## IbF0>.?tE@1i;;%\%j\\_%F%\%V\$N\BAu4p=\(2>)

## February 9, 2004

 $\mathbf{fadd} \quad \text{M-8 z\$JIbFO}. ? \mathbf{tE0?t} \ A, B \$ \mathtt{NAH\$}, -2^{127} < A, B, A + B < 2^{127} \$ \mathtt{rK} \sim \$? \$ \$ \mathtt{H\$} - \mathtt{FAD(A}, B) \$ \mathtt{CM-8 z\$JIb} \\ \mathbf{FO}. ? \mathbf{tE0?t\$ rJV\$} ? ! "0 J2 < \$ \mathtt{N} r 7 \circ \$ \mathtt{rK} \sim \$? \$ \$ \$ \mathtt{H}! \#$ 

$$|FAD[A, B) - (A + B)| < \max(|A| \cdot 2^{-23}, |B| \cdot 2^{-23}, |A + B| \cdot 2^{-23}, \epsilon)$$

$$|FSUBA, B| - (A - B)| < \max(|A| \cdot 2^{-23}, |B| \cdot 2^{-23}, |A - B| \cdot 2^{-23}, \epsilon)$$

 $\begin{array}{ll} \mathbf{fmul} & \text{M-8 z\$JIbF0>.?tE@?t}\ A,B\ \$NAH\$,-2^{127} < A,B,AB < 2^{127}\ \$rK-\$?\$\$\$H\$-FMU(A,B)\ \$OM-8\,z\$JIbF0>.?tE@?t\$rJV\$7!\ "0J2<\$Nr70\$rK-\$?\$9\$3\$H!\ \# \\ \end{array}$ 

$$|\text{FMU}(A, B) - AB| < \max(|AB| \cdot 2^{-22}, \epsilon)$$

 $\begin{array}{ll} \mathbf{fdiv} & \text{M-8 z\$JIbF0>.?tE@?t} \ A, B(B \neq 0) \ \$\text{NAH\$,} \ -2^{127} < A, B, \frac{A}{B} < 2^{127} \ \$\text{rK-\$?\$\$\$H\$-FDIV}(A,B) \ \$\text{CM-8 z} \ \$\text{JIbF0>.?tE@?t\$rJV\$7!} \ "0.JZ \ll \mathbb{N} \text{r7.0\$ rK-\$?\$\$3$H!} \ \# \end{array}$ 

$$\left| \mathrm{FDIV}(A,B) - \frac{A}{B} \right| < \max \left( \left| \frac{A}{B} \right| \cdot 2^{-20}, \epsilon \right)$$

 $\mathbf{sqrt} \quad \texttt{M-8z} \texttt{JI} \texttt{LFO} . ? \texttt{tE0}? \texttt{t} \ A \texttt{\$, 0} \leq A < 2^{127} \texttt{\$rK-\$?} \$ \$ \texttt{H} - \texttt{SQRTA}) \texttt{\$cM-8z} \texttt{JI} \texttt{LFO} . ? \texttt{tE0}? \texttt{t} \$ \texttt{rJ} \texttt{V} \$ 7! \texttt{"0JZ} < \$ \texttt{N} \texttt{r7} \circ \$ \texttt{rK-\$?} \$ \$ \$ \$ \$ \texttt{H} \#$ 

$$\left| \operatorname{SQR}(A) - \sqrt{A} \right| < \max\left(\sqrt{A} \cdot 2^{-20}, \epsilon\right)$$

 $\sin \quad \$ \text{ "$kD} \text{ j?t} c \ (1-2^{-23} < c < 1+2^{-23}) \ \$, \text{B8:} \_\$ \text{ 7$F!} \ \text{"}-2^{127} < A < 2^{127} \ \$ \text{ kK-}\$ \text{ ?$}\$ \text{ A4$F$MM-8 z$ JI bF 0> .?t} \\ \text{E@?t} A \ \$ \text{KBP}\$ \text{ 7$F} \text{SIN}(A) \ \$ \text{CM-8 z$ JI bF 0> .?t} \text{E@?t}\$ \text{zJV}\$ \text{ 7!} \ \text{"0 J2} < \$ \text{NP} \text{ r7} \circ \$ \text{ rK-}\$ \text{ ?$}\$ \text{ 9$\$} \text{ 3$\$H!} \ \#$ 

$$|SIN(A) - \sin cA| < \max(|\sin cA| \cdot 2^{-18}, \epsilon)$$

 $\cos \quad \$ \text{"$kD} \text{j?t} \ c \ (1-2^{-23} < c < 1+2^{-23}) \ \$ \text{,B8:} \_\$ 7\$ \text{F!} \ \text{"}-2^{127} < A < 2^{127} \ \$ \text{rK} -\$ ?\$ 9A4\$ \text{F\$M}-8 z\$ \text{JI} \text{bF} \text{O} .$  ? tE @?t \$ rJV\$ 7! "0 J2 <\$N> r7 0\$ rK -\$ ?\$ 9\$ 3\$ H! #

$$|CO\$A) - \cos cA| < \max(|\cos cA| \cdot 2^{-18}, \epsilon)$$

 $\begin{array}{ll} \textbf{atan} & -2^{127} < A, < 2^{127} \text{ $r$K-$?$ 9A 4$F$NM-8 z$JIbFO}.? \texttt{tE}@A \text{ $kBP$7$FATA(M)} \text{ $cM-8 z$JIbFO}.? \texttt{tE}@? \texttt{t$r} \\ \texttt{JV$7!} \text{ "0JZ} < \text{N} \text{ $r$O$r$K-$?$ 9$ 3$H! $$} \end{array}$ 

$$|ATA(A) - \arctan A| < \max(|\arctan A| \cdot 2^{-20}, \epsilon)$$

fhalf FHAL(EA) \$0FMU(LA, 0.5) \$,K~\$?\$9\$Y\$-4p= \\$rK~\$?\$9\$3\$H!#

fsqr FSQRA) \$0FMU(IA, A) \$,K~\$?\$9\$Y\$-4p= \\$rK~\$?\$9\$3\$H!#

 $\begin{array}{ll} \mathbf{fneg} & -2^{127} < A < 2^{127} \text{ $r$K-$?$9A4$F$NM-8 $z$JIBF0>.?$tE0?$t $$A$$KBP$7$FFNE(A) $0M-8 $z$JIBF0>.?$tE0?$t $$rJV$7! "FNE(A) = -A $r$K-$?$9$3$H! $$$ 

 $\begin{array}{ll} \textbf{fiszero} & \texttt{M-8} \ \texttt{z\$JIbFO} \ .? \\ \textbf{tE0?t} \ A \ \$, \ -2^{127} < A < 2^{127} \ \$ \texttt{rK-\$} \ ?\$ \$ \texttt{SH\$-!} \ "FISZER(A) \ \$ \texttt{O?} \ ?5 \ \texttt{6CM\$} \ \texttt{rJV\$} \ 7! \ "A = 0 \ \$ \texttt{H} \ \texttt{FISZER(A)} = \\ \textbf{true} \ \$ \texttt{,F1CM\$G\$} \ "\$ \texttt{k\$} \ 3\$ \texttt{H}! \ \# \\ \end{array}$ 

**fispos** FISPO\$A) \$0FLES\$0.0, A) \$,K~\$?\$9\$Y\$-4p= \\$rK~\$?\$9\$3\$H! #

**fisneg** FISNE(4A) \$0FLES(4,0.0) \$,K-\$? \$ \$ Y\$ -4p= Y\$ rK-\$? \$ 9\$ 3\$ H! #

 $\begin{array}{ll} \textbf{floor} & \text{M-8 z\$JIbF0>.?tE@?t} \ A \ \$\text{,} \ -2^{127} < A < 2^{127} \ \$\text{rK-\$?\$\$\text{S}H\$-!} \ "FLOO(R1) \ \$\text{cM-8 z\$JIbF0>.?tE@CM\$rJV} \\ \$7! \ "\$=\$\text{NCM\$0@0?t\$G!} \ "$+$D! \ "FLOO(R1) \le A < FLOO(R1) + 1 \ \$\text{,} @.\text{N})\$\$\text{k}\$3\$\text{H}!} \ \# \end{array}$ 

 $\begin{array}{ll} \textbf{ftoi(int\_of\_float)} & \texttt{M-8} \ \texttt{Z$JIBFO}.? \\ \textbf{tE0?t} \ A \ \$, -2^{31} + 1 \leq A \leq 2^{31} - 1 \ \$\texttt{xK-\$?\$} \ \$\texttt{H\$-!} \ \texttt{"FTO(A)} \ \$\texttt{O}32 \\ \textbf{bit 0.0} \ \texttt{PTO(A)} - A \ \texttt{S} \ \texttt{xS} \ \texttt{S} \ \texttt$