QUANTUM STATES FOR TWO QUBIT SYSTEMS

Question 1

We define a state $|\psi\rangle = \alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|10\rangle + \delta\,|11\rangle$ to be a valid quantum state if $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Which of the following equations describe a valid quantum state?

(a) Example: $\frac{1}{2}\,|00\rangle + \frac{1}{2}\,|01\rangle + \frac{1}{2}\,|10\rangle + \frac{1}{2}\,|11\rangle$

$$\alpha = \frac{1}{2}, \beta = \frac{1}{2}, \gamma = \frac{1}{2}, \delta = \frac{1}{2}$$

$$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\frac{1}{2})^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2 = 1$$

Since $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$, $|\psi\rangle$ is a valid quantum state.

(b) $|\psi\rangle = \frac{\sqrt{7}}{5}\,|00\rangle + \frac{\sqrt{7}}{\sqrt{5}}\,|01\rangle + \frac{3}{5}\,|10\rangle + \frac{\sqrt{2}}{5}\,|11\rangle$

(c) $|\psi\rangle = |00\rangle$

(d) $|\psi\rangle = |00\rangle + |01\rangle + |10\rangle + |11\rangle$

(e) $|\psi\rangle = \frac{3}{5}|00\rangle + \frac{4}{5}|01\rangle$

MEASUREMENT

Question 2

For the following, quantum states, what would the qubits most likely collapse to?

(a) Example: $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$

This state is in an equal superposition of the four states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.$Therefore, we can equally expect the states to be:$
$FirstQubit: 0, SecondQubit: 0$
$FirstQubit: 0, SecondQubit: 1$
$FirstQubit: 1, SecondQubit: 0$
$FirstQubit: 1, SecondQubit: 1$

(b) $|\psi\rangle = |00\rangle$

(c) $|\psi\rangle = \frac{1}{\sqrt{2}}|0> + \frac{1}{\sqrt{2}}|1>$

(d) $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

(e) For $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$*, if we measure the first qubit to result in 0, what can we say about the second qubit?*

(f) For $|\psi\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$*, if we measure the first qubit to result in 0, what can we say about the second qubit?*

(g) For $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$*, if we measure the first qubit to result in 1, what can we say about the second qubit?*

(h) For $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$*, if we measure the first qubit to result in 0, what can we say about the second qubit?*

ENTANGLEMENT: BELL STATES

Question 3

Prepare bell states starting with the states $|00\rangle, |01\rangle, |10\rangle, |11\rangle . What is the resulting state?$