





NPTEL ONLINE CERTIFICATION COURSES

Course Name: Deep Learning

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Department: E & ECE, IIT Kharagpur

Topic

Lecture 27: Back propagation Learning – Examples

CONCEPTS COVERED

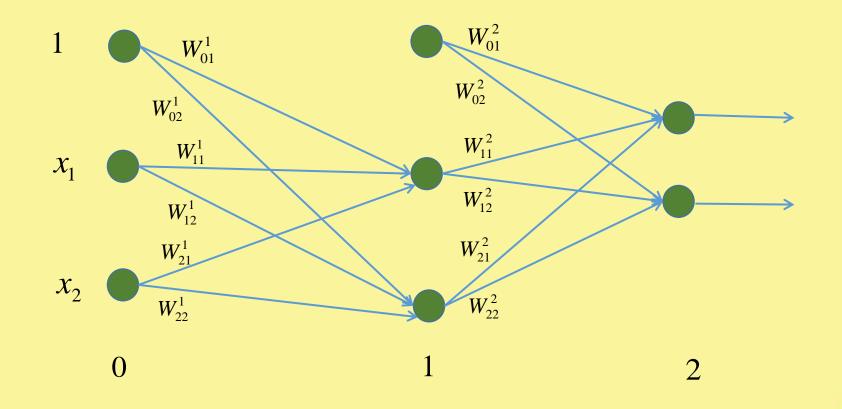
Concepts Covered:

- ☐ Back Propagation Learning in MLP
- ☐ Back Propagation Learning Network Level
- Back Propagation Node Level





Backpropagation at Network Level

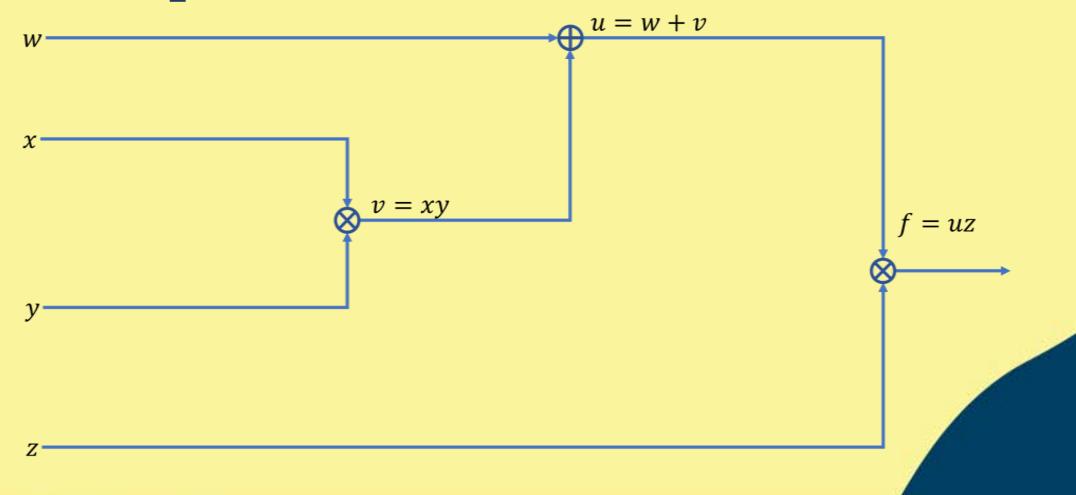




Back Propagation Learning at Node Level

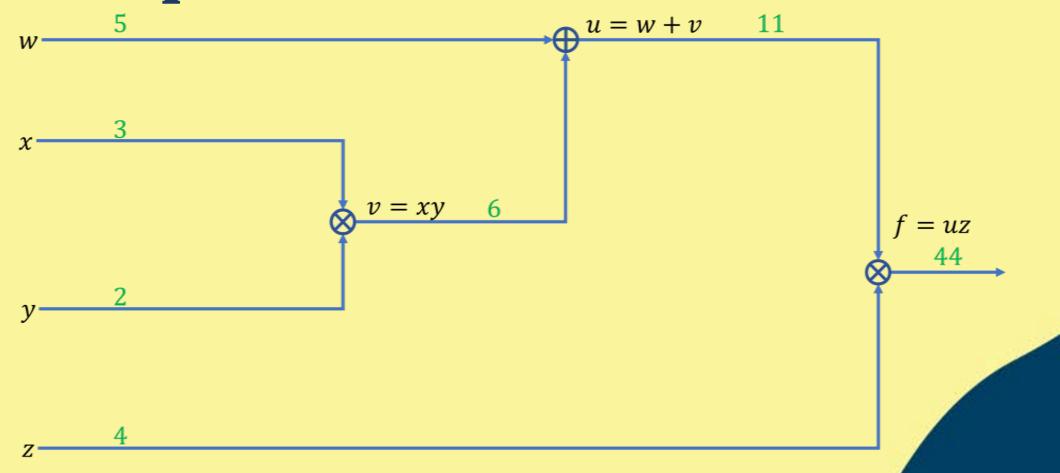


Example: Node Architecture



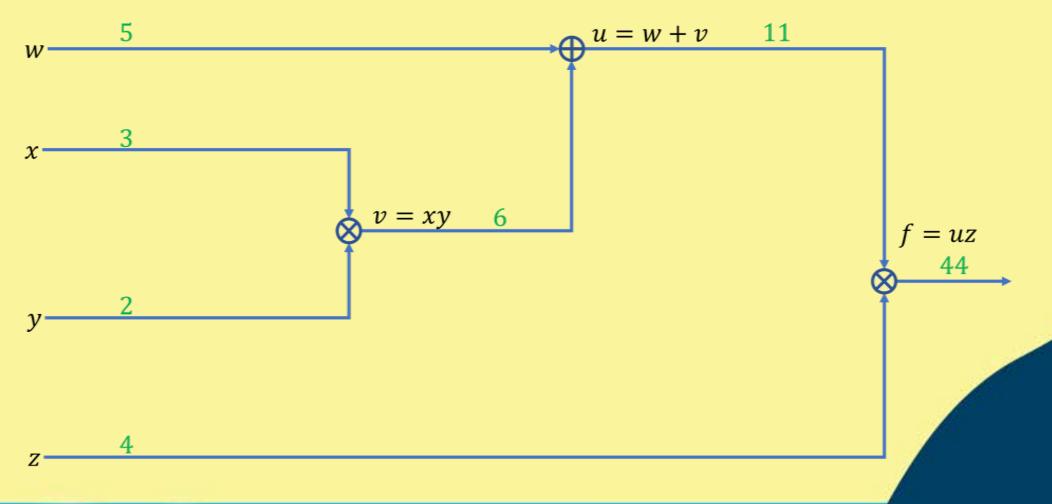


Example: Forward Pass





Example: Backpropagation





Back propagation: Pseudo Code

```
# Set Input
w=5; x=3; y=2; z=4
```

Forward Pass

```
v = x*y

u = w+v

f = u+z
```

Backward Pass

```
dfdu = z

dfdz = u

dfdw = 1*dfdu # dudw = 1

dfdv = 1*dfdu # dudv = 1

dfdx = y*dfdv # dvdx = y

dfdy = x*dfdv # dvdy = x
```





Back propagation: Pseudo Code

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dfdw = 1*dfdu  # dudw = 1
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```





Example: Calculate Gradients

$$w = \frac{5}{4} + \frac{\partial f}{\partial w} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial w} = \frac{d}{du}(uz) \cdot \frac{d}{dw}(w+v) = z.(1+0) = z = 4$$

$$w: 5 \rightarrow 5.001 (\Delta w = 0.001)$$

$$f: 44 \rightarrow 44.004 (\Delta f = 0.004)$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial v} \cdot \frac{\partial v}{\partial x} = \frac{d}{du}(uz) \cdot \frac{d}{dv}(w+v) \cdot \frac{d}{dx}(xy)$$

$$= z. (0+1) \cdot y = zy = 8$$

$$x: 3 \rightarrow 3.001 (\Delta x = 0.001)$$

$$f: 44 \rightarrow 44.008 (\Delta f = 0.008)$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial v} \cdot \frac{\partial v}{\partial y} = \frac{d}{du}(uz) \cdot \frac{d}{dv}(w+v) \cdot \frac{d}{dx}(xy)$$

$$= z. (0+1) \cdot x = zx = 12$$

$$y: 2 \rightarrow 2.001 (\Delta y = 0.001)$$

$$f: 44 \rightarrow 44.012 (\Delta f = 0.012)$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(uz) = u = 11$$

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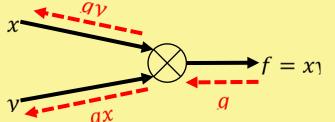
$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(uz) = u = 11$$

 $f: 44 \rightarrow 44.011 \ (\Delta f = 0.011)$

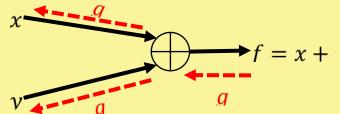
f = uz



Understanding Gradient Backward



$$\frac{\partial L}{\partial f} = \frac{\partial f}{\partial x} = y; \quad \frac{\partial f}{\partial y} = x; \quad \frac{\partial L}{\partial x} = \frac{\partial L}{\partial f}. \quad \frac{\partial f}{\partial x} = gy; \quad \frac{\partial L}{\partial y} = \frac{\partial L}{\partial f}. \quad \frac{\partial f}{\partial y} = gx$$



$$\frac{\partial L}{\partial f} = \frac{\partial f}{\partial x} = 1; \quad \frac{\partial f}{\partial y} = 1; \quad \frac{\partial L}{\partial x} = \frac{\partial L}{\partial f} \cdot \frac{\partial f}{\partial x} = g; \quad \frac{\partial L}{\partial x} = \frac{\partial L}{\partial f} \cdot \frac{\partial f}{\partial x} = g$$

$$Case - I: x > y; f = max(x, y) = x$$

$$f = \max(x, 1)$$

$$\frac{\partial f}{\partial x} = 1; \frac{\partial f}{\partial y} = 0; \frac{\partial L}{\partial x} = \frac{\partial L}{\partial f} \cdot \frac{\partial f}{\partial x} = g \quad \frac{\partial L}{\partial y} = \frac{\partial L}{\partial f} \cdot \frac{\partial f}{\partial y} = 0$$

$$Case - II: x < y; f = max(x, y) = y$$

$$\frac{\partial L}{\partial f} = g$$

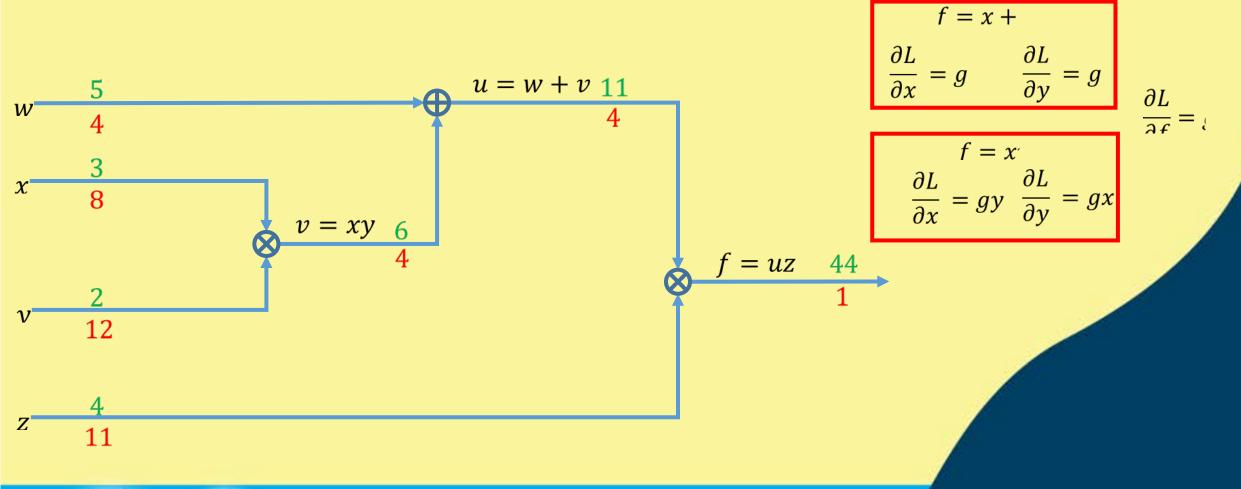
Case – II:
$$x < y$$
; $f = max(x, y) = y$

$$\frac{\partial L}{\partial f} = g$$

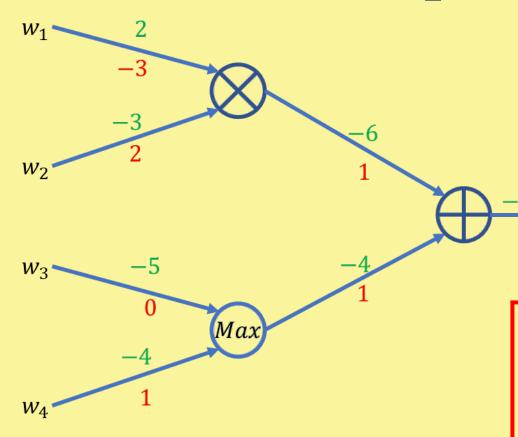
$$\frac{\partial f}{\partial x} = 0$$
; $\frac{\partial f}{\partial y} = 1$; $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial f} \cdot \frac{\partial f}{\partial x} = 0$; $\frac{\partial L}{\partial y} = \frac{\partial L}{\partial f} \cdot \frac{\partial f}{\partial y} = g$



Previous example: different approach



Another Example



$$f = x + \frac{\partial L}{\partial x} = g \qquad \frac{\partial L}{\partial y} = g$$

$$f = x^{2}$$

$$\frac{L}{x} = gy \frac{\partial L}{\partial x} = gx$$

$$\frac{\partial L}{\partial f} = \xi$$

$$f = \max(x, y)$$

$$\frac{\partial L}{\partial x} = g \text{ if } x > y \qquad \frac{\partial L}{\partial y} = g \text{ if } y > x$$

$$= 0 \text{ otherwise} \qquad = 0 \text{ otherwise}$$









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Thank you