System of linear Equations (Non homogeneous $AX=B$) (5)
homogeneous $AX=0$

Pb) Solve
$x+y+z=4, \; 2x+5y-2z=3, \; x+7y-7z=5$

Matrix form $AX=B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & -2 \\ 1 & 7 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 & 1 & 4 \\ 2 & 5 & -2 & 3 \\ 1 & 7 & -7 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 6 & -8 & 1 \end{bmatrix}$$

R_2	:	2	5	-2	3
R_1	:	1	1	1	4
$2R_1$	:	2	2	2	8
$R_2 \to R_2 - 2R_1$	:	0	3	-4	-5

R_3	:	1	7	-7	5
R_1	:	1	1	1	4
$R_3 \to R_3 - R_1$	:	0	6	-8	1

$R_3 \to R_3 - 2R_2$

R_3	:	0	6	-8	1
R_2	:	0	3	-4	-5
$2R_2$	:	0	6	-8	-10
$R_3 \to R_3 - 2R_2$	:	0	0	0	11

$$\sim \begin{array}{c} \begin{array}{cccc} A & & & B \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 0 & 0 & 11 \end{array}\right] \end{array}$$

$\rho(A) = 2, \quad \rho(AB) = 3$

$\therefore \rho(A) \neq \rho(AB)$

No solution.

2) Solve $x+y+z=9$, $2x+5y+7z=52$, ~~$2x+5y+7z=52$~~ $2x+y-z=0$

$AX=B$

$$\begin{bmatrix}1&1&1\\2&5&7\\2&1&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}9\\52\\0\end{bmatrix} \rightarrow (1)$$

$$[AB] = \begin{bmatrix}1&1&1&9\\2&5&7&52\\2&1&-1&0\end{bmatrix}$$

	R_2	:	2	5	7	52
	R_1	:	1	1	1	9
$-$	$2R_1$	:	2	2	2	18
	$R_2 \to R_2-2R_1$	:	0	3	5	34

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 2R_1$

	R_3	:	2	1	-1	0
	R_1	:	1	1	1	9
$-$	$2R_1$	:	2	2	2	18
	$R_3 \to R_3-2R_1$	:	0	-1	-3	-18

$$\sim \begin{bmatrix}1&1&1&9\\0&3&5&34\\0&-1&-3&-18\end{bmatrix}$$

$R_3 \to 3R_3 + R_2$

	R_3	:	0	-1	-3	-18
	$3R_3$	:	0	-3	-9	-54
$+$	R_2	:	0	3	5	34
	$R_3 \to 3R_3+R_2$	:	0	0	-4	-20

$$\sim \begin{array}{c} \begin{array}{cc} A & \quad\quad B \end{array} \\ \begin{bmatrix}1&1&1&9\\0&3&5&34\\0&0&-4&-20\end{bmatrix} \end{array}$$

$\rho(A)=3$, $\rho(AB)=3$, $n=3$ = number of variables (or) unknowns.

$\therefore \rho(A)=\rho(AB)=n$

system of linear eqns has unique solutions

from (1) $\Rightarrow$ $AX = B$

$$\underset{3\times3}{\begin{bmatrix}1&1&1\\0&3&5\\0&0&-4\end{bmatrix}}\underset{3\times1}{\begin{bmatrix}x\\y\\z\end{bmatrix}}=\begin{bmatrix}9\\34\\-20\end{bmatrix}$$

$$\begin{bmatrix}x+y+z\\0x+3y+5z\\0x+0y-4z\end{bmatrix}=\begin{bmatrix}9\\34\\-20\end{bmatrix}$$

$x+y+z=9$
$3y+5z=34$
$-4z=-20$
$z=+5$
$3y+5(+5)=34$
$3y+25=34$
$3y=34-25=9$
$y=3$

$x+y+z=9$
$x+3+5=9$
$x+8=9$
$x=1$

$$X=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\3\\5\end{bmatrix}$$

varification

$x+y+z=1+3+5=9$
$2x+5y+7z=2+15+35=52$
$2x+y-z=2+2-5=0$

Pb) Show that the eqns $x+y+z=6$, $x+2y+3z=14$, $x+4y+7z=30$ are consistent and solve them.

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} \rightarrow (1)$$

$$AB = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & \cancel{16}\ 24 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 3R_2$

$$\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & \cancel{8} \end{array}\right] \end{array}$$

$\rho(A) = 2$, $\rho(AB) = 2$

here $n = 3$

$\rho(A) = \rho(AB) < 2$

System of eqns have infinite solutions

From (1)

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1x + 1y + 1z \\ 0x + 1y + 2z \\ 0x + 0y + 0z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$$

$x + y + z = 6$

$y + 2z = 8$

Let $z = K$

Take $y + 2(K) = 8$

$y = 8 - 2K$

Take $x + y + z = 6$

$x + 8 - 2K + K = 6$

$x + 8 - K = 6$

$x = 6 - 8 + K = K - 2$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} K-2 \\ 8-2K \\ K \end{bmatrix}$$

R_3	:	0	3	6	24
$3R_2$	:	0	3	6	24
$\ominus$					
$R_3 \rightarrow R_3 - 3R_2$	:	0	0	0	0

pb) $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$. Discuss for what values of λ, μ. The eqns are have i) no solution ii) unique iii) infinite solution.

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$.

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{array}{c} \quad A \qquad\qquad\quad B \\ \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right] \end{array}$$

This is Echelon form

1) if $\lambda = 3$, $\mu \neq 10$ then $\overset{2}{\rho(A)} \neq \overset{3}{\rho(AB)}$.
no solution.

2) if $\lambda \neq 3$, μ = any value then $\overset{3}{\rho(A)} = \overset{3}{\rho(AB)} = \overset{3}{n}$.
The system of eqns has unique solution.

3) if $\lambda = 3$, $\mu = 10$ then $\overset{2}{\rho(A)} = \overset{2}{\rho(AB)} < \overset{3}{n}$.
The system of eqns has infinite solutions.