NPTEL ONLINE CERTIFICATION COURSES

Course Name: Deep Learning

Faculty Name: Prof. P. K. Biswas

Department : E & ECE, IIT Kharagpur

Topic

Lecture 06: Discriminant Function and Decision Surface

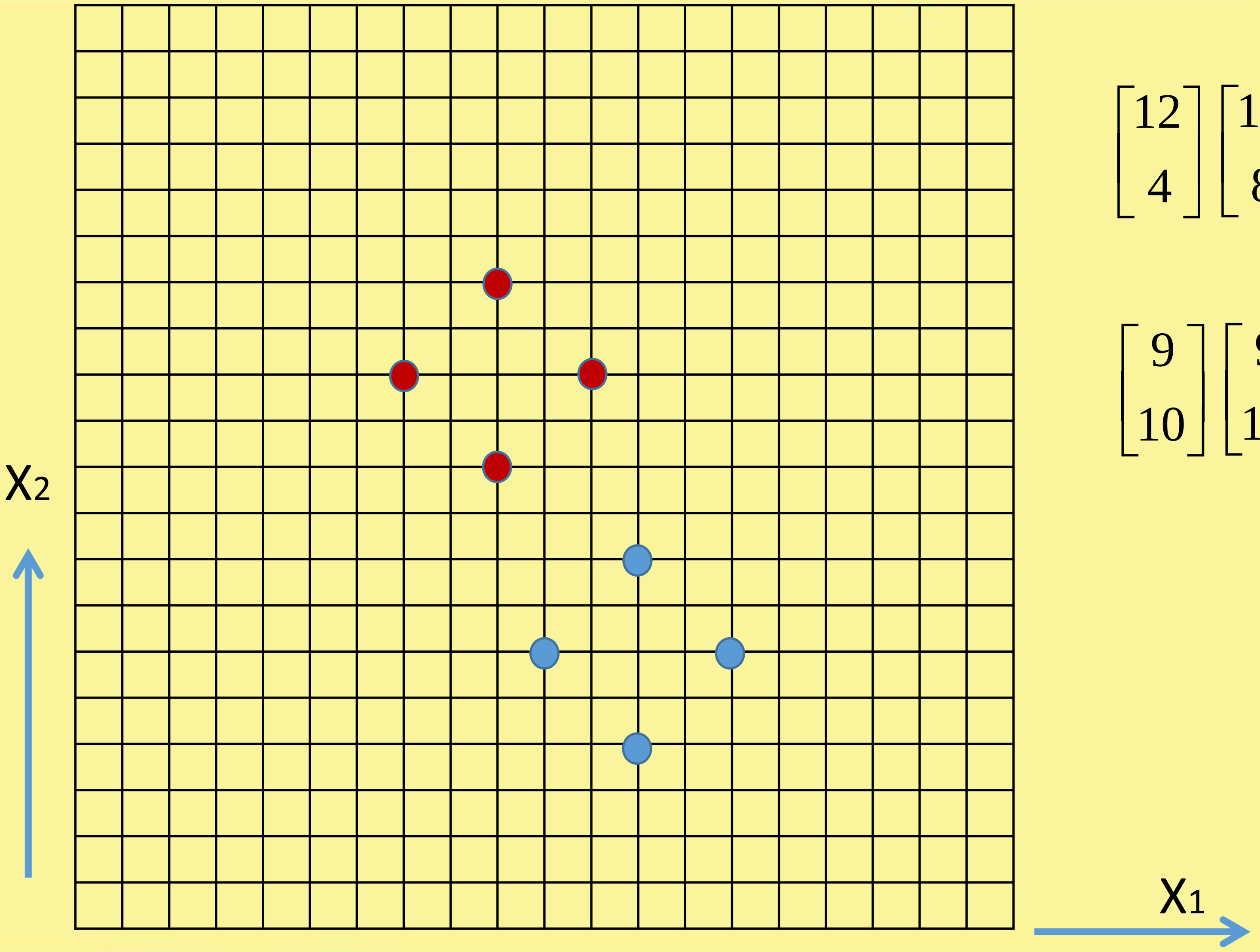
CONCEPTS COVERED

Concepts Covered:

- ❑ Bayes Minimum Error Classifier
- ❑ Bayes Minimum Risk Classifier
- ❑ Discriminant Function
- ❑ Decision Boundary

Discriminant Function

Discriminant Function under Multivariate Normal Distribution



$$\begin{bmatrix}12\\4\end{bmatrix}\begin{bmatrix}12\\8\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix}\begin{bmatrix}14\\6\end{bmatrix}\Rightarrow \omega_1$$

$$\begin{bmatrix}9\\10\end{bmatrix}\begin{bmatrix}9\\14\end{bmatrix}\begin{bmatrix}7\\12\end{bmatrix}\begin{bmatrix}11\\12\end{bmatrix}\Rightarrow \omega_2$$

$$\begin{bmatrix}12\\4\end{bmatrix}\begin{bmatrix}12\\8\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix}\begin{bmatrix}14\\6\end{bmatrix} \Rightarrow \omega_1 \qquad \mu_1 = \frac{1}{4}\left[\begin{bmatrix}12\\4\end{bmatrix}+\begin{bmatrix}12\\8\end{bmatrix}+\begin{bmatrix}10\\6\end{bmatrix}+\begin{bmatrix}14\\6\end{bmatrix}\right] = \begin{bmatrix}12\\6\end{bmatrix}$$

$$[X_1-\mu_1][X_1-\mu_1]^t = \begin{bmatrix}0\\-2\end{bmatrix}\begin{bmatrix}0 & -2\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 4\end{bmatrix} = M_1$$

$$[X_2-\mu_1][X_2-\mu_1]^t = \begin{bmatrix}0\\2\end{bmatrix}\begin{bmatrix}0 & 2\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 4\end{bmatrix} = M_2$$

$$[X_3-\mu_1][X_3-\mu_1]^t = \begin{bmatrix}-2\\0\end{bmatrix}\begin{bmatrix}-2 & 0\end{bmatrix} = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = M_3$$

$$[X_4-\mu_1][X_4-\mu_1]^t = \begin{bmatrix}2\\0\end{bmatrix}\begin{bmatrix}2 & 0\end{bmatrix} = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = M_4$$

$$\Sigma_1 = \frac{1}{4}[M_1+M_2+M_3+M_4]$$

$$= \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 2I$$

$$\begin{bmatrix} 9 \\ 10 \end{bmatrix} \begin{bmatrix} 9 \\ 14 \end{bmatrix} \begin{bmatrix} 7 \\ 12 \end{bmatrix} \begin{bmatrix} 11 \\ 12 \end{bmatrix} \Rightarrow \omega_2 \qquad \mu_2 = \frac{1}{4}\left[\begin{bmatrix} 9 \\ 10 \end{bmatrix} + \begin{bmatrix} 9 \\ 14 \end{bmatrix} + \begin{bmatrix} 7 \\ 12 \end{bmatrix} + \begin{bmatrix} 11 \\ 12 \end{bmatrix}\right] = \begin{bmatrix} 9 \\ 12 \end{bmatrix}$$

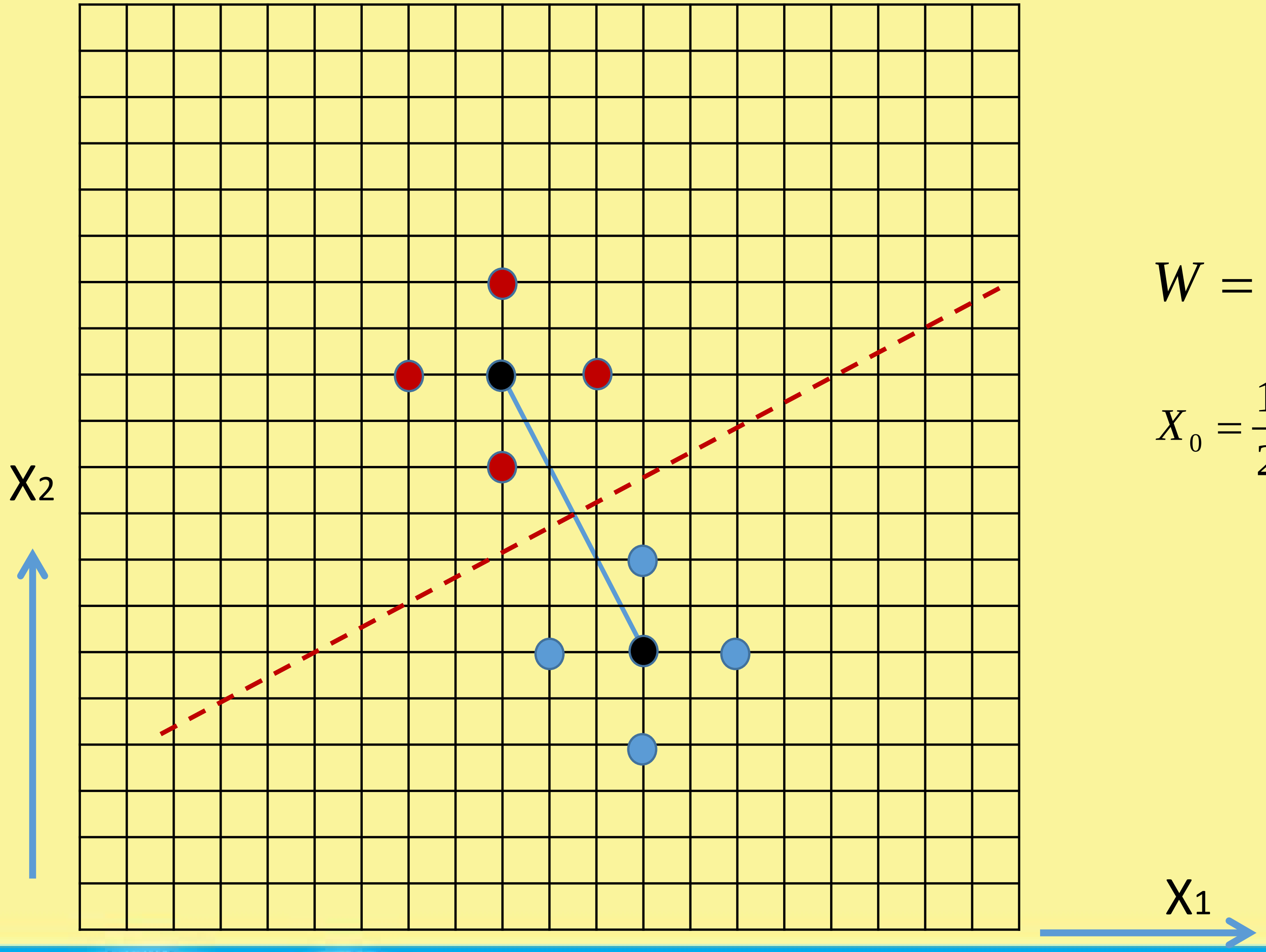
$$\Sigma_2 = 2I$$

$$\Sigma_1 = \Sigma_2 = 2I \approx \sigma^2 I$$

Where

$$\sigma = \sqrt{2}$$

Decision Surface



$$W^t(X - X_0) = 0$$

$$W = \mu_2 - \mu_1$$

$$X_0 = \frac{1}{2}(\mu_1 + \mu_2) - \frac{\sigma^2}{\left\|\mu_1 - \mu_2\right\|^2} \ln \frac{P(\omega_1)}{P(\omega_2)} (\mu_1 - \mu_2)$$

Discriminant Function under Multivariate Normal Distribution

NPTEL ONLINE CERTIFICATION COURSES

Thank you