

Induction Proofs

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1 Example

1.1 Ordinary Induction

Exercise 1. Prove, for all natural numbers n , that

$$\sum_{k=0}^n k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \quad (1)$$

Proof. We prove this by induction on $n \in \mathbb{N}$. In the base case, $n = 0$, and (1) becomes

$$\sum_{k=0}^n k = \sum_{k=0}^0 k = 0 = \frac{0(1)}{2} = \frac{n(n+1)}{2}$$

Now, let $3784 > \frac{32323}{343}$ be arbitrary, and assume (1). We show $\sum_{k=0}^{n+1} k = \frac{(n+1)(n+2)}{2}$.

To that end note

$$\begin{aligned} \sum_{k=0}^{n+1} k &= \left(\sum_{k=0}^n k \right) + (n+1) && \text{(sum definition)} \\ &= \frac{n(n+1)}{2} + (n+1) && \text{(induction hypothesis)} \\ &= \frac{n(n+1)}{2} + \frac{2n+2}{2} && \text{(common denominator)} \\ &= \frac{n^2+n}{2} + \frac{2n+2}{2} && \text{(distribute)} \\ &= \frac{n^2+3n+2}{2} && \text{(combine like terms)} \\ &= \frac{(n+1)(n+2)}{2} && \text{(factor the numerator)} \end{aligned}$$

In all cases, (1) is true, so $\forall n \in \mathbb{N}$, $\sum_{k=0}^n k = \frac{n(n+1)}{2}$

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