Assignment 4.1

(1) $E(2) = 2$ as Expected value of constant is always the constant c.

$E(X) = \mu = \sum_{i=1}^{4} x_i P(x_i)$

$= 0\cdot(0.2) + 1\cdot(0.1) + 2\cdot(0.4) + 3\cdot(0.3)$

$= 0.1 + 0.8 + 0.9$

$\therefore E(X) = 1.8$

$\therefore E(2x) = 2E(X) = 2(1.8) = 3.6$

$V(x) = E(x-\mu)^2 = \sum_{i=1}^{4} (x_i-\mu)^2 P(x_i)$

$= (0-\mu)^2 P(0) + (1-\mu)^2 P(1) + (2-\mu)^2 P(2) + (3-\mu)^2 P(3)$

$= \frac{2\mu^2}{10} + (1-2\mu+\mu^2)\frac{1}{10} + (4-4\mu+\mu^2)\frac{4}{10} + (9-6\mu+\mu^2)\frac{3}{10}$

$= \frac{1}{10}(\cancel{\mu^2+\mu^2+\mu^2}\ 2\mu^2 + 1 - 2\mu + \mu^2 + 16 - 16\mu + 4\mu^2 + 27 - 18\mu + 3\mu^2$

$= \frac{1}{10}(10\mu^2 - 36\mu + 44) = \frac{2}{10}(5\mu^2 - 18\mu + 22)$

$= \frac{2}{10}(5(1.8)^2 - 18(1.8) + 22) = \frac{2}{10}\left(\cancel{5\left(\frac{18}{10}\right)^2 - \frac{18\cdot 18}{10} + \frac{220}{10}}\right)$

$= \frac{2}{10}(5(3.24) - 32.4 + 22)$

$= \frac{2}{10}(16.2 - 32.4 + 22) = \frac{2}{10}(5.8) = 1.16$

$\therefore V(X) = 1.16.$

(2) Mean(x) i.e $E(X) = \sum_{i=0}^{11} x_i (P(x_i))$ ——(i)

$= 3(0.03) + 4(0.05) + 5(0.07) + 6.(0.10) + 7(0.14) + 8(0.20) + 9(0.18) + 10(0.12) + 11(0.07) + 12(0.03) + 13(0.01)$

$= 0.09 + 0.2 + 0.35 + 0.6 + 0.98 + 1.6 + 1.62 + 1.2 + 0.77 + 0.36 + 0.13 = 7.9$

∴ Expected actual patient life is 7.9 years

$Var(x) = E(x - E(x))^2 = \sum_{i=0}^{11} (x_i - 7.9)^2 P(x_i)$

$= (3-7.9)^2 \frac{3}{100} + (4-7.9)^2 \frac{5}{100} + (5-7.9)^2 \frac{7}{100} + (6-7.9)^2 \frac{1}{100} + (7-7.9)^2 \frac{14}{100} + (8-7.9)^2 \frac{20}{100} + (9-7.9)^2 \frac{18}{100} + (10-7.9)^2 \frac{12}{100} + (11-7.9)^2 \frac{7}{100} + (12-7.9)^2 \frac{3}{100} + (13-7.9)^2 \frac{1}{100}$

$= \frac{1}{100} \left((-4.9)^2 3 + (3.9)^2 5 + (2.9)^2 7 + (1.9)^2 + (0.9)^2 14 + (0.1)^2 \cdot 20 + (1.1)^2 18 + (2.1)^2 \cdot 12 + (3.1)^2 \cdot 7 + (4.1)^2 3 + (5.1)^2 \right.$

$= \frac{1}{100} \left(\left(\frac{49}{10}\right)^2 3 + \left(\frac{39}{10}\right)^2 5 + \left(\frac{29}{10}\right)^2 7 + \left(\frac{19}{10}\right)^2 + \left(\frac{9}{10}\right)^2 14 + \left(\frac{1}{10}\right)^2 20 + \left(\frac{11}{10}\right)^2 18 + \left(\frac{21}{10}\right)^2 \cdot 12 + \left(\frac{31}{10}\right)^2 7 + \left(\frac{41}{10}\right)^2 3 + \left(\frac{51}{10}\right)^2 \right.$

$= \frac{1}{100} \frac{1}{100} \left((49)^2 3 + (39)^2 5 + (29)^2 7 + (19)^2 + (9)^2 14 + 20 + (11)^2 18 + (21)^2 \cdot 12 + (31)^2 \cdot 7 + (41)^2 3 + (51)^2 \right.$

$= \frac{1}{10000} \left(7203 + 7605 + 5887 + 361 + 1134 + 20 + 2178 + 5292 + 6727 + 5043 + 2601 \right.$

$= \frac{44001}{10000} = 4.4$

$\therefore Var(x) = 4.4.$

③(a) Since Sum (PMD) = 1

$K = 1 - (0.1 + 0.3 + 0.3 + 0.1)$

$= 1 - (0.8) = 0.2$

$\therefore K = 0.2$

(b) $E(x) = \mu = \sum_{i=0}^{4} x_i P(x_i)$

$= (-4)\cdot(0.1) + (-2)\cdot(0.3) + 0\cdot(0.2) + 2(0.3) + 4\cdot(0.1)$

$= -0.4 - 0.6 + 0.6 + 0.4$

$= 0$

(c) $V(x) = E(x - \mu)^2 = \sum_{i=0}^{4} (x_i - \mu)^2 P(x_i)$

$= (-4)^2\cdot 0.1 + (-2)^2\cdot(0.3) + (0)^2(0.2) + (2)^2\cdot(0.3) + (4)^2\cdot(0.1)$

$= 16\cdot 0.1 + 4\cdot 0.3 + 4(0.3) + 16\cdot 0.1$

$= 1.6 + 1.2 + 1.2 + 1.6$

$= 5.6$

(d) $S.D = \sqrt{\sigma^2} = \sqrt{V(x)} = \sqrt{5.6} = 2.36$

(e) $E(2x + 5) = 2E(x) + 5 = 2\cdot 0 + 5 = 5$