NPTEL ONLINE CERTIFICATION COURSES

# Course Name: Deep Learning

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## Topic

Lecture 07: Discriminant Function and Decision Surface - II

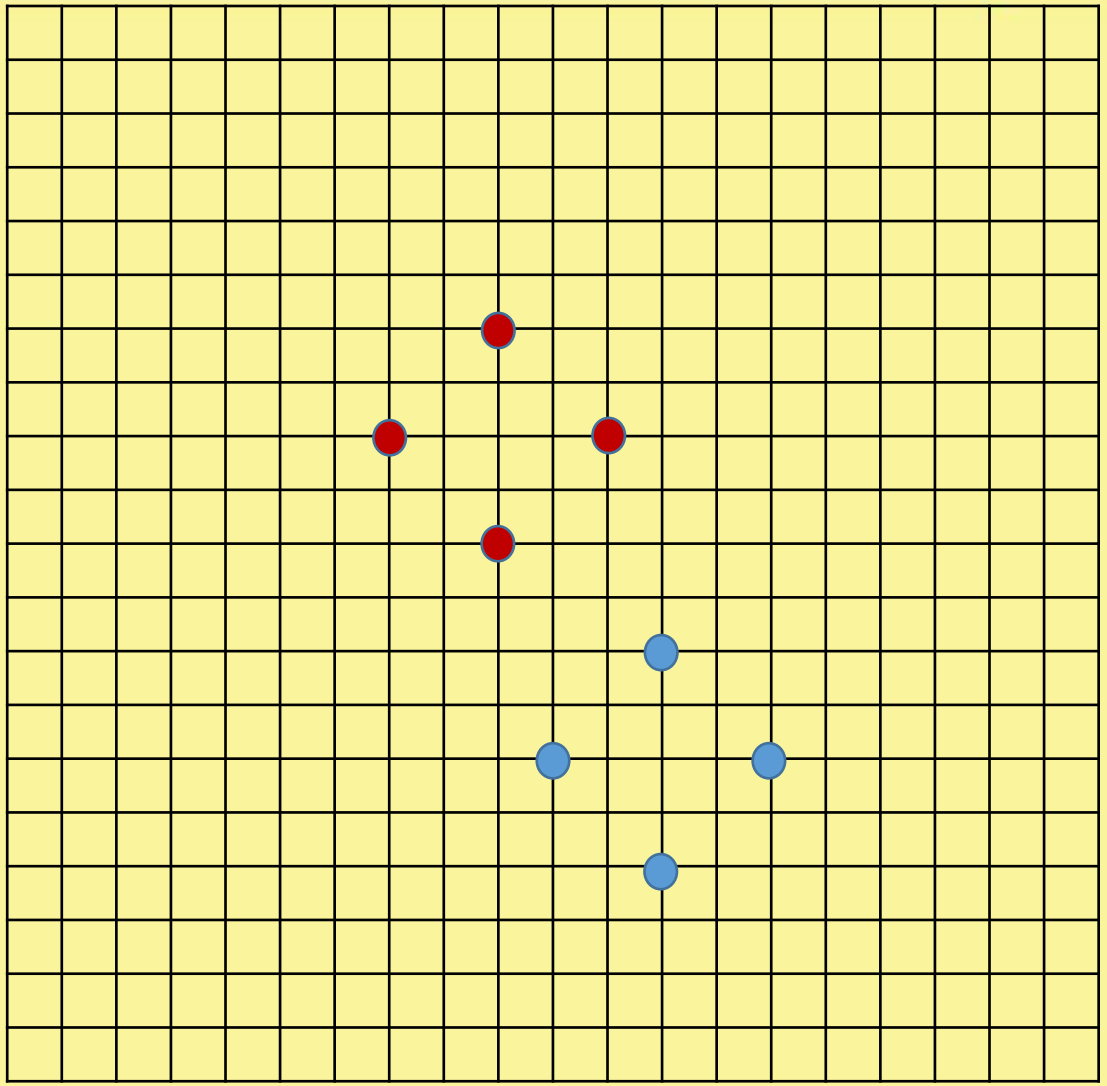
CONCEPTS COVERED

Concepts Covered:

- Discriminant Function under Multivariate Normal Distribution
- Decision Boundary under Various Cases of Covariance Matrices
- Examples

# Discriminant Function under Multivariate Normal Distribution

# Decision Surface




$$\begin{bmatrix}12\\4\end{bmatrix}\begin{bmatrix}12\\8\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix}\begin{bmatrix}14\\6\end{bmatrix} \Rightarrow \omega_1$$

$$\begin{bmatrix}9\\10\end{bmatrix}\begin{bmatrix}9\\14\end{bmatrix}\begin{bmatrix}7\\12\end{bmatrix}\begin{bmatrix}11\\12\end{bmatrix} \Rightarrow \omega_2$$

# Decision Surface

$$\begin{bmatrix}12\\4\end{bmatrix}\begin{bmatrix}12\\8\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix}\begin{bmatrix}14\\6\end{bmatrix} \Rightarrow \omega_1$$

$$\mu_1 = \frac{1}{4}\left[\begin{bmatrix}12\\4\end{bmatrix}+\begin{bmatrix}12\\8\end{bmatrix}+\begin{bmatrix}10\\6\end{bmatrix}+\begin{bmatrix}14\\6\end{bmatrix}\right] = \begin{bmatrix}12\\6\end{bmatrix}$$

$$[X_1-\mu_1][X_1-\mu_1]^t = \begin{bmatrix}0\\-2\end{bmatrix}[0 \quad -2] = \begin{bmatrix}0 & 0\\0 & 4\end{bmatrix} = M_1$$

$$[X_2-\mu_1][X_2-\mu_1]^t = \begin{bmatrix}0\\2\end{bmatrix}[0 \quad 2] = \begin{bmatrix}0 & 0\\0 & 4\end{bmatrix} = M_2$$

$$[X_3-\mu_1][X_3-\mu_1]^t = \begin{bmatrix}-2\\0\end{bmatrix}[-2 \quad 0] = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = M_3$$

$$[X_4-\mu_1][X_4-\mu_1]^t = \begin{bmatrix}2\\0\end{bmatrix}[2 \quad 0] = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = M_4$$

$$\Sigma_1 = \frac{1}{4}[M_1+M_2+M_3+M_4]$$

$$= \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 2I$$

# Decision Surface

$$\begin{bmatrix}9\\10\end{bmatrix}\begin{bmatrix}9\\14\end{bmatrix}\begin{bmatrix}7\\12\end{bmatrix}\begin{bmatrix}11\\12\end{bmatrix} \Rightarrow \omega_2 \qquad \mu_2 = \frac{1}{4}\left[\begin{bmatrix}9\\10\end{bmatrix}+\begin{bmatrix}9\\14\end{bmatrix}+\begin{bmatrix}7\\12\end{bmatrix}+\begin{bmatrix}11\\12\end{bmatrix}\right] = \begin{bmatrix}9\\12\end{bmatrix}$$

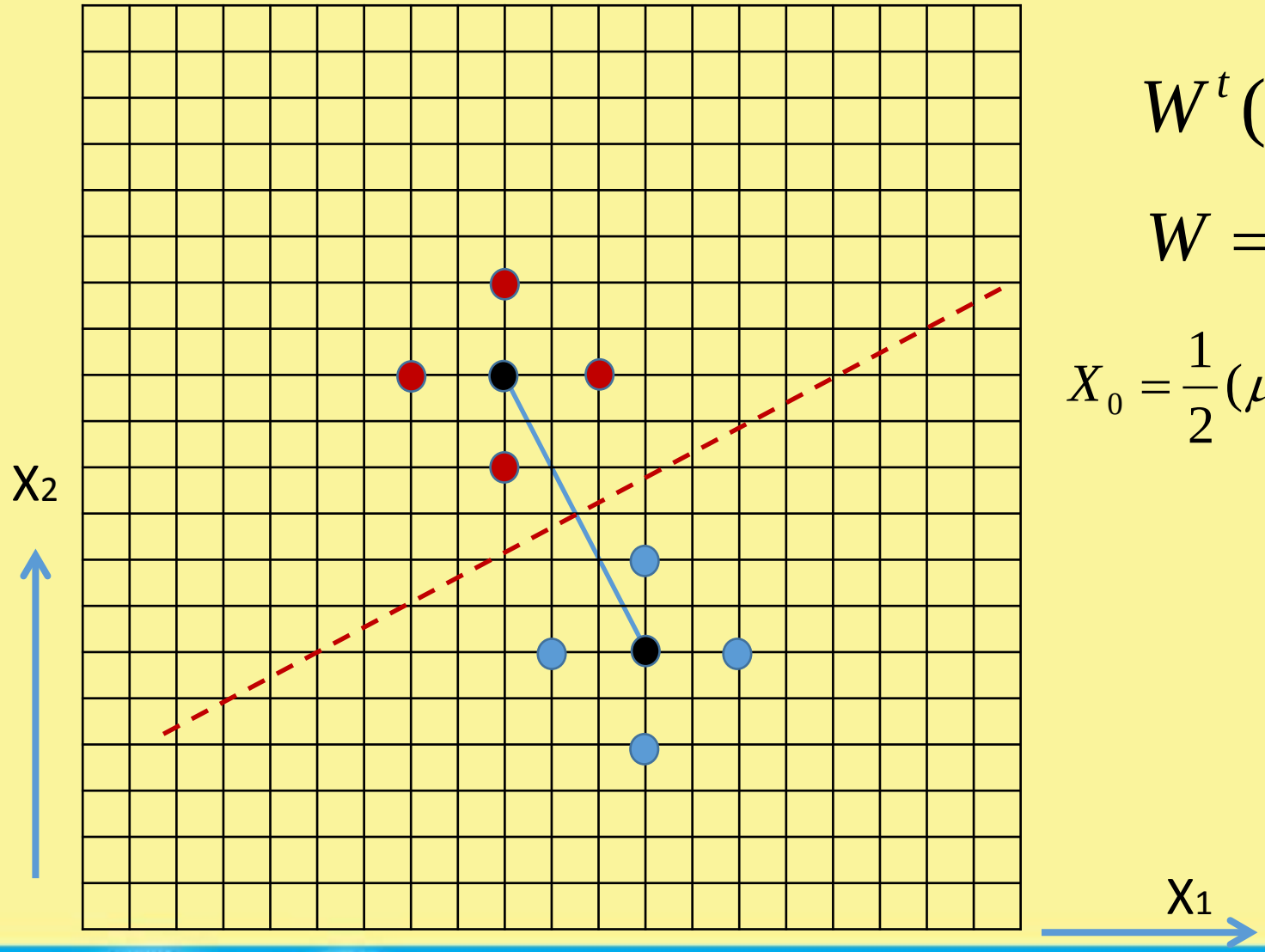
$$\Sigma_2 = 2I$$

$$\Sigma_1 = \Sigma_2 = 2I \approx \sigma^2 I$$

Where

$$\sigma = \sqrt{2}$$

# Decision Surface




$$W^t(X - X_0) = 0$$

$$W = \mu_2 - \mu_1$$

$$X_0 = \frac{1}{2}(\mu_1 - \mu_2) - \frac{\sigma^2}{\|\mu_1 - \mu_2\|^2} \ln \frac{P(\omega_1)}{P(\omega_2)}(\mu_1 - \mu_2)$$

# Discriminant Function under Multivariate Normal Distribution

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*Thank you*