Homogeneous D.Es.

A differential Equation (D.E) of the form

$$\boxed{\frac{dy}{dx} = \frac{f(x,y)}{\phi(x,y)}} \longrightarrow (*)$$

is called a "homogeneous equation" if each term of $f(x,y)$ and $\phi(x,y)$ are homogeneous equations of the same ~~order~~ degree.

Def:

Homogeneous functions: A function $f(x,y)$ is said to be homogeneous function of degree K (in x and y) if and only if (iff) ($\Leftrightarrow$)

$$\boxed{f(\lambda x, \lambda y) = \lambda^K f(x,y)}$$

Ex: $f(x,y) = \sqrt{x+4y}$

$$f(\lambda x, \lambda y) = \sqrt{\lambda x + 4\lambda y} = (\lambda)^{1/2}\sqrt{x+4y} =$$

$$\Rightarrow \boxed{f(\lambda x, \lambda y) = \lambda^{1/2} f(x,y)} \quad \boxed{\lambda > 0,} \boxed{K \in \mathbb{R}}$$

$\therefore$ $f(x,y)$ is homogeneous of degree $1/2$.

Working Rule

put $\boxed{y = vx} \longrightarrow (1)$

$$\frac{dy}{dx} = v\cdot(1) + x\cdot\frac{dv}{dx}$$

$$\boxed{\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}} \longrightarrow (2)$$

put (1) & (2) in (*), we get

$$\boxed{v + x\cdot\frac{dv}{dx} = \frac{f(x,vx)}{\phi(x,vx)}}$$

By using Variable separable method. we get the solution of eq (*).

Problems

(P1) Solve $(x^2+xy)\,dy = (x^2+y^2)\,dx$

(S) Given $(x^2+xy)\,dy = (x^2+y^2)\,dx$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy} \longrightarrow (1)$$

Clearly (1) is homogeneous D.E. So.

Let $y = vx \Rightarrow \boxed{\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}} \rightarrow (2)$

substituting (2) in (1), we get

$$(1) \Rightarrow \quad v + x\cdot\frac{dv}{dx} = \frac{x^2+(vx)^2}{x^2+x(vx)}$$

$$\Rightarrow \quad v + x\cdot\frac{dv}{dx} = \frac{\cancel{x^2}(1+v^2)}{\cancel{x^2}(1+v)}$$

$$\Rightarrow \quad x\cdot\frac{dv}{dx} = \frac{1+v^2}{1+v} - v$$

$$\Rightarrow \quad x\cdot\frac{dv}{dx} = \frac{1+v^2-v-v^2}{1+v}$$

$$\Rightarrow \quad x\cdot\frac{dv}{dx} = \frac{1-v}{1+v}$$

$$\Rightarrow \quad \int \frac{1+v}{1-v}\,dv = \int \frac{1}{x}\,dx$$

$$\because \int \frac{f'(x)}{f(x)}\,dx = \log f(x) + c$$

$$\Rightarrow \quad \int\left(\frac{1}{1-v} + \frac{v}{1-v}\right)dv = \log x + \log c$$

$$\Rightarrow \quad \int \frac{1}{1-v}\,dv + \int \frac{v}{1-v}\,dv = \log(xc)$$

$$\Rightarrow \quad -\log(1-v) - \int \frac{v}{v-1}\,dv = \log(xc)$$

$$\Rightarrow \quad -\log(1-v) - \left[\int \frac{v-1+1}{v-1}\,dv\right] = \log(xc)$$

$$\Rightarrow \quad -\log(1-v) - \left[\int dv + \int \frac{1}{v-1}\,dv\right] = \log(xc)$$

$$\Rightarrow -\cancel{\log(1-v)} - v - \log(v-1) = \log(xC)$$

$$\Rightarrow -v - \log(v-1) = \log(xC) + \log(1-v)$$

$$\Rightarrow v + \log(v-1) + \log(xC) + \log(1-v) = 0$$

$$\Rightarrow v + \log\left[(v-1)(xC)(1-v)\right] = 0$$

$$\Rightarrow v + \log\left[\left(y/x - 1\right)(xC)(1 - y/x)\right] = 0$$

$$\Rightarrow v + \log\left[\cancel{x}C\frac{(y-x)}{\cancel{x}}\frac{(x-y)}{x}\right] = 0$$

$$\Rightarrow v + \log\left[\frac{C}{x}(y-x)(x-y)\right] = 0$$

$$\Rightarrow \frac{y}{x} + \log\left[\frac{(x-y)^2}{x}\cdot(-C)\right] = 0$$

$$\Rightarrow \frac{y}{x} + \log\left(\frac{(x-y)^2}{x}\cdot C\right) = 0$$

$$\Rightarrow \boxed{\frac{y}{x} + 2\log(x-y) \mp \log\left(\frac{c}{x}\right) = 0}$$

(P) Solution of $x\frac{dy}{dx} = y(\log y - \log x + 1)$ which of the following are true

(a) $y = xe^{Cx}$ (b) $y = e^{Cx}$ (c) $y = xA^{x}$ (d) $y = xe^{x^2 C}$

(5). $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$$\frac{x}{y}\frac{dy}{dx} = \log y - \log x + 1$$

$$\frac{x}{y}\cdot\frac{dy}{dx} = \log\left(\frac{y}{x}\right) + 1$$

$$\frac{dy}{dx} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right) + 1\right] \longrightarrow (1)$$

put $y = vx \Rightarrow \boxed{\frac{y}{x} = v}$

$$\Rightarrow \boxed{\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}} \longrightarrow (2)$$

substituting (2) in (1),

$$(1) \Rightarrow v + x\cdot\frac{dv}{dx} = v\left[\log(v) + 1\right]$$

$$x\cdot\frac{dv}{dx} = v\left[\log(v) + 1\right] - v$$

$$x\frac{dv}{dx} = v\log v + v - v$$

$$x\frac{dv}{dx} = v\log v$$

$$\int \frac{1}{v\log v}\,dv = \int \frac{1}{x}\,dx$$

$$\Rightarrow \int \left(\frac{1}{v}\right)\Big/\log v \; dv = \log x + \log c$$

$$\Rightarrow \log(\log v) = \log(xC)$$

$$\Rightarrow \log v = xC$$

$$\Rightarrow v = e^{xC}$$

$$\Rightarrow \frac{y}{x} = e^{xC} \qquad [\because \text{from } y = vx]$$

$\boxed{A = e^{C}}$

$$\Rightarrow \boxed{y = xe^{xC}} \text{ and } y = x(e^{C})^{x}$$

$$\Rightarrow \boxed{y = xA^{x}}$$

(P) Solve the differential equation $(x^2+4y^2+xy)dx - x^2dy = 0$

(S) Given $(x^2+4y^2+xy)dx - x^2dy = 0$

$\Rightarrow (x^2+4y^2+xy)dx = x^2dy$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{x^2+4y^2+xy}{x^2}} \longrightarrow (1)$$

Let $y = vx \Rightarrow \boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}} \longrightarrow (2)$

sub. (2) in (1), we get

$$v + x.\frac{dv}{dx} = \frac{x^2+4(vx)^2+x(vx)}{x^2}$$

$$v + x.\frac{dv}{dx} = \frac{x^2(1+4v^2+v)}{x^2}$$

$$v + x.\frac{dv}{dx} = 1+4v^2+v$$

$$x.\frac{dv}{dx} = 1+4v^2$$

$$\frac{1}{1+4v^2}dv = \frac{1}{x}dx$$

$$\int \frac{1}{1+4v^2}dv = \int \frac{1}{x}dx$$

$$\int \frac{1}{1+(2v)^2}dv = \log x + \log c$$

$$\Rightarrow \int \frac{1}{1+x^2}\left(\frac{dx}{2}\right) = \log(xc)$$

By put $2v = x$

$2\frac{dv}{dx} = 1 \Rightarrow \boxed{dv = \frac{dx}{2}}$

$$\Rightarrow \frac{1}{2}\int \frac{1}{1+x^2}dx = \log(xc)$$

$\because \int \frac{1}{1+x^2}dx = \tan^{-1}(x)$

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{x}{1}\right) = \log xc$$

$$\Rightarrow \frac{\tan^{-1}(2v)}{2} = \log xc$$

$$\Rightarrow \tan^{-1}(2v) = 2\log xc$$

$$\Rightarrow \tan^{-1}(2(y/x)) = 2\log xc$$

$$\Rightarrow \boxed{\tan^{-1}\left(\frac{2y}{x}\right) = 2\log xc}$$

P) Solve the initial value problem $(3xy+y^2)dx+(x^2+xy)dy=0$, $y(1)=1$

S) Given D.E.

$$(3xy+y^2)dx+(x^2+xy)dy=0$$

$$\boxed{\frac{dy}{dx}=\frac{-(3xy+y^2)}{x^2+xy}} \rightarrow (1)$$

Let $y=vx$

$$\boxed{\frac{dy}{dx}=v+x\cdot\frac{dv}{dx}} \rightarrow (2)$$ substituting (2) in (1), we get,

$$(1) \Rightarrow v+x\cdot\frac{dv}{dx}=\frac{-[3x(vx)+(vx)^2]}{x^2+x(vx)}$$

$$\Rightarrow v+x\cdot\frac{dv}{dx}=\frac{-x^2(3v+v^2)}{x^2(1+v)}$$

$$\Rightarrow v+x\cdot\frac{dv}{dx}=\frac{-(3v+v^2)}{1+v}$$

$$\Rightarrow x\cdot\frac{dv}{dx}=\frac{-(3v+v^2)-v(1+v)}{1+v}$$

$$\Rightarrow x\cdot\frac{dv}{dx}=\frac{-3v-v^2-v-v^2}{1+v}$$

$$\Rightarrow x\cdot\frac{dv}{dx}=\frac{-4v-2v^2}{1+v}$$

$$\Rightarrow x\cdot\frac{dv}{dx}=\frac{-2v(2+v)}{1+v}$$

$$\Rightarrow \frac{1+v}{v(2+v)}dv=\frac{-2}{x}dx$$

$$\Rightarrow \int\frac{1+v}{2+v}dv=-\int\frac{2}{x}dx$$

$$\Rightarrow \int\frac{1}{2+v}dv+\int\frac{v}{v+2}dv=-2\log x+\log c$$

$$\Rightarrow \log(v+2)+\int\frac{v+2-2}{v+2}dv=$$

$$\frac{1+v}{v(2+v)}=\frac{A}{v}+\frac{B}{2+v}$$

$$1+v=A(v+2)+Bv$$

$v \rightarrow 1=A+B$

const. $\rightarrow 1=2A \Rightarrow \boxed{A=\frac{1}{2}}$

$\boxed{B=\frac{1}{2}}$

$$\Rightarrow \frac{1+v}{v(2+v)}dv=\frac{-2}{x}dx$$

$$\int\frac{1+v}{v(2+v)}dv=\int\frac{-2}{x}dx$$

$$\int\left(\frac{\left(\frac{1}{2}\right)}{v}+\frac{\left(\frac{1}{2}\right)}{2+v}\right)dv=-2\log x+\log c$$

$$\frac{1}{2}\left[\int\frac{1}{v}dv+\int\frac{1}{v+2}dv\right]=-2\log x+\log c$$

$$\Rightarrow \frac{1}{2}[\log v+\log(v+2)]=-2\log x+\log c$$

$$\Rightarrow \log v+\log(v+2)=-4\log x+\log c$$

$$\Rightarrow \log(v(v+2))=-4\log x+\log c$$

$$\Rightarrow \log(v(v+2))=\log\left(\frac{1}{x^4}\right)+\log c$$

$$\Rightarrow \boxed{v(v+2)=\frac{c}{x^4}} \rightarrow (3)$$

substitute $\boxed{v=y/x}$ in (3),

$$(3) \Rightarrow \frac{y}{x}(y/x+2)=\frac{c}{x^4}$$

$$\frac{y}{x}\left(\frac{y+2x}{x}\right)=\frac{c}{x^4}$$

$$\Rightarrow \boxed{y(y+2x)=\frac{c}{x^2}}$$

Given $y(1)=1 \Rightarrow$

$$1(1+2(1))=\frac{c}{1} \Rightarrow \boxed{c=3}$$

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$\therefore$ The required solution is

$$\boxed{y(y+2x) = \frac{B}{x^2}}$$

Equation Reducible to Homogeneous equation:-

Consider $\boxed{\dfrac{dy}{dx} = \dfrac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}} \longrightarrow (1)$ If $\boxed{\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}}$ then

put $\Longrightarrow$ $\boxed{x = X+h \quad \text{and} \quad y = Y+k}$

$dx = dX \quad \text{and} \quad dy = dY$

$(1) \Rightarrow$

$$\frac{dY}{dX} = \frac{a_1(X+h)+b_1(Y+k)+c_1}{a_2(X+h)+b_2(Y+k)+c_2}$$

$$= \frac{a_1X+b_1Y+(a_1h+b_1k+c_1)}{a_2X+b_2Y+(a_2h+b_2k+c_2)} \longrightarrow (2)$$

We chose h and k, so that eqn (2) is homogeneous

$$\Rightarrow \frac{dY}{dX} = \frac{a_1X+b_1Y+(a_1h+b_1k+c_1)}{a_2X+b_2Y+(a_2h+b_2k+c_2)}$$

Taking; $\boxed{\begin{aligned} a_1h+b_1k+c_1=0 \\ a_2h+b_2k+c_2=0 \end{aligned}}$

$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

Case(i):- $\boxed{\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} \Rightarrow a_1b_2 - a_2b_1 \neq 0}$

Then $\boxed{\dfrac{dY}{dX} = \dfrac{a_1X+b_1Y}{a_2X+b_2Y}}$ which is homogeneous

Ex: $\dfrac{dy}{dx} = \dfrac{x-2y+5}{2x+y-1} \longrightarrow (1)$

clearly $\dfrac{1}{2} \neq \dfrac{-2}{1} \quad \left[\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}\right]$

$\therefore$ Case (i) is satisfied

Now put $x = X+h$ and $y = Y+k$

$dx = dX$ and $dy = dY$

$$(1) \Rightarrow \frac{dY}{dX} = \frac{X+h-2(Y+k)+5}{2(X+h)+Y+k-1} = \frac{X+h-2Y-2k+5}{2X+2h+Y+k-1}$$

$$\Rightarrow \boxed{\frac{dY}{dX} = \frac{X-2Y+(h-2k+5)}{2X+Y+(2h+k-1)}} \longrightarrow (2)$$

By taking, $h-2K+5=0$, $2h+K-1=0$ in ②, we get

$$② \Rightarrow \boxed{\frac{dY}{dX} = \frac{X-2Y}{2X+Y}} \longrightarrow ③$$

solving the equations

$2\times(h-2K+5)=0$

$2h+K-1=0$

$(-)$ $-5K+11=0 \Rightarrow \boxed{K=+\frac{11}{5}}$

$2h+\frac{11}{5}-1=0 \Rightarrow 2h+\frac{6}{5}=0$

$\Rightarrow \boxed{h=\frac{-3}{5}}$

clearly ③ is homogeneous D.E. So

put $Y=VX \Rightarrow \frac{dY}{dX} = V+X\cdot\frac{dV}{dX}$

$\therefore x = X+h$

$\Rightarrow \boxed{X = x+\frac{3}{5}}$

$Y = y-K$

$\Rightarrow \boxed{Y = y-\frac{11}{5}}$

$$\therefore ③ \Rightarrow \frac{dY}{dX} = \frac{X-2Y}{2X+Y}$$

$$\Rightarrow V+X\cdot\frac{dV}{dX} = \frac{X-2(VX)}{2X+VX}$$

$$\Rightarrow V+X\cdot\frac{dV}{dX} = \frac{X(1-2V)}{X(2+V)}$$

$$\Rightarrow V+X\cdot\frac{dV}{dX} = \frac{1-2V}{2+V}$$

$$\Rightarrow X\cdot\frac{dV}{dX} = \frac{1-2V}{2+V}-V$$

$$\Rightarrow X\cdot\frac{dV}{dX} = \frac{1-2V-2V-V^2}{2+V}$$

$$\Rightarrow X\cdot\frac{dV}{dX} = \frac{1-V^2-4V}{V+2}$$

$$\Rightarrow \frac{V+2}{1-V^2-4V}\,dV = \frac{1}{X}\,dX$$

$$\Rightarrow \int\frac{V+2}{1-V^2-4V}\,dV = \int\frac{1}{X}\,dX$$

$$\Rightarrow \frac{-1}{2}\int\frac{-2V-4}{1-V^2-4V}\,dV = \log X+\log C$$

$$\Rightarrow \boxed{\frac{-1}{2}\log(1-4V-V^2) = \log XC}$$

Now replace $V=\frac{Y}{X}$,

$$\Rightarrow \frac{-1}{2}\log\left[1-\frac{4Y}{X}-\left(\frac{Y}{X}\right)^2\right] = \log XC$$

$$\Rightarrow \frac{-1}{2}\log\left[1-\frac{4Y}{X}-\frac{Y^2}{X^2}\right] = \log XC$$

$$\Rightarrow \frac{-1}{2}\log\left[\frac{X^2-4XY-Y^2}{X^2}\right] = \log XC$$

$$\Rightarrow \log\left[\left(\frac{X^2-4XY-Y^2}{X^2}\right)^{-1/2}\right] = \log XC$$

$$\Rightarrow \log\left[\frac{(X^2)^{1/2}}{\sqrt{X^2-4XY-Y^2}}\right] = \log XC$$

$$\Rightarrow \log\left[\frac{X}{\sqrt{X^2-4XY-Y^2}}\right] = \log XC$$

$$\Rightarrow \log X - \log\left(\sqrt{X^2-4XY-Y^2}\right) = \log X+\log C$$

$$\Rightarrow -\log\left(\sqrt{X^2-4XY-Y^2}\right) = \log C$$

$$\Rightarrow \log\left(\frac{1}{\sqrt{X^2-4XY-Y^2}}\right) = \log C$$

$$\Rightarrow \frac{1}{\sqrt{X^2-4XY-Y^2}} = C$$

$$\Rightarrow \frac{1}{\sqrt{(x+\frac{3}{5})^2-4(x+\frac{3}{5})(y-\frac{11}{5})-(y-\frac{11}{5})^2}} = 0$$

$\left[\because X = x+\frac{3}{5},\ Y = y-\frac{11}{5}\right]$

Case(ii): Consider $\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2} \longrightarrow (*)$

When $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \lambda,} \Rightarrow \boxed{\begin{array}{l} a_1 = a_2\lambda \\ b_1 = b_2\lambda \end{array}}$

$$(*) \Rightarrow \frac{dy}{dx} = \frac{a_2\lambda x + b_2\lambda y + c_1}{a_2x+b_2y+c_2} = \frac{\lambda(a_2x+b_2y)+c_1}{a_2x+b_2y+c_2}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\lambda(a_2x+b_2y)+c_1}{a_2x+b_2y+c_2}} \longrightarrow (1)$$

put $a_2x+b_2y = t$ in (1).

$$\Rightarrow a_2 + b_2 \cdot \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow b_2 \cdot \frac{dy}{dx} = \frac{dt}{dx} - a_2$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{b_2}\left(\frac{dt}{dx} - a_2\right)}$$

$$\therefore (1) \Rightarrow \frac{1}{b_2}\left(\frac{dt}{dx} - a_2\right) = \frac{\lambda(t)+c_1}{t+c_2}$$

$$\Rightarrow \frac{dt}{dx} - a_2 = b_2\left[\frac{\lambda(t)+c_1}{t+c_2}\right]$$

$$\Rightarrow \boxed{\frac{dt}{dx} = a_2 + b_2\left[\frac{\lambda(t)+c_1}{t+c_2}\right]}$$

we can solve this by using Variable separable method

Ex: Solve $\frac{dy}{dx} = \frac{3x+2y+1}{6x+4y+5}$

(S) Given $\frac{dy}{dx} = \frac{3x+2y+1}{6x+4y+5} \longrightarrow (1)$

clearly $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow \boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2}}$ hence case (ii) is satisfied.

$$(1) \Rightarrow \frac{dy}{dx} = \frac{3x+2y+1}{2(3x+2y)+5}$$

Now put $3x+2y = t$

$$3 + 2 \cdot \frac{dy}{dx} = \frac{dt}{dx}$$

$$2 \cdot \frac{dy}{dx} = \frac{dt}{dx} - 3$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2}\left[\frac{dt}{dx} - 3\right]}$$

By substituting t and $\frac{dy}{dx}$ in (1) we get,

$$\frac{1}{2}\left[\frac{dt}{dx} - 3\right] = \frac{t+1}{2t+5}$$

$$\frac{dt}{dx}-3=\frac{2t+2}{2t+5}$$

$$\Rightarrow \frac{dt}{dx}=\frac{2t+2}{2t+5}+3$$

$$\frac{dt}{dx}=\frac{2t+2+6t+15}{2t+5}$$

$$\frac{dt}{dx}=\frac{8t+17}{2t+5}$$

$$\Rightarrow \int \frac{2t+5}{8t+17}\,dt=\int dx$$

$$\Rightarrow \frac{1}{4}\int \frac{8t+20}{8t+17}\,dt=\int dx$$

$$\Rightarrow \frac{1}{4}\left[\int \frac{8t+17+3}{8t+17}\,dt\right]=x+c$$

$$\Rightarrow \frac{1}{4}\left[\int dt+\int \frac{3}{8t+17}\,dt\right]=x+c$$

$$\Rightarrow \frac{1}{4}\left[t+\frac{3}{8}\int \frac{8}{8t+17}\,dt\right]=x+c$$

$$\Rightarrow \frac{1}{4}\left[t+\frac{3}{8}\log(8t+17)\right]=x+c$$

$$\Rightarrow \frac{1}{4}(3x+2y)+\frac{3}{32}\log\left(8(3x+2y)+17\right)=x+c$$

$$\Rightarrow \boxed{8(3x+2y)+3\log(24x+16y+17)=32(x+c)}$$

Case (iii): $\frac{dy}{dx}=\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$

If $\boxed{a_1=b_2 \text{ and } b_1=a_2 \text{ (or) } a_1=-b_1 \text{ and } b_1=-a_2}$

Ex: $\frac{dy}{dx}=\frac{x+2y+3}{2x+y+1}$ Here $a_1=1$, $b_1=2$, $a_2=2$, $b_2=1$ $\Rightarrow \boxed{a_1=b_2,\ a_2=b_1}$

*→ Apply Componendo and Dividendo (C & D)

$$\frac{dy+dx}{dy-dx}=\frac{x+2y+3+2x+y+1}{x+2y+3-2x-y-1}=\frac{3x+3y+4}{-x+y+2}=\frac{3(x+y)+4}{y-x+2}$$

$$\Rightarrow \frac{d(x+y)}{d(y-x)} = \frac{3(x+y)+2}{y-x+2}$$

$$\Rightarrow \int \frac{1}{3(x+y)+2}\, d(x+y) = \int \frac{1}{(y-x)+2}\, d(y-x)$$

$$\Rightarrow \frac{1}{3}\log[3(x+y)+4] = \log[(y-x)+2] + c$$

$$\Rightarrow \log[3(x+y)+4] = 3\log[(y-x)+2] + 3c$$

$$\Rightarrow \boxed{\log[3(x+y)+4] = 3\log[(y-x)+2] + C}$$