform of $x(n) = \begin{cases} 0 & n > 0 \\ 1 & n \leq 0 \end{cases}$

Sol → $$Z\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n}.$$

$$= \sum_{n=-\infty}^{0} z^{-n} + \sum_{n=0}^{\infty} 0.$$

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$$= \sum_{n=-\infty}^{0} z^{-n}$$

$$= 1 + z + z^2 + \cdots z^n + \cdots \infty$$

$$= \frac{1}{1-z} \quad \underline{\underline{Ans}}$$

Q Find z-transform of $Z\left[\frac{1}{n}\right]$

Sol → $Z\left[\frac{1}{n}\right] = \sum_{n=\phi}^{\infty} \frac{1}{n} \cdot z^{-n}$

$$= \sum_{n=1}^{\infty} \frac{1}{n} z^{-n}$$

$$= \frac{1}{z} + \frac{1}{2z^2} + \frac{1}{3z^3} + \cdots$$

$$= -\log\left(1 - \frac{1}{z}\right).$$

$$= -\log\left(\frac{z-1}{z}\right)$$

$$= \log\left(\frac{z}{z-1}\right). \quad \underline{\underline{Ans}}$$

$$\because \log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} + \cdots$$

Q find z-transform of $Z\left[\cos\frac{n\pi}{2}\right]$

Sol → $Z\left[\cos\frac{n\pi}{2}\right] = \sum_{n=0}^{\infty} \cos\frac{n\pi}{2} z^{-n}$

$$= \cos 0 + \cos\frac{\pi}{2} \cdot \frac{1}{z} + \cos\pi \cdot \frac{1}{z^2} + \cos\frac{3\pi}{2} \cdot \frac{1}{z^3} + \cos 2\pi \cdot \frac{1}{z^4} + \cdots$$

$$\Rightarrow 1 - \frac{1}{z^2} + \frac{1}{z^4} + - -$$

$$\Rightarrow \left(1 + \frac{1}{z^2}\right)^{-1} \quad \underline{\underline{\text{Ans}}}$$

$$\because (1+x)^{-1} = 1 - x + x^2 - x^3 + - -$$

Q find z-transform of $Z\left[\sin\frac{n\pi}{2}\right]$

Sol → $$Z\left\{\sin\frac{n\pi}{2}\right\} = \sum_{n=0}^{\infty} \sin\frac{n\pi}{2} \, z^{-n}.$$

$$= \sin 0 + \sin\frac{\pi}{2}\cdot\frac{1}{z} + \sin\pi\cdot\frac{1}{z^2} + \sin\frac{3\pi}{2}\cdot\frac{1}{z^3}$$

$$+ \sin 2\pi\cdot\frac{1}{z^4} + - - .$$

$$= 0 + \frac{1}{z} + 0 - \frac{1}{z^3} + 0 + \frac{1}{z^5} + -$$

$$= \frac{1}{z}\left[1 - \frac{1}{z^2} + \frac{1}{z^4} + -\right]$$

$$= \frac{1}{z}\left[\left(1 + \frac{1}{z^2}\right)^{-1}\right] \quad \underline{\underline{\text{Ans}}}$$

Convolution of Sequence

The Convolution of two sequence $\{x(n)\}$ and $\{y(n)\}$ is defined by.
$$\omega(n) = \sum_{k=-\infty}^{\infty} x(k)\, y(n-k)$$

Convolution Theorem :-

$$Z[\omega(n)] = W(z) = Z\{x(n)\} \cdot Z\{y(n)\}$$
$$= X(z) \cdot Y(z).$$
$$= Y(z) \cdot X(z).$$

Convolution Theorem for Inverse Z-transform

If $f(n)$ and $g(n)$ be causel sequence, then

$$Z[f(n) \cdot g(n)] = Z\{f(n)\} \cdot Z\{g(n)\} = F(z) \cdot G(z)$$

and $$Z^{-1}[F(n) \cdot G(n)] = f(n) \cdot g(n).$$
$$= \sum_{k=0}^{n} f(n-k)\, g(k)$$
$$= Z^{-1}\{F(z)\} \cdot Z^{-1}\{G(z)\}$$

Q find the Inverse z-transform of $\frac{z^2}{(z-a)^2}$.

Sol → $Z^{-1}\left(\frac{z^2}{(z-a)^2}\right) = Z^{-1}\left(\frac{z}{(z-a)} \cdot \frac{z}{(z-a)}\right)$

$= Z^{-1}\left[\frac{z}{z-a}\right] \cdot Z^{-1}\left[\frac{z}{z-a}\right]$

$= a^n \cdot a^n \quad \left[\because Z[a^n] = \frac{z}{z-a}\right]$

$= \sum_{k=0}^{n} a^{n-k} a^k$.

$= a^n \sum_{k=0}^{n} a^{-k} \cdot a^k$

$= a^n \sum_{k=0}^{n} 1$.

$= a^n \left(1 + 1 + \ldots 1\right]$

$= a^n (n+1)$. Ans

Q Find $Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right]$

Sol → $Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right] = Z^{-1}\left[\frac{z}{\left(z-\frac{1}{2}\right)} \cdot \frac{z}{\left(z-\frac{1}{4}\right)}\right]$

$= Z^{-1}\left[\frac{z}{z-\frac{1}{2}}\right] \cdot Z^{-1}\left[\frac{z}{z-\frac{1}{4}}\right]$

$$= \left(\frac{1}{2}\right)^n \cdot \left(\frac{1}{4}\right)^n$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{n-k} \left(\frac{1}{4}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{-k} \left(\frac{1}{2}\right)^{2k}$$

$$= \left(\frac{1}{2}\right)^n \cdot \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \left[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + - \right]$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1 - \left(\frac{1}{2}\right)^{n+1}}{1 - \frac{1}{2}}\right]$$

$$= \left(\frac{1}{2}\right)^{n-1} \left[1 - \left(\frac{1}{2}\right)^{n+1}\right]$$

$$= \left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{2}\right)^{2n} \quad \text{Ans}$$

Long Division Method to find Z^{-1}.

$$Z\{x(n)\} = \sum_{n=0}^{\infty} x(n) z^{-n} = X(z).$$

Q $Z^{-1}\left(\frac{10z}{(z-1)(z-2)}\right)$

Sol → $Z^{-1}\left(\frac{10z}{z^2-3z+2}\right)$

$$= Z^{-1}\left[\frac{10z}{z^2(1-3z^{-1}+2z^{-2})}\right]$$

$$= Z^{-1}\left[\frac{10z^{-1}}{1-3z^{-1}+2z^{-2}}\right]$$

$$\begin{array}{r|l}
 & 10z^{-1}+30z^{-2}+70z^{-3}+150z^{-4} \\
\hline
1-3z^{-1}+2z^{-2} & \cancel{10z^{-1}} \\
 & -\cancel{10z^{-1}} \underset{+}{-} 30z^{-2} \underset{-}{+} 20z^{-3} \\
\hline
 & \cancel{30z^{-2}} - 20z^{-3} \\
 & -\cancel{30z^{-2}} \underset{+}{-} 90z^{-3} \underset{-}{+} 60z^{-4} \\
\hline
 & \cancel{70z^{-3}} - 60z^{-4} \\
 & -\cancel{70z^{-3}} \underset{+}{-} 210z^{-4} \underset{-}{+} 140z^{-5} \\
\hline
 & \cancel{150z^{-4}} - 140z^{-5} \\
 & \cancel{150z^{-4}} - 450z^{-5} + 300z^{-6} \\
\hline
\end{array}$$

Quoitent $= 10z^{-1}+30z^{-2}+70z^{-3}+150z^{-4}$ —

As we know, $\sum_{n=0}^{\infty} x(n)z^{-n} = x(0)z^0 + x(1)z^{-1}$
$+ x(2)z^{-2} + x(3)x^{-3} + x(4)z^{-4} + $ —

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On Comparing both eq.

$x(0) = 0$, $x(1) = 10$, $x(2) = 30$.

$x(3) = 70$, $x(4) = 150$.

$x(n) = \{0, 10, 30, 70, 150 \ldots\}$

$= 10\{0, 1, 3, 7, 15 \ldots\}$

$x(n) = 10\{2^n - 1\}$ Ans

Q $Z^{-1}\left[\dfrac{z^2+2z}{z^2+2z+4}\right]$

Sol → $Z^{-1}\left[\dfrac{z^2+2z}{z^2+2z+4}\right] = Z^{-1}\left[\dfrac{z^2(1+2z^{-1})}{z^2(1+2z^{-1}+4z^{-2})}\right]$

$$\begin{array}{r l}
 & 1 - 4z^{-2} + 8z^{-3} - 32z^{-5} \\
1+2z^{-1}+4z^{-2}) & 1 + 2z^{-1} \\
 & \underline{\cancel{1} + 2z^{-1} + 4z^{-2}} \\
 & -4z^{-2} \\
 & -4z^{-2} - 8z^{-3} - 16z^{-4} \\
 & \underline{+ \qquad + \qquad +} \\
 & 8z^{-3} + 16z^{-4} \\
 & \underline{8z^{-3} + 16z^{-4} + 32z^{-5}} \\
 & -32z^{-5} \\
 & \underline{-32z^{-5} + -}
\end{array}$$

Quoitent $= 1 - 4z^{-2} + 8z^{-3} - 32z^{-5}$

On Comparing it with $x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4} + \ldots$

$x(0) = 1, \quad x(1) = 0, \quad x(2) = -4$

$x(3) = 8 \quad x(4) = 0 \quad , x(5) = -32$

$x(n) = \{1, 0, -4, 8, 0, -32 \ldots\}$

Ans

Partial fraction Method to find Z-inverse

Q $x(z) = \dfrac{z}{z^2 + 7z + 10}$

Sol → $x(z) = \dfrac{z}{z^2 + 7z + 10}$

$\dfrac{x(z)}{z} = \dfrac{1}{(z+2)(z+5)} = \dfrac{A}{(z+5)} + \dfrac{B}{(z+2)}$

$\dfrac{A(z+2) + B(z+5)}{(z+2)(z+5)} = \dfrac{1}{(z+2)(z+5)}$

On Comparing:

$A(z+2) + B(z+5) = 1$

Putting $z = -2$

$$A(-2+2) + B(-2+5) = 1$$

$$\Rightarrow B = \frac{1}{3}.$$

Putting $z = -5$

$$A(-5+2) + B(-5+5) = 1$$

$$\Rightarrow A = \frac{-1}{3}.$$

$$\Rightarrow \frac{x(z)}{z} = \frac{-1}{3}\frac{1}{(z+5)} + \frac{1}{3}\frac{1}{(z+2)}$$

$$\Rightarrow x(z) = \frac{-z}{3(z+5)} + \frac{z}{3(z+2)}$$

Now, $x(n) = Z^{-1}[X(z)]$

$$= Z^{-1}\left[\frac{-1}{3}\left(\frac{z}{z+5}\right) + \frac{1}{3}\left(\frac{z}{z+2}\right)\right]$$

$$= \frac{-1}{3}Z^{-1}\left(\frac{z}{z+5}\right) + \frac{1}{3}Z^{-1}\left(\frac{z}{z+2}\right).$$

$$= \frac{-1}{3}(-5)^n + \frac{1}{3}(-2)^n.$$

$$\Rightarrow \left(\frac{1}{3}\right)\left((-2)^n - (-5)^n\right) \quad \underline{\text{Ans}}$$

Q $x(z) = \frac{z^2+z}{(z-1)(z^2+1)}$.

Sol → $\frac{x(z)}{z} = \frac{(z+1)}{(z-1)(z^2+1)}$

$= \frac{(z+1)}{(z-1)(z-i)(z+i)}$. --- (1)

$= \frac{A}{(z-1)} + \frac{B}{(z-i)} + \frac{C}{(z+i)}$ --- (2)

On comparing (1) & (2).

$z+1 = A(z-i)(z+i) + B(z-1)(z+i) + C(z-1)(z-i)$

On Putting $z = 1$

$1+1 = A(1-i)(1+i) + 0 + 0$

$\Rightarrow 2 = A(1+1)$

$\Rightarrow A = 1$

On putting $z = i$

$i+1 = 0 + B(i-1)(i+i) + 0$

$\Rightarrow i+1 = B(2i^2 - 2i)$.

$\Rightarrow i+1 = B(-2-2i)$.

$\Rightarrow i+1 = -2B - 2Bi$

$\Rightarrow B = \frac{-1}{2}$

Putting $z = -i$

$$-i+1 = 0 + 0 + C(-2i)(-i-1)$$

$$\Rightarrow -i+1 = C[+2i^2+2i]$$

$$\Rightarrow -i+1 = C(-2+2i)$$

$$\Rightarrow -i+1 = -2C(-i+1).$$

$$\Rightarrow C = \frac{-1}{2}$$

$$\Rightarrow \frac{X(z)}{z} = \frac{1}{z-1} + \left(\frac{-1}{2}\right)\left(\frac{1}{z-i}\right) + \left(\frac{-1}{2} \cdot \frac{1}{z+i}\right)$$

$$\Rightarrow X(z) = \frac{z}{z-1} - \frac{1}{2}\frac{z}{z-i} - \frac{1}{2} \cdot \frac{z}{z+i}.$$

$$x(n) = Z^{-1}(X(z))$$

$$\Rightarrow x(n) = Z^{-1}\left[\frac{z}{z-1} - \frac{1}{2}\frac{z}{z-i} - \frac{1}{2} \cdot \frac{z}{z+i}\right]$$

$$x(n) = (1)^n - \frac{1}{2}(i)^n - \frac{1}{2}(-i)^n$$ Ans

Cauchy-Residue Theorem to find inverse z-transform

$$x(n) = \frac{1}{2\pi i}\oint_c X(z) z^{n-1} dz = \sum R^+$$

Sum of Residue

Q Find $Z^{-1}\left\{\dfrac{z^2}{(z-a)(z-b)}\right\}$ by using Residue Theorem.

Sol → $$x(n) = \frac{1}{2\pi i}\oint_c \frac{z^2}{(z-a)(z-b)} z^{n-1}\,dz$$

$$= \frac{1}{2\pi i}\oint_c \frac{z^{n+1}}{(z-a)(z-b)}\,dz$$

For pole,

$$(z-a)(z-b) = 0$$

$\Rightarrow$ $z=a$, $z=b$ are two poles.

Residue Formula :-

① Residue of $f(z)$ at its simple pole $z=z_0$ is

$$\lim_{z\to z_0}(z-z_0)\,f(z).$$

② Residue of $f(z)$ at its n^{th} order pole is

$$\frac{1}{(n-1)!}\lim_{z\to z_0}\frac{d^{n-1}}{dx^{n-1}}(z-z_0)^n f(z).$$

Now, at $z=a$.

$$R_1^+ = \lim_{z\to a}(z-a)\frac{z^{n+1}}{(z-a)(z-b)}$$

$$= \frac{a^{n+1}}{a-b}$$

At $z=b$.

$$R_2^+ = \lim_{z\to b}(z-b)\frac{z^{n+1}}{(z-a)(z-b)}$$

$$= \frac{b^{n+1}}{b-a}.$$

$$X(n) = \sum R^{+}$$

$$= R_1 + R_2$$

$$= \frac{a^{n+1}}{a-b} + \frac{b^{n+1}}{b-a}$$

$$x(n) = \frac{1}{(a-b)}\left[a^{n+1} - b^{n+1}\right]$$ Ans

Q $Z^{-1}\left\{\frac{z(z+1)}{(z-1)^3}\right\}$

Sol → $x(n) = \frac{1}{2\pi i}\oint_c \frac{z(z+1)}{(z-1)^3} z^{n-1}\, dz$

$$= \frac{1}{2\pi i}\oint_c \frac{z^n(z+1)}{(z-1)^3}\, dz$$

For pole :-

$$(z-1)^3 = 0$$

$$\Rightarrow z = 1 \text{ (order 3)}$$

Residue at $z = 1$ (order 3)

$$= \frac{1}{(3-1)!}\lim_{z\to 1}\frac{d^2}{dx^2}(z-1)^3\frac{z^n(z+1)}{(z-1)^3}$$

$$= \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dx^2}\left(z^{n+1} + z^n\right)$$

$$= \frac{1}{2!}\lim_{z\to 1}(n+1)n z^{n-1} + n(n+1)z^{n-2}.$$

$$\Rightarrow \frac{1}{2}\left[n(n+1) + n(n+1)\right]$$ Ans

Application of z-transform to solve finit difference eq:-

$$\Rightarrow Z\{y(n+2)\} = z^2 Y(z) - z^2 y(0) - z y(1).$$

$$z\{y(n+1)\} = zY(z) - z\, y(0)$$

where, $Y(z) = Z[y(n)]$

and where. $y(n)$ be the sequence.

Q Solve $y_{n+2} - 4y_{n+1} + 4y_n = 0$ given $y_0 = 1$ and $y_1 = 0$.

Sol→ $y_{n+2} - 4y_{n+1} + 4y_n = 0$

Taking z-transform on both sides of difference eq.

$$z[y_{n+2}] - 4z[y_{n+1}] + 4z[y(n)] = z[0]$$

$$\Rightarrow z^2 Y(z) - z^2 y(0) - z y(1) - 4[zY(z) - z\, y(0)] + 4z[y(n)] = 0$$

$$\Rightarrow z^2 y(z) - z^2 y(0) - z y(1) - 4zY(z) + 4zy(0) + 4Y(z) = 0$$

$[\because z[y(n)] = Y(z)]$

$$\Rightarrow z^2\left[Y(z) - y(0) - \frac{y(1)}{z}\right] - 4[z\{Y(z) - y(0)\}] + 4Y(z) = 0$$

Using initial condition $y(0) = 1$ and $y(1) = 0$

$$z^2[Y(z) - 1 - 0] + 4[zY(z) - z] + Y(Y(z)] = 0$$

$$= [z^2 - 4z + 4] Y(z) = z^2 - 4z.$$

$$Y(z) = \frac{z^2 - 4z}{z^2 - 4z + 4} = \frac{z^2 - 4z}{(z-2)^2}$$

$$y(n) = \frac{1}{2\pi i} \oint_C Y(z) z^{n-1} dz$$

$$= \frac{1}{2\pi i} \oint_C \frac{z(z-4)}{(z-2)^2} z^{n-1} dz$$

$$= \frac{1}{2\pi i} \oint_C \frac{z^n(z-4)}{(z-2)^2}$$

$\Rightarrow$ Pole is $z = 2$ (order 2)

$\Rightarrow$ Residue at $z = 2$ (order 2)

$$= \frac{1}{1!} \lim_{z \to 2} \frac{d}{dz} (z-2)^2 \frac{z^n(z-4)}{(z-2)^2}$$

$$= \lim_{z \to 2} \frac{d}{dz} (z^{n+1} - 4z^n)$$

$$= \lim_{z \to 2} [(n+1) z^n - 4n z^{n-1}]$$

$$= 2^n (n+1) - 4n 2^{n-1}$$

$$= 2^n [n + 1 - 2n]$$

$$= 2^n [1 - n] \quad \text{Ans}$$

Q. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given $y_0 = y_1 = 0$.

Sol → $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$

Taking z- transform on both side.

$$z[y_{n+2}] + 6z[y_{n+1}] + 9[z(y_n)] = z[2^n]$$

$$\Rightarrow z^2Y(z) - z^2y(0) - zy(1) + 6[zY(z) - zy(0)] + 9Y(z) = \frac{z}{z-2}$$

$$\Rightarrow z^2Y(z) + 6zY(z) + 9[Y(z)] = \frac{z}{z-2}$$

$$\Rightarrow (z^2 + 6z + 9)Y(z) = \frac{z}{z-2}$$

$$\Rightarrow Y(z) = \frac{z}{(z-2)(z+3)^2}.$$

Using partial fraction method

$$\frac{Y(z)}{z} = \frac{1}{(z-2)(z+3)^2} = \frac{A}{(z-2)} + \frac{B}{(z+3)} + \frac{C}{(z+3)^2}$$

$$\Rightarrow 1 = A(z+3)^2 + B(z-2)(z+3) + C(z-2) \quad \text{—(1)}$$

Putting $z = 2$

$$1 = A(2+3)^2 + 0 + 0$$

$$\Rightarrow A = \frac{1}{25}$$

Putting $z = -3$

$$1 = 0 + 0 + C(-5)$$

$$\Rightarrow C = -\frac{1}{5}$$

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also, $A + B = 0$

$\Rightarrow B = -A$

$\Rightarrow B = \frac{-1}{25}$

$$\Rightarrow \frac{Y(z)}{z} = \frac{+1}{25}\frac{1}{(z-2)} + \frac{-1}{25}\left(\frac{1}{z+3}\right) - \frac{1}{5}\frac{1}{(z+3)^2}$$

$$\Rightarrow Y(z) = \frac{+1}{25}\frac{z}{(z-2)} - \frac{1}{25}\frac{z}{z+3} - \frac{1}{5}\frac{z}{(z+3)^2} \quad \text{—} (2)$$

Taking Inverse on both side

$$z^{-1}[Y(z)] = \frac{1}{25}z^{-1}\left(\frac{z}{z-2}\right) - \frac{1}{25}z^{-1}\left(\frac{z}{z+3}\right) - \frac{1}{5}z^{-1}\left(\frac{z}{(z+3)^2}\right)$$

$$\Rightarrow y(n) = \frac{1}{25}2^n - \frac{1}{25}(-3)^n - \frac{1}{5}n(-3)^{n-1}$$

$$\left[\because z[n(-3)^n] = \frac{-3z}{(z+3)^2}\right]$$

$$\Rightarrow y(n) = \frac{1}{25}\left[2^n - (-3)^n + \frac{5}{3}n(-3)^n\right]$$

Ans

Try Yourself :- solve $x(n+1) - 2x(n) = 1$ given, $x(0) = 0$