Solve $(x+2y)(dx-dy) = dx+dy$

$(x+2y)(dx) - (x+2y)dy = dx+dy$

$dx(x+2y-1) - dy(x+2y+1) = 0$

$dx(x+2y-1) = dy(x+2y+1)$

$$\frac{dy}{dx} = \frac{x+2y-1}{x+2y+1} \longrightarrow ①$$

It is not in homogeneous form. But we can convert it into homogeneous D.E. By using the formulas.

In ①, $a = 1,\ b = 2$, $a' = 1\ \ b' = 2$ $\Rightarrow$ $ab' - a'b = 2-2 = 0$

∴ case(i) is failed.

put $x+2y = z \Rightarrow 1 + 2\frac{dy}{dx} = \frac{dz}{dx}$

$2\frac{dy}{dx} = \frac{dz}{dx} - 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{dz}{dx} - 1\right)$

$\therefore ① \Rightarrow \frac{1}{2}\left(\frac{dz}{dx} - 1\right) = \frac{z-1}{z+1}$

$\frac{dz}{dx} - 1 = 2\left(\frac{z-1}{z+1}\right)$

$\frac{dz}{dx} = 1 + \frac{2(z-1)}{z+1} = \frac{z+1+2z-2}{z+1} = \frac{3z-1}{z+1}$

$\frac{z-1}{z-1} + \frac{1}{3z-1}$

$1 + \frac{1}{3z-1}$

$\frac{z+1}{3z-1} = \frac{1+\frac{1}{z-1}}{3z-1}$

$\frac{3z-1 - 2z+2}{3z-1}$

$1 - \frac{(2z-1)}{3z-1}$

$1-$

$\Rightarrow \frac{1}{3} + \frac{4}{3}\left(z-\frac{1}{3}\right)$

$\frac{1}{3} + \frac{4}{3}$

$\frac{dz}{dx} = \frac{3z-1}{z+1}$

$\left(\frac{z+1}{3z-1}\right) dz = dx \Rightarrow \int\left(\frac{1}{3} + \frac{4}{3}\cdot\frac{1}{3z-1}\right) dz = dx$

$\Rightarrow \frac{z+1}{3z-1} = \frac{z+1}{3\left(z-\frac{1}{3}\right)} = \frac{1}{3}\left(\frac{z-\frac{1}{3}+\frac{4}{3}}{z-1/3}\right) = \frac{1}{3}\left(\frac{z-\frac{1}{3}}{z-\frac{1}{3}}\right.$

$+\frac{4}{3}\left(z-\frac{1}{3}\right)$

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$$\int \frac{1}{3} + \int \frac{4}{3} \cdot \frac{1}{3z-1}\, dz = \int dx$$

$$\frac{1}{3} z + \frac{4}{3} \cdot \left(\frac{1}{3}\right) \log(3z-1) = x+c$$

$$\frac{z}{3} + \frac{4}{9} \log(3z-1) = x+c.$$

$$3z + 4\log(3z-1) = 9x+9c$$

$$3(x+2y) + 4\log(3(x+2y)-1) = 9x+9c$$

$$3x+6y + 4\log(3x+6y-1) = 9x+9c$$

$$4\log(3x+6y-1) = 6x-6y+9c$$

$$\boxed{2\log(3x+6y-1) = 3x-3y+c.}$$

Ⓟ solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3} \longrightarrow ①$

$a=1,\ b=2,\ a'=2,\ b'=1$

$ab' - ba' = (1)(1) - (2)(2) = -3 \neq 0.$

∴ case (i) is satisfied

∴ put $x = X+h,\quad y = Y+k$

$dx = dX,\quad dy = dY$

$$① \Rightarrow \frac{dX}{dY} = \frac{X+2Y-3}{2X+Y-3} = \frac{(X+h)+(2(Y+k)-3}{2(X+h)+(Y+k)-3}$$

$$= \frac{X+2Y+(h+2k-3)}{2X+Y+(2h+k-3)}$$

Here $\boxed{\begin{array}{l} h+2k=3 \\ 2h+k=3 \end{array}}$

by solving this system we get $\boxed{h<k=1}$

$\therefore \frac{dy}{dx} = \frac{x+2y}{2x+y}$ in homogeneous

put $y = vx. \Rightarrow \frac{dy}{dx} = v + x.\frac{dv}{dx}$

$$v + x\cdot\frac{dv}{dx} = \frac{x+2xv}{2x+vx} = \frac{1+2v}{2+v}$$

$$x\cdot\frac{dv}{dx} = \frac{1+2v}{v+2} - v = \frac{1+2v-v(v+2)}{v+2}$$

$$x\cdot\frac{dv}{dx} = \frac{1+2v-v^2-\cancel{2v}}{v+2}$$

$$x.\frac{dv}{dx} = \frac{1-v^2}{v+2}$$

$$\frac{v+2}{1-v^2}dv = \frac{1}{x}dx$$

$$\int\frac{v+2}{1-v^2}dv = \int\frac{1}{x}dx$$

$$\int\frac{v+1+1}{(1-v)(1+v)}dv = \int\frac{1}{x}dx$$

$$\int\frac{1}{1-v} + \frac{1}{1-v^2}.dv = \int\frac{1}{x}dx$$

$$-\log(1-v) + \int\frac{1}{1-v^2}dv = \log x + \log c$$

$$-\log(1-v) + \frac{1}{2}\log\left(\frac{1+v}{1-v}\right) = \log(xc)$$

$$-2\log(1-v) + \log\left(\frac{1+v}{1-v}\right) = \log(x^2c^2)$$

$$\int\frac{v}{1-v^2}dv + \int\frac{2}{1-v^2}dv = \log x + \log c$$

$\log\left(\frac{1+v}{1-v}\right)$.

$\frac{v+1+1}{(1-v)(1+v)}$

$$-\frac{1}{2}\log(1-v^2) + \frac{2}{2}\log\left(\frac{1+v}{1-v}\right) = 2\log(xc)$$

$+\frac{1}{(v-1)(v+1)}$

$\frac{v+1}{(1-v)(v+1)}$

$$\log(1-v^2) + \log\left(\frac{1+v}{1-v}\right) = 2\log(xc)$$

$\log\left(\frac{1}{(1-v)^2}\right)$

$\frac{1}{1+v} + \frac{1}{(1-v)}$

$\log\left(\frac{(1+v)}{(1-v)} \frac{1+v}{1-v} \cdot \frac{1}{1-v^2}\right) = \log(x^2c^2)$

$-\log(1-v) +$

$\log\left(\frac{1+v}{1-v}\right) - 2\log(1-v)^2$

$\log\left(\frac{(1+v)^2}{(1-v)^2}\right.$. $x(1-v)(1+v)\log\left(\frac{1+v}{1-v}\right)$

$$\Rightarrow \log\left(\frac{1+v}{1-v}\right)+\log\left(1/(1-v)^2\right) = \log(x^2c^2)$$

$$\Rightarrow \log\left[\left(\frac{1+v}{1-v}\right)\cdot\frac{1}{(1-v)^2}\right] = \log(x^2c^2)$$

$$\Rightarrow \frac{1+v}{(1-v)^3} = x^2c^2$$

$x = X+h$

$X = x-h$

$$\Rightarrow \frac{1+Y/X}{(1-Y/X)^3} = \overset{X^2}{\cancel{(x-h)^2}}c^2$$

$$\Rightarrow \frac{X+Y}{X\left(\frac{X-Y}{X}\right)^3} = \overset{X^2}{\cancel{(x-1)^2}}c^2$$

$$\Rightarrow \frac{X+Y}{\cancel{X}(X-Y)^3}\cdot\cancel{X^2} = \cancel{x^2}\overset{X^2}{\cancel{(x-1)^2}}c^2$$

$$\Rightarrow \frac{\cancel{X^2}(X+Y)}{(X-Y)^3} = \cancel{(x-1)^2c^2}$$

$$\Rightarrow \frac{X+Y}{(X-Y)3} = c^2 \Rightarrow \boxed{X+Y = c^2\ (X-Y)^3}\ \checkmark$$

$$x-h+y-k = c^2\ (x-h-y-k)^3$$

$$x-1+y-1 = c^2\ (x-1-y\cancel{+}1)^3$$

$$\boxed{x+y-2 = c^2\,(x-y)^3}\ \checkmark$$

Dec 13 NET (P)

$y:\mathbb{R}\to\mathbb{R}$ satisfy the I.V.P $y'(t) = 1-y^2(t)$, $y(0)=0$

then (i) $y(t_1) = 1$ for some $t_1 \in \mathbb{R}$

(ii) $y(t) > -1$ $\forall\, t\in\mathbb{R}$

(iii) y is strictly increasing in $\mathbb{R}$

(iv) y is increasing in $(0,1)$ and decreasing in $(1,\infty)$