Rank of matrix (Echelon form)

Let A is ~~n x n~~ matrix.

Zero Row : All elements are zeros in row then the row is called zero row. Ex [0 0 0 0]

Non zero Row: At leaft one element is non zero element in row Then the row is called non zero row.

Ex: [0 0 1 0]

Rank of matrix A = number of non zero rows of A

Rank is denoted by P(A) (or) r.

Rows are denoted by $\overline{R} = R1, R_2, R_3, R_4 \cdots$

Elementary Transformation of matrix: The following operations, three of which refer to rows and Three to columns are known as elementary transformations.

i) The interchange of any two rows (Columns)

ii) The multiplication of any row (Columns) by a non zero number.

iii) The addition of a constant multiple of the elements of any row (Column) to the corresponding elements of any other row (Column).

Echelon form;-

Ex: 1) $A = \begin{bmatrix} 3 & 4 & 5 \\ (7) & 2 & 1 \\ (3) & 5 & 6 \end{bmatrix}_{3\times3}$

3) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ (3) & 2 & 5 & 5 \\ (3) & (2) & 1 & 4 \end{bmatrix}_{3\times4}$

2) $A \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ (3) & 4 & 2 & 1 \\ (4) & (2) & 3 & 2 \\ (1) & (5) & (2) & 3 \end{bmatrix}_{4\times4}$

① Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ into echelon form and find its Rank.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ (2) & 4 & 3 & 2 \\ (3) & (2) & 1 & 3 \\ (6) & (8) & (7) & 5 \end{bmatrix} \begin{matrix} \to R_1 \\ 2 \to R_2 \\ 3 \to R_3 \\ 5 \to R_4 \end{matrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & (-4) & -8 & 3 \\ 0 & (-4) & (-11) & 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$R_2 : 2\ 4\ 3\ 2$

$2R_1 : \underline{2}\ \underline{4}\ \underline{6}\ \underline{0}$ (−)

$R_2 \to R_2 - 2R_1 : 0\ 0\ -3\ 2$

$R_3 : 3\ 2\ 1\ 3$

$3R_1 : \underline{3}\ \underline{6}\ \underline{9}\ \underline{0}$ (−)

$R_3' \to R_3 - 3R_1 : 0\ -4\ -8\ 3$

$R_4 : 6\ 8\ 7\ 5$

$6R_1 : \underline{6}\ \underline{12}\ \underline{18}\ \underline{0}$ (−)

$R_4 \to R_4 - 6R_1 : 0\ -4\ -11\ 5$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

R_4	:	0	−4	−11	5
R_2	:	0	−4	−8	3
⊖		−	+	+	−
$R_4 \to R_4 - R_2$	:	0	0	−3	2

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

R_4	:	0	0	−3	2
R_3	:	0	0	−3	2
⊖				+	−
$R_4 \to R_4 - R_3$	:	0	0	0	0

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is Echelon form.

Rank of A = number of non zero rows

$= \underline{\underline{3}}$

(2) $A = \begin{bmatrix} 5 & 3 & 14 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$

$R_3 \to 5R_3 - R_1$

$\sim \begin{bmatrix} 5 & 3 & 14 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & -8 & -4 & 0 \end{bmatrix}$

R_3 :	1	−1	2	0
$5R_3$:	5	−5	10	0
R_1 :	5	3	14	0
⊖	−	−	−	−
$R_3 \to 5R_3 - R_1$:	0	−8	−4	0

$\sim \begin{bmatrix} 0 & -8 & -4 & 0 \\ 5 & 3 & 14 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 12 & 8 \end{bmatrix}$

R_3 :	0	−8	−4	0
$8R_2$:	0	8	16	8
⊕		+	+	+
$R_3 \to R_3 + 8R_2$:	0	0	12	8

This is Echelon form

Rank of A = number of non zero rows

= 3

(3) $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 + R_1$

R_2 : 2 −1 1 0

$2R_1$: 2 −4 0 2

(−)

$R_2 \to R_2 - 2R_1$: 0 3 1 −2

R_3 : 3 −3 1 1

$3R_1$: 3 −6 0 3

(−)

$R_3 \to R_3 - 3R_1$: 0 3 1 −2

R_4 : −1 −1 −1 1

R_1 : 1 −2 0 1

(+)

$R_4 \to R_4 + R_1$: 0 −3 −1 2

$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 3 & 1 & -2 \\ 0 & -3 & -1 & 2 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 + R_2$

R_3 : 0 3 1 −2

R_2 : 0 3 1 −2

(−)

$R_3 \to R_3 - R_2$: 0 0 0 0

$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

This is Echelon form.

R_4 : 0 −3 −1 2

R_2 : 0 3 1 −2

(+)

$R_4 \to R_4 - R_2$: 0 0 0 0

Rank = number of non zero rows

= 2

(4)

$$A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 + 2R_1$

$R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix}$$

$R_3 \to 2R_3 - 11R_2$

$R_4 \to R_4 + 2R_2$

$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & +1 & 0 \end{bmatrix}$$

$R_4 \to 6R_4 + R_3$

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) = 4$

R_2 : 1 1 −1 0

R_1 : −1 −3 3 −1

(+)

$R_2 \to R_2 - R_1$: 0 −2 2 −1

R_3 : 2 −5 2 −3

$2R_1$: 2 6 6 2

(+)

$R_3 \to R_3 - 2R_1$: 0 −11 8 −5

R_4 : −1 1 0 1

R_1 : −1 −3 3 −1

(−)

$R_4 \to R_4 - R_1$: 0 4 −3 2

R_3 : 0 −11 8 −5

$2R_3$: 0 −22 16 −10

R_2 : 0 −2 2 −1

$11R_2$: 0 22 22 11

(−)

$R_3 \to 2R_3 - 11R_2$: 0 0 −6 1

R_4 : 0 4 −3 2

$2R_2$: 0 4 4 2

(+)

$R_4 \to R_4 - 2R_2$: 0 0 1 0

$6R_4$: 0 0 6 0

R_3 : 0 0 −6 1

$R_4 \to 6R_4 + R_3$: 0 0 0 1