```
clear;clc
```

Problem 1

Load open loop LTI state space

```
load ol_sys.mat
size(ol_sys)
```

```
State-space model with 3 outputs, 3 inputs, and 6 states.
```

Q should be N by N

```
q = 1;
Q = q * transpose(C) * C;
```

R should be M by M

```
R = 2E-7*diag([0.9 1.2 1.3]);
```

LQR

```
G = lqr(ol_sys,Q,R);
```

Initial Condition is

```
x0 = [1;0;0.5;0;-1;0];
```

Construct full state feedback close loop system

```
cl_sys = ss(A-B*G,[],C - D * G,[]);
```

Problem 2

Simulate for 10 seconds

```
[y_cl,t_cl,x_cl] = initial(cl_sys,x0,10); %for 10s
S_lqr = lsiminfo(y_cl,t_cl,'SettlingTimeThreshold',0.05); % 5% seltting times
```

Compute maximum sellting time.

```
for c = 1:3
    SettlingTimeArray_lqr(c) = S_lqr(c).SettlingTime;
end


disp(['LQR SettlingTimes = ', num2str(SettlingTimeArray_lqr),' s'])
```

```
LQR SettlingTimes = 7.604      6.7253      6.6969 s
```

Compute maximum control torque.

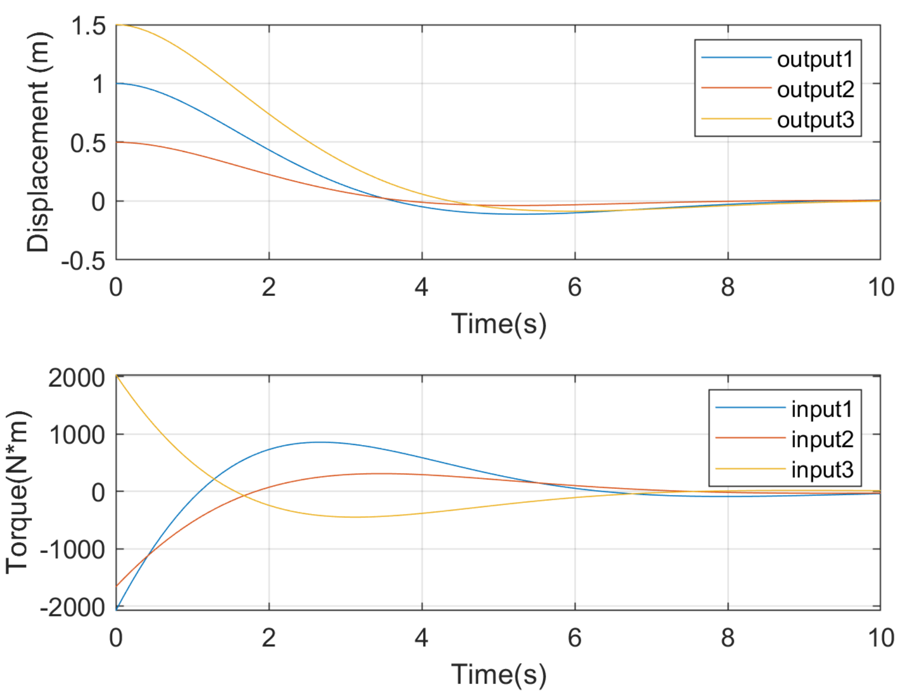
```
[m,n] = size(x_cl);
for c = 1 : m
u(:,c) = - G * x_cl(c,:)';
end
%compute max input - control torque
umax_lqr = max(abs(u(:)));
disp(['umax =', num2str(umax_lqr),' N*m'])
```

```
umax =2077.8856 N*m
```

```
figure
```

Generate the output and input plots

```
subplot(2,1,1)
plot(t_cl,y_cl)
legend('output1','output2','output3')
grid on
xlabel('Time(s)')
ylabel('Displacement (m)')
set(gca,'fontsize',11) % change font size
subplot(2,1,2)
plot(t_cl,u)
legend('input1','input2','input3')
grid on
hold off
xlabel('Time(s)')
ylabel('Torque(N*m)')
set(gca,'fontsize',11) % change font size
```



Problem 3

Oberservability

```
Obverbility = rank(obsv(ol_sys))
```

```
Obverbility = 6
```

The entire system is observable.

Controllability for each single **output**, and store the rank into an array.

```
for c = 1:3
    sub_sys = ol_sys(c,:);
    obsv_sub_sys(c) = rank(obsv(sub_sys));
```

```
end
obsv_sub_sys
```

```
obsv_sub_sys = 1×3
     2     4     6
```

The with system with first or second output is **NOT** observable

```
obsv_sub_sys(4) = rank(obsv(ol_sys([1 2],:)))
```

```
obsv_sub_sys = 1×4
     2     4     6     4
```

Sub-system with combination of output 1 and 2 is **NOT** observable.

```
obsv_sub_sys(5) = rank(obsv(ol_sys([2 3],:)))
```

```
obsv_sub_sys = 1×5
     2     4     6     4     6
```

Sub-system with combination of output 2 and 3 is observable

```
obsv_sub_sys(6) = rank(obsv(ol_sys([1 3],:)))
```

```
obsv_sub_sys = 1×6
     2     4     6     4     6     6
```

Sub-system with combination of output 1 and 3 is observable

Problem 4

Define the oberser poles and place the poles,

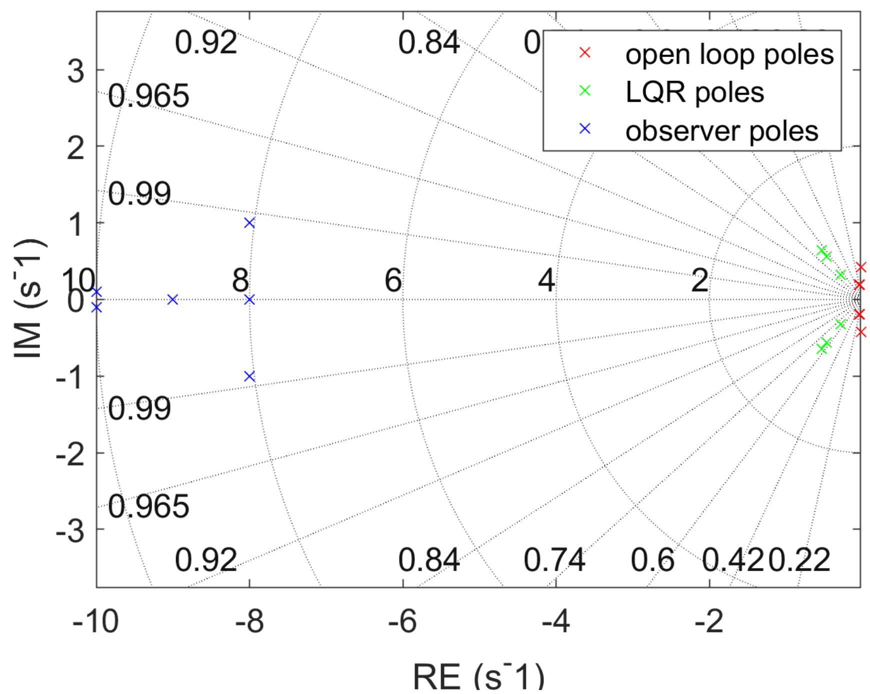
```
des_obs_poles = [-8 ...
                 -9 ...
                 -10+0.1i ...
                 -10-0.1i ...
                 -8+1i ...
                 -8-1i];
K = place(A',C',des_obs_poles)';

figure
plot(pole(ol_sys),'x','color','r')
hold on
plot(pole(cl_sys),'x','Color','g')
plot(des_obs_poles,'x','Color','b')
hold off

xlabel('RE (s^-1)')
ylabel('IM (s^-1)')

set(gca,'fontsize',14) % change font size

axis equal
legend('open loop poles','LQR poles','observer poles','AutoUpdate','off')
sgrid
```



Problem 5

Augmented state space system is

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -\mathrm{BG} \\ \mathrm{KC} & \hat{A} - K\hat{C} - \hat{B}G - K\left(D - \hat{D}\right)G \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

$$\begin{bmatrix} y \\ \hat{y} \\ u \end{bmatrix} = \begin{bmatrix} C & -\mathrm{DG} \\ 0 & \hat{C} - \hat{D}G \\ 0 & -G \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

Output u for plotting

we also assume

$A = \hat{A}$

$B = \hat{B}$

$C = \hat{C}$

$D = \hat{D}$

Construct LTI object with only state observer and no state feedback

Define a gain of zeros

```
G_Zeros = zeros(size(G));

A_aug1 = [A -B*G_Zeros; K*C (A-K*C-B*G_Zeros)];
C_aug1 = [C -D*G_Zeros;zeros(size(C)) C - D*G_Zeros; zeros(3,6) -G_Zeros];
obv_sys1 =ss(A_aug1,[],C_aug1,[]);
```

Augmented Initial condition

```
x0_aug1 = [x0;zeros(6,1)];
[y_obs1,t_obs1,x_obs1] = initial(obv_sys1,x0_aug1,1); %simulate for 1 second
```

Seperate output, estimated output and

```
yo1 = y_obs1(:,1:3);
yo1_hat = y_obs1(:,4:6);
uo1 = y_obs1(:,7:9);
```

New IC for 2nd stage

```
x0_aug2 = x_obs1(end,:);
```

Construct LTI model with state observer and full state feedback

```
A_aug2 = [A -B*G; K*C (A-K*C-B*G)];
C_aug2 = [C -D*G;zeros(size(C)) C - D*G;zeros(3,6) -G];
obv_sys2 =ss(A_aug2,[],C_aug2,[]);
```

simulate for 9 second

```
[y_obs2,t_obs2,x_obs2] = initial(obv_sys2,x0_aug2,9); %simulate for 9 seconds
```

Seperate output, estimated output and

```
yo2 = y_obs2(:,1:3);
yo2_hat = y_obs2(:,4:6);
uo2 = y_obs2(:,7:9);

```

Combine output, estimated output, input and time

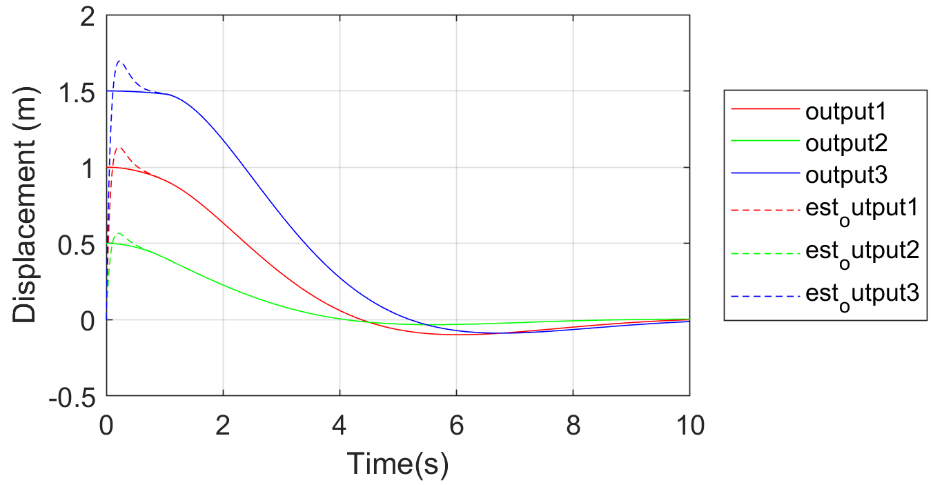
```
yo = [yo1;yo2];
yo_hat = [yo1_hat;yo2_hat];
uo = [uo1;uo2];
to = [t_obs1;t_obs2+1];
```

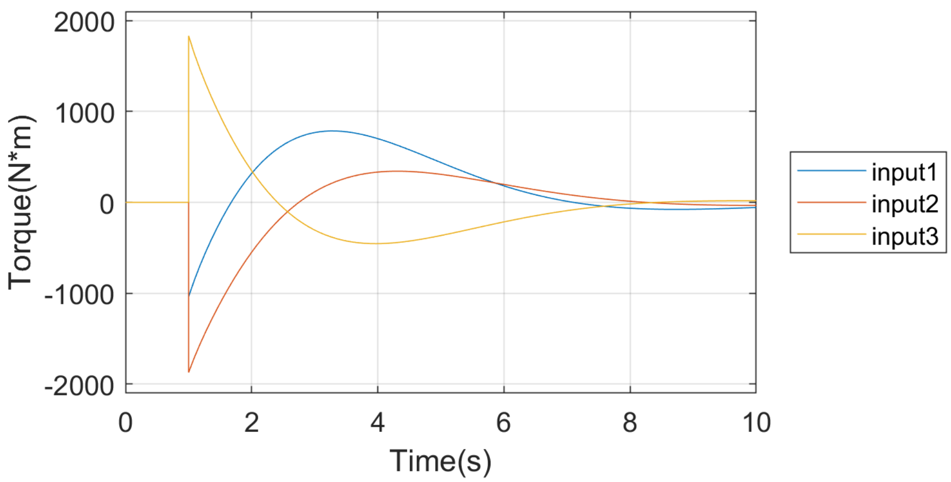
plot

```
NameArray = {'Color'};
ValueArray = {'red','green','blue'}';

subplot(2,1,1);
p1 = plot(to,yo);
set(p1,NameArray,ValueArray)
hold on
p2 = plot(to,yo_hat,'--');
set(p2,NameArray,ValueArray)
hold off
legend('output1','output2','output3','est_output1','est_output2','est_output3','Location','eastoutside')
xlabel('Time(s)')
ylabel('Displacement (m)')
set(gca,'fontsize',12) % change font size
grid on

subplot(2,1,2)
plot(to,uo)
ylim([-2100 2100])
legend('input1','input2','input3','Location','eastoutside')
xlabel('Time(s)')
ylabel('Torque(N*m)')
grid on
set(gca,'fontsize',12) % change font size
set(gcf,'Units','Normalized','OuterPosition',[0 0 0.4 0.75])
```





Compute settling time

```
S_obs = lsiminfo(yo,to,'SettlingTimeThreshold',0.05);
```

Compute maximum sellting time.

```
for c = 1:3
    SettlingTimeArray_obs(c) = S_obs(c).SettlingTime;
end

%find max settling time
% maxSettlingTime_obs = max(SettlingTimeArray_obs);
disp(['maxSettlingTime = ', num2str(SettlingTimeArray_obs),' s'])
```

```
maxSettlingTime = 7.9588      6.9109      6.8998 s
```

Compute maximum input

```
umax_obs = max(abs(uo(:)));
disp(['umax_obs =', num2str(umax_obs),' N*m'])
```

```
umax_obs =1872.2938 N*m
```

Problem 6

Buiilt the augmented Q and R for LQI

```
Qi = [Q zeros(6,3); zeros(3,6), diag([1 1 1])];
% Ri = 2E-7*diag([1 1 1.3]); %Ri is M by M
Ri = 2E-7*diag([0.9 1.2 1.3]);
```

Angumented gain includes the first 6 state feedback gains and 3 integral gains

```
Ga = lqi(ol_sys,Qi,Ri);
```

Separate state feedback gains and integral gains

```
G0 = Ga(:,1:6);
Gi = Ga(:,7:9);
```

Problem 7

```
REF_sys = tf(eye(3)); % Static gain identity matrix
SUM_sys1 = tf(eye(3)); % Static gain identity matrix
SUM_sys2 = tf(eye(3)); % Static gain identity matrix
G0_sys = tf(-G0); % Static gain G0 matrix (negated)
GI_sys = tf(-Gi); % Static gain GI matrix (negated)
```

Converting the 3x3 identity matrix into a cell array to serve as the matrix of numerator coefficients.

[1 0] represents the polynomial 1*s+0 for all denominators:

```
INTE_sys = tf(num2cell(eye(3)),[1 0]);
```

Define the observer system to have the input and output we need

$$\left[\dot{\hat{x}}\right] = [A - \mathrm{KC}]\hat{x} + [B \quad K]\begin{bmatrix} u \\ y \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 1 \\ C \end{bmatrix}\hat{x} + \begin{bmatrix} 0 \\ D \end{bmatrix}u$$

```
A_aug3 = [A - K*C];
B_aug3 = [B K];
C_aug3 = [eye(6);C];

OBS_sys =ss(A_aug3,B_aug3,C_aug3,[]);

set(OBS_sys,'StateName',{'x1_hat','x2_hat','x3_hat','x4_hat','x5_hat','x6_hat'});
set(OBS_sys,'InputName',{'u1','u2','u3','y1','y2','y3'});
set(OBS_sys,'OutputName',{'x1_hat','x2_hat','x3_hat','x4_hat','x5_hat','x6_hat',...
    'y1_hat','y2_hat','y3_hat'});
```

use append to join dynamic system models of the elements of your block diagram

```
blksys = append(ol_sys,OBS_sys,REF_sys,...
                SUM_sys1,INTE_sys,G0_sys,GI_sys,SUM_sys2);
connections = [1 28 0; %OL_sys
               2 29 0;
               3 30 0;
               ...
               4 28 0; %OBS_sys
               5 29 0;
               6 30 0;
               7 1 0;
               8 2 0;
               9 3 0;
```

```
            ...
            13 13 -1; %SUM1_sys
            14 14 -2;
            15 15 -3;
            ...
            16 16 0; %INTE_sys
            17 17 0;
            18 18 0;
            ...
            19 4 0; %G0_sys
            20 5 0;
            21 6 0;
            22 7 0;
            23 8 0;
            24 9 0;
            ...
            25 19 0; %GI_sys
            26 20 0;
            27 21 0;
            ...
            28 22 25; %SUM2_sys
            29 23 26;
            30 24 27];
```

Input is the input of REF_sys

```
inputs = [10 11 12];
```

output is the real output, estimated output and the input(for the ease of plotting)

```
outputs = [1 2 3 10 11 12 28 29 30]; %[y y_est u]
Connected_Sys = connect(blksys,connections,inputs,outputs);
```

Problem 8

Construct augmented initial condition

0s for initial observer states and initial integrator states

```
x0_aug3 = [x0;zeros(9,1)];
```

Construct a time vector

```
Tfinal = 20; %final sim time (s)
Ts = 0.1; %data sample period (s)
Time = Ts * [0:round(Tfinal/Ts)]; %time vector (s)
```

Define referene input

```
SA_ref = 0.5;
dAB_ref = 0;
dBC_ref = 1;
```

Reference matrix has 3 columns and one row for each time

```
for c = 1:length(Time)
reference(c,:) = [SA_ref dAB_ref dBC_ref];
end
```

Simulate

```
[y8,t8,x8] = lsim(Connected_Sys,reference,Time,x0_aug3);
```

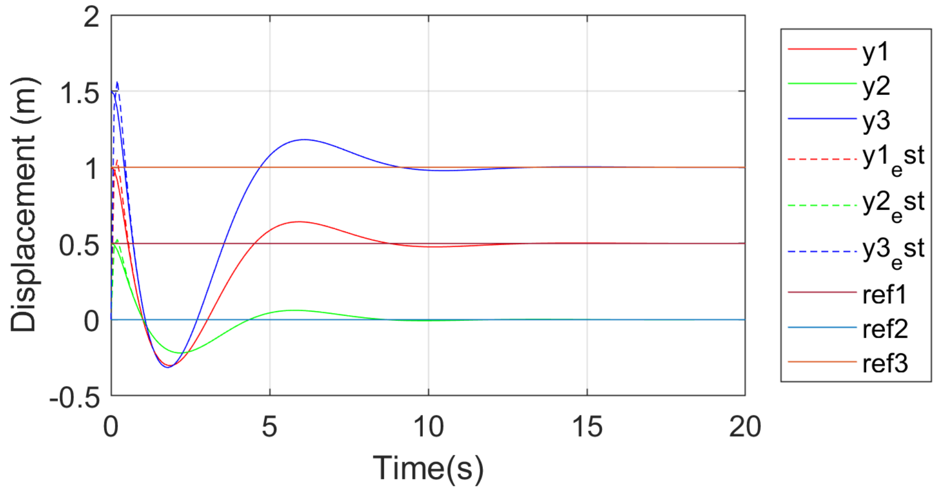
extract the real output, estimated output and the input

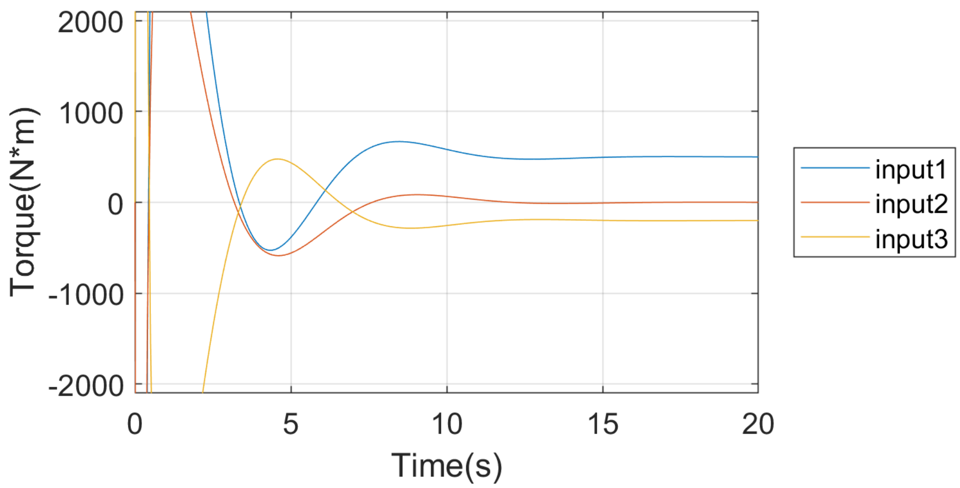
```
y8_real = y8(:,1:3);
y8_est = y8(:,4:6);
u8 = y8(:,7:9);
```

plot

```
figure
subplot(2,1,1)
p7 = plot(t8,y8_real);
set(p7,NameArray,ValueArray)
hold on
p8 = plot(t8,y8_est,'--');
set(p8,NameArray,ValueArray)
p9 = plot(t8,reference);
xlabel('Time(s)')
ylabel('Displacement (m)')
grid on
legend('y1','y2','y3','y1_est','y2_est','y3_est','ref1','ref2','ref3','Location','eastoutside')
hold off
set(gcf,'Units','Normalized','OuterPosition',[0 0 0.4 0.75])
set(gca,'fontsize',13) % change font size

subplot(2,1,2)
plot(t8,u8)
xlabel('Time(s)')
ylabel('Torque(N*m)')
grid on
set(gca,'fontsize',13) % change font size
ylim([-2100 2100])
legend('input1','input2','input3','Location','eastoutside')
```





Compute maximum sellting time.

```
S8 = lsiminfo(y8_real,t8,'SettlingTimeThreshold',0.05);
for c = 1:3
    SettlingTimeArray8(c) = S8(c).SettlingTime;
end

%find max settling time
maxSettlingTime = max(SettlingTimeArray8) ;
disp(['maxSettlingTime8 = ', num2str(maxSettlingTime),' s'])
```

```
maxSettlingTime8 = 7.9806 s
```