DC circuits

* **Introduction:-** Electric circuit theory & electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications and instrumentation are based on electric circuit theory. circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general and because of the applied mathematics, physics & topology involved

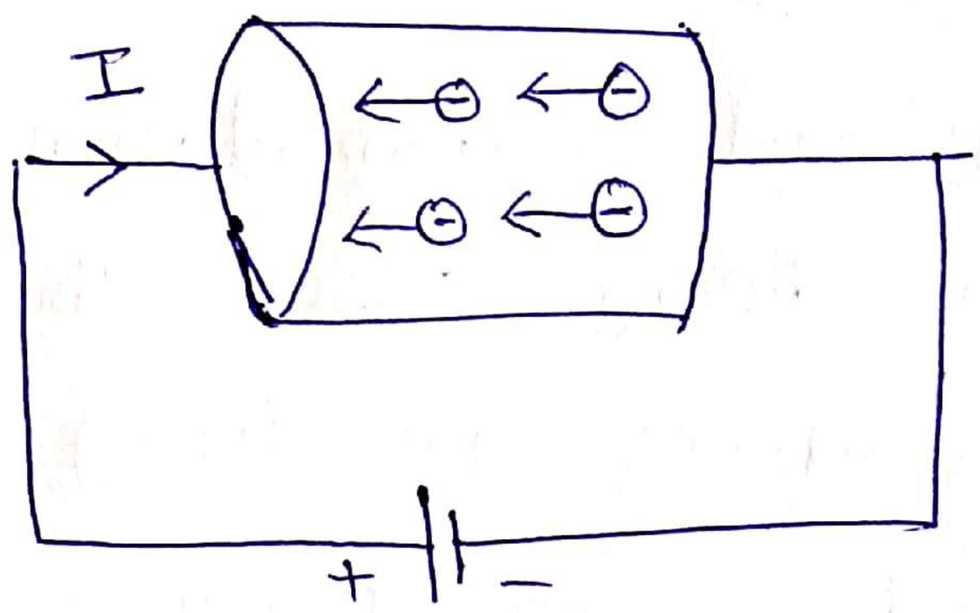
* An electric circuit is an interconnection of electrical elements.

* **Charge & Current:-**

→ (Electric) charge is the most basic quantity in an electric circuit.

→ charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

* Charge of an Electron $\Rightarrow e = -1.602 \times 10^{-19} C$
* The law of conservation of charge states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the Electric charges in a System does not change.



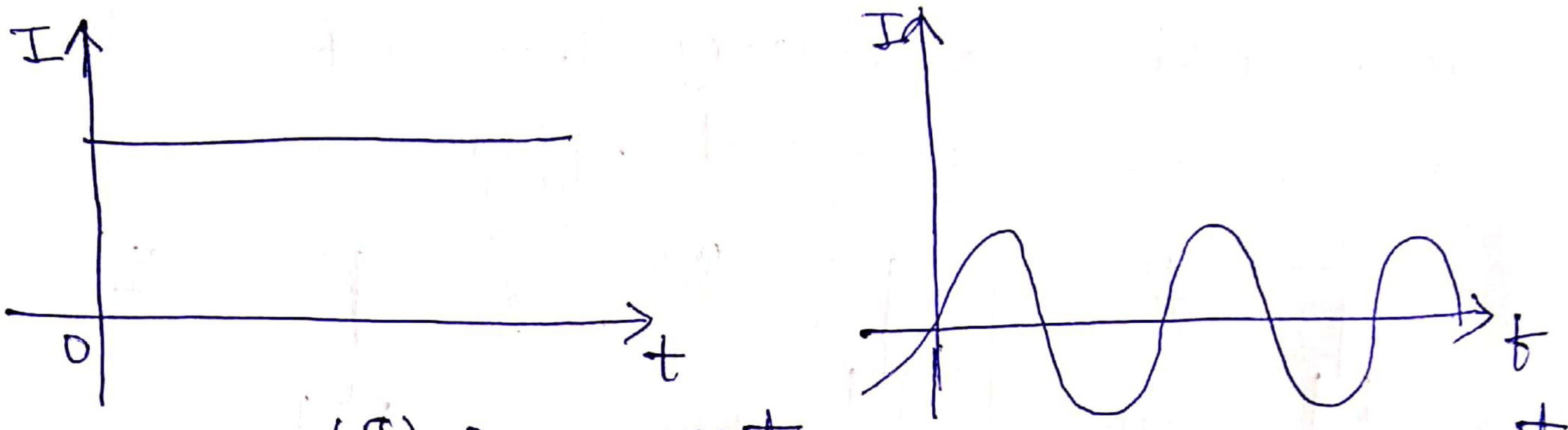
← Electric current due to flow of Electronic charge in a conductor

when a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move, positive charges move in one direction while negative charges move in the opposite direction. This motion of charge creates Electric current. It is conventional to take the current flow as the movement of positive charges, that is opposite to the flow of negative charges. Current flow in metallic conductors is due to negatively charged Electrons.

→ Electric Current is the time rate of change of charge, measured in amperes (A).

$$\boxed{I = \frac{Q}{t}} \quad \text{or} \quad \boxed{i = \frac{dq}{dt}}$$

1 ampere = 1 coulomb/second



(a) D.C current (b) A.C current

fig. Two types of current

→ A direct Current (dc) is a current that remains constant with time.

→ An alternating current (ac) is a current that Varies Sinuosoidally with time.

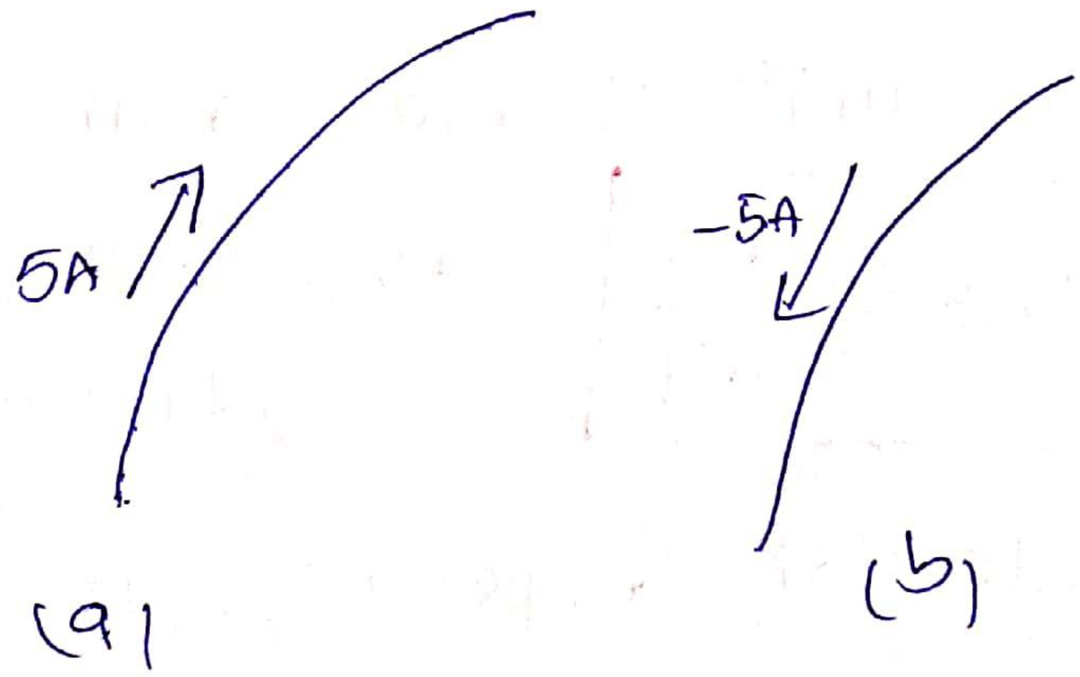


fig. conventional current flow

(a) positive current flow

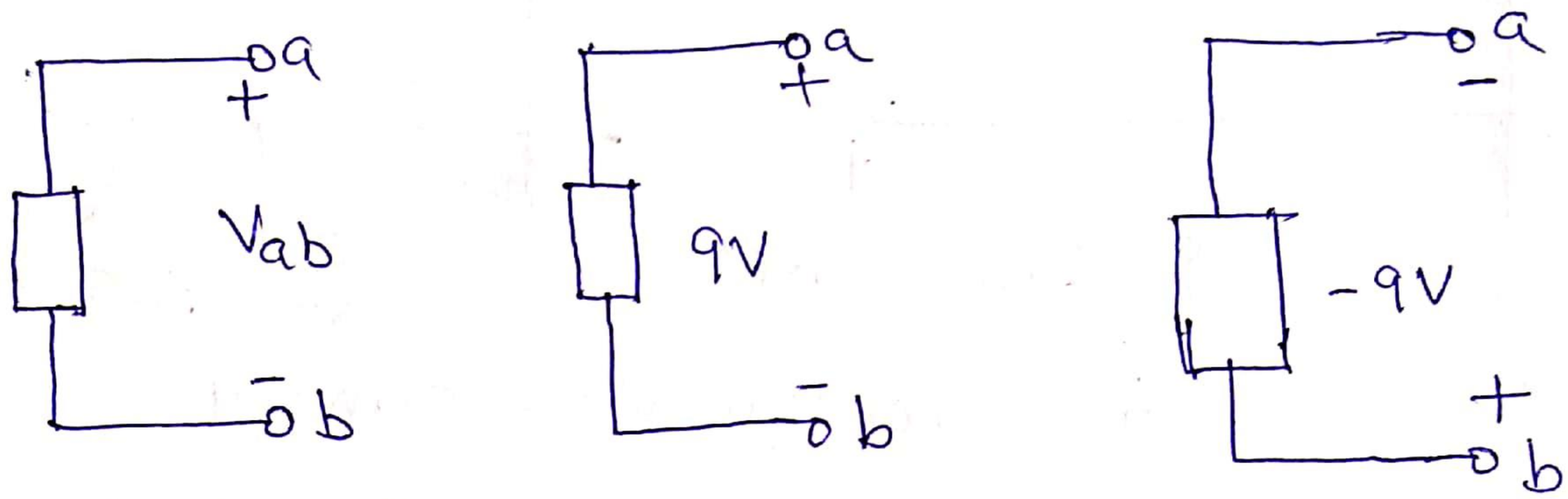
(b) negative current flow

* How much charge is represented by 4600 electrons

→ Each Electron has -1.602×10^{-19} C.

Hence 4600 will have -1.602×10^{-19} C/e × 4,600 electron

* **Voltage:-** To move Electron in a conductor in a particular direction requires some work or Energy transfer. This work is performed by an External Electromotive force (e.m.f) typically represented by a battery. This emf is also known as Voltage or potential difference.



* polarity of Voltage Vab
* point 'a' is 9v above point 'b'
* point 'b' is 'a' −9v above point

→ The voltage Vab b/w two points 'a' & 'b' in an Electric Circuit is the Energy (or work) needed to move a unit charge from 'a' to 'b'.

$$\therefore V_{ab} = \frac{dw}{dq}$$

1V ⇒ 1 Joule/coulomb
⇒ 1 newton meter/coulomb

→ Voltage (or potential difference) is the Energy required to move a unit charge through an Element, measured in Volts (V).

$$V_{ab} = -V_{ba}$$

(when polarities reversed)

* power & Energy:-

→ power is the time rate of Expending or absorbing Energy, measured in watts (W)

$$\boxed{\therefore P = \frac{dw}{dt}}$$

$$P = \frac{dw}{dq} \times \frac{dq}{dt} = V \times I \qquad \boxed{\therefore P = Vi}$$

→ The power absorbed or Supplied by an Element is the product of the voltage across the Element and current through it. If the power has a +ve Sign, power is being delivered to or absorbed by an Element. If the power has a -ve sign, power is being supplied by the Element.

→ $P = +Vi$ or $Vi > 0$, the Element is absorbing power.

→ $P = -Vi$ or $Vi < 0$, the Element is releasing or supplying power.

* passive Sign Convention is Satisfied when the current Enters through the positive terminal of an Element & $P = +Vi$. If the current Enters through the negative terminal, $P = -Vi$.

* **Types of Elements:-** Two types of Elements found in Electrical Circuits.

a) Active elements (Energy sources): The Elements which are capable of generating or delivering the Energy are called active Elements

Ex:- Generators, Batteries

b) passive Elements (loads): The Elements which are capable of receiving the Energy are called passive Elements

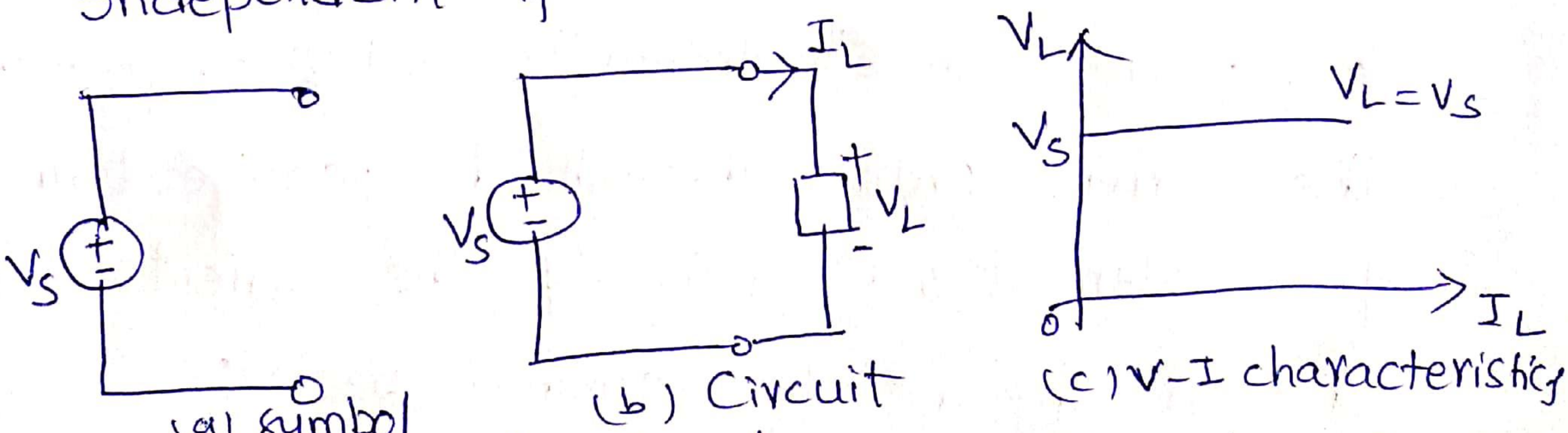
Ex:- Resistors, Capacitors & Inductors

a) Active Elements (Energy Sources):- There are two kinds of sources

(1) Independent Sources

(2) Dependent Sources

(1) Independent Sources :- An ideal Independent Source is an active Element that provides a specified voltage or current that is completely independent of other circuit Elements



(a) symbol (b) Circuit (c) V-I characteristics

* fig. Ideal Independent voltage Source

* Practical Independent Voltage Source:-

Practically, Every voltage Source has Some Series resistance across its terminals Known as internal resistance and is represented by R_{se}. For Ideal Voltage Source $R_{se} = 0$. But in practical voltage Source have Small amount of R_{se}, due to this voltage across the terminals decreases with increase in current. $V_L = V_S - I_L R_{se}$

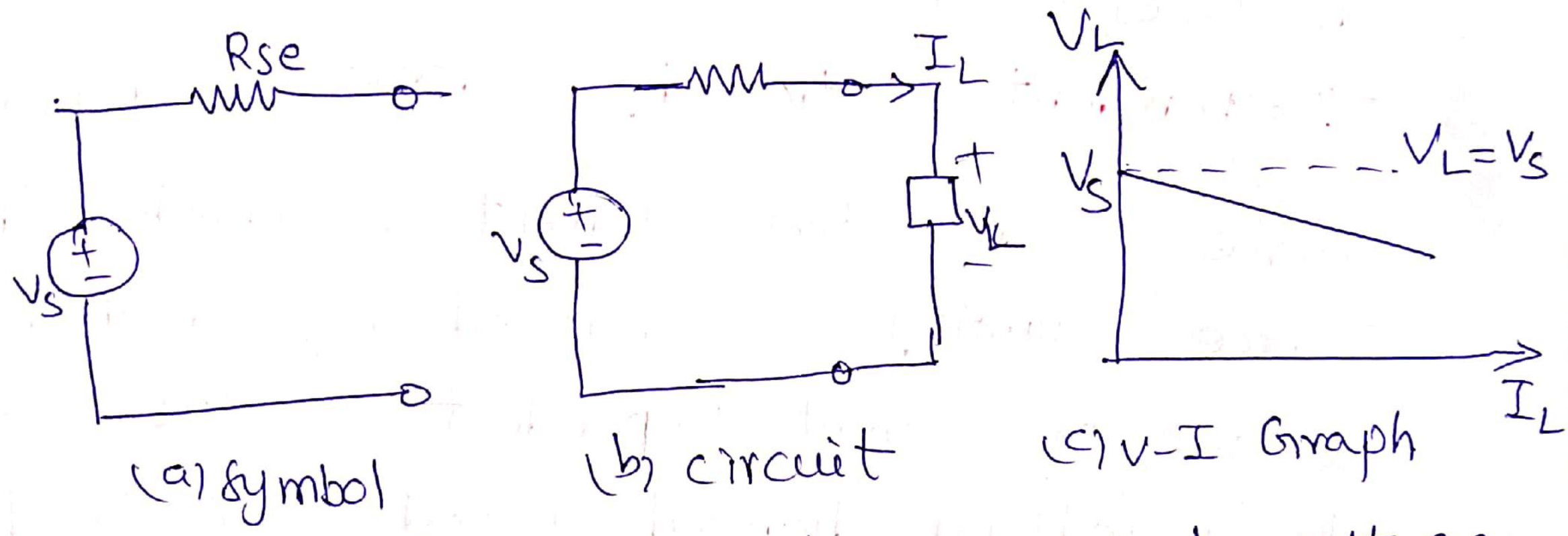
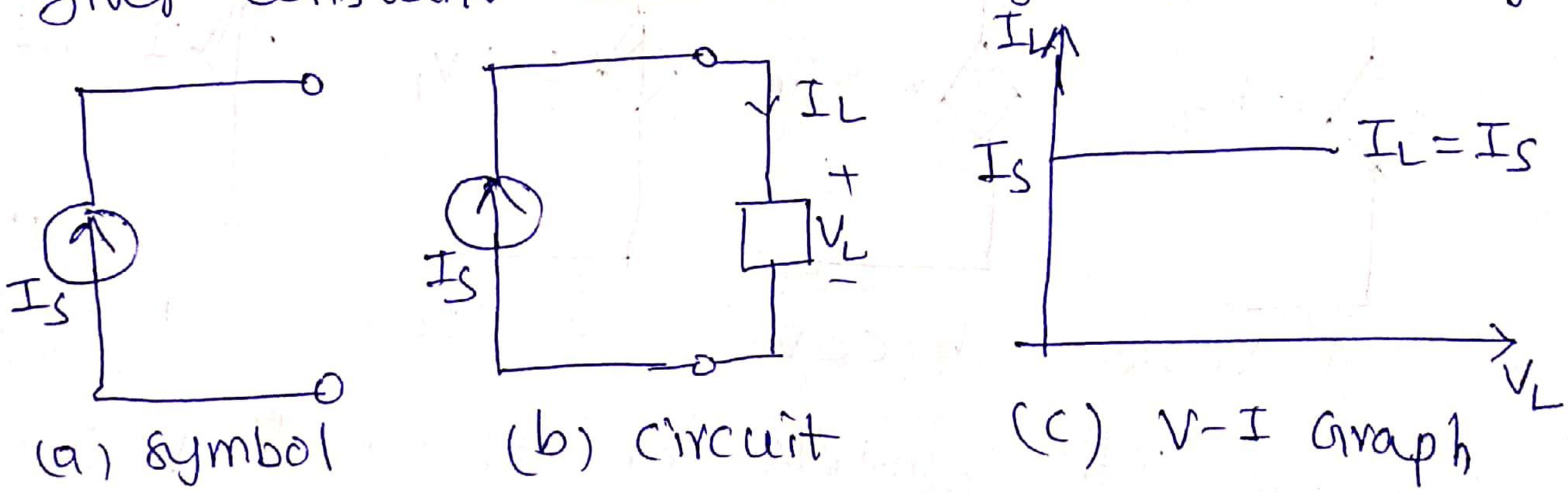


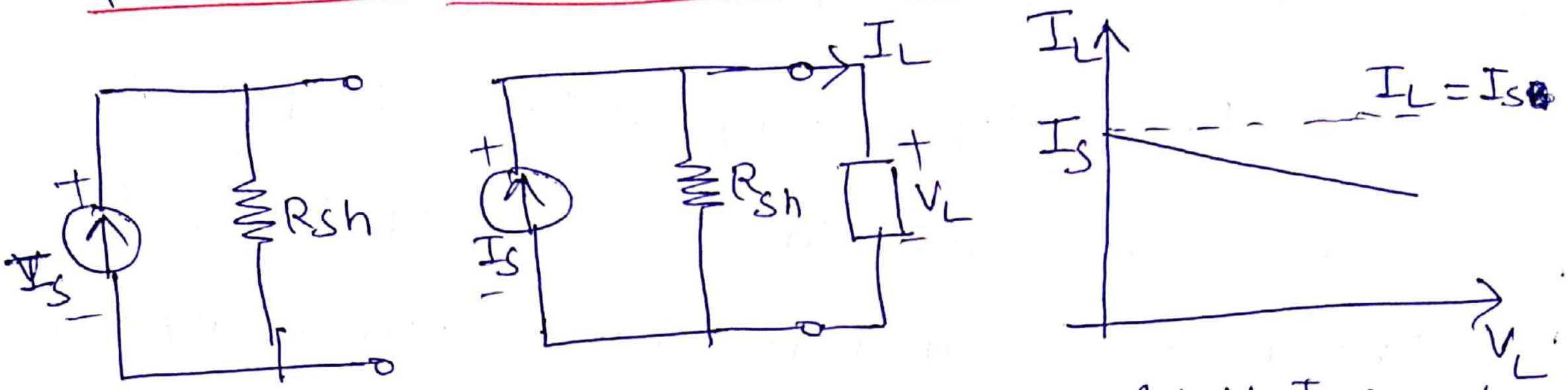
fig. Practical Independent Voltage Source.

* Ideal Independent Current Source:-

which gives constant current through its terminals



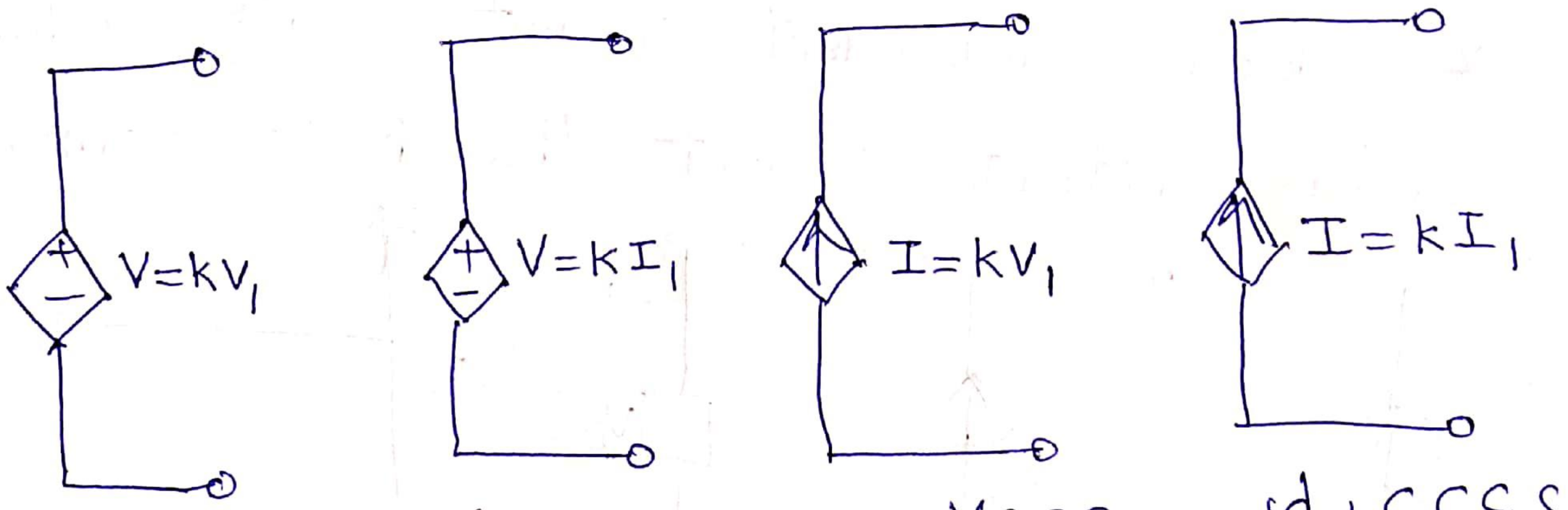
* Practical Independent current Source:-



(a) Symbol (b) circuit (c) V-I Graph

For practical independent current source has some shunt resistance across its terminals (ideally ∞). due to thy current (I_L) decreases with increase in voltage ($I_L = I_s - I_{sh}$)

* **Dependent (Controlled) Sources:-** A dependent source is an active element in which the source quantity is controlled by another voltage or current. Dependent sources are designated by diamond shaped symbols. '4' types of dependent sources

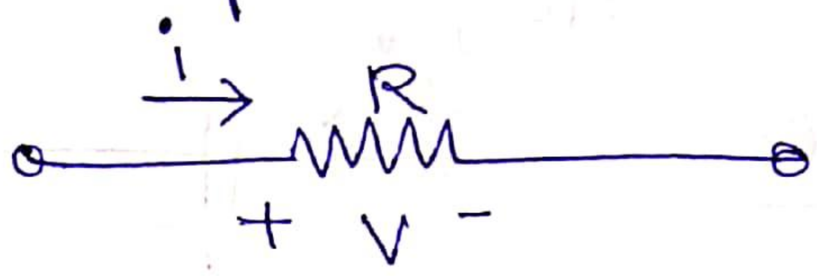


(a) VCVS (b) CCVS (c) VCCS (d) CCCS

* fig. symbols for dependent voltage & source & dependent current source

* Passive Elements:-

→ **Resistor:-** The ability of a conducting material to resist (oppose) the flow of current, is known as resistance and is represented by symbol 'R', measured in ohms. The circuit element used to model the current resisting behavior of a material is called the resistor.



← Symbol for resistor

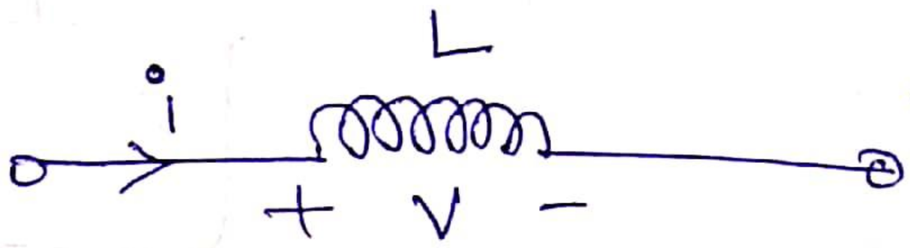
The resistance of a conducting material depends on geometrical shape

ρ – resistivity
l – length
A – Area

$* R = \frac{\rho l}{A} \Rightarrow G = \frac{1}{R}$ (conductance) → mho

$* \rho = \frac{RA}{l} \Rightarrow \sigma = \frac{1}{\rho}$ (conductivity) → $mho.m^{-1}$

→ **Inductor:-**



← Symbol of inductor

A wire of certain length, when twisted into a coil becomes a basic inductor. If current is made to pass through an inductor, an Electro-magnetic field is formed. A change in the magnitude of the current changes the Electro-magnetic field (EMF). Increase in current Expands the fields, & decrease in current reduces it.

∴ A change in current produces change in EMF, which induces a voltage across the coil

$$V \propto \frac{di}{dt} \rightarrow \boxed{V = L\frac{di}{dt}}$$

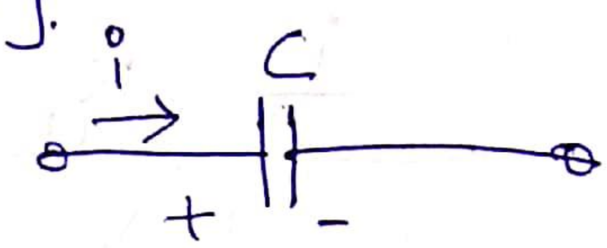
Inductance, Measured in Henry

$$\boxed{i(t) = \frac{1}{L}\int_0^t V.dt + i(0)}$$

Initial current in coil

Suppose constant current through inductor which gives '0' voltage across it. [Inductor acts as Short circuit to dc].

* Capacitor:-



Symbol for capacitor, Measured in Farads.

As we apply voltage source 'V' to capacitor, the source deposits a positive charges '+q' on one plate & a negative charges -q on the other. The capacitor is said to store the electric charge.

$$q \propto V \Rightarrow q = CV \qquad \boxed{C = \frac{q}{V}} \qquad \boxed{C = \frac{\epsilon A}{d}}$$

The capacitance is the ratio of the charge 'q' per plate to the applied voltage 'V'

$$i \propto \frac{dq}{dt} \Rightarrow \boxed{i = C\frac{dv}{dt}}$$

$$\boxed{V(t) = \frac{1}{C}\int_0^t i.dt + V(0)}$$

Initial voltage across capacitor

Suppose constant voltage across the capacitor gives '0' current through it. [Capacitor acts as open circuit to dc]

* DC Circuits *

* **Ohm's Law:-** ohm's law states that the voltage(V) across a resistor is directly proportional to the current (i) flowing through the resistor.

i.e $\boxed{V \propto i}$

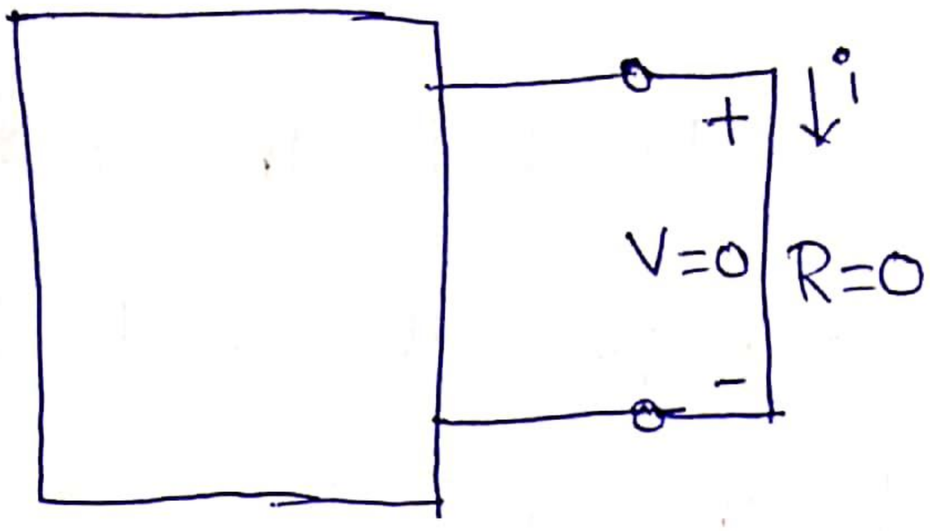
$V = iR$

where 'R' is a proportionality constant called resistance of the resistor measured in ohms (Ω).

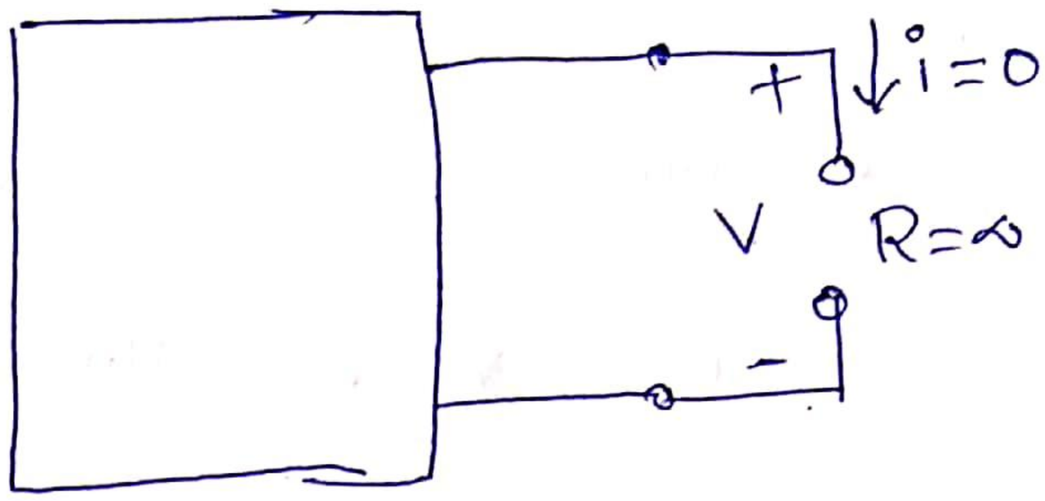
"The resistance 'R' of an Element denotes its ability to resist the flow of Electric Current."

As Current flows from higher potential to lower potential for $V = iR$. If Current flows from a lower potential to a higher potential, $V = -iR$

The value of 'R' can change from zero to infinity, it is important that we consider the two extreme possible values of R. An Element with $R = 0$ is called a short circuit (s.c). For short circuit $V = iR = 0$. Showing that the voltage is zero but the current could be anything



(a) Short circuit (R=0)



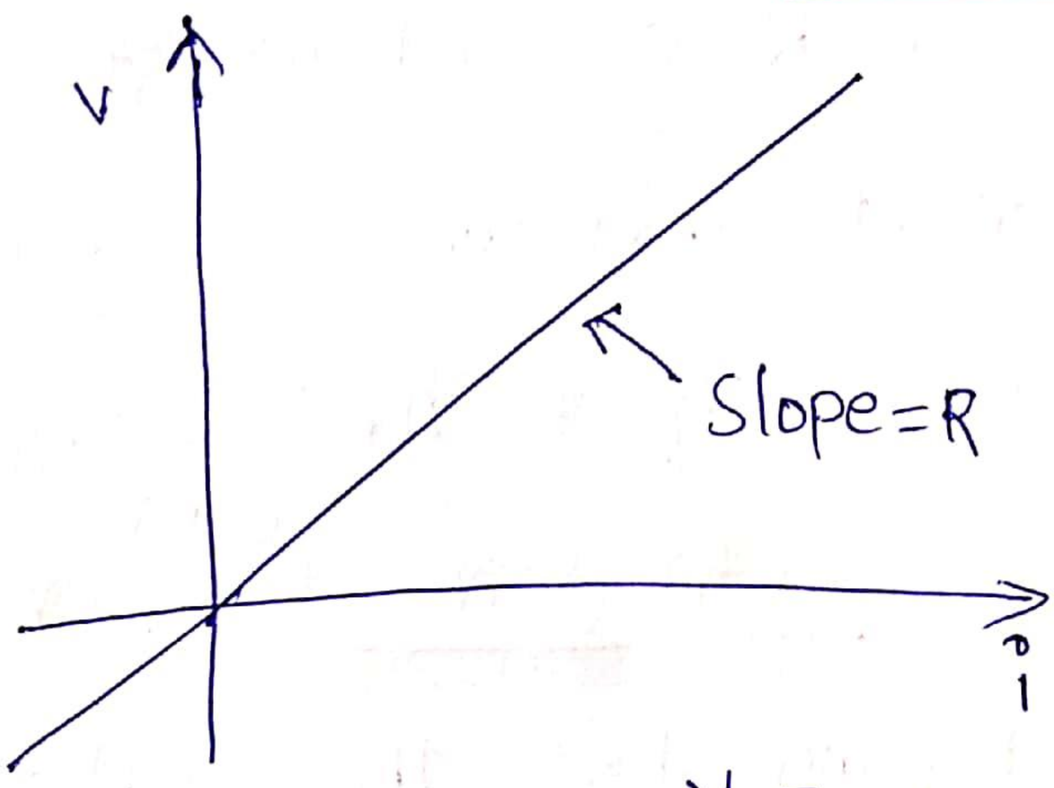
(b) open circuit (R=∞)

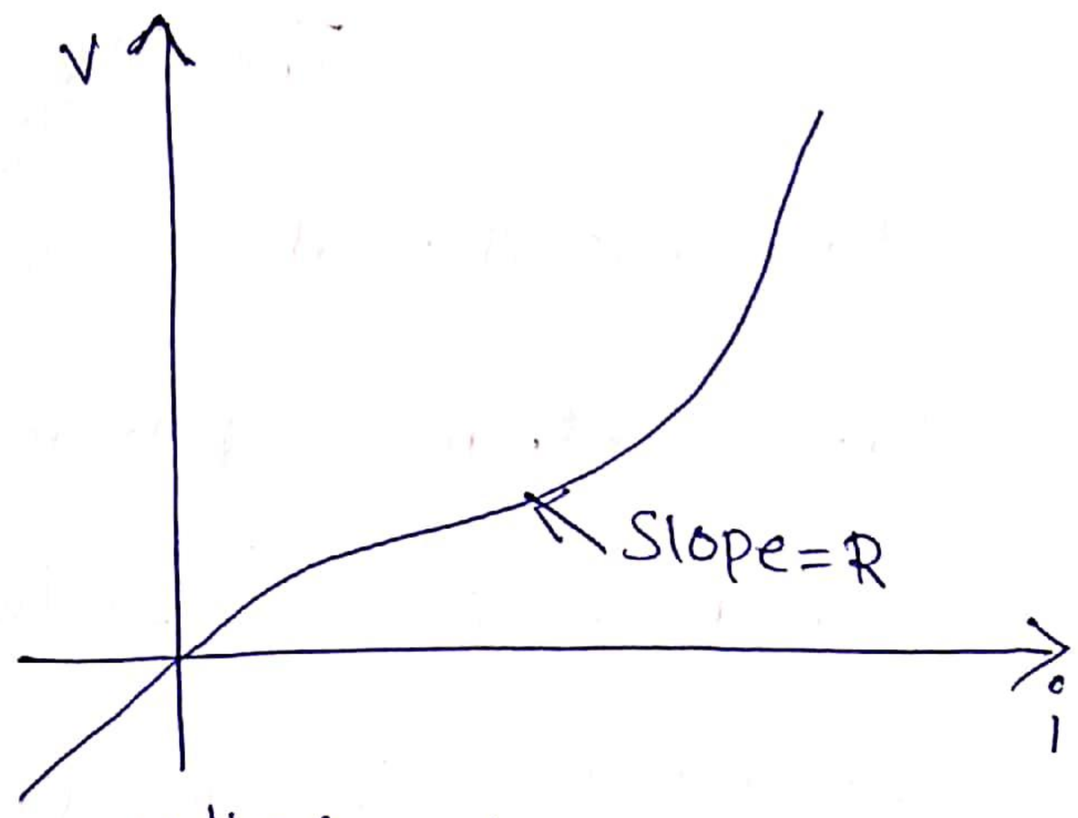
Similarly an Element with $R=\infty$ is known as open circuit (o.c). For an open circuit

$$i = \lim_{R\to\infty} \frac{V}{R} = 0$$

. Indicating that the current is zero through the voltage could be anything

* A s.c is a circuit Element with resistance approaching zero
* An o.c is a circuit Element with resistance approaching infinity





V-I Characteristics

~~(a) V-I~~

(a) a linear resistor

(b) a Non linear resistor

* NODES, BRANCHES & LOOPS :-

→ A Branch represents a single Element such as voltage Source or a resistor

→ A Node represents or is the point of Connection between two or more branches

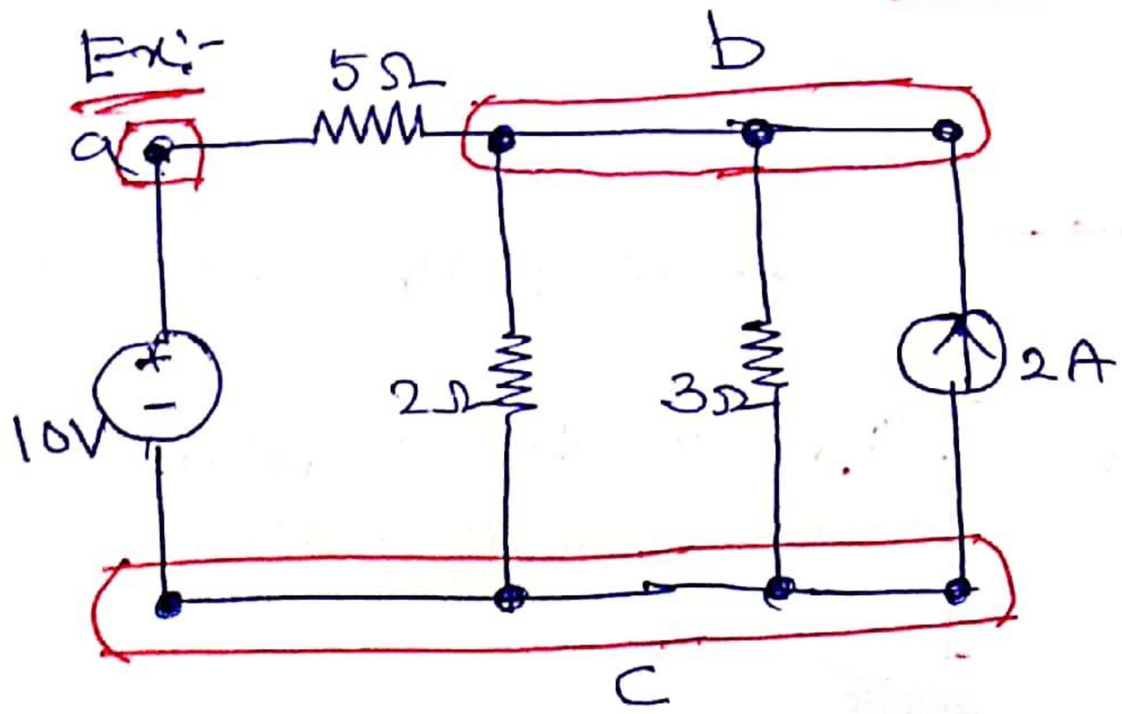
→ A loop is any closed path in a circuit. It is formed by starting at a node, passing through a set of nodes, and returning to the Starting node without passing through any node more than once.

"A loop is said to be independent if it contains a branch which is not in any other loop."

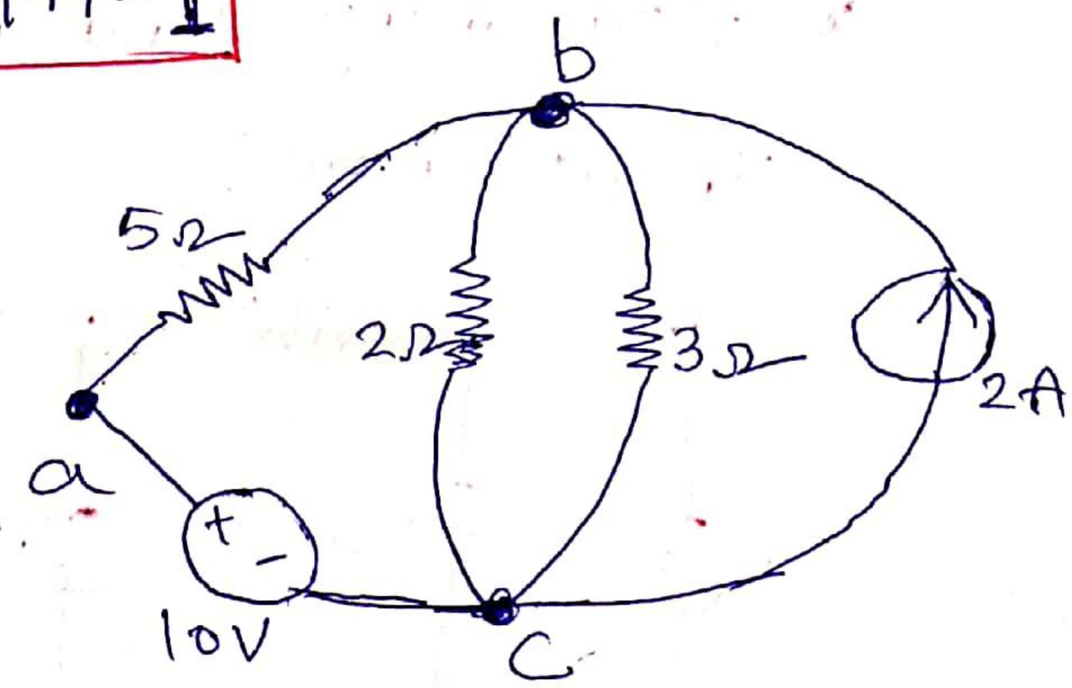
A network with 'b' branches, 'n' nodes and 'l' independent loops will satisfy the fundamental theorem of network topology

$$\boxed{b = l + n - 1}$$

Ex:-



(a)



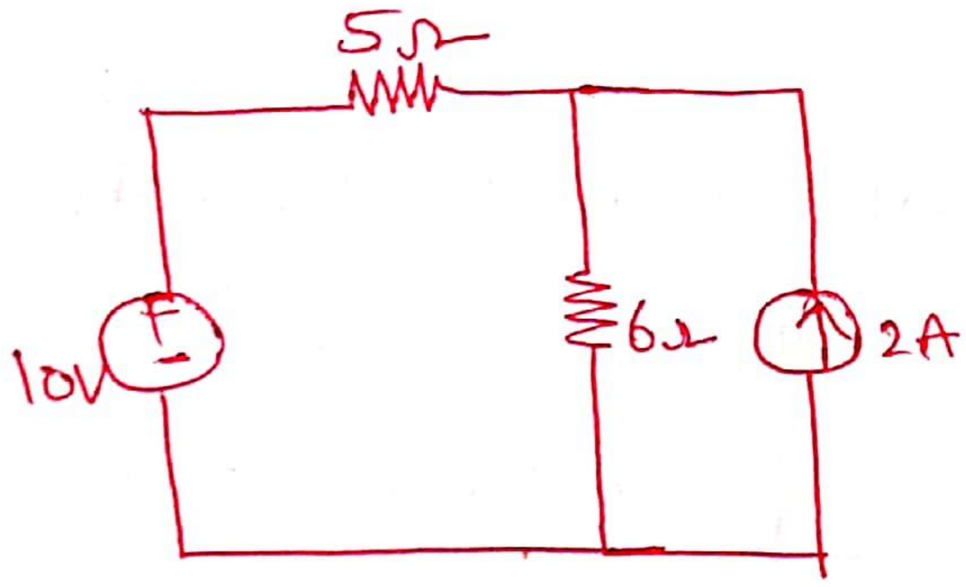
(b)

In above Example Branches: 5
Nodes : 3 (a, b & c)
loops : 6 (3 of them independent)

* Two or more Elements are in series if they cascaded or connected sequentially and consequently carry the same current.

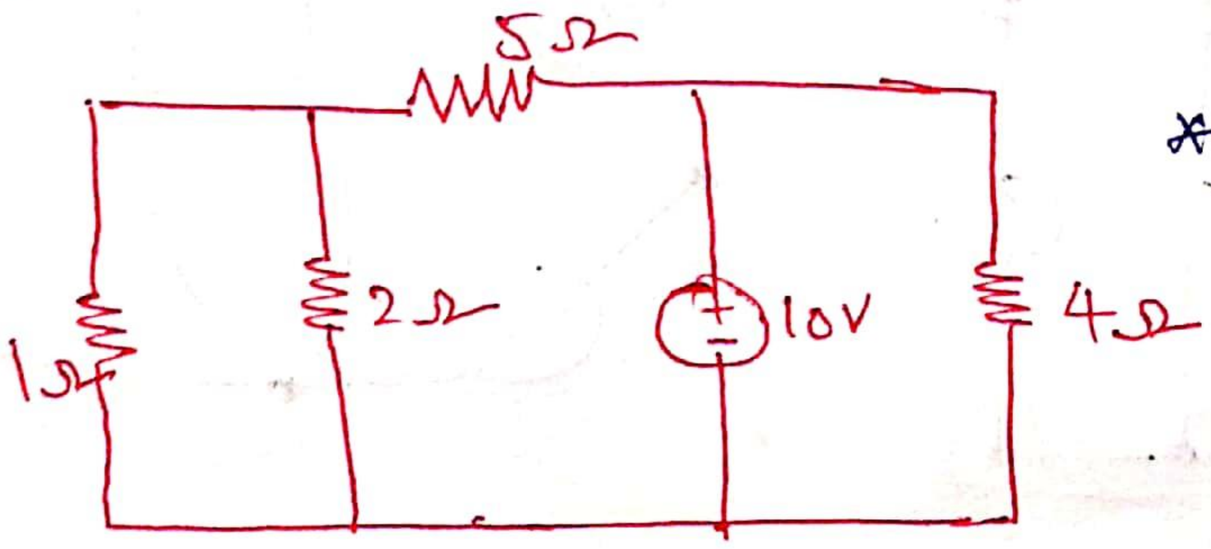
* Two or more Elements in parallel if they connected to the same two nodes and consequently have the same voltage across them.

* Determine the No. of branches and nodes in the ckt shown.



* No. of branches = No. of Elements = 4
* No. of Nodes = 3

* How many branches & nodes does the following circuit have?

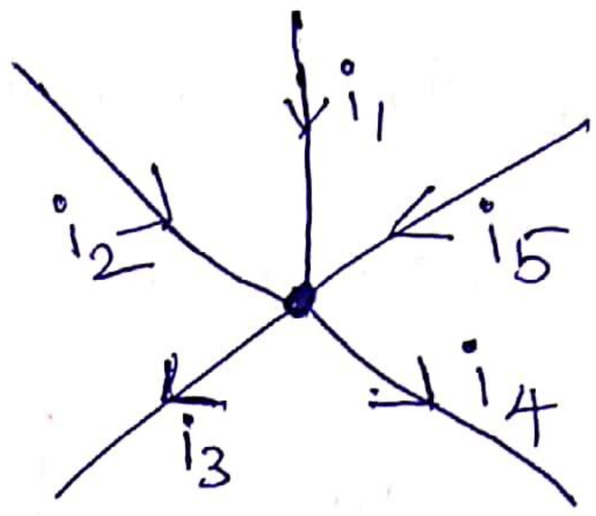


* No. of branches = 5
* No. of nodes = 3

* Kirchoff's Laws:-

→ **Kirchoff's Current Law (KCL):-** In the network, the algebraic sum of the currents at a particular node is equal to zero.

Explanation:-



* connection
incoming current = +ve
outgoing current = -ve
(or)
Vice versa

For above example i_1, i_2 & i_5 entering & i_3 & i_4 leaving

Then $i_1 + i_2 - i_3 - i_4 + i_5 = 0$

$$\boxed{\therefore \; i_1 + i_2 + i_5 = i_3 + i_4}$$

Alternative statement of KCL "The sum of the currents entering at a node is equal to the sum of the currents leaving that node."

* Sum of incoming current = Sum of outgoing current

→ KCL follows law of conservation of charge.

* sum of incoming charges at node = Sum of outgoing charges

Ex:- $\boxed{q_1 + q_2 + q_5 = q_3 + q_4}$

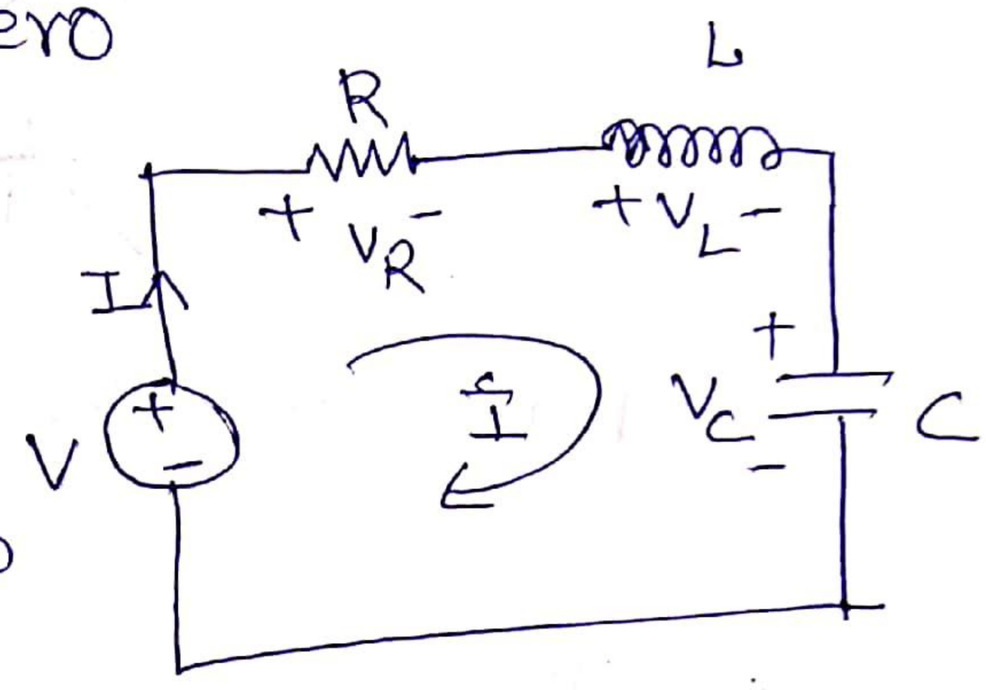
* Features of KCL:-

→ Applicable for any lumped ckt

→ Independent of Nature of Element

→ Follows law of Conservation of charge

* Kirchoff's Voltage Law:-(KVL) In network the algebraic sum of the branch voltages around the loop is Equal to zero



Explanation:- By Applying KVL to the loop we get $-V + V_R + V_L + V_C = 0$

$$\boxed{V = V_R + V_L + V_C}$$

Alternative Statement " the algebraic sum of the voltage sources in a closed loop is Equal to the sum of the voltage drops in that loop

* Features of KVL:-

→ Applicable to any lumped ckt

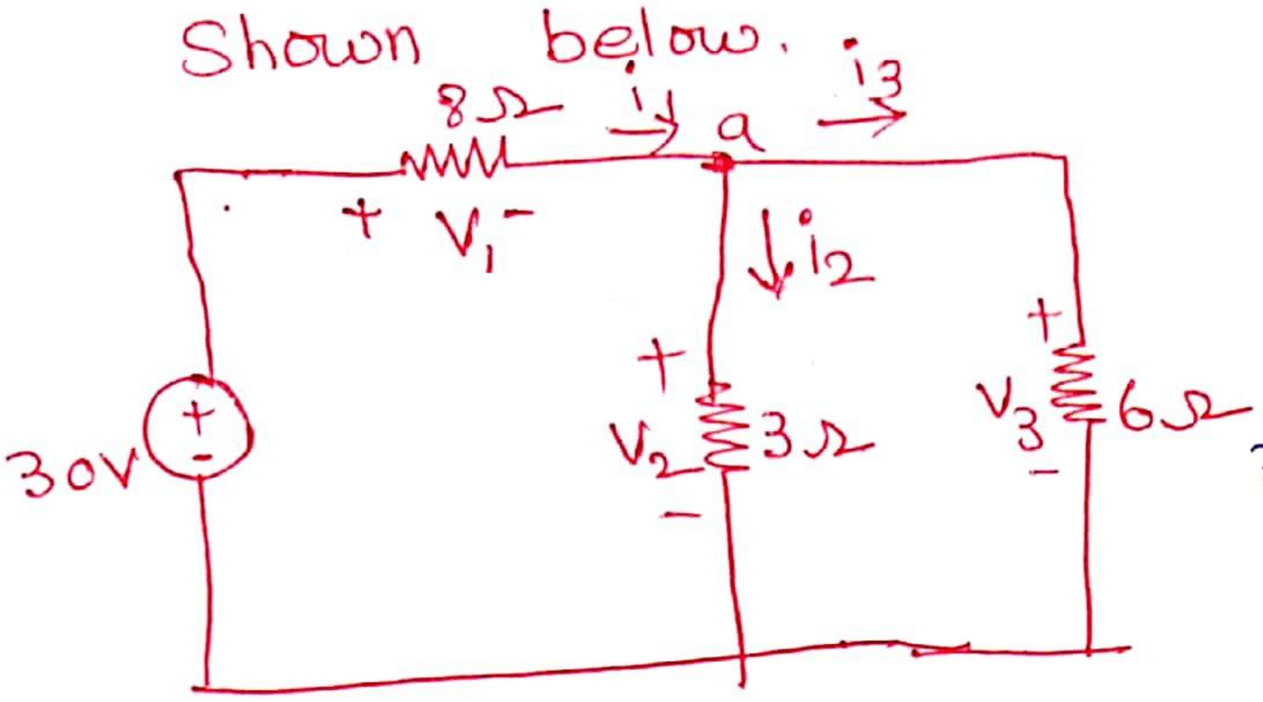
→ Independent of Nature of Element

→ Follows law of Conservation of Energy.

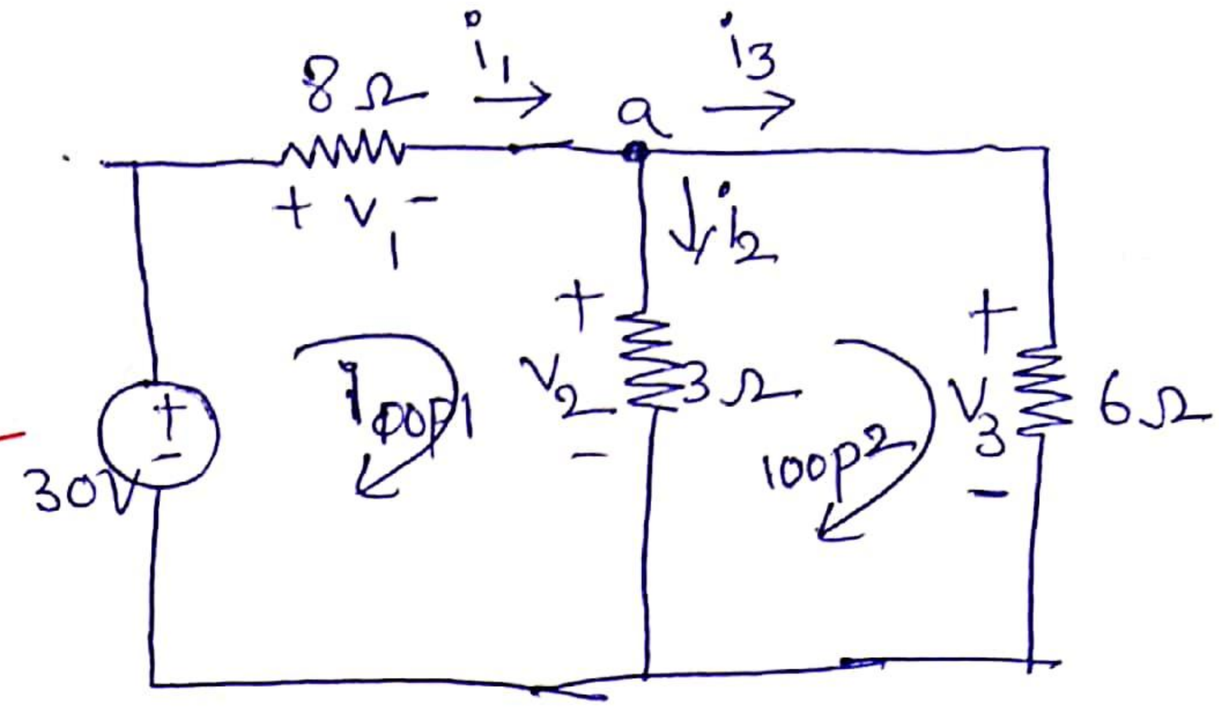
* Energy transferred = Energy absorbed

* Problems on KCL & KVL:-

Find the currents & voltages in the circuit shown below.



(a)



(b)

Sol:- we apply ohm's law and kirchoff's laws.

By ohm's law $V_1 = 8i_1$, $V_2 = 3i_2$, $V_3 = 6i_3 \rightarrow (1)$

At node 'a', applying KCL we get

$$\cancel{i_1 + i_2 =} \quad i_1 = i_2 + i_3 \text{ (or)}$$

$$i_1 - i_2 - i_3 = 0 \longrightarrow (2)$$

Applying KVL to loop 1 we get

$-30 + V_1 + V_2 = 0$ But $V_1 = 8i_1$ & $V_2 = 3i_2$

Substituting we get

$$-30 + 8i_1 + 3i_2 = 0$$

$$8i_1 = 30 - 3i_2$$

$$\boxed{i_1 = \frac{30 - 3i_2}{8}} \longrightarrow (3)$$

Applying KVL to loop 2 we get

$$V_3 - V_2 = 0$$

$$\therefore V_3 = V_2 \longrightarrow (4)$$

From ① $V_2 = 3i_2$ & $V_3 = 6i_3$

$$V_2 = V_3 \Rightarrow 3i_2 = 6i_3$$

$$i_2 = 2i_3$$

$$\therefore i_3 = \frac{i_2}{2} \rightarrow ⑤$$

Sub ③ & ⑤ in ② we get.

$$\frac{30-3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

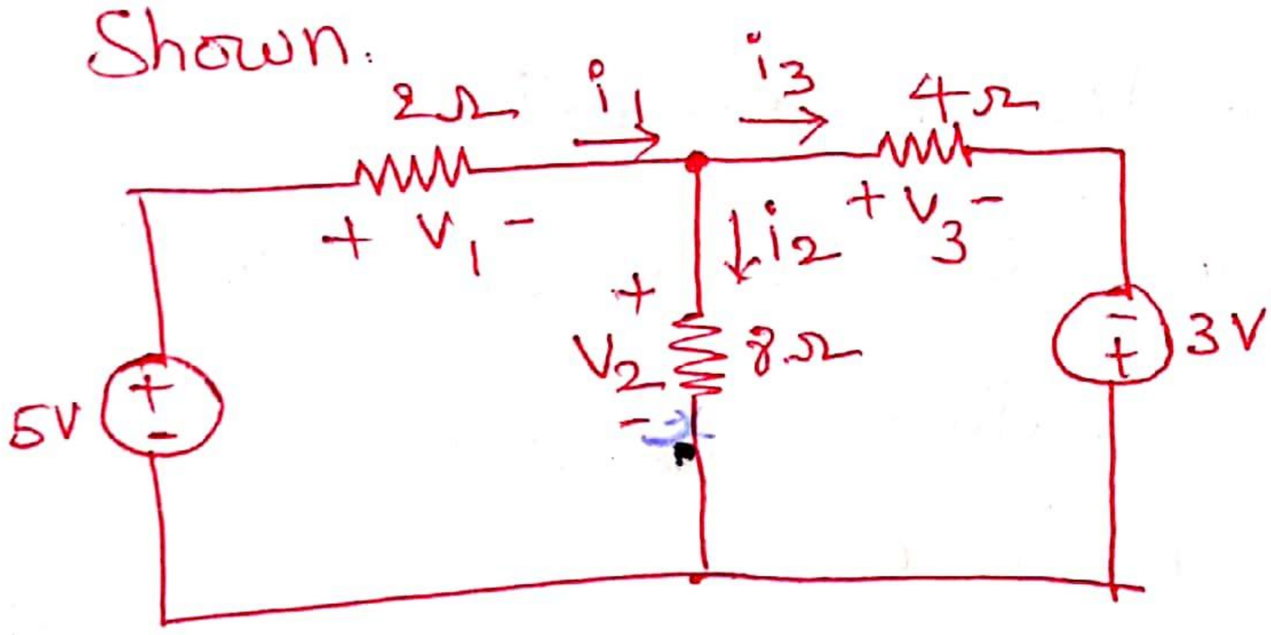
$$i_2 = 2A.$$

From the value of i_2, we can use Eqn. ① to ⑤ to obtain

$i_1 = 3A$, $i_3 = 1A$, $V_1 = 24V$, $V_2 = 6V$ & $V_3 = 6V$

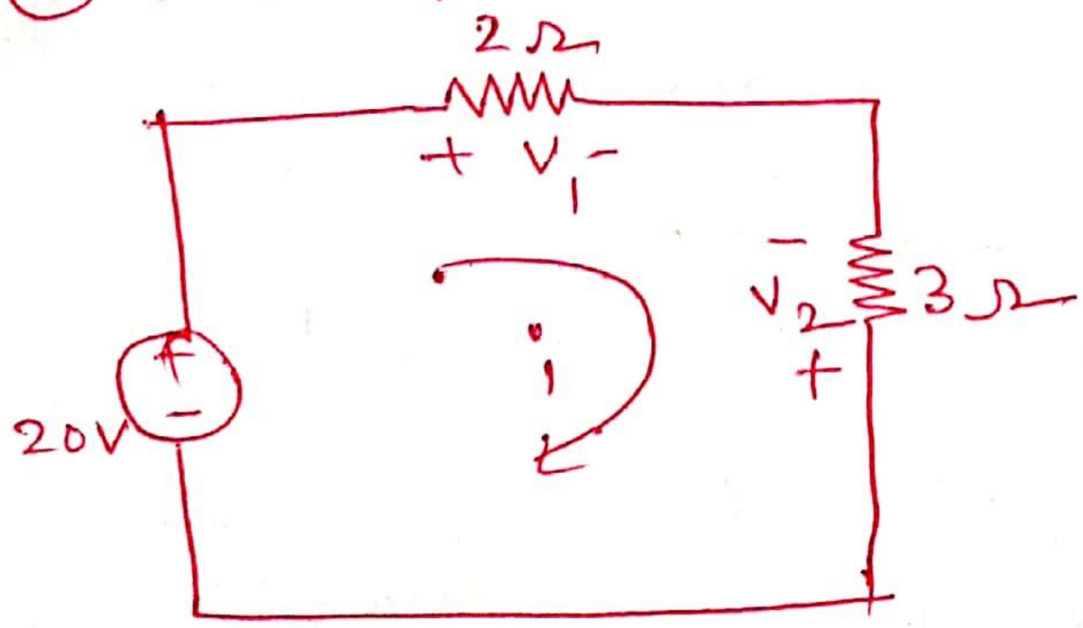
* Practice problem:-

① Find the currents & voltages in the ckt shown.



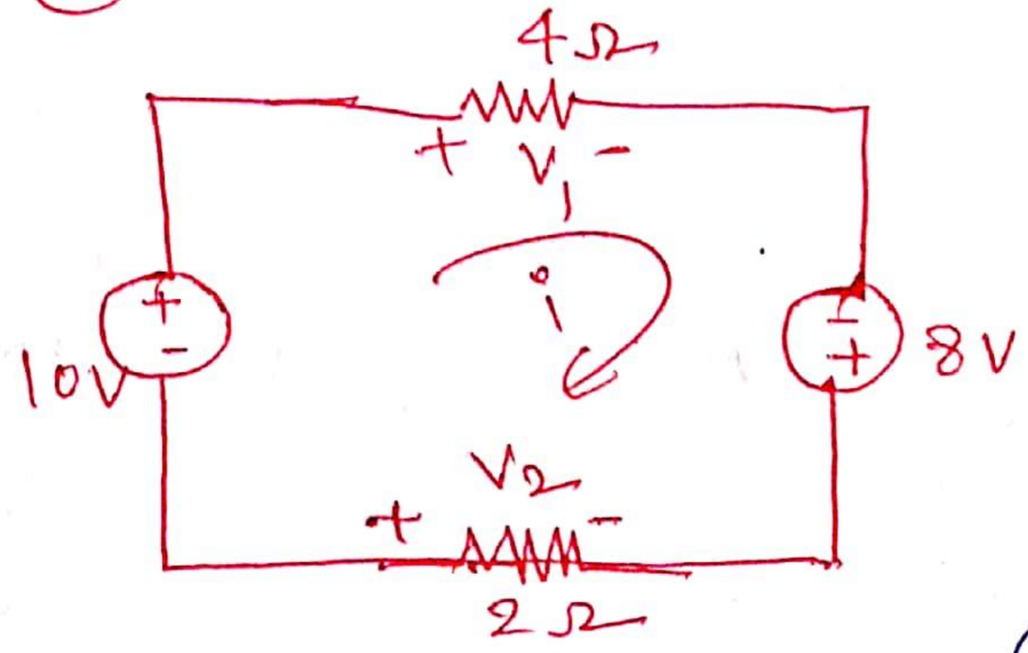
Ans:- $V_1 = 3V$, $V_2 = 2V$, $V_3 = 5V$; $i_1 = 1.5A$, $i_2 = 0.25A$, $i_3 = 1.25A$

② Find V_1 & V_2 in ckt



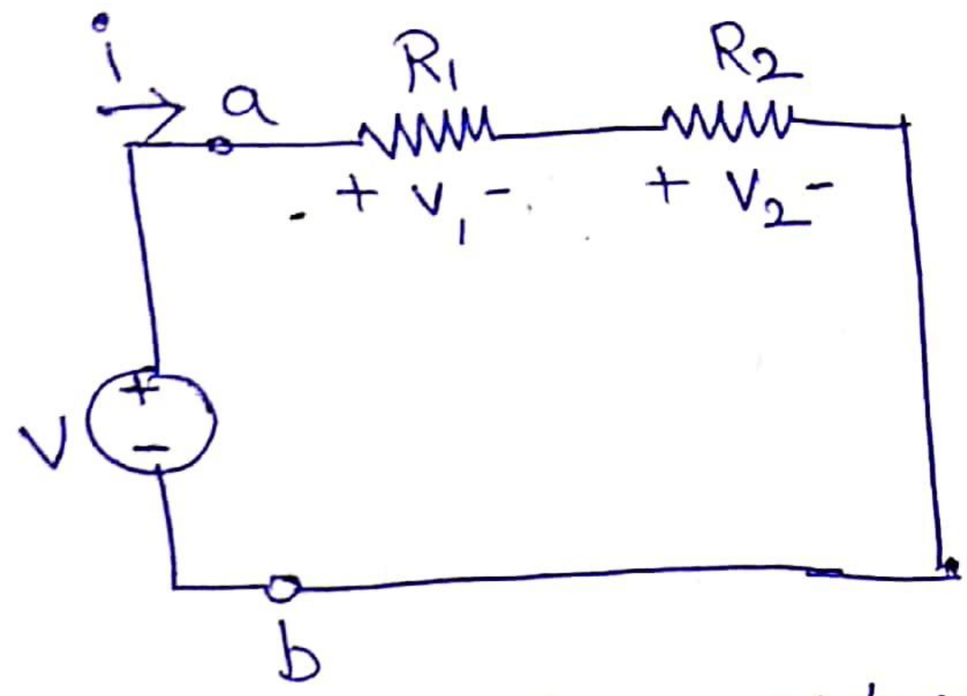
Ans:- $V_1 = 8V$, $V_2 = -12V$

③ Find V_1 & V_2 in ckt

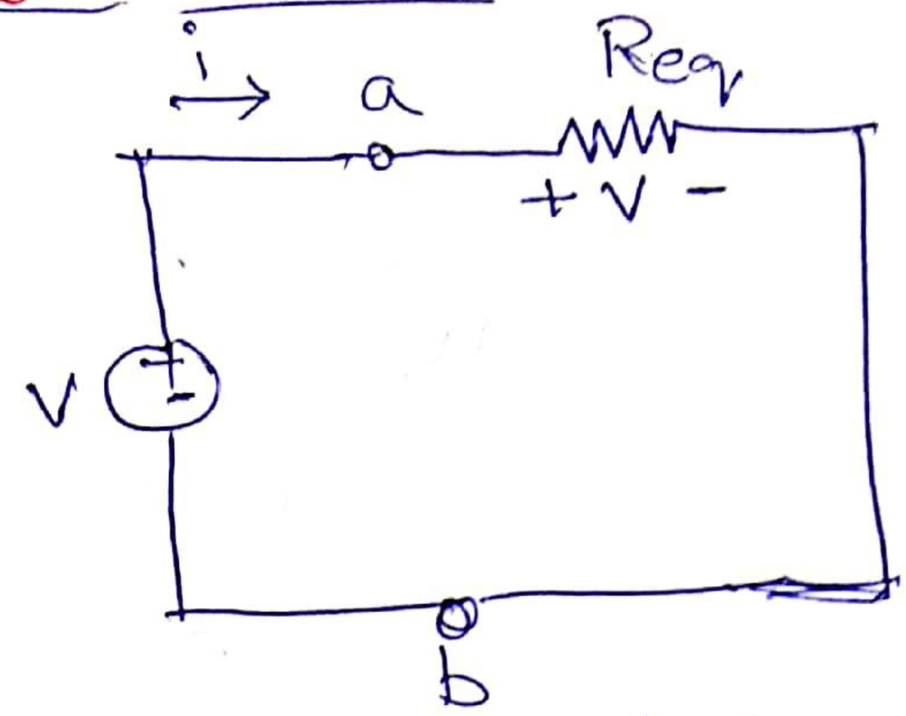


Ans:- $V_1 = 36V$, $V_2 = -18V$

* Series Resistors & Voltage Division :-



(a) Single-loop ckt with two resistors in Series



(b) Equivalent circuit of fig (a)

→ Applying ohm's law in fig (a) for each resistor we get $V_1 = iR_1$ & $V_2 = iR_2 \longrightarrow (1)$

→ Applying KVL to the loop, we get

$$-V + V_1 + V_2 = 0 \Rightarrow V = V_1 + V_2 \longrightarrow (2)$$

* $V = i(R_1 + R_2)$ → After substituting V_1 & V_2

$$\boxed{i = \frac{V}{R_1 + R_2}} \longrightarrow (3)$$

For fig (b) we write * $V = iR_{eq}$

where $R_{eq} = R_1 + R_2 \longrightarrow (4)$

* Equating both Equations

Note:- The Equivalent resistance of any number of resistors connected in series is the sum of the individual resistances

If 'N' resistors in series, then

$$\boxed{R_{eq} = R_1 + R_2 + \cdots R_N = \sum_{n=1}^{N} R_n}$$

— To determine the voltage across Each resistor,

Substitute ③ in ①

$$V_1 = \left(\frac{R_1}{R_1+R_2}\right)V \quad \& \quad V_2 = \left(\frac{R_2}{R_1+R_2}\right)V$$

Voltage Division principle

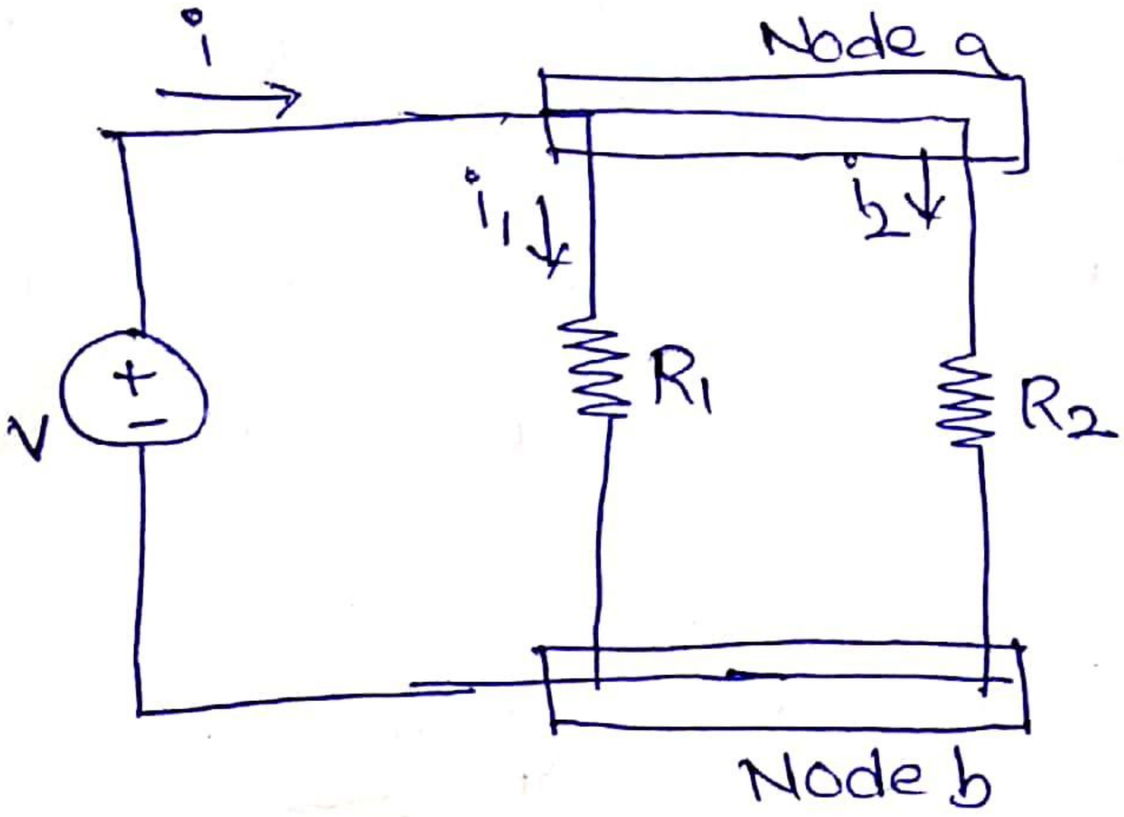
fig (a) called voltage divider circuit

*NOTE:- In general if a voltage divider has 'N' resistors ($R_1, R_2 --- R_N$) in series with the source voltage V, the nth resistor (R_n) will have a drop of

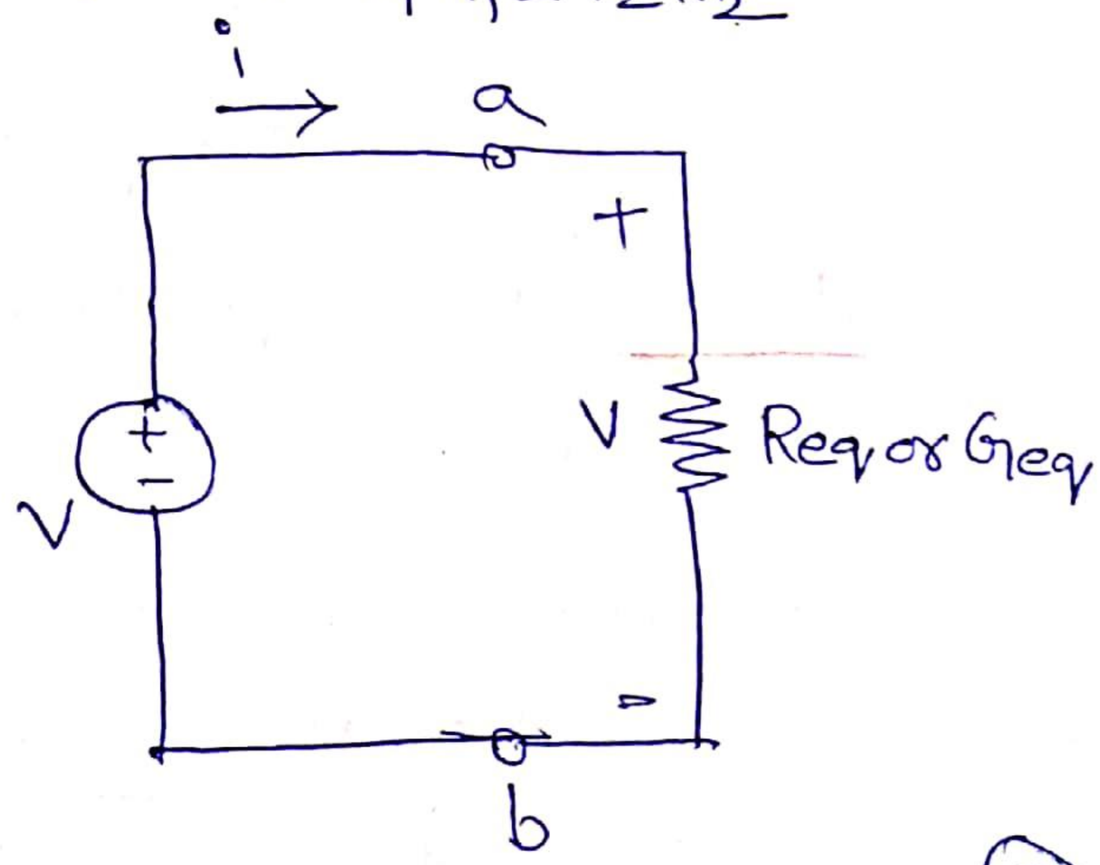
$$\boxed{V_n = \left(\frac{R_n}{R_1+R_2+---R_N}\right)V}$$

* parallel Resistors & Current Division:-

If two resistors are Connected in parallel and therefore have the Same Voltage across them. from ohm's law $V = i_1R_1 = i_2R_2$



(a) Two resistors in parallel



(b) Equivalent ckt.

from fig. (a) $i_1 = \frac{V}{R_1}$, $i_2 = \frac{V}{R_2}$ → (1)

Applying KCL at node 'a' gives the total current i as $i = i_1 + i_2$ → (2)

Sub (1) in (2) we get

$$i = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}} \rightarrow (3)$$

where R_{eq} is the Equivalent resistance of the resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow (4)$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}} \rightarrow (5)$$

NOTE:- The Equivalent resistance of two parallel resistors is Equal to the product of their resistances divided by their Sum

In general with 'N' resistors parallel The Equivalent resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \rightarrow (6)$$

$$\boxed{G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N}$$

Equivalent conductance

* Conductances in parallel behave as a single conductance whose value is Equal to the sum of the individual conductances

The Equivalent conductance G_{eq} of N resistors in series given as

$$\boxed{\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \cdots \frac{1}{G_N}} \rightarrow (7)$$

→ Given the total current 'i' entering node 'a', then how to obtain i_1 & i_2? we know that the Equivalent resistor has the same voltage

$$V = i R_{eq} = i\left(\frac{R_1 R_2}{R_1 + R_2}\right) \rightarrow (8)$$

Sub. (8) in (1) we get

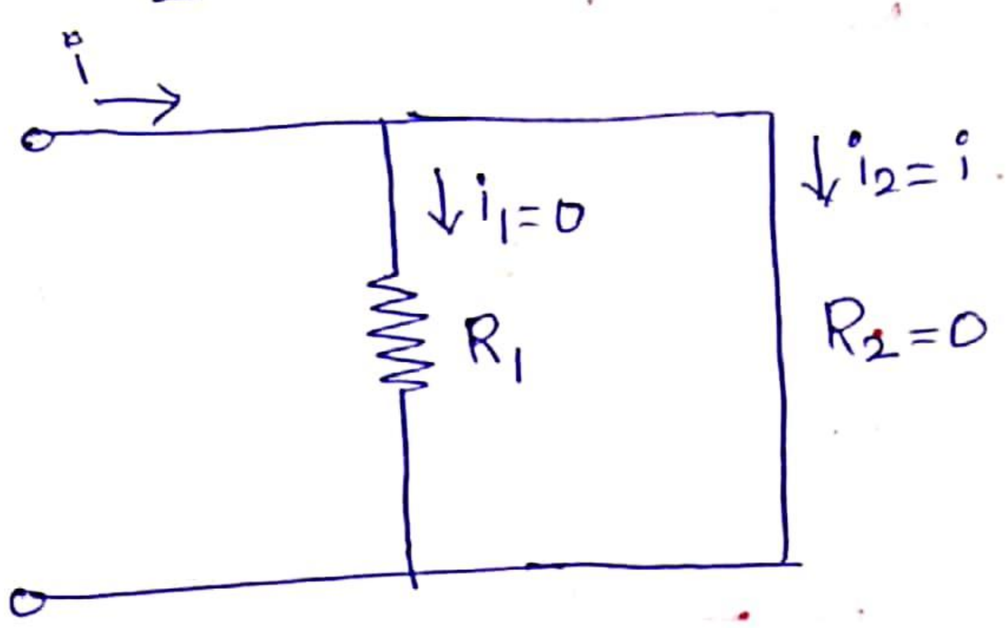
$$i_1 = \frac{V}{R_1} = i\left(\frac{\cancel{R_1} R_2}{R_1 + R_2}\right) \times \frac{1}{\cancel{R_1}} = \left(\frac{R_2}{R_1 + R_2}\right) i$$

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i \quad \text{|||ly} \quad i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i$$

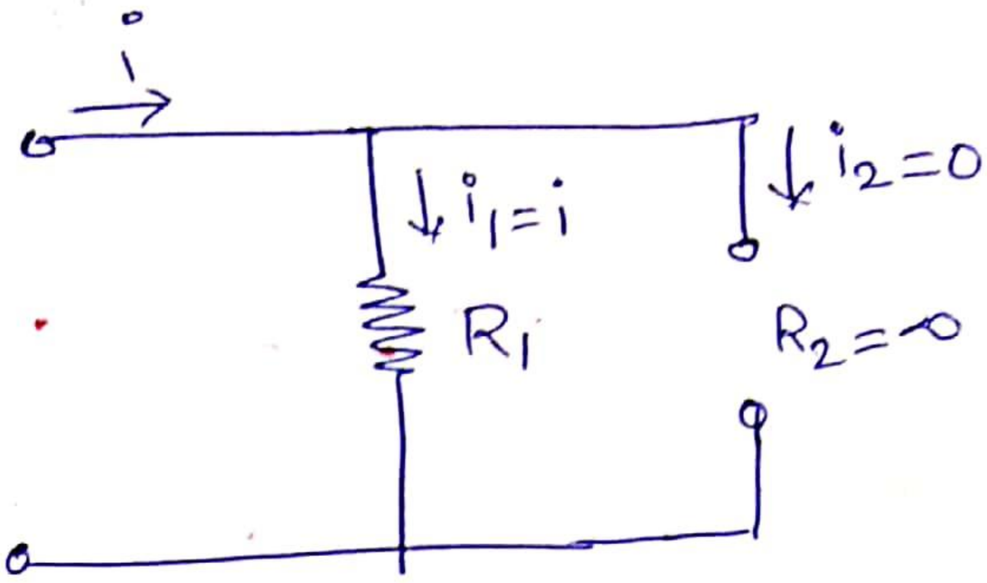
Principle of current division or Rule

The ckt in fig (a) called "current divider"

* Case (i) one of resistor is zero say $R_2 = 0$, that is R_2 is a short circuit



(a) short circuit [case (i)]



(b) open circuit [case (ii)]

For case (i) R_2 is a short circuit, its resistance is equal to zero $\therefore R_2 = 0$

when $R_2 = 0 \Rightarrow i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i = 0$

$$\boxed{i_1 = 0}$$

Then entire current flows through the short circuited path.

$$\boxed{\therefore R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = 0} \quad (\because R_2 = 0)$$

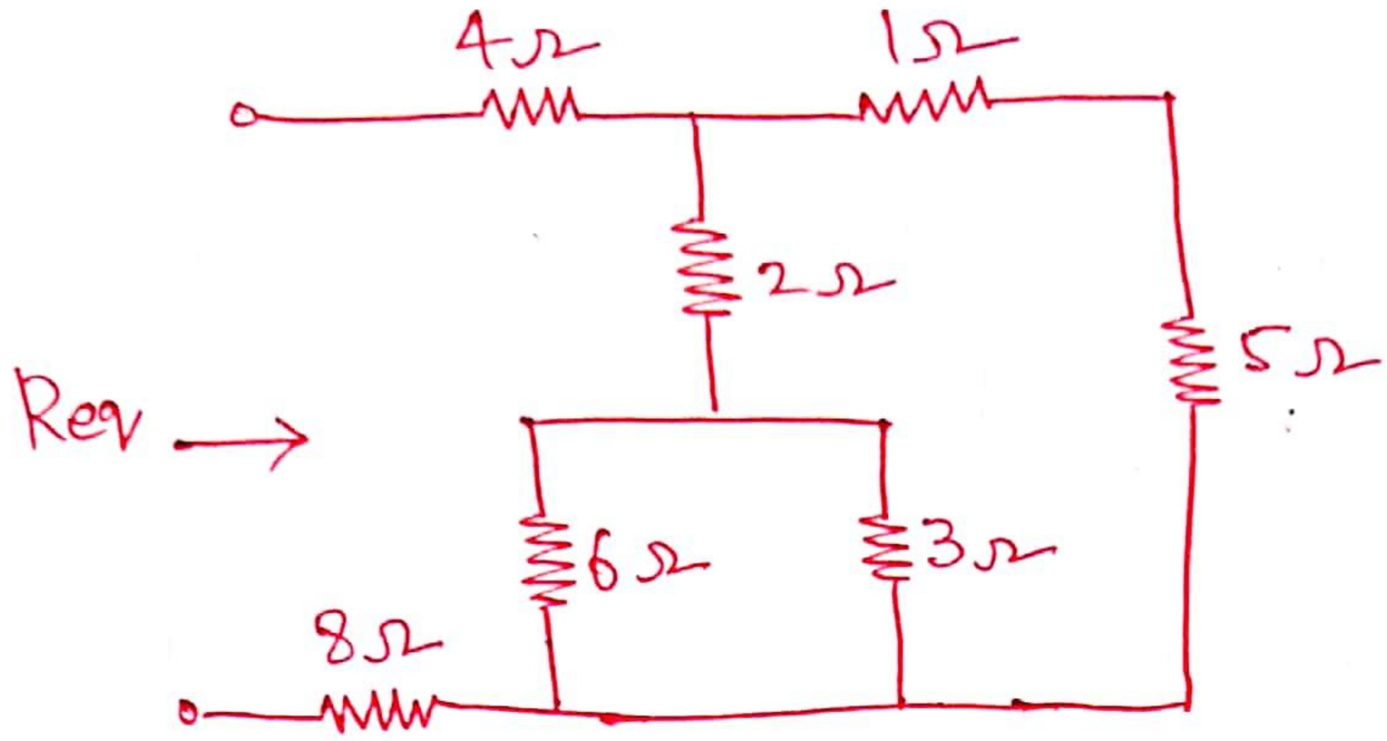
* Case (ii) Suppose $R_2 = \infty$, that is R_2 is an open circuit. The current still flows through the path of least resistance R_1.

$$\therefore R_{eq} = \frac{R_1 \cancel{R_2}}{\cancel{R_2}\left(1 + \frac{R_1}{R_2}\right)} = \lim_{R_2 \to \infty} \frac{R_1}{1 + \frac{R_1}{R_2}} = R_1$$

$$i_1 = \lim_{R \to \infty} \left[\frac{1}{1 + \frac{R_1}{R_2}}\right] i \qquad \boxed{R_{eq} = R_1} \Rightarrow i_1 = \left[\frac{\cancel{R_2}}{\cancel{R_2}\left(1 + \frac{R_1}{R_2}\right)}\right] i$$

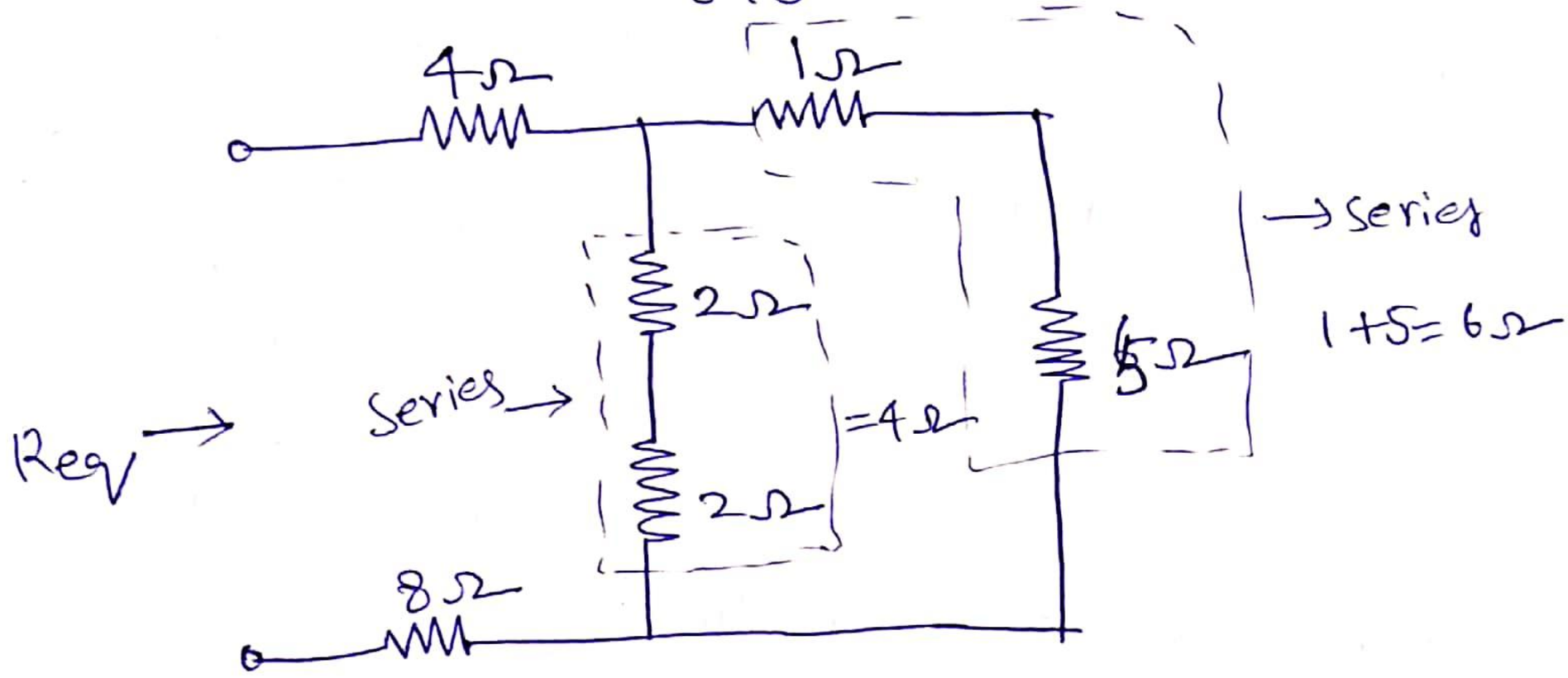
$$\boxed{i_1 = i}$$

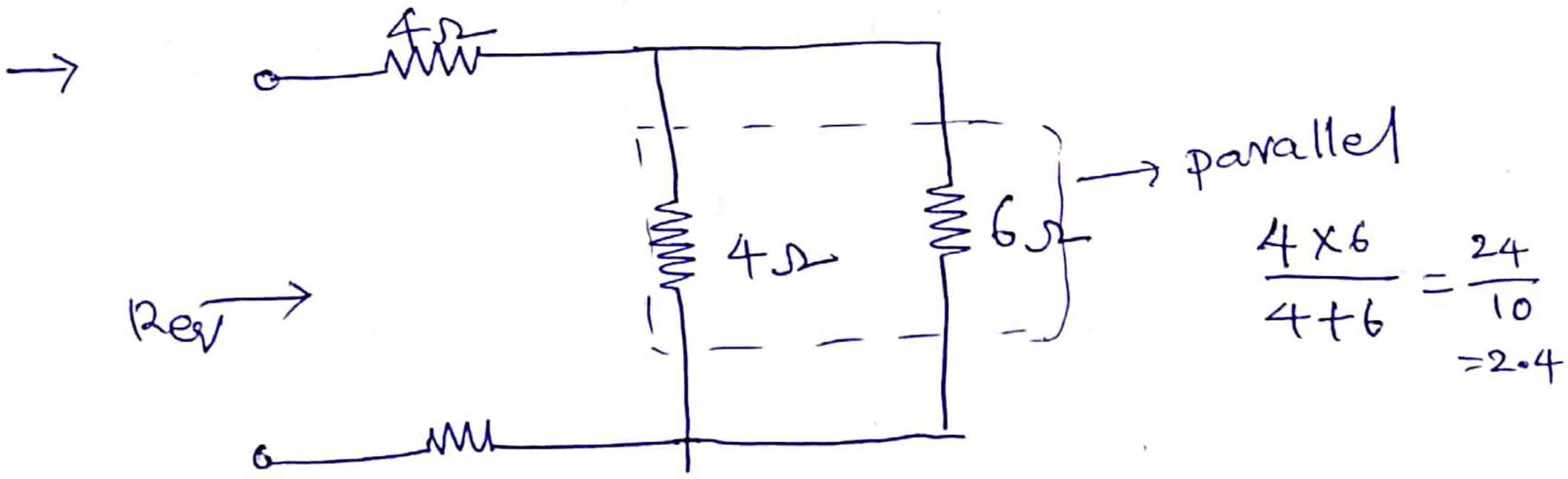
* find Req for the circuit shown below



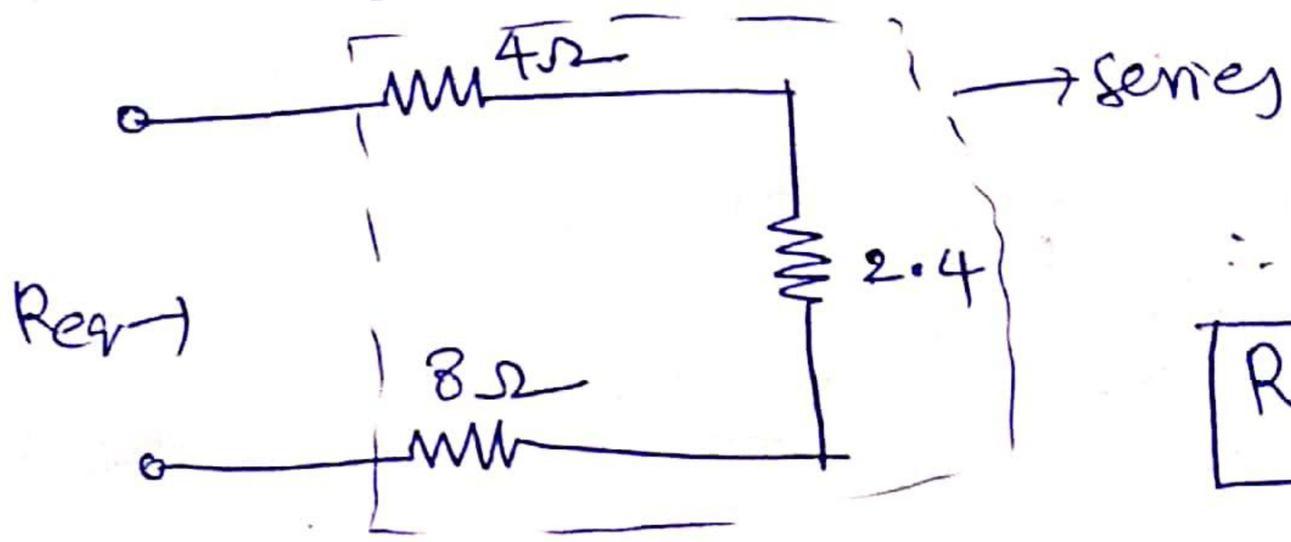
Sol: → The 6Ω & 3Ω resistors are in parallel

$$6\Omega \| 3\Omega = \frac{6 \times 3}{6+3} = 2\Omega$$





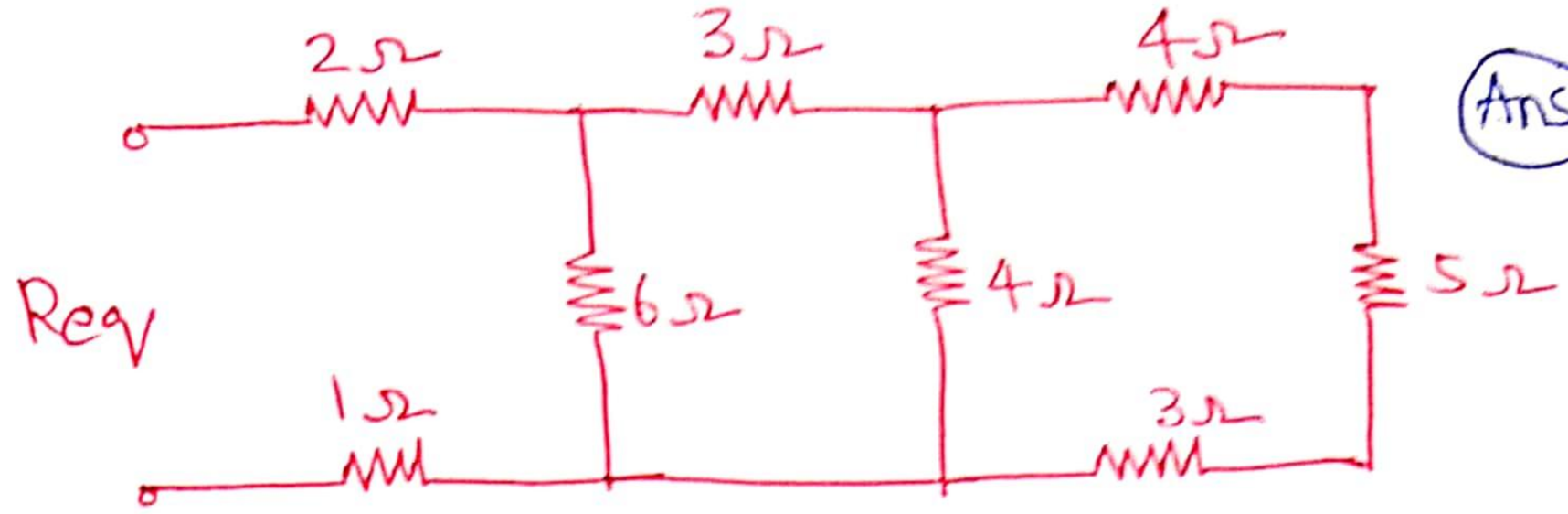
$$\frac{4 \times 6}{4+6} = \frac{24}{10} = 2.4$$



$\therefore Req = 4\Omega + 2.4\Omega + 8\Omega$

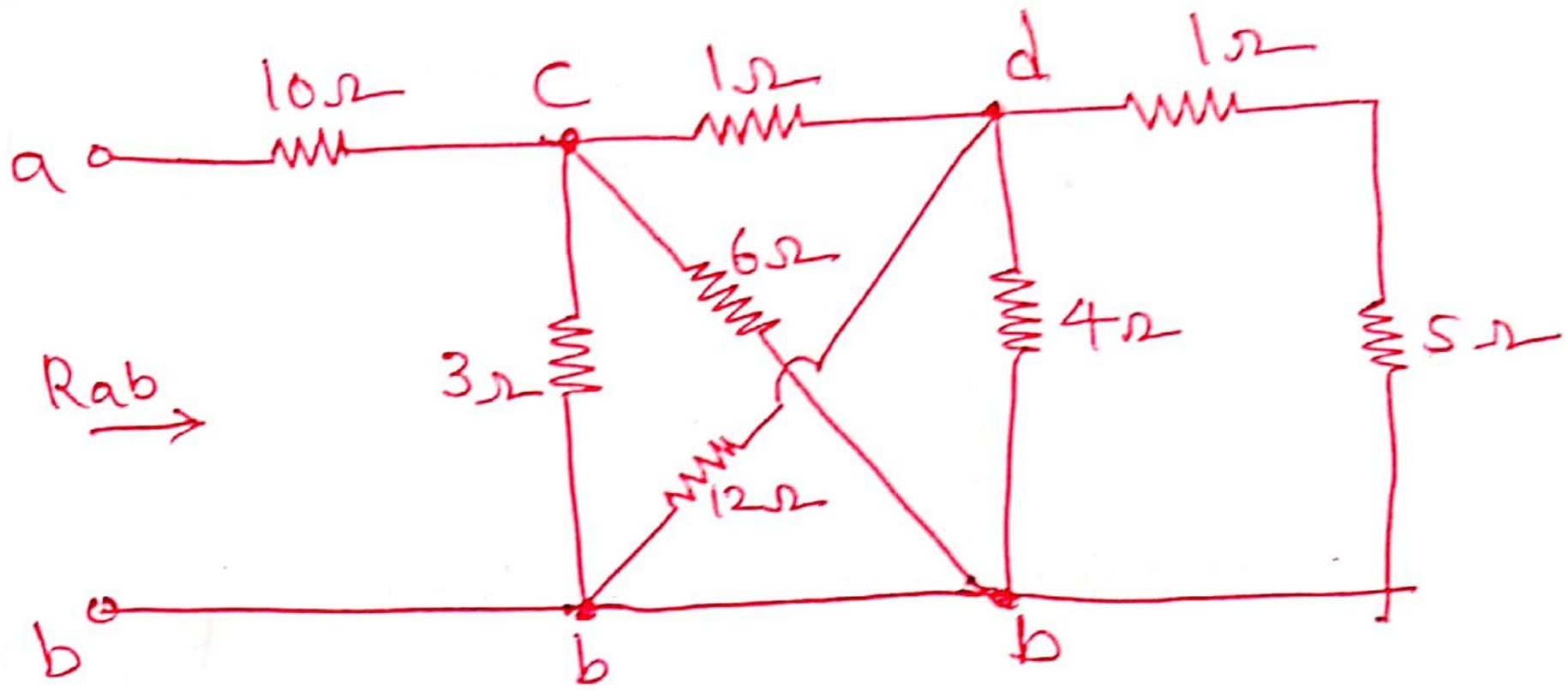
$$\boxed{Req = 14.4\Omega}$$

* Find Req for the Circuit Shown below



Ans:- Req = 6Ω

* Find the Equivalent resistance Rab in the Circuit Shown below



Sol- 3Ω & 6Ω resistors are in parallel because they connected to Same nodes b & c.

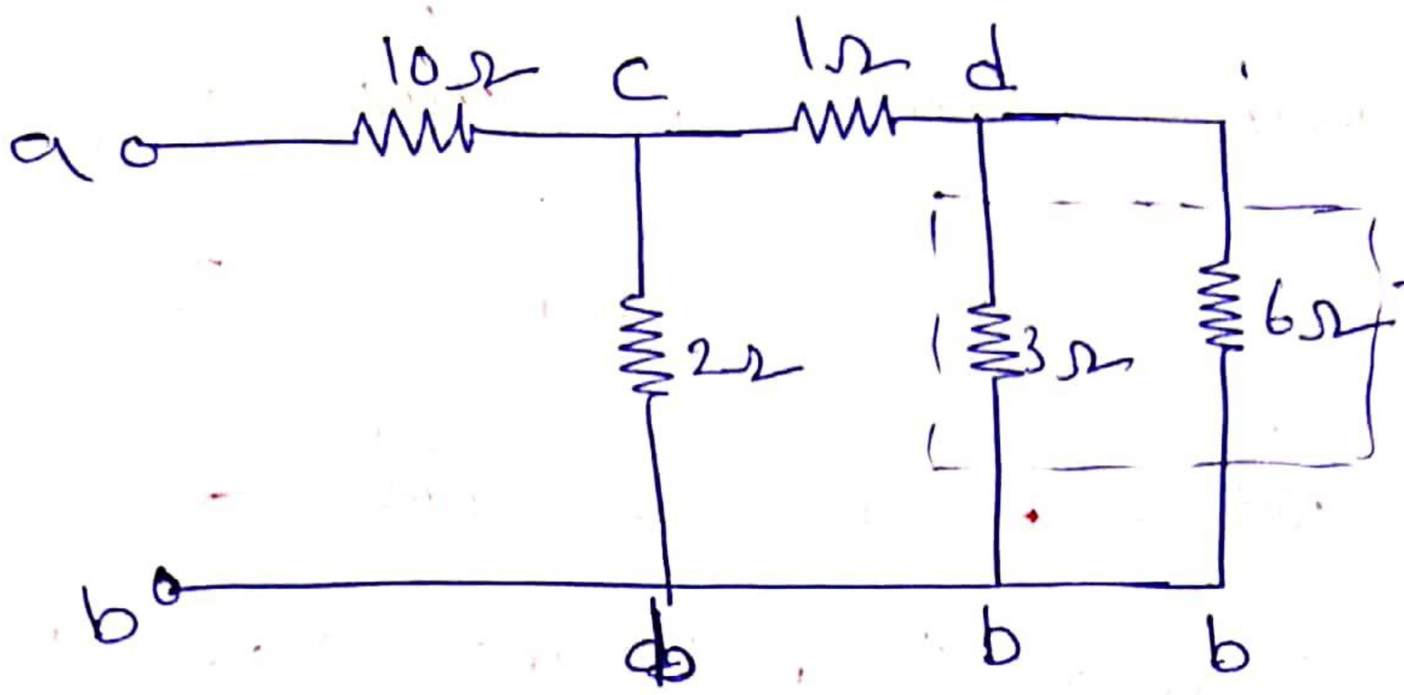
$$\therefore 3\Omega \parallel 6\Omega = \frac{3\times 6}{3+6} = 2\Omega$$

||ly 12Ω & 4Ω resistors are in parallel because they connected to same nodes b & d.

$$\therefore 4\Omega \parallel 12\Omega = \frac{12\times 4}{12+4} = 3\Omega$$

1Ω & 5Ω in series $1+5 = 6\Omega$

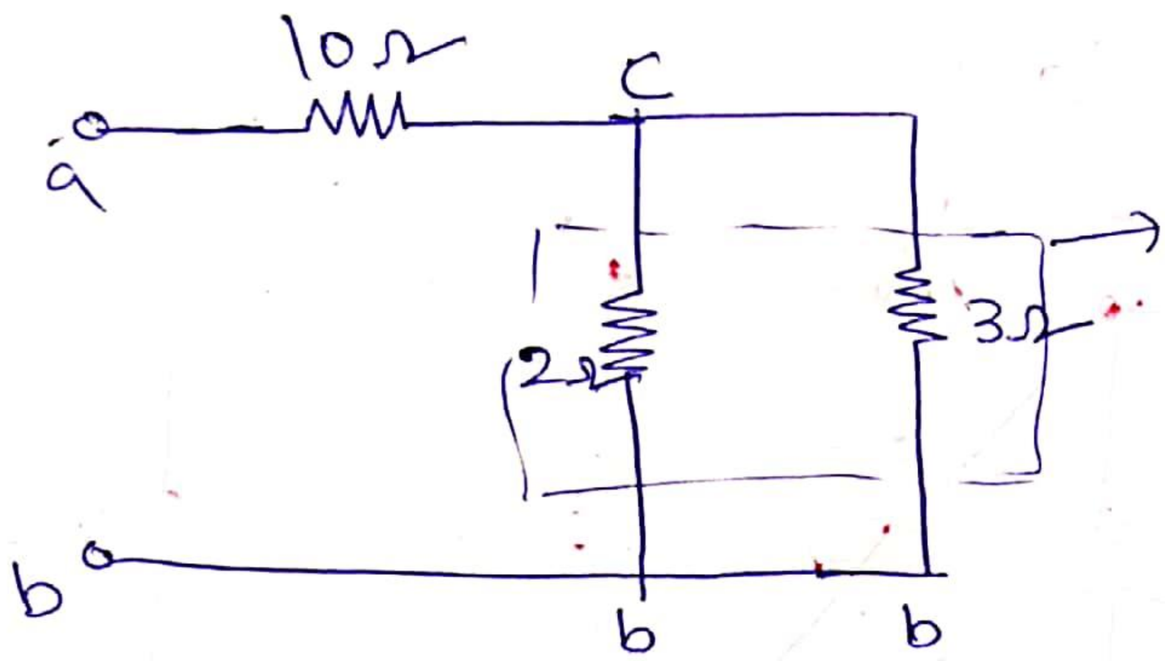
∴ The reduced circuit is



→ parallel

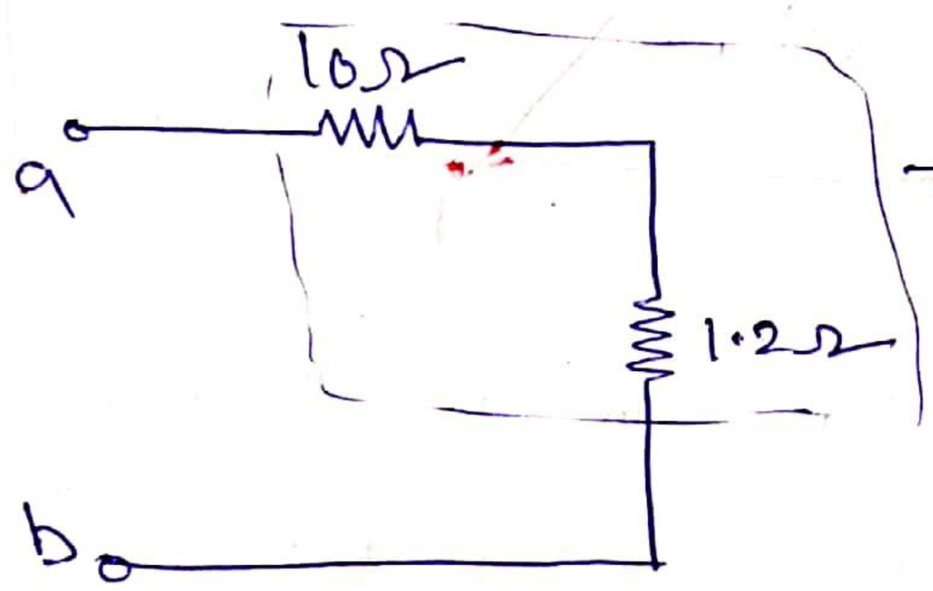
$\frac{3 \times 6}{3+6} = \frac{18}{9}$

$\Rightarrow 2\Omega$



→ parallel

$\frac{2 \times 3}{2+3} = 1.2\Omega$

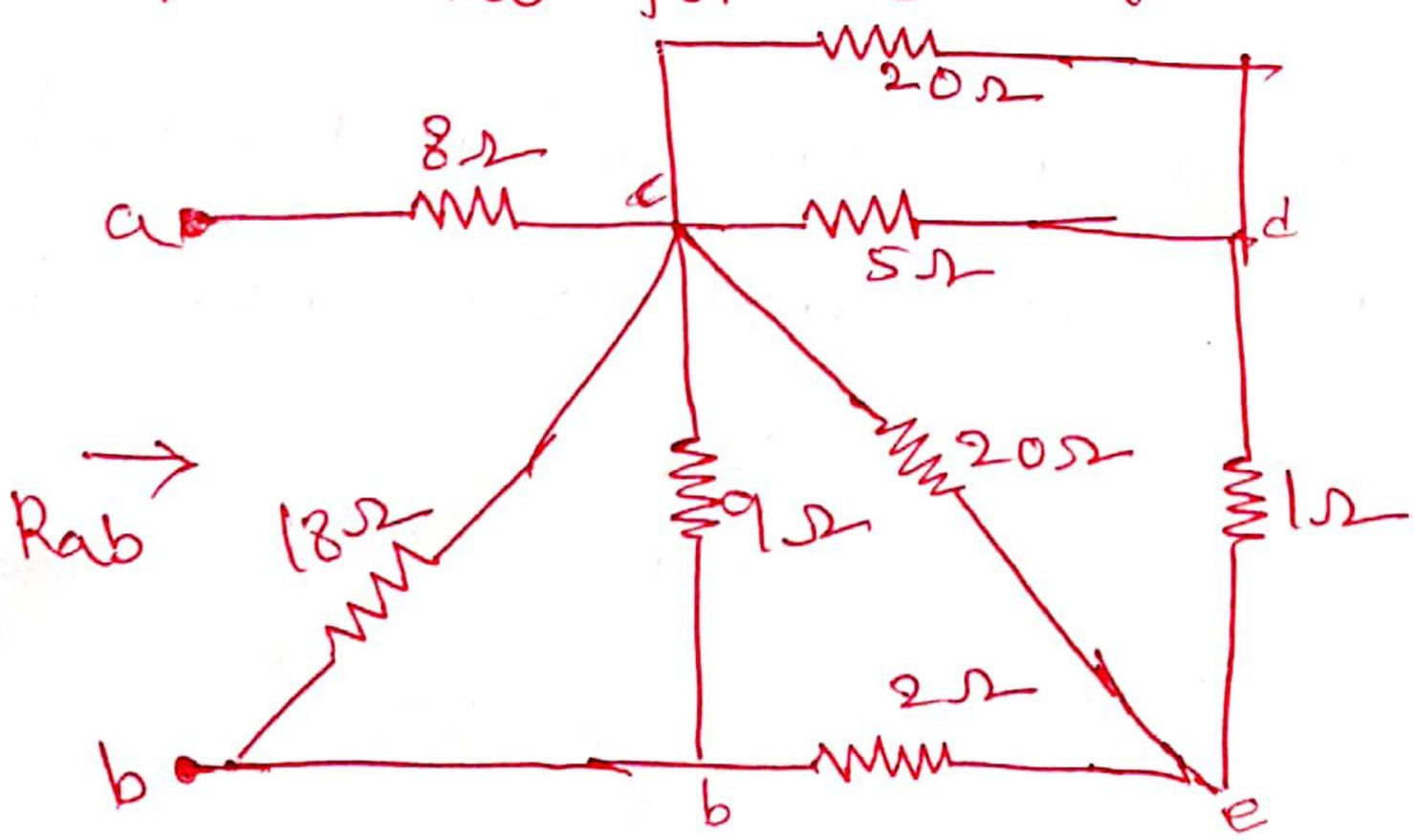


→ series

∴ $R_{ab} = 10 + 1.2\Omega$

$\boxed{R_{ab} = 11.2\Omega}$

* Find Rab for the ckt shown below



$R_{ab} = 11\Omega$

* Inductors in Series:-

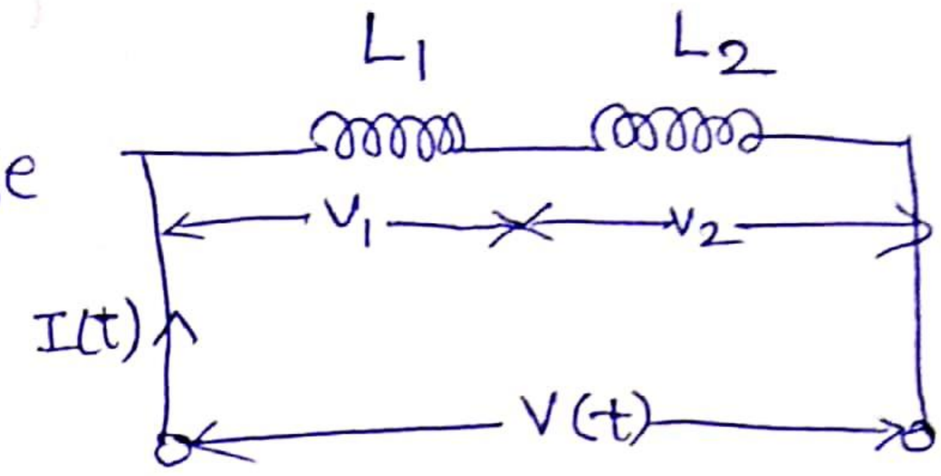


fig. Inductors in Series

Let a time-varying voltage $V(t)$ be applied across them. It is Evident that a time-varying Current $I(t)$ results, and voltages V_1, V_2 develop across the inductors, with instantaneous polarities

From fig. we have $V_1 = L_1 \frac{dI}{dt}$, $V_2 = L_2 \frac{dI}{dt}$

By applying KVL, $V(t) = V_1 + V_2$

$$V(t) = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$

$$V(t) = (L_1 + L_2) \frac{dI}{dt}$$

But. $V(t) = L_{eq} \frac{dI}{dt}$

$$L_{eq} \cancel{\frac{dI}{dt}} = (L_1 + L_2) \cancel{\frac{dI}{dt}}$$

$$\boxed{L_{eq} = L_1 + L_2}$$

* Inductors in parallel:-

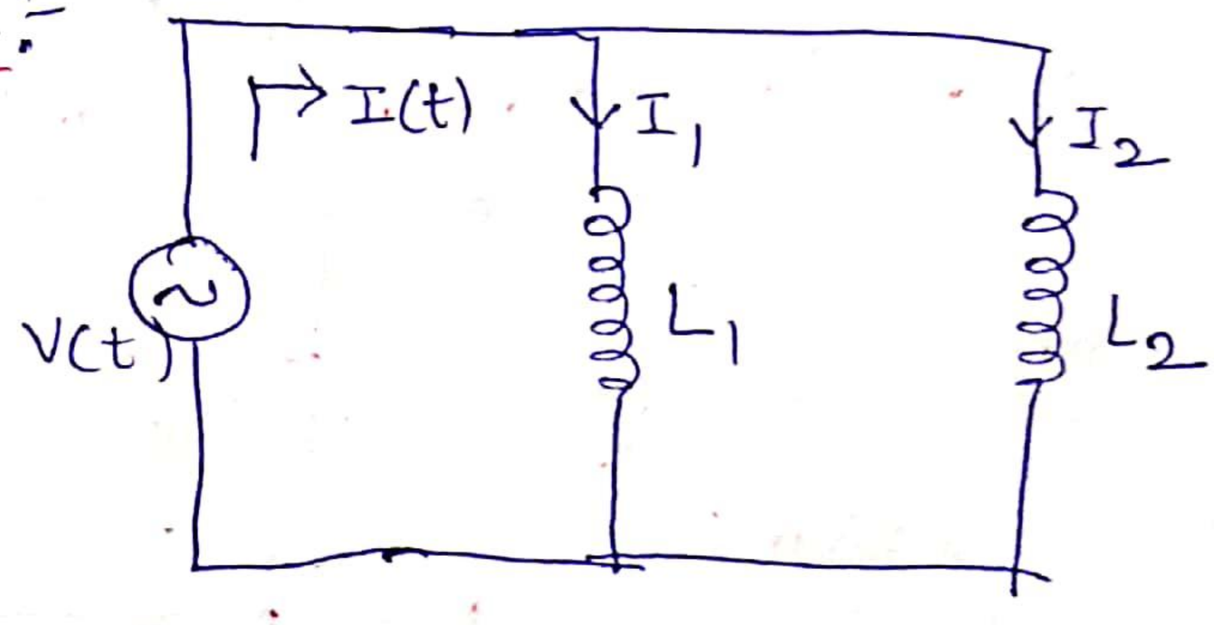


fig. Inductors in parallel

Applying KCL, we can write

$$I(t) = I_1 + I_2$$

But $I_1 = \frac{1}{L_1}\int V.dt$; $I_2 = \frac{1}{L_2}\int V.dt$

$$\frac{1}{L_{eq}}\int V\,dt = \frac{1}{L_1}\int V\,dt + \frac{1}{L_2}\int V\,dt$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \quad (\text{or})$$

$$\boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$$

Thus it is seen that

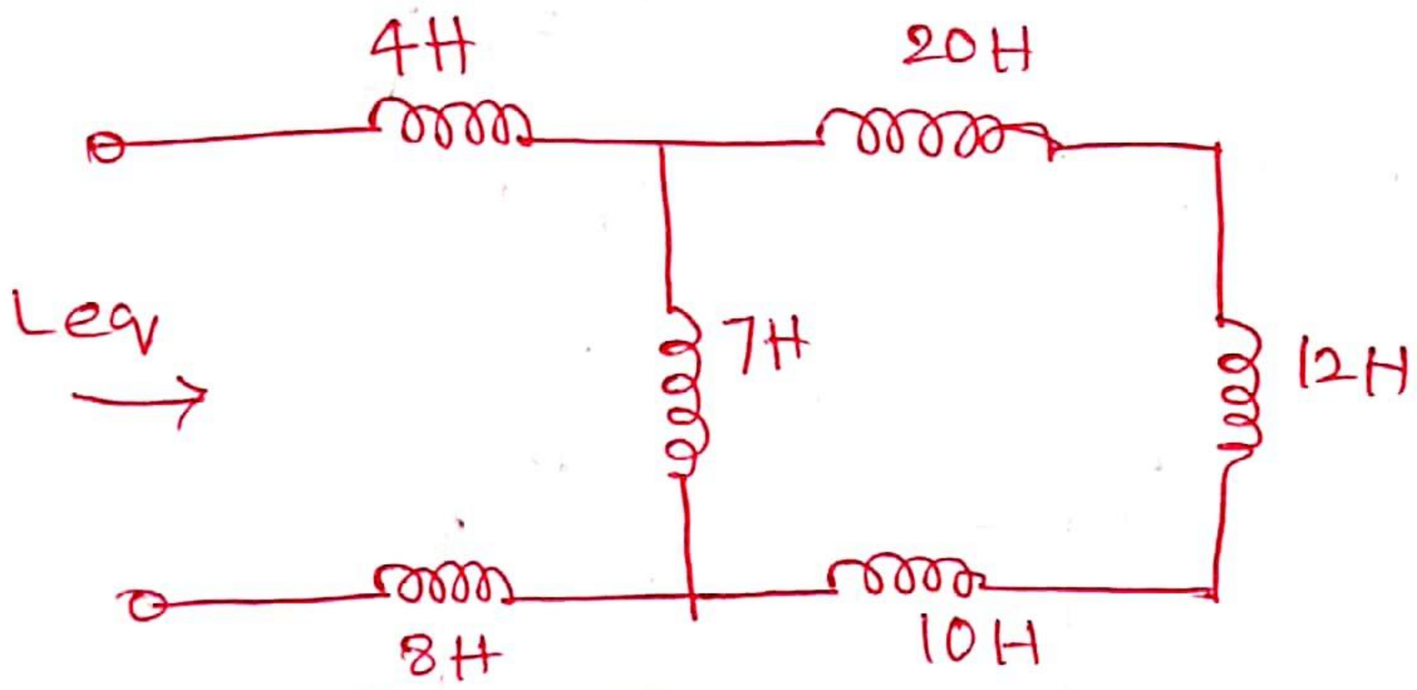
(i) $L_{eq} = \Sigma L$ for series combination of inductors

(ii) $\frac{1}{L_{eq}} = \Sigma \frac{1}{L}$ for parallel " "

* Problems:-

* ~~Capacit~~

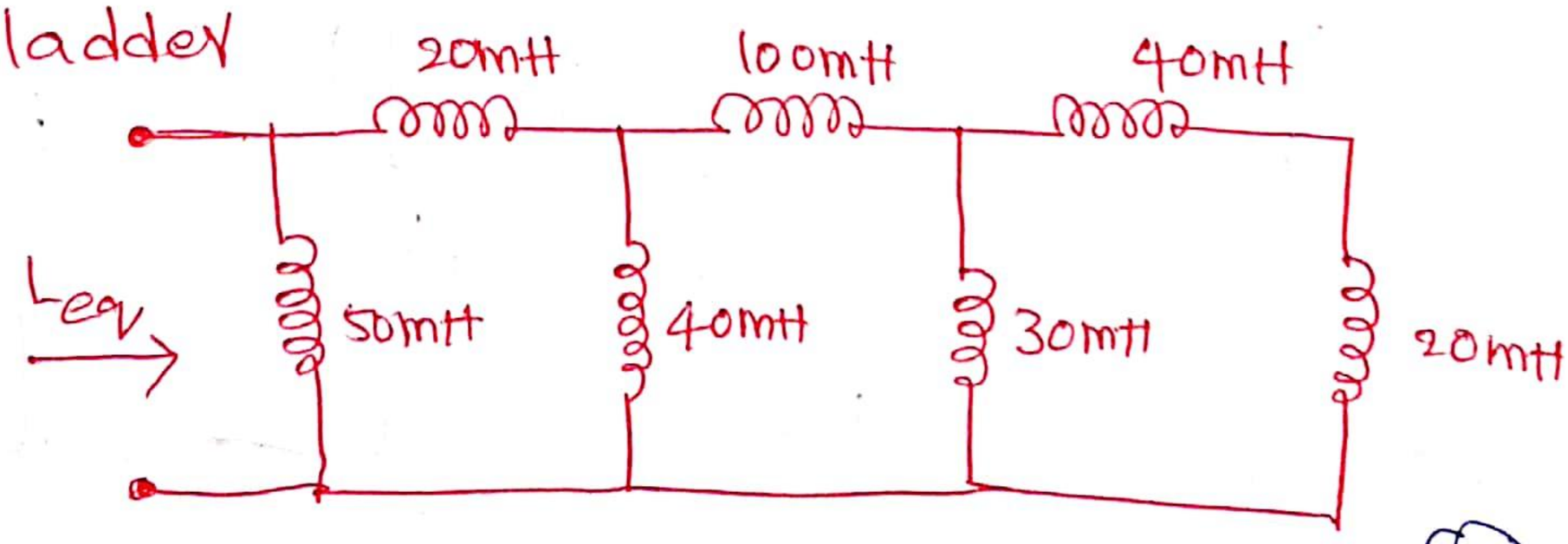
→ Calculate equivalent inductance of the circuit shown



Ans:-

$L_{eq} = 18\,H$

→ Calculate the equivalent inductance for the inductive ladder



Ans:-

$L_{eq} = 25\,mH$

* Capacitors in ~~parallel~~ Series:-

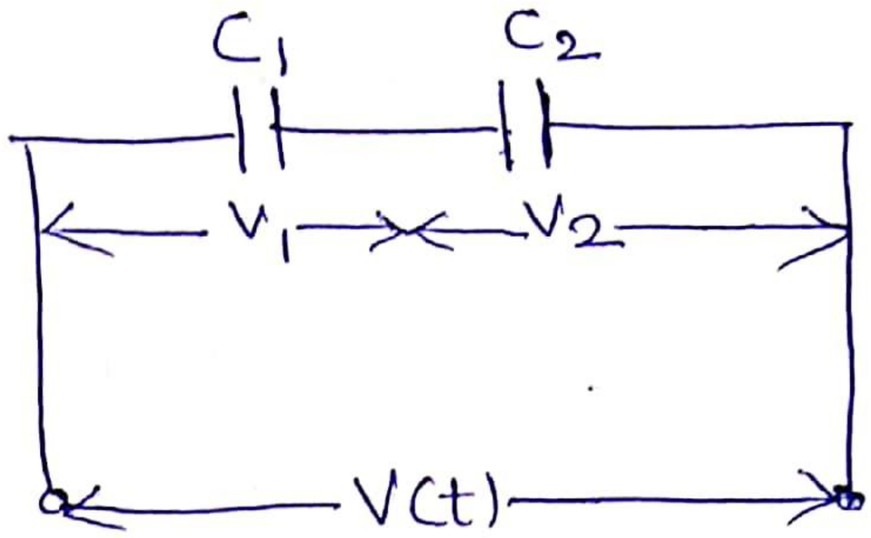


*fig. Capacitors in Series

Consider Several Capacitors connected in Series. The same charge 'Q' accumulates on Each one of the Capacitor plates when a voltage v(t) is applied. Since the capacitances are different, different Voltage develop across the Capacitors.

By KVL, we have $V = V_1 + V_2 \longrightarrow (1)$

But $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \longrightarrow (2)$$

If now the Capacitors are replaced by an Equivalent Capacitor C_{eq}, we have $V = \frac{Q}{C_{eq}} \rightarrow (3)$

Sub (3) in (2)

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\boxed{\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}} \text{ (or)}$$

$$\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

* capacitors in parallel:-

Let $v(t)$ be the voltage applied across capacitors C_1 & C_2 connected in parallel. Let Q_1, Q_2, are the charges which accumulate on the capacitor plates

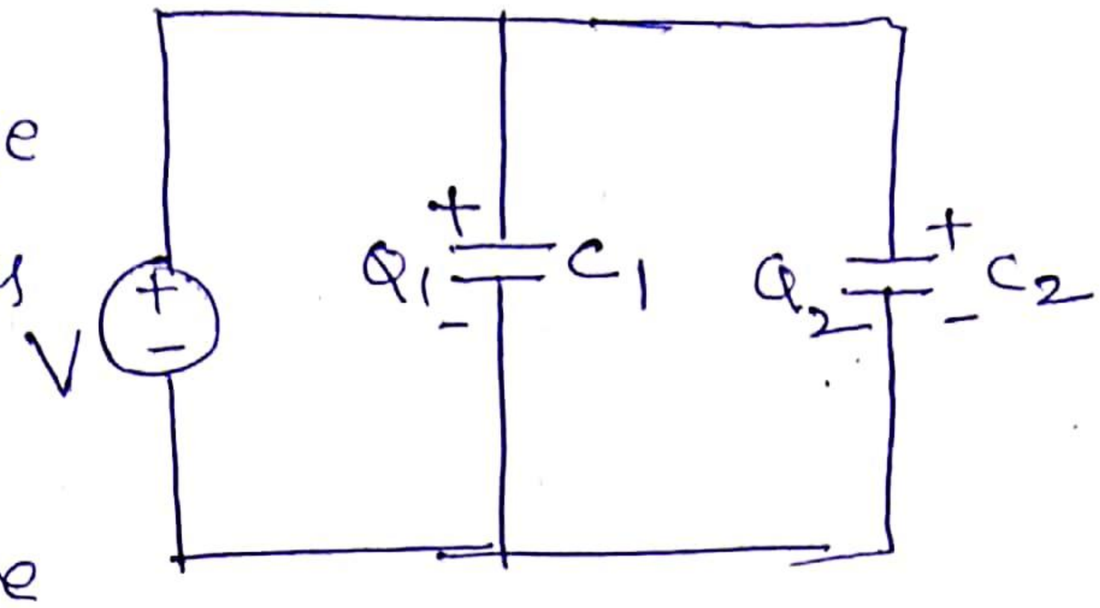


fig. Capacitors in parallel

In general we have $Q = CV$

$\therefore Q_1 = C_1 V$ & $Q_2 = C_2 V$

$\therefore$ The total charge on all capacitors is given by $Q = V(C_1 + C_2) \rightarrow (1)$

If the capacitors are replaced by a Single Equivalent capacitor C_{eq}, we get

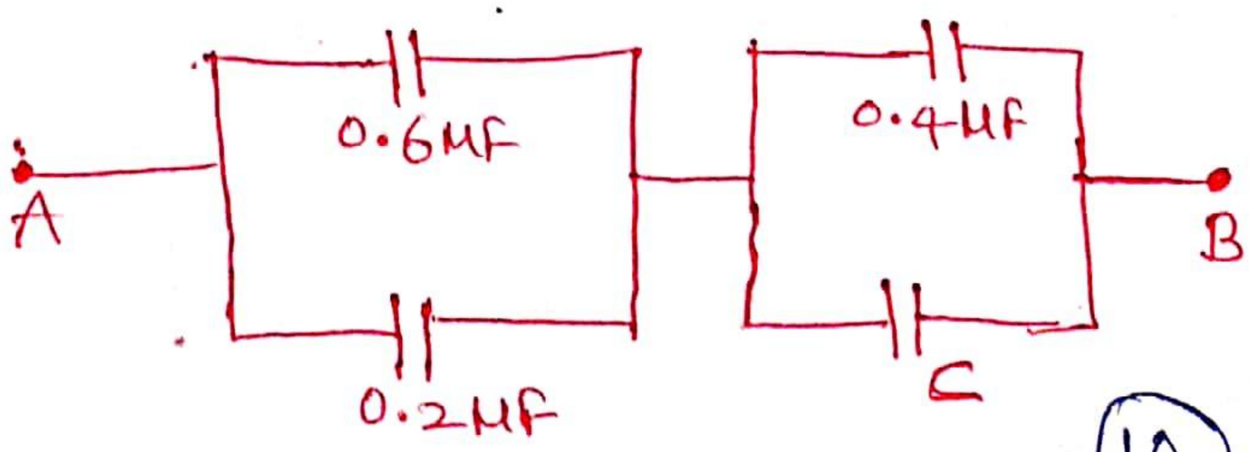
$$Q = C_{eq} V \rightarrow (2)$$

$$(1) = (2) \Rightarrow \boxed{C_{eq} = C_1 + C_2}$$

*Note:- (i) $\frac{1}{C_{eq}} = \Sigma \frac{1}{C}$ for Capacitors in series

(ii) $C_{eq} = \Sigma C$ " " parallel

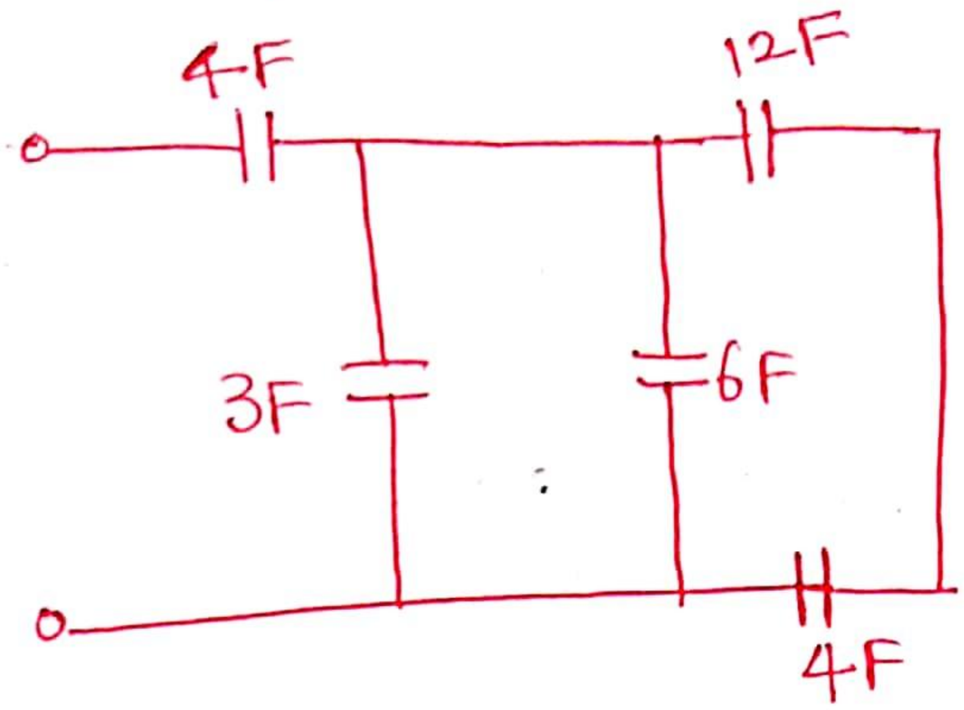
* Find the value of 'c' which gives an Equivalent Capacitances of 0.5μF b/w terminals A & B



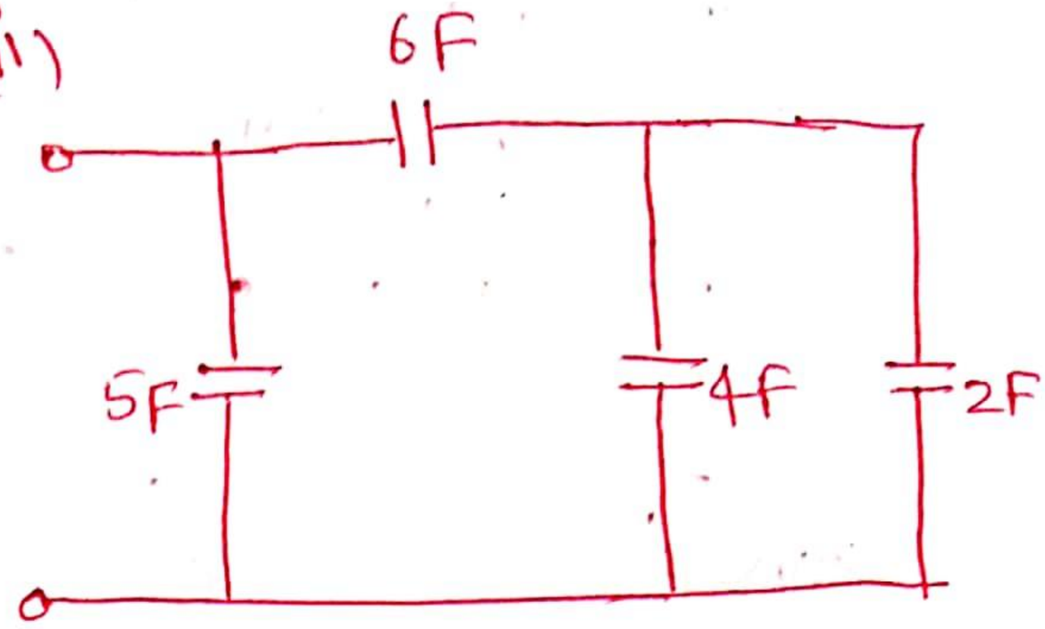
Ans:- $C = 0.933\,\mu F$

* Determine the Equivalent Capacitance for Each of the circuit shown

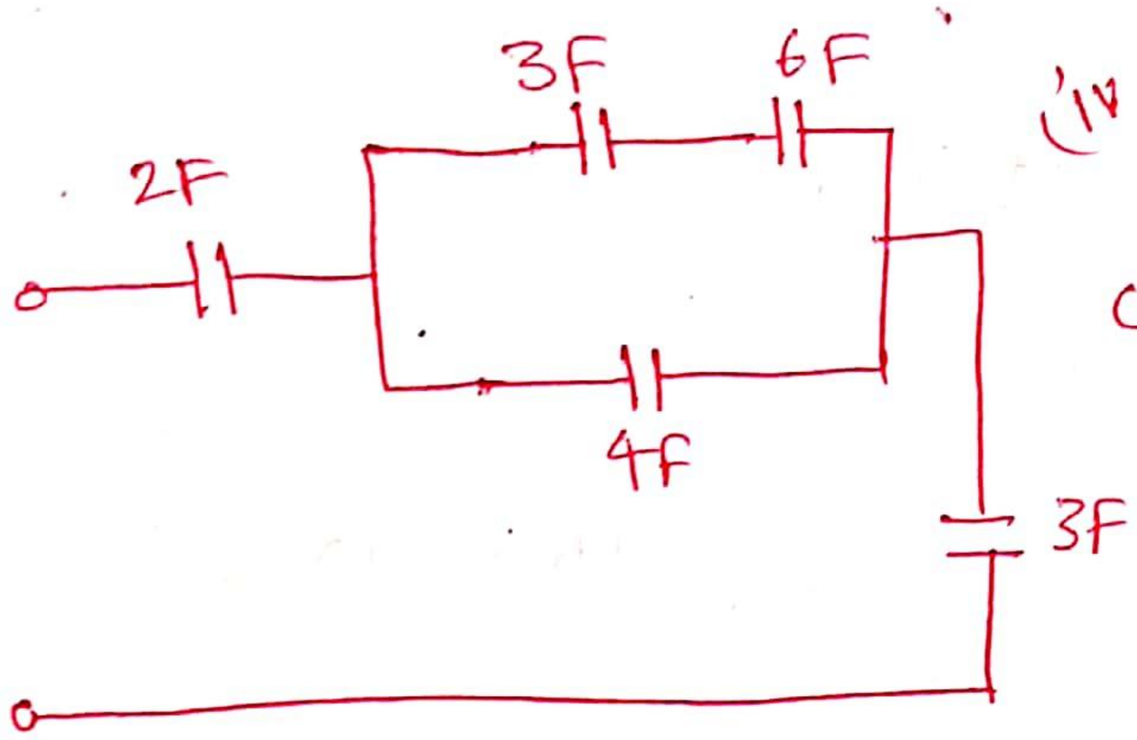
(i)



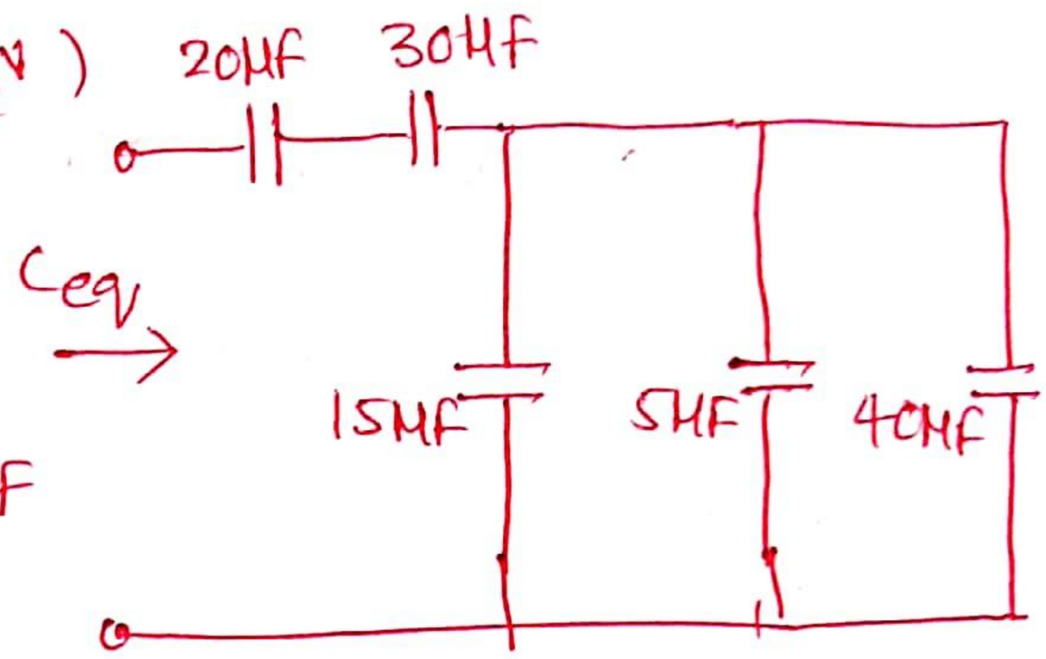
(ii)



(iii)

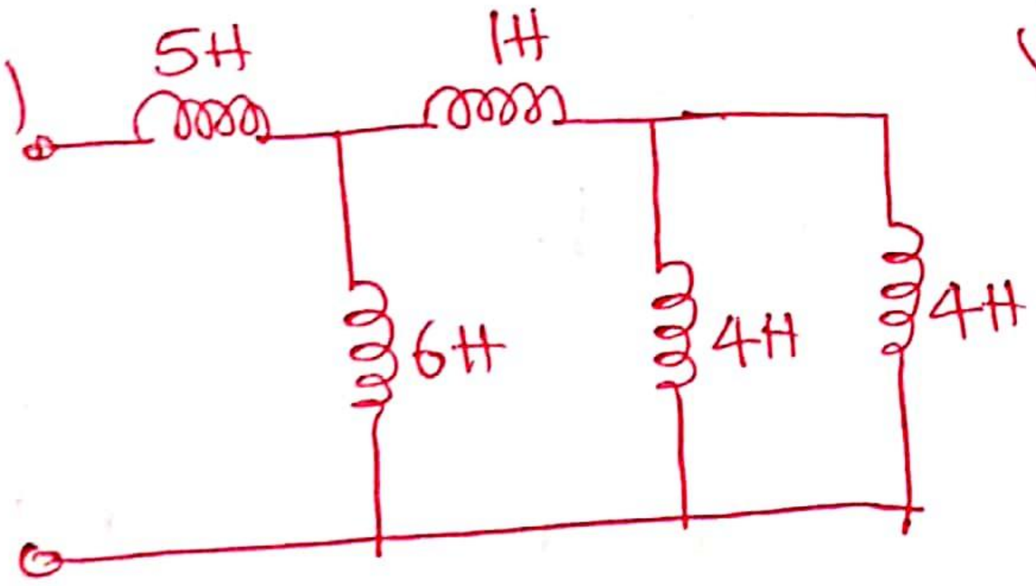


(iv)

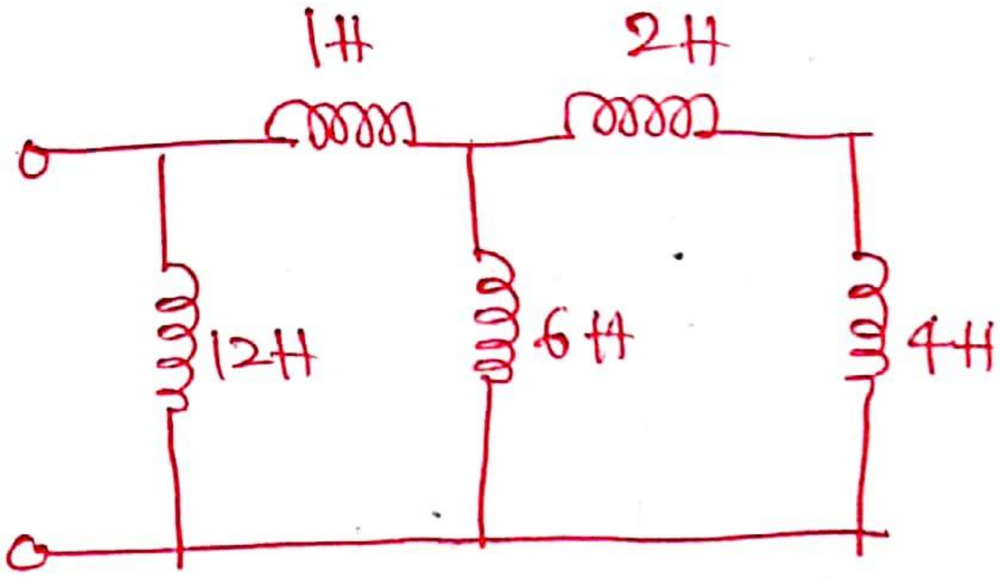


* Determine the Equivalent inductance for Each circuit shown below.

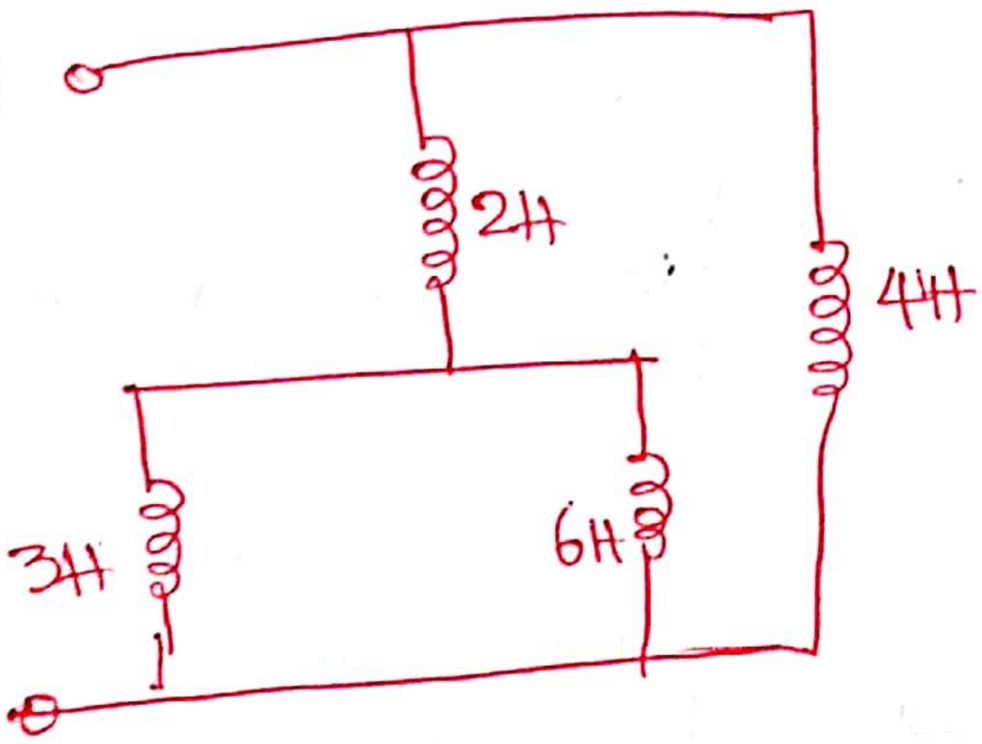
(i)



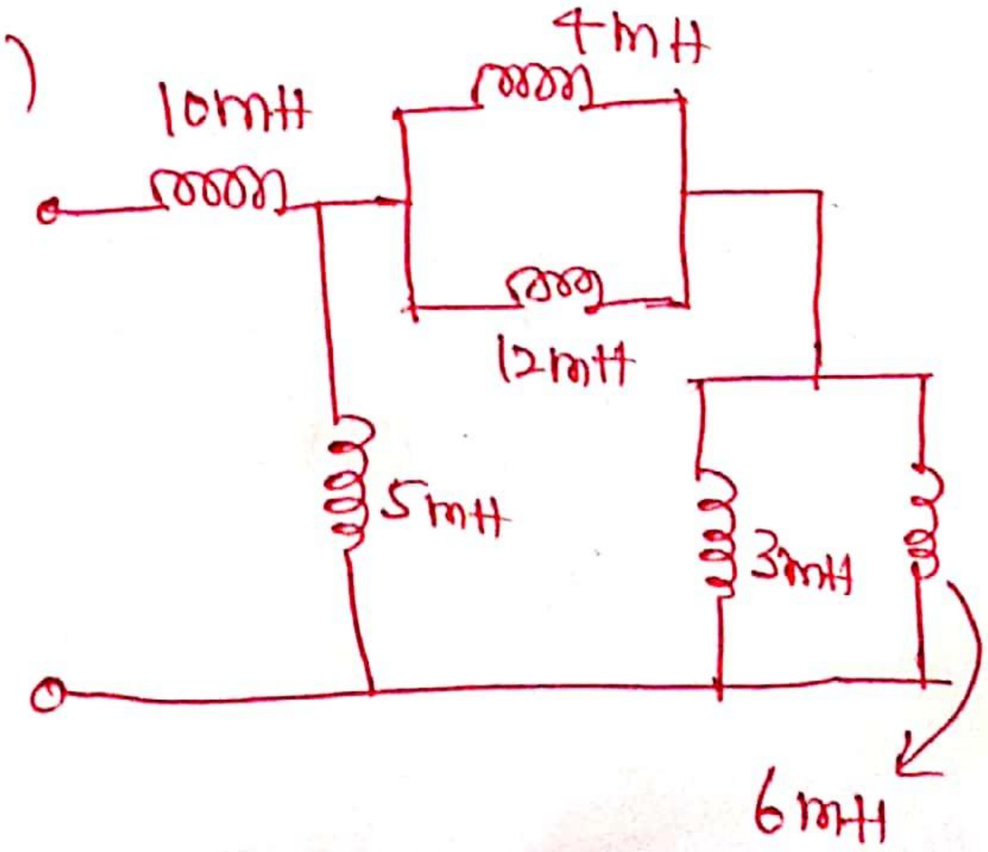
(ii)



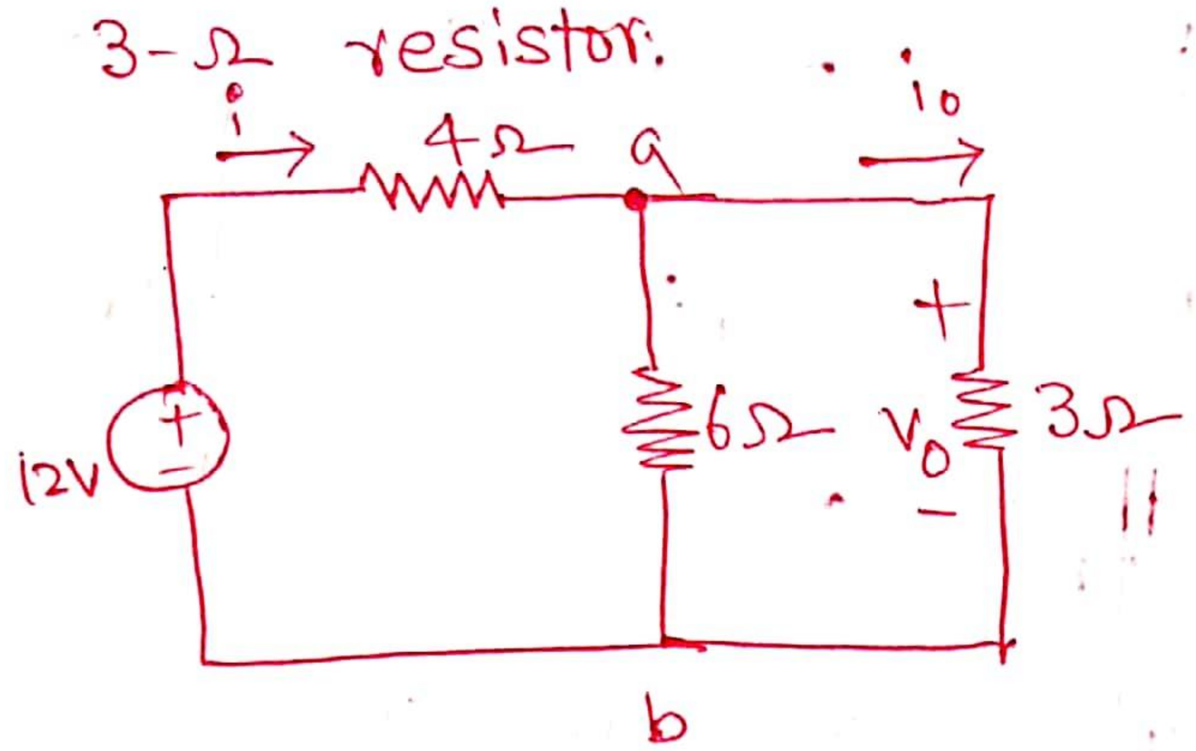
(iii)



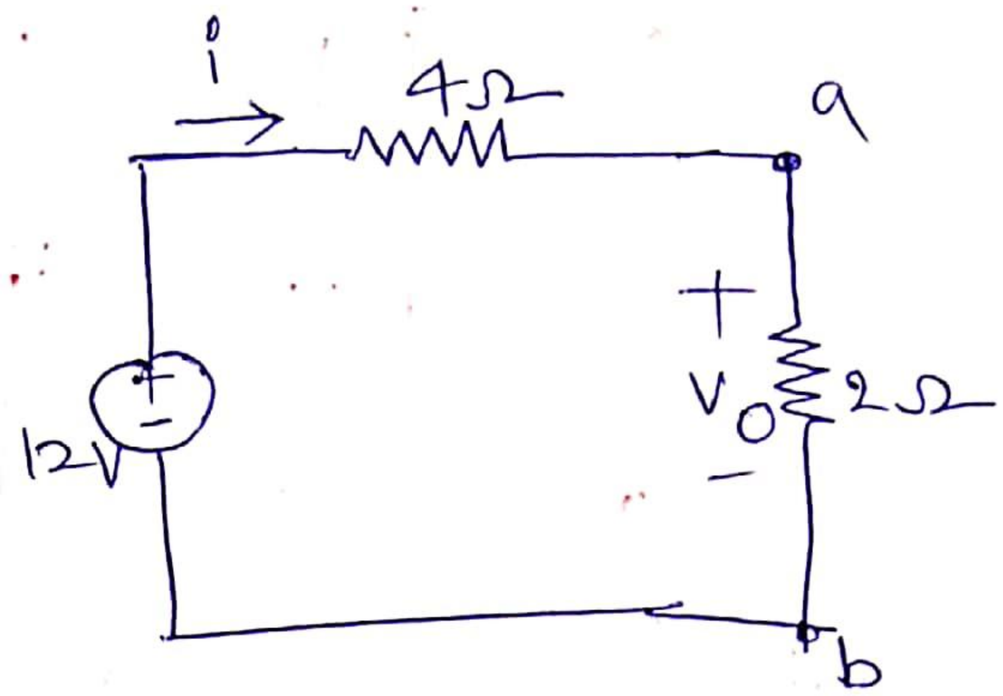
(iv)



* For the circuit shown below, calculate i_o & V_o and calculate the power dissipated in the 3-Ω resistor.



(a) Original circuit (given)



(b) Equivalent ckt

Sol:- we can obtain V_o in two ways.

* One way is to apply ohm's law to get

$$i = \frac{12}{4+2} = \frac{12}{6} = 2A \quad \text{(Before this)}$$

From fig(a) $6\Omega \times 3\Omega$ in parallel

$$\therefore 6\Omega \| 3\Omega = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega$$

* Another way to obtain V_o

$$i = 2A \Rightarrow V_o = 2i = 2 \times 2 = 4V$$

By using voltage divider rule

$$V_o = \frac{2}{6} \times 12 = 4V$$

i_o also two ways

By Applying ohms Law

$$V_o = i_o 3 = 4 = 3i_o$$

$$\boxed{i_o = \frac{4}{3} A}$$

By current divider rule

$$i_o = \left(\frac{6}{6+3}\right) i = \frac{4}{3} A$$

power dissipated by 3Ω resistor $(P) = V_o I_o = 4 \times \frac{4}{3} = 5.333W$

** Delta - star transformation:-

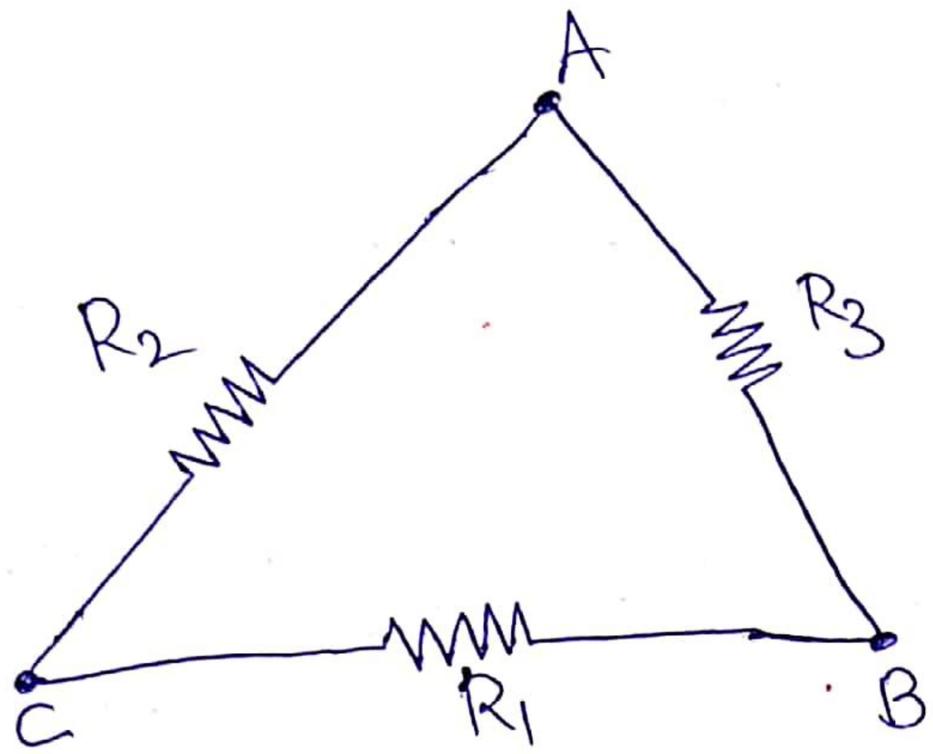
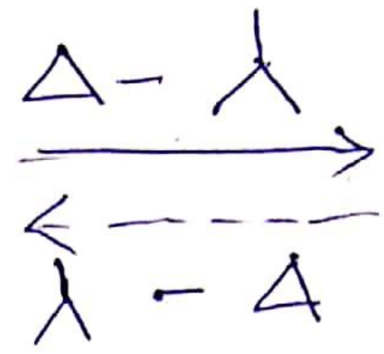


fig (a)



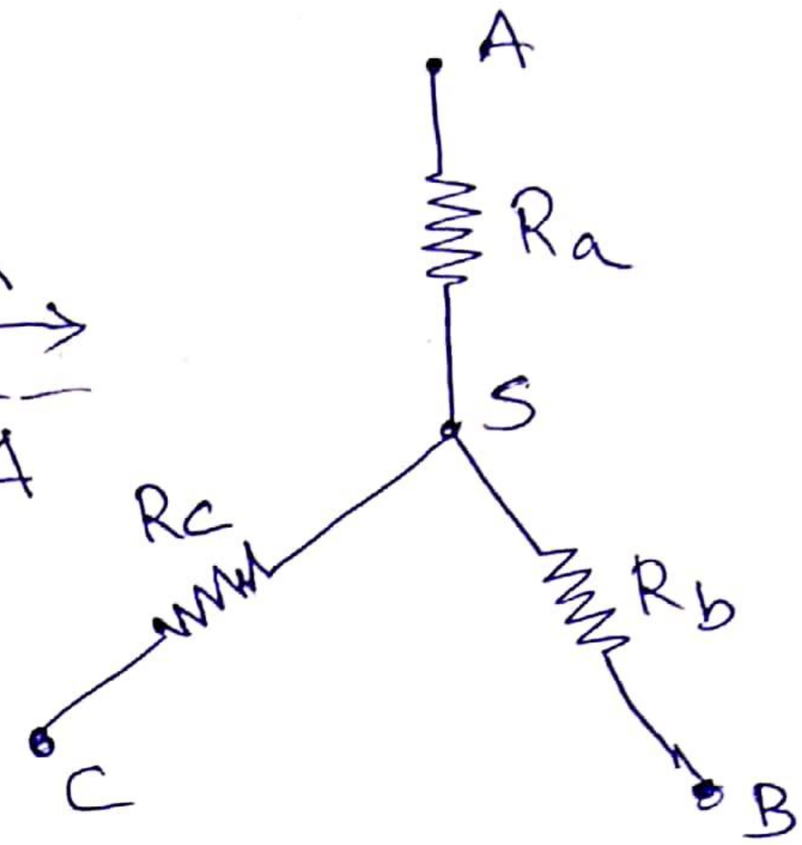


fig (b)

→ '3' resistors R_1, R_2 & R_3 connected in a closed mesh or delta to '3' terminals A, B & C.

It is possible to replace these delta-connected resistors by '3' resistors R_a, R_b & R_c connected respectively b/w the same terminals A, B & C and a common point 'S' in fig (b). Such an arrangement is said to be star-connected.

If we consider terminals A & B in fig (a) we have a circuit having a resistance R_3 in parallel with a circuit having resistances R_1 & R_2 in series. Hence

$$R_{AB} = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \rightarrow (1)$$

From fig (b) $R_{AB} = R_a + R_b \rightarrow (2)$

$$\therefore R_a + R_b = \frac{R_1R_3 + R_2R_3}{R_1+R_2+R_3} \longrightarrow (3)$$

Similarly $$R_b + R_c = \frac{R_1R_2 + R_1R_3}{R_1+R_2+R_3} \longrightarrow (4)$$

$$R_c + R_a \text{ or } R_a + R_c = \frac{R_1R_2 + R_2R_3}{R_1+R_2+R_3} \longrightarrow (5)$$

Substracting (4) from (3)

$\cancel{R_b - R_a + R_c}$ –

$$R_a + \cancel{R_b} - \cancel{R_b} - R_c = \frac{R_1R_3 + R_2R_3}{R_1+R_2+R_3} - \frac{R_1R_2 + R_1R_3}{R_1+R_2+R_3}$$

$$R_a - R_c = \frac{\cancel{R_1R_3} + R_2R_3 - R_1R_2 - \cancel{R_1R_3}}{R_1+R_2+R_3}$$

$$R_a - R_c = \frac{R_2R_3 - R_1R_2}{R_1+R_2+R_3} \longrightarrow (6)$$

Adding (5) & (6)

$$R_a + \cancel{R_c} + R_a - \cancel{R_c} = \frac{R_1R_2 + R_2R_3}{R_1+R_2+R_3} + \frac{R_2R_3 - R_1R_2}{R_1+R_2+R_3}$$

$$2R_a = \frac{\cancel{R_1R_2} + 2R_2R_3 - \cancel{R_1R_2}}{R_1+R_2+R_3}$$

$$\boxed{R_a = \frac{R_2R_3}{R_1+R_2+R_3}} \longrightarrow (7)$$

Similarly $R_b = \dfrac{R_3 R_1}{R_1+R_2+R_3} \rightarrow (8)$; $R_c = \dfrac{R_1 R_2}{R_1+R_2+R_3} \rightarrow (9)$

The Equivalent Star resistance connected to a given terminal is Equal to the product of the two delta resistances Connected to the same terminal divided by the sum of the delta resistances

* Star-delta transformation:- Let us next consider how to replace the star-connected network of fig(b) by the Equivalent delta-connected network of fig. (a)

Dividing Equation (7) by Equation (8)

$$\frac{R_a}{R_b} = \frac{R_2 \cancel{R_3}}{\cancel{R_1+R_2+R_3}} \times \frac{\cancel{R_1+R_2+R_3}}{\cancel{R_3} R_1}$$

$$\frac{R_a}{R_b} = \frac{R_2}{R_1}$$

$$\therefore R_2 = \frac{R_1 R_a}{R_b} \rightarrow (10)$$

Similarly dividing Equation (7) by (9)

$$\frac{R_a}{R_c} = \frac{R_2 R_3}{\cancel{R_1+R_2+R_3}} \times \frac{\cancel{R_1+R_2+R_3}}{R_1 \cancel{R_2}}$$

$$\frac{R_a}{R_c} = \frac{R_3}{R_1}$$

$$\therefore R_3 = \frac{R_1 R_a}{R_c} \longrightarrow (11)$$

Substituting equation (10) & (11) in (7)

$$R_a = \frac{\frac{R_1 R_a}{R_b} \times \frac{R_1 R_a}{R_c}}{R_1 + \frac{R_1 R_a}{R_b} + \frac{R_1 R_a}{R_c}}$$

~~Sol~~

$$R_a = \frac{R_1 R_a \times R_1 R_a}{\cancel{R_b R_c}} \times \frac{\cancel{R_b R_c}}{R_1 R_b R_c + R_1 R_a R_c + R_1 R_a R_b}$$

$$\cancel{R_a} = \frac{1}{\cancel{R_1}} \left[\frac{\cancel{R_1 R_a} \times R_1 R_a}{R_b R_c + R_c R_a + R_a R_b} \right]$$

$$R_1 R_a = R_b R_c + R_c R_a + R_a R_b$$

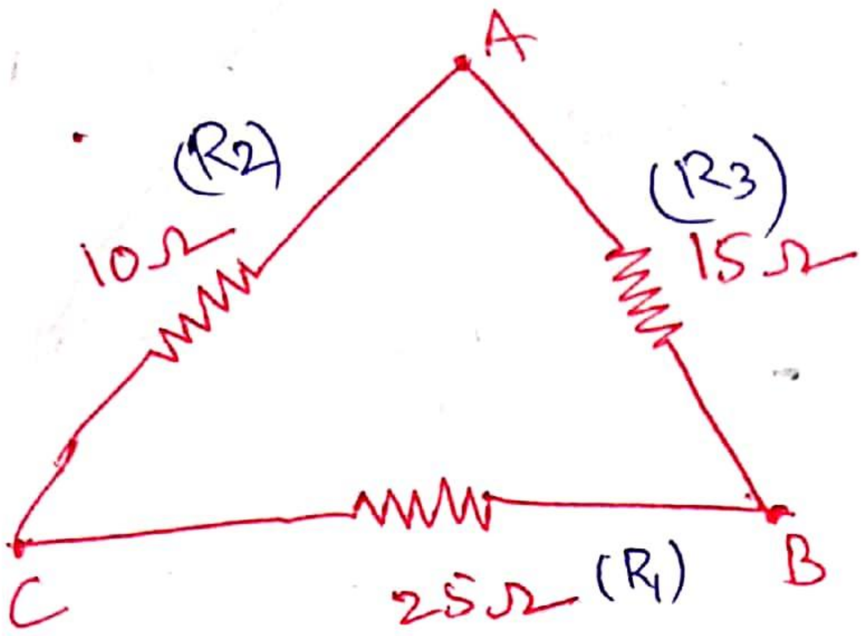
$$\boxed{R_1 = R_b + R_c + \frac{R_b R_c}{R_a}} \longrightarrow (12)$$

Similarly $\boxed{R_2 = R_c + R_a + \frac{R_c R_a}{R_b}} \longrightarrow (13)$

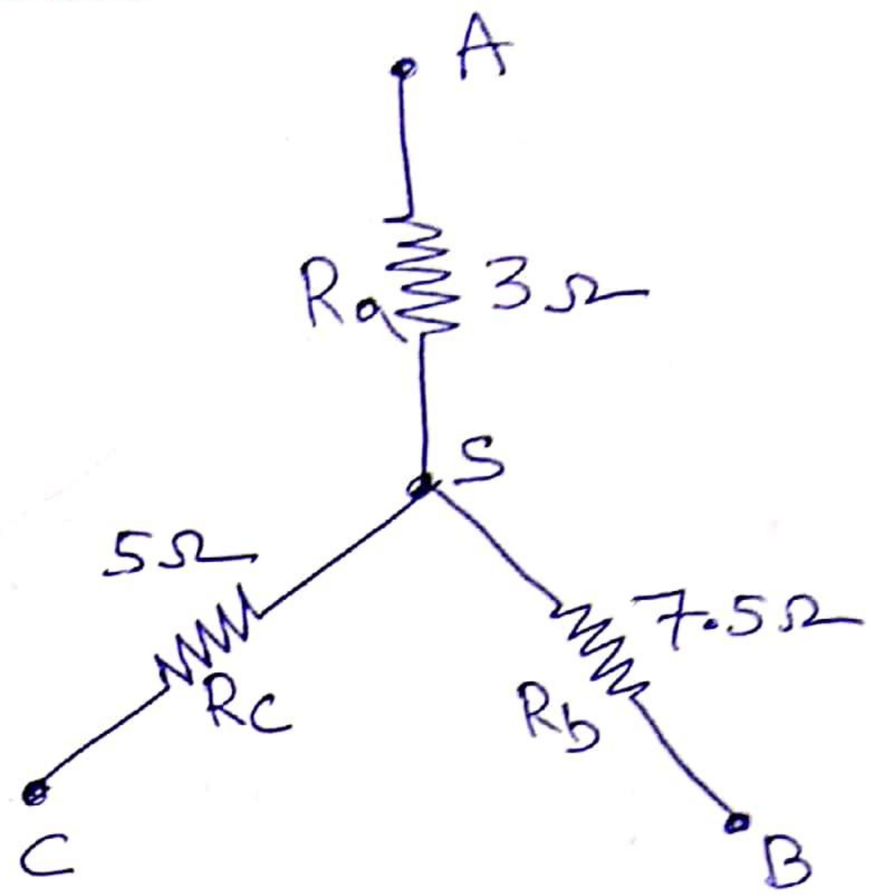
$$\boxed{R_3 = R_a + R_b + \frac{R_a R_b}{R_c}}$$

$\therefore$ The equivalent delta resistance b/w two terminals is the sum of two star resistances connected to those terminals plus the product of the same two star resistances divided by the third star resistance. (13)

* Convert the Δ network to an equivalent Y network or Star network



(a) original N/W



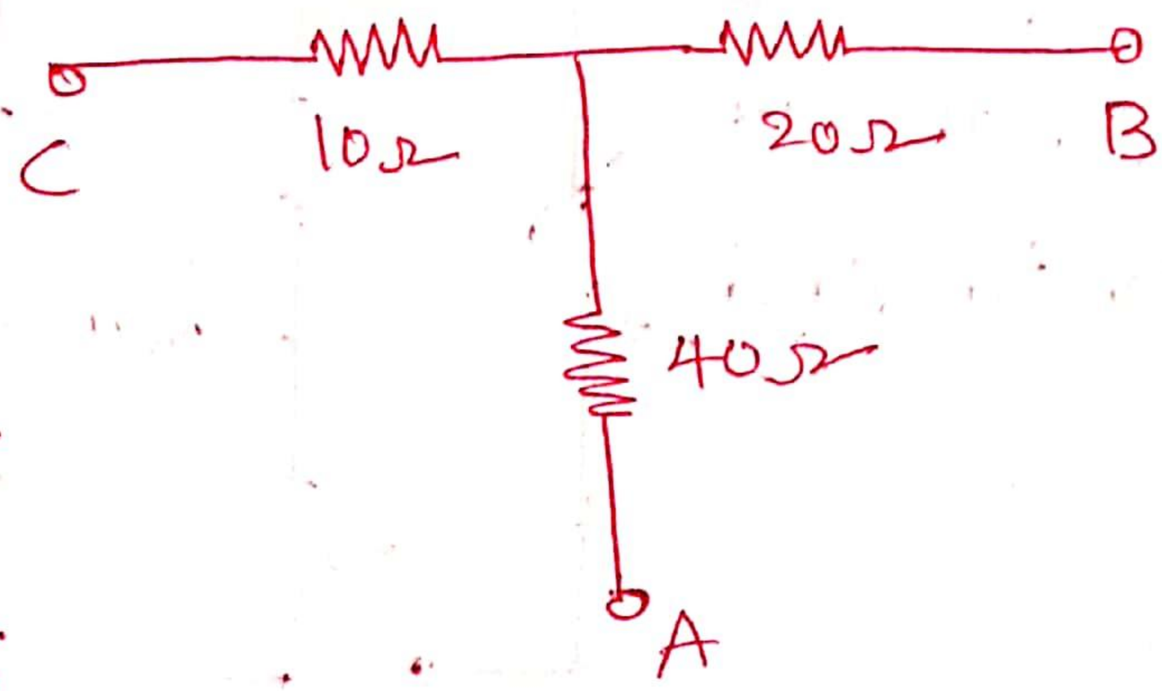
Sol:

$$R_a = \frac{R_2 R_3}{R_1+R_2+R_3} = \frac{10\times15}{10+15+25} = \frac{150}{50} = 3\Omega$$

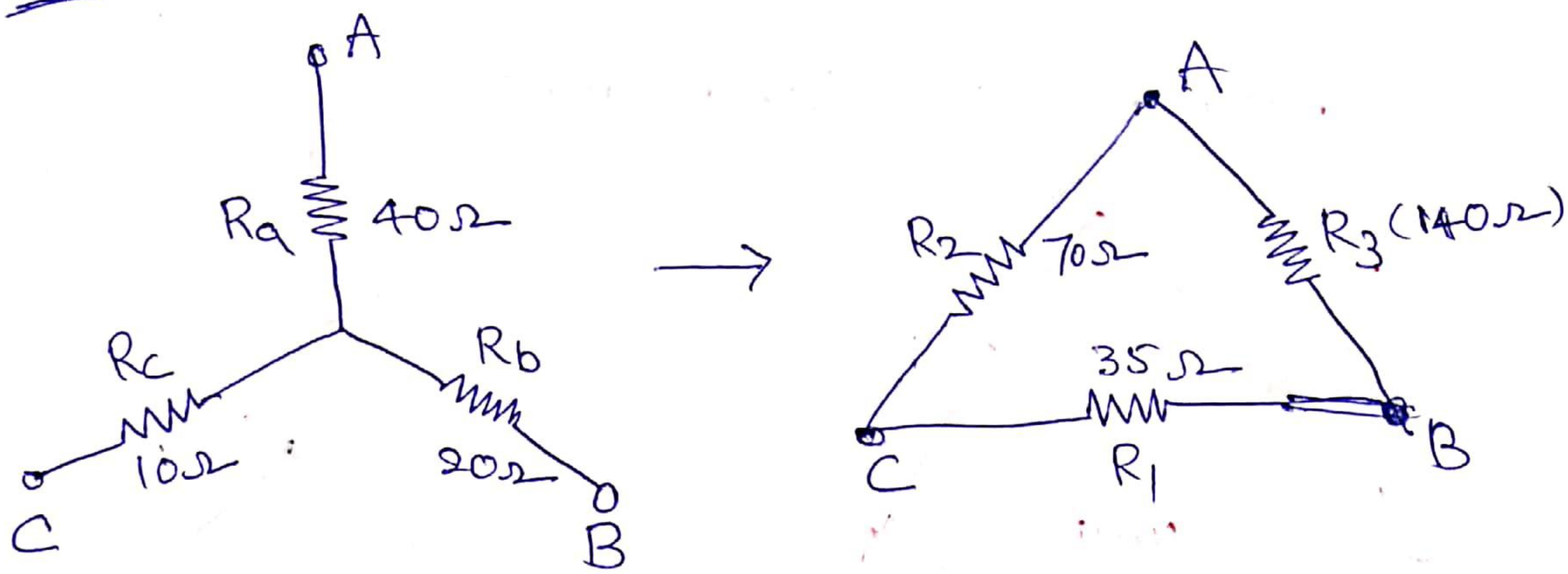
$$R_b = \frac{R_1 R_3}{R_1+R_2+R_3} = \frac{25\times15}{50} = 7.5\Omega$$

$$R_c = \frac{R_1 R_2}{R_1+R_2+R_3} = \frac{25\times10}{50} = 5\Omega$$

* Transform the wye network or star n/w to delta network.



Sol:- Above circuit can be redrawn as



From formulae's we have

$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a} = 20 + 10 + \frac{10 \times 20}{40}$$

$$R_1 = 35\Omega$$

Similarly $R_2 = R_c + R_a + \frac{R_c R_a}{R_b} = 50 + \frac{10 \times 40}{20}$

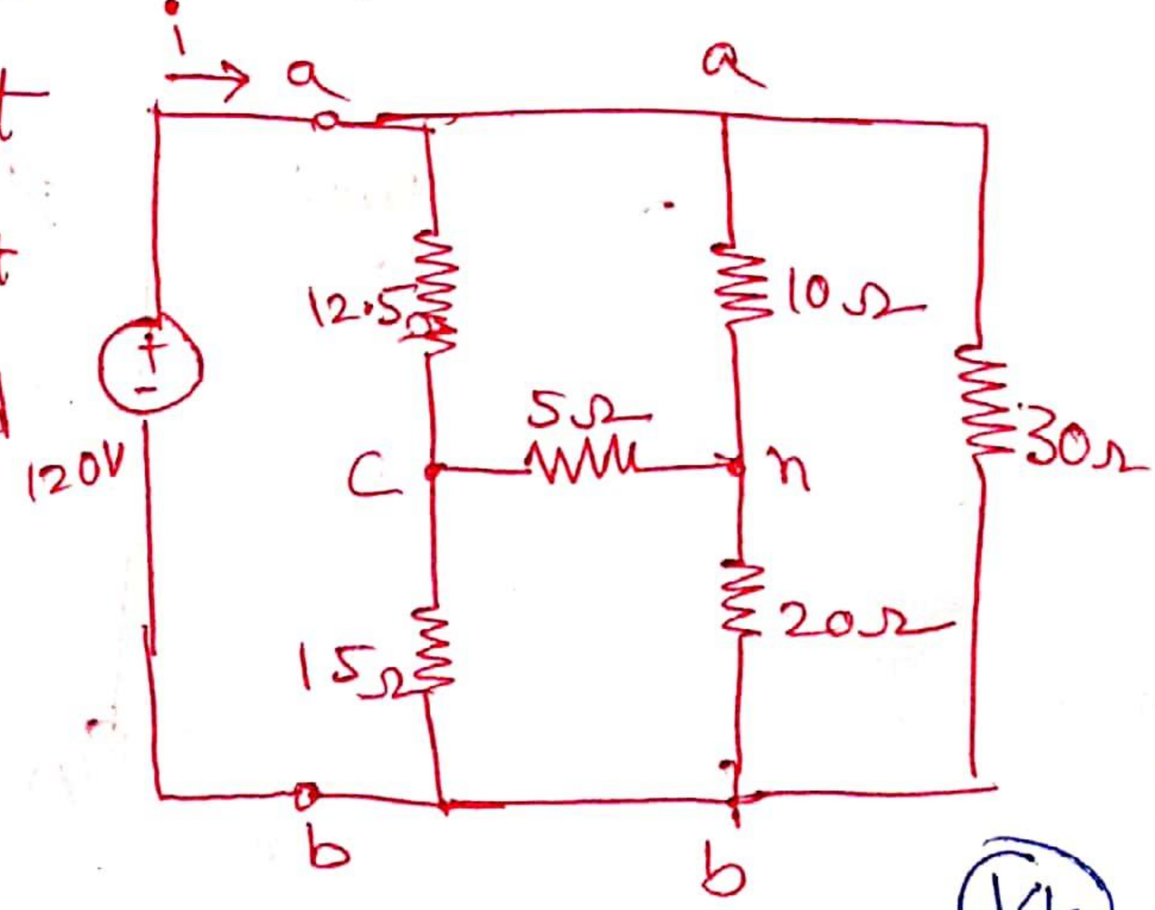
$$R_2 = 70\Omega$$

$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c} = 60 + \frac{40 \times 20}{10}$$

$$\Rightarrow 60 + 80 = 140\Omega$$

$$R_3 = 140\Omega.$$

* Obtain the Equivalent resistance Rab for the ckt shown and use it to find current i



Sol:- Let 5Ω, 10Ω & 20Ω resistor form a Star connection convert them in delta connection we get let $R_a = 5\Omega$, $R_b = 10\Omega$ & $R_c = 20\Omega$

$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a} = 30 + \frac{20 \times 10}{5}$$

$$\Rightarrow 30 + 40 = 70\Omega$$

$$R_2 = R_a + R_c + \frac{R_a R_c}{R_b} = 25 + \frac{5 \times 20}{10}$$

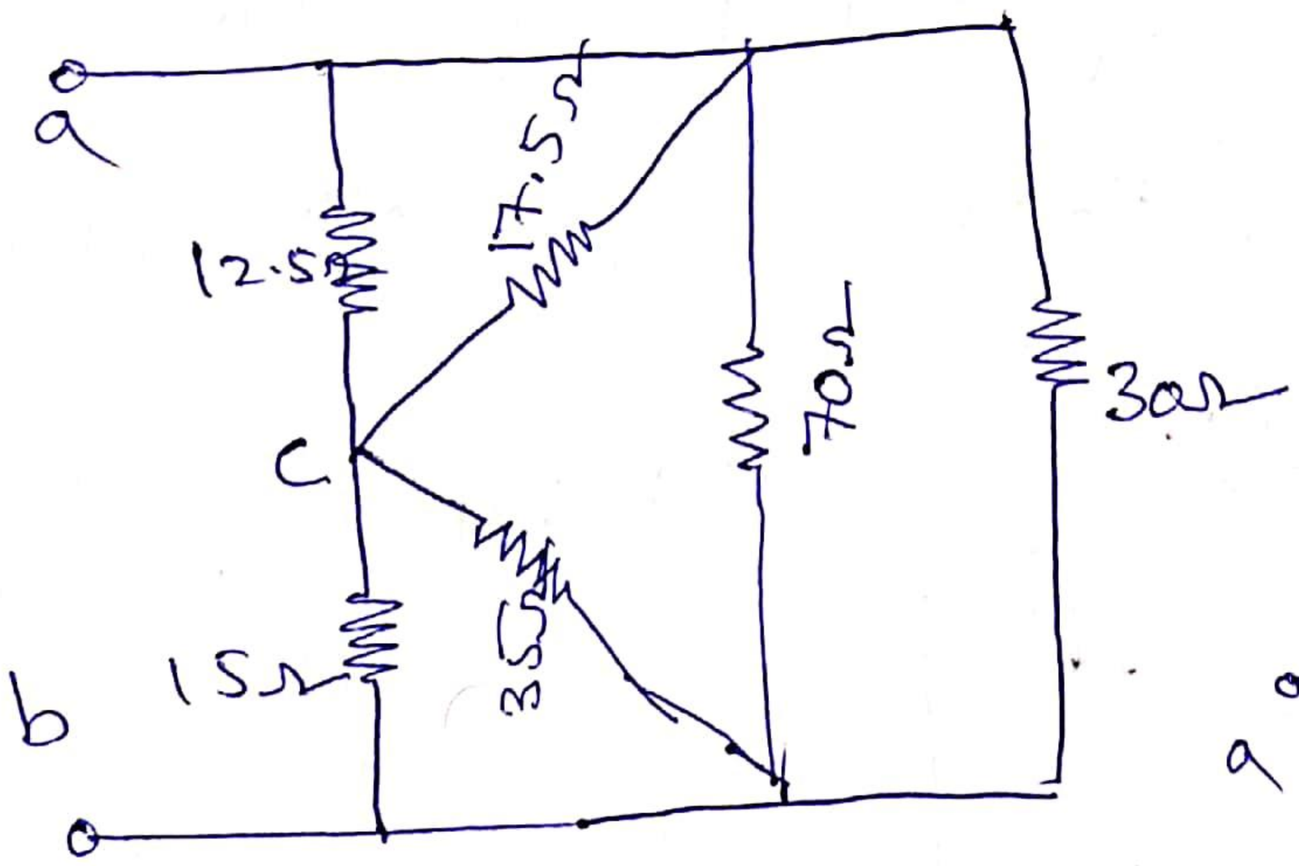
$$\Rightarrow 25 + 10 = 35\Omega$$

$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c} = 15 + \frac{5 \times 10}{20}$$

$$\Rightarrow 15 + 2.5\Omega$$

$$R_3 = 17.5\Omega$$

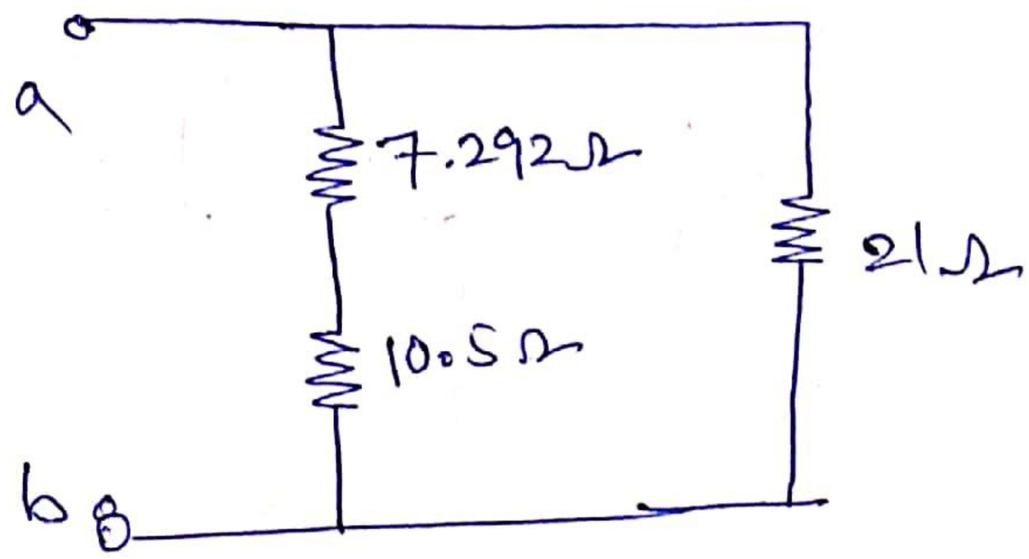
Then reduced circuit or is (removing Voltage source



$$70 \| 30 = \frac{70 \times 30}{70 + 30} = 21\Omega$$

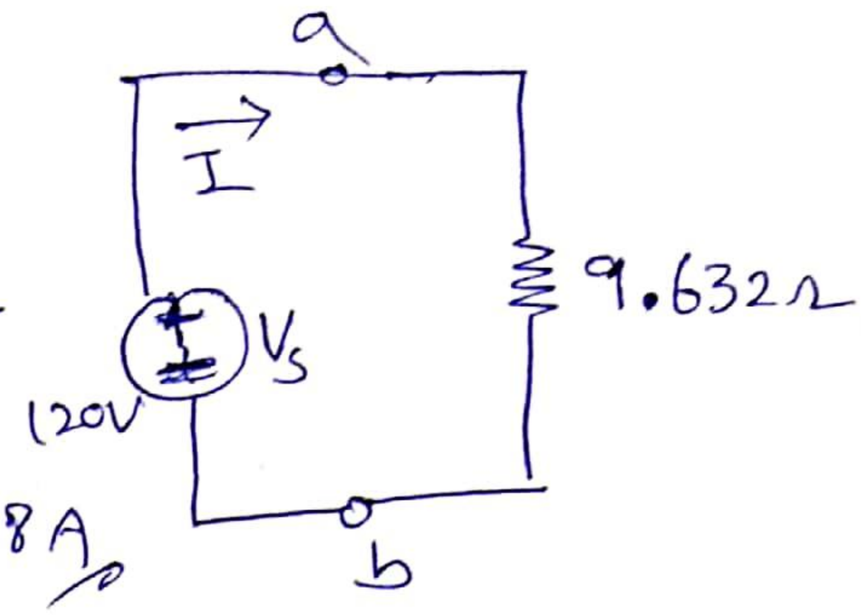
$$12.5 \| 17.5 = 7.2917\Omega$$

$$15 \| 35 = 10.5\Omega$$



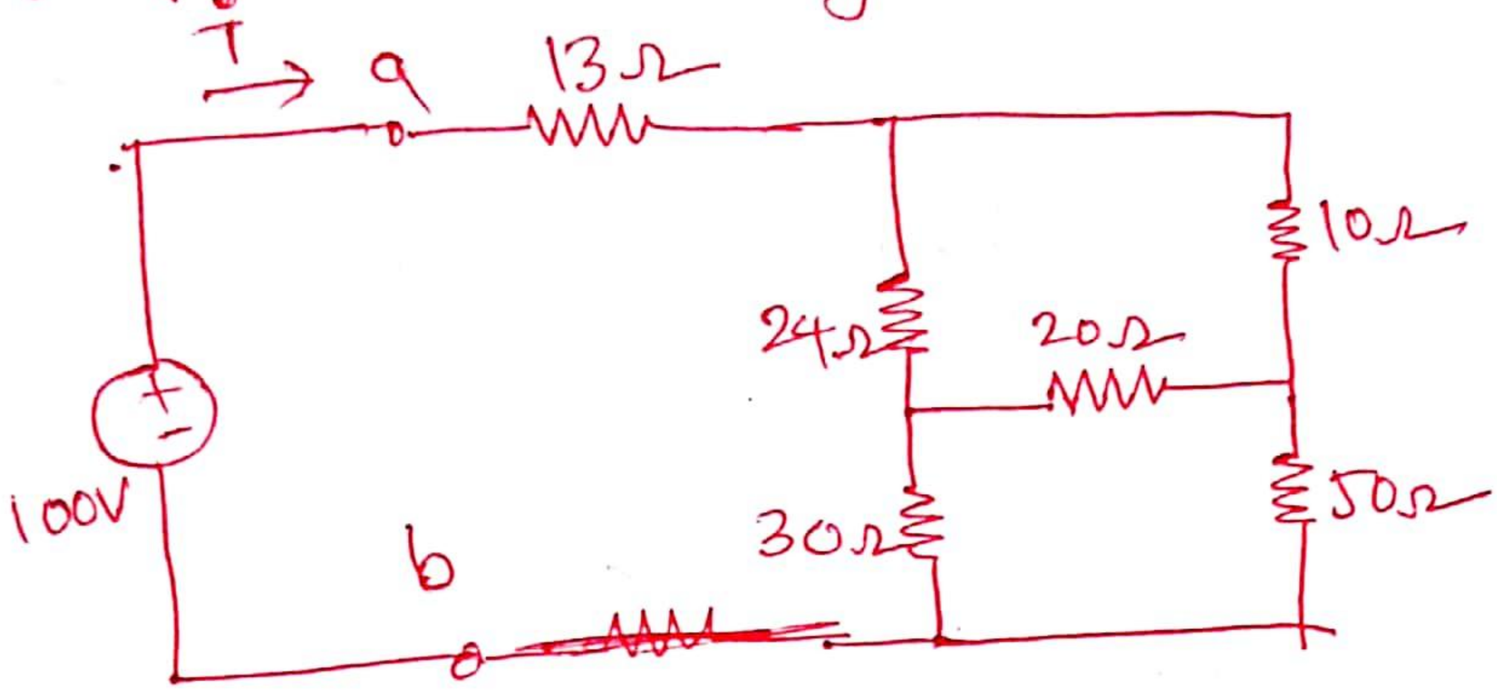
$7.292 + 10.5 = 17.792$

$\Rightarrow 17.792 \| 21 = 9.632\,\Omega$



$$I = \frac{V_s}{R_{ab}} = \frac{120}{9.632} = 12.458\,A$$

* For the bridge network below find R_{ab} & i.



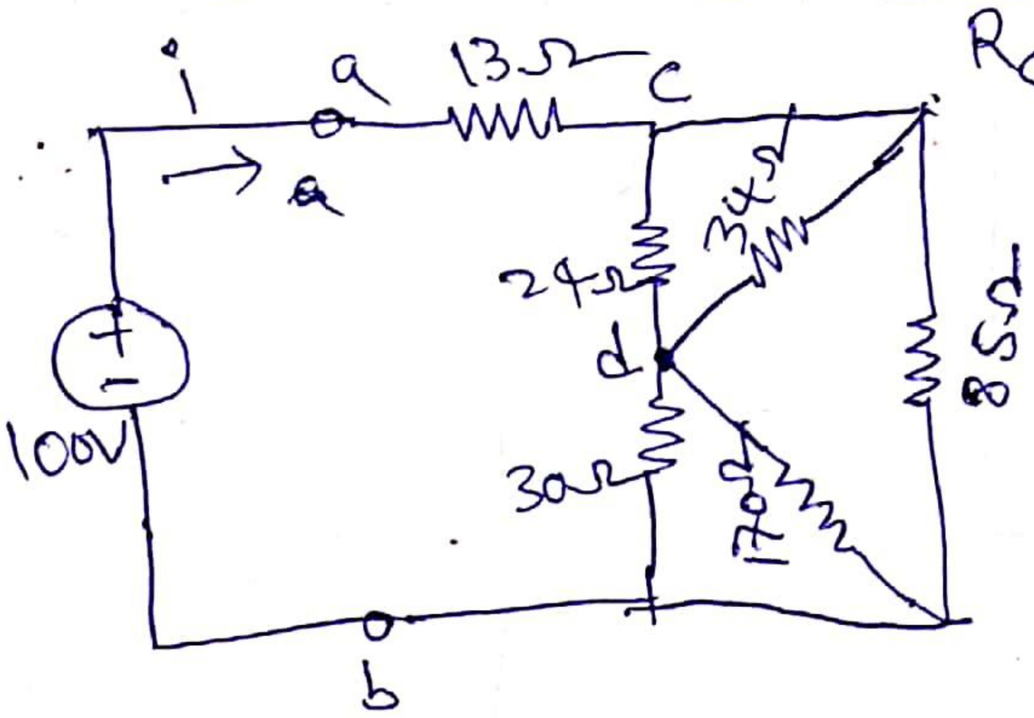
Sol:- let 20Ω, 10Ω & 50Ω forms a star-connection

$\therefore R_a = 20\,\Omega,\ R_b = 10\,\Omega$ & $R_c = 50\,\Omega$

$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a} = 60 + \frac{\cancel{10}^{1} \times \cancel{50}^{25}}{\cancel{20}_{2}} = 85\,\Omega$$

$$R_2 = R_a + R_c + \frac{R_a R_c}{R_b} = 70 + \frac{\cancel{20}^{2} \times 50}{\cancel{10}_{1}} = 170\,\Omega$$

$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c} = 30 + \frac{\cancel{20}^{4} \times \cancel{10}}{\cancel{50}} = 34\,\Omega$$



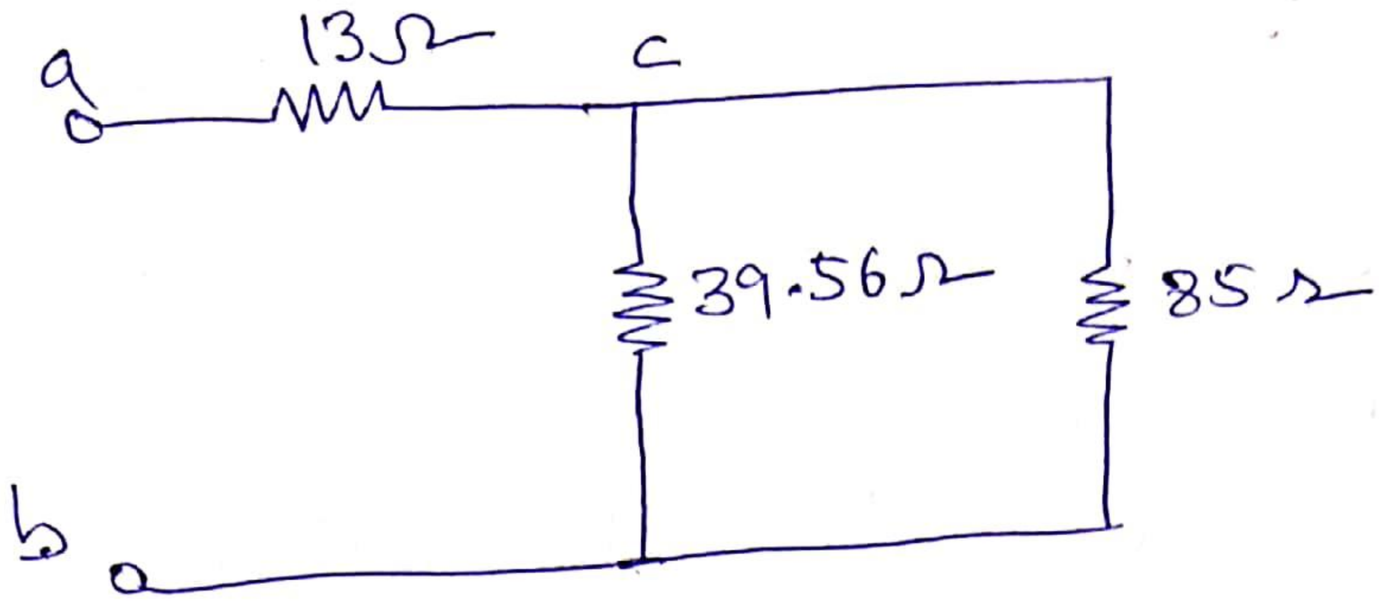
$$\rightarrow 24 \| 34 = \frac{24 \times 34}{24 + 34}$$

$$= \frac{816}{58} = 14.06\,\Omega$$

$$\rightarrow 30 \| 170 = \frac{30 \times 170}{30 + 170}$$

$$\Rightarrow \frac{5100}{200} = 25.5\,\Omega$$

$\Rightarrow 14.06 + 25.5 = 39.56\,\Omega$



39.561185

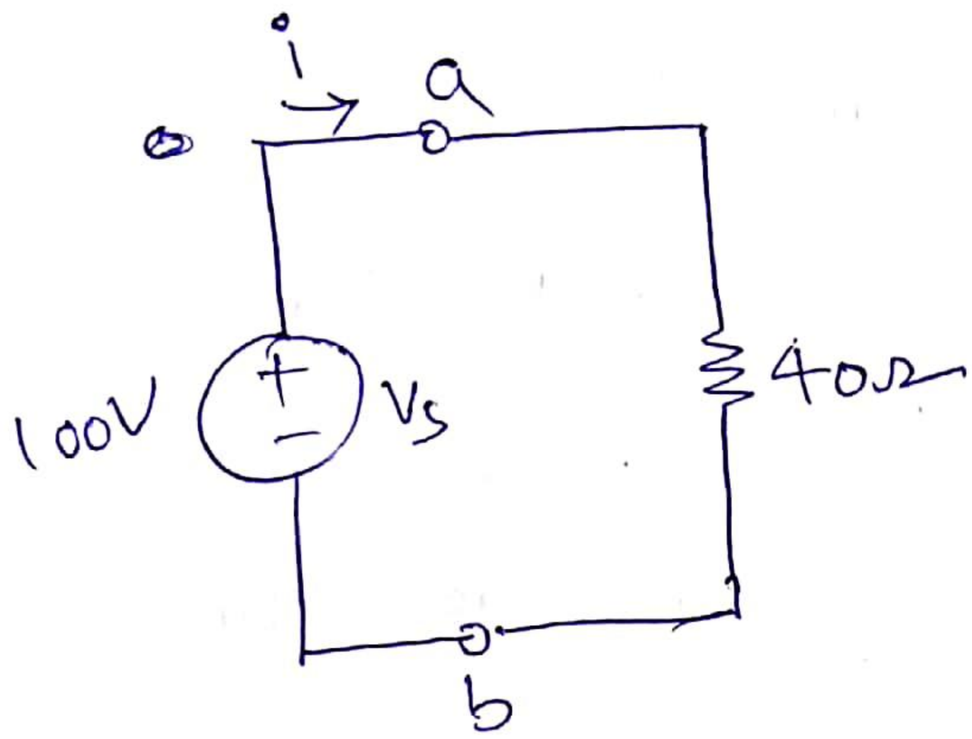
$\Rightarrow \dfrac{3362.6}{124.56}$

$\Rightarrow 26.99\,\Omega$

$\simeq 27\,\Omega$

$27 + 13 = 40\,\Omega$

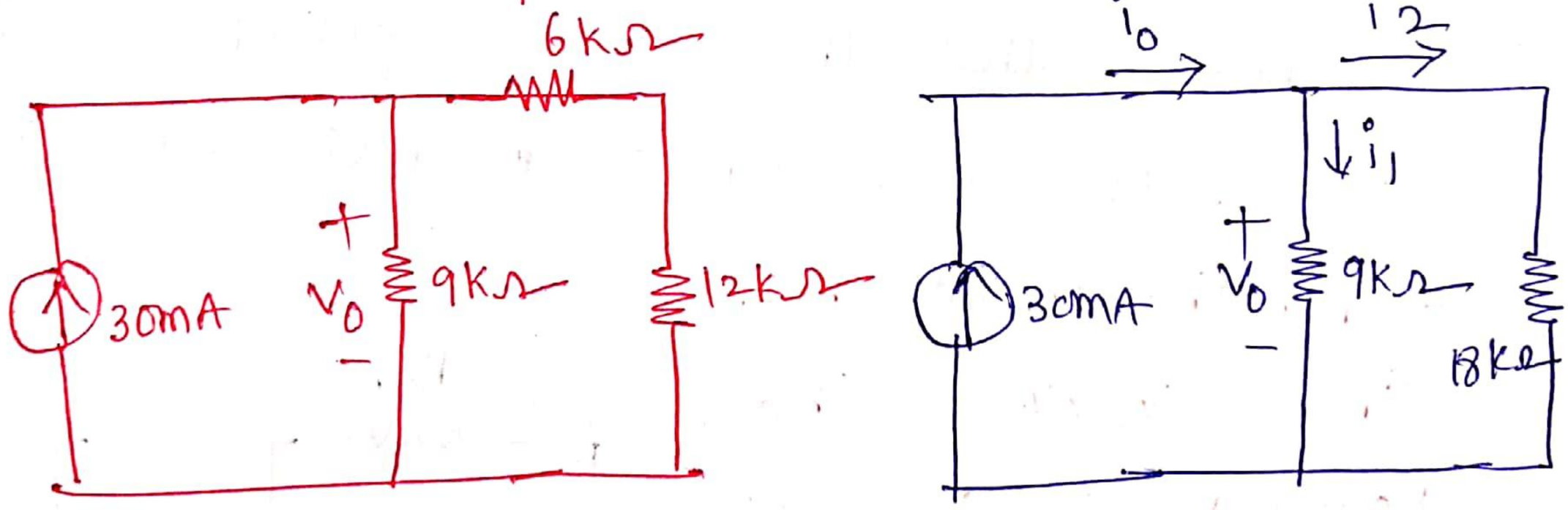
$\boxed{R_{ab} = 40\,\Omega}$



$i = \dfrac{V_s}{R_{ab}} = \dfrac{100}{40}$

$\boxed{i = 2.5\,A}$

* For the circuit shown below determine (a) the voltage v_o, (b) the power supplied by the current source (c) the power absorbed by each resistor.



(a) Original circuit (b) Equivalent circuit

Sol:- The 6kΩ & 12kΩ in series ⇒ 18kΩ

for fig (b) applying the current division to find i_1 & i_2.

$$i_1 = \left[\frac{18K}{(18+9)K}\right] 30mA = 20mA \qquad i_2 = \left[\frac{9K}{(9+18)K}\right] 30\,mA = 10mA$$

— Notice that the voltage across 9kΩ & 18kΩ resistors is the same.

$$\therefore V_o = 9K i_1 = 18K i_2 = 180V.$$

* power supplied by the source is

$$P_o = V_o i_o = 180 \times 30m = 5.4W$$

* Power absorbed by 12kΩ resistor

$$P = i_2^2 R = (10m)^2 \times 12K = 1.2W$$

* power absorbed by 6kΩ resistor

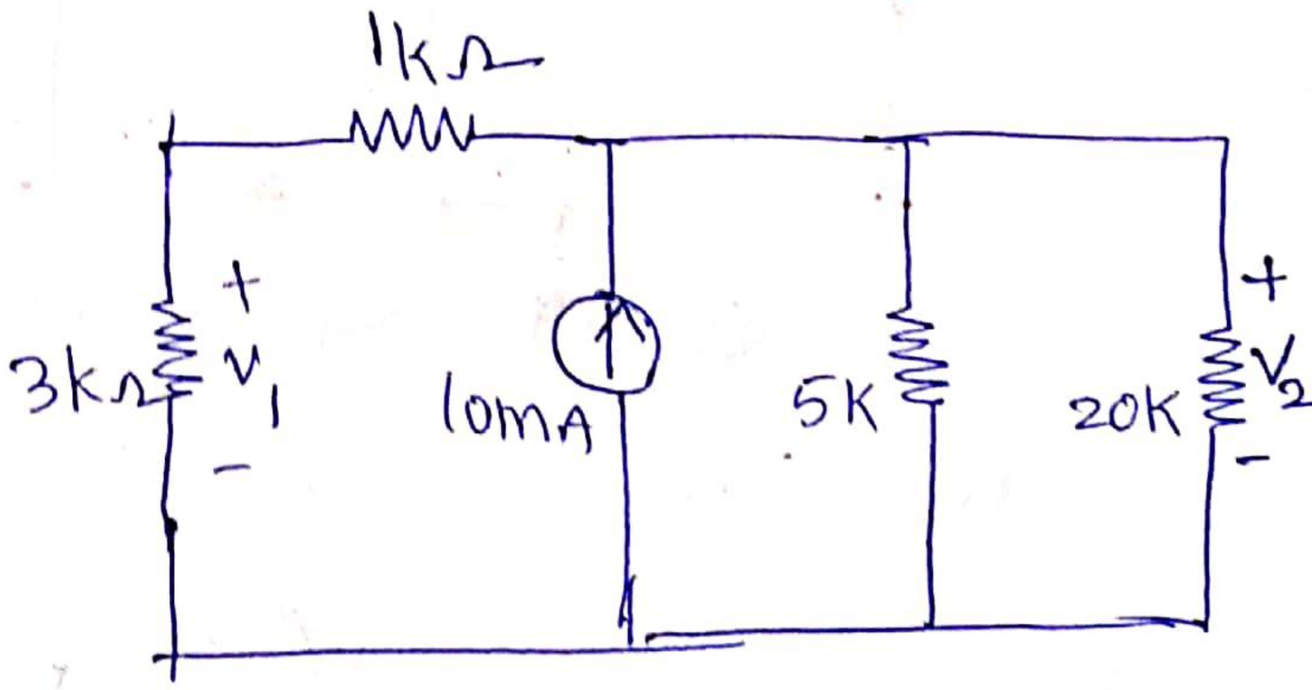
$$P = i_2^2 R = (10\,m)^2 \times 6K = 0.6W$$

* power absorbed by 9kΩ resistor

$$P = i_1^2 R = (20 \times 10^{-3})^2 \times 9K = 3.6\,W$$

→ Notice that the power supplied (5.4W) Equals to the power absorbed (1.2 + 0.6 + 3.6 = 5.4W)

* For the circuit shown find (a) V_1 & V_2 (b) the power dissipated by the 3kΩ & 20kΩ resistors (c) power supplied by the current source

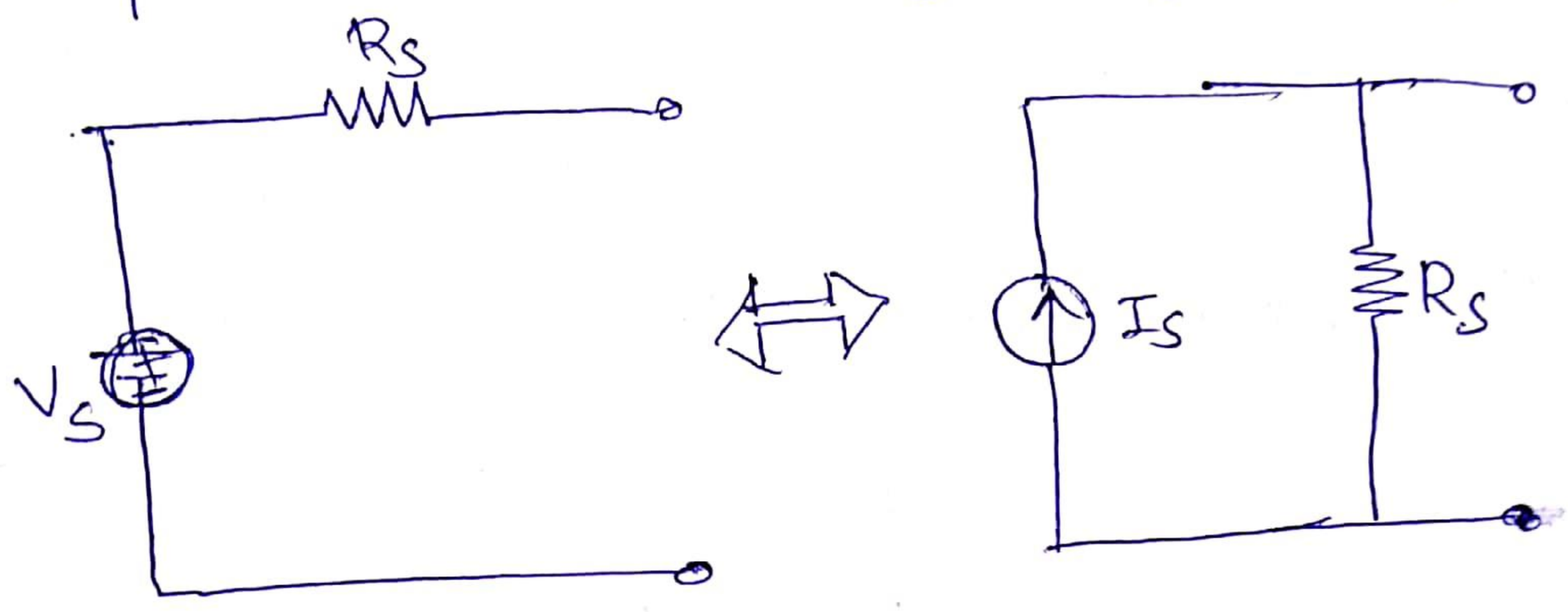


Ans.. (a) 15V, 20V (b) 75mW, 20mW, (c) 200mW

* Source Conversion:-

→ Voltage source has some series resistance with source. (Ideally series resistance = 0)

→ Current source has some shunt resistance (R_{sh}) in parallel with source (Ideally $R_{sh} = \infty$)



fig(a) Voltage source fig(b) Current source

fig. (b) is equivalent to fig(a) if

$$I_S = \frac{V_S}{R_S} \longrightarrow (1)$$

→ The resistances in both sources is R_S

$$\therefore V_S = I_S R_S \longrightarrow (2)$$

Verification

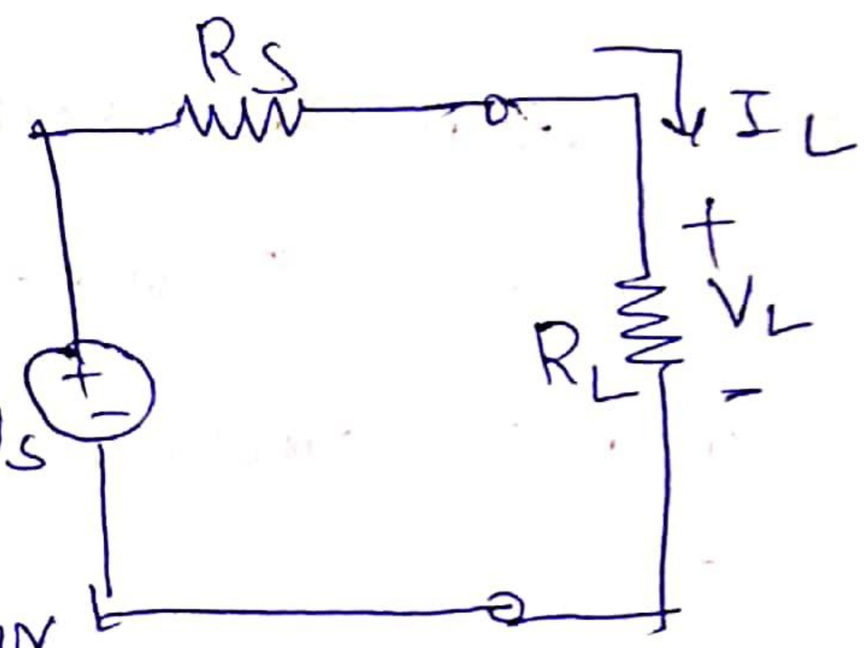


fig (c)

→ from fig. (c) Voltage across R_L ⇒ $V_L = \left(\frac{R_L}{R_L + R_S}\right) V_S \longrightarrow (3)$

→ Current through the resistor R_L is given as

$$I_L = \frac{V_S}{R_S + R_L} \longrightarrow (4)$$

→ Next consider fig.(d) The current source is connected to the same load R_L.

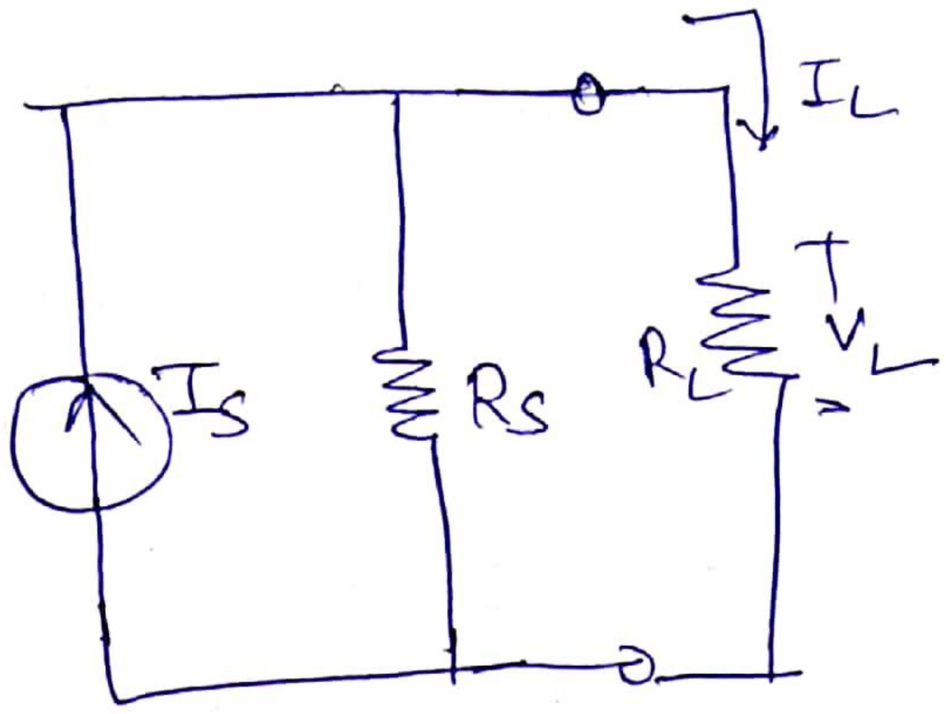


fig.(d)

→ The current through the resistor R_L is given by $I_L = \left(\frac{R_s}{R_s+R_L}\right) I_s$ → (5)

But when converting the source, we get

$I_s = \frac{V_s}{R_s}$ & sub (1) in (5) we get

$$I_L = \frac{\cancel{R_s}}{R_s+R_L} \times \frac{V_s}{\cancel{R_s}} = \frac{V_s}{R_s+R_L} \rightarrow (6)$$

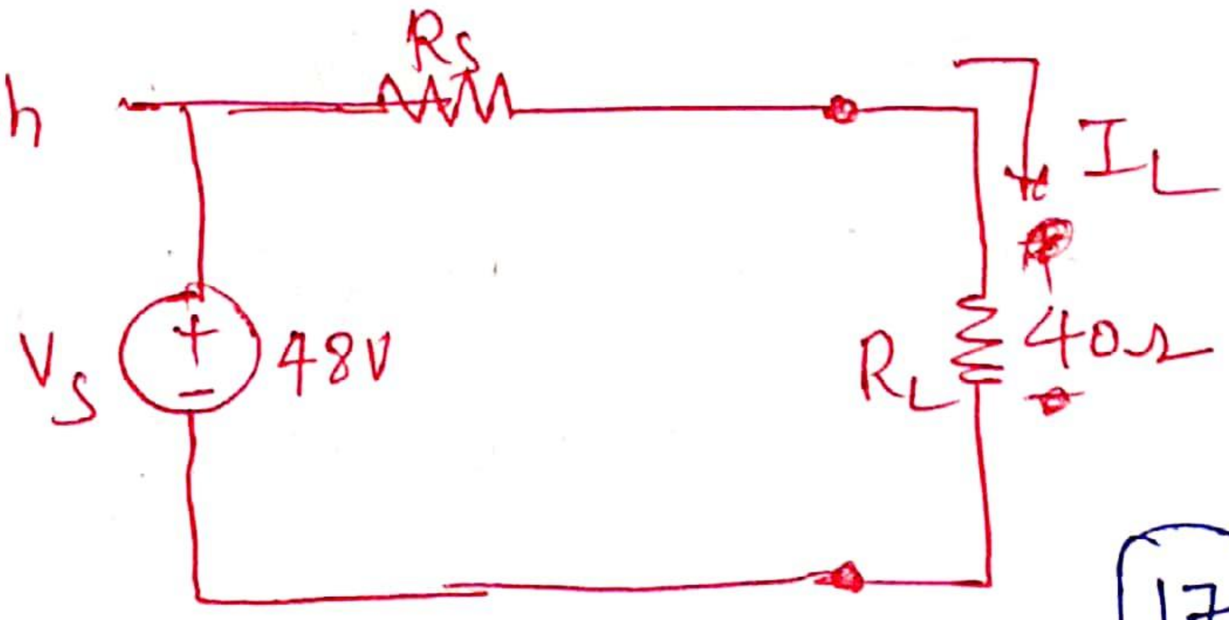
fig.(d): The voltage across resistor is given by

$$V_L = I_L R_L = \left(\frac{V_s}{R_s+R_L}\right) R_L$$

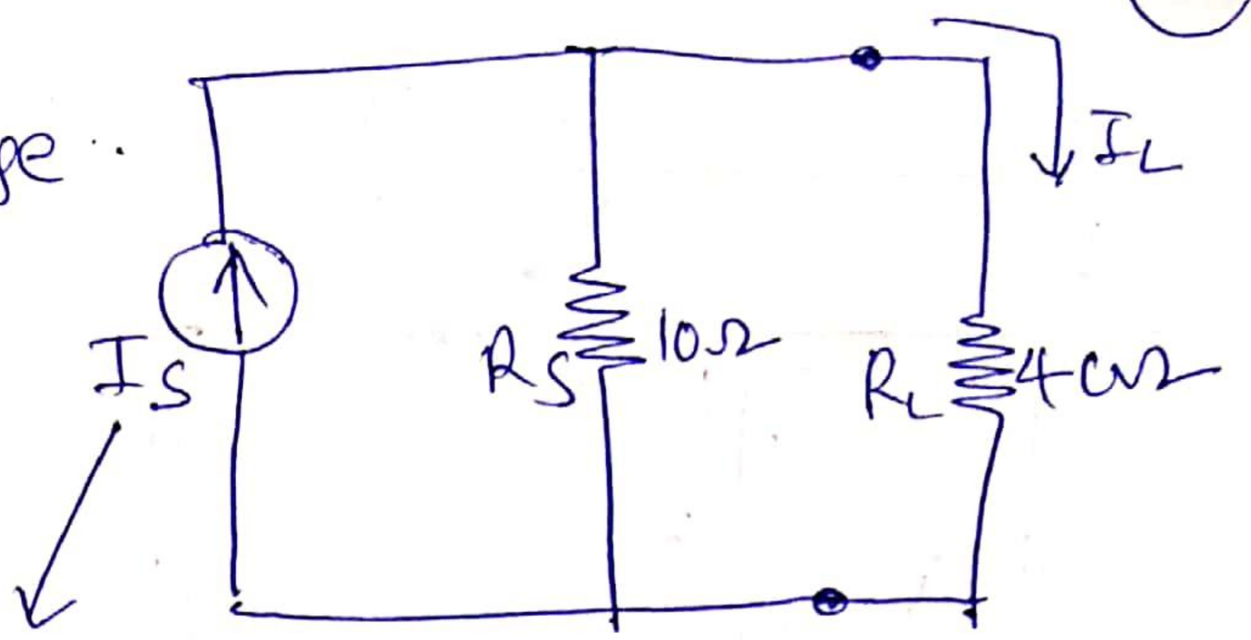
$$\boxed{V_L = \left(\frac{R_L}{R_L+R_s}\right) V_s} \rightarrow (7)$$

* Equation (3) & (7) are same.

* Convert the voltage source of following fig. into a current & verify the current ~~I_L~~ through load is same for each source



Sol:- Given circuit voltage Source Converted to Current Source



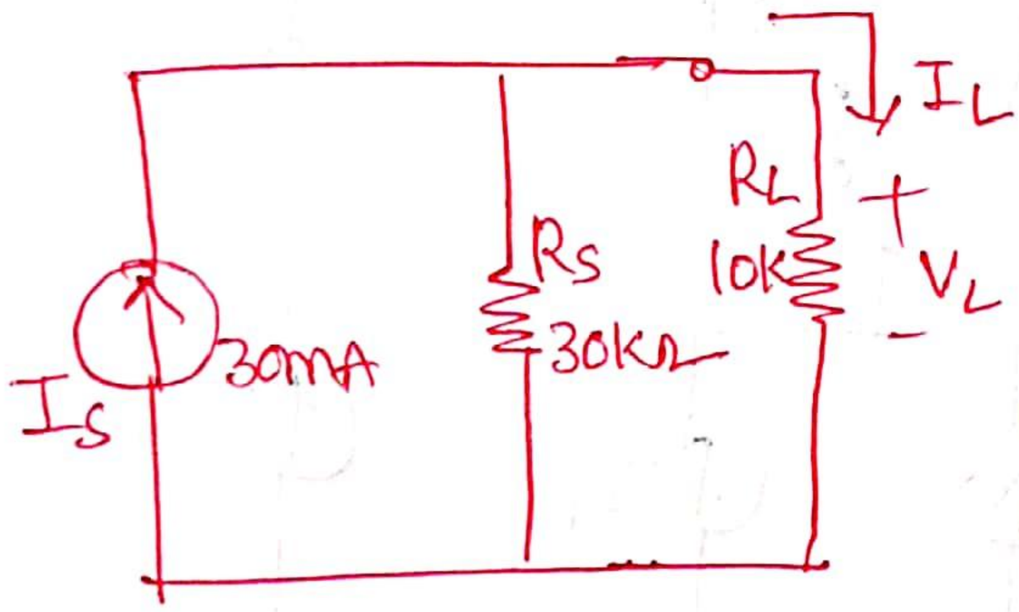
$$I_S = \frac{48V}{10\Omega} = 4.8A$$ fig (a)

→ For given circuit, current through the load is given as $I_L = \frac{48}{10+40} = 0.96A$ → (1)

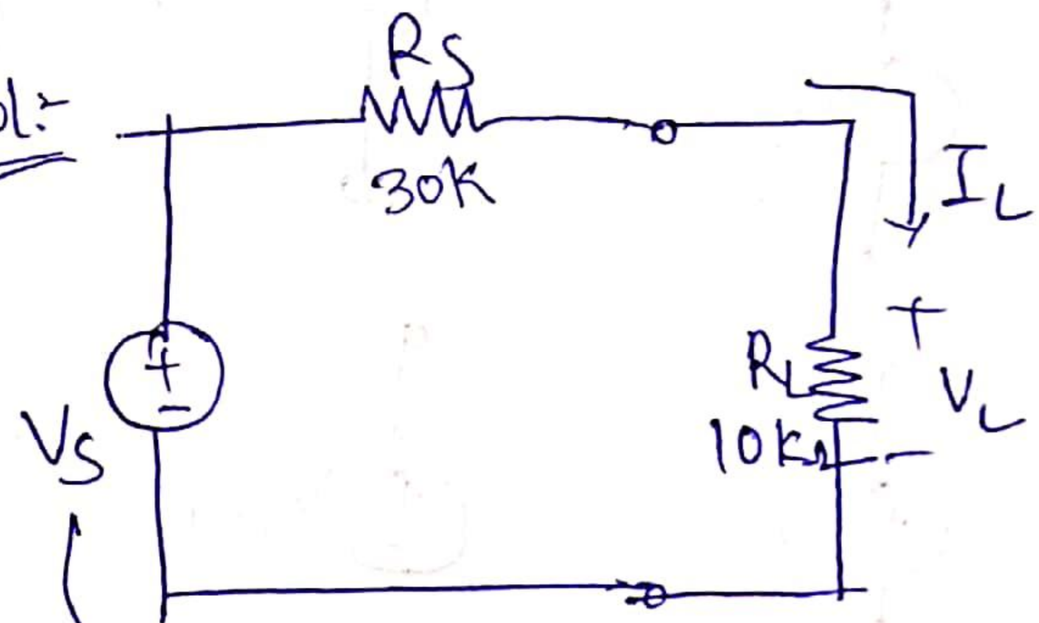
→ From fig (b) also $I_L = \frac{10}{50} \times 4.8 = 0.96A$ → (2)

(1) = (2)

* Convert the Current Source Shown below to Voltage Source and verify the load voltages



Sol:-



$V_S = I_S R_S = 900V$

$I_L = \left(\frac{30K}{40K}\right) 30m$

$I_L = 22.5mA$

$V_L = I_L R_L$

$V_L = 22.5m \times 10K$

$\boxed{V_L = 225V}$

$V_L = \left(\frac{10K}{40K}\right) 900$

$\boxed{V_L = 225V}$

* Series voltage sources & parallel current sources:-

Ex:-

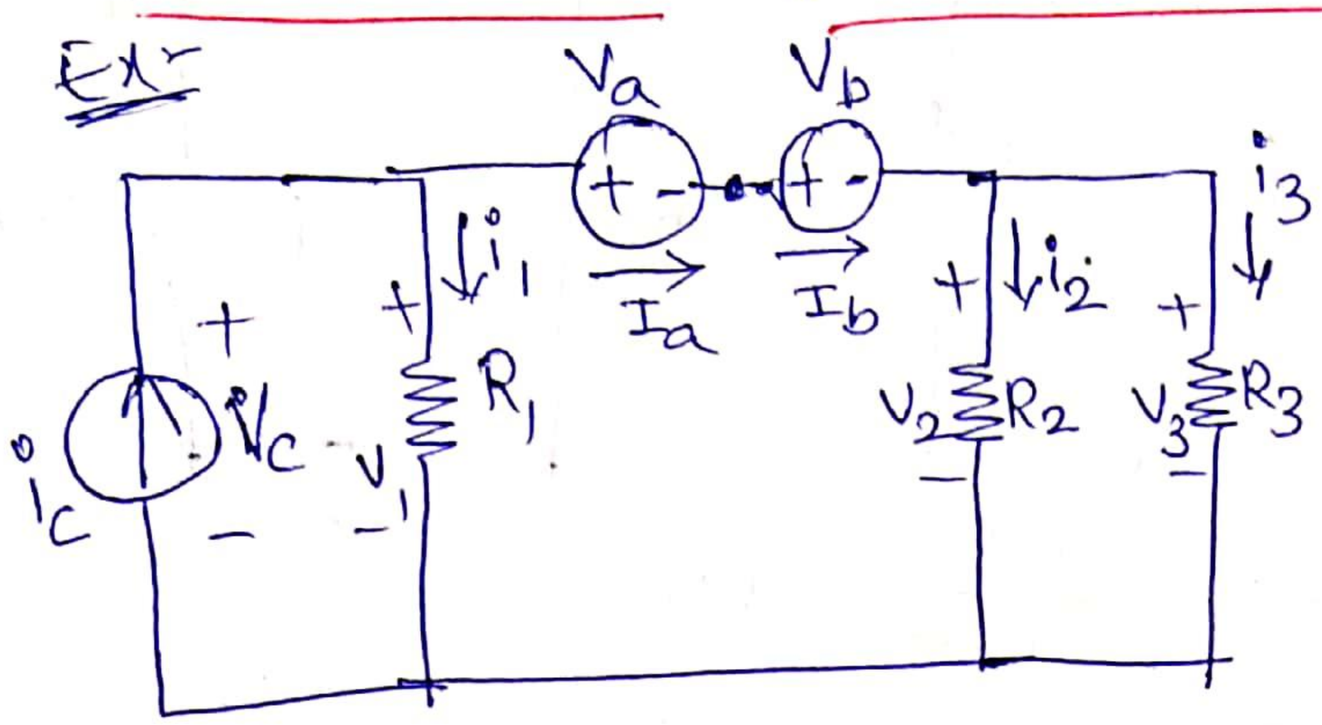


fig (a) voltage sources in series

$$\boxed{V_s = V_a + V_b}$$

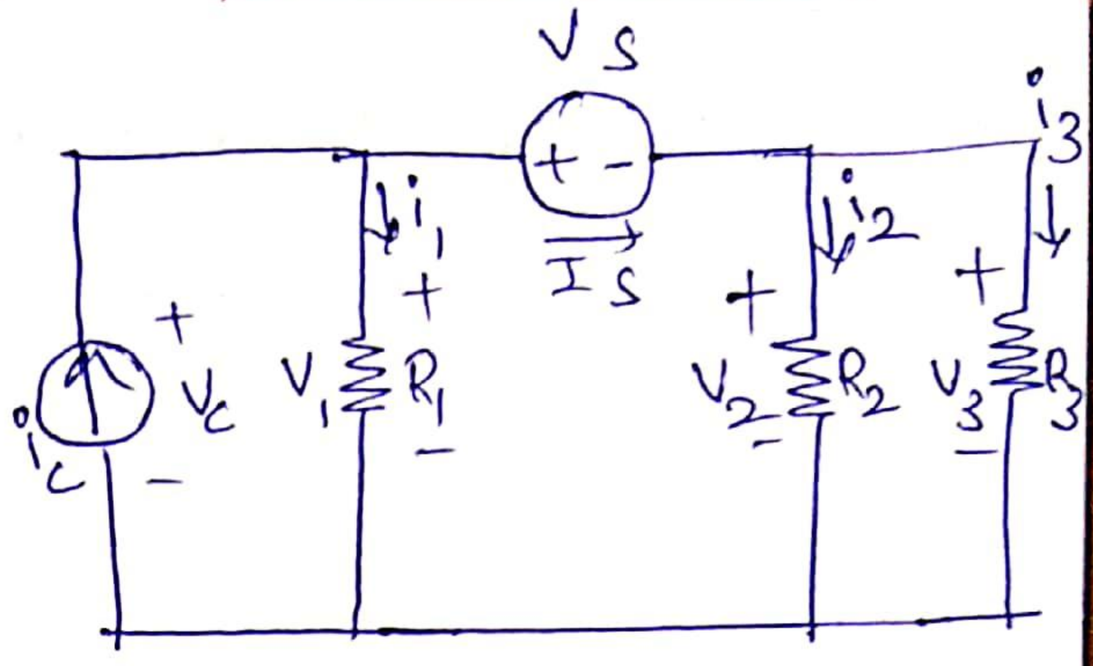
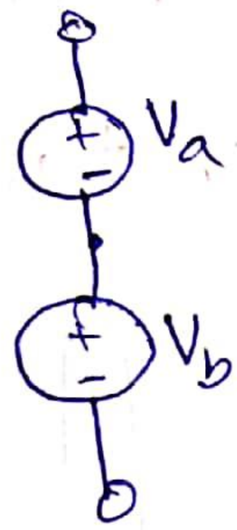
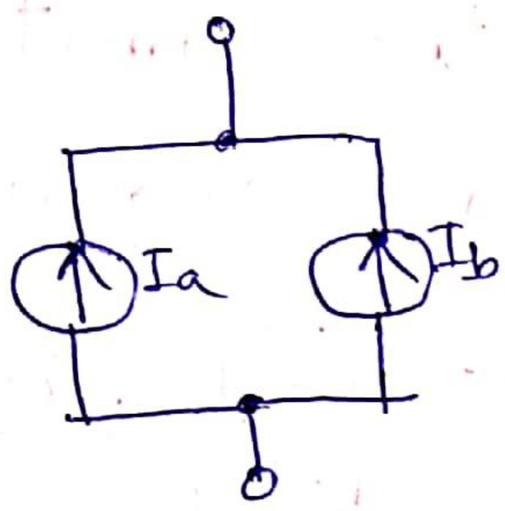
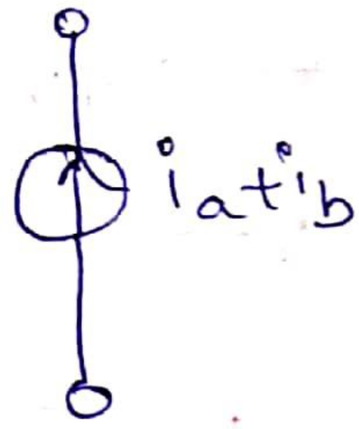
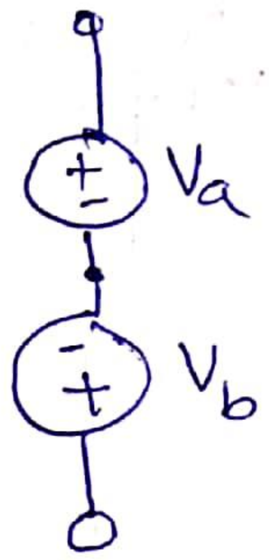
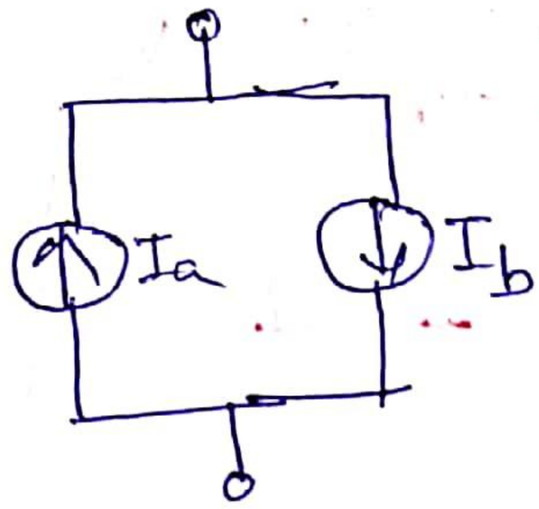
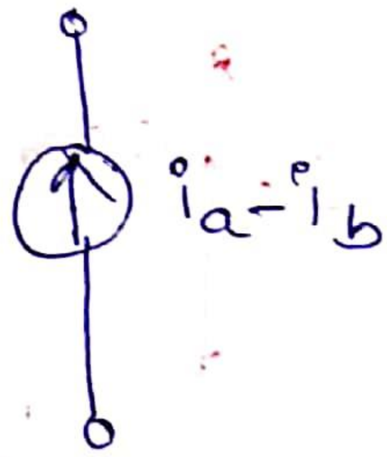
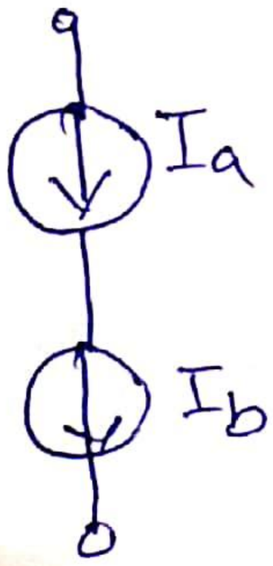
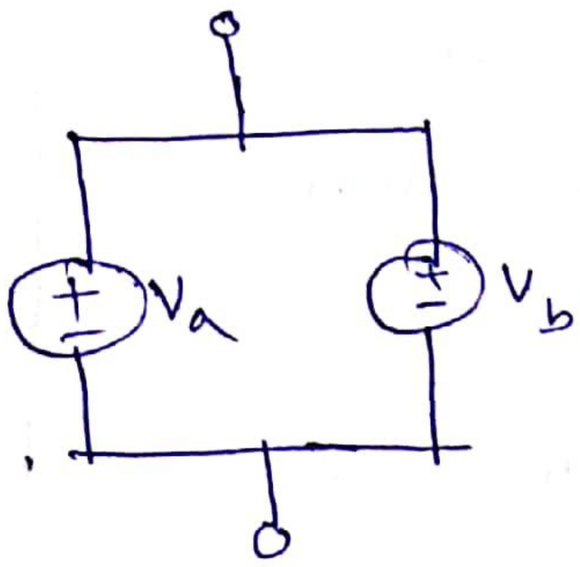


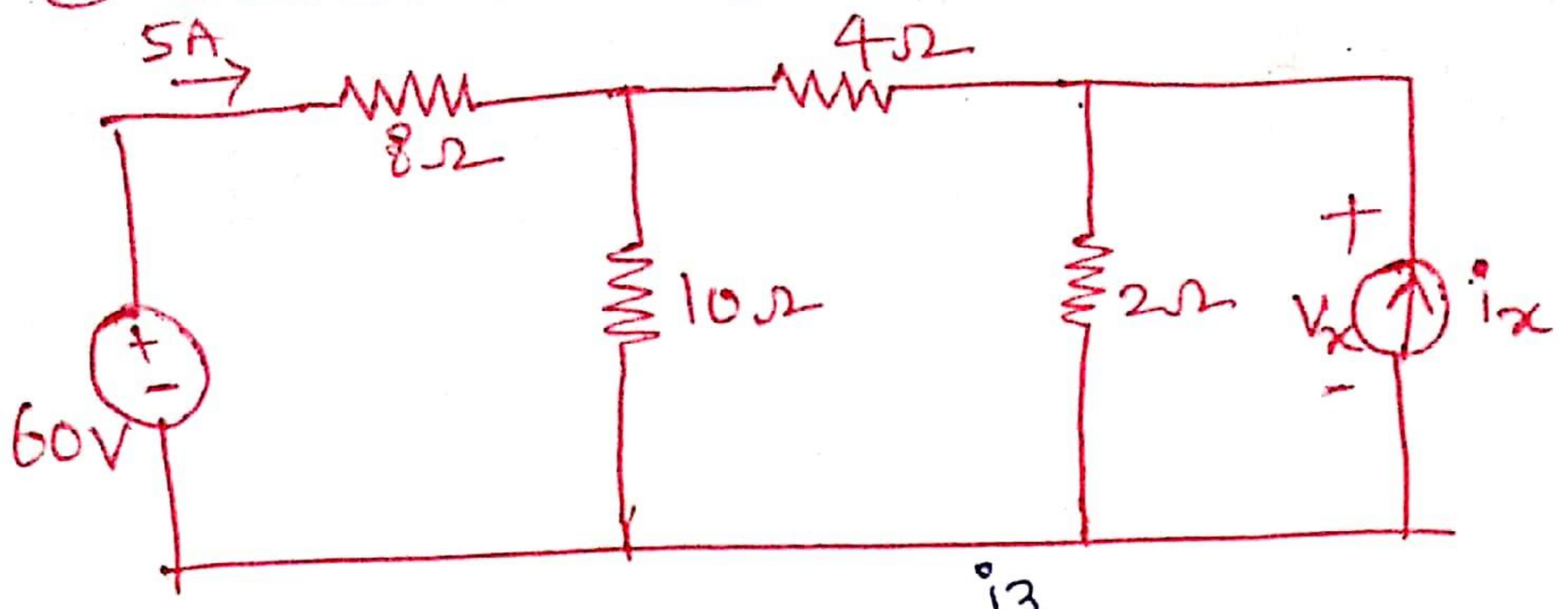
fig. (b) Equivalent Circuit

* parallel & series voltage and current sources:-

	Circuit	Equivalent circuit	circuit	Equivalent ckt
1)				
2)				
3)		Not allowed		Not allowed

* problems *

① Determine V_x in the Circuit shown below



Sol:-

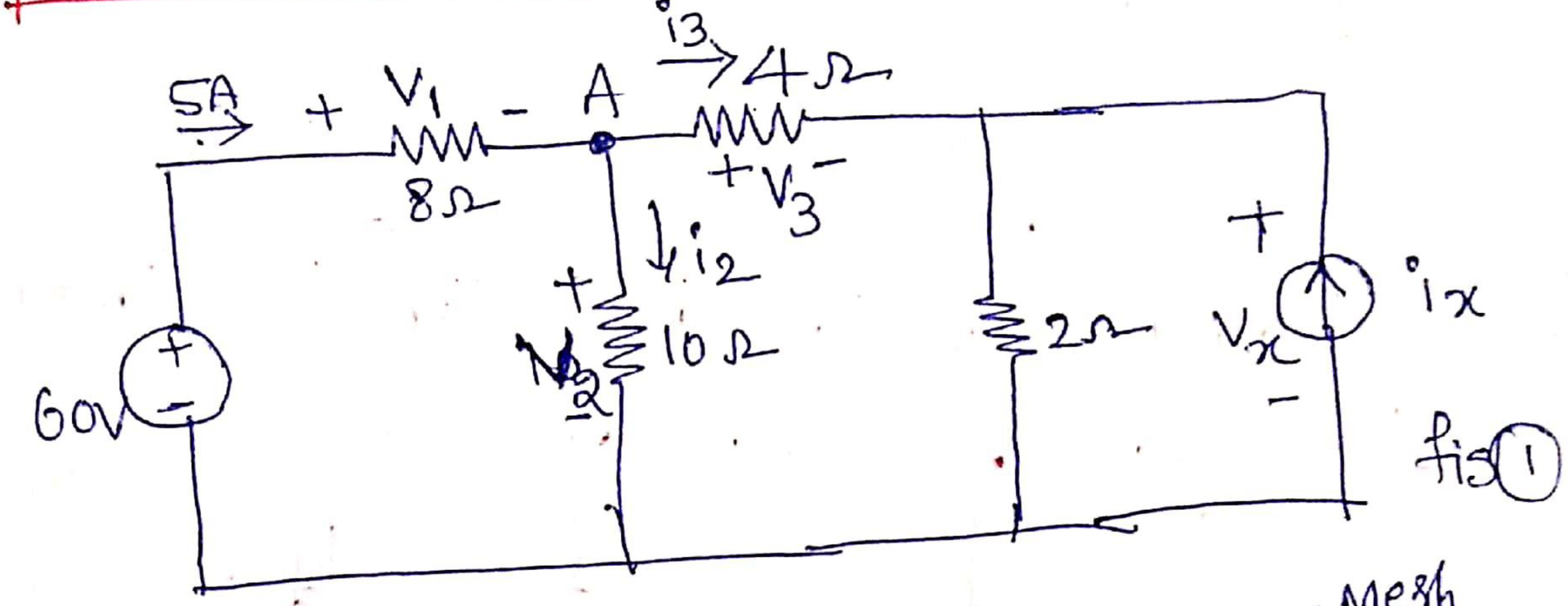


fig ①

→ Applying KVL for the first loop/Mesh we get

$$-60 + V_1 + V_2 = 0 \rightarrow (1)$$

→ Applying KVL for second mesh/loop we get

$$-V_2 + V_3 + V_x = 0 \rightarrow (2)$$

From fig: $V_1 = 5 \times 8 = 40V$ → sub in (1)

$$-60 + 40 + V_2 = 0$$

$$\boxed{V_2 = +20V}$$

From Equ (2) $V_x - V_2 + V_3 = 0$

$$V_x = 20 - V_3 \rightarrow (3) \quad \boxed{\because V_2 = 20V}$$

we need V_3 → for this set i_3

Apply KCL at node 'A' we get

$$5 = i_2 + i_3$$

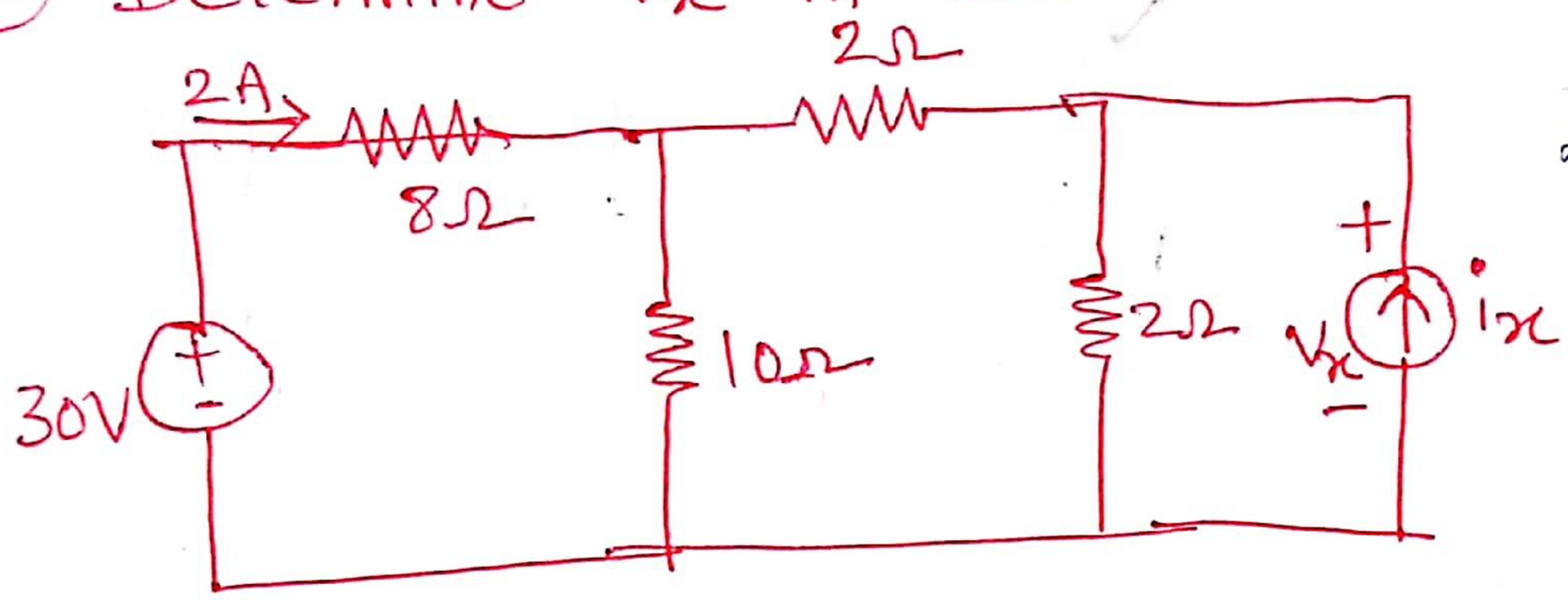
$$i_3 = 5 - i_2 = 5 - \frac{V_2}{10} = 5 - \frac{20}{10} = 5 - 2 = 3A$$

$$\therefore V_3 = i_3 R_3 = 3 \times 4 = 12V \rightarrow (4)$$

Sub. Eqn (4) in (3) we get

$$\boxed{V_x = 20 - 12 = 8V}$$

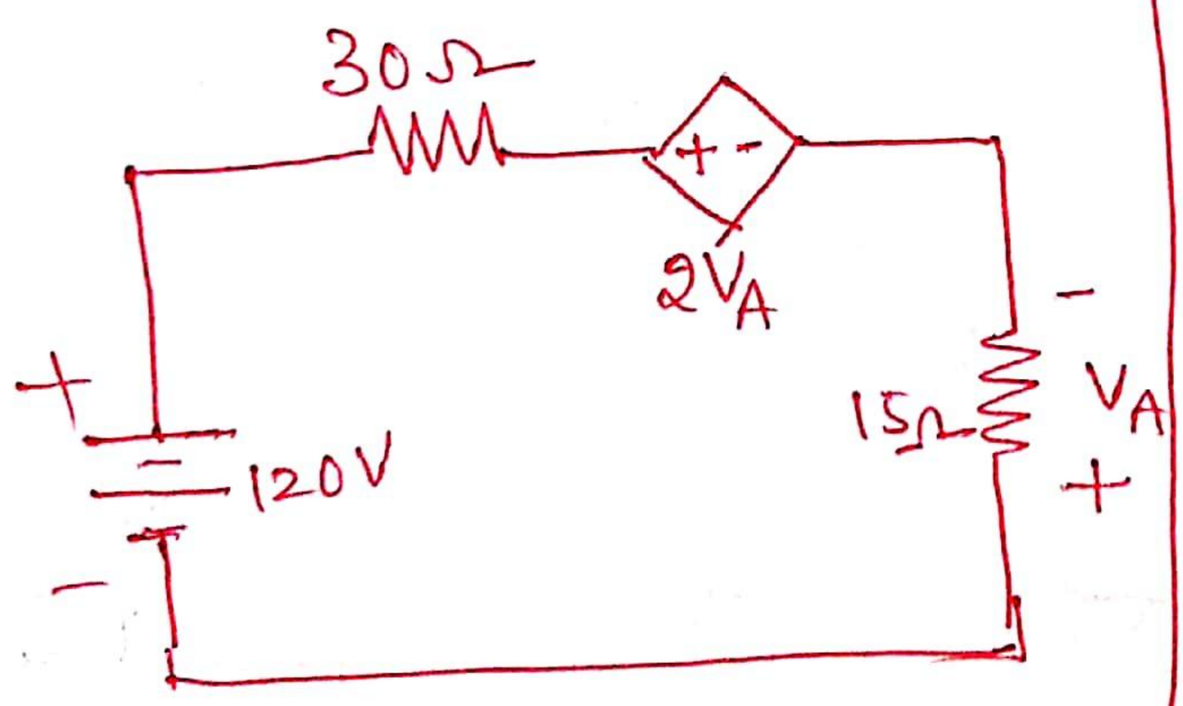
(*2) Determine V_x in the Circuit shown,



* Ans:

$V_x = 12.8V$

(3) Compute the power absorbed in each Element for the Circuit shown.



* Ans:- $P_{120V} = -960W$; $i = 8A$

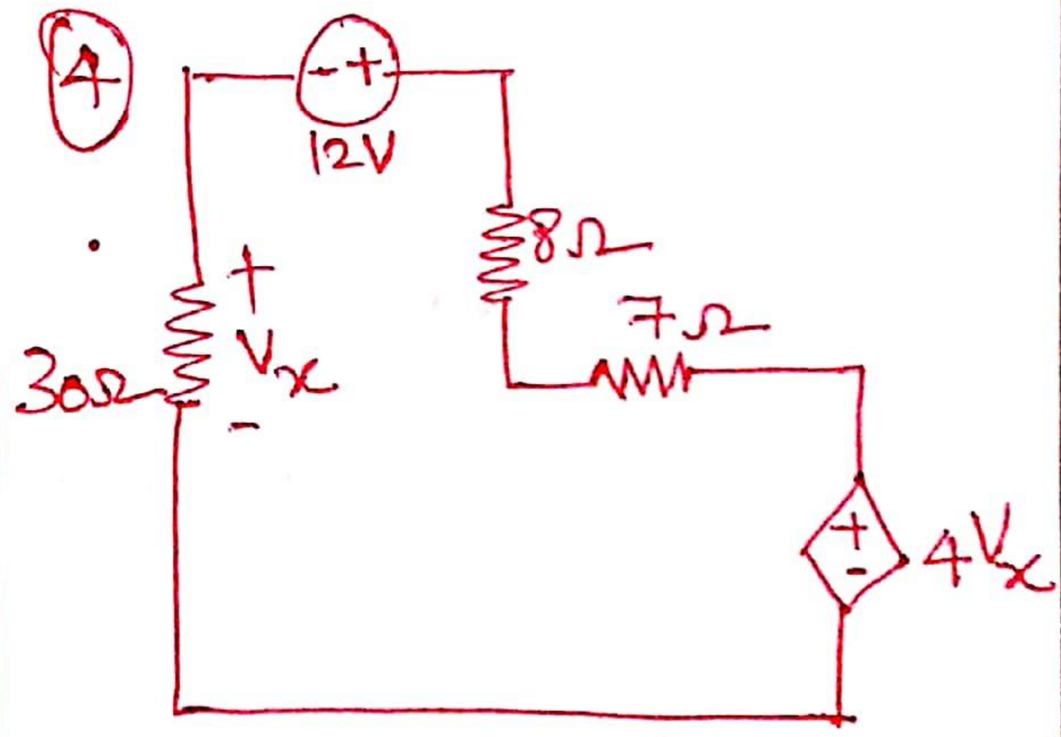
$P_{30\Omega} = 1920W$

$P_{dep} = -1920W$

$P_{15\Omega} = 960W$

$\boxed{\Sigma P_{absorbed} = \Sigma P_{supplied}}$

(4)



$P_{30\Omega} = 0.768W$

$P_{12V} = 1.92W$

$P_{8\Omega} = 0.208W$

$P_{7\Omega} = 0.1792W$

$P_{dep} = -3.072W$

5) Find the voltage, current & power associated with each element in the circuit

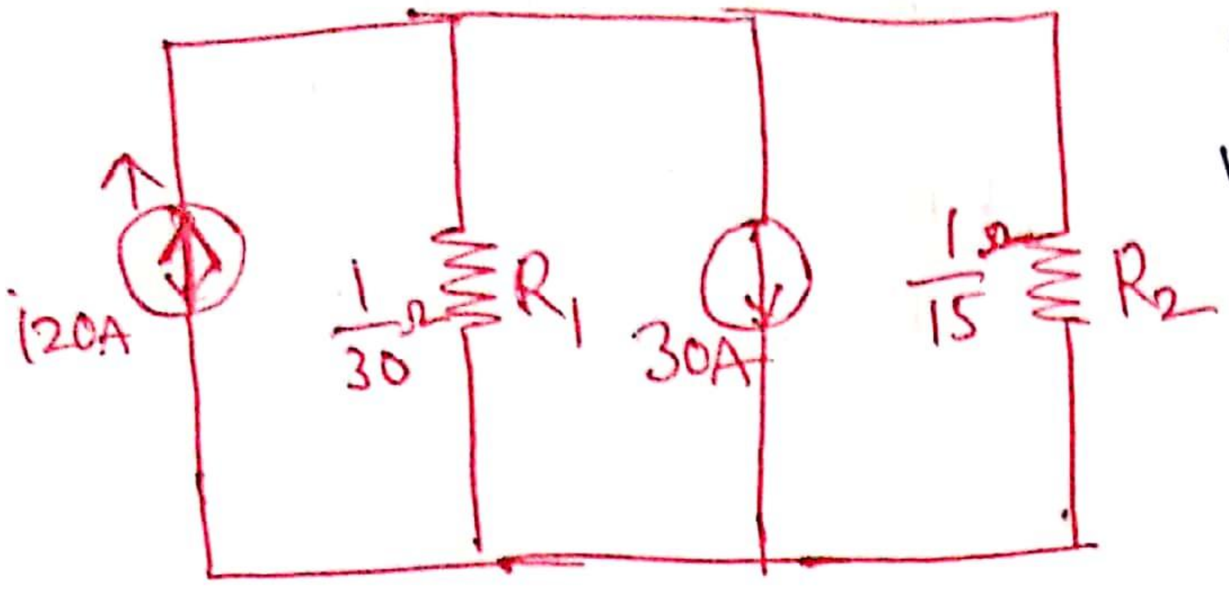


fig.(a)

→ Applying KCL for fig.(a) we get

$$i_1 + i_2 + 30 = 120$$

$$i_1 + i_2 = 90 \rightarrow (1)$$

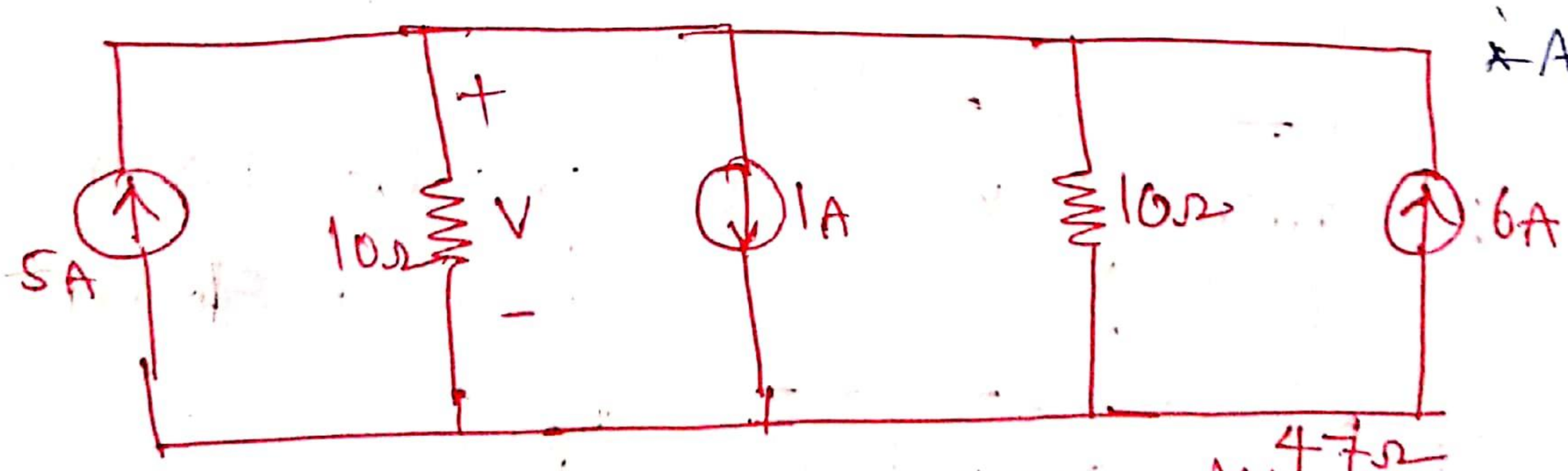
But $i_1 = 30V$ & $i_2 = 15V \Rightarrow 45V = 90 \Rightarrow V = 2V$

$\therefore$ $i_1 = 60A$ & $i_2 = 30A$

→ $P_{R1} = P_{\frac{1}{30}\Omega} = 120W$; $P_{R2} = P_{\frac{1}{15}\Omega} = 60W$

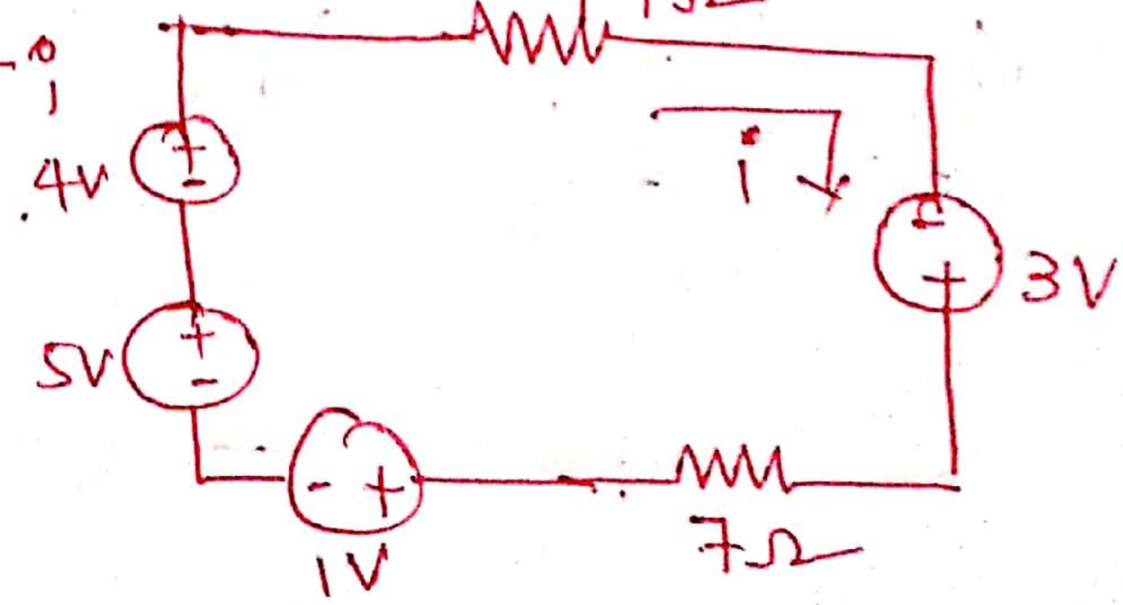
$P_{120A} = 120(-2) = -240W$; $P_{30A} = 30 \times 2 = 60W$

6) Determine V in the circuit shown



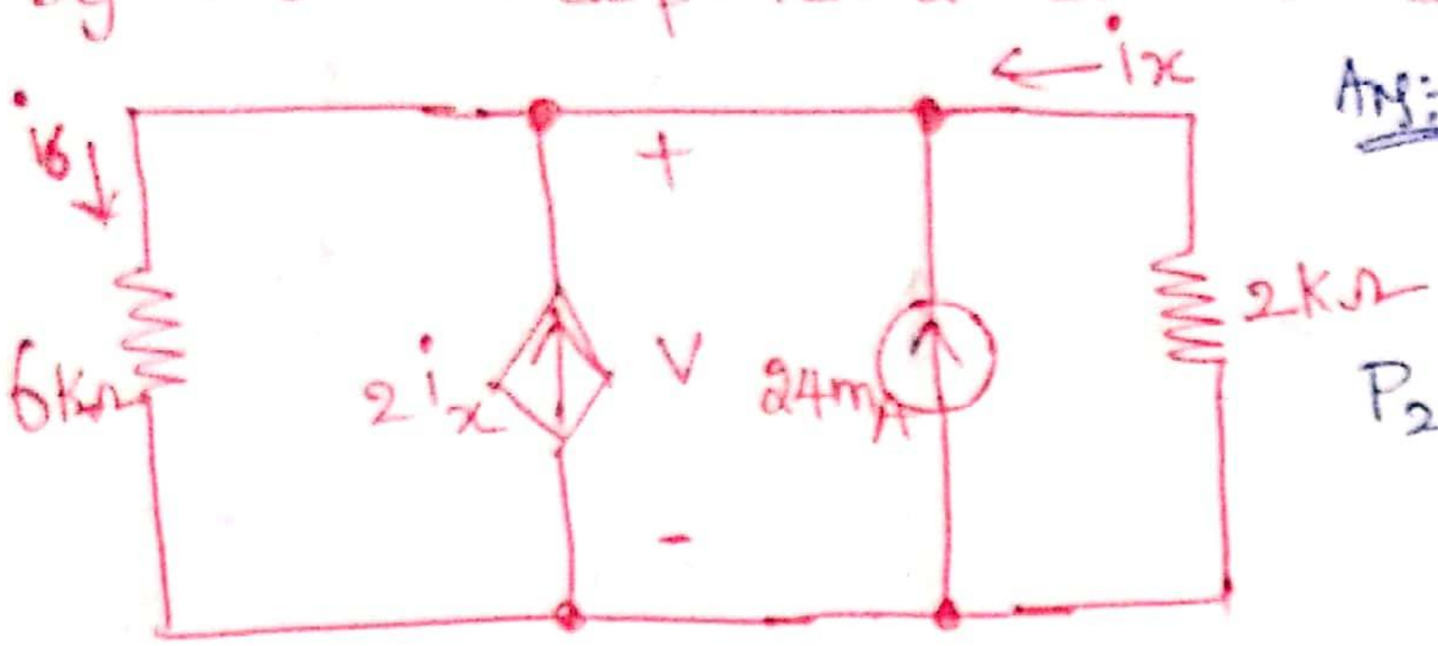
Ans: 50V

7) Determine the current i



Ans:- ~~54A~~ $i = \frac{11}{54}$ A

(8) Determine the value of 'V' & power supplied by the independent current source



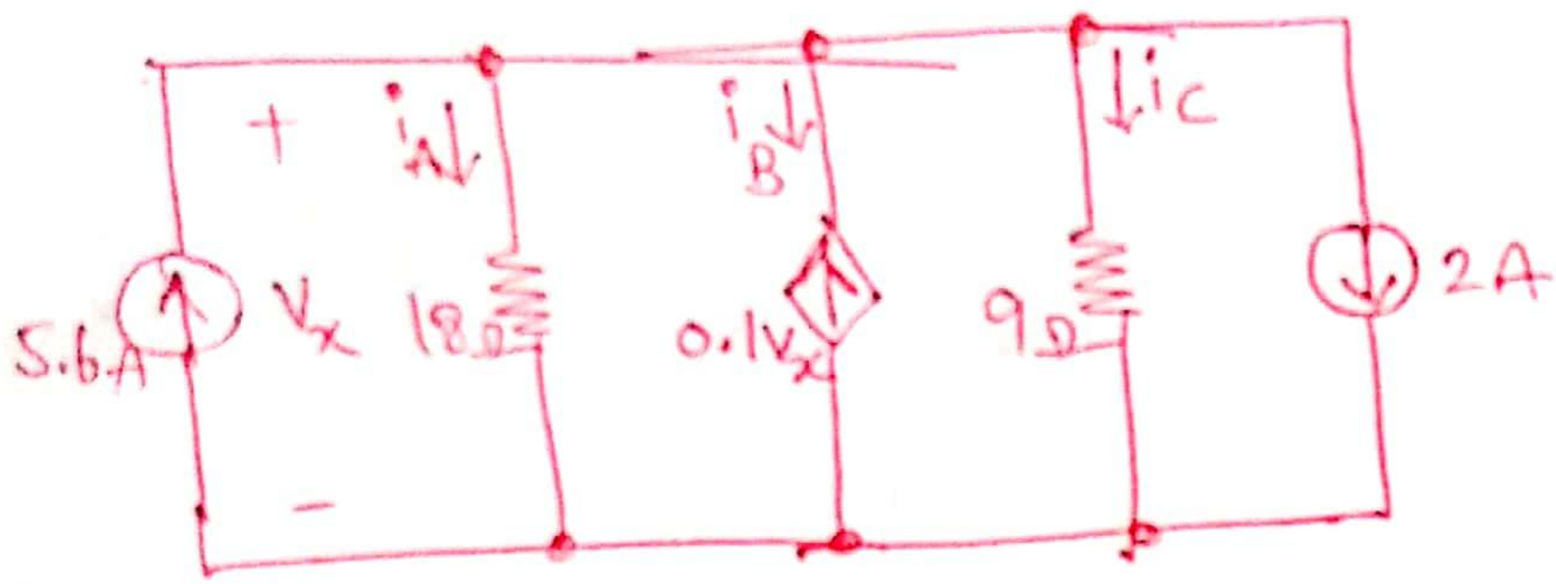
Ans:- $i = 24mA$

$V = 14.4V$

$P_{24mA} = 14.4 \times 0.024$

$\Rightarrow 0.3456W$

(9) For the single-node pair ckt, find i_A, i_B & i_C

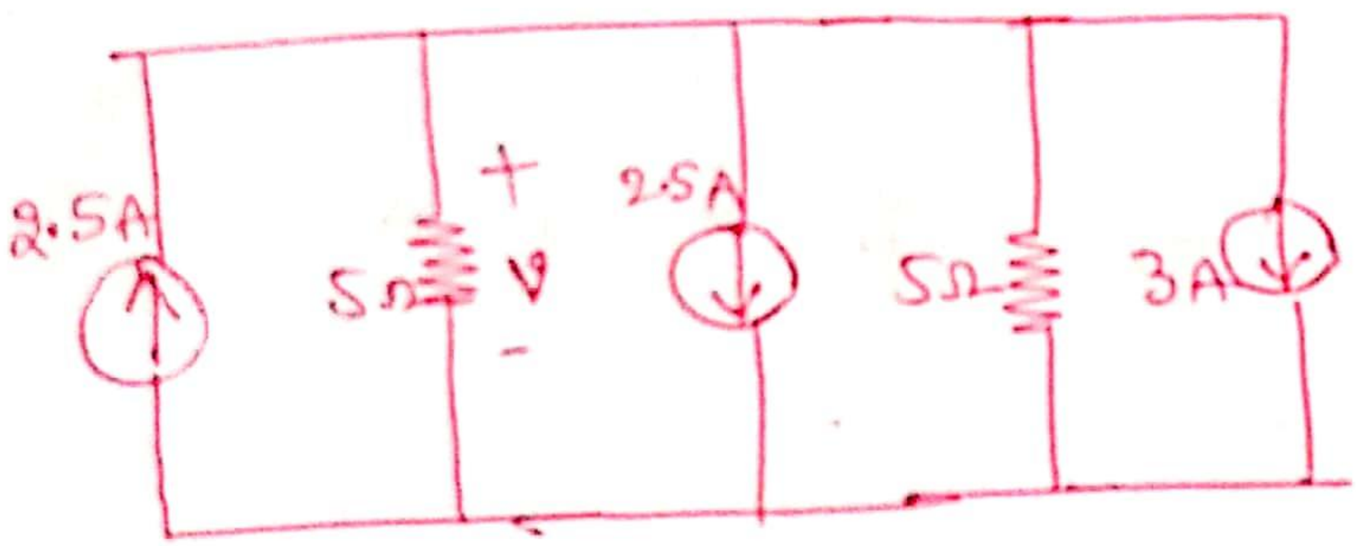


Ans:- $i_A = 3A$,

$i_B = -5.4A$

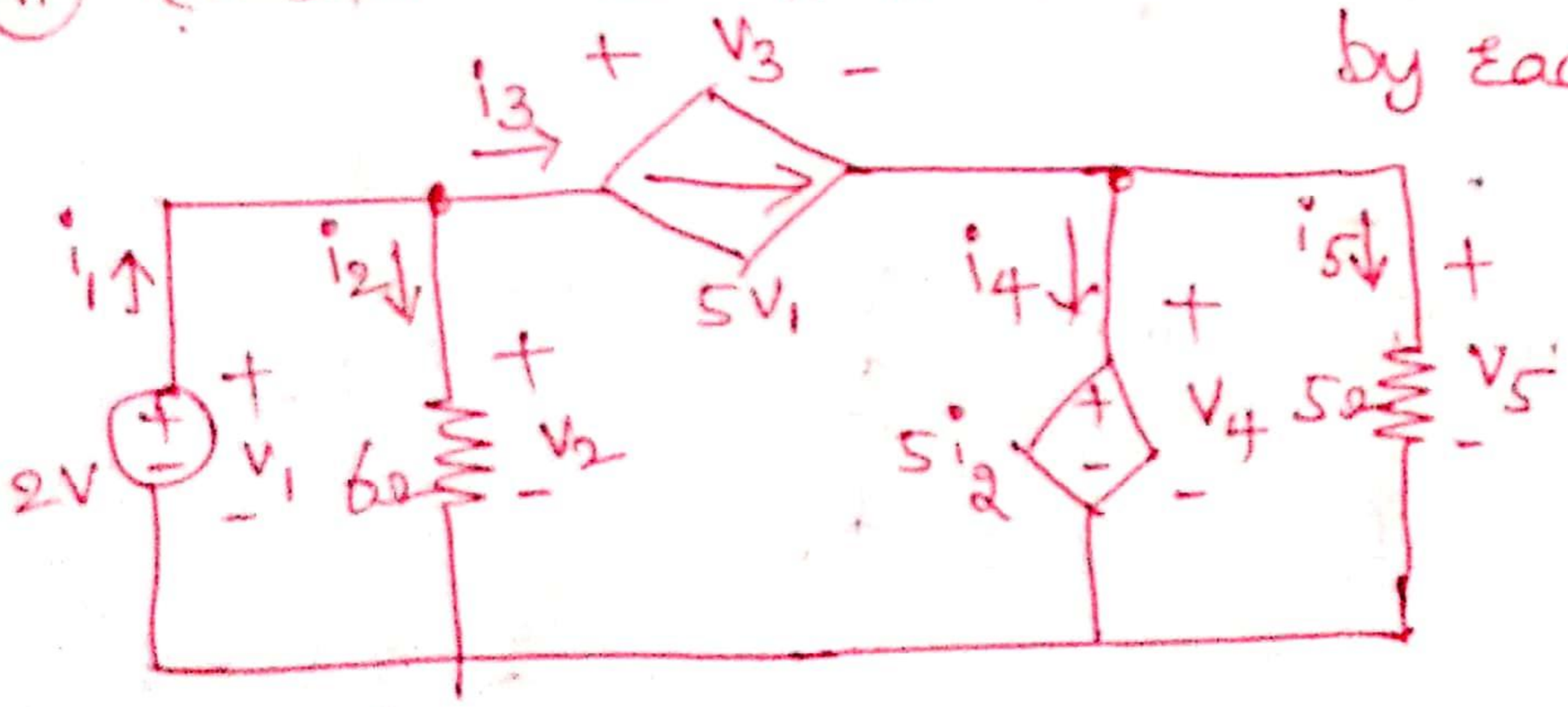
$i_C = 6A$

(10) Determine 'v' in the ckt shown below



Ans:- $v = 7.5V$

(11) (a) Determine $i_1, v_1, \ldots$ (b) Calculate power absorbed by each element



* Methods of Analysis:-

Nodal Analysis:- Nodal analysis provides a general procedure for analyzing circuits using "node voltages" as the circuit variables.

Given a circuit with 'n' nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

→ Steps to Determine Node voltages:

1. Select a node as the reference node. Assign voltages $V_1, V_2, V_3 \ldots V_{n-1}$ to the remaining (n-1) nodes. The voltages are referenced with respect to the "reference node".
2. Apply KCL to each of the (n-1) nonreference nodes. Use ohm's law to express the "branch currents" in terms of node voltages
3. Solve the resulting simultaneous equations to obtain the unknown "node voltages".

The reference node is commonly called the ground. Since it is assumed to have zero potential. (a) Common ground (b) ground (c) chasis ground.

[The No. of nonreference nodes is equal to the number of independent equations]

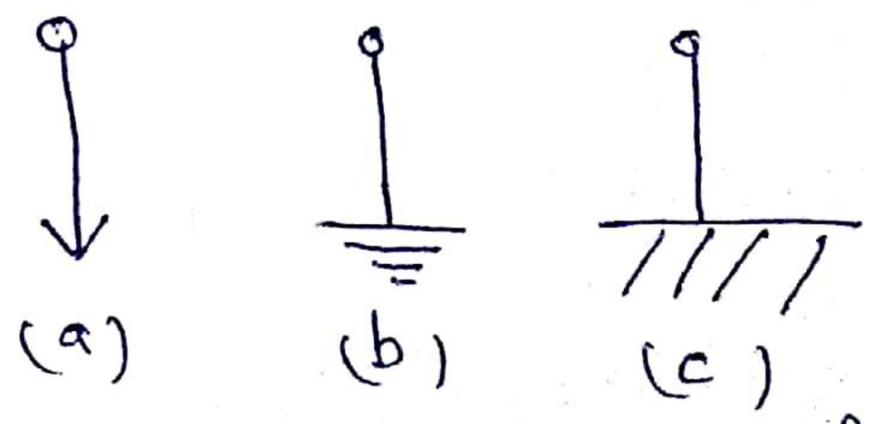
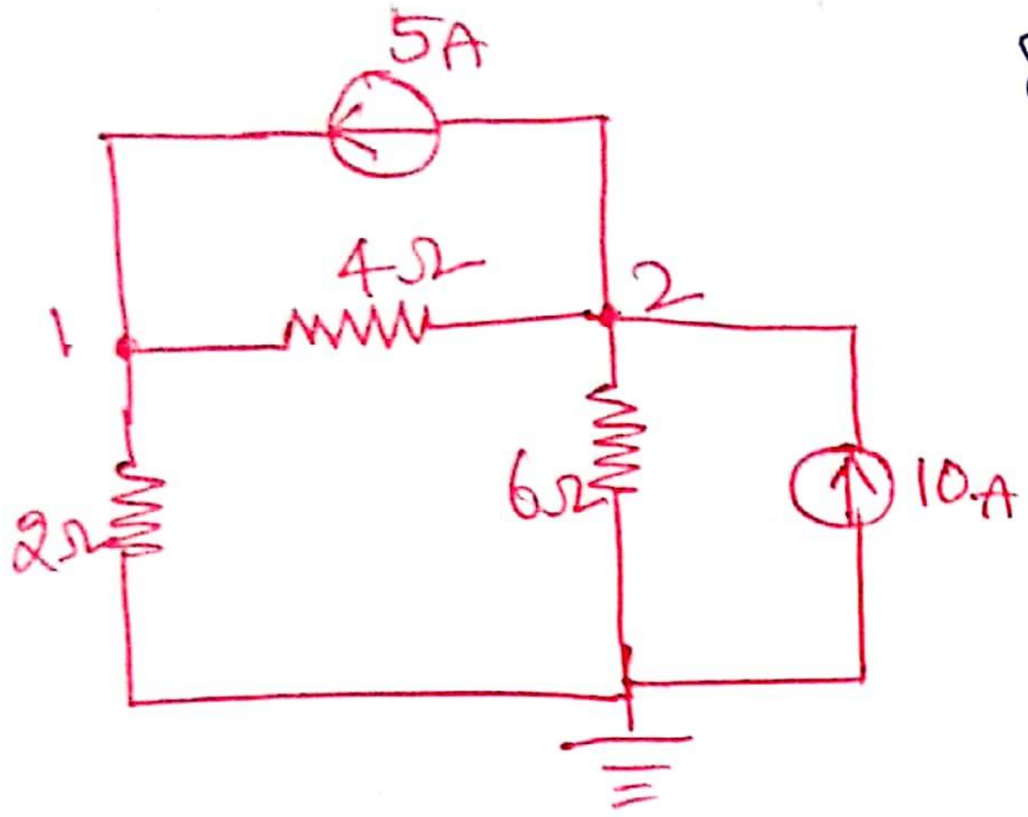


fig: Common symbols for reference node

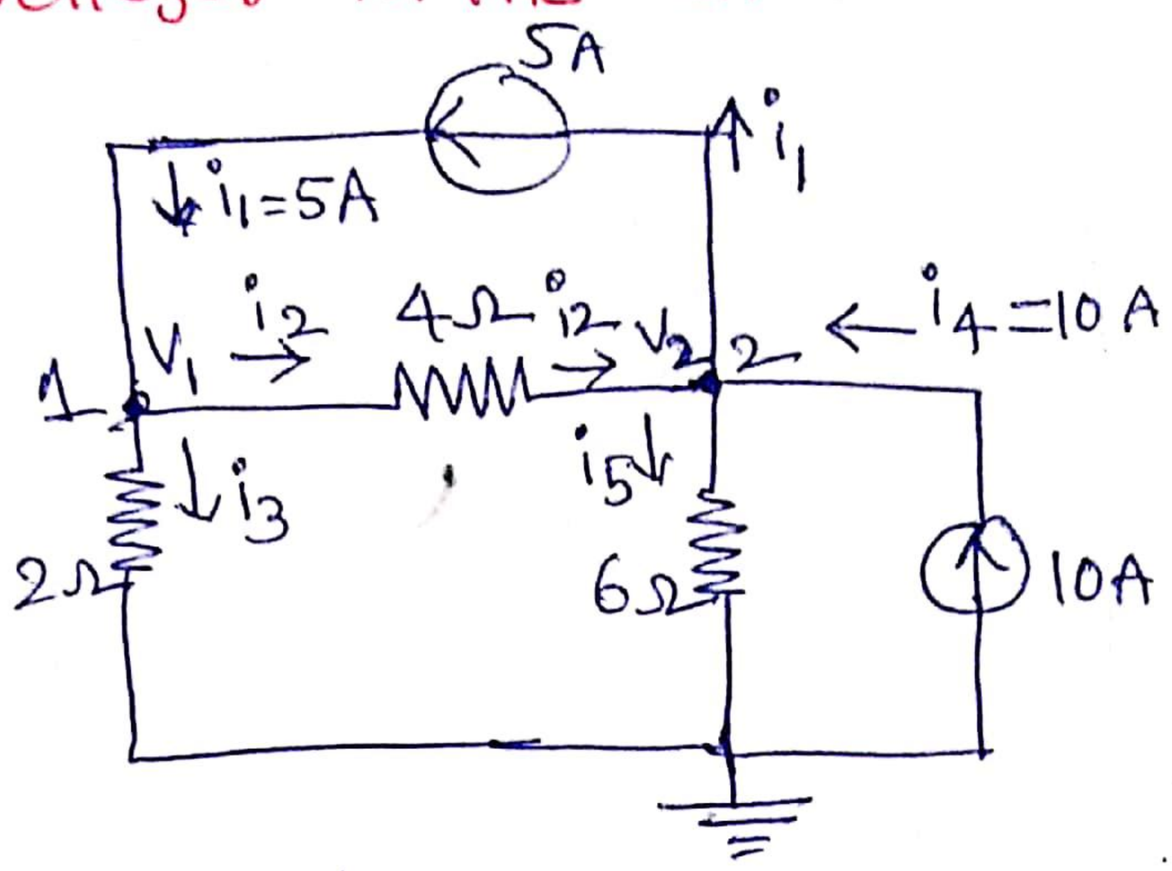
→ Current flows from a higher potential to a lower potential in a resistor

$$\boxed{i = \frac{V_{higher} - V_{lower}}{R}}$$

* Calculate the node voltages in the circuit shown



Sol:-



→ At node 1, applying KCL & ohm's law gives

$$i_1 = i_2 + i_3 \Rightarrow 5 = \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2}$$

Simplifying we get $3V_1 - V_2 = 20 \rightarrow (1)$

→ At node 2, apply KCL & ohm's law gives

$$i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{V_1 - V_2}{4} + 10 = 5 + \frac{V_2 - 0}{6}$$

Simplifying we get $-3V_1 + 5V_2 = 60 \rightarrow (2)$

Solving (1) & (2) gives node voltages V_1 & V_2

Method 1:- Using the Elimination technique

$(1) + (2) \Rightarrow 4V_2 = 80 \Rightarrow \boxed{V_2 = 20V.} \rightarrow$ sub in (1)

$\Rightarrow \boxed{V_1 = 13.33V}$

i.e $3V_1 - 20 = 20$

$3V_1 = 20 + 20 = 40 \Rightarrow V_1 = \frac{40}{3} = 13.33V$

*Method 2:- Using Cramer's rule, we can get values of V_1 & V_2 by writing Equations in matrix form

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

→ The determinant of the Matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

$$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100+60}{12} = 13.33\,V$$

$$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180+60}{12} = 20\,V$$

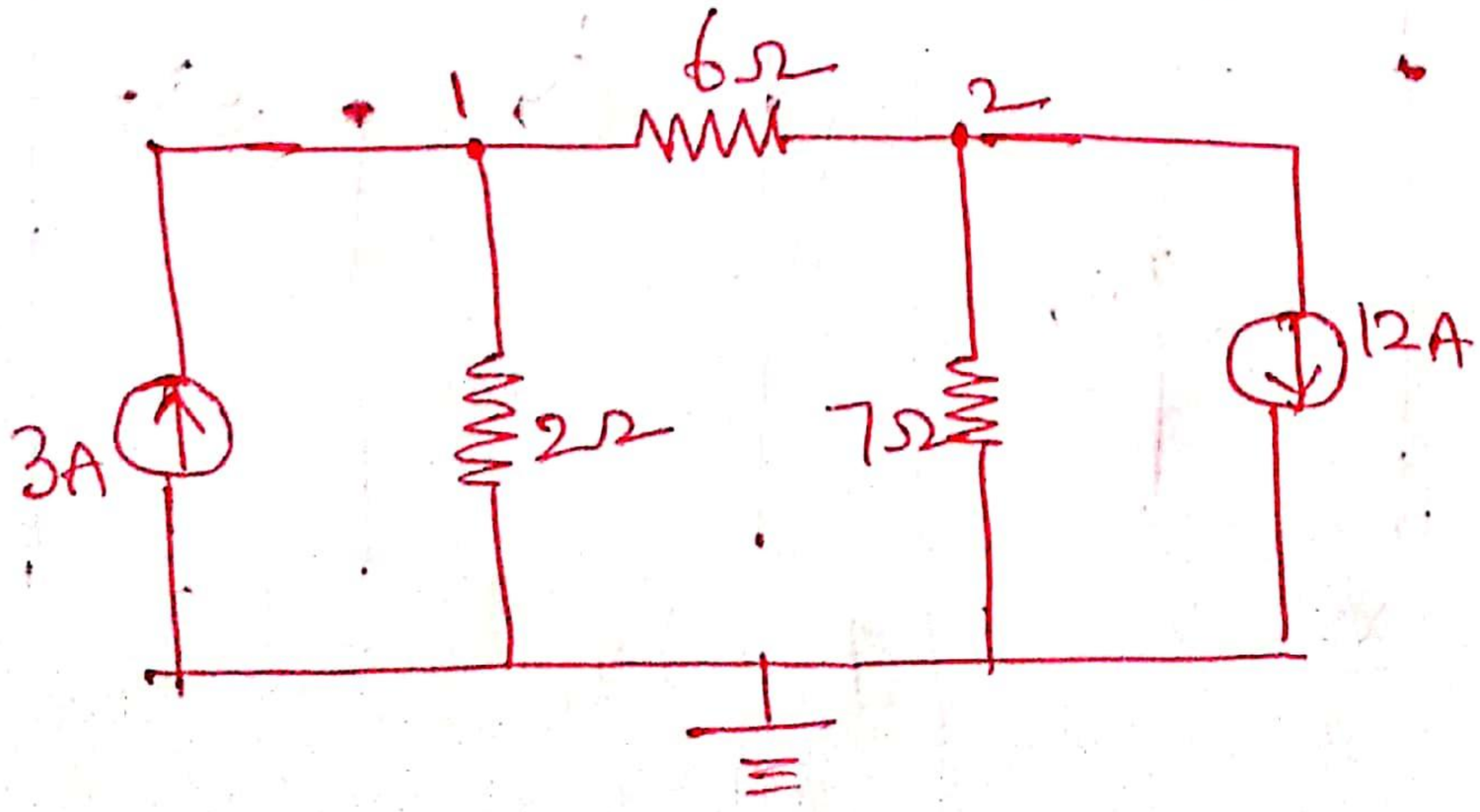
By substituting V_1 & V_2 we get currents

$$i_1 = 5A, \quad i_2 = \frac{V_1 - V_2}{4} = -1.6668A, \quad i_3 = \frac{V_1}{2} = 6.66A$$

$$i_4 = 10A, \quad i_5 = \frac{V_2}{6} = 3.33A$$

The fact that i_2 is negative shows that the current flows in the direction opposite to the one assumed.

* Obtain the node voltages in the Circuit Shown

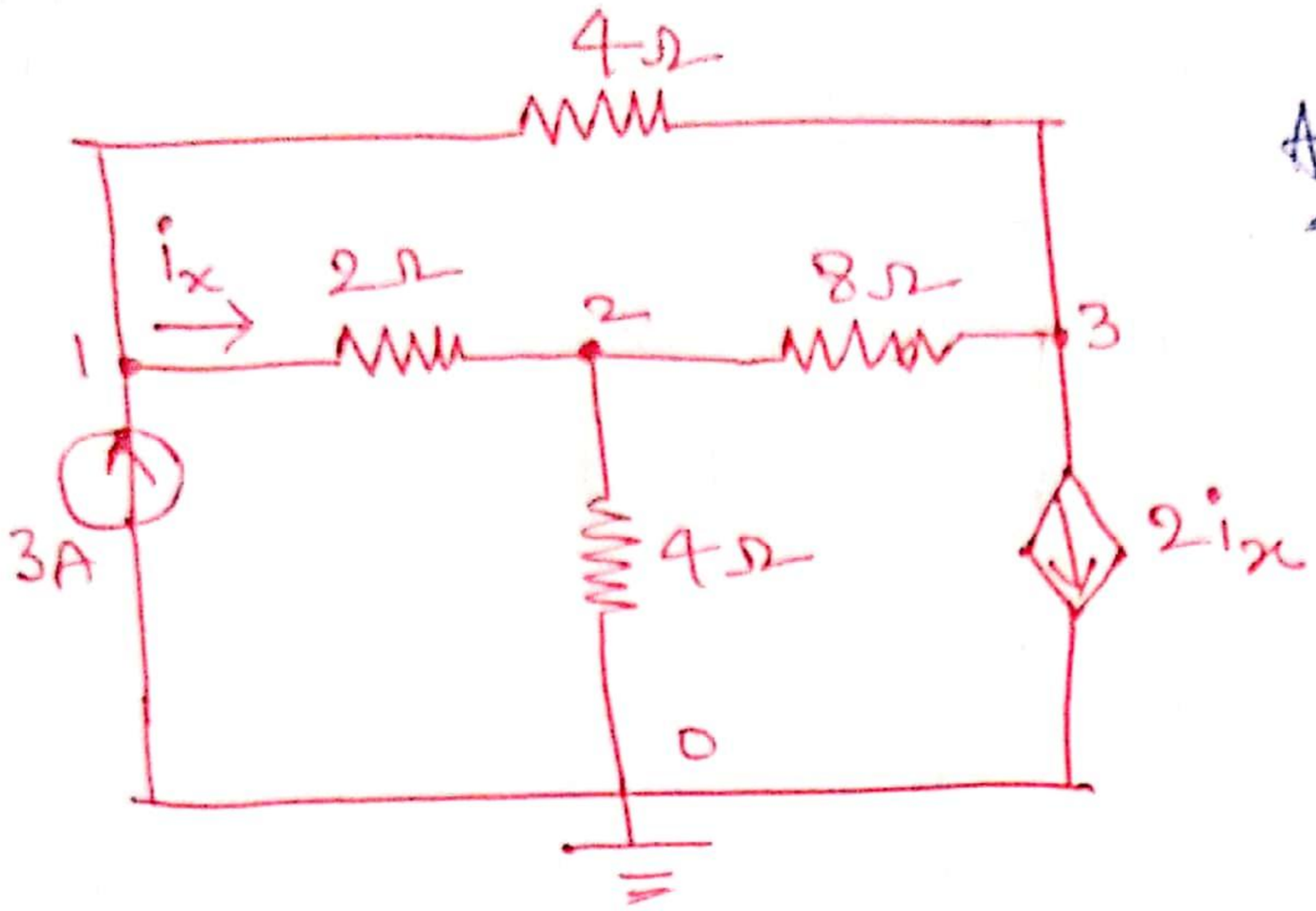


Ans:-

$V_1 = -6V$,

$V_2 = -42V$

* Determine the voltages at the nodes.

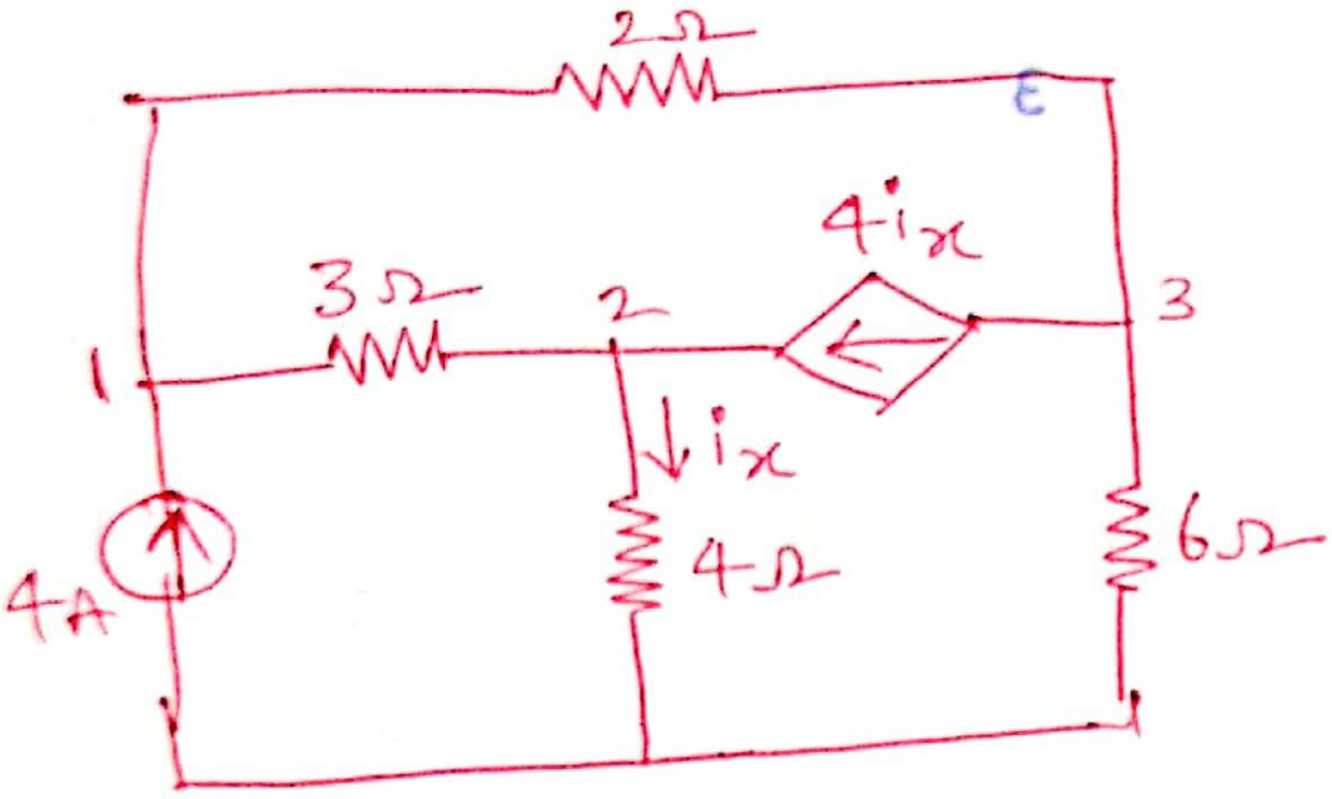


Ans:- $V_1 = 4.8V$,

$V_2 = 2.4V$,

$V_3 = -2.4V$

* Determine the voltages at the three nonreference nodes in the circuit

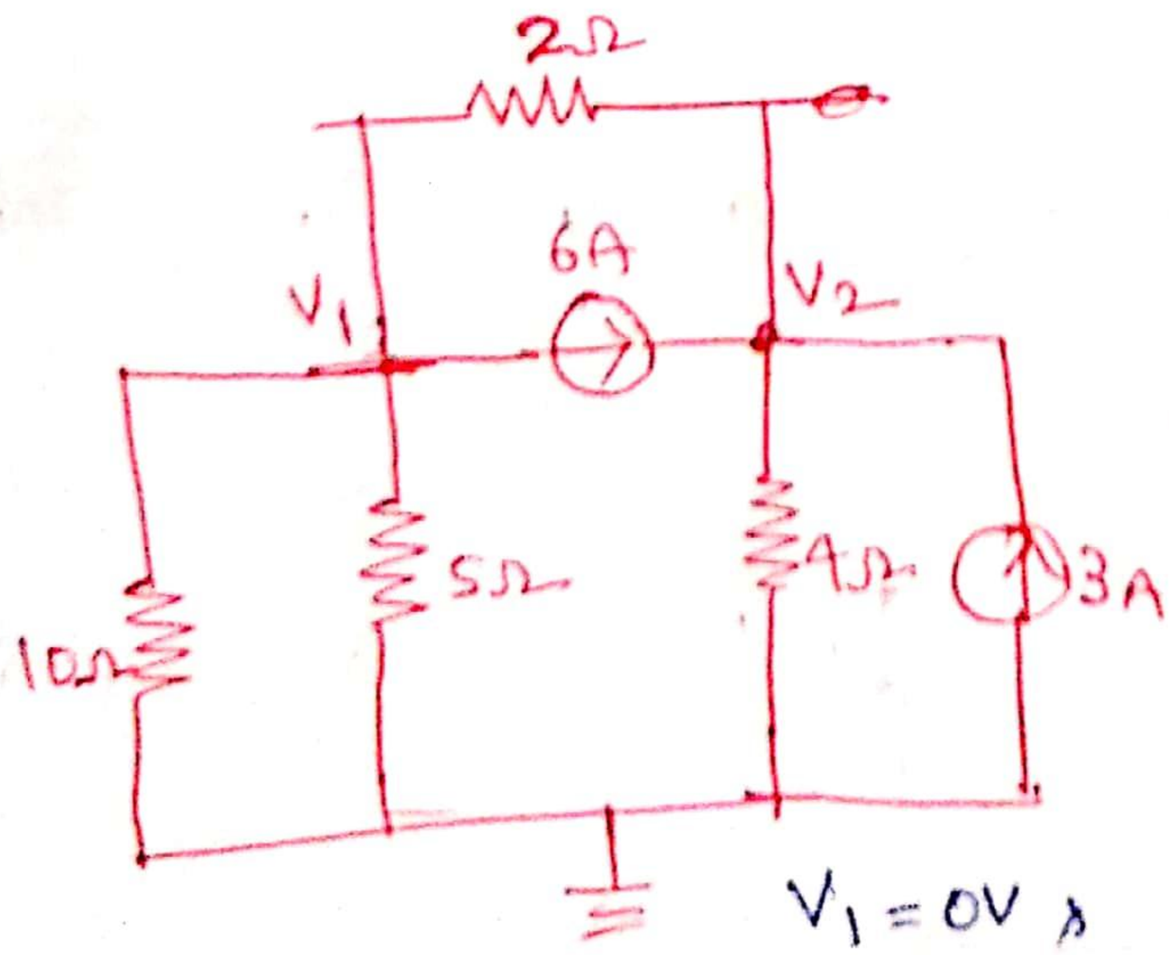


Ans:- $V_1 = 3V$,

$V_2 = -25.6V$ &

$V_3 = 62.4V$

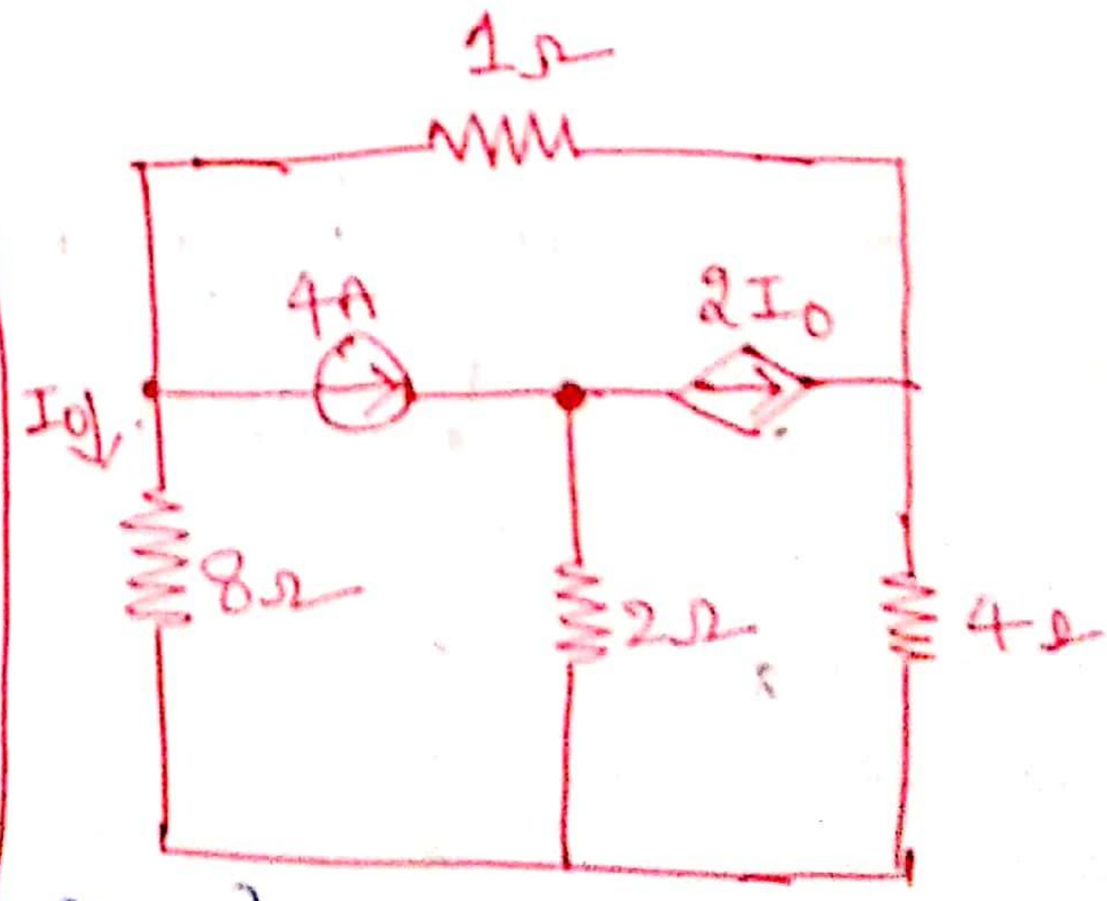
* For the circuit shown below obtain V_1 & V_2



$V_1 = 0V$ &

$V_2 = 12V$

* Find I_o in the Circuit shown.



$I_o \Rightarrow -4A$ $\left\{ \begin{array}{l} V_1 = -32V,\ V_2 = 24V \ \& \\ V_3 = -32V \end{array} \right.$

(ii) Nodal Analysis with Voltage Sources:-

case1:- If a voltage source is connected b/w the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source.

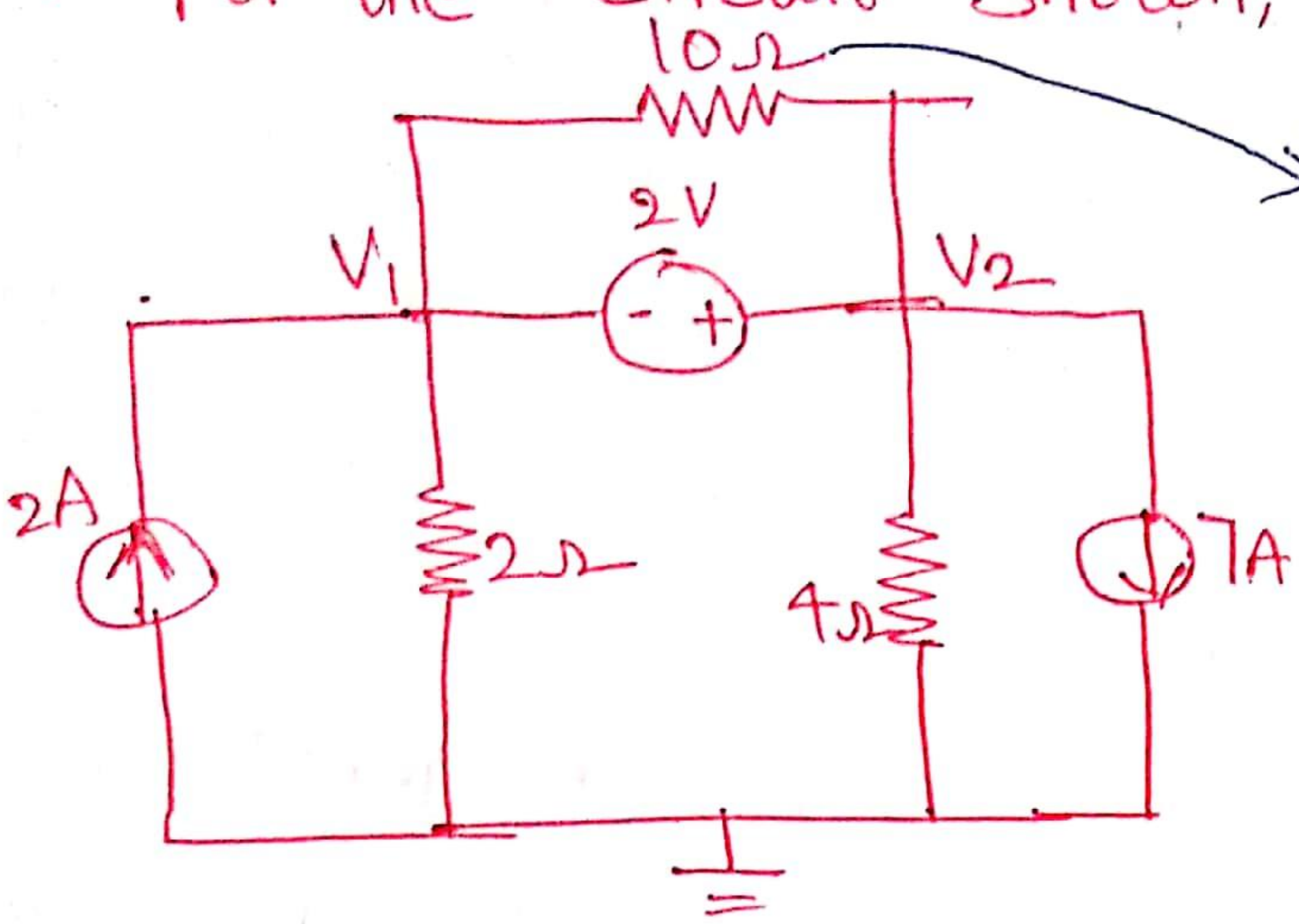
case 2:- If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or super node; we apply both KCL & KVL to determine the node voltages.

A super node is formed by enclosing a (dependent or independent) voltage source connected b/w two nonreference nodes and any elements connected in parallel with it.

* **Following properties of a supernode:-**

1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages
2. A supernode has no voltage of its own.
3. A supernode requires the application of both KCL & KVL [KCL must be satisfied at a supernode like any other node.]

* For the circuit shown, find node voltages.



10Ω resistor does not make any difference because it is connected across the supernode

Sol:- The super node contains the 2-V source, nodes 1 & 2 and 10Ω resistor, Applying KCL to the supernode we get (fig a)

$$i_1 + i_2 + 7 = 2 \rightarrow (1)$$

→ Expressing i_1 & i_2 in terms of node voltages

$$2 = \frac{V_1 - 0}{2} + \frac{V_2 - 0}{4} + 7 \Rightarrow 2V_1 + V_2 + 28 = 8$$

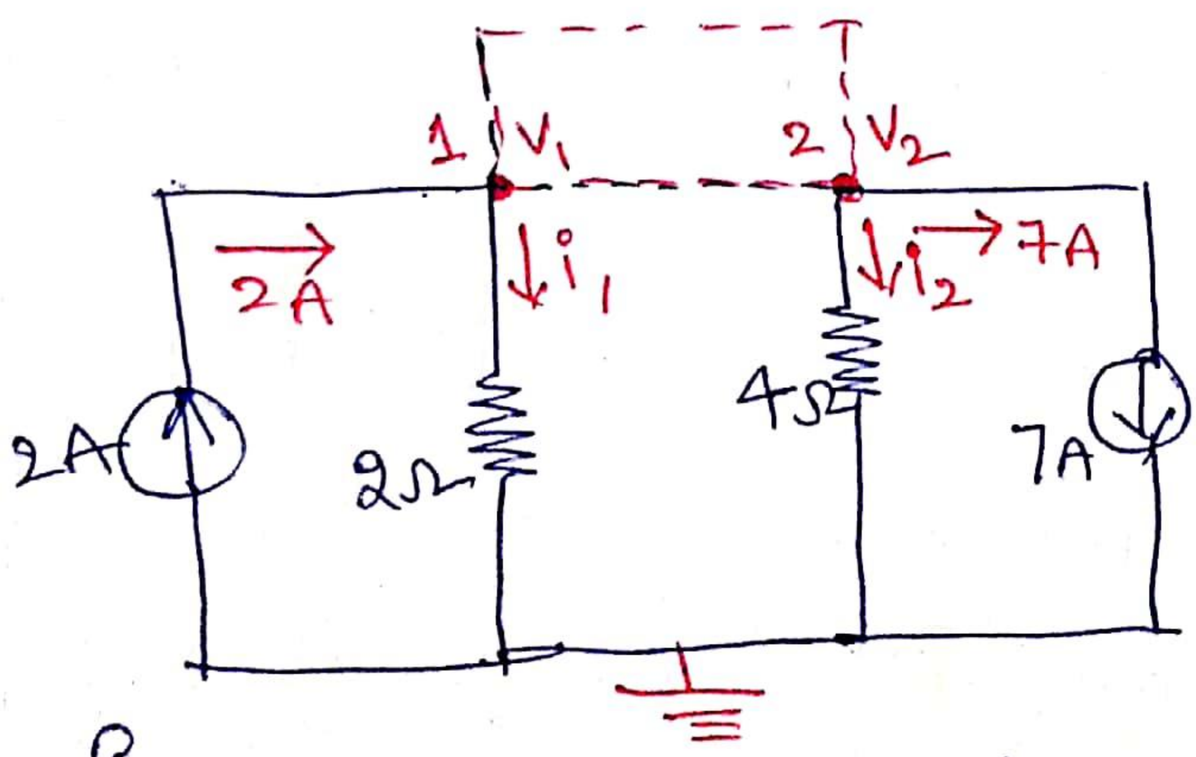
$$V_2 = -20 - 2V_1 \rightarrow (2)$$

→ Apply KVL to the circuit fig. (b) we get

$$-V_1 - 2 + V_2 = 0 \Rightarrow V_2 = V_1 + 2 \rightarrow (3)$$

Sub (3) in (2) we get $V_1 = -7.333\,V$ &

$V_2 = -5.333\,V$



fig(a) KCL to supernode

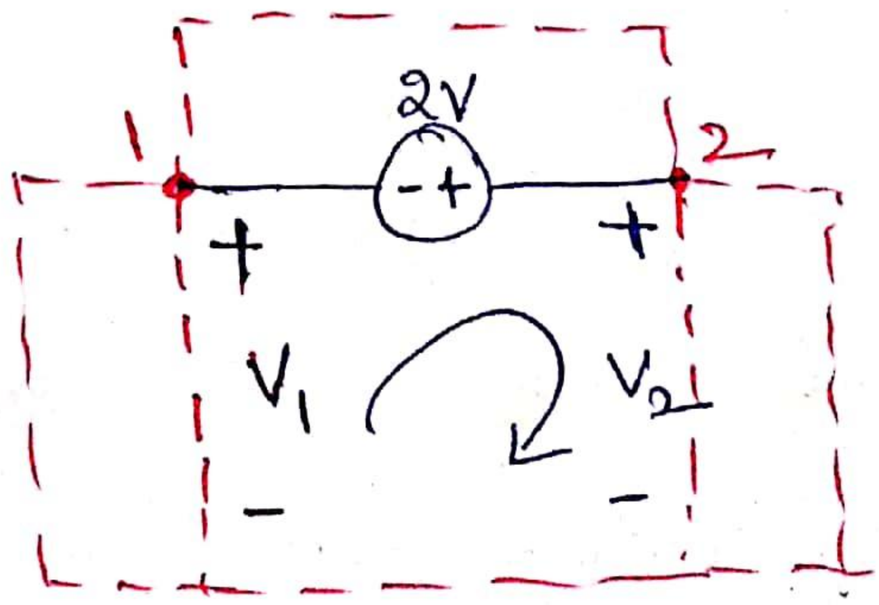
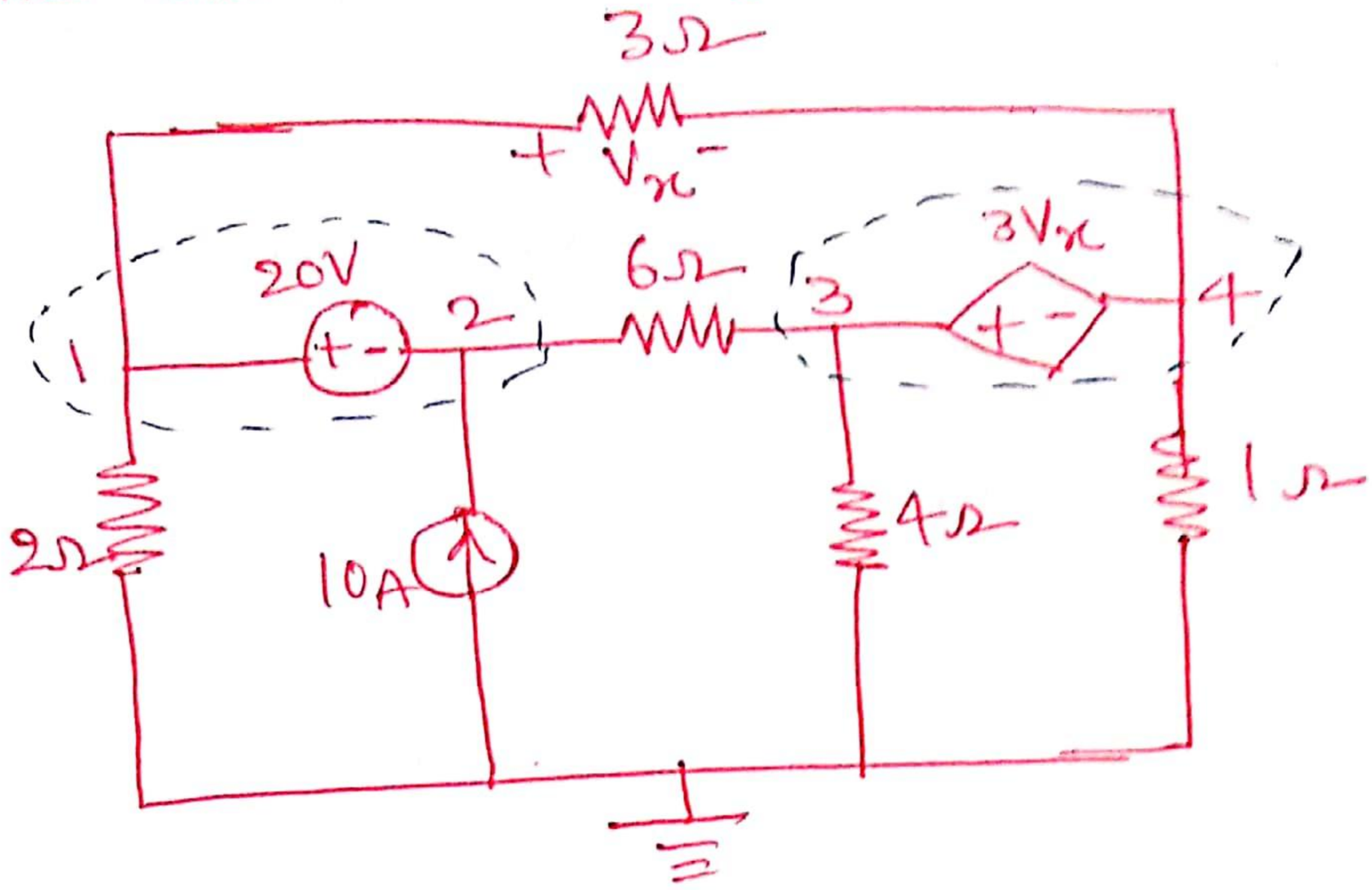


fig.(b) KVL to the loop

* Find the node voltages in the circuit



Sol:- Nodes 1 & 2 form a super node & 3 & 4 also forms a supernode. We apply KCL to the two super nodes in fig(a). At supernode 1-2,

$$i_3 + 10 = i_1 + i_2$$

Expressing interms of the node voltages

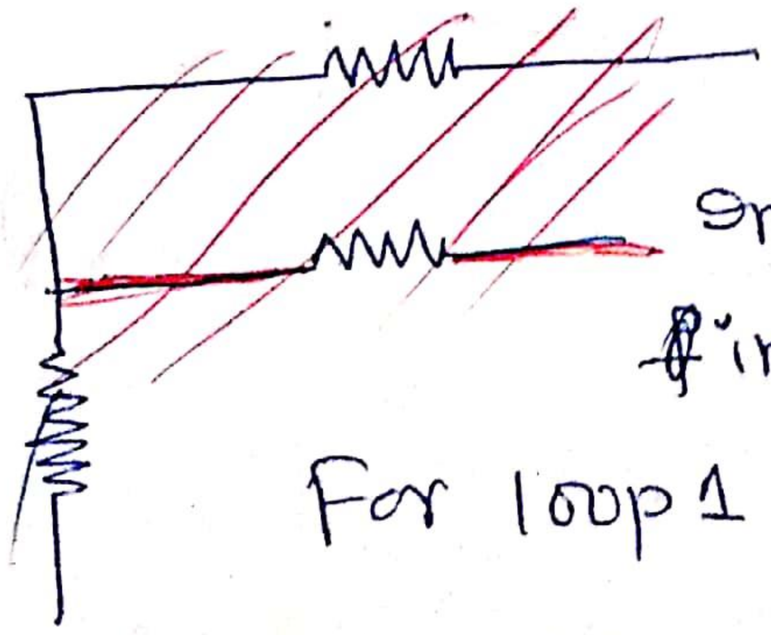
$$\frac{V_3 - V_2}{6} + 10 = \frac{V_1 - V_4}{3} + \frac{V_1}{2}$$

$$\therefore 5V_1 + V_2 - V_3 - 2V_4 = 60 \longrightarrow (1)$$

At supernode 3-4 $\Rightarrow i_1 = i_3 + i_4 + i_5$

$$\Rightarrow \frac{V_1 - V_4}{3} = \frac{V_3 - V_2}{6} + \frac{V_4}{1} + \frac{V_3}{4}$$

$$\therefore 4V_1 + 2V_2 - 5V_3 - 16V_4 = 0 \longrightarrow (2)$$



Now apply KVL to the branches involving the voltage sources shown in fig (b)

For loop 1 $\Rightarrow -V_1 + 20 + V_2 = 0 \Rightarrow V_1 - V_2 = 20 \longrightarrow (3)$

loop 2 $\Rightarrow -V_3 + 3V_x + V_4 = 0$

But $V_x = V_1 - V_4$ so that $3V_1 - V_3 - 2V_4 = 0 \rightarrow (4)$

For loop 3 $V_x - 3V_x + 6i_3 - 20 = 0$

But $6i_3 = V_3 - V_2$ & $V_x = V_1 - V_4$, Hence

loop 3 Equation becomes $-2V_1 - V_2 + V_3 + 2V_4 = 20 \rightarrow (5)$

we can Eliminate one node voltage so that we solve '3' simultaneous Equations instead of '4'.

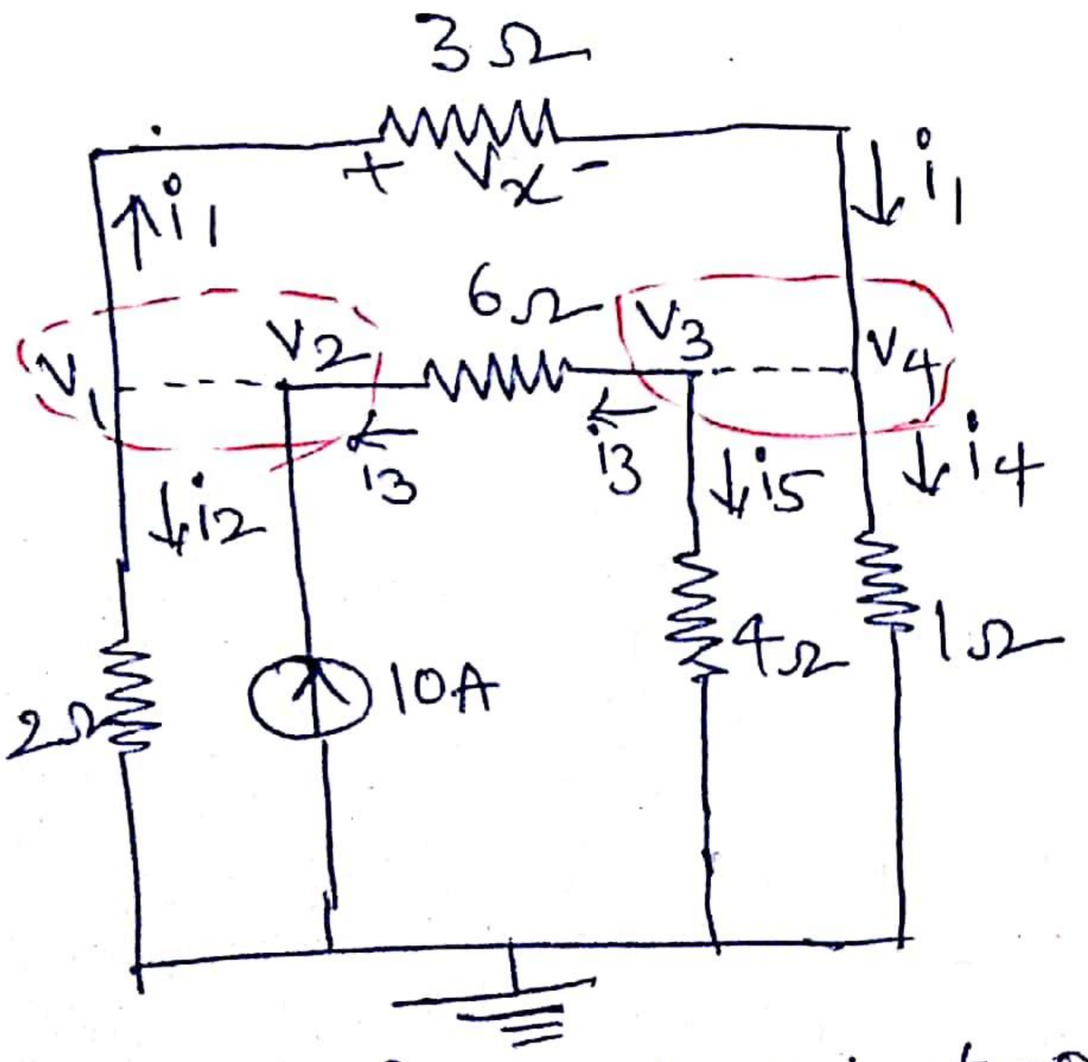
From Equ (3) $V_2 = V_1 - 20 \rightarrow$ sub in (1) & (2) gives

$6V_1 - V_3 - 2V_4 = 80 \rightarrow (6)$ &

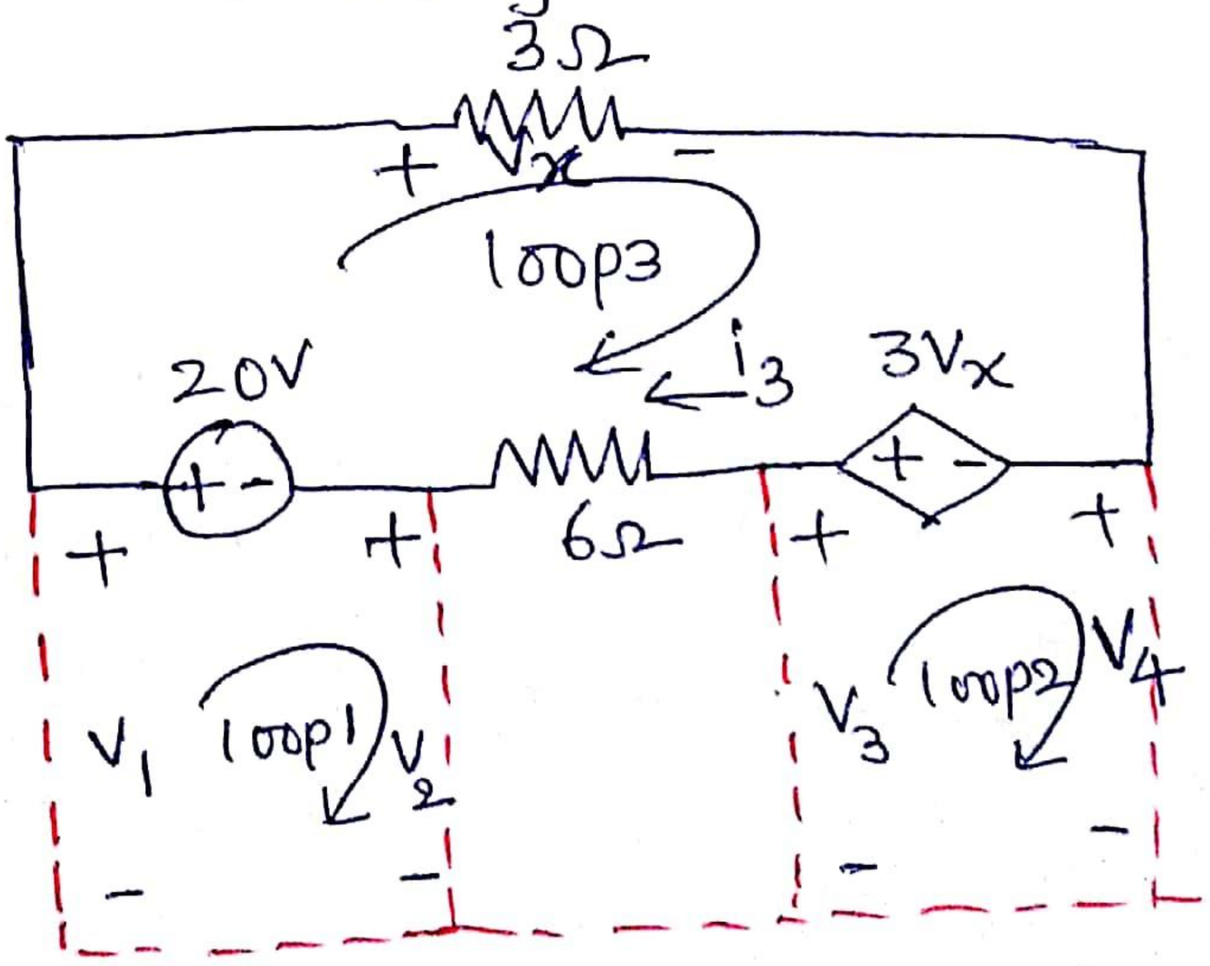
$6V_1 - 5V_3 - 16V_4 = 40 \rightarrow (7)$

solving Equations (4), (6) & (7) using Cramer's rule we get $V_1 = 26.67V$, $V_3 = 173.33V$ & $V_4 = -46.67V$ &

$V_2 = V_1 - 20 = 6.667V$. [writing equations in matrix form & apply cramer's rule, $\Delta = -18$, $\Delta_1 = -480$, $\Delta_3 = -3120$, $\Delta_4 = 840$]

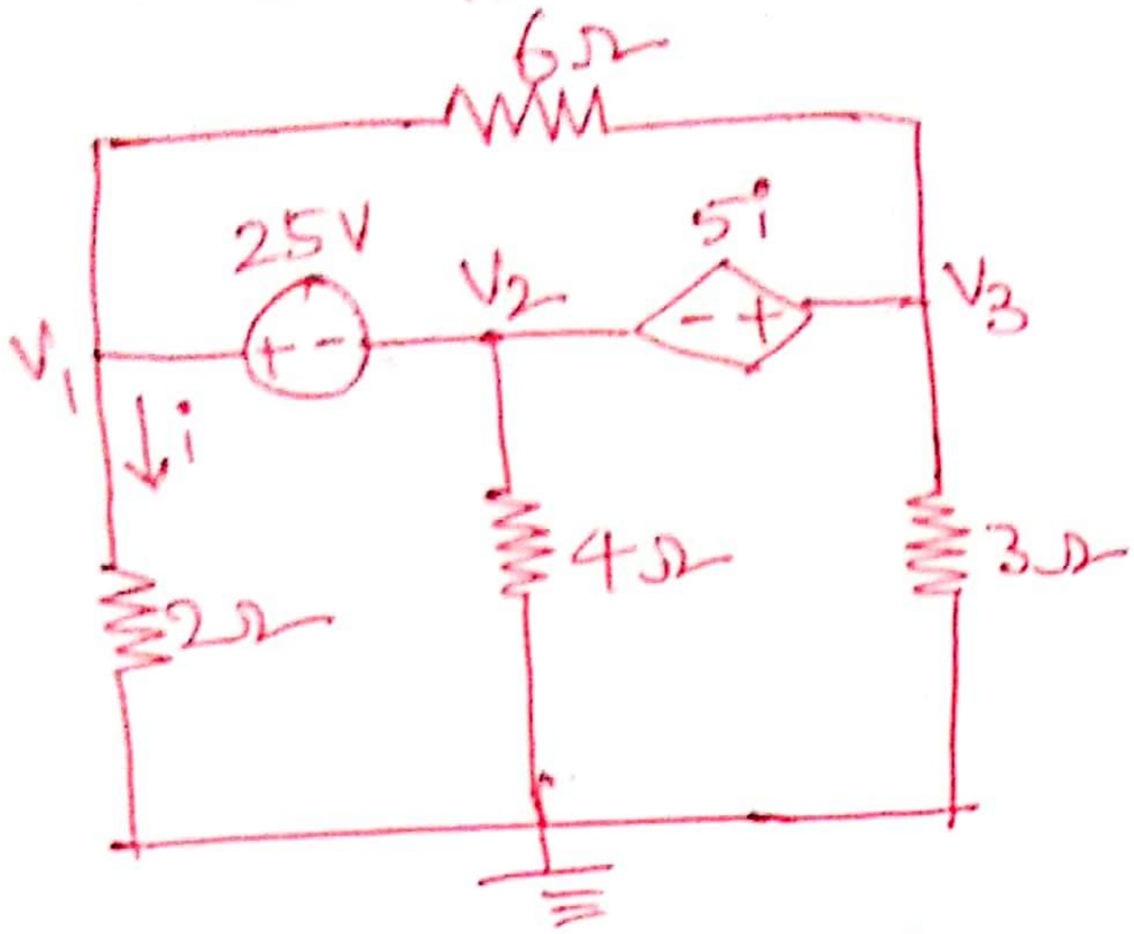


(a) fig (a) KCL to two supernodes



(b) KVL to the loops

* Find V_1, V_2 & V_3 in the circuit shown

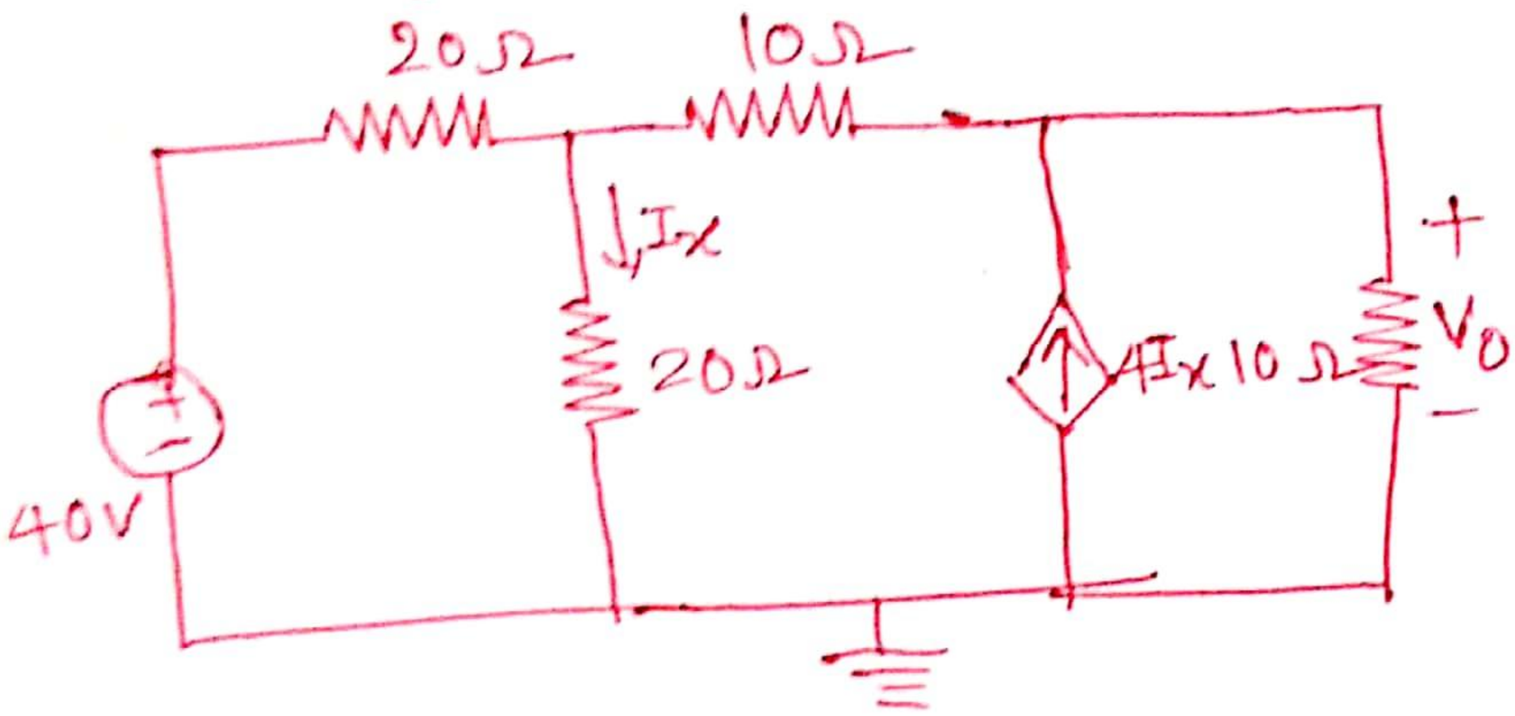


Ans:- $V_1 = 7.608V$,

$V_2 = -17.39V$ &

$V_3 = 1.6305V$

* Using nodal analysis, determine V_o in the ckt.



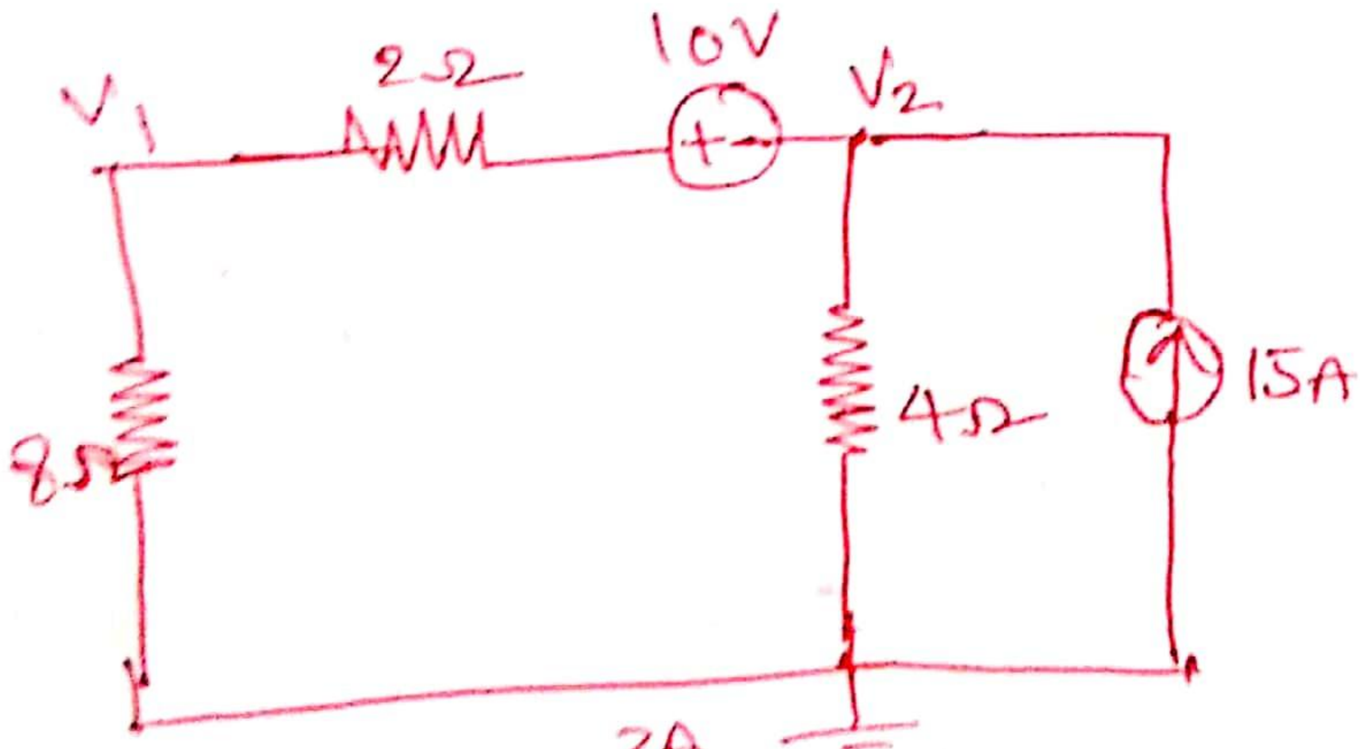
$V_1 = 40V$,

$V_2 = 40V$ &

$V_3 = 60V$ &

$V_o = 60V$

* Calculate V_1 & V_2 in the ckt. Using Nodal analysis



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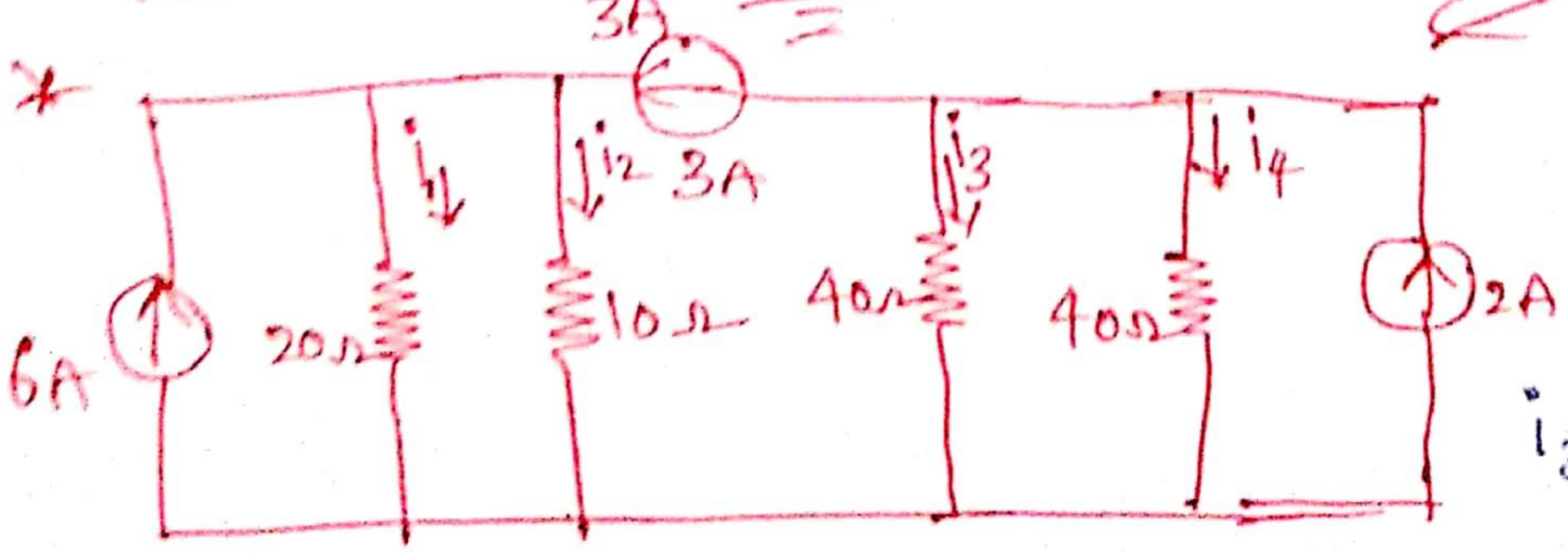


Calculate i_1, i_2, i_3 & i_4

$i_1 = 3A$,

$i_2 = 6A$

$i_3 = i_4 = -0.5A$

(iii) Mesh Analysis without Current Sources:-

A "Mesh" is a "loop" which does not contain any other "loops" within it.

In the mesh analysis of a circuit with 'n' meshes, we take the following three steps.

→ Steps to determine Mesh currents:-

1. Assign mesh currents $i_1, i_2 - - - i_n$ to the 'n' meshes
2. Apply KVL to Each of the 'n' meshes. Use ohm's law to Express the voltages in terms of the mesh currents.
3. Solve the resulting 'n' simultaneous Equations to get the mesh currents.

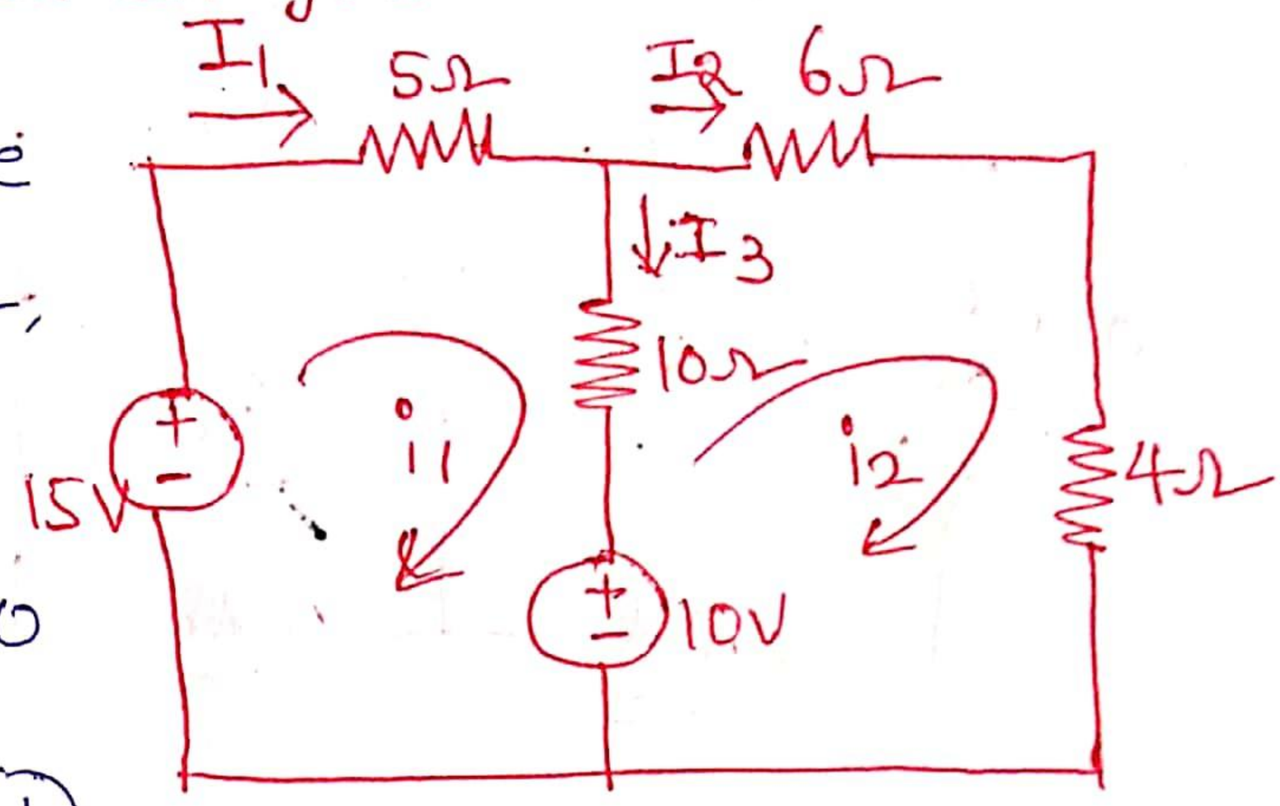
[loop is a closed path with no node passed more than once]

→ Nodal analysis applies "KCL" to find "unknown Voltages" in a given circuit.

→ Mesh analysis applies "KVL" to find "unknown currents" in a given circuit.

→ The direction of the mesh current is arbitrary (clockwise or counterclockwise)

* For the circuit shown, find the branch currents I_1, I_2 & I_3 using Mesh analysis.



Sol:- We first obtain the mesh currents using KVL;

For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

$$\Rightarrow 3i_1 - 2i_2 = 1 \longrightarrow (1)$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

$$\Rightarrow i_1 - 2i_2 = -1 \longrightarrow (2) \text{ (or) } i_1 = 2i_2 - 1 \longrightarrow (2)$$

Method 1:- Sub (2) in (1) we get

By simplifying $\boxed{i_2 = 1A}$

from Equ (2) $\boxed{i_1 = 2 - 1 = 1A}$

Thus $\boxed{I_1 = i_1 = 1A, \quad I_2 = i_2 = 1A, \quad I_3 = i_1 - i_2 = 0}$

Method 2:- Using cramer's rule in Matrix form

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

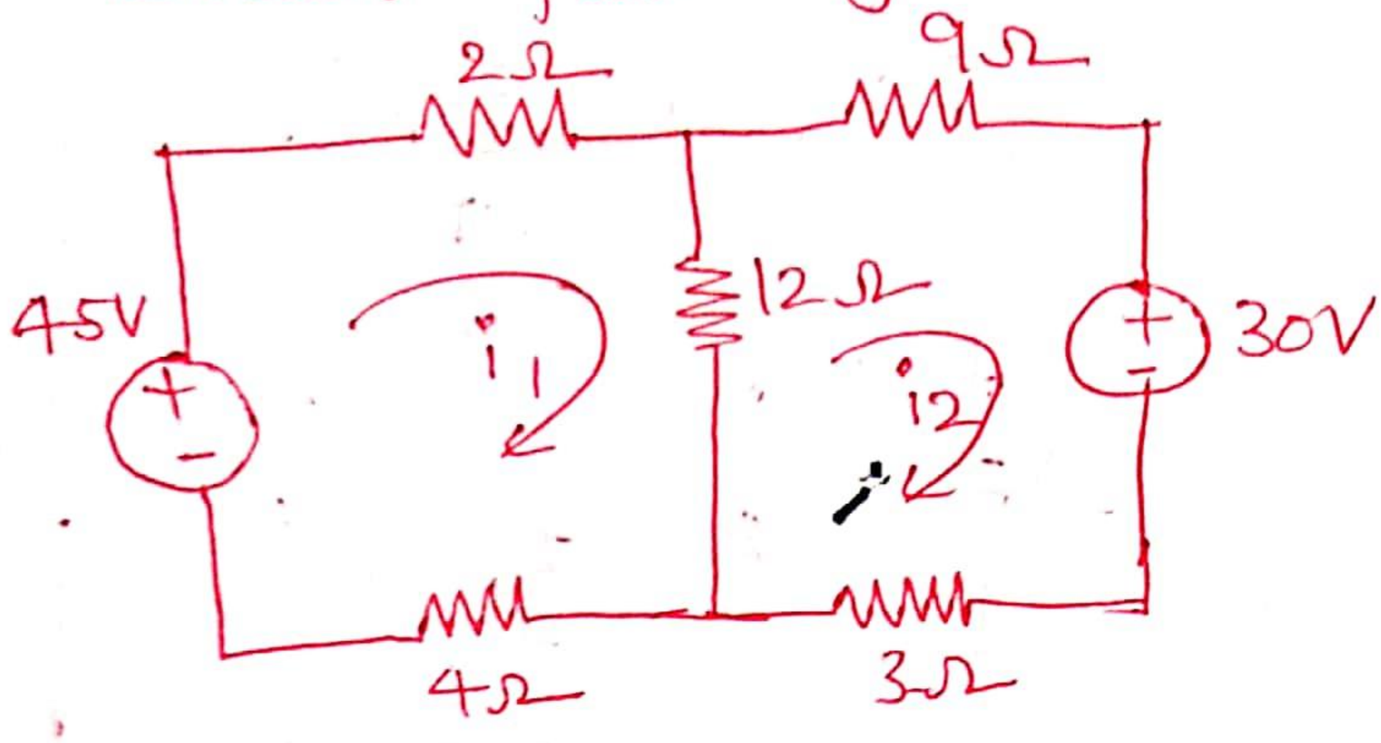
we obtain determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \quad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 4$$

$$\therefore i_1 = \frac{\Delta_1}{\Delta} = \frac{4}{4} = 1A \ \& \ i_2 = \frac{\Delta_2}{\Delta} = \frac{4}{4} = 1A.$$

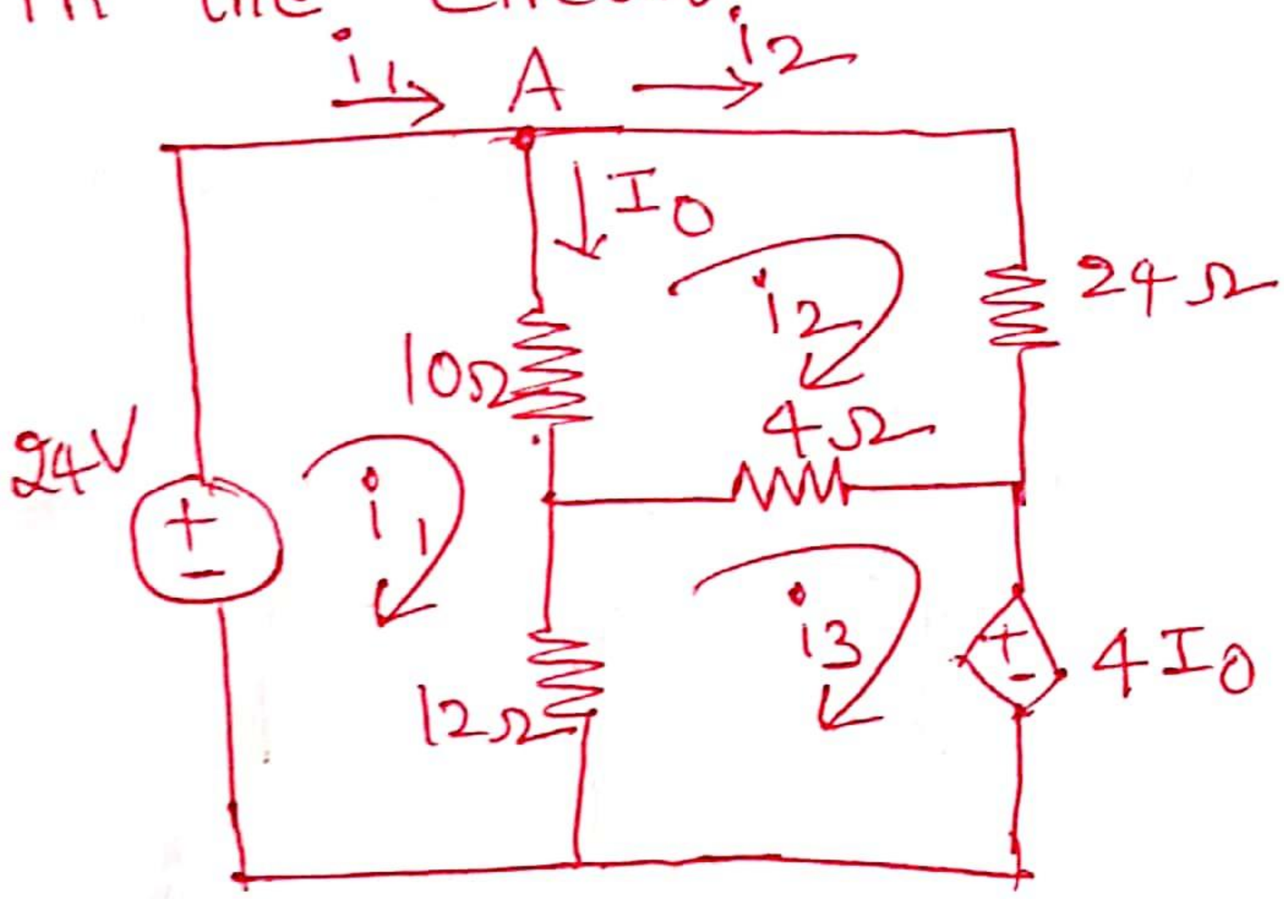
* Calculate the mesh currents i_1 & i_2 of the circuit following.



Ans:- $i_1 = 2.5A$,

$i_2 = 0A$

* Use Mesh analysis to find the current I_0 in the circuit.



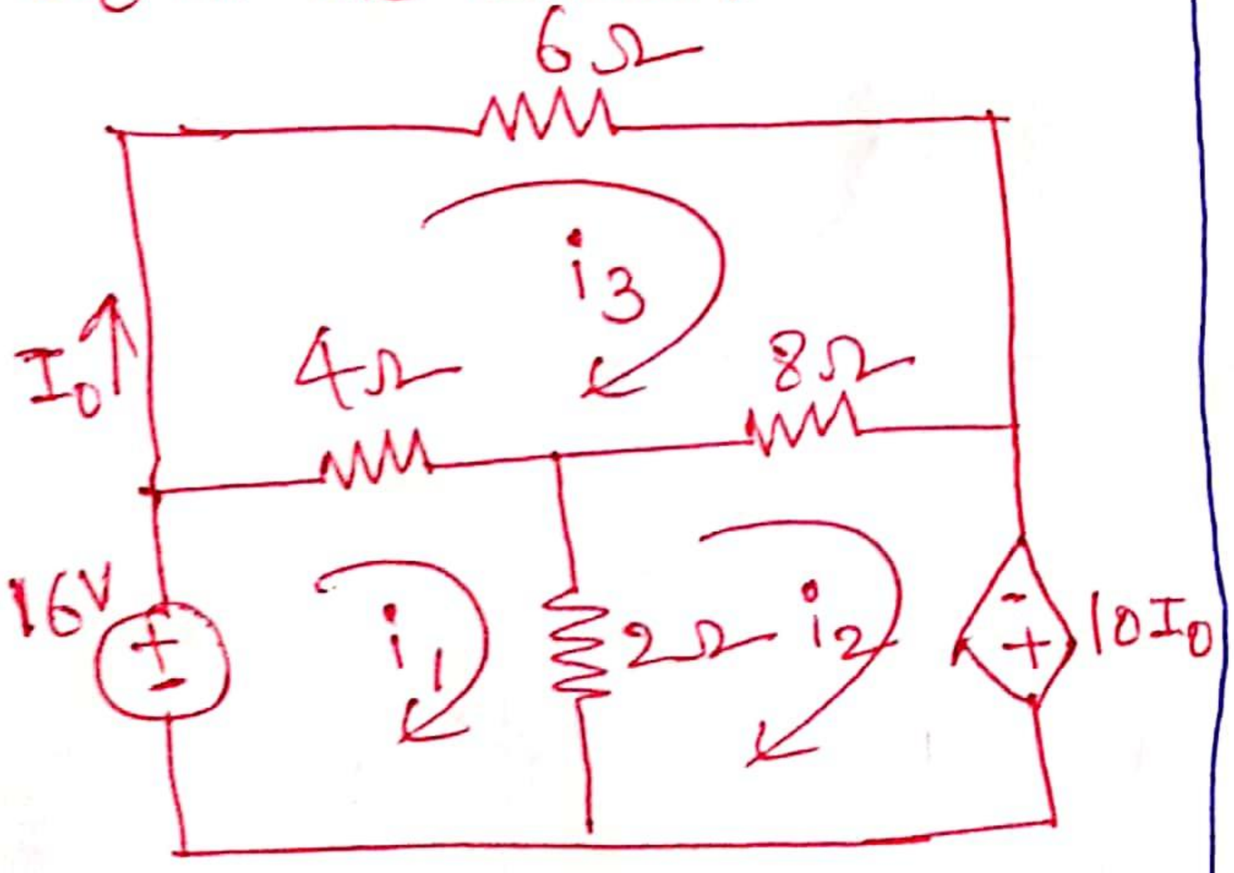
$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25A$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75A$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5A$$

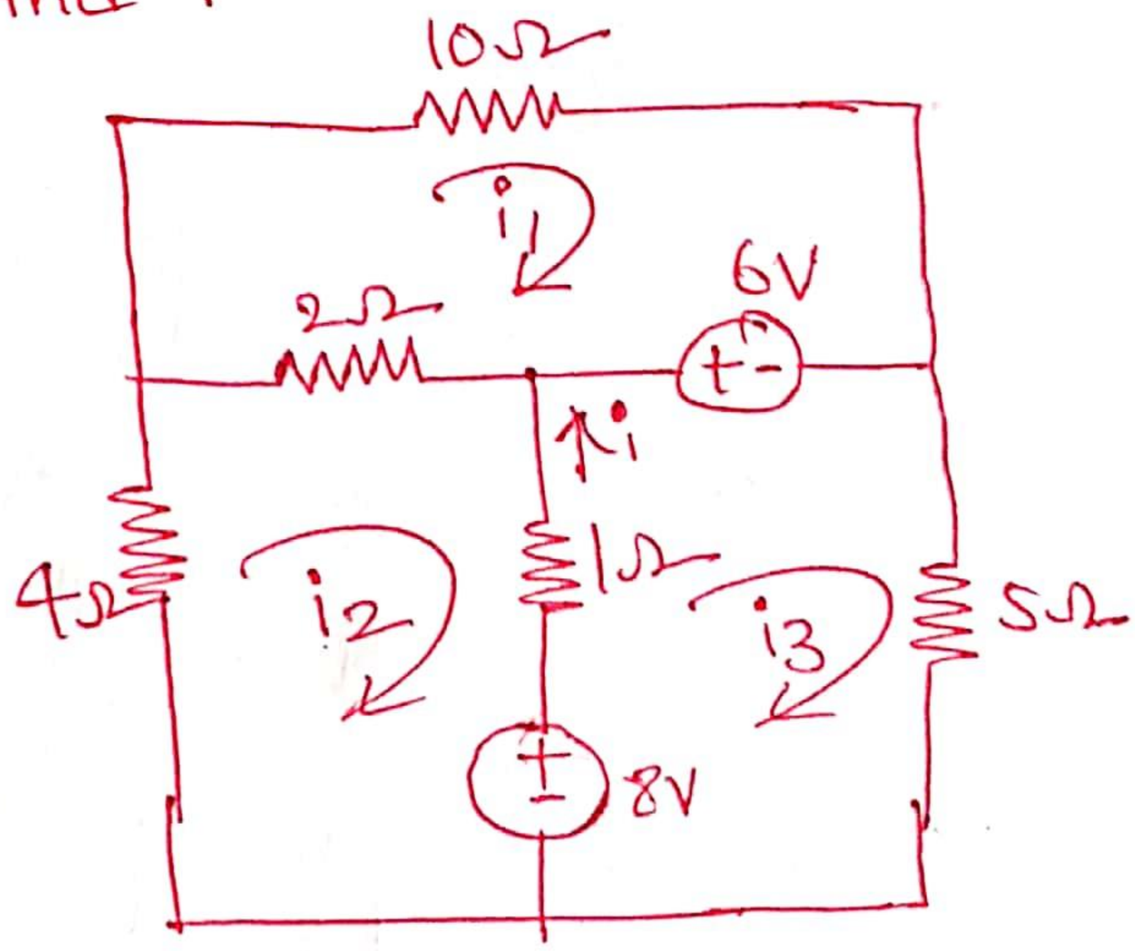
$$I_0 = i_1 - i_2 = 1.5A$$

* Use Mesh analysis, find I_0 in the circuit



* Ans :- −4A

* Apply Mesh analysis to find 'i'

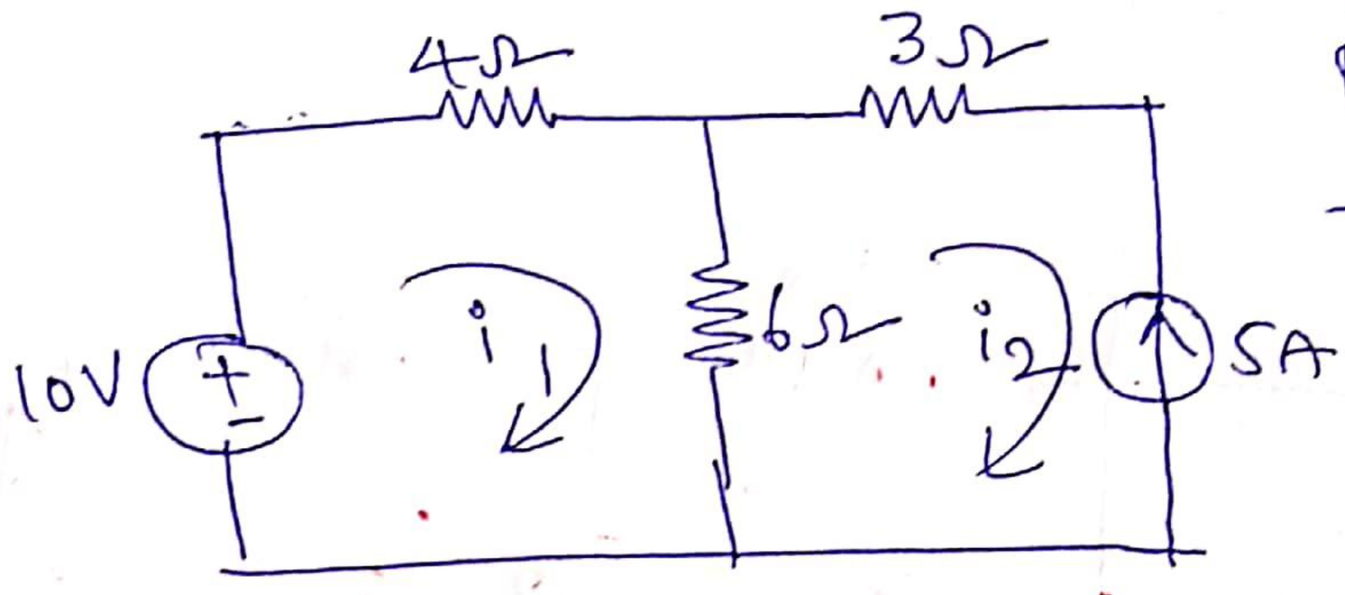


Ans:-

(iv) Mesh Analysis with current Source :-

Applying Mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But, because of presence of the current sources reduces the number of Equations.

case 1 :- when a current source Exists only in one mesh, write a mesh Equation for the other mesh in the usual way, [Ex:- we set $i_2 = -5A$]



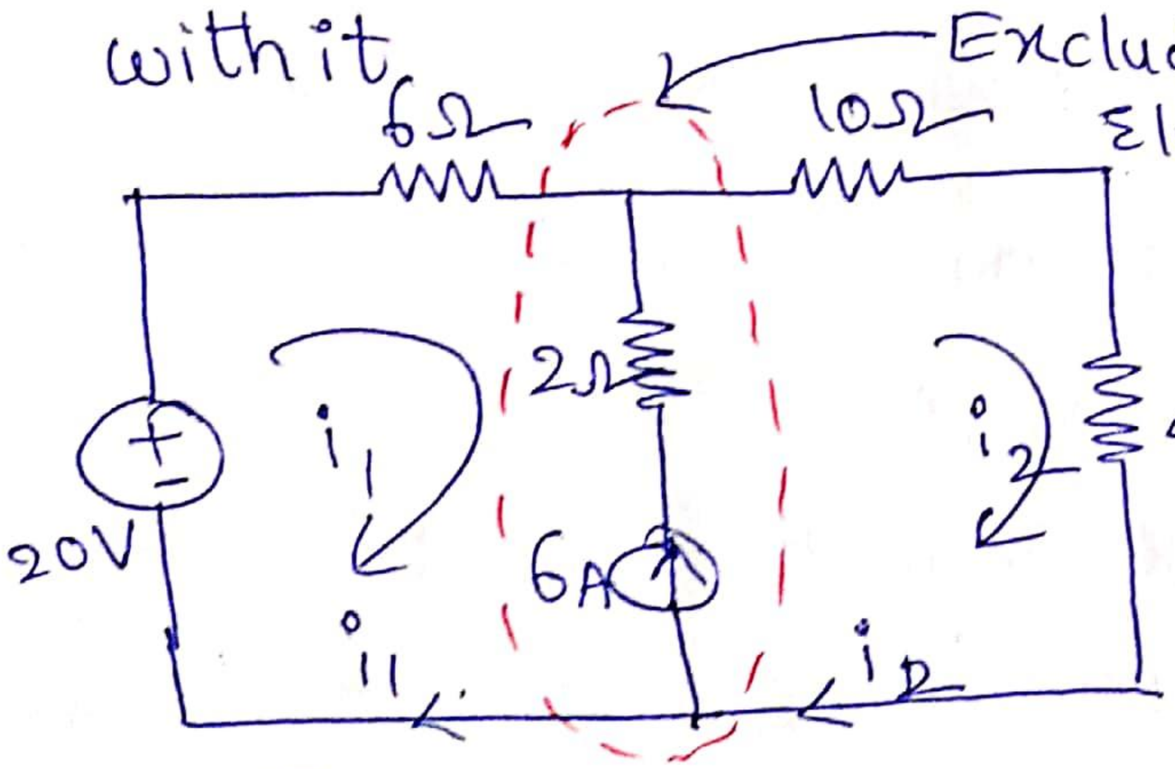
For Mesh 1 :-

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

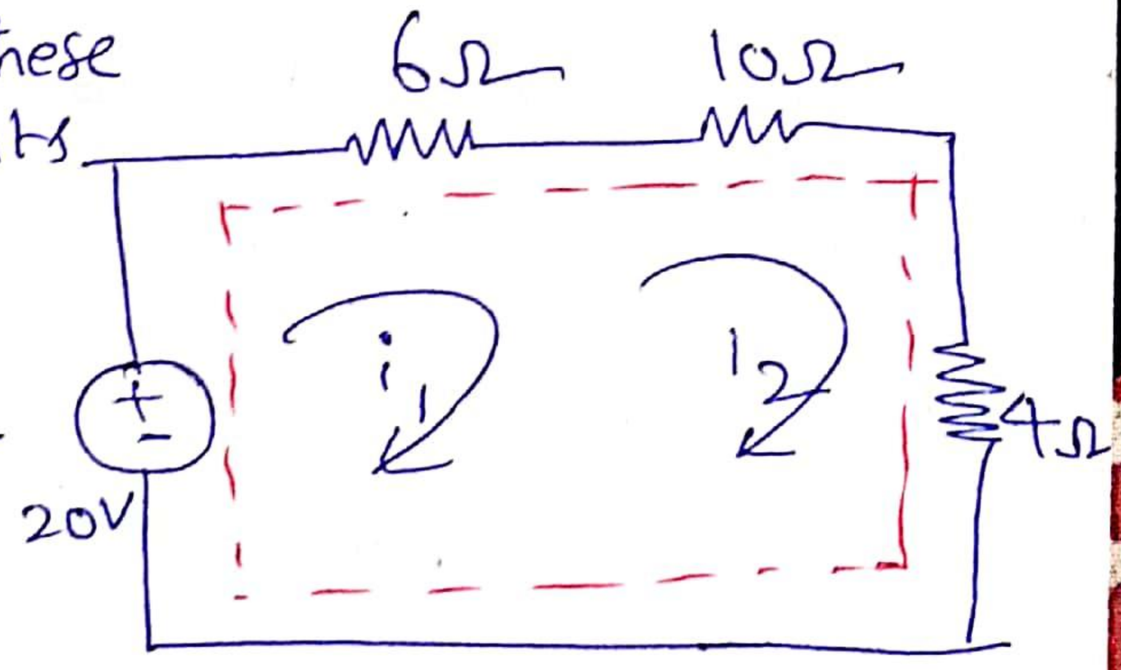
$$i_2 = -5$$

$$\Rightarrow i_1 = -2A.$$

case 2 :- when current source Exists b/w two meshes. we create a supermesh by Excluding the current source & any Elements connected in series with it.



* Two Meshes having common current source

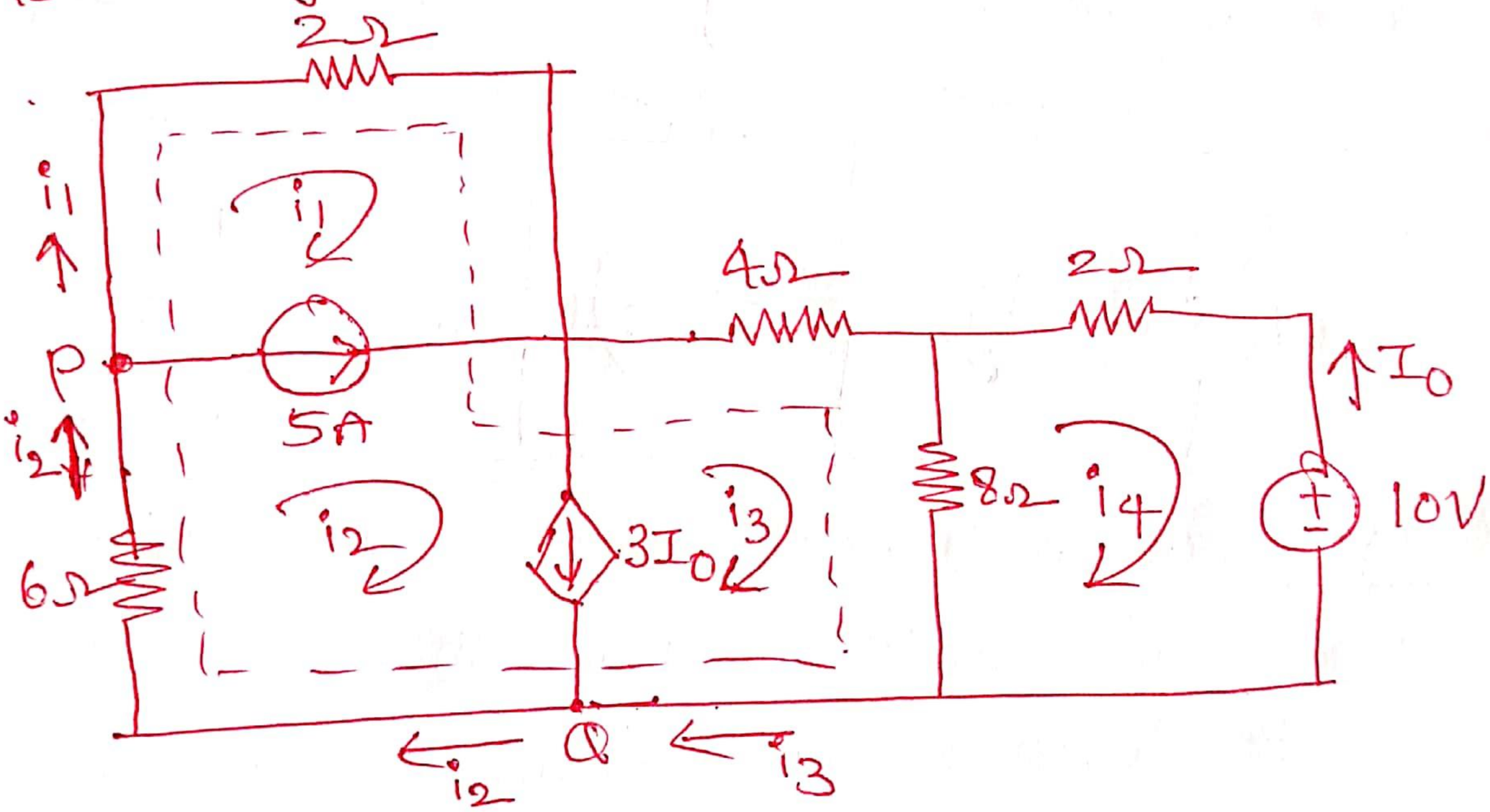


* A supermesh is created by Excluding the current source

* properties of a supermesh:-

1. The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.
2. A supermesh has no current of its own
3. A supermesh requires the application of both KVL & KCL

* For the circuit shown, find i_1 to i_4 using Mesh analysis.



Sol:- Note that meshes 1 & 2 form a supermesh since they have an independent current source in common. Also meshes 2 & 3 form another supermesh because they have a dependent current source in common. The two supermeshes intersect & form a large supermesh.

→ Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or,

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \longrightarrow (1)$$

→ For the independent current source, we apply KCL at node 'P'. $i_2 = i_1 + 5 \longrightarrow (2)$

→ For the dependent current source, we apply KCL at node 'Q', $i_2 = i_3 + 3I_0$

But $I_0 = -i_4$

$$\therefore i_2 = i_3 - 3i_4 \longrightarrow (3)$$

→ Applying KVL in mesh '4'

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

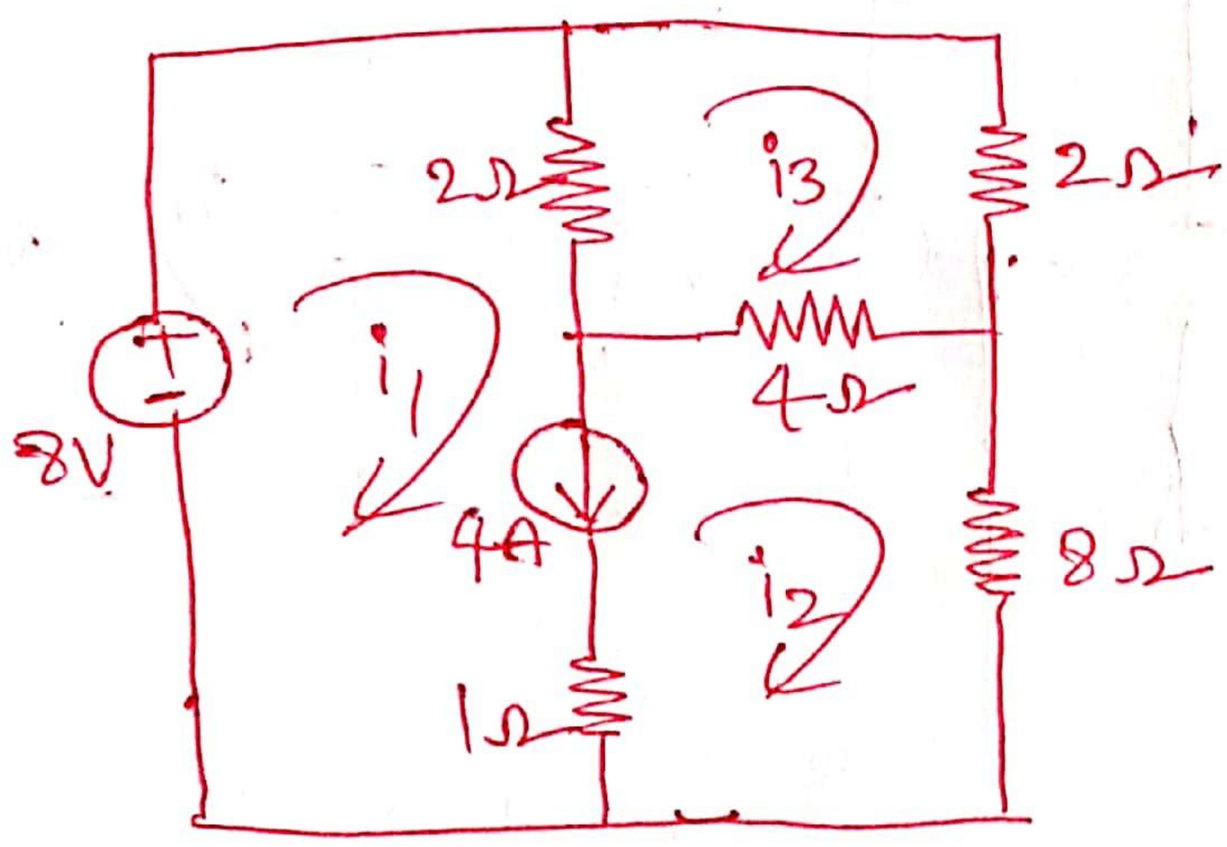
or,

$$5i_4 - 4i_3 = -5 \longrightarrow (4)$$

By solving (1) to (4) we get

$i_1 = -7.5A$, $i_2 = -2.5A$, $i_3 = 3.93A$ & $i_4 = 2.143A$

* Use Mesh analysis to find i_1, i_2 & i_3

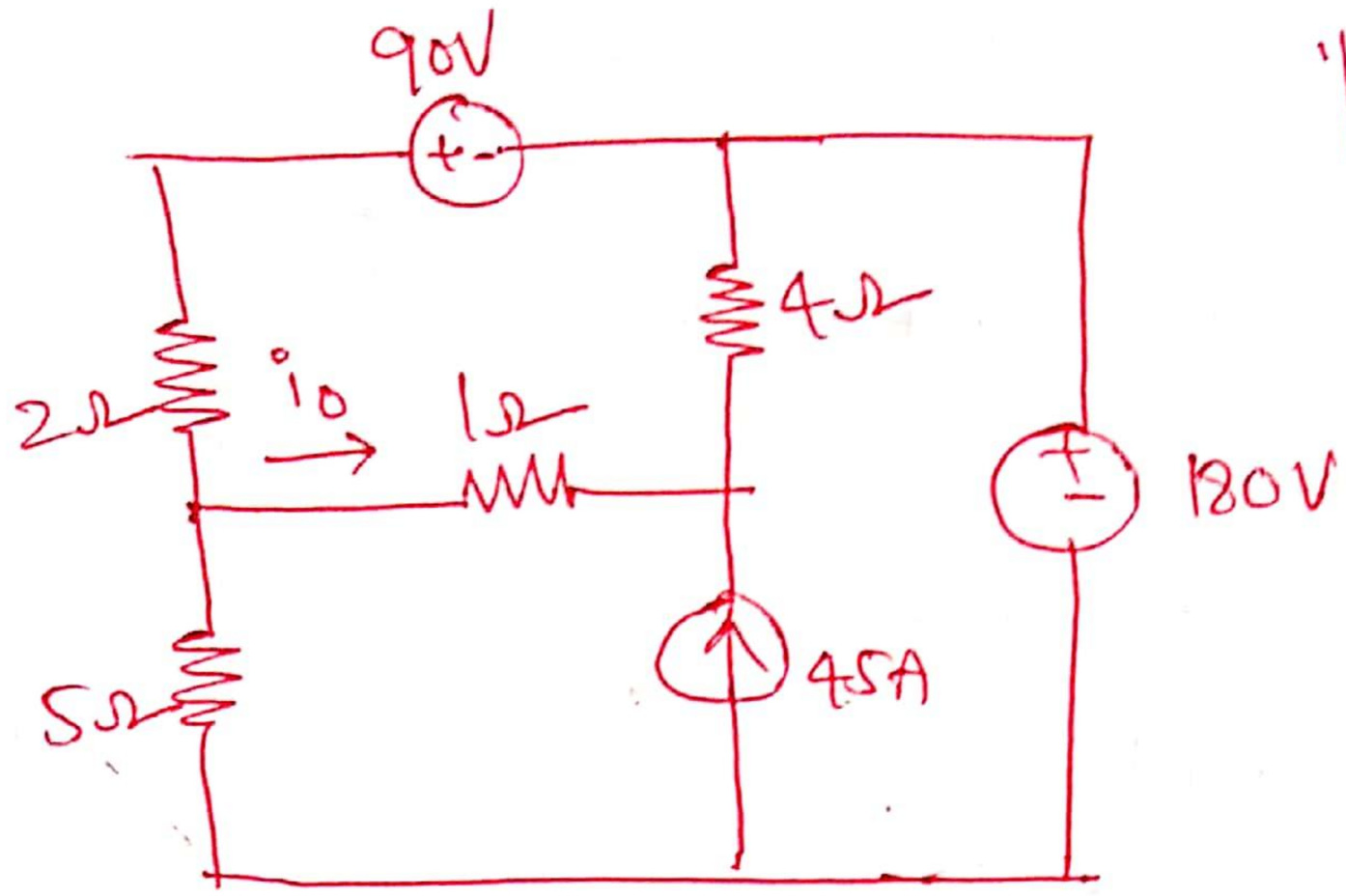


Ans:- $i_1 = 4.632A$,

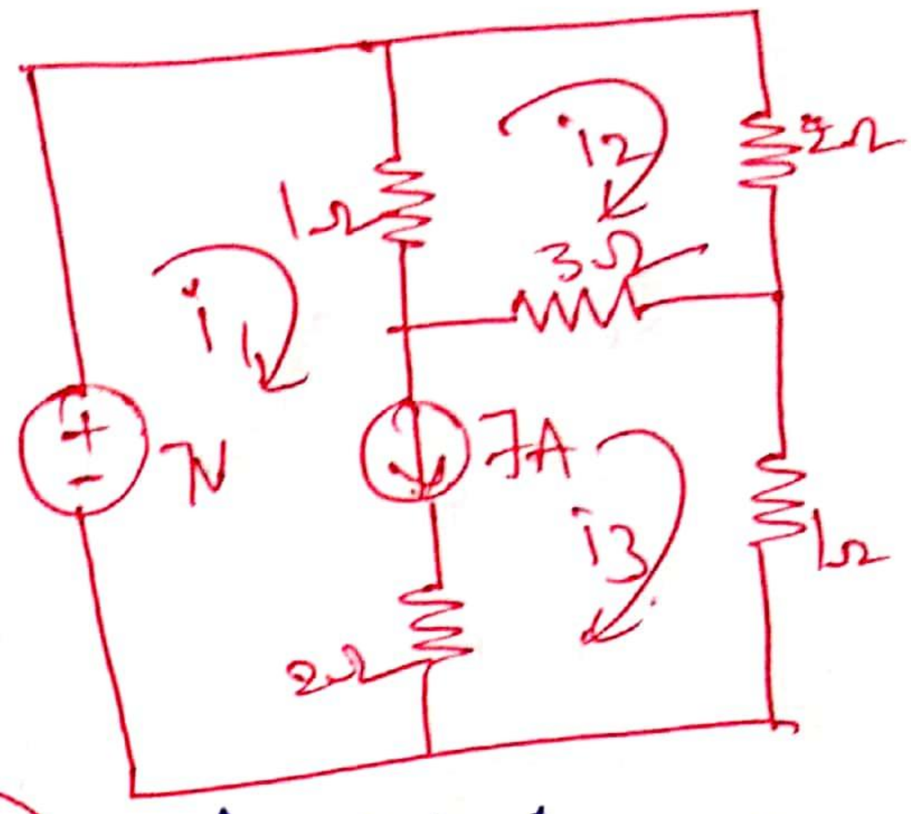
$i_2 = 631.6mA$,

$i_3 = 1.4736A$

* Use Mesh analysis to obtain i_o in the ckt



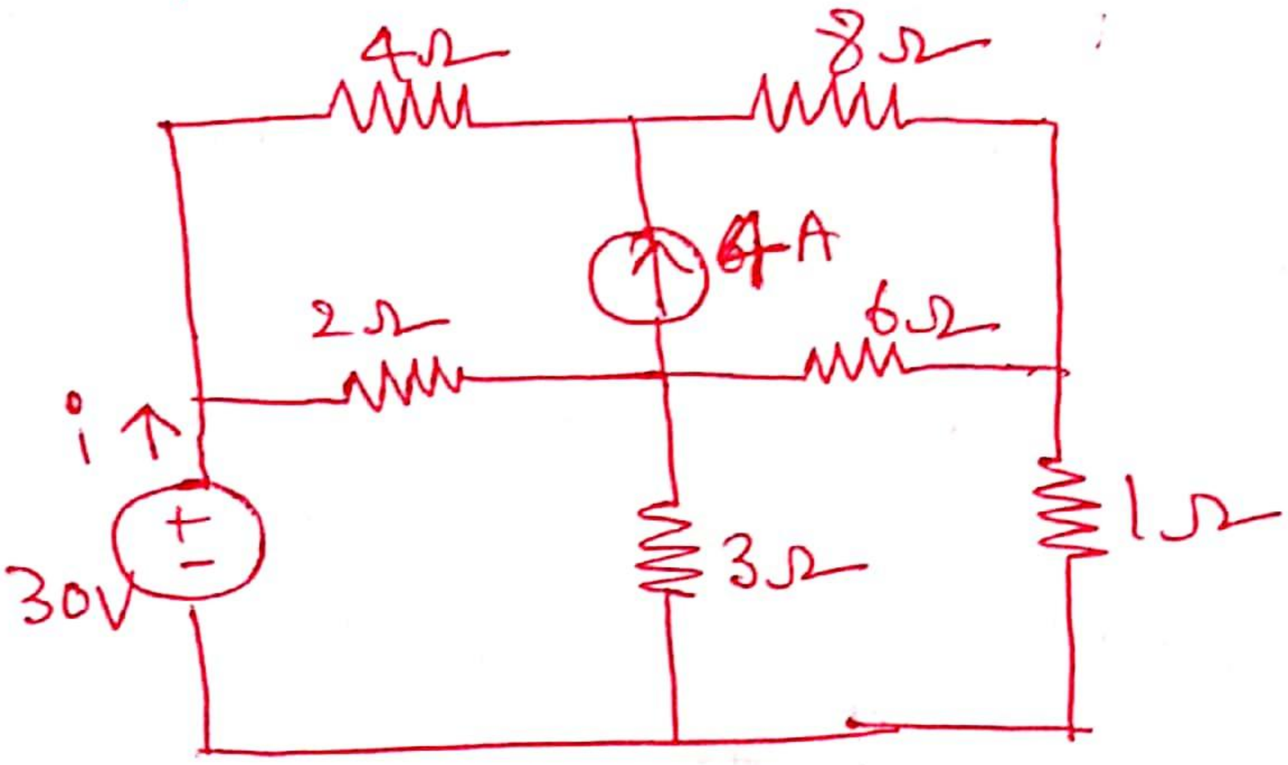
* Find '3' Mesh current



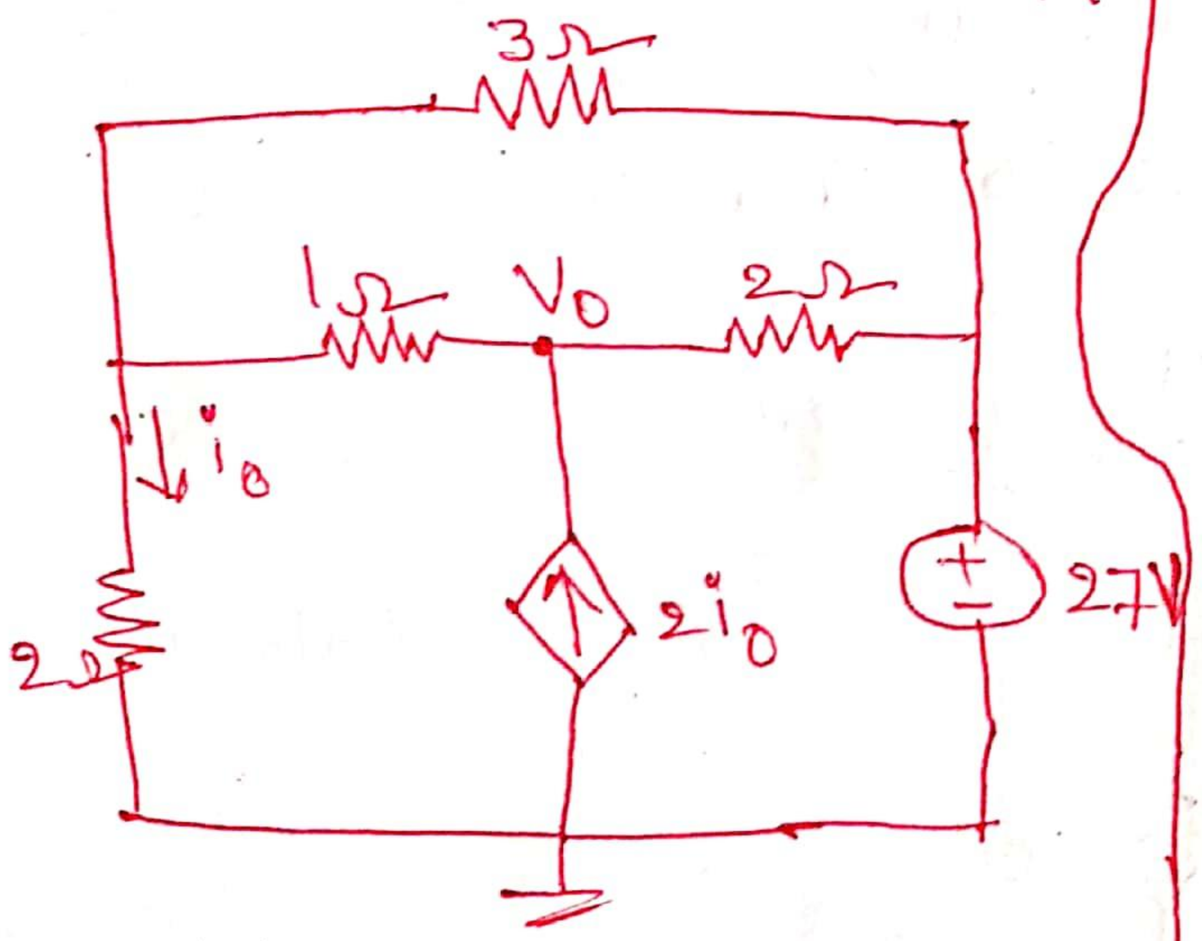
$i_1 = 9A$, $i_2 = 2.5A$

$i_3 = 2A$

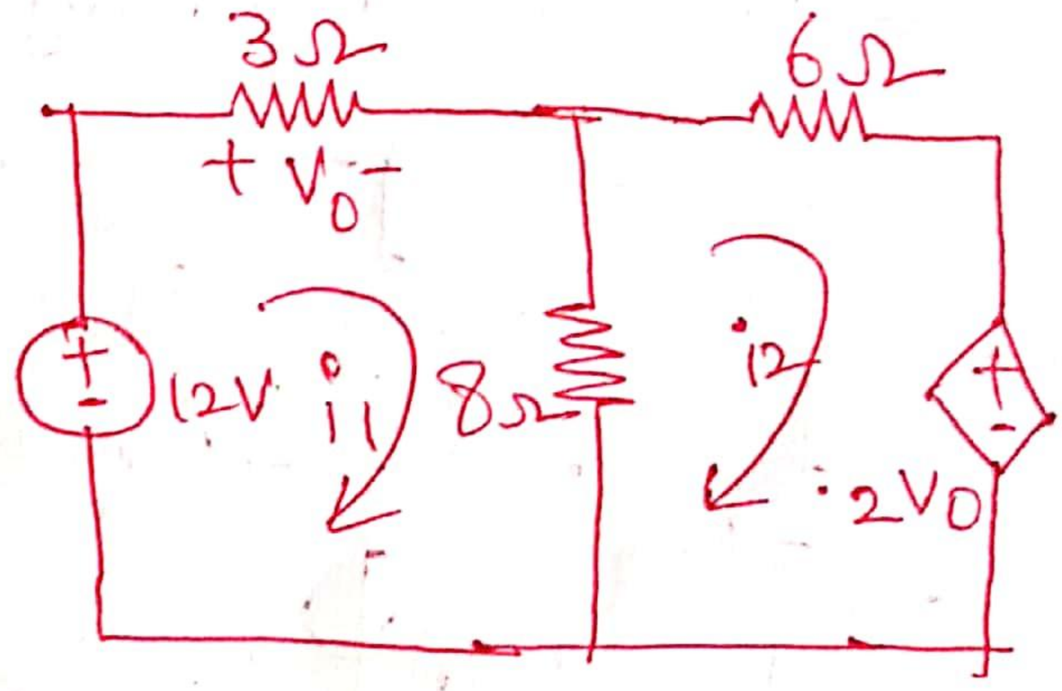
* Find current 'i' in the circuit



* Find V_o & i_o in th ckt



* Calculate the mesh currents i_1 & i_2



$i_2 = \frac{12}{69} = 0.173A$

$i_1 = 1.2174A$

* Network Theorems:-

1. Superposition Theorem:-

*Statement:- In any linear network the voltage across (or current through) an Element is the algebraic sum of the voltages across (or currents through) that Element due to Each independent Source acting alone

→ Two points keep in mind while applying super position principle/theorem to any linear circuit

* we consider one independent source at a time while all other independent sources are turned off. This implies we replace Every voltage Source by 0V (or a short circuit) & Every Current Source by 0A (or an open circuit).

* Dependent ~~Vo~~ Sources are left intact because they are controlled by circuit variable.

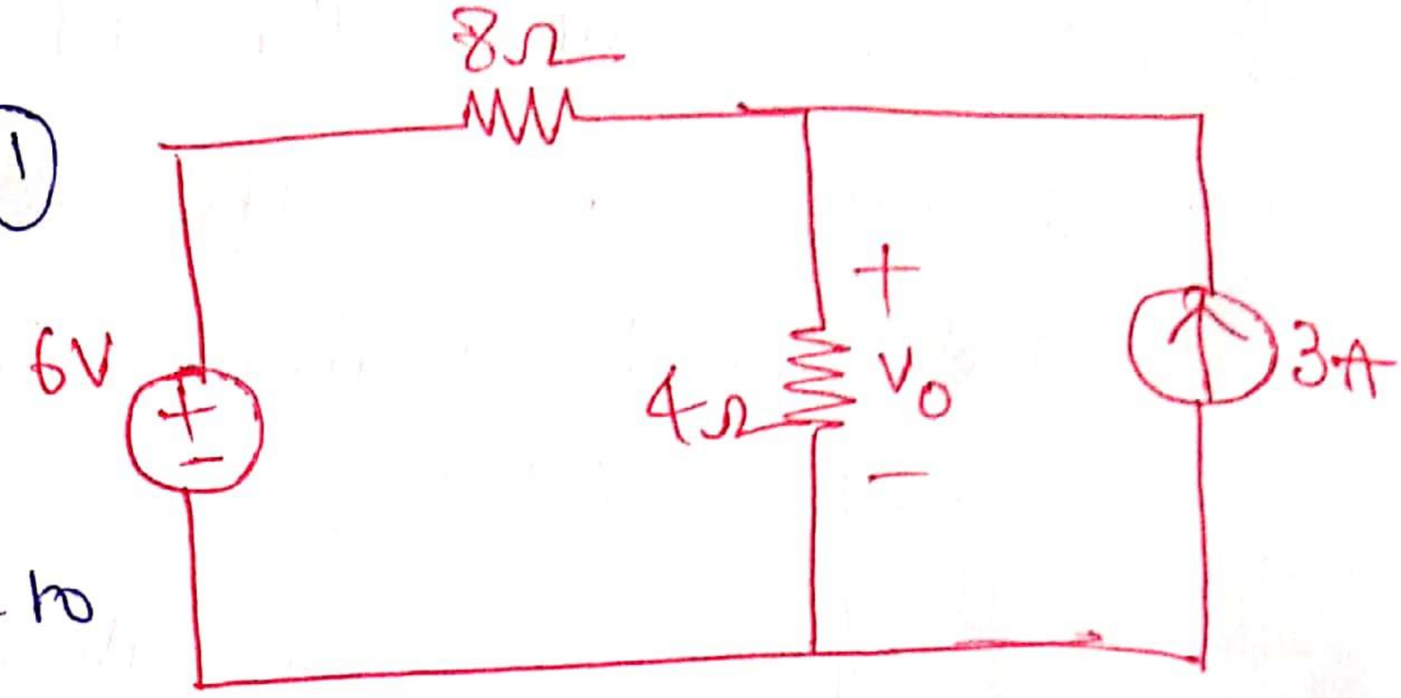
* steps to apply superposition principle:-

(i) Turnoff all independent Sources Except one Source. Find the output (voltage or current) due to that active Source.

(ii) Repeat step(i) for Each of the other independent Sources

(ii) find the total contribution by adding algebraically all the contributions due to the independent sources.

Ex: Use the superposition theorem to find V_o in the circuit.



Sol: Let $V_o = V_1 + V_2$ → (1)

where V_1 & V_2 are the contributions due to the 6V voltage source & 3A current source respectively. To obtain V_1, we set the current source to zero (fig (a)) & to obtain V_2 we set voltage source to zero shown in fig (b)

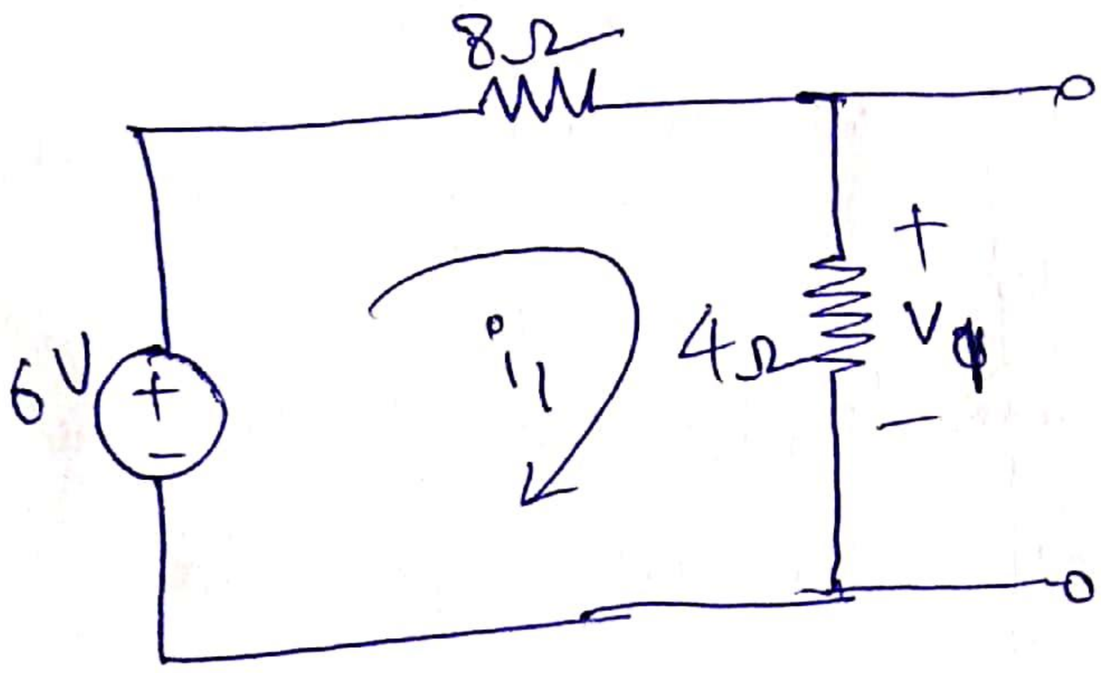


fig. (a)

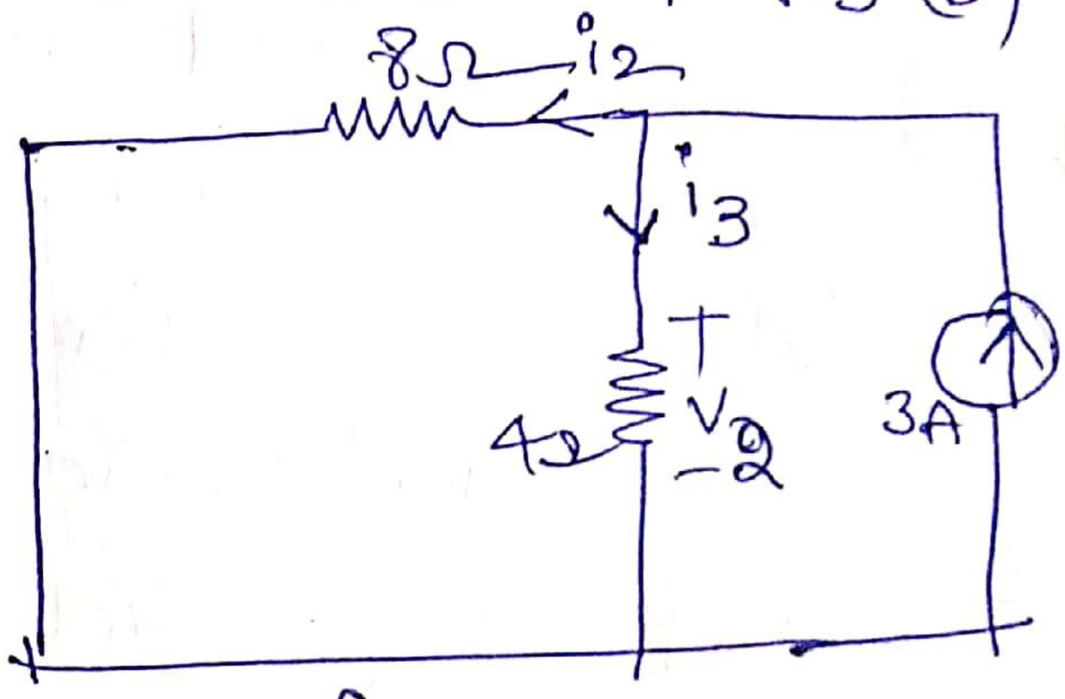


fig (b.)

Applying KVL to fig. (a) we get

$-6 + 8i_1 + 4i_1 = 0 \Rightarrow -6 + 12i_1 = 0 \Rightarrow i_1 = 0.5A$

$\therefore V_1 = i_1 \times 4 = 0.5 \times 4 = 2V$ (or)

By using voltage division $V_1 = \frac{4}{12} \times 6 = 2V$ → (2)

For fig (b) using current division

$$i_3 = \frac{8}{4+8}(3) = 2A$$

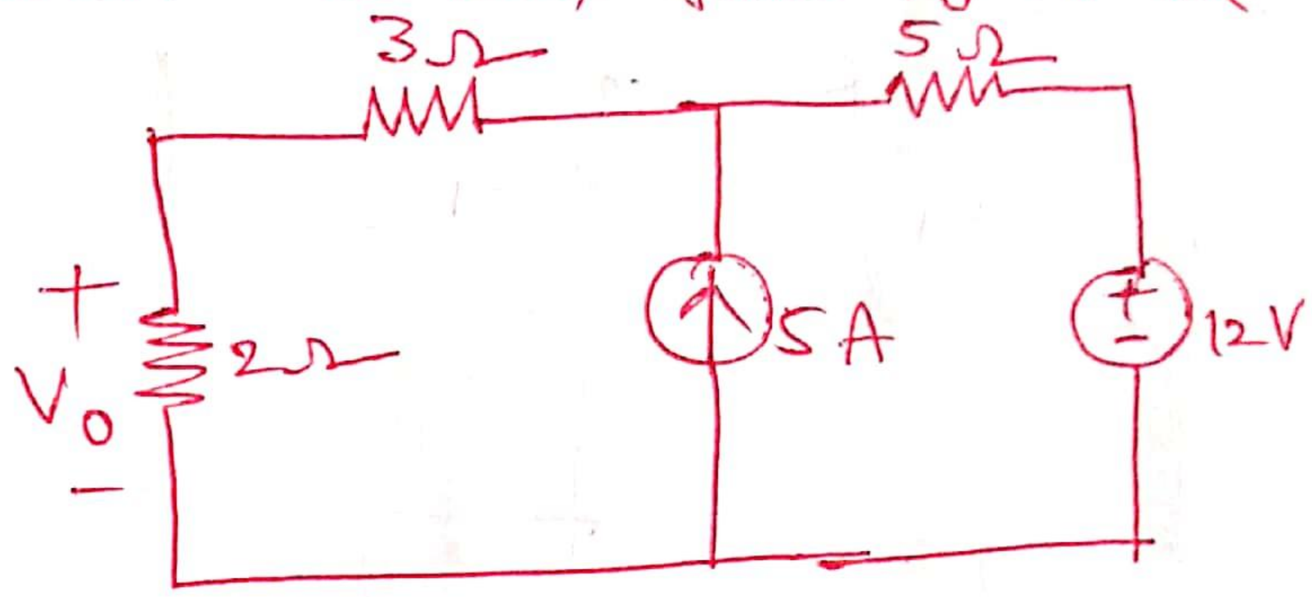
Hence $V_2 = 4i_3 = 8V \rightarrow (3)$

Sub. Equations (2) & (3) in (1) we get

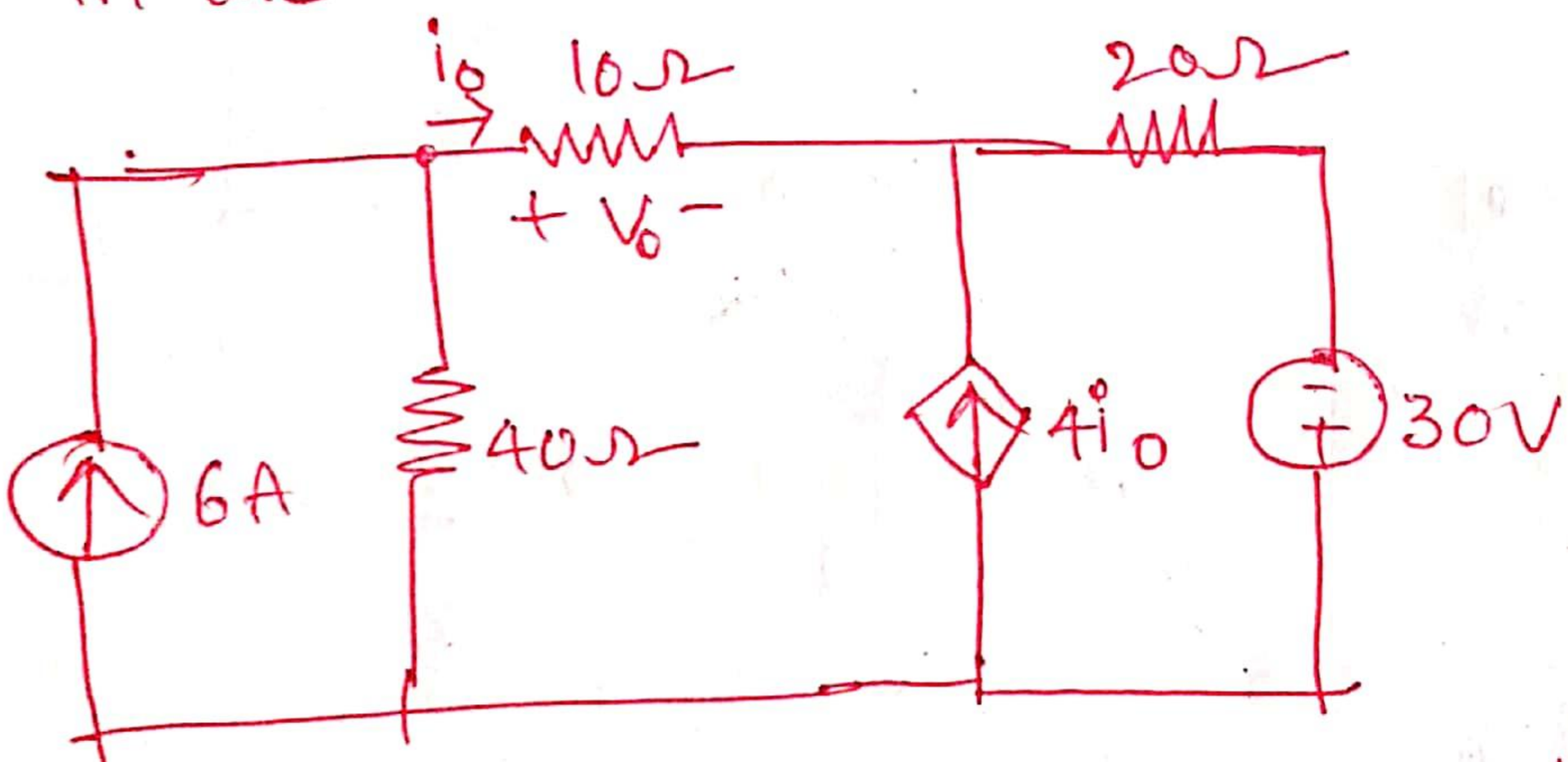
$$V_0 = 2+8 = 10V$$

* Using the superposition theorem, find V_0 in the circuit

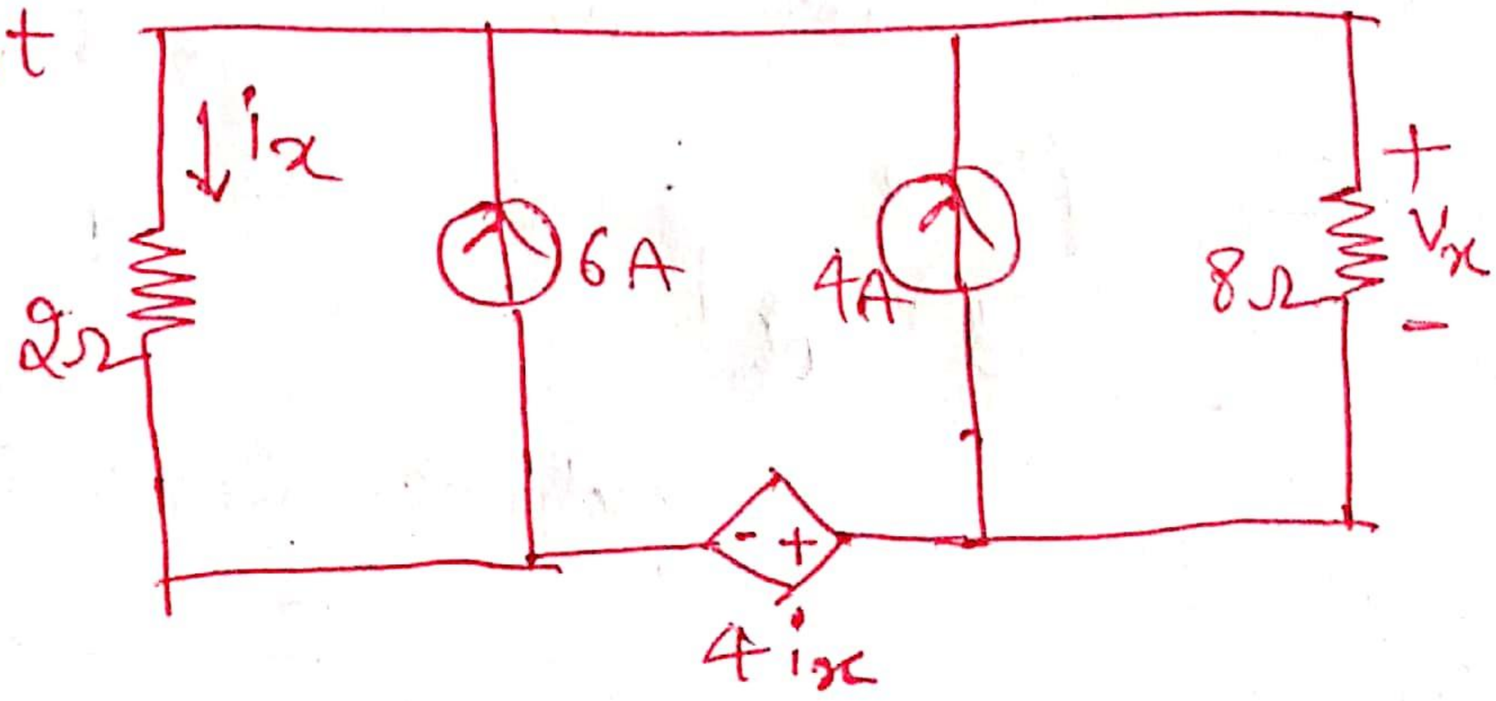
Ans:- 7.4V



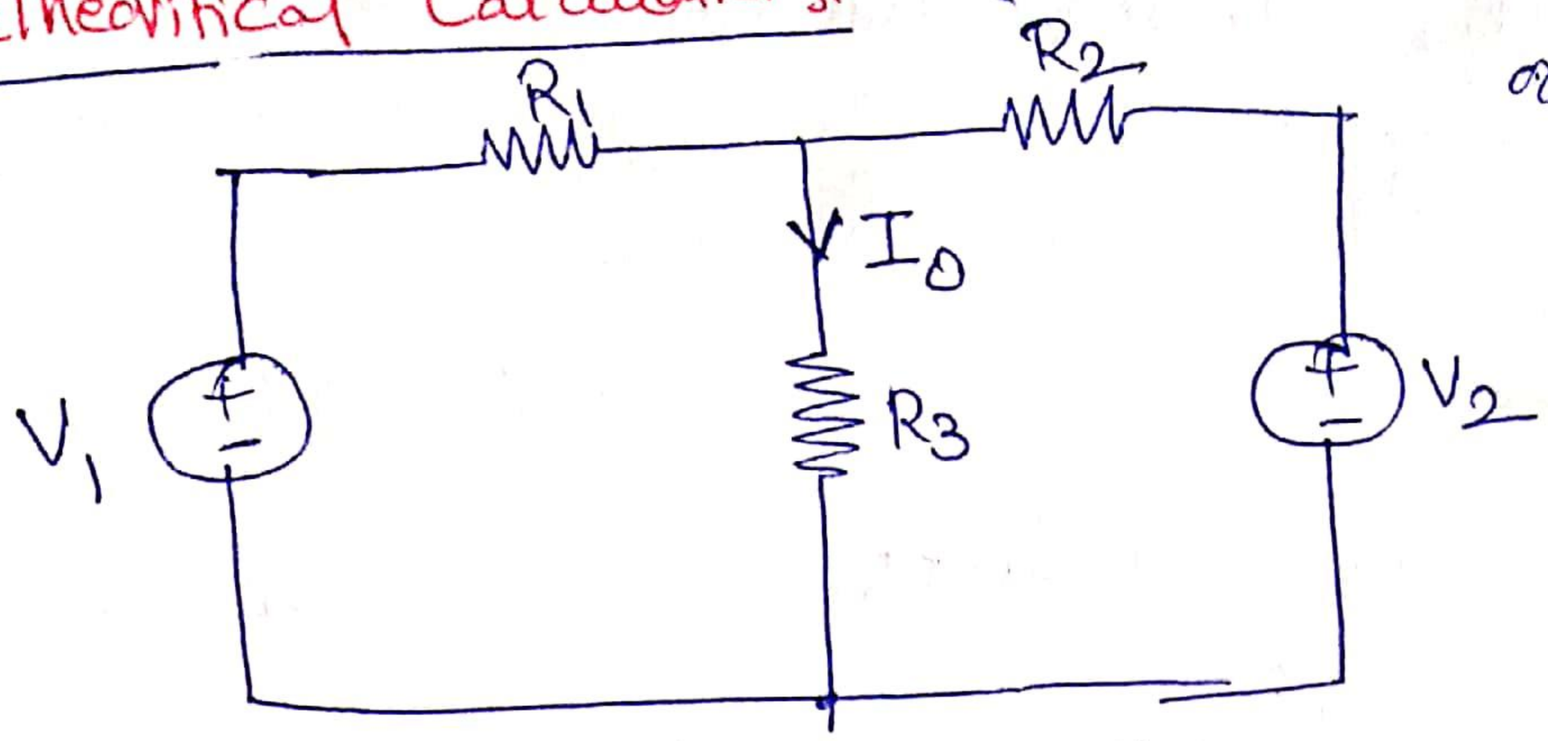
* Use the superposition principle to find i_0 & V_0 in the circuit



* Use the super position principle to solve for V_x in the circuit

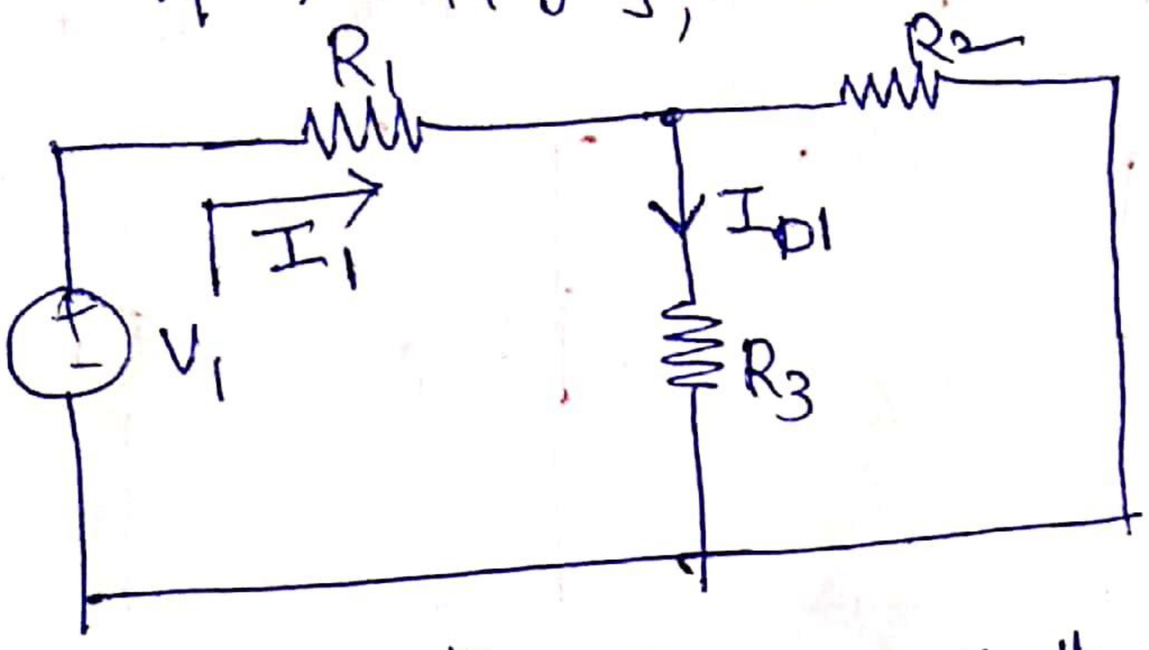


* ~~use~~ Theoritical Calculations:- (we can also use Nodal or Mesh Analysis)



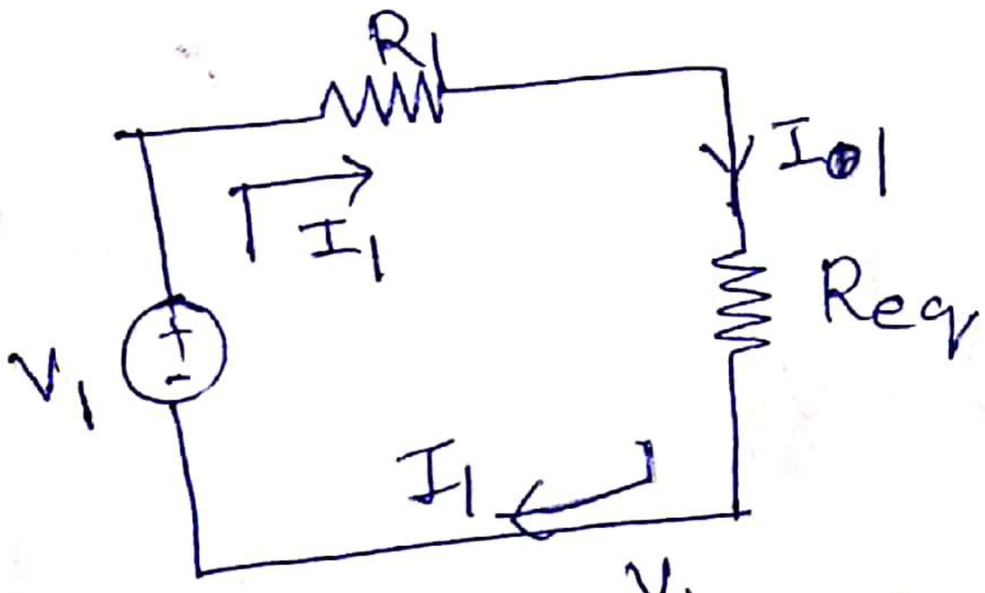
case(i)

$V_1 \rightarrow$ Applying, $V_2 = 0$



R_2 & R_3 parallel

let $R_{eq} = R_2 \| R_3$



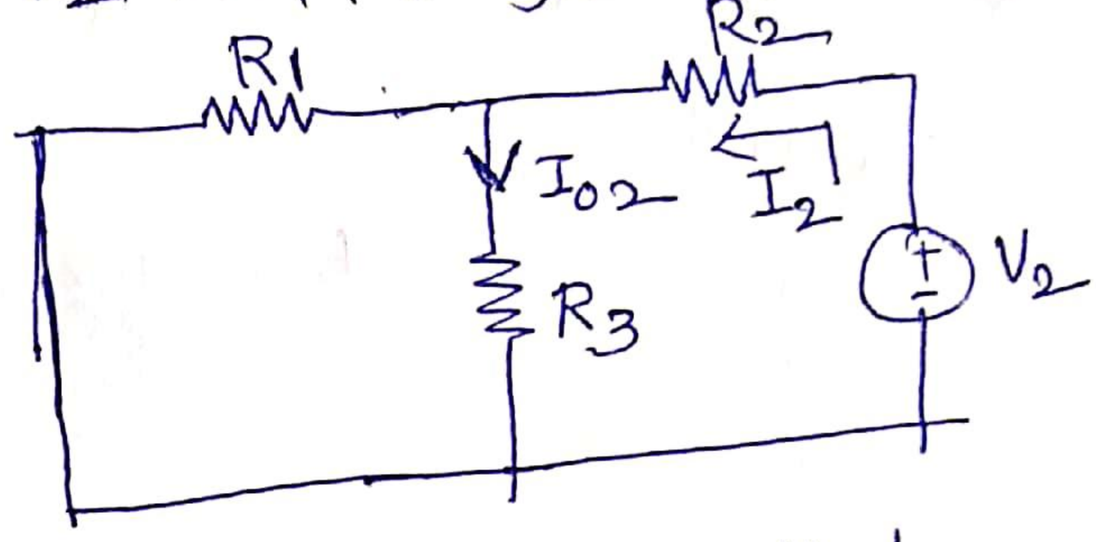
* $I_1 = \dfrac{V_1}{R_1 + R_{eq}}$ (using ohm's Law)

* By using current division

$$I_{01} = \left(\frac{R_2}{R_3 + R_{eq}}\right) I_1$$

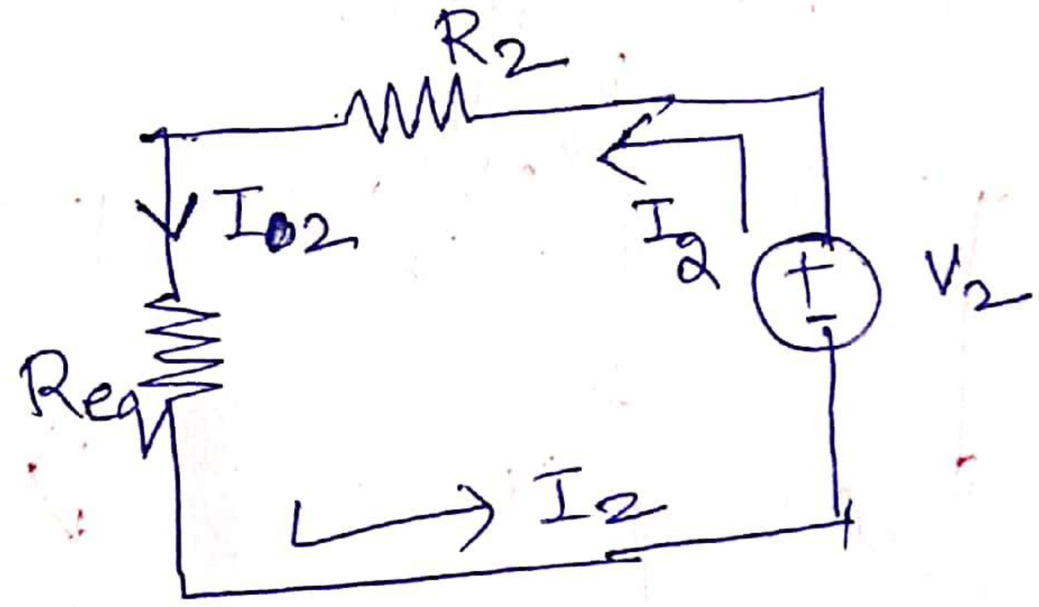
case(ii)

$V_2 \rightarrow$ Applying, $V_1 = 0$



R_1 & R_3 parallel

let $R_{eq} = R_1 \| R_3$



* $I_2 = \dfrac{V_2}{R_2 + R_{eq}}$ (using ohm's Law)

* By using current division

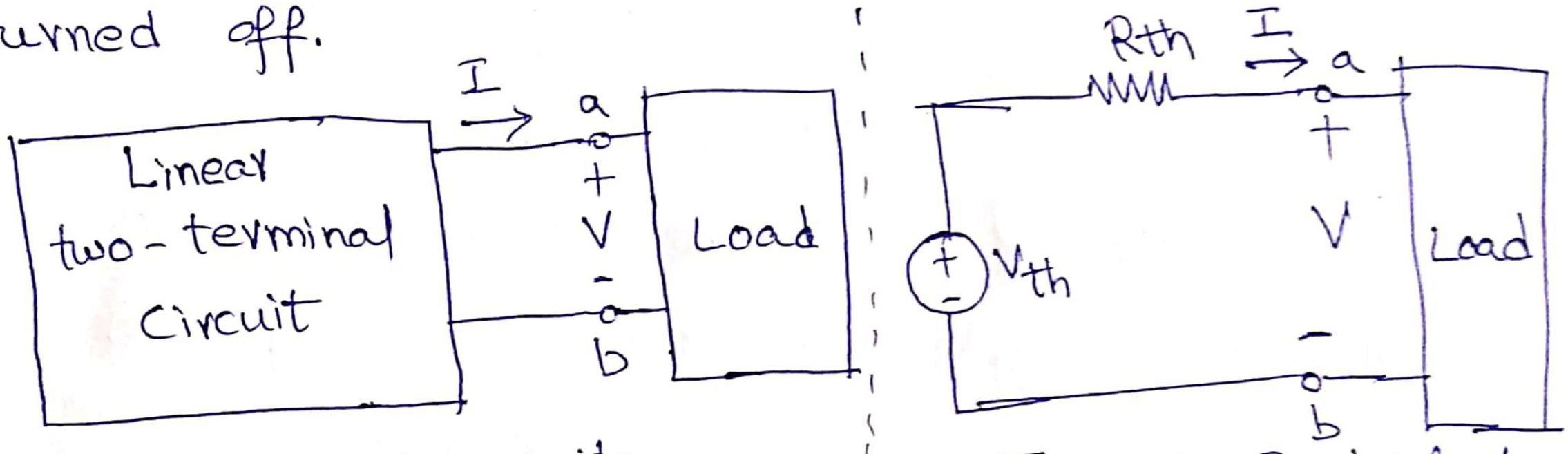
$$I_{02} = \left(\frac{R_1}{R_1 + R_{eq}}\right) I_2$$

Adding I_{01} & I_{02} gives I_0

$$\boxed{\therefore I_0 = I_{01} + I_{02}}$$

* Thevenin's Theorem:-

→ **Statement:-** A linear two-terminal circuit can be replaced by an Equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}, where V_{th} is the open circuit voltage at the terminals & R_{th} is the input or Equivalent resistance at the terminals when the independent sources are turned off.

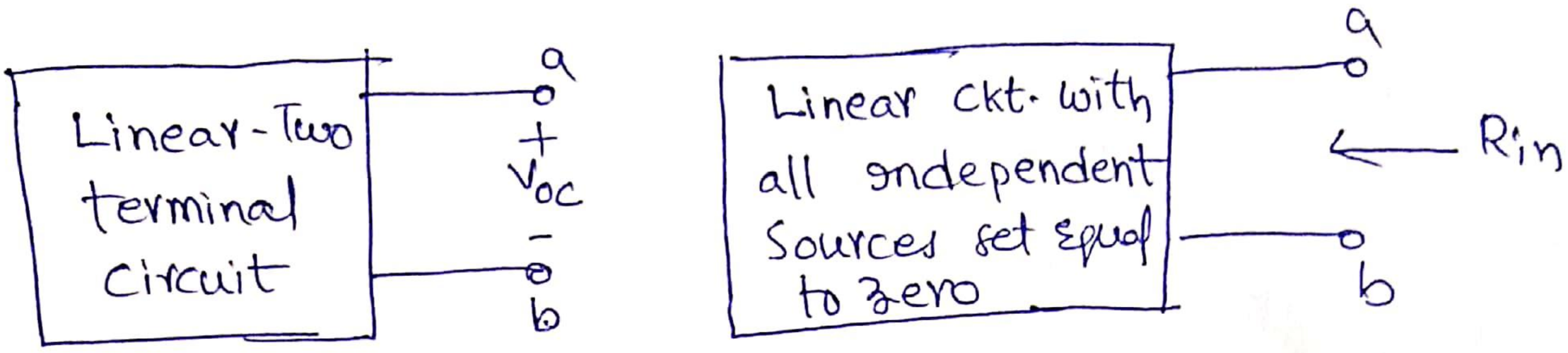


1.(a) original circuit 1.(b) Thevenin Equivalent ckt

→ we need to find V_{th} & R_{th}. Let the above two circuits are Equivalent. Two circuits are said to be Equivalent if they have the same voltage-current relation at their terminals. If the two terminals a-b are made to open-circuited (by removing the load), no current flows, so that the open circuit voltage across the terminals a-b in fig 1(a) must be Equal to the voltage source V_{th} in fig. 1(b), since the two circuits are Equivalent.

Thus V_{th} is the open-circuit voltage across the terminals as shown in fig. 2(a) that is

$$\boxed{V_{Th} = V_{oc}}$$



$V_{Th} = V_{oc}$
fig 2(a)

$R_{Th} = R_{in}$
fig 2(b)

fig(2) find V_{Th} & R_{Th}.

Again, with the load disconnected and terminals a-b open-circuited, we turn off all independent sources. The input resistance (or Equivalent resistance) of the dead circuit at the terminals a-b in fig. 1(a) must be Equal to R_{th} in fig. 1(b) because the two circuits are Equivalent. Thus R_{Th} is the input resistance at the terminals when the independent sources are turned off as shown in fig. 2(b).

$$\boxed{R_{Th} = R_{in}}$$

Thevenin's theorem is very important in circuit analysis. A large circuit may be replaced by a single independent voltage source and a single resistor

→ In finding the Thevenin resistance R_{Th}, we need to consider two cases

Case 1:- If the N/w has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the N/w looking b/w terminals a-b in fig. 2(b)

Case 2:- If the N/w has dependent sources, we turn off all independent sources. As with Superposition, dependent sources are not to be turned off because they are controlled by circuit variables. we apply a voltage source "V_0" at the terminals "a & b" & determine the resulting current i_0. Then $R_{Th} = \frac{V_0}{i_0}$ as shown in fig. 3(a). Alternatively, we may insert a current source i_0 at the terminals a-b as shown in fig. 3(b). and find the terminal voltage V_0. Again $R_{Th} = \frac{V_0}{i_0}$. Either of two approaches gives same result. In Either approach we may assume any value of V_0 & i_0.

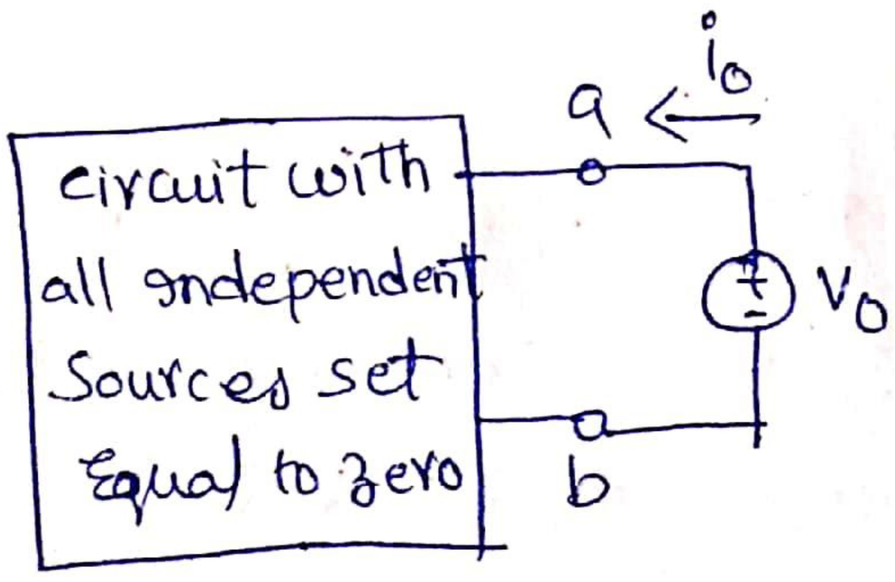


fig. 3(a) $R_{Th} = \frac{V_0}{i_0}$

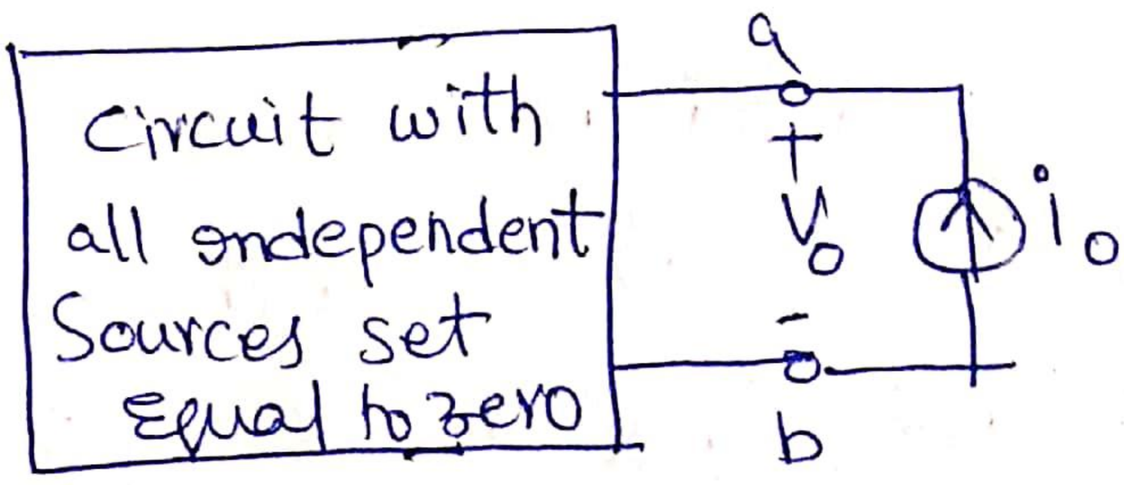


fig. 3(b) $R_{Th} = \frac{V_0}{i_0}$

→ A linear circuit with a variable load can be replaced by the Thevenin Equivalent, Exclusive of the load. The Equivalent network behaves the same way Externally as the original circuit. consider a linear circuit terminated by a load R_L. shown in fig. 4(a). The current I_L through the load and voltage V_L across the load are Easily determined once the Thevenin Equivalent of the circuit at the load terminals is obtain, shown in fig. 4(b)

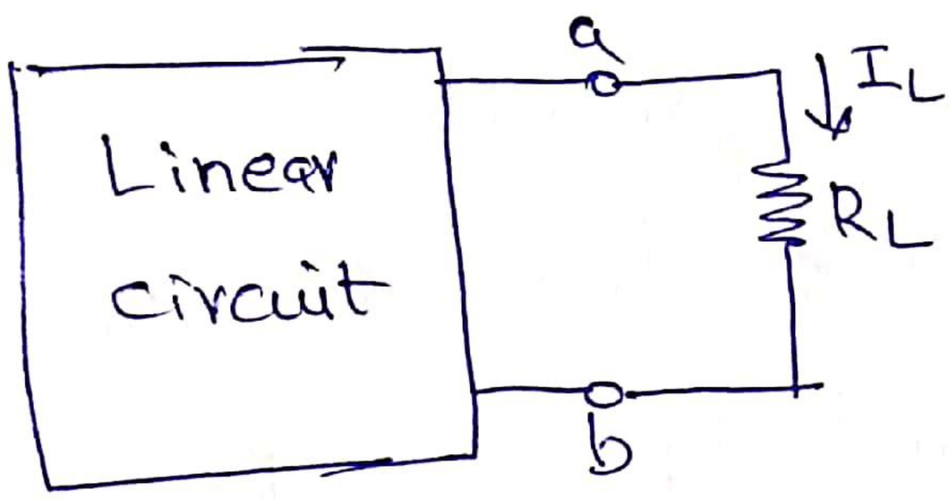


fig 4(a). original ckt with load

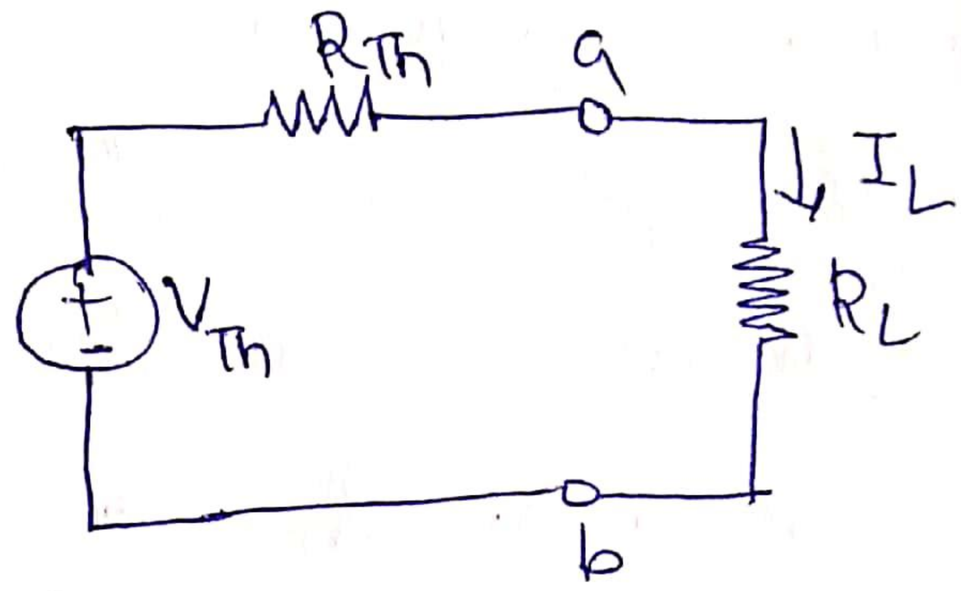
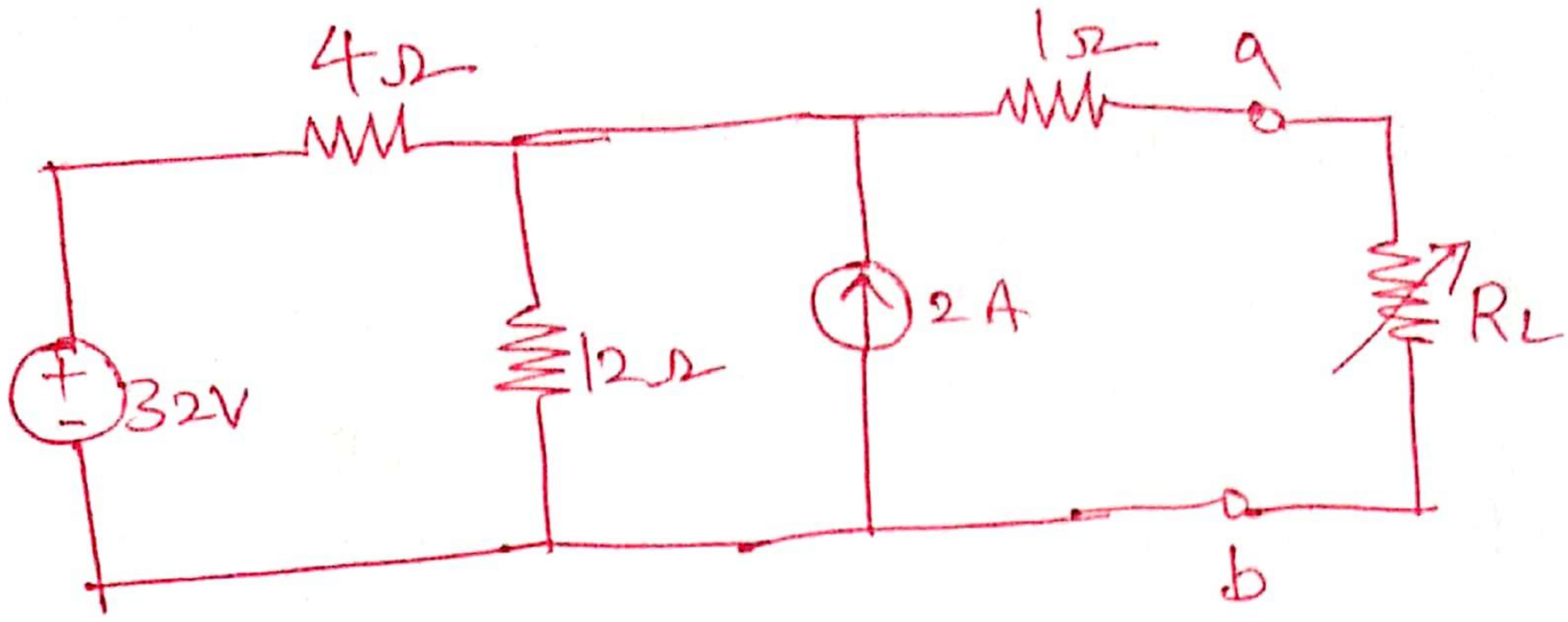


fig. 4(b). Thevenin Equivalent with load.

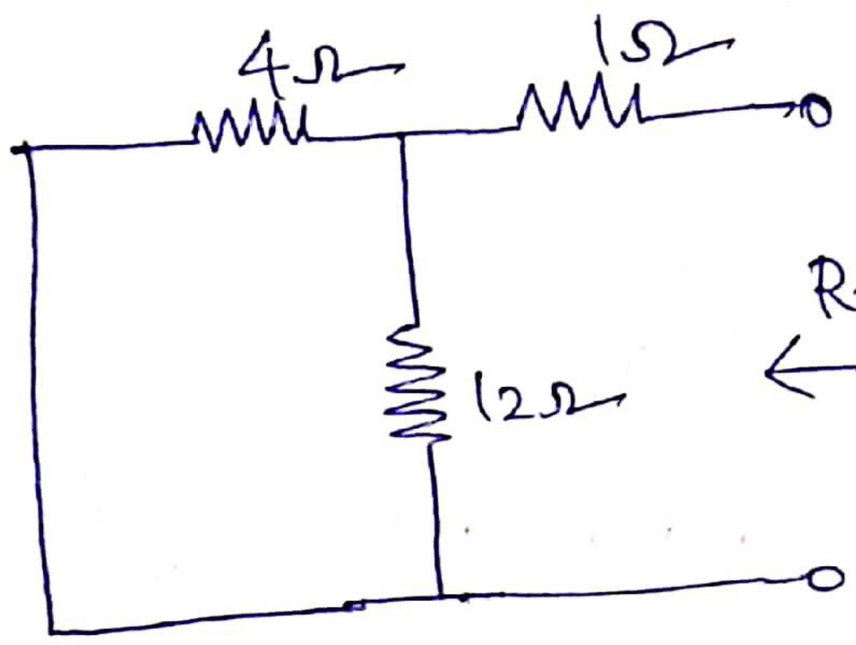
From fig. 4(b) $I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$ A

$$V_L = I_L R_L = \frac{R_L}{R_{Th} + R_L} V_{Th}.$$ (Voltage Divider)

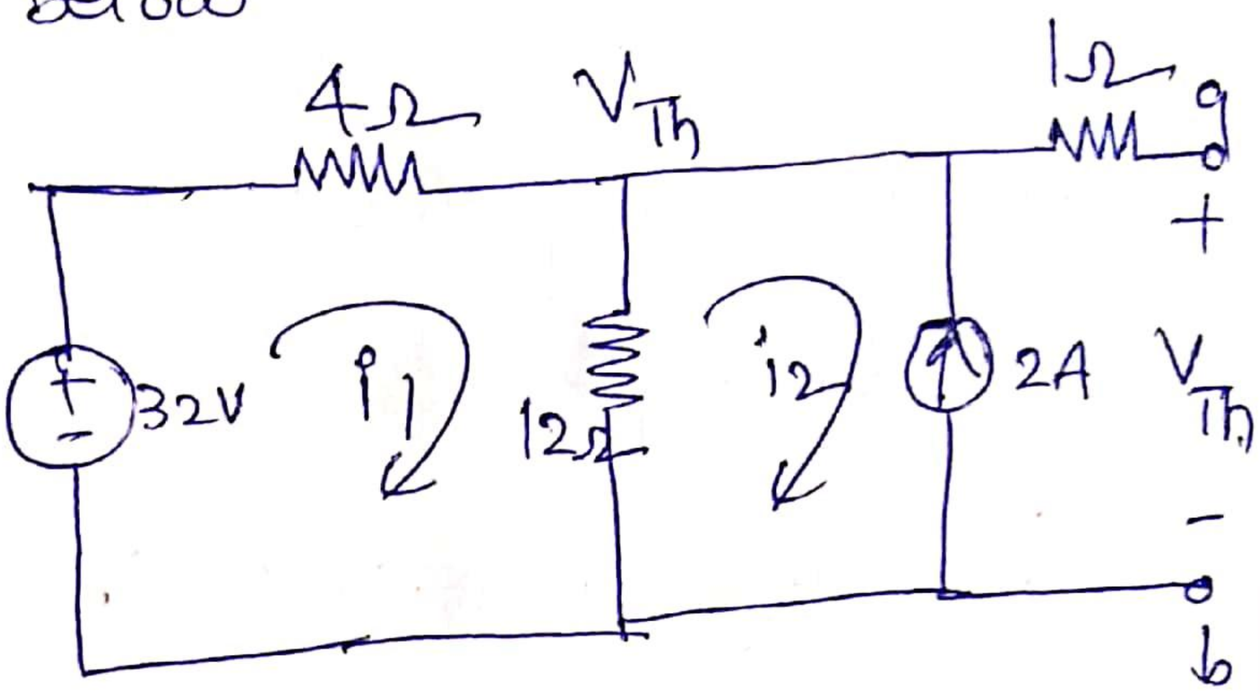
Ex: Find the Thevenin Equivalent circuit of the circuit shown to the left of the terminals a-b. Then find the current through $R_L = 6, 16$ & $36\,\Omega$.



Sol:- To find R_{Th} by turning off the 32-V voltage source (replacing it by short circuit) and the 2A current source (replacing it by open circuit). The circuit becomes shown below



(a) finding R_{Th}



(b) finding V_{Th}.

$\therefore R_{Th} = (4 \| 12) + 1 = 4\Omega$

To find V_{Th}, consider circuit in fig. (b). Applying Mesh analysis to the two loops

loop2:- $i_2 = -2A$

loop1:- $-32 + 4i_1 + (i_1 - i_2)12 = 0$,

Solving for i_1, we get $i_1 = 0.5A$.

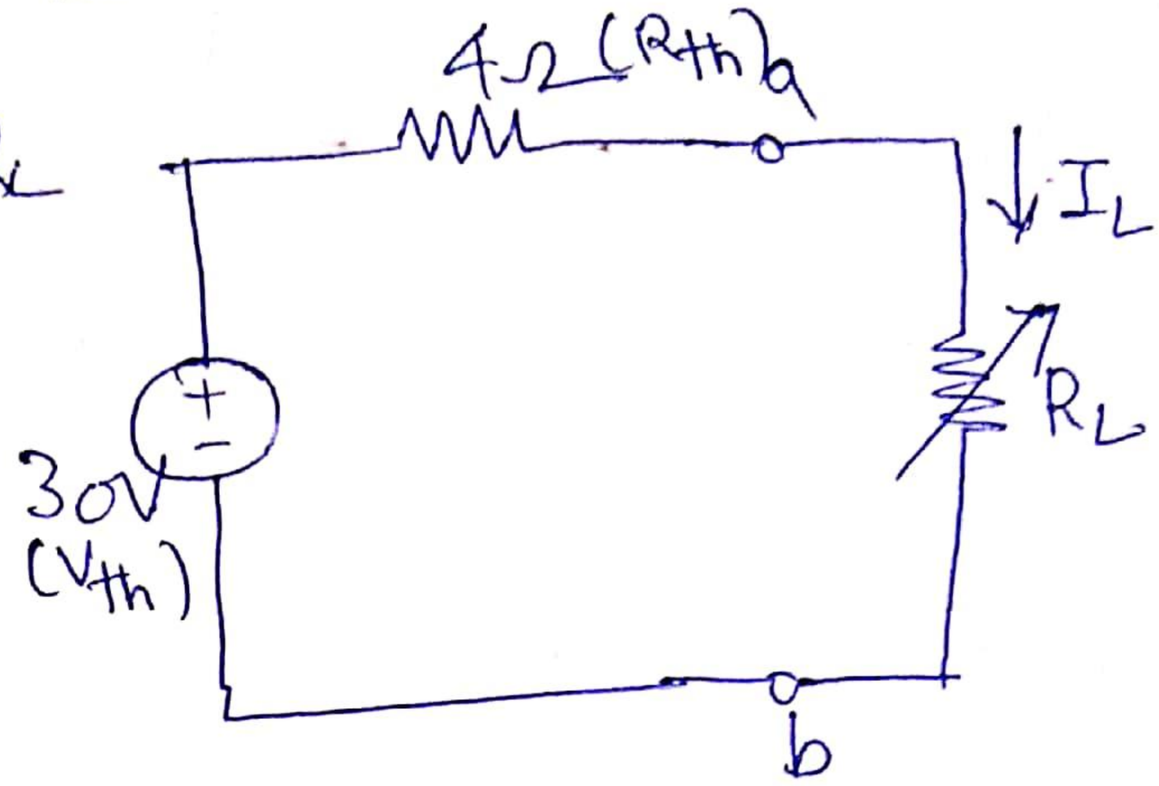
Thus $V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30V$.

- Alternatively it is Easier to use nodal analysis we ignore the 1Ω resistor since no current flows through it.

→ At the top node KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12} \Rightarrow V_{Th} = 30V$$

The current through R_L is $I_L = \frac{V_{Th}}{R_{Th} + R_L}$



→ when $R_L = 6\,\Omega$

$$I_L = \frac{30}{10} = 3A$$

* Thevenin Equivalent ckt

→ when $R_L = 16\,\Omega \rightarrow I_L = \frac{30}{20} = 1.5A$

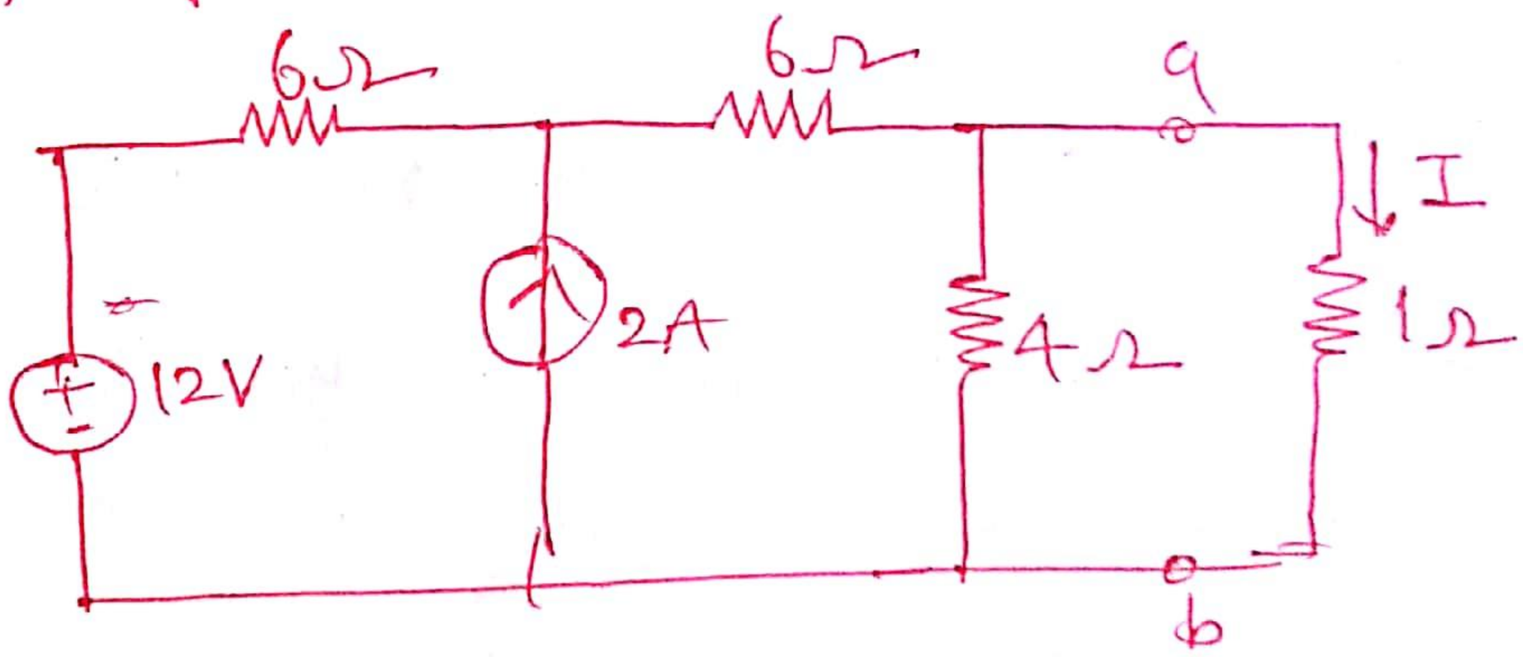
→ when $R_L = 36\,\Omega \rightarrow I_L = \frac{30}{40} = 0.75A$

* Using Thevenin's Theorem, find the Equivalent circuit to the left of the terminals a-b. Find I

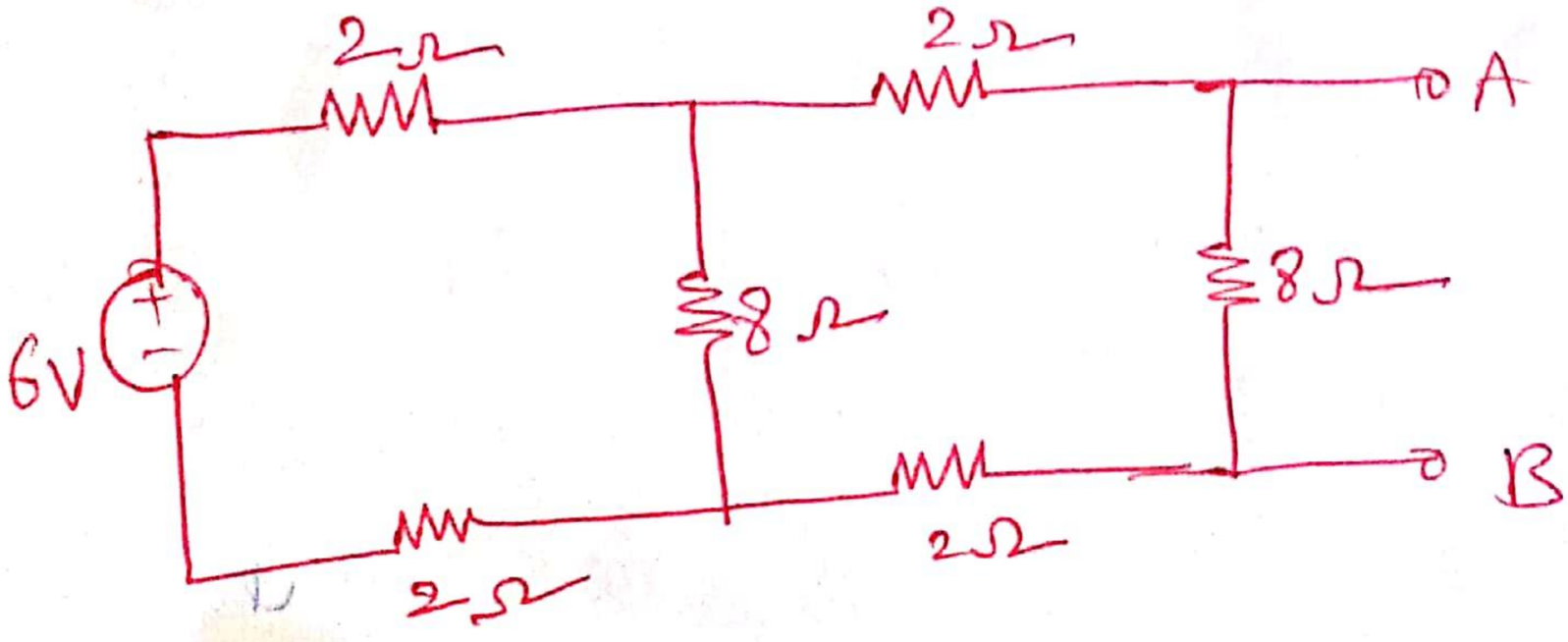
$V_{Th} = 6V$

$R_{Th} = 3\,\Omega$

$I = 1.5A$



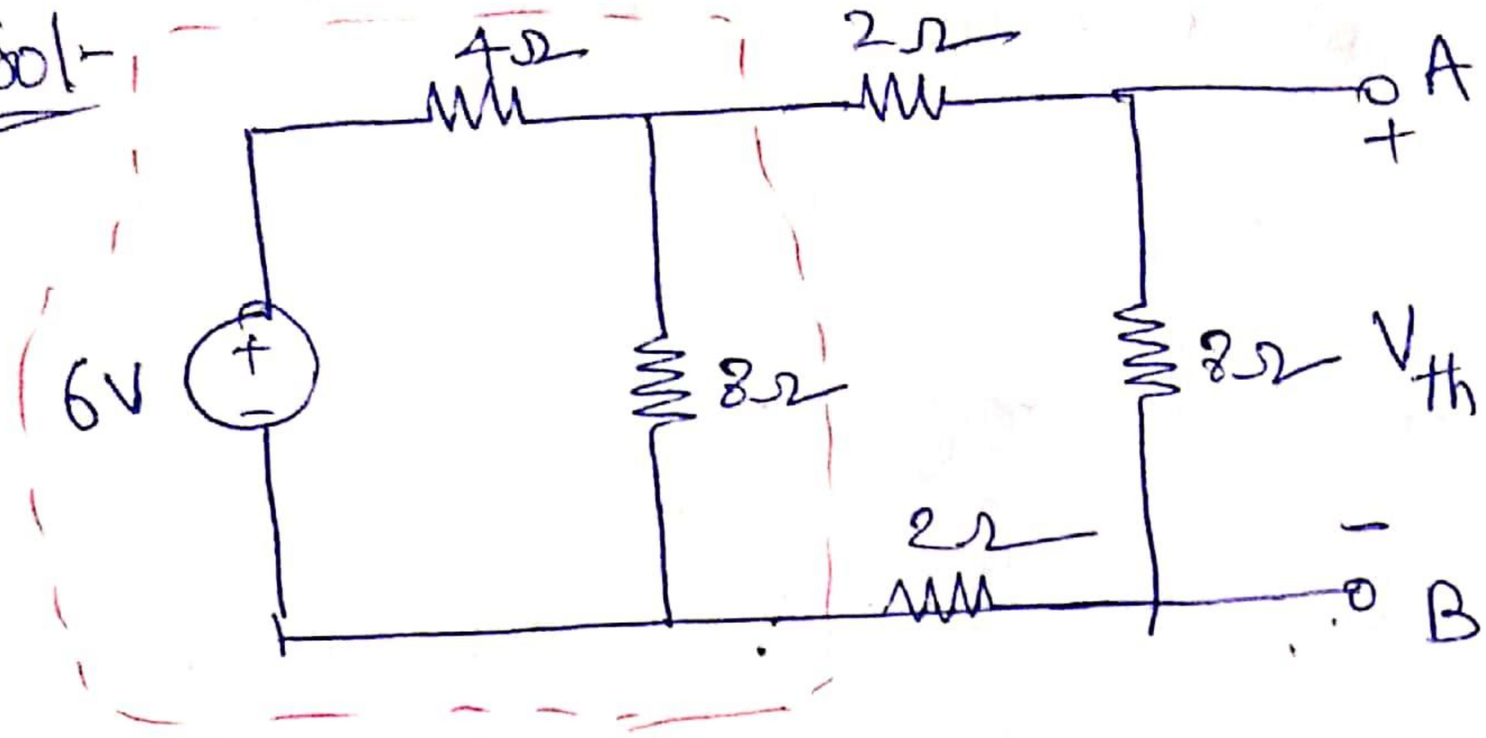
* Find V_{Th} & R_{Th} across AB



$V_{Th} = \frac{24}{11} V$

$R_{Th} = \frac{40}{11}\,\Omega$

Sol:-

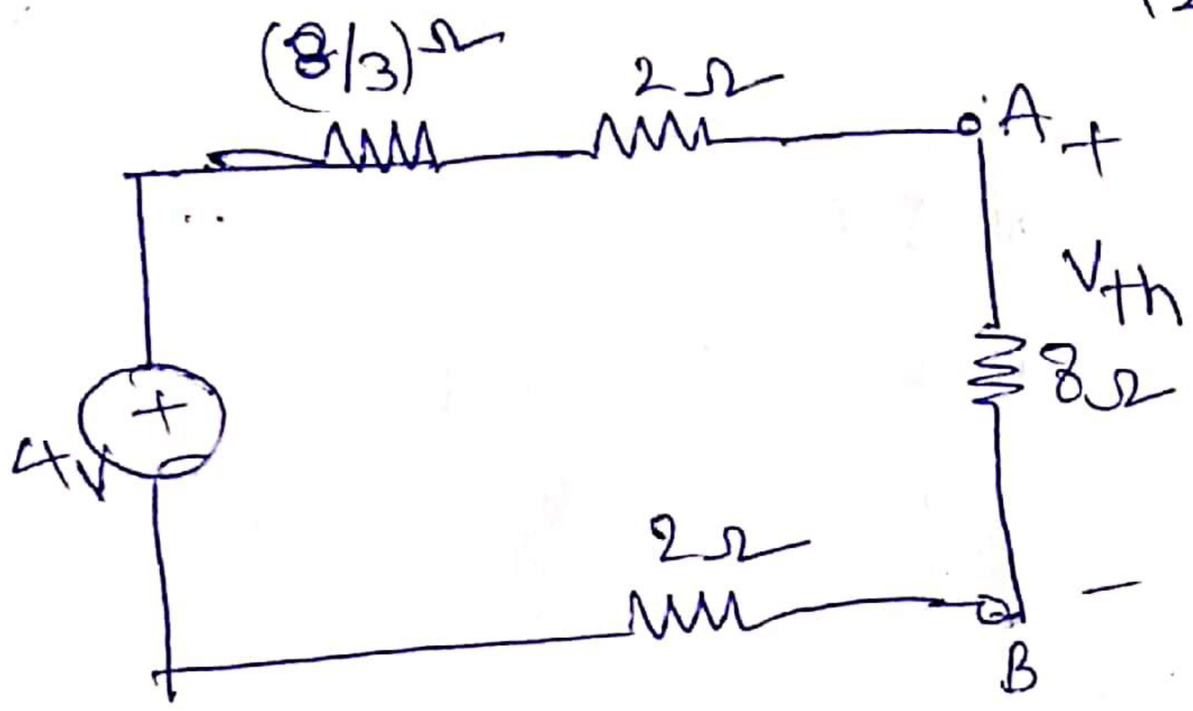


Since 2Ω & 2Ω resistor is in series

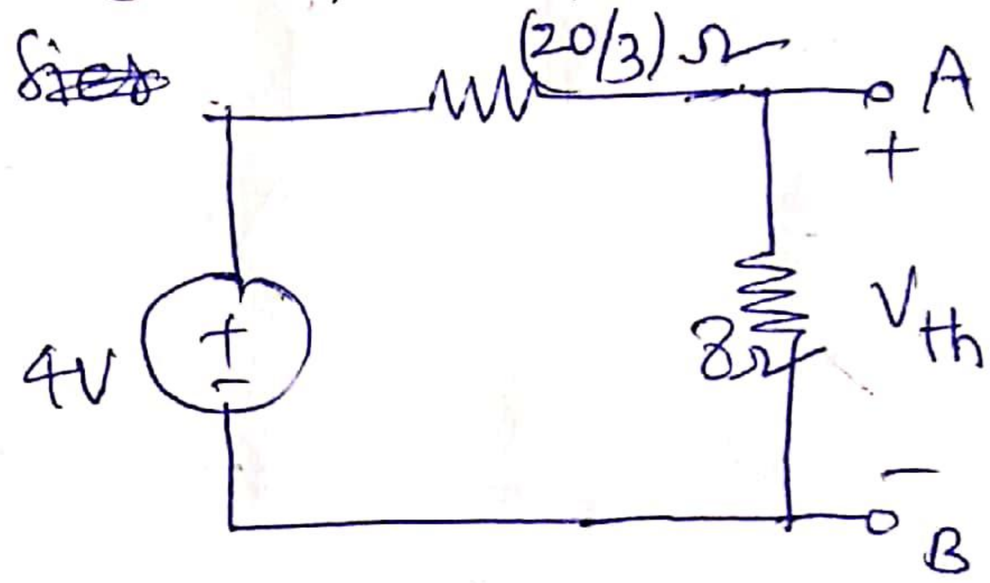
Voltage across 8Ω resistor $\Rightarrow \frac{\cancel{8}^{4} \times 6}{\cancel{12}_{2}} = 4V$

4Ω & 8Ω are in parallel

$$4 \| 8 = \frac{32}{12} = \frac{8}{3}$$

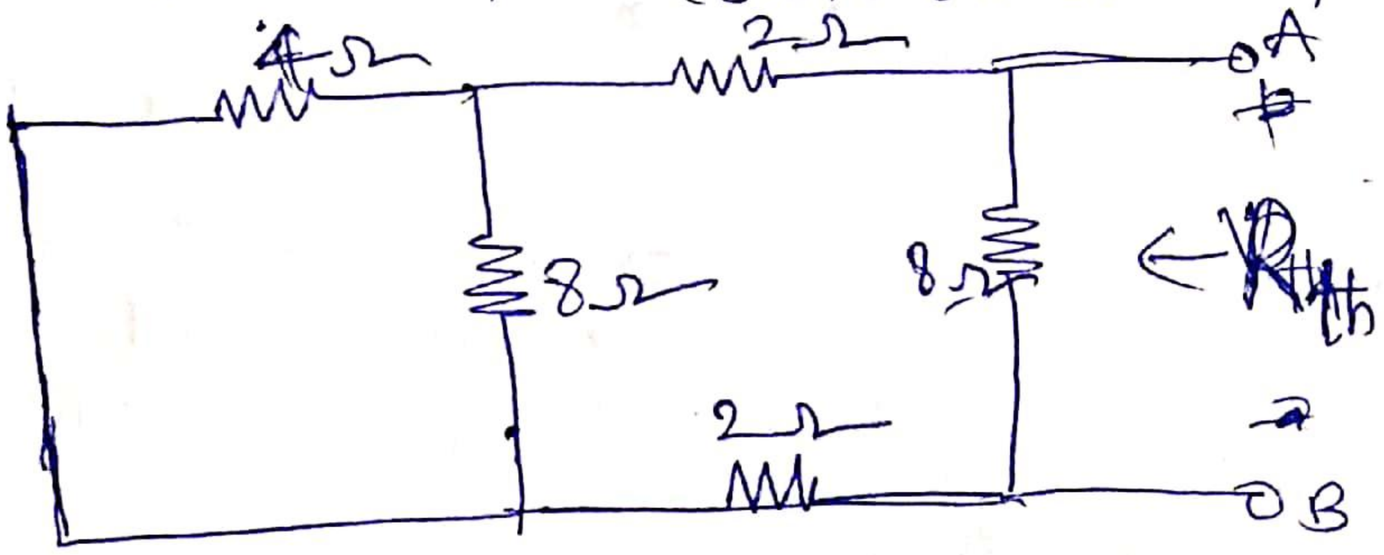


(8/3)Ω, 2Ω & 2Ω in series

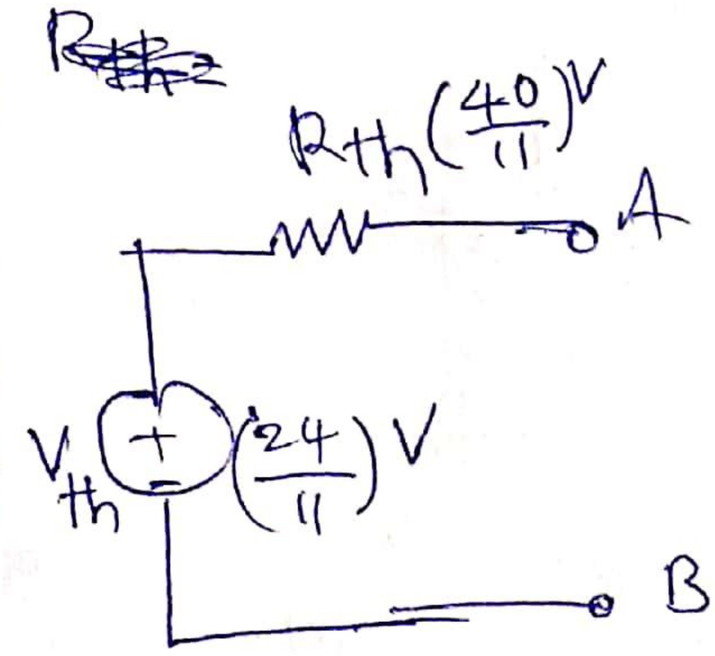


$$\therefore V_{th} = \frac{8 \times 4}{\frac{20}{3} + 8} = \frac{24}{11} V$$

For find R_{th} :- (short 6V source)



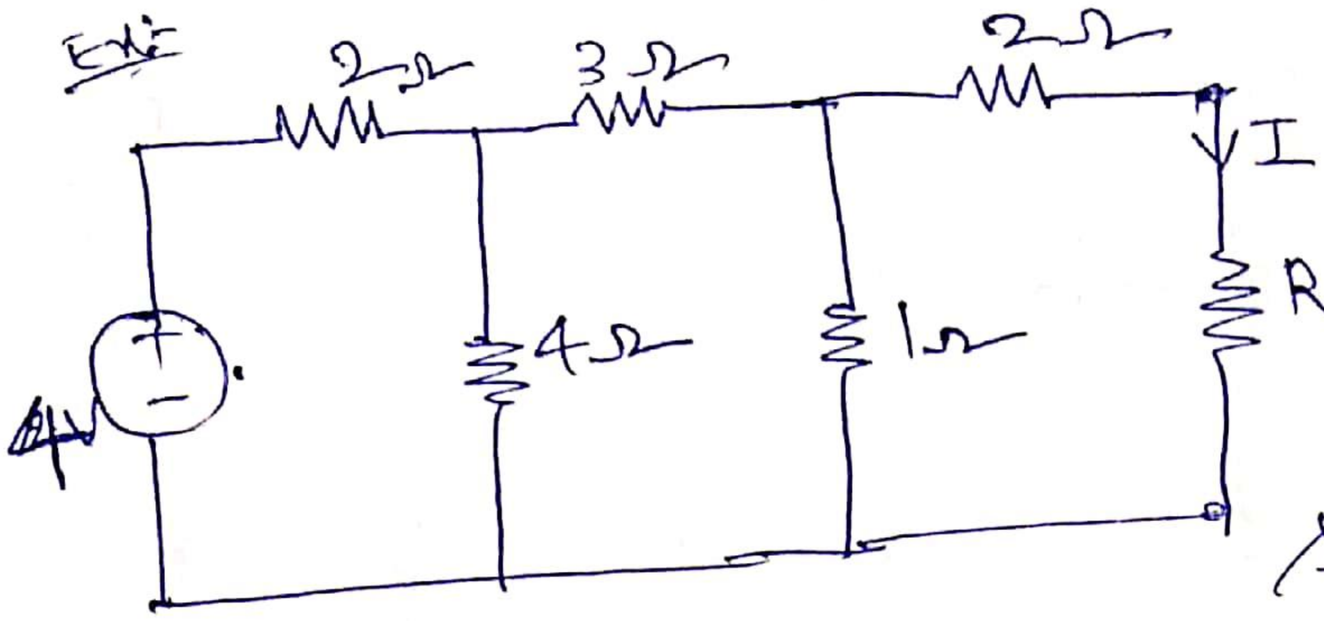
$$R_{th} = \left(\frac{8}{3} + 4\right) \| 8 = \frac{40}{11} \Omega$$



* Method to find Thevenin Equivalent circuit:-

Case-1:- A Network with all are independent sources

Ex:-



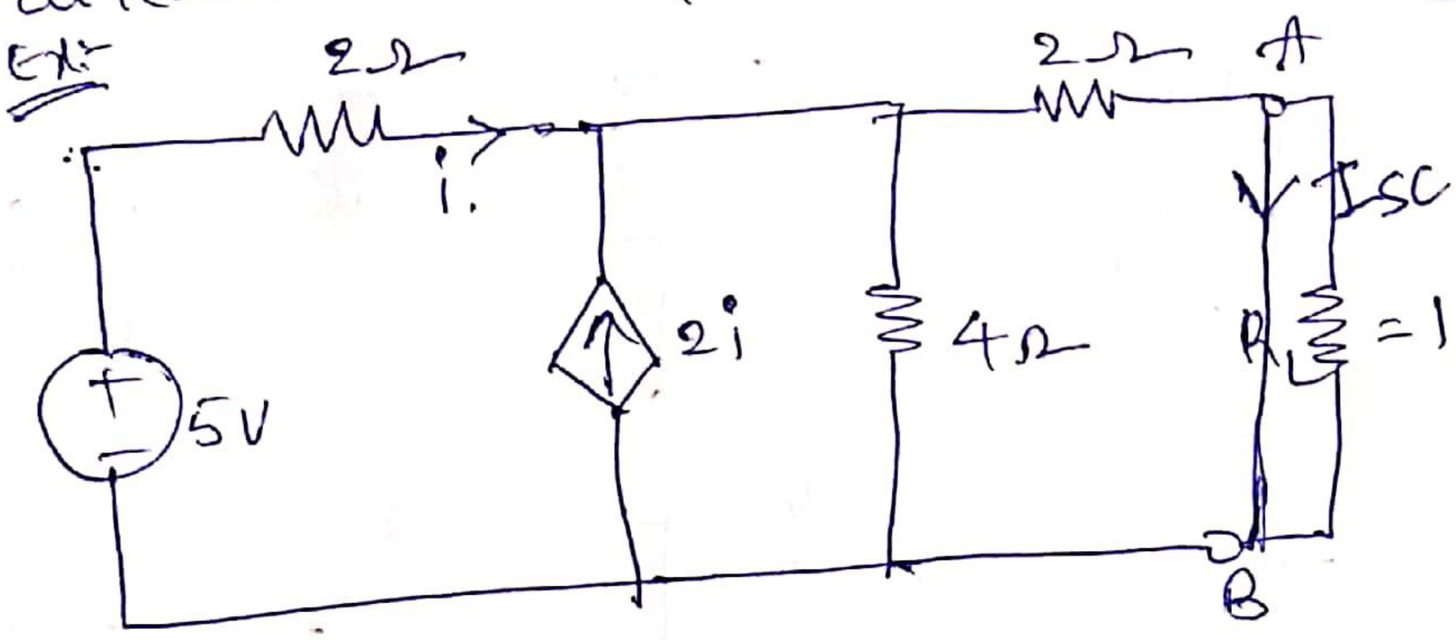
⟺ Remove R_L

→ find V_{th} across R_L

→ find R_{th} by Short circuit voltage source & open circuit current source

Case-2:- Atleast one independent source and atleast one dependent source.

Ex:-



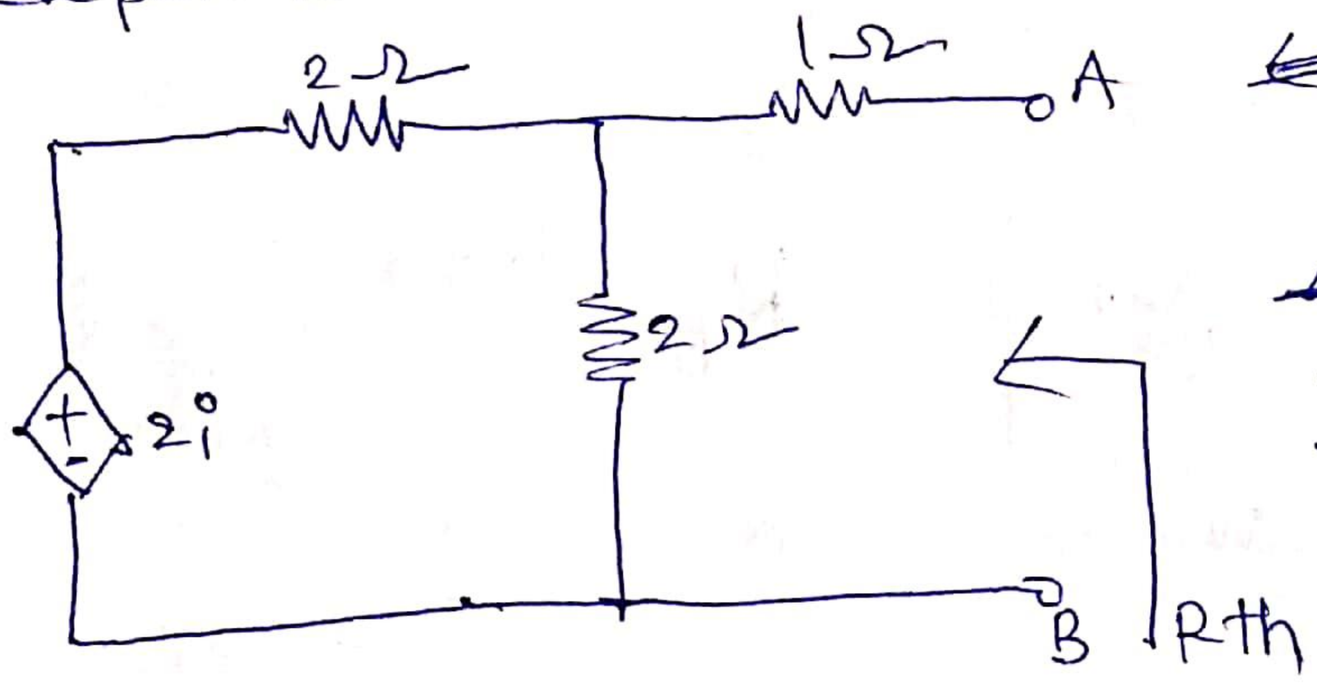
→ find V_{th} by removing R_L

→ I_{SC} (short circuiting terminal (AB) we get I_{SC}

$$\therefore R_{th} = \frac{V_{th}}{I_{SC}}$$

I_{SC} is called Norton's current (I_N)

case-3:- No independent source but atleast one dependent source



⟺ This circuit is called dead circuit

→ we cannot find V_{th}

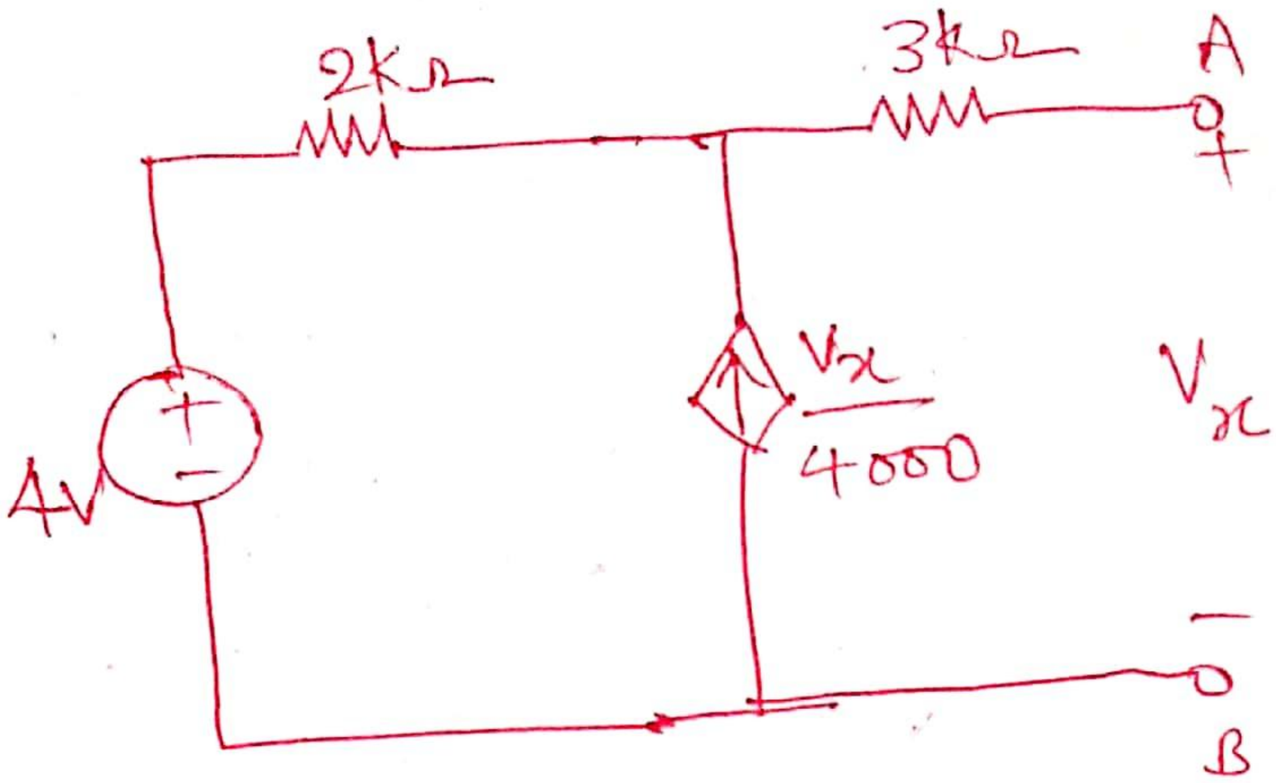
→ we cannot find I_{SC}

→ R_{th} we can find

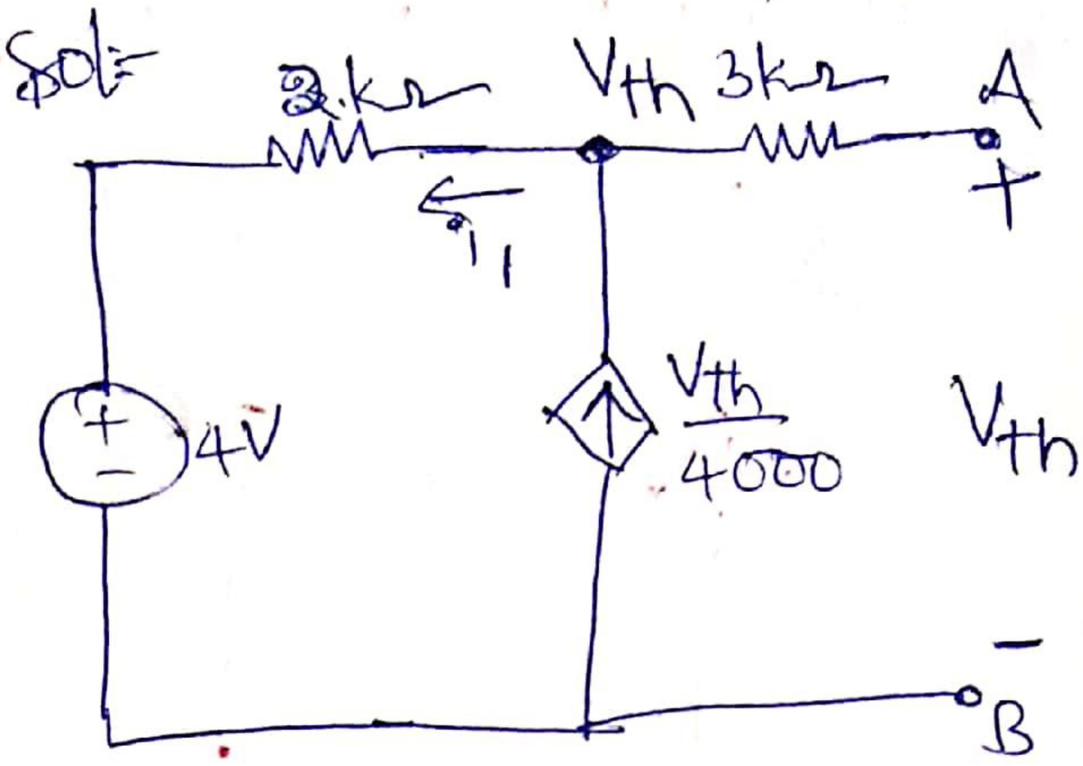
we can find R_{th} by applying some test Voltage (V) at AB, then Measure current 'I'.

$$\therefore R_{th} = \frac{V}{I}$$

* Find Thevenin Equivalent Circuit (case 2)



Sol:-



fig(a.)

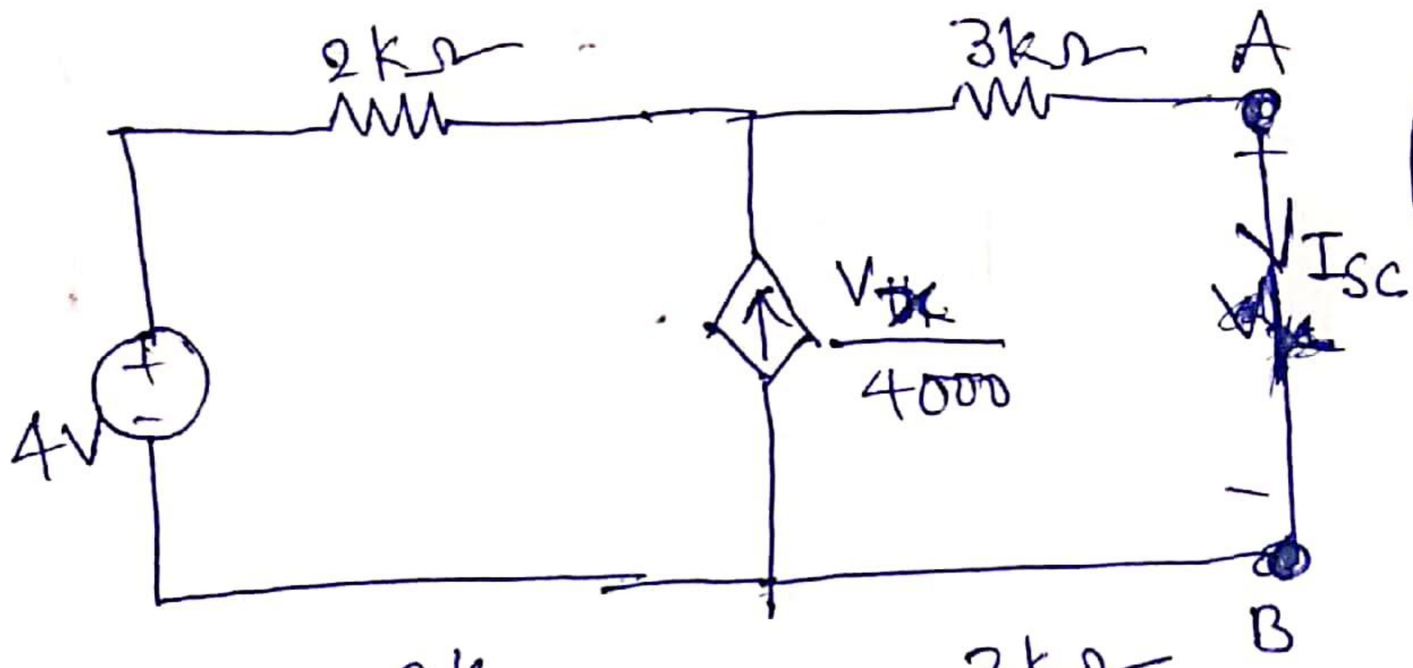
To find V_{th}:-

Apply KCL at V_{th} (Nodal Equations) [current through 3kΩ is 0]

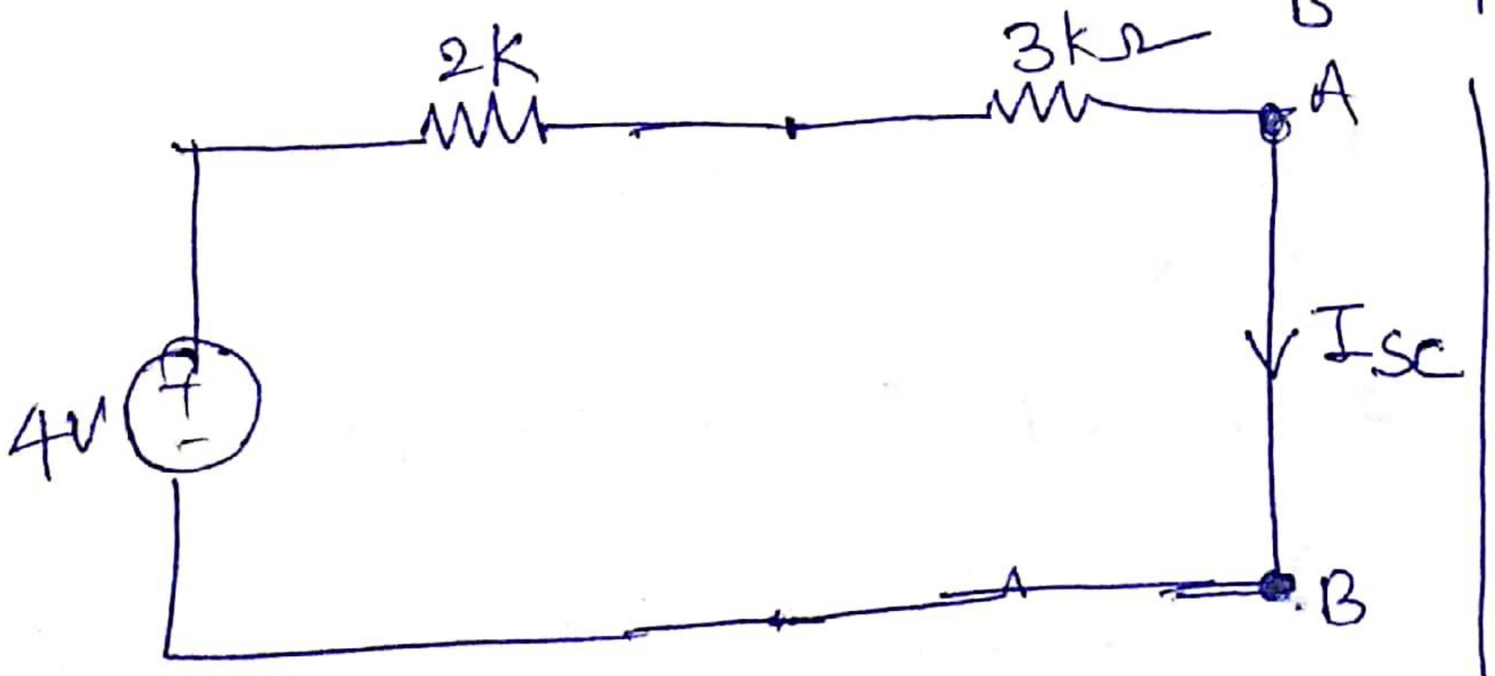
$$i_1 - \frac{V_{th}}{4000} = 0$$

$$\frac{V_{th}-4}{2k} - \frac{V_{th}}{4k} = 0 \Rightarrow \text{Solving } V_{th} \Rightarrow V_{th} = 8V.$$

To find I_{sc}:- we cannot find R_{th} directly at terminals A B because of dependent source appears in ckt. To find I_{sc}, short circuit the AB terminals



← If we short ckt voltage across AB is '0' i.e $V_x = 0$, then $\frac{V_x}{4000} = 0$ (open ckt)

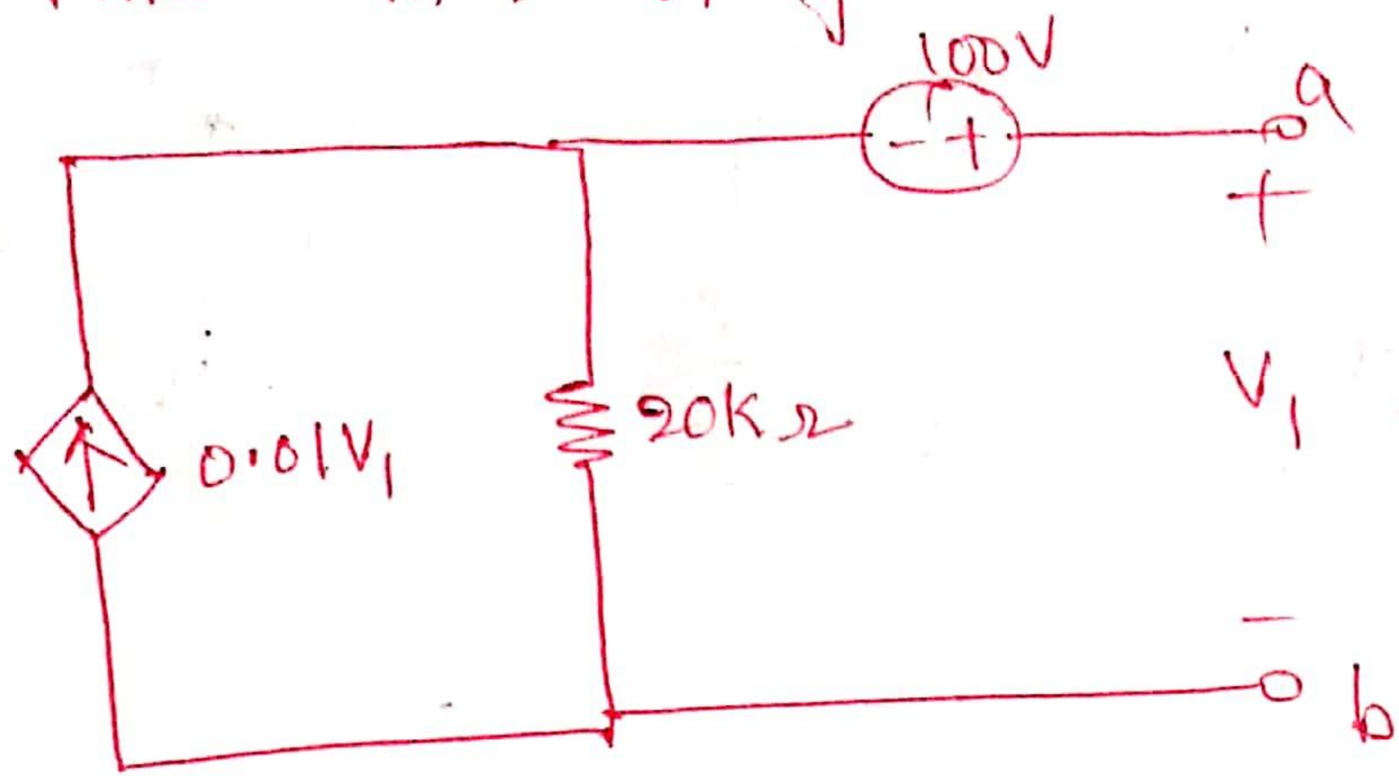


$$I_{sc} = \frac{4}{2k+3k} = 0.8mA$$

$$\therefore R_{th} = \frac{V_{th}}{I_{sc}} = \frac{8}{0.8 \times 10^{-3}}$$

$$R_{th} = 10k\Omega$$

* Find V_{th} & R_{th} of the following circuit (case-2)



[In case-2 we cannot find R_{th} directly]

$\left[\therefore R_{th} = \frac{V_{th}}{I_{sc}}\right]$

Sol:-

* Apply KVL fig (a) [for V_{th}]

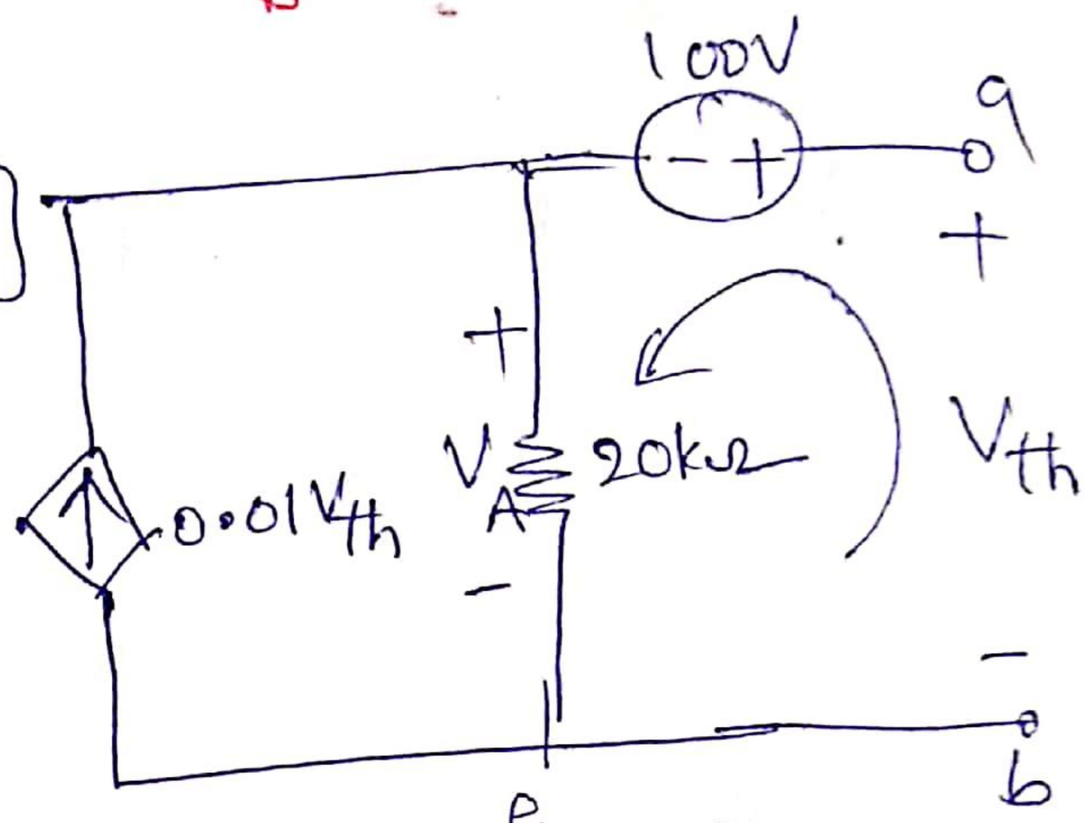


fig. (a)

$$-V_{th} + 100 + V_A = 0$$

$$V_{th} = 100 + V_A$$

$$V_{th} = 100 + 0.01V_{th} \times 20K$$

$$\boxed{\therefore V_{th} = \frac{-100}{199}\ V}$$

* for I_{sc} Short ckt 'ab'

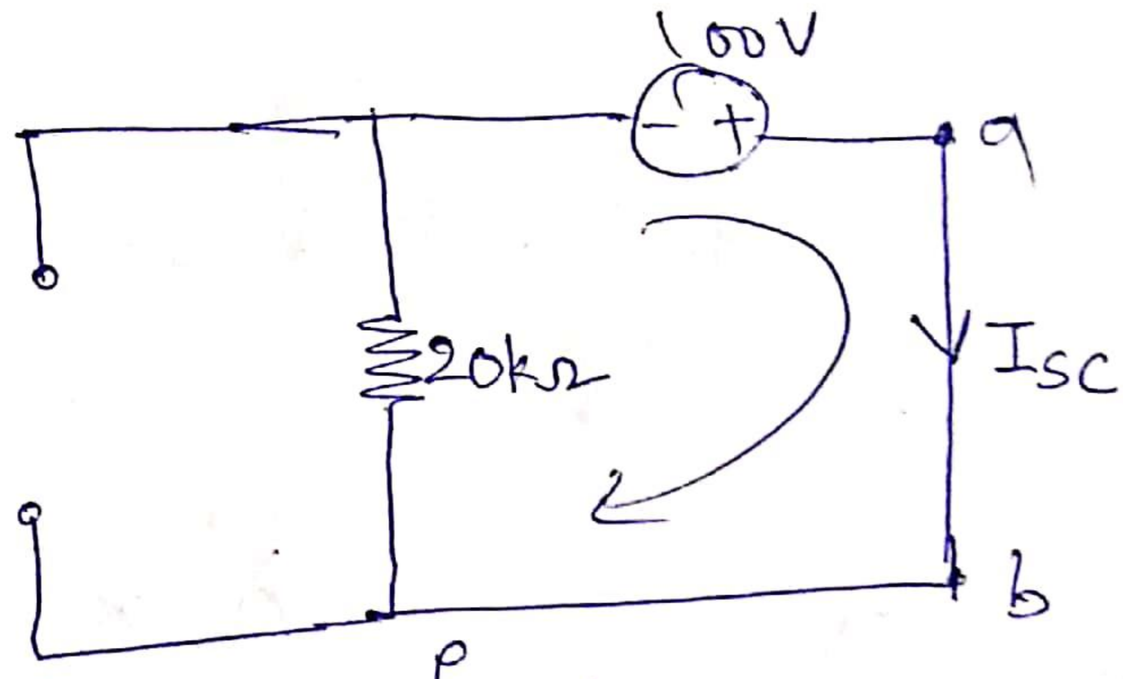


fig (b)

Short ckt ab results $V_1 = 0V$ as $V_1 = 0V$ then current source also 0A (open ckt).

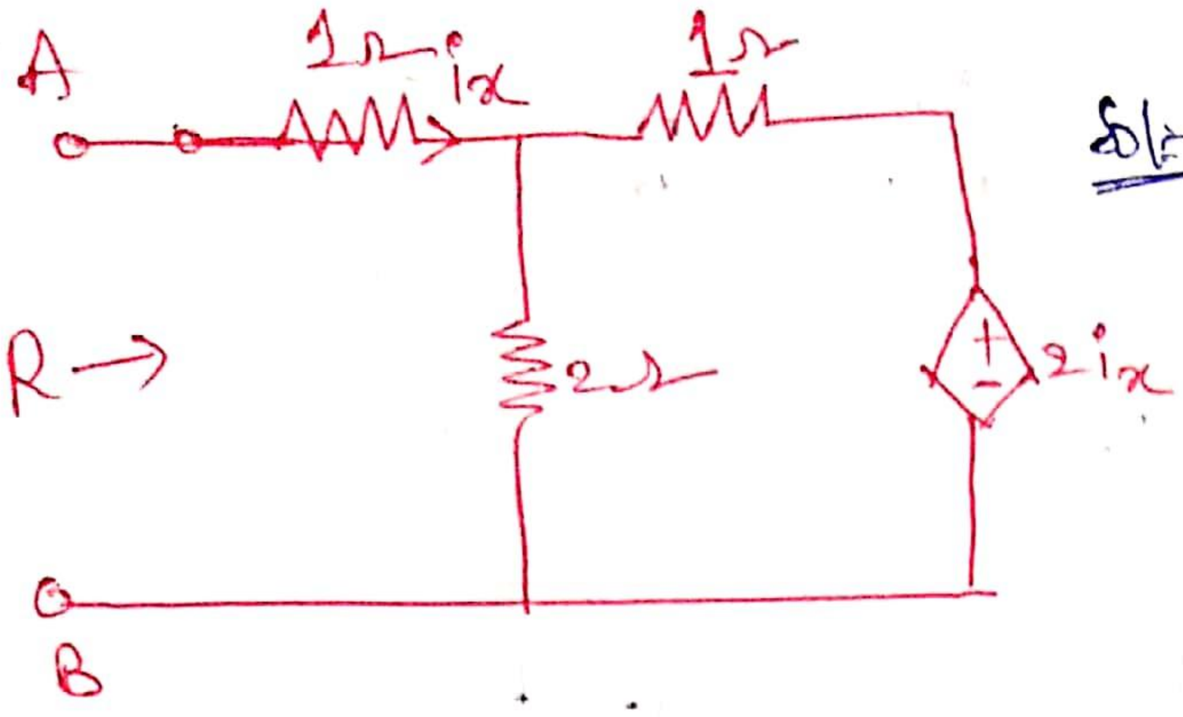
$$I_{sc} = \frac{100}{20k} = 5mA$$

$$\therefore R_{th} = \frac{-100}{199 \times 5\times10^{-3}} = -100.5\ \Omega$$

R_{th} is negative results (resistance is negative)

the given network is a active network

* Find 'R' across AB terminal:



(case-3)

Sol:-

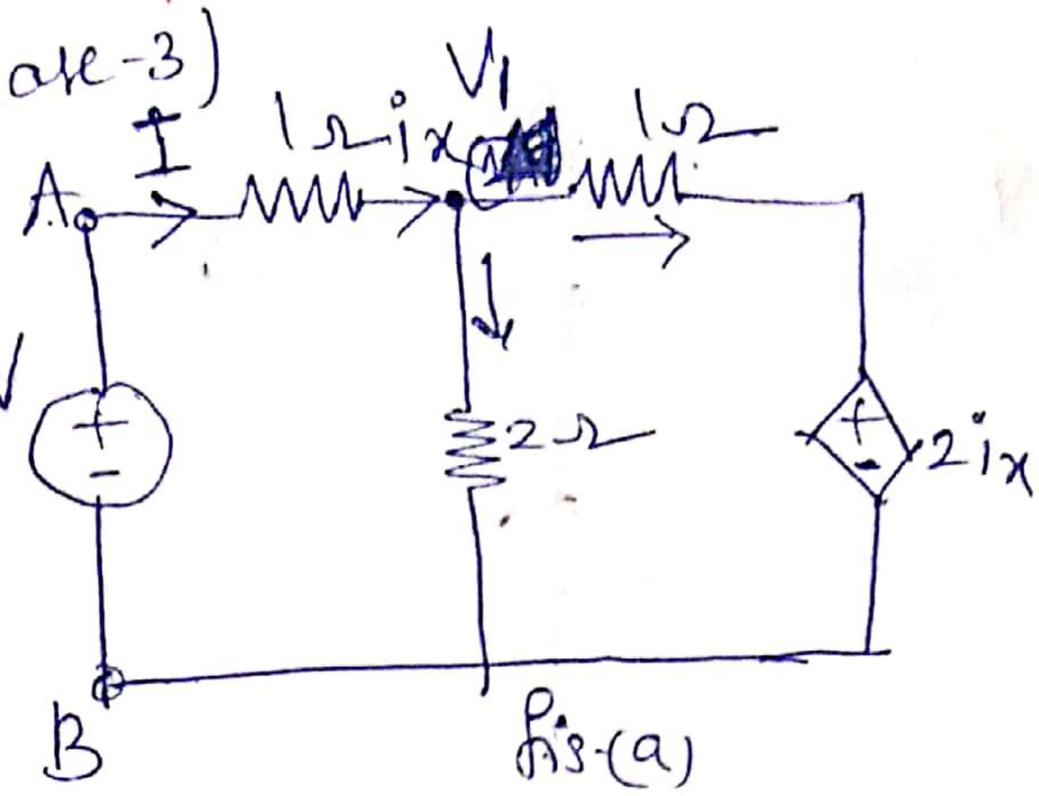


fig-(a)

→ we cannot find V_{th}, I_{sc} but we can find R_{th} by applying some test voltage at AB terminals (fig(a))

Applying KCL at node '1' we get

$$i_x = \frac{V_1}{2} + \frac{V_1 - 2i_x}{1\Omega}, \quad \text{let } i_x = I$$

$$I = \frac{V_1}{2} + \frac{V_1 - 2I}{1}$$

$$I = \frac{V_1 + 2V_1 - 4I}{2} \Rightarrow 2I = 3V_1 - 4I$$

$$6I = 3V_1 \rightarrow (1)$$

from fig.(a) $I = \frac{V - V_1}{1\Omega} \rightarrow (2)$

(or)

$$V_1 = V - I \rightarrow (2)$$

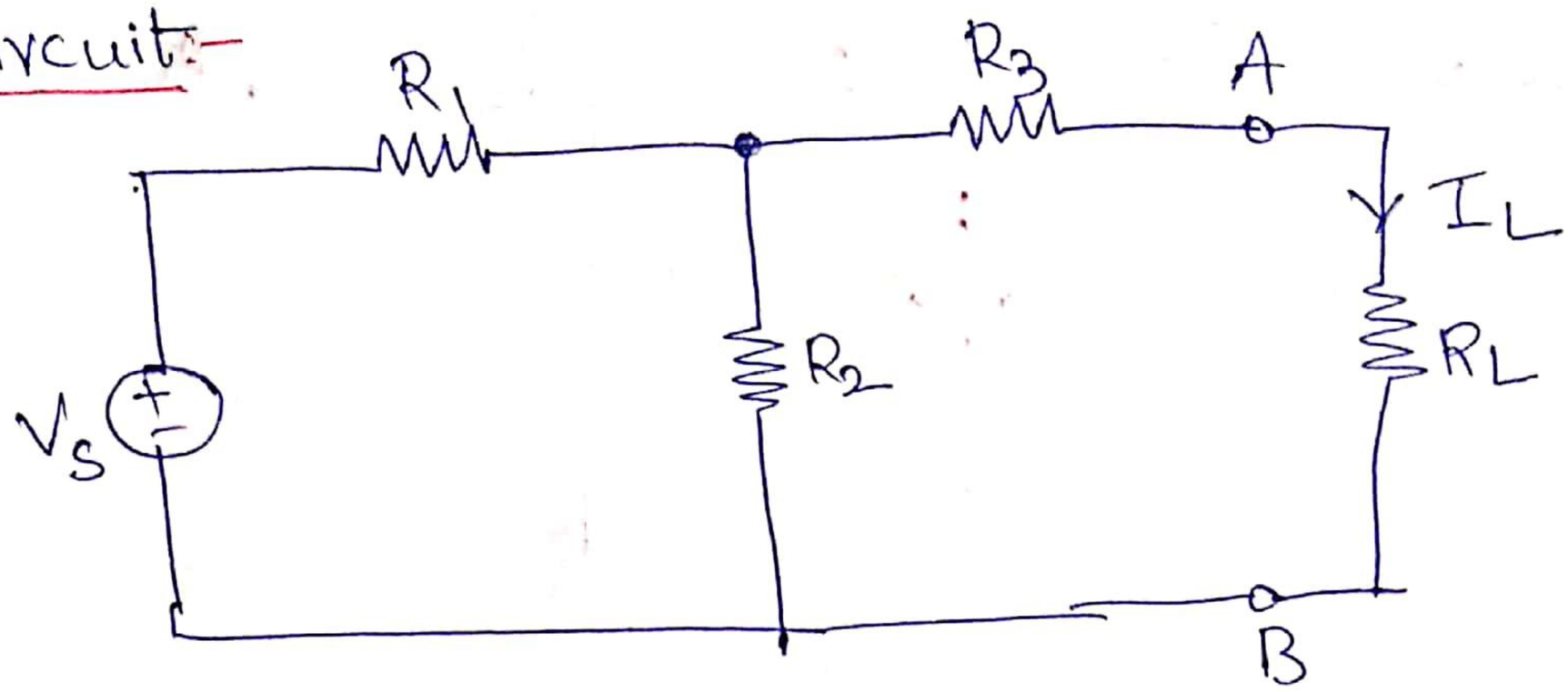
sub (2) in (1) we get

$$6I = 3V - 3I$$

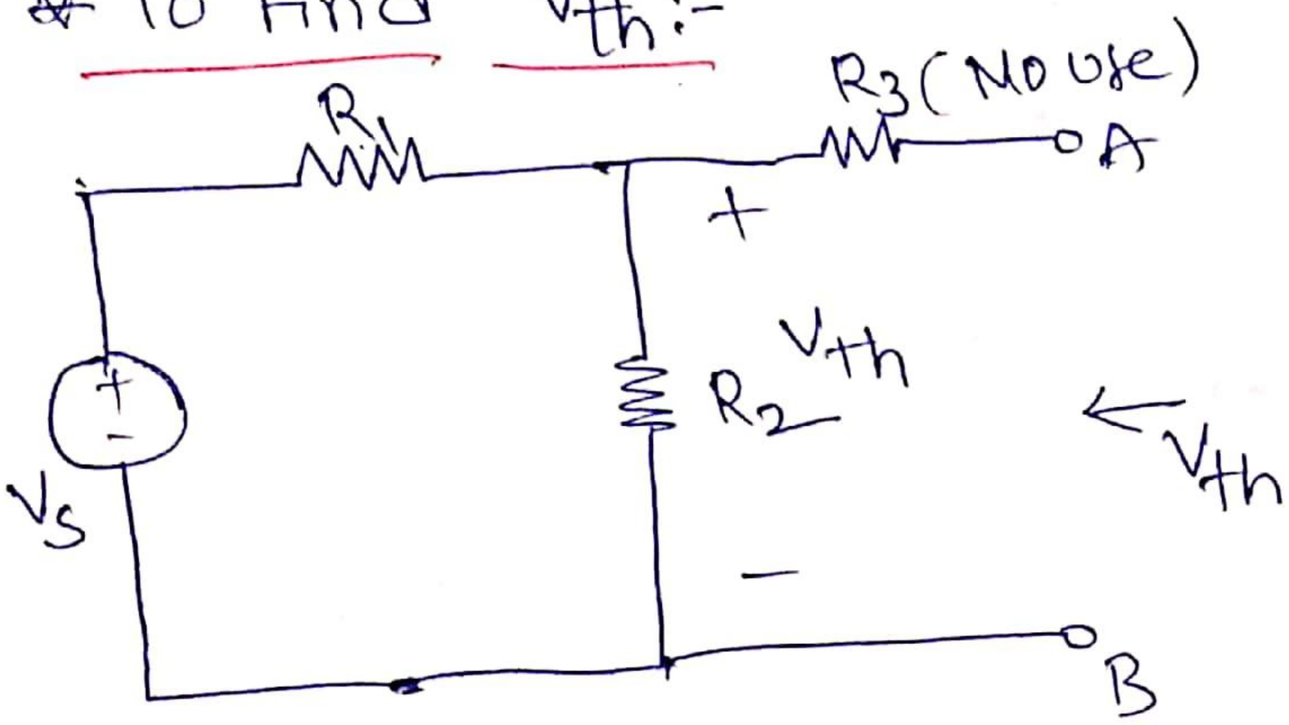
$$9I = 3V \Rightarrow \frac{V}{I} = R_{th} = 3\Omega$$

* Thevenin's Theorem :- (Theoritical Calculations)

Circuit:-



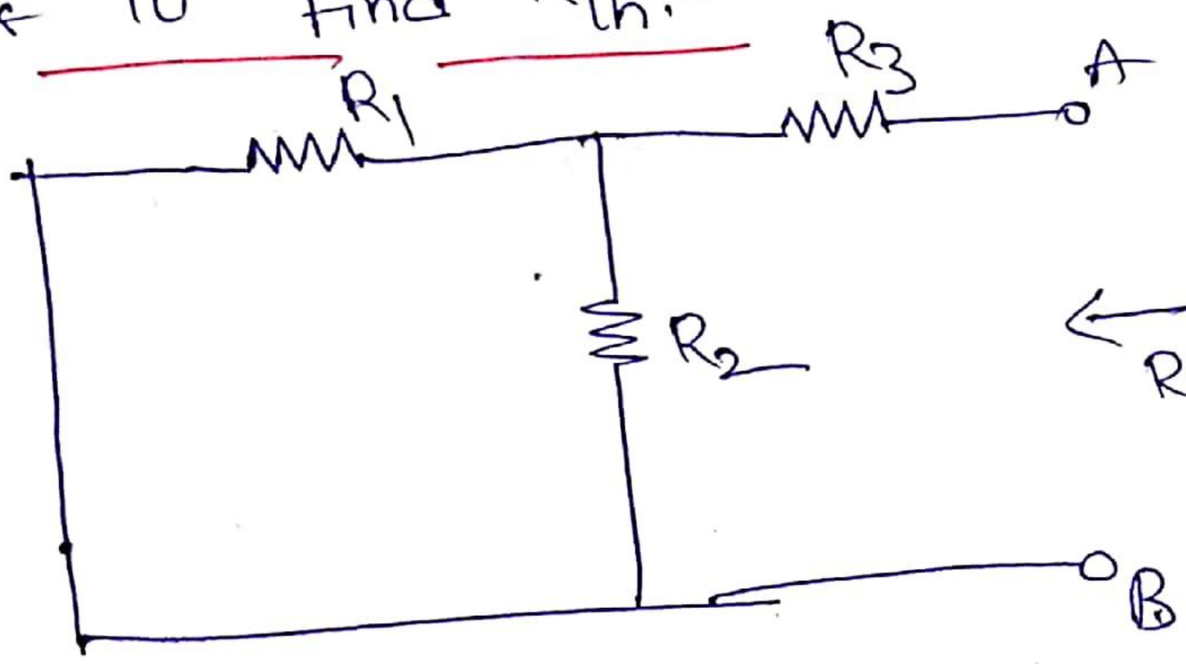
* To find V_{th}:-



Using voltage divider Rule

$$V_{th} = \left(\frac{R_2}{R_1 + R_2}\right) V_S$$

* To find R_{th}:-



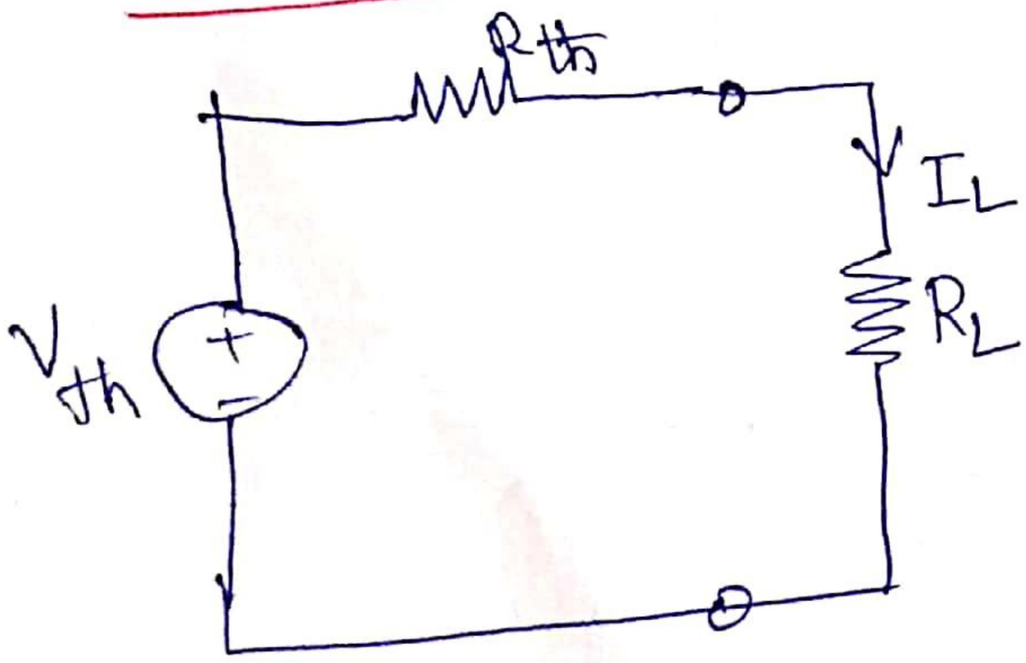
$$R_{th} = (R_1 \parallel R_2) + R_3$$

$$\downarrow$$

$$I_{sc} = \frac{V_{th}}{R_{th}}$$

[We can also find I_{sc} by short circuiting AB terminals $\rightarrow R_{th} = \frac{V_{th}}{I_{sc}}$]

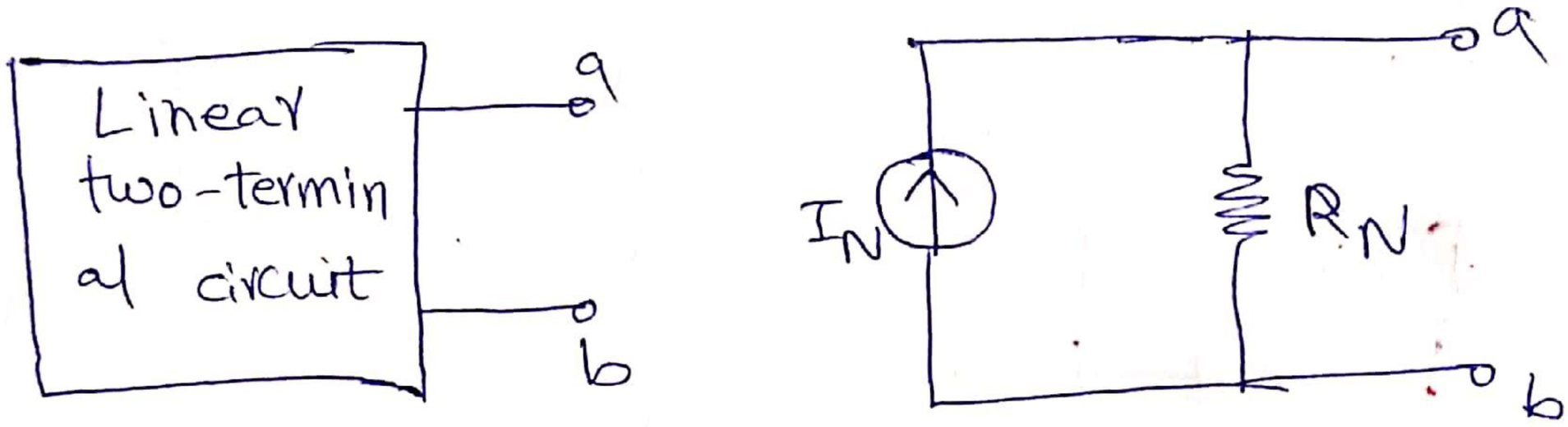
* To find I_L:-



$$\therefore I_L = \frac{V_{th}}{R_{th} + R_L}$$

* Norton's Theorem:-

* **Statement:-** A linear two terminal circuit can be replaced by an Equivalent circuit consisting of a current source 'I_N' in parallel with resistor 'R_N', where I_N is the short circuit current through the terminals and R_N is the input or Equivalent resistance at the terminals when the independent sources are turned off.



(a) original circuit (b) Norton Equivalent ckt

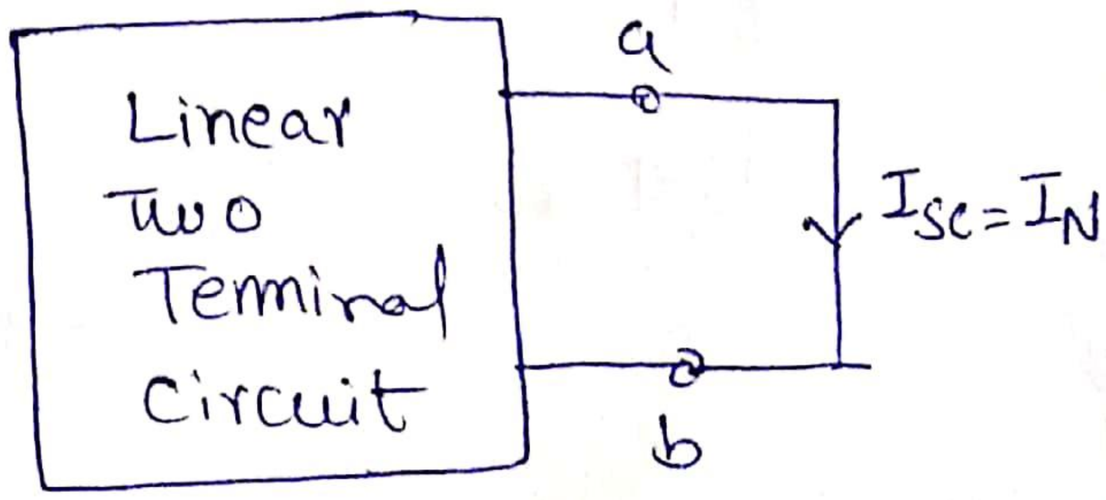
$$\boxed{R_N = R_{Th}}$$

* To find R_N, it is the same way how we find R_{th} in Thevenin's theorem

* To find the Norton Current I_N, we determine the short circuit current flowing from terminal a to b.

$$\therefore I_N = I_{sc}.$$

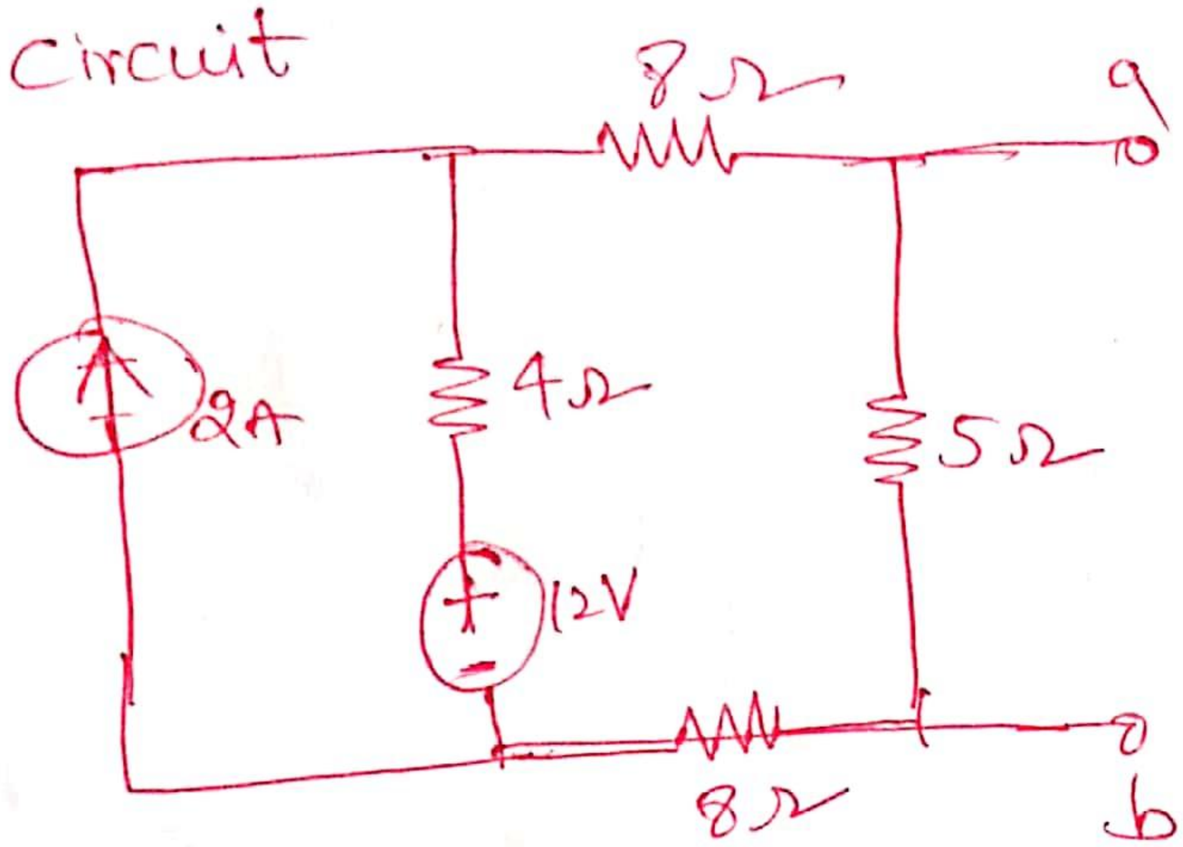
$$\boxed{\therefore I_N = \frac{V_{Th}}{R_{Th}}}$$



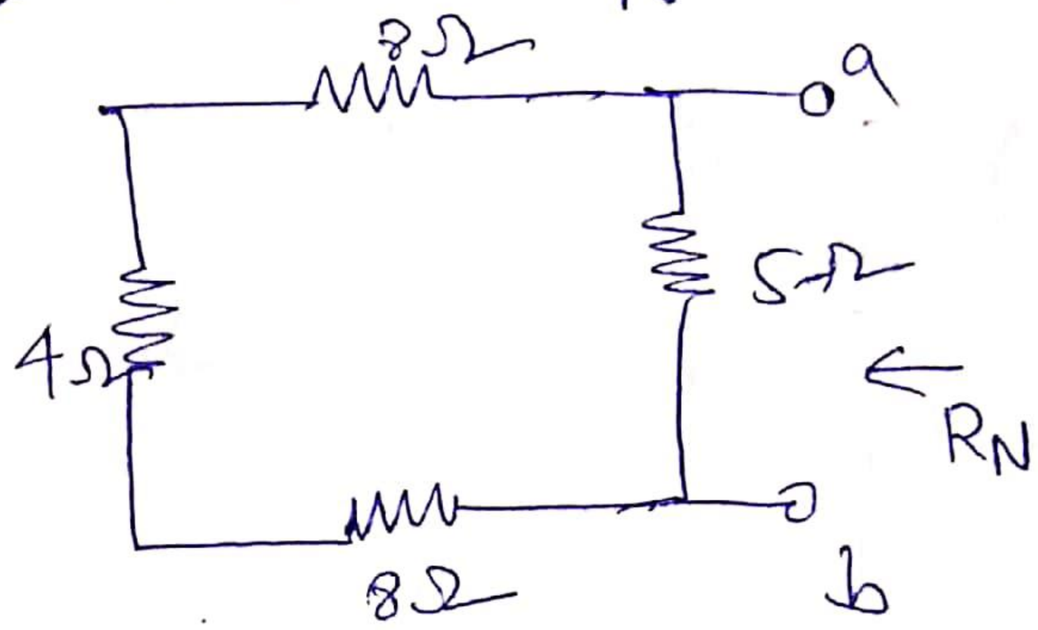
* Thevenin & Norton Equivalent circuits are related by a "source transformation". Source transformation is often called "Thevenin - Norton transformation".

$$\boxed{V_{Th} = V_{oc}, \quad I_N = I_{sc} \quad \& \quad R_{th} = \frac{V_{oc}}{I_{sc}} = R_N}$$

* Find the Norton Equivalent circuit of the following circuit



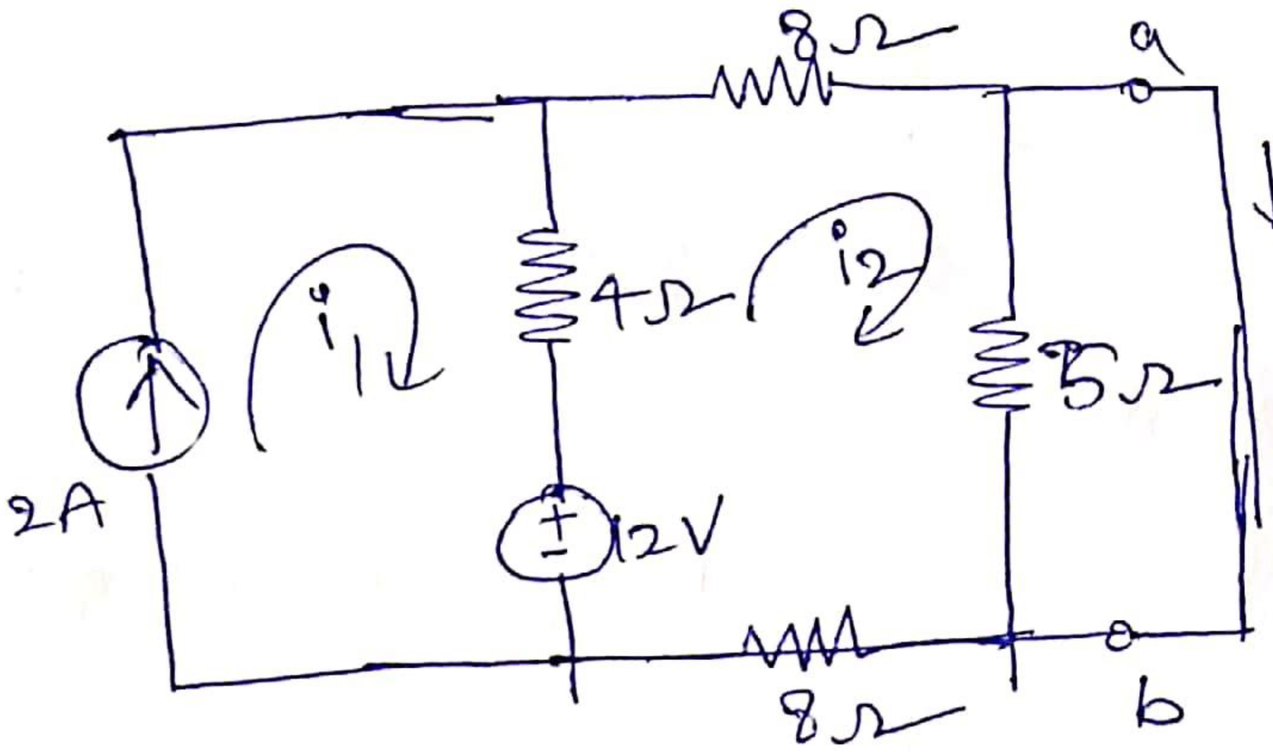
Sol: * To find R_N



$R_N = R_{ab} = (4+8+8) \| 5$

$$\boxed{R_N = 4\Omega}$$

* To find I_N,



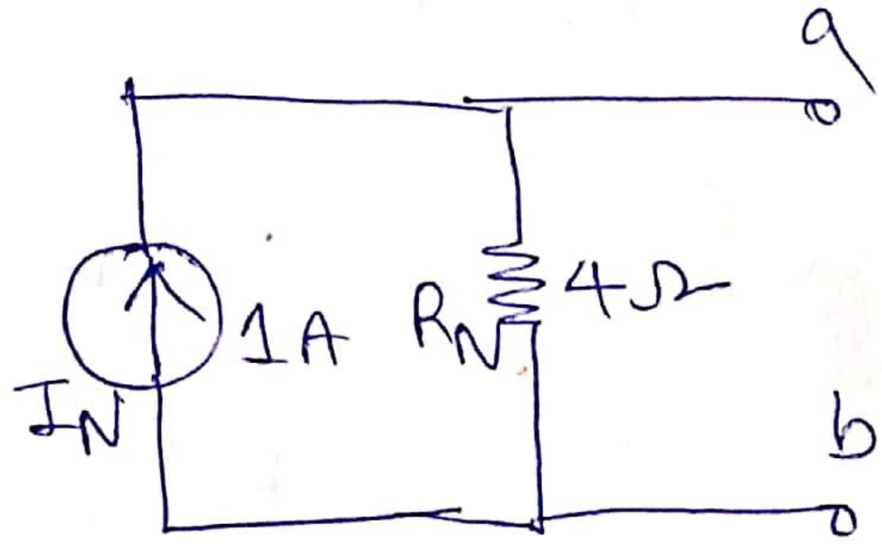
$I_{sc} = I_N$ [5Ω resistor is no use because of short circuited]

→ From fig. $i_1 = 2A$ (loop 1)

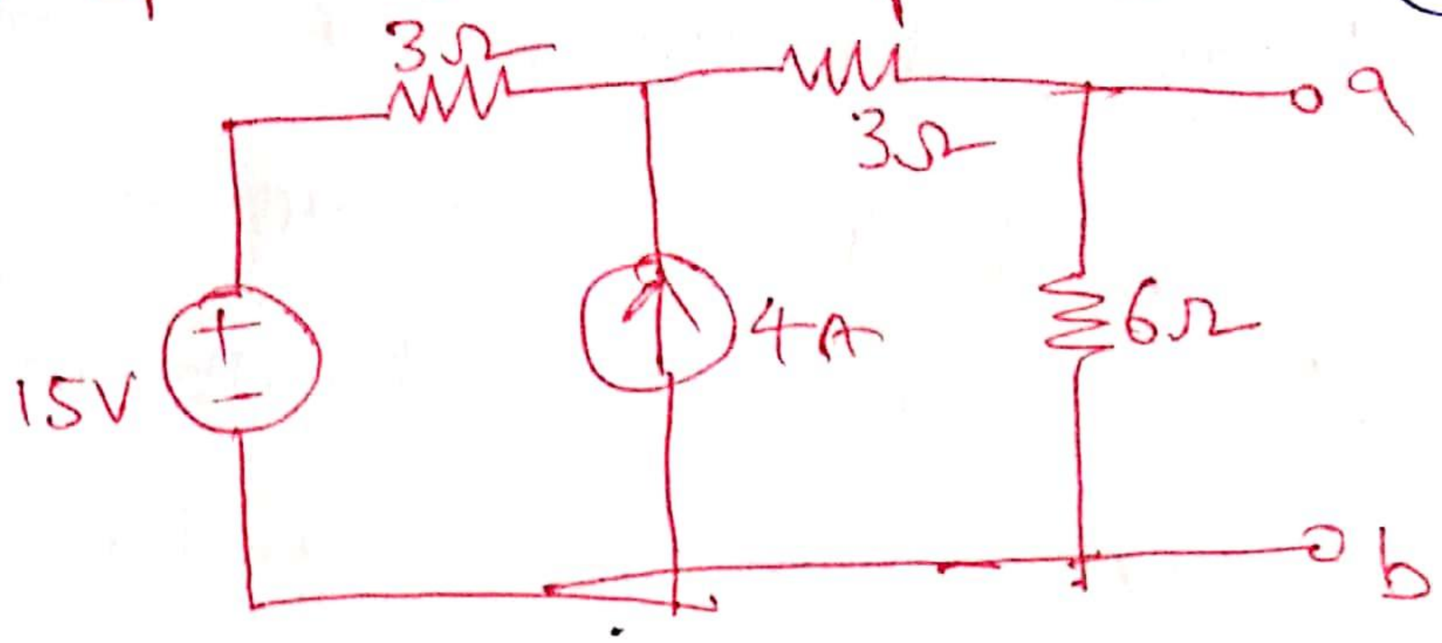
→ loop 2:- $20 i_2 - 4 i_1 - 12 = 0$

$\therefore i_2 = 1A = I_{sc} = I_N$

$$\boxed{I_N = 1A}$$



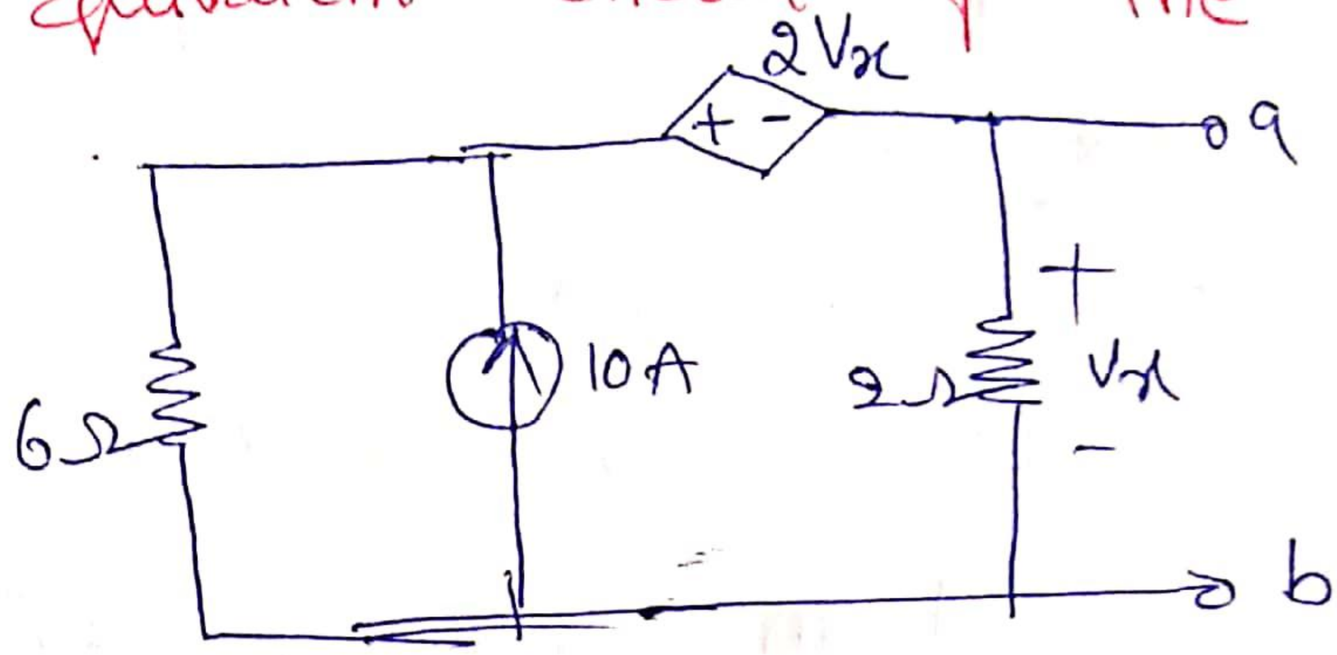
* Find the Norton Equivalent circuit of the following circuit.



Ans:- $R_N = 3\Omega$,

$I_N = 4.5A$

* Find the Norton Equivalent circuit of the following circuit.



Ans.- $R_N = 1\Omega$

$I_N = 10A$

* Maximum power Transfer Theorem:-

→ The Thevenin Equivalent circuit is useful in finding the max. power a linear circuit can deliver to a load.

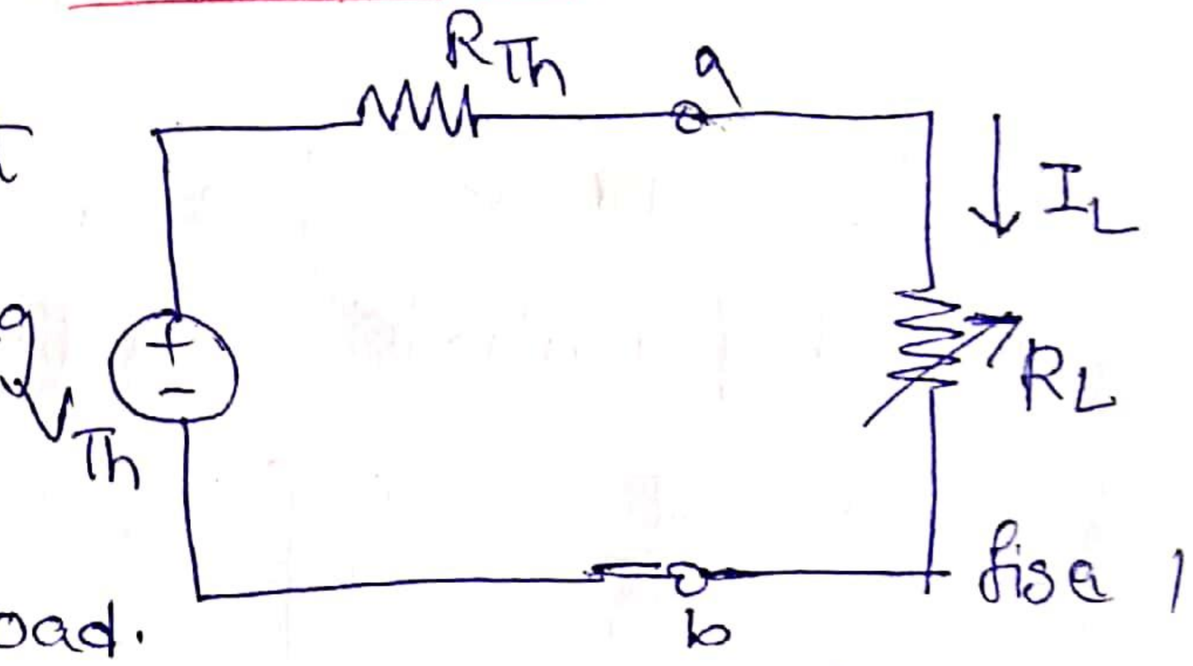
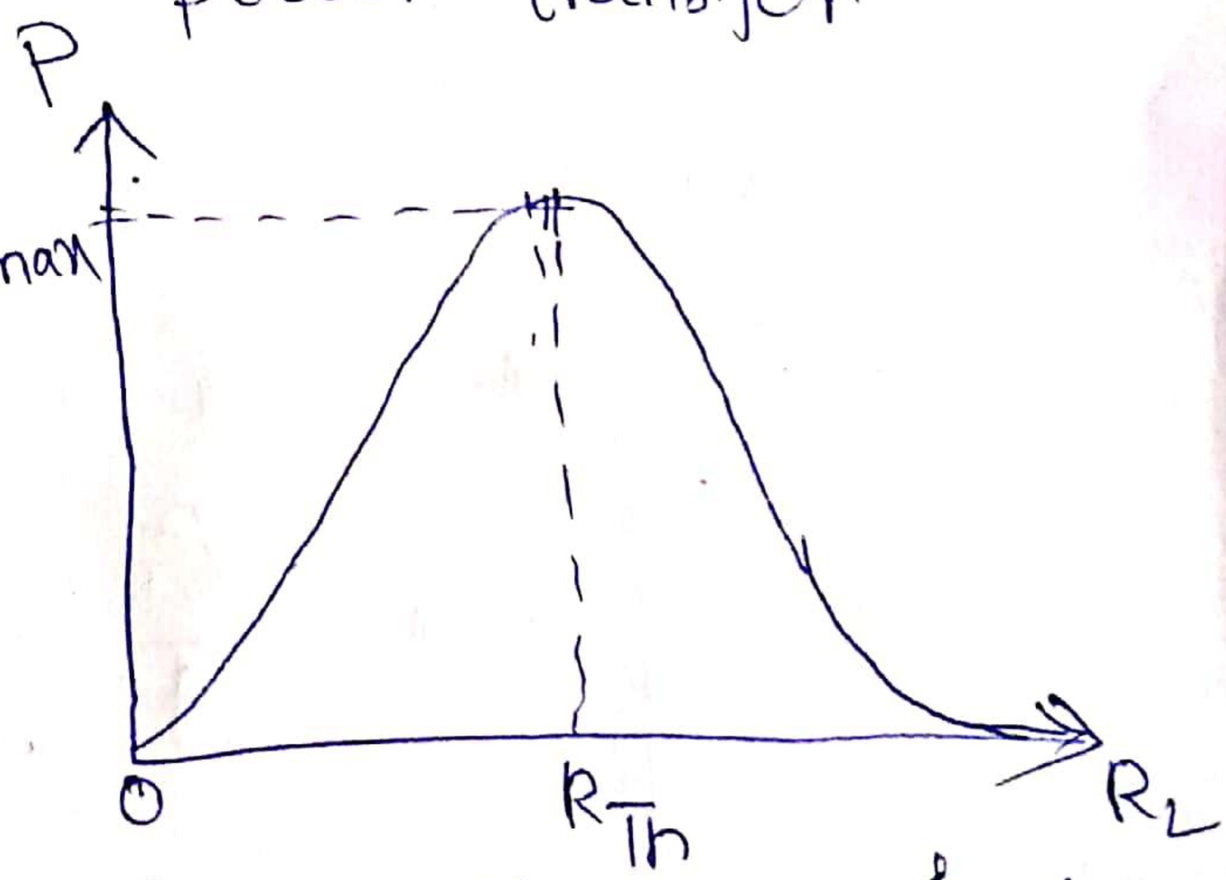


fig a

* Circuit used for Max. power transfer.

→ For a given ckt, V_{th} & R_{th} are fixed. By varying the R_L (load resistance) the power delivered to the load is plotted, is shown in fig. (b)



* Power Vs R_L

fig (b)

From fig. (b) the power is small for small or larger values of R_L but maximum for some value of R_L b/w 0 & ∞. we want to show this max. power occurs when R_L is Equal to R_{th}. This is known as Maximum power transfer theorem.

** Maximum power is transfered to the load when the load resistance equals the Thevenin's resistance as seen from the load ($R_L = R_{Th}$).

From fig. (a) $P = I_L^2 R_L$

$$P = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L \quad \left[\because I_L = \frac{V_{Th}}{R_{Th}+R_L}\right] \longrightarrow (1)$$

→ To prove the maximum power transfer theorem we differentiate Equ. (1) w.r.t R_L and set to zero

$$\therefore \frac{dP}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = 0$$

$$\Rightarrow V_{Th}^2 \left[\frac{(R_{Th}+R_L-2R_L)}{(R_{Th}+R_L)^3}\right] = 0$$

This implies $R_{Th} - R_L = 0 \Rightarrow \boxed{R_L = R_{Th}} \longrightarrow (2)$

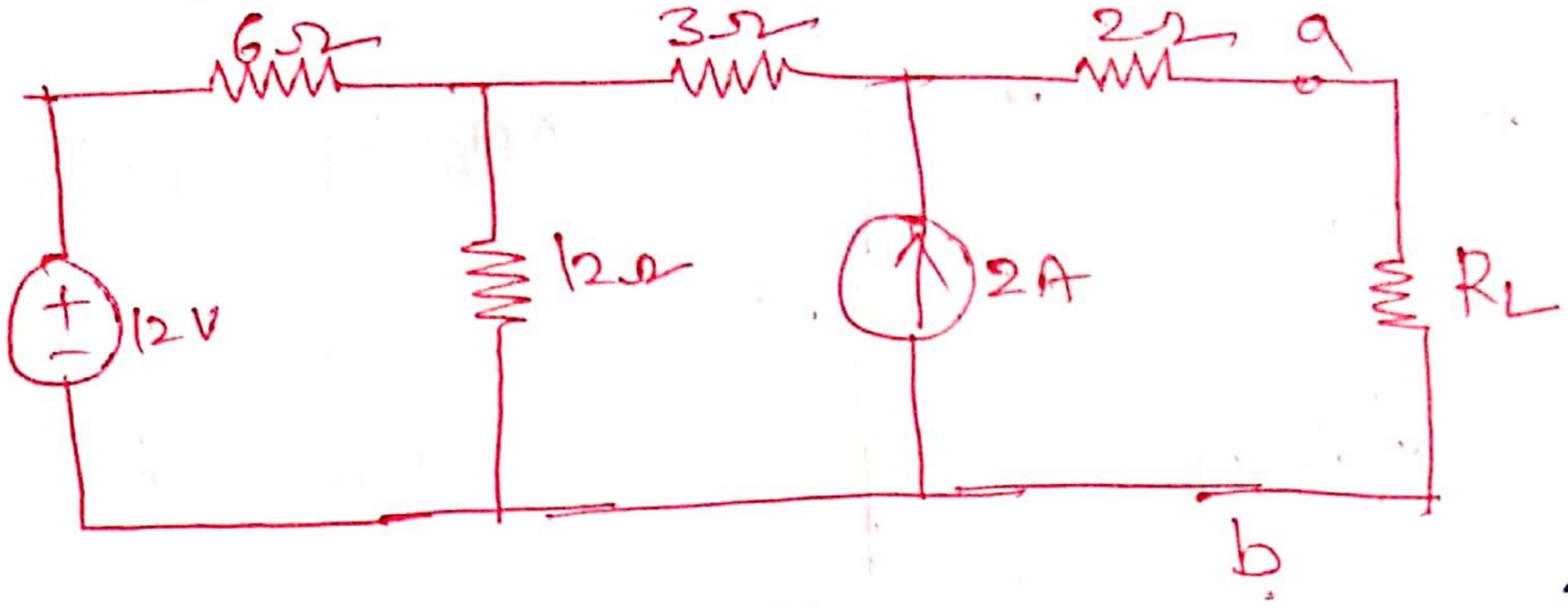
→ The max. power transfered to load by sub (2) in (1)

$$\boxed{P_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

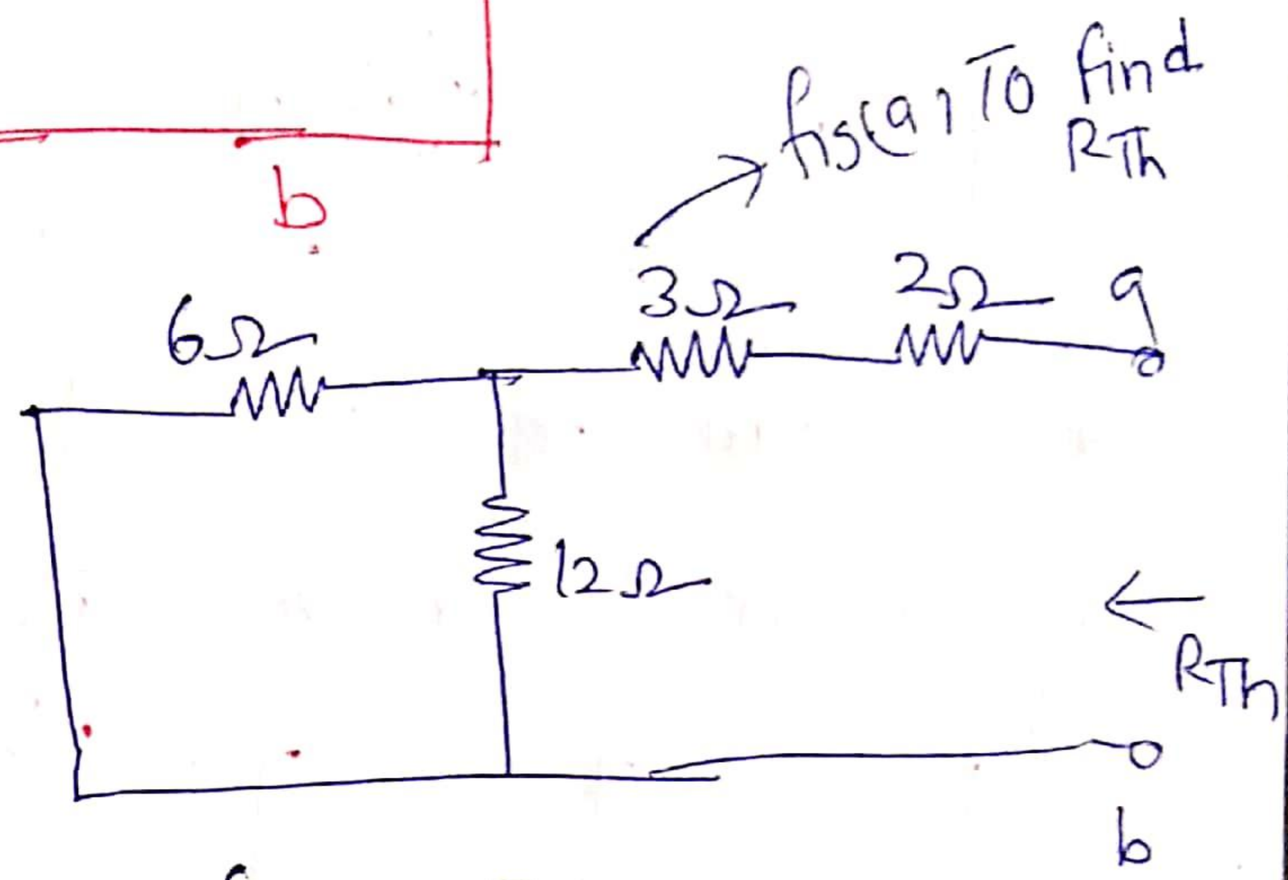
→ Applies only when $R_L = R_{Th}$.

→ For $R_L \neq R_{Th}$ we need to compute separately

* Find the value of R_L for maximum power transfer in the circuit. Find the max. power



Sol:- * To find R_N or R_{Th} :-



$$R_{Th} = 2 + 3 + (6 \| 12)$$

$$\boxed{R_{Th} = 9\Omega}$$

* To find V_{Th}:-

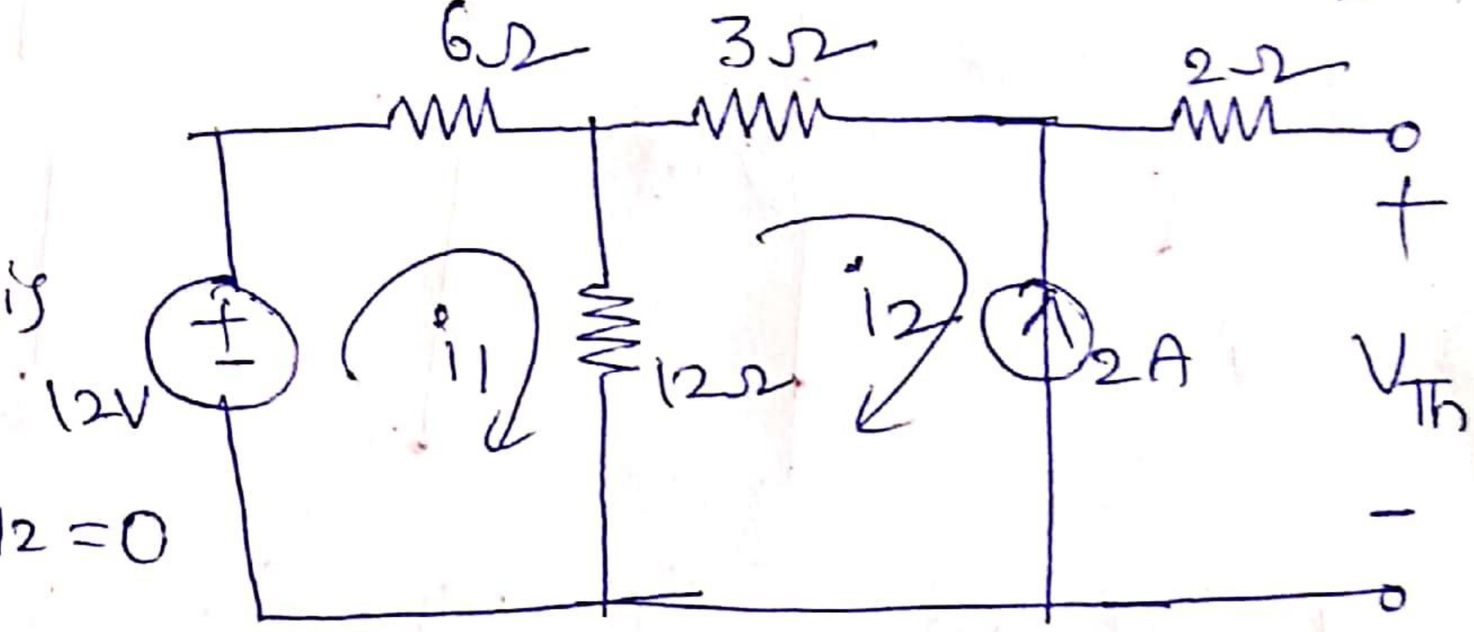


fig.(b) To find V_{Th}

Using Mesh analysis

loop 1:-

$$-12 + 6i_1 + (i_1 - i_2)12 = 0$$

From loop 2; $i_2 = -2A$

$$\therefore i_1 = -2/3 A$$

→ Applying KVL to outer loop we get

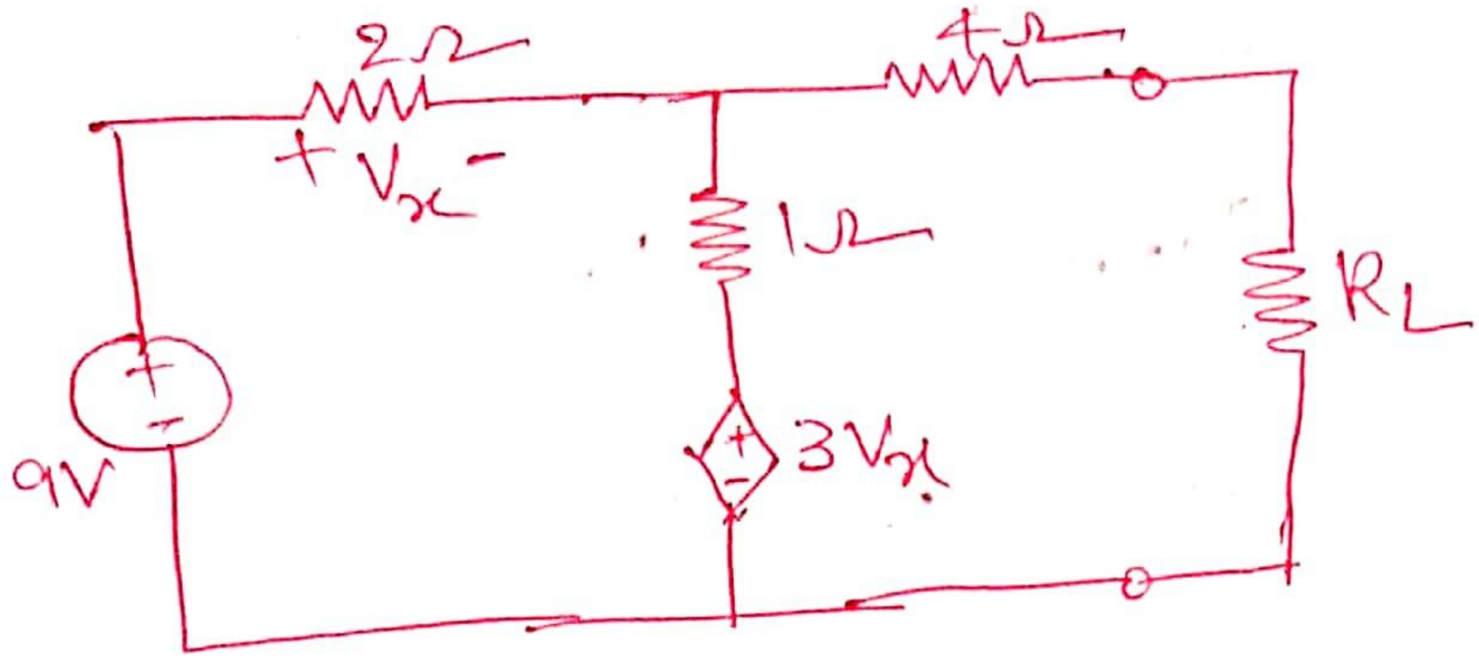
$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$$

$$\boxed{\therefore V_{Th} = 22V}$$

For Max. power Transfer $R_L = R_{Th} = 9\Omega$

Max. power is $P_{max} = \dfrac{V_{Th}^2}{4R_L} = \dfrac{22^2}{2 \times 9} = 13.44W$

* Determine the value of R_L that will draw the maximum power from the ckt. Calculate max. power

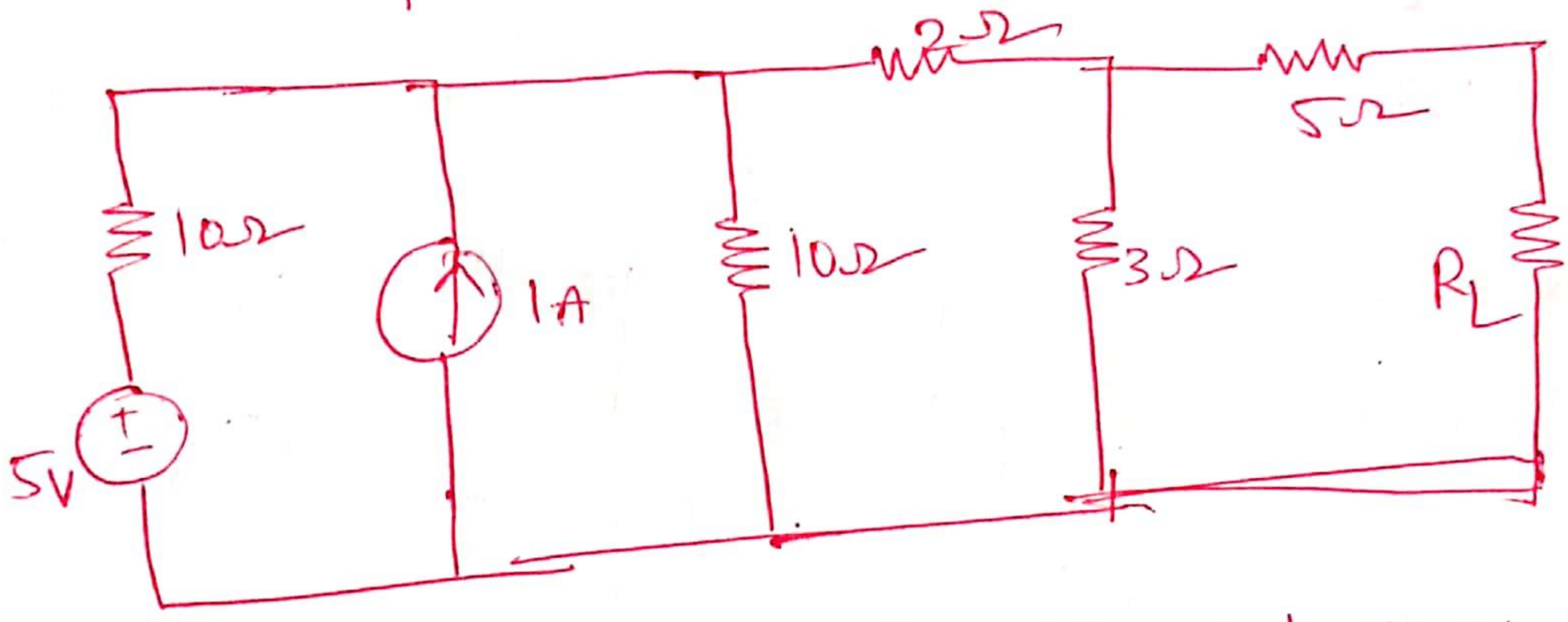


Ans:- $R_L = 4.222\,\Omega$

$P_{max} = 2.901\,W$

$V_{th} = 7V$

* obtain the value of R_L that will draw the maximum power from the ckt. Calculate max. power



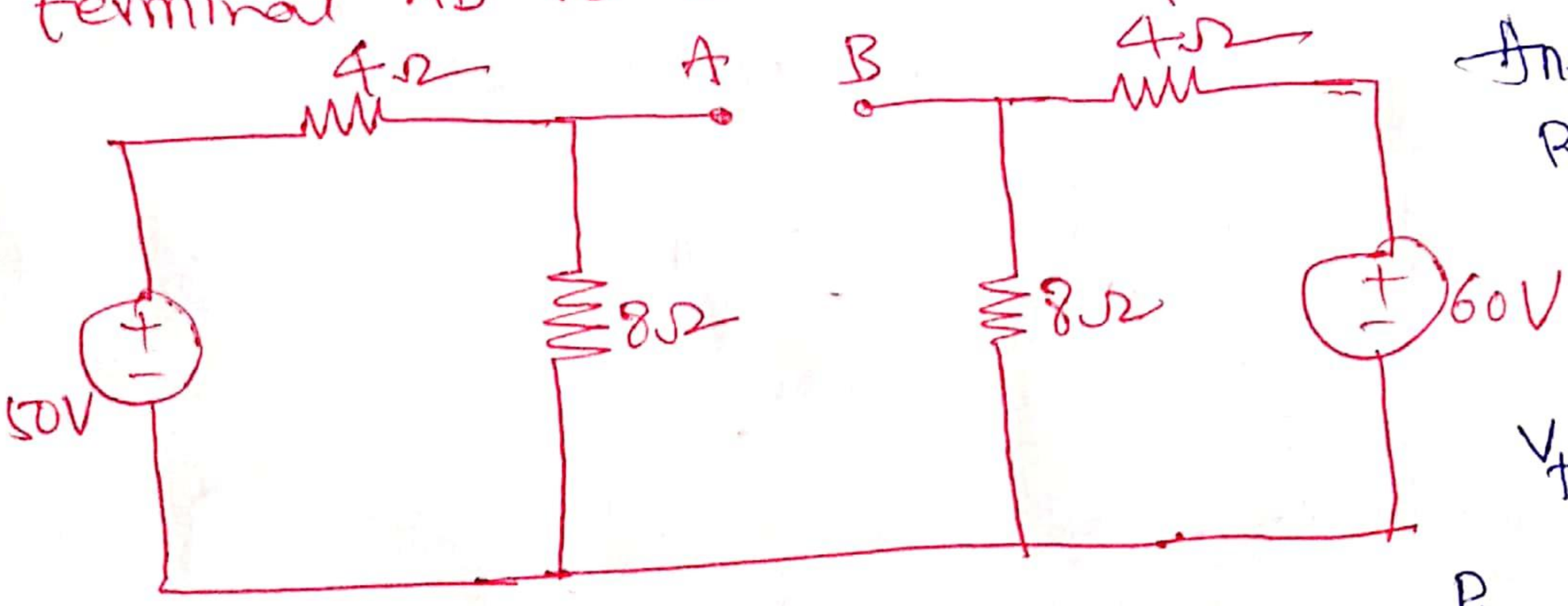
Ans:-

$R_{th} = 7.1\,\Omega$

$V_{th} = 2.25V$

$P_{max} = 0.178\,W$

* For the following Circuit what value of load resistance should be connected across the terminal AB to receive max. power



Ans:-

$R_{th} = 5.33\,\Omega$

$\Rightarrow R_L$

$V_{th} = V_A - V_B$

$V_{th} = -6.67V$

$P_{max} = 2.08W$