Investigación Operativa Práctica 1

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Parte 1

Simplex

Formato Entrada

```
examen_parcial_1_2023_2024 = """

argmax z = 0x1 + 1x2

x1 + 6x2 <= 14

0.4x1 + 0.3x2 >= 3

1x1 + 0x2 >= 1

0x1 + 1x2 >= 1

"""

examen_parcial_2_2022_2023 = """

argmax z = x1 + 2x2

-1x1+ x2 <= -1

x1 + x2 <= 2
"""
```

```
#Codigo para sacar la matriz
for i, line in enumerate(lines):
    match = re.findall(r'x(\d+)', line)
    if line.startswith("argmax") or line.startswith("argmin"):
       line.replace("argmax", "")
       if line.startswith("argmin"):
           change z = True
       line.replace("argmin", "")
       line = line.replace("z =", "").strip()
       parsed line = [1] # valor de la z
       for j in range(1, num variables + 1):
           match = re.search(rf'([+-]?\d*\.?\d*)x{j}', line)
           coeff = float(match.group(1)) if match and match.group(1) else 1
           parsed line.append(-coeff)
       # terminamos el reglon de las z
       parsed line.extend([0] * (len(lines) - 1))
       parsed line.append(0)
       if "<=" in line:
           line, rhs = line.split("<=")
           rhs = float(rhs.strip())
       elif ">=" in line:
           line, rhs = line.split(">=")
           rhs = -float(rhs.strip())
           if 0 not in lineas a cambiar signo:
               lineas a cambiar signo.append(0)
           lineas a cambiar signo.append(i)
       parsed line = [0] # empienzan en
```

Formato Clase

```
self.si = len(self.matriz[0]) - 2 - self.ni
self.diccionario = {f'x{i}': deepcopy(self.matriz[1:, i]) for i in range(1, self.ni + 1)}
self.diccionario.update({f's{i}': deepcopy(self.matriz[1:, self.ni + i]) for i in range(1, self.si + 1)
self.diccionario.update({'reglon_de_las_z': deepcopy(self.matriz[0][1:-1])})
self.diccionario.update({'terminos_independientes': deepcopy(self.matriz[1:, -1])})
self.diccionario.update({'restricciones': deepcopy(self.matriz[1:, 1:-1])})
self.reglon_de_las_z = deepcopy(self.matriz[0][1:-1])
self.terminos_independientes = deepcopy(self.matriz[1:, -1])
self.restricciones = deepcopy(self.matriz[1:, 1:-1])
self.base = {f's{i+1}': deepcopy(self.diccionario['s' + str(i+1)]) for i in range(self.si)}
```

No es lo más eficiente... Pero es claro

Empecemos...

```
def resolucion(self, imprimir = False):
    # comprobar si 0 esta en la solucion, es decir si cumple las restriccione:
    if np.all(self.terminos_independientes >= 0):
        return self.simplex_primal(imprimir)
    else:
        self.simplex_dual(imprimir)
        return self.simplex_primal(imprimir)
```

Simplex Dual

```
while np.any(np.array(self.terminos independientes) <= -le-5) and vueltas > 0:
   vueltas -= 1
   if np.all(np.array(self.terminos independientes) >= -le-5) or np.all(np.isclose(self.reglon de las z[:self.ni], 0, atol=le-6)):
       break
   print('-----')
   self.iteraciones += 1
   print('Iteracion:', self.iteraciones)
   # Mascara para que los terminos independientes sean negativos
   negative mask = self.terminos independientes < -1e-5
   negative values = self.terminos independientes[negative mask]
   if negative values.size > 0:
       columna salida = np.argmin(negative values)
       columna salida = np.where(negative mask)[0][columna salida]
   non zero mask = (np.abs(self.region de las z) > tolerance) & (np.abs(self.restricciones[columna salida, :]) > tolerance)
   division result = np.where(non zero mask,
                               self.reglon de las z / self.restricciones[columna salida, :],
                              np.inf)
```

Simplex Dual

```
columna entrada = np.argmin(abs(division result))
if columna entrada < self.ni:
    variable entrada = 'x' + str(columna entrada + 1)
    variable entrada = 's' + str(columna entrada - self.ni + 1)
variable salida = list(self.base.keys())[columna salida]
if imprimir:
    print('Variable de salida:', variable salida)
    print('Variable de entrada:', variable entrada)
# sacamos las columnas de la base, los nombres estan en self.base
B = self.base
B[variable entrada] = self.diccionario[variable entrada]
del B[variable salida]
# Mantenemos el orden de las variables, primero las x1, x2, x3, ... y luego las s1, s2, s3, ...
B = {k: B[k] for k in sorted(B, key=self.orden variables)}
self.base = B
```

Simplex Dual

```
inversa B = np.linalg.inv(np.array(list(B.values())).T)
# coeficientes reglon de las z para variables básicas
indices base = [self.orden variables(k) - 1 for k in B.keys()]
CB = np.array(-self.diccionario['reglon de las z'][indices base])
# Realizamos la multiplicacion
self.restricciones = np.dot(inversa B, self.diccionario['restricciones'])
self.terminos independientes = np.dot(inversa B, self.diccionario['terminos independientes'])
self.matriz[1:, 1:-1] =deepcopy(self.restricciones)
self.reglon de las z = np.dot(CB, self.matriz[1:, 1:-1]) + self.diccionario['reglon de las z']
self.matriz[0][1:-1] = deepcopy(self.reglon de las z)
self.matriz[1:, -1] = deepcopy(self.terminos independientes)
# ver en que base estamos y sacar los valores de la columna de terminos independientes
for i, k in enumerate(self.base.keys()):
    self.base[k] = deepcopy(self.diccionario[k])
self.matriz[0][-1] = np.dot(CB, self.terminos independientes)
```

Simplex Primal

Casi lo mismo...

```
columna entrada = np.argmin(self.reglon de las z)
if columna entrada < self.ni:
   variable entrada = 'x' + str(columna entrada + 1)
else:
   variable entrada = 's' + str(columna entrada - self.ni + 1)
# mascara para que las restricciones sean mayores a 0
positive mask = np.abs(self.restricciones[:, columna entrada]) > le-5
division result = np.where(
   positive mask,
   np.abs(self.terminos independientes / self.restricciones[:, columna entrada]),
   np.inf
columna salida = np.argmin(division result)
variable salida = list(self.base.keys())[columna salida]
if imprimir:
   print('Variable de entrada:', variable entrada)
   print[['Variable de salida:', variable salida]]
```

Ejemplos Simplex

```
examen_final_2023_2024 = """

argmax z = 3x1 + 2x2

3x1 + x2 <= 10

x1 + 4x2 <= 12

"""

Simplex(examen_final_2023_2024).resolucion(True)

$\sqrt{0.0s}$

Iteracion: 0

MATRIZ

[[ 1. -3. -2. 0. 0. 0.]

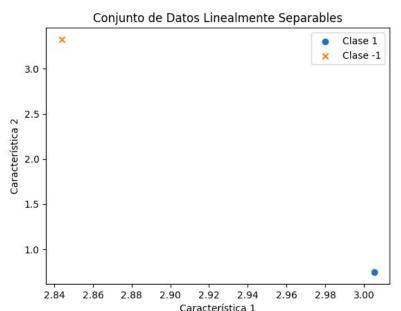
[ 0. 3. 1. 1. 0. 10.]

[ 0. 1. 4. 0. 1. 12.]]
```

```
SIMPLEX PRIMAL
Iteracion: 1
Variable de entrada: x1
Variable de salida: s1
Base: {'x1': array([3., 1.]), 's2': array([0., 1.])}
matriz:
[[ 1.
        0. -1. 1. 0.
                               10. ]
[ 0. 1. 0.333 0.333 0. 3.333]
        0. 3.667 -0.333 1. 8.667]]
Iteracion: 2
Variable de entrada: x2
Variable de salida: s2
Base: {'x1': array([3., 1.]), 'x2': array([1., 4.])}
matriz:
[[ 1.
        0. 0.
                    0.909 0.273 12.364]
[0. 1. -0. 0.364 -0.091 2.545]
 [ 0.
                    -0.091 0.273 2.364]]
(np.float64(12.363636363636363),
dict keys(['x1', 'x2']),
array([2.545, 2.364]))
```

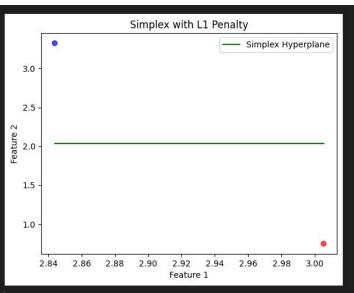
SVM

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make blobs
# Generar un conjunto de datos linealmente separable
X, y = make blobs(n samples= 2, centers=2, n features=2, random state=0)
y = 2 * y - 1
# Visualizar los datos
plt.scatter(X[y == 1][:, 0], X[y == 1][:, 1], label='Clase 1', marker='o')
plt.scatter(X[y == -1][:, 0], X[y == -1][:, 1], label='Clase -1', marker='x')
plt.title('Conjunto de Datos Linealmente Separables')
plt.xlabel('Característica 1')
plt.ylabel('Característica 2')
plt.legend()
plt.show()
```



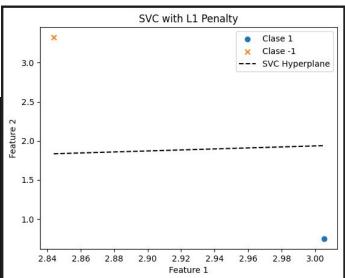
SVM SIMPLEX

```
tabla = np.stack([X[:, 0], X[:, 1], y], axis=1)
# Definir las variables y restricciones para Simplex
                                                                                       3.0
restricciones = "argmin z = 1x1 + 1x2 + 0x3 \n"
for i, fila in enumerate(tabla):
                                                                                       2.5
    z1, z2, label = fila
                                                                                     Feature 2
    restricciones += f"{label * z1}x1 + {label * z2}x2 + {label}x3 >= {1}\n"
                                                                                       2.0
maximo, variables, valores = Simplex(restricciones).resolucion(True)
dicc = dict(zip(variables, valores))
dicc = \{k: dicc.get(k, 0) \text{ for } k \text{ in } ['x1', 'x2', 'x3']\}
                                                                                       1.5
# Calcular el hiperplano de Simplex
                                                                                       1.0
w1, w2, b = dicc['x1'], dicc['x2'], dicc['x3']
# Crear el gráfico
if w2 == 0:
    x hyperplane = -b / w1
    plt.axvline(x=x hyperplane, color='green', linestyle='--', label='Decision Boundary')
    xx = np.linspace(X[:, 0].min(), X[:, 0].max(), 100)
    yy = (-w1 / w2) * xx - b / w2
    plt.plot(xx, yy, label='Simplex Hyperplane', color='green')
```

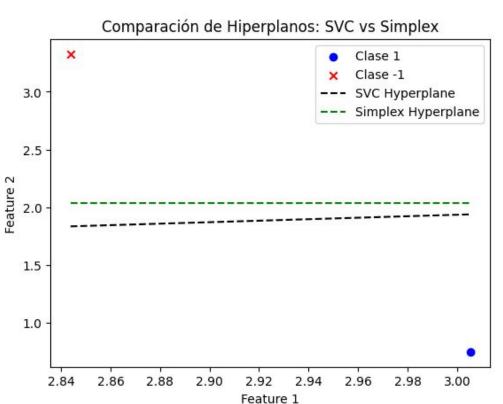


SVM SKLEARN

```
from sklearn.svm import LinearSVC as SVC
                                                                                  Feature
# Crear un clasificador SVM con regularización L1
clf = SVC(penalty='l1')
clf.fit(X, y)
                                                                                    1.5 -
# Obtener los coeficientes y el término independiente
w = clf.coef [0] # Coefficients
                                                                                    1.0
b = clf.intercept [0] # Intercept
# Crear el gráfico
plt.scatter(X[y == 1][:, 0], X[y == 1][:, 1], label='Clase 1', marker='o')
plt.scatter(X[y == -1][:, 0], X[y == -1][:, 1], label='Clase -1', marker='x')
print('Coeficientes SVC', clf.coef , 'Intercept', clf.intercept )
if w[1] == 0:
   x \text{ hyperplane} = -b / w[0]
   plt.axvline(x=x hyperplane, color='black', linestyle='--', label='Decision Boundary')
    xx = np.linspace(X[:, 0].min(), X[:, 0].max(), 100)
    yy = (-w[0] / w[1]) * xx - b / w[1]
    plt.plot(xx, yy, 'k--', label='SVC Hyperplane')
```



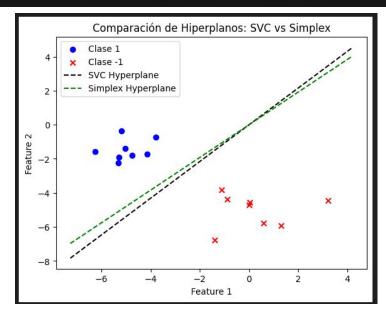
SVM Comparativa

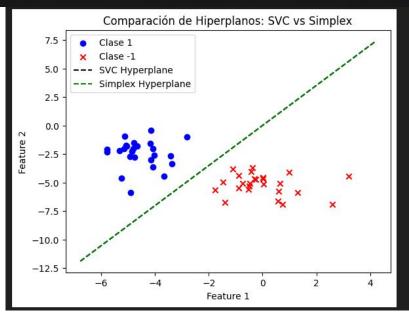


SVM Comparativa Valor Absoluto

```
restricciones = "argmin z = 1x1 + -1x2 + 1x3 + -1x4 + 0x5\n"

for i, fila in enumerate(tabla):
    z1, z2, label = fila
    restricciones += f"{label * z1}x1 + {-label * z1}x2 + {label * z2}x3 + {-label * z2}x4 + {label}x5 >= 1\n'
```





Parte 2

Ataques Adversariales

```
class MNISTModel(torch.nn.Module):
   def init (self):
        super(). init ()
        self.conv1 = torch.nn.Conv2d(1, 32, 3, 1)
        self.conv2 = torch.nn.Conv2d(32, 64, 3, 1)
        self.max pool2d = torch.nn.MaxPool2d((2, 2))
        self.dropout1 = torch.nn.Dropout2d(0.25)
        self.conv3 = torch.nn.Conv2d(64, 128, 3, 1)
        self.conv4 = torch.nn.Conv2d(128, 256, 3, 1)
        self.max pool2d2 = torch.nn.MaxPool2d((2, 2))
        self.dropout2 = torch.nn.Dropout2d(0.5)
        self.flatten = torch.nn.Flatten()
        self.fcl = torch.nn.Linear(4096, 128)
        self.dropout3 = torch.nn.Dropout(0.5)
        self.fc2 = torch.nn.Linear(128, 10)
```

Arquitectura empleada:

- Capas convolucionales:
 (entrada, filtro, tamaño, salida)
- Max Pooling: reduce el espacio de dimensión
- Dropout: apaga neuronas al azar durante el entrenamiento
- Flatten
- Linear: Capa densa

```
def forward(self, x):
    x = self.conv1(x)
    x = torch.relu(x)
    x = self.conv2(x)
    x = torch.relu(x)
    x = self.max pool2d(x)
    x = self.dropout1(x)
    x = self.conv3(x)
    x = torch.relu(x)
    x = self.conv4(x)
    x = torch.relu(x)
    x = self.max pool2d2(x)
    x = self.dropout2(x)
    x = self.flatten(x)
    x = self.fcl(x)
    x = torch.relu(x)
    x = self.dropout3(x)
    output = self.fc2(x)
    return output
```

```
# funcion de perdida y optimizador
criterion = torch.nn.CrossEntropyLoss()
optimizer = torch.optim.Adam(model.parameters(), lr=0.001)
```

CrossEntropyLoss contiene la función de activación SoftMax

Adam: método de descenso de gradiente estocástico

Activación Relu

```
X_train = X_train.reshape(-1, 1, 28, 28)
X_test = X_test.reshape(-1, 1, 28, 28)
# normalizamos
X_train = X_train / 255
X_test = X_test / 255
```

Normalizamos el dataset u lo entrenamos con torch:

- Se pone a cero los gradientes
- Se hace la predicción
- Se calcula la pérdida y los gradientes
- Se ajusta el modelo

```
model.train()
running loss = 0.0
for i, (data, target) in enumerate(train loader):
    # Move data and target to GPU
    data, target = data.to(device), target.to(device)
    # zero the parameter gradients
    optimizer.zero grad()
    # forward + backward + optimize
    output = model(data)
    loss = criterion(output, target)
    loss, backward()
    optimizer.step()
    running loss += loss.item()
print(f'Epoch: {epoch}, Training loss: {running loss / le
```

Evaluamos el modelo mientras se entrena.

Early stopping

Guardar el mejor modelo

```
model.eval()
with torch.no grad():
    test loss = 0
    correct = 0
    for data, target in test loader:
        data, target = data.to(device), target.to(device)
        output = model(data)
        test loss += criterion(output, target).item()
        pred = output.data.max(1, keepdim=True)[1]
        correct += pred.eq(target.data.view as(pred)).sum()
    test loss /= len(test loader.dataset)
    if test loss < best loss:
        best loss = test loss
        best model = model.state dict()
    print(f'Epoch: {epoch}, Test set: Average loss: {test lo
if test loss < best loss:
    intentos = 10
if test loss > best loss:
    intentos -= 1
    if intentos == 0:
        break
```

```
from prettytable import PrettyTable
def count parameters(model):
    table = PrettyTable(["Modules", "Parameters"])
    total params = 0
    for name, parameter in model.named parameters():
        if not parameter.requires grad:
            continue
        params = parameter.numel()
        table.add row([name, params])
        total params += params
    print(table)
    print(f"Total Trainable Params: {total params}")
    return total params
count parameters (model)
```

```
Modules
                 Parameters
 convl.weight
                   288
  convl.bias
                    32
 conv2.weight
                  18432
  conv2.bias
                    64
 conv3.weight
                  73728
  conv3.bias
                   128
 conv4.weight
                  294912
  conv4.bias
                   256
  fc1.weight
                  524288
   fc1.bias
                   128
  fc2.weight
                   1280
   fc2.bias
                    10
Total Trainable Params: 913546
```

2. SVM Núcleo gaussiano

```
# importamos SVM
   from sklearn.svm import SVC
   from sklearn.metrics import accuracy score
   from tensorflow.keras.datasets import mnist
   classifier = SVC(kernel='rbf', random state=42)
   # Cargamos el dataset
   (x train, y train), (x test, y test) = mnist.load data()
   x train = x train / 255.00
   x \text{ test} = x \text{ test} / 255.0
   # entrenamos con mnist
   classifier.fit(x train.reshape(-1, 784), y train)
   y pred svm = classifier.predict(x test.reshape(-1, 784))
   print(f'Accuracy: {accuracy score(y test, y pred svm) * 100:.2f}%')

√ 3m 54.9s

Accuracy: 97.92%
```

Kernel trick permite el uso de clasificadores lineales para problemas no lineales

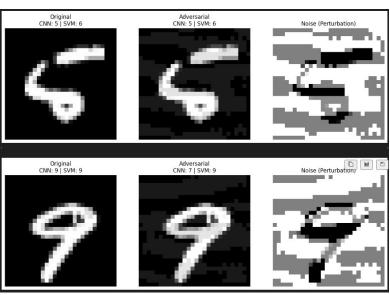
3. Ataques FGSM

```
def fgsm_attack(image, epsilon, data_grad):
    # Obtener el signo del gradiente
    sign_data_grad = data_grad.sign()
    # Crear la imagen perturbada
    perturbed_image = image + epsilon * sign_data_grad
    # Clip values to [0, 1]
    perturbed_image = torch.clamp(perturbed_image, 0, 1)
    return perturbed_image
```

El objetivo es maximizar la pérdida. Encuentra cuánto contribuye cada pixel al valor de pérdida y agrega una perturbación en consecuencia

4. Generación de algunas imágenes adversariales

```
epsilon = 0.3
model.eval()
for i in range(total samples):
    # Obtenemos la imagen v la etiqueta
    image = X test[i].unsqueeze(0).to(device)
   label = y test[i].item()
   # Predecimos la imagen original con el modelo CNN
    output = model(image)
    , cnn original pred = torch.max(output, 1)
   cnn original pred = cnn original pred.item()
    # Se calcula el gradiente de la imagen
    image.requires grad = True
    output = model(image)
   loss = criterion(output, torch.tensor([label]).to(device))
   loss.backward()
    data grad = image.grad.data
    # Generamos la imagen adversaria
   perturbed image = fgsm attack(image, epsilon, data grad)
    # Predecimos con el modelo CNN
    output = model(perturbed image)
    , cnn adversarial pred = torch.max(output, 1)
   cnn adversarial pred = cnn adversarial pred.item()
    # Predecimos con el modelo SVM
   svm original pred = classifier.predict(image.detach().numpy().reshape(1, 784))[0]
   sym adversarial pred = classifier.predict(perturbed image.cpu().detach().reshape(1, 784))[0]
   visualize images(image, perturbed image, cnn original pred, cnn adversarial pred, svm original pred,
    aciertos += cnn adversarial pred == label
print(f'Aciertos: {aciertos}/{total samples}')
```



5. Error adversariales

```
if not os.path.exists('adversariales'):
    os.makedirs('adversariales')
    for i in range(10):
        os.makedirs(f'adversariales/{i}')
epsilon = 0.1
model.eval()
for i, (data, target) in enumerate(tgdm(test loader)):
    data, target = data.to(device), target.to(device)
    data.requires grad = True
    output = model(data)
    loss = criterion(output, target)
    model.zero grad()
    loss.backward()
    data grad = data.grad.data
    perturbed data = fgsm attack(data, epsilon, data grad)
    for j in range(perturbed data.size(0)):
        clase = target[j].item()
        save image(perturbed data[j], f'adversariales/{clase}/adversarial {i * test loader.batch size + j}.png')
```

5. Error adversariales

```
if not os.path.exists('adversariales'):
    os.makedirs('adversariales')
    for i in range(10):
        os.makedirs(f'adversariales/{i}')
epsilon = 0.1
model.eval()
for i, (data, target) in enumerate(tgdm(test loader)):
    data, target = data.to(device), target.to(device)
    data.requires grad = True
    output = model(data)
    loss = criterion(output, target)
    model.zero grad()
    loss.backward()
    data grad = data.grad.data
    perturbed data = fgsm attack(data, epsilon, data grad)
    for j in range(perturbed data.size(0)):
        clase = target[j].item()
        save image(perturbed data[j], f'adversariales/{clase}/adversarial {i * test loader.batch size + j}.png')
```

5. Error adversariales

```
cnn adversarial preds = []
  svm adversarial preds = []
  path = 'adversariales'
  adversarial dataset = torchvision.datasets.ImageFolder(path,
      transform=transforms.Compose([transforms.ToTensor(),
                transforms.Grayscale(num output channels=1)])
  adversarial loader = torch.utils.data.DataLoader(adversarial dataset, batch size=1000, shuffle=False)
  y test = adversarial dataset.targets
   model.eval()
   for data, target in tqdm(adversarial loader):
       data, target = data.to(device), target.to(device)
      with torch.no grad():
          adversarial pred cnn = model(data).argmax(1)
          cnn adversarial preds.extend(adversarial pred cnn.cpu().numpy())
      adversarial pred sym = classifier.predict(data.cpu().detach().numpy().reshape(-1, 784))
       svm adversarial preds.extend(adversarial pred svm)
   # Calculate the accuracy on adversarial images
  cnn adversarial accuracy = accuracy score(y test, cnn adversarial preds)
  svm adversarial accuracy = accuracy score(y test, svm adversarial preds)
  print(f'CNN Adversarial Accuracy: {cnn adversarial accuracy * 100:.2f}%')
  print(f'SVM Adversarial Accuracy: {svm adversarial accuracy * 100:.2f}%')
 √ 1m 1.3s
              | 10/10 [01:01<00:00, 6.13s/it]
CNN Adversarial Accuracy: 84.95%
SVM Adversarial Accuracy: 88.94%
```

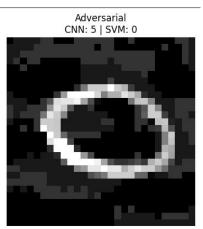
6. Entrena adversarialmente

```
model.eval()
  cnn preds = 0
  svm preds = 0
  with torch.no grad():
      for data, target in tgdm(test loader all):
          data, target = data.to(device), target.to(device)
          output = model(data)
          pred = output.argmax(1)
          svm pred = classifier.predict(data.cpu().detach().numpy().reshape(-1, 784))
          cnn preds += pred.eq(target).sum().item()
          svm preds += np.sum(svm pred == target.cpu().numpy())
  cnn accuracy = cnn preds / len(test loader all.dataset)
  svm accuracy = svm preds / len(test loader all.dataset)
  print(f'CNN Accuracy: {cnn accuracy * 100:.2f}%')
  print(f'SVM Accuracy: {svm accuracy * 100:.2f}%')
 / 1m 56.1s
              | 20/20 [01:56<00:00, 5.81s/it]
CNN Accuracy: 98.44%
SVM Accuracy: 93.69%
```

```
path = 'adversariales train'
path test = 'adversariales'
image transform = transforms.Compose([
    transforms.ToTensor(),
   transforms.Grayscale(num output channels=1)
label transform = transforms.Lambda(lambda y: torch.tensor(y, dtype=torch.int64))
adversarial dataset = torchvision.datasets.ImageFolder(
   transform=image transform,
   target transform=label transform
adversarial dataset test = torchvision.datasets.ImageFolder(
    path test,
   transform=image transform,
   target transform=label transform
(x train original, y train original), (x test original, y test original) = mnist.load data()
x train original = x train original / 255.0
x test original = x test original / 255.0
x train original = torch.tensor(x train original, dtype=torch.float32).unsqueeze(1) # Shape
x test original = torch.tensor(x test original, dtype=torch.float32).unsqueeze(1)
y train original = torch.tensor(y train original, dtype=torch.int64)
y test original = torch.tensor(y test original, dtype=torch.int64)
train original = TensorDataset(x train original, y train original)
test original = TensorDataset(x test original, y test original)
train dataset all = ConcatDataset([train original, adversarial dataset])
test dataset all = ConcatDataset([test original, adversarial dataset test])
train loader all = DataLoader(train dataset all, batch size=1000, shuffle=True)
test loader all = DataLoader(test dataset all, batch size=1000, shuffle=False)
```

6. Vs Doblemente adversariales





FIN