Spelling Correction: Edit Distance

Pawan Goyal

CSE, IITKGP

Week 2: Lecture 1

Spelling Correction

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I am writing this email on behaf of ...

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The user typed ‘behaf’.

Which are some close words?

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Isolated word error correction

- Pick the one that is closest to ‘behaf’

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- Pick the one that is closest to ‘behaf’
- How to define ‘closest’?

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- Pick the one that is closest to 'behaf'
- How to define 'closest'?
- Need a **distance metric**

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The user typed ‘behaf’.

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- behalf
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Isolated word error correction

- Pick the one that is closest to ‘behaf’
- How to define ‘closest’?
- Need a **distance metric**
- The simplest metric: **edit distance**

Edit Distance

- The minimum edit distance between two strings

Edit Distance

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- Is the minimum number of editing operations

Edit Distance

- The minimum edit distance between two strings
- Is the minimum number of editing operations
  - Insertion
  - Deletion
  - Substitution

Example

Edit distance from ‘intention’ to ‘execution’

Example

Edit distance from ‘intention’ to ‘execution’

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

Minimum Edit Distance

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

Minimum Edit Distance

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I N T E * N T I O N
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- If each operation has a cost of 1 (Levenshtein)
  - Distance between these is 5

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I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

- If each operation has a cost of 1 (Levenshtein)
  - Distance between these is 5
- If substitution costs 2 (alternate version)
  - Distance between these is 8

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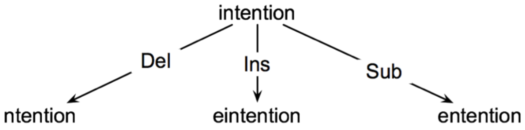

Minimum Edit as Search

How to navigate?

- The space of all edit sequences is huge

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- Lot of distinct paths end up at the same state
- Don't have to keep track of all of them
- Keep track of the shortest path to each state

Defining Minimum Edit Distance Matrix

For two strings

- X of length n
- Y of length m

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We define $D(i,j)$

- the edit distance between $X[1..i]$ and $Y[1..j]$
- i.e., the first i characters of X and the first j characters of Y

Defining Minimum Edit Distance Matrix

For two strings

- X of length n
- Y of length m

We define $D(i,j)$

- the edit distance between $X[1..i]$ and $Y[1..j]$
- i.e., the first i characters of X and the first j characters of Y

Thus, the edit distance between X and Y is $D(n,m)$

Computing Minimum Edit Distance

Dynamic Programming

- A tabular computation of $D(n,m)$

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- Solving problems by combining solutions to subproblems

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  - Compute $D(i,j)$ for small i,j
  - Compute larger $D(i,j)$ based on previously computed smaller values
  - Compute $D(i,j)$ for all i and j till you get to $D(n,m)$

Dynamic Programming Algorithm

Initialization

```
D(i,0) = i
D(0,j) = j
```

Recurrence Relation:

```
For each  i = 1…M
      For each  j = 1…N
```

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} \end{cases}$$

Termination:

```
D(N,M) is distance
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The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

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The Edit Distance Table

N	9	8	9	10	11	12	11	10	9	**8**
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

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  - We often need to align characters of the two strings to each other
- We do this by keeping a “backtrace”
- Every time we enter a cell, remember where we came from
- When we reach the end,
  - Trace back the path from the upper right corner to read off the alignment

The Edit Distance Table

N	9									
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Minimum Edit with Backtrace

n	9	↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙←↓ 12	↓ 11	↓ 10	↓ 9	**↙ 8**	
o	8	↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↓ 10	↓ 9	**↙ 8**	← 9	
i	7	↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	**↙ 8**	← 9	← 10	
t	6	↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	**↙ 8**	← 9	← 10	←↓ 11	
n	5	↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	**↙←↓ 8**	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙↓ 10	
e	4	↙ 3	← 4	**↙← 5**	**← 6**	← 7	←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	
t	3	↙←↓ 4	**↙←↓ 5**	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙ 7	←↓ 8	↙←↓ 9	↓ 8	
n	2	**↙←↓ 3**	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↓ 7	↙←↓ 8	↙ 7	
i	**1**	↙←↓ 2	↙←↓ 3	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙ 6	← 7	← 8	
#	**0**	1	2	3	4	5	6	7	8	9	
	#	**e**	**x**	**e**	**c**	**u**	**t**	**i**	**o**	**n**	

Adding Backtrace to Minimum Edit

Base conditions:

`D(i,0) = i`   `D(0,j) = j`

Termination:

`D(N,M) is distance`

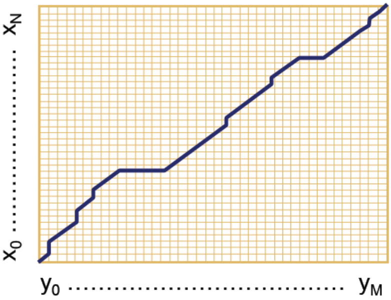
Recurrence Relation:

```
For each  i = 1…M
     For each  j = 1…N
```

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 & \text{deletion} \\ D(i,j-1) + 1 & \text{insertion} \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} & \text{substitution} \end{cases}$$

$$ptr(i,j) = \begin{cases} \text{LEFT} & \text{insertion} \\ \text{DOWN} & \text{deletion} \\ \text{DIAG} & \text{substitution} \end{cases}$$

The distance matrix



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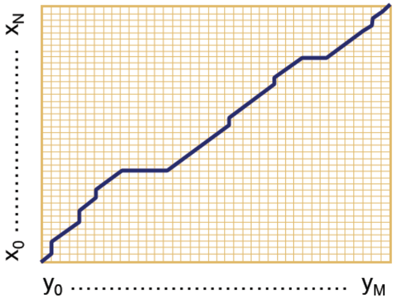
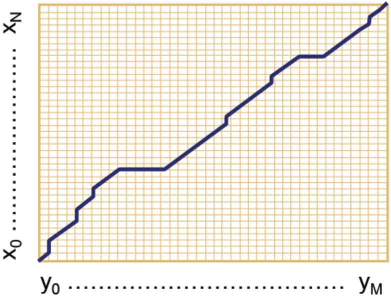


Every non-decreasing path from (0,0) to (M,N) corresponds to an alignment of two sequences.

The distance matrix



Every non-decreasing path from (0,0) to (M,N) corresponds to an alignment of two sequences.

An optimal alignment is composed of optimal sub-alignments.

Result of Backtrace

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

Performance

Time

Performance

Time

$O(nm)$

Space

Performance

Time

$O(nm)$

Space

$O(nm)$

Backtrace

Performance

Time

$O(nm)$

Space

$O(nm)$

Backtrace

$O(n+m)$

Weighted Edit Distance, Other variations

Pawan Goyal

CSE, IITKGP

Week 2: Lecture 2

Weighted Edit Distance

Why to add weights to the computation?

- Some letters are more likely to be mistyped.

Confusion Matrix for Spelling Errors

sub[X, Y] = Substitution of X (incorrect) for Y (correct)

X	Y (correct)																									
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
c	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
e	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	4	0	0	3
l	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
o	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
p	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	2	43	0	0	4	0	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
y	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
z	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	0	2	21	3	0	0	0	0	3	0

Keyboard Design

Weighted Minimum Edit Distance

Initialization:

```
D(0,0) = 0
D(i,0) = D(i-1,0) + del[x(i)];     1 < i ≤ N
D(0,j) = D(0,j-1) + ins[y(j)];     1 < j ≤ M
```

Recurrence Relation:

$$D(i,j) = \min \begin{cases} D(i-1,j) & + \ del[x(i)] \\ D(i,j-1) & + \ ins[y(j)] \\ D(i-1,j-1) & + \ sub[x(i),y(j)] \end{cases}$$

Termination:

```
D(N,M) is distance
```

How to modify the algorithm with transpose?

Transpose

- $transpose(x,y) = (y,x)$
- Also known as metathesis

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Modification to the dynamic programmic algorithm

$$D[i][j] = min \begin{cases} D(i-1,j)+1 & (deletion) \\ D(i,j-1)+1 & (insertion) \\ D(i-1,j-1)+ & \begin{cases} 1 & if(x[i] \neq y[j])(substitution) \\ 0 & otherwise \end{cases} \\ D(i-2,j-2)+1 & (x[i] = y[j-1] \text{ and } x[i-1] = y[j] \\ & (transposition) \end{cases}$$

How to find dictionary entries with smallest edit distance?

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Naïve Method

Compute edit ditance from the query term to each dictionary term – an exhaustive search

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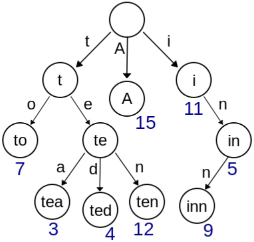

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- For a word of length 9, alphabet of size 36, this will lead to 114,324 terms to search for
- For Chinese alphabet size is 70,000 (Unicode Han Characters)

How to find dictionary entries with smallest edit distance?

Symmetric Delete Spelling Correction

- Generate terms with an edit distance ≤ 2 (deletes) from each dictionary term (offline)
- Generate terms with an edit distance ≤ 2 (deletes) from the input terms and search in dictionary

How to find dictionary entries with smallest edit distance?

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Number of deletes within edit distance ≤ 2 for a word of length 9 will be 45

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Number of deletes within edit distance ≤ 2 for a word of length 9 will be 45

A further check is required to remove the false positives

Spelling Correction

Spelling Correction

Types of spelling errors: Non-word Errors

- behaf $\rightarrow$ behalf

Spelling Correction

Types of spelling errors: Non-word Errors

- behaf $\rightarrow$ behalf

Types of spelling errors: Real-word Errors

- **Typographical errors:** three $\rightarrow$ there
- **Cognitive errors (homophones):** piece $\rightarrow$ peace, too $\rightarrow$ two

Non-word spelling errors

Non-word spelling error detection

- Any word not in a dictionary is an error
- The larger the dictionary the better

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Non-word spelling error detection

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Non-word spelling error correction

- Generate candidates: real words that are similar to the error word
- Choose the best one:
  - Shortest weighted edit distance
  - Highest noisy channel probabliity

Real word spelling errors

For each word w, generate candidate set

- Find candidate words with similar pronunciations
- Find candidate words with similar spelling
- Include w in candidate set

Real word spelling errors

For each word w, generate candidate set

- Find candidate words with similar pronunciations
- Find candidate words with similar spelling
- Include w in candidate set

Choosing best candidate

- Noisy Channel

Noisy Channel Model for Spelling Correction

Pawan Goyal

CSE, IITKGP

Week 2: Lecture 3

Noisy Channel

We see an observation x of the misspelled word

Find the correct word w

$$\hat{w} = \arg\max_{w \in V} P(w|x)$$

Noisy Channel

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$$\hat{w} = \underset{w \in V}{\arg\max} P(w|x)$$

$$= \underset{w \in V}{\arg\max} \frac{P(x|w)P(w)}{P(x)}$$

Noisy Channel

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$$= \arg\max_{w \in V} \frac{P(x|w)P(w)}{P(x)}$$

$$= \arg\max_{w \in V} P(x|w)P(w)$$

Non-word spelling error: acress

Words with similar spelling

Small edit distance to error

Words with similar pronunciation

Small edit distance of pronunciation to error

Non-word spelling error: acress

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Damerau-Levenshtein edit distance

Minimum edit distance, where edits are:

Non-word spelling error: acress

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Minimum edit distance, where edits are:

Insertion, Deletion, Substitution,

Non-word spelling error: acress

Words with similar spelling

Small edit distance to error

Words with similar pronuncitation

Small edit distance of pronunciation to error

Damerau-Levenshtein edit distance

Minimum edit distance, where edits are:
Insertion, Deletion, Substitution,
Transposition of two adjacent letters

Words within edit distance 1 of acress

Error	Candidate Correction	Correct Letter	Error Letter	Type
acress	actress	t	–	deletion
acress	cress	–	a	insertion
acress	caress	ca	ac	transposition
acress	access	c	r	substitution
acress	across	o	e	substitution
acress	acres	–	s	insertion
acress	acres	–	s	insertion

Candidate generation

- 80% of errors are within edit distance 1
- Almost all errors within edit distance 2

Candidate generation

- 80% of errors are within edit distance 1
- Almost all errors within edit distance 2

Allow deletion of space or hyphen

- thisidea $\rightarrow$ this idea
- inlaw $\rightarrow$ in-law

Computing error probability: confusion matrix

- del[x,y]: count (xy typed as x)
- ins[x,y]: count (x typed as xy)
- sub[x,y]: count (x typed as y)
- trans[x,y]: count(xy typed as yx)

Computing error probability: confusion matrix

- del[x,y]: count (xy typed as x)
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- trans[x,y]: count(xy typed as yx)

Insertion and deletion are conditioned on previous character

Channel model

$$P(x|w) = \begin{cases} \dfrac{\text{del}[w_{i-1}, w_i]}{\text{count}[w_{i-1} w_i]}, & \text{if deletion} \\ \dfrac{\text{ins}[w_{i-1}, x_i]}{\text{count}[w_{i-1}]}, & \text{if insertion} \\ \dfrac{\text{sub}[x_i, w_i]}{\text{count}[w_i]}, & \text{if substitution} \\ \dfrac{\text{trans}[w_i, w_{i+1}]}{\text{count}[w_i w_{i+1}]}, & \text{if transposition} \end{cases}$$

Channel model for acress

Candidate Correction	Correct Letter	Error Letter	x|w	P(x|word)
actress	t	-	c|ct	.000117
cress	-	a	a|#	.00000144
caress	ca	ac	ac|ca	.00000164
access	c	r	r|c	.000000209
across	o	e	e|o	.0000093
acres	-	s	es|e	.0000321
acres	-	s	ss|s	.0000342

Noisy channel probability for acress

Candidate Correction	Correct Letter	Error Letter	x|w	P(x|word)	P(word)	10^9 *P(x|w)P(w)
actress	t	–	c|ct	.000117	.0000231	2.7
cress	–	a	a|#	.00000144	.000000544	.00078
caress	ca	ac	ac|ca	.00000164	.00000170	.0028
access	c	r	r|c	.000000209	.0000916	.019
across	o	e	e|o	.0000093	.000299	2.8
acres	–	s	es|e	.0000321	.0000318	1.0
acres	–	s	ss|s	.0000342	.0000318	1.0

Using a bigram language model

- " ... versatile acress whose ..."

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- Counts from the Corpus of Contemporary American English with add-1 smoothing

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- P("versatile actress whose") = 0.000021 * 0.0010 = 210 x 10^{-10}

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- P("versatile actress whose") = 0.000021 * 0.0010 = 210 x 10^{-10}
- P("versatile across whose") = 0.000021 * 0.000006 = 1 x10^{-10}

Real-word spelling errors

- The study was conducted mainly **be** John Black
- The design **an** construction of the system ...

Real-word spelling errors

- The study was conducted mainly **be** John Black
- The design **an** construction of the system ...

25-40% of spelling errors are real words

Noisy channel for real-word spell correction

Given a sentence $X = w_1, w_2, w_3 \ldots, w_n$

- Candidate (w_1) = $\{w_1, w'_1, w''_1, w'''_1, \ldots\}$
- Candidate (w_2) = $\{w_2, w'_2, w''_2, w'''_2, \ldots\}$
- Candidate (w_3) = $\{w_3, w'_3, w''_3, w'''_3, \ldots\}$

Noisy channel for real-word spell correction

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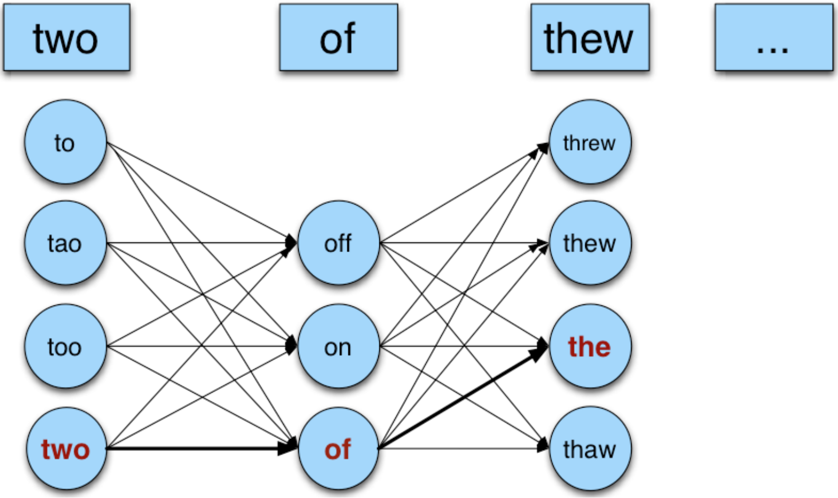
- Candidate (w_3) = $\{w_3, w'_3, w''_3, w'''_3, \dots\}$

Choose the sequence W that maximizes $P(W|X)$

Noisy channel for real-world spell correction



Simplification: One error per sentence

Choose among all possible sentences with one word replaced

two of thew

- w_1, w''_2, w_3 two **off** thew
- w_1, w_2, w'_3 two of **the**
- w'''_1, w_2, w_3 **too** of thew

Simplification: One error per sentence

Choose among all possible sentences with one word replaced

two of thew

- w_1, w''_2, w_3 two **off** thew
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Choose the sequence W that maximizes $P(W|X)$

Getting the probability values

Noisy Channel

$$\hat{W} = \arg\max_{W \in S} P(W|X)$$

where X is the observed sentence and S is the set of all the possible sequences from the candidate set

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- Same as for non-word spelling correction

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P(X|W)

- Same as for non-word spelling correction
- Also require proabability for no error $P(w|w)$

Probability of no error

What is the probability for a correctly typed word? P("the"|"the")

Probability of no error

What is the probability for a correctly typed word? P("the"|"the")

It may depend on the source text under consideration

- 1 error in 10 words $\rightarrow$ 0.9
- 1 error in 100 words $\rightarrow$ 0.99

Computing $P(W)$

Use Language Model

- Unigram
- Bigram
- ...

N-gram Language Models

Pawan Goyal

CSE, IITKGP

Week 2: Lecture 4

Context Sensitive Spelling Correction

Context Sensitive Spelling Correction

The office is about fifteen minuets from my house

Context Sensitive Spelling Correction

The office is about fifteen minuets from my house

min·u·et *noun* \ˌmin-yə-ˈwet\

: a slow, graceful dance that was popular in the 17th and 18th centuries

: the music for a minuet

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: a slow, graceful dance that was popular in the 17th and 18th centuries

: the music for a minuet

Use a Language Model

P(about fifteen **minutes** from) > P(about fifteen **minuets** from)

Speech Recognition

- P(I saw a van) $>>$ P(eyes awe of an)

Probablilistic Language Models: Applications

Speech Recognition

- P(I saw a van) $>>$ P(eyes awe of an)

Machine Translation

Which sentence is more plausible in the target language?

- P(**high** winds) > P(**large** winds)

Probablilistic Language Models: Applications

Speech Recognition

- P(I saw a van) $>>$ P(eyes awe of an)

Machine Translation

Which sentence is more plausible in the target language?

- P(**high** winds) > P(**large** winds)

Other Applications

- Context Sensitive Spelling Correction
- Natural Language Generation
- ...

Completion Prediction

- Language model also supports predicting the completion of a sentence.
  - Please turn off your cell ...
  - Your program does not ...

Completion Prediction

- Language model also supports predicting the completion of a sentence.
  - Please turn off your cell ...
  - Your program does not ...
- *Predictive text input* systems can guess what you are typing and give choices on how to complete it.

Probabilistic Language Modeling

- **Goal:** Compute the probability of a sentence or sequence of words:

$$P(W) = P(w_1, w_2, w_3, \ldots, w_n)$$

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- **Related Task:** probability of an upcoming word:

$$P(w_4 | w_1, w_2, w_3)$$

Probabilistic Language Modeling

- **Goal:** Compute the probability of a sentence or sequence of words:

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- **Related Task:** probability of an upcoming word:

$$P(w_4 | w_1, w_2, w_3)$$

- A model that computes either of these is called a **language model**

Computing $P(W)$

How to compute the joint probability

P(about, fifteen, minutes, from)

Computing $P(W)$

How to compute the joint probability

P(about, fifteen, minutes, from)

Basic Idea

Rely on the Chain Rule of Probability

The Chain Rule

Conditional Probabilities

$$P(B|A) = \frac{P(A,B)}{P(A)}$$

The Chain Rule

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The Chain Rule

Conditional Probabilities

$$P(B|A) = \frac{P(A,B)}{P(A)}$$

$$P(A,B) = P(A)P(B|A)$$

More Variables

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

The Chain Rule

Conditional Probabilities

$$P(B|A) = \frac{P(A,B)}{P(A)}$$

$$P(A,B) = P(A)P(B|A)$$

More Variables

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

The Chain Rule in General

$$P(x_1,x_2,\ldots,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)\ldots P(x_n|x_1,\ldots,x_{n-1})$$

Probability of words in sentences

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i | w_1 w_2 \dots w_{i-1})$$

P("about fifteen minutes from") =

Probability of words in sentences

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i | w_1 w_2 \ldots w_{i-1})$$

P("about fifteen minutes from") =

P(about) x P(fifteen | about) x P(minutes | about fifteen) x P(from | about fifteen minutes)

Estimating These Probability Values

Count and divide

$$\text{P(office} \mid \text{about fifteen minutes from)} = \frac{\textit{Count}\text{ (about fifteen minutes from office)}}{\textit{Count}\text{ (about fifteen minutes from)}}$$

Estimating These Probability Values

Count and divide

$$P(\text{office} \mid \text{about fifteen minutes from}) = \frac{\mathit{Count}\,(\text{about fifteen minutes from office})}{\mathit{Count}\,(\text{about fifteen minutes from})}$$

What is the problem

We may never see enough data for estimating these

Markov Assumption

Simplifying Assumption: Use only the previous word

P(office | about fifteen minutes from) $\approx$ P(office | from)

Markov Assumption

Simplifying Assumption: Use only the previous word

P(office | about fifteen minutes from) $\approx$ P(office | from)

Or the couple previous words

P(office | about fifteen minutes from) $\approx$ P(office | minutes from)

Markov Assumption

More Formally: kth order Markov Model

Chain Rule:

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i | w_1 w_2 \ldots w_{i-1})$$

Markov Assumption

More Formally: kth order Markov Model

Chain Rule:

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i | w_1 w_2 \ldots w_{i-1})$$

Using Markov Assumption: only k previous words

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i | w_{i-k} \ldots w_{i-1})$$

Markov Assumption

More Formally: kth order Markov Model

Chain Rule:

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i | w_1 w_2 \dots w_{i-1})$$

Using Markov Assumption: only k previous words

$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i | w_{i-k} \dots w_{i-1})$$

We approximate each component in the product

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx P(w_i | w_{i-k} \dots w_{i-1})$$

N-Gram Models

P(office | about fifteen minutes from)

An N-gram model uses only $N-1$ words of prior context.

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- Unigram: P(office)
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Markov model and Language Model

N-Gram Models

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Markov model and Language Model

An N-gram model is an $N-1$-order Markov Model

N-Gram Models

- We can extend to trigrams, 4-grams, 5-grams
- In general, an insufficient model of language:

N-Gram Models

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- In general, an insufficient model of language:
  language has long-distance dependencies:
  "The computer which I had just put into the machine room on the fifth floor **crashed**."

N-Gram Models

- We can extend to trigrams, 4-grams, 5-grams
- In general, an insufficient model of language:
  language has long-distance dependencies:
  "The computer which I had just put into the machine room on the fifth floor **crashed**."
- In most of the applications, we can get away with N-gram models

Estimating N-grams probabilities

Estimating N-grams probabilities

Maximum Likelihood Estimate

Value that makes the observed data the "most probable"

$$P(w_i|w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

Estimating N-grams probabilities

Maximum Likelihood Estimate

Value that makes the observed data the "most probable"

$$P(w_i|w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An Example

$$P(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s>I am here </s>
<s>who am I </s>
<s>I would like to know </s>

An Example

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Estimating bigrams

P(I|<s>) =
P(</s>|here) =
P(would | I) =
P(here | am) =
P(know | like) =

An Example

$$P(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s>I am here </s>
<s>who am I </s>
<s>I would like to know </s>

Estimating bigrams

P(I|<s>) = 2/3
P(</s>|here) =1
P(would | I) = 1/3
P(here | am) = 1/2
P(know | like) = 0

Bigram counts from 9222 Restaurant Sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Computing bigram probabilities

Normlize by unigrams

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Computing bigram probabilities

Normlize by unigrams

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Bigram Probabilities

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Computing Sentence Probabilities

P(<s> I want english food </s>)

= P(I | <s>) x P(want | I) x P(english | want) x P(food | english) x P(</s> | food)

Computing Sentence Probabilities

P(<s> I want english food </s>)

= P(I | <s>) x P(want | I) x P(english | want) x P(food | english) x P(</s> | food)
= 0.000031

What knowledge does n-gram represent?

- P(english|want) = .0011
- P(chinese|want) = .0065
- P(to|want) = .66
- P(eat | to) = .28
- P(food | to) = 0
- P(want | spend) = 0
- P (i | <s>) = .25

Practical Issues

Everything in log space

- Avoids underflow

Practical Issues

Everything in log space

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- Adding is faster than multiplying

Practical Issues

Everything in log space

- Avoids underflow
- Adding is faster than multiplying

$$log(p_1 \times p_2 \times p_3 \times p_4) = logp_1 + logp_2 + logp_3 + logp_4$$

Handling zeros

Use smoothing

Language Modeling Toolkit

SRILM

http://www.speech.sri.com/projects/srilm/

Google N-grams

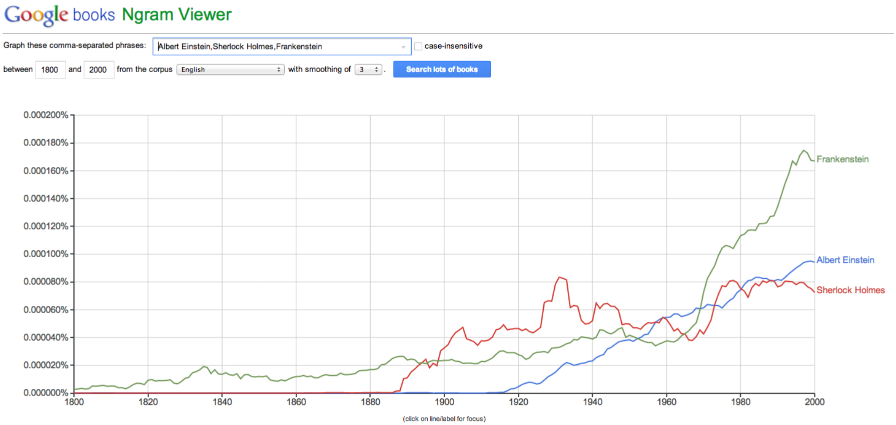
Number of tokens: 1,024,908,267,229
Number of sentences: 95,119,665,584
Number of unigrams: 13,588,391
Number of bigrams: 314,843,401
Number of trigrams: 977,069,902
Number of fourgrams: 1,313,818,354
Number of fivegrams: 1,176,470,663

`http://googleresearch.blogspot.in/2006/08/all-our-n-gram-are-belong-to-you.html`

Example from the 4-gram data

serve as the inspector 66
serve as the inspiration 1390
serve as the installation 136
serve as the institute 187
serve as the institution 279
serve as the institutional 461

Google books Ngram Data



Evaluation of Language Models, Basic Smoothing

Pawan Goyal

CSE, IITKGP

Week 2: Lecture 5

Evaluating Language Model

Does it prefer good sentences to bad sentences?

Assign higher probability to real (or frequently observed) sentences than ungrammatical (or rarely observed) ones

Evaluating Language Model

Does it prefer good sentences to bad sentences?

Assign higher probability to real (or frequently observed) sentences than ungrammatical (or rarely observed) ones

Training and Test Corpora

- Parameters of the model are trained on a large corpus of text, called **training set**.
- Performance is tested on a disjoint (held-out) **test data** using an **evaluation metric**

Extrinsic evaluation of N-grams models

Comparison of two models, A and B

- Use each model for one or more tasks: *spelling corrector, speech recognizer, machine translation*
- Get accuracy values for A and B
- Compare accuracy for A and B

Intrinsic evaluation: Perplexity

Intuition: The Shannon Game

How well can we predict the next word?

Intrinsic evaluation: Perplexity

Intuition: The Shannon Game

How well can we predict the next word?

- I always order pizza with cheese and ...
- The president of India is ...
- I wrote a ...

Intrinsic evaluation: Perplexity

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Unigram model doesn't work for this game.

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Intuition: The Shannon Game

How well can we predict the next word?

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- The president of India is ...
- I wrote a ...

Unigram model doesn't work for this game.

A better model of text

is one which assigns a higher probability to the actual word

Perplexity

The best language model is one that best predics an unseen test set

Perplexity ($PP(W)$)

Perplexity is the inverse probability of the test data, normalized by the number of words:

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$$PP(W) = P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}}$$

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Perplexity is the inverse probability of the test data, normalized by the number of words:

$$PP(W) = P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}}$$

Applying chain Rule

$$PP(W) = \left(\prod \frac{1}{P(w_i | w_1 \ldots w_{i-1})} \right)^{\frac{1}{N}}$$

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Applying chain Rule

$$PP(W) = \left(\prod \frac{1}{P(w_i|w_1 \ldots w_{i-1})} \right)^{\frac{1}{N}}$$

For bigrams

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&= \left(\frac{1}{10} \right)^{-1} \\
&= 10
\end{aligned}
$$

Lower perplexity = better model

WSJ Corpus

Training: 38 million words

Test: 1.5 million words

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N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

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Unigram perplexity: 962?

The model is as confused on test data as if it had to choose uniformly and independently among 962 possibilities for each word.

The Shannon Visualization Method

Use the language model to generate word sequences

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```
<s> I
    I want
      want to
           to eat
              eat Chinese
                  Chinese food
                          food  </s>
I want to eat Chinese food
```

Shakespeare as Corpus

- N = 884,647 tokens, V = 29,066
- Shakespeare produced 300,000 bigram types out of $V^2 = 844$ million possible bigrams.

Approximating Shakespeare

Unigram

To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have
Every enter now severally so, let
Hill he late speaks; or! a more to leg less first you enter
Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like

Bigram

What means, sir. I confess she? then all sorts, he is trim, captain.
Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.
What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman?

Trigram

Sweet prince, Falstaff shall die. Harry of Monmouth's grave.
This shall forbid it should be branded, if renown made it empty.
Indeed the duke; and had a very good friend.
Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.

Quadrigram

King Henry.What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;
Will you not tell me who I am?
It cannot be but so.
Indeed the short and the long. Marry, 'tis a noble Lepidus.

Problems with simple MLE estimate: zeros

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Training set

- ... denied the allegations
- ... denied the reports
- ... denied the claims
- ... denied the request

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Zero probability n-grams

- P(offer | denied the) = 0

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Zero probability n-grams

- P(offer | denied the) = 0
- The test set will be assigned a probability 0
- And the perplexity can't be computed

Language Modeling: Smoothing

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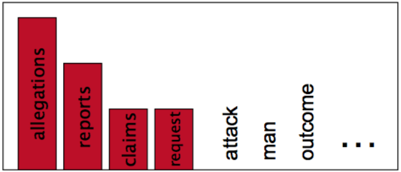
With sparse statistics

P(w | denied the)
3 allegations
2 reports
1 claims
1 request
7 total



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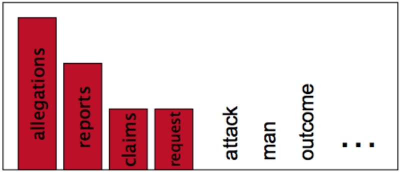
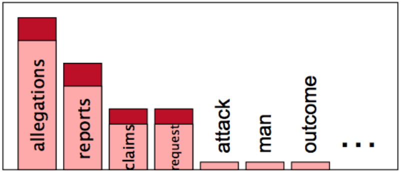


Steal probability mass to generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other

7 total



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- Just add one to all the counts!
- MLE estimate for bigram: $P_{MLE}(w_i|w_{i-1}) = \frac{c(w_{i-1},w_i)}{c(w_{i-1})}$
- Add-1 estimate: $P_{Add-1}(w_i|w_{i-1}) = \frac{c(w_{i-1},w_i)+1}{c(w_{i-1})+V}$

Reconstituted counts as effect of smoothing

Effective bigram count ($c^*(w_{n-1}w_n)$)

$$\frac{c^*(w_{n-1}w_n)}{c(w_{n-1})} = \frac{c(w_{n-1}w_n)+1}{c(w_{n-1})+V}$$

Comparing with bigrams: Restaurant corpus

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

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i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

More general formulations: Add-k

$$P_{Add-k}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i) + k}{c(w_{i-1}) + kV}$$

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Unigram prior smoothing:

$$P_{UnigramPrior}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i) + mP(w_i)}{c(w_{i-1}) + m}$$

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A good value of k or m?

Can be optimized on held-out set