

QUANTUM STATES FOR SINGLE QUBIT SYSTEMS

Question 1

We define a state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to be a quantum state if $|\alpha|^2 + |\beta|^2 = 1$. Which of the following equations describe a quantum state?

(a) Example: $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$$\alpha = \frac{1}{\sqrt{2}}, \beta = \frac{1}{\sqrt{2}}$$

$$\alpha^2 + \beta^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

Since $\alpha^2 + \beta^2 = 1$, $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ is a valid quantum state.

(b) $|\psi\rangle = \frac{1}{4}|0\rangle + \frac{3}{4}|1\rangle$

(c) $|\psi\rangle = |0\rangle + |1\rangle$

(d) $|\psi\rangle = \frac{5}{13}|0\rangle + \frac{12}{13}|1\rangle$

(e) $|\psi\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$

(f) $|\psi\rangle = |1\rangle$

(g) $|\psi\rangle = \frac{1}{4}|0\rangle$

(h) $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$

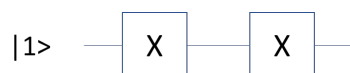
(i) $|\psi\rangle = \frac{\sqrt{7}}{4}|0\rangle + \frac{\sqrt{5}}{4}|1\rangle$

QUANTUM GATES AND MEASUREMENT

Question 2

What is the resulting state for each of the circuits below?

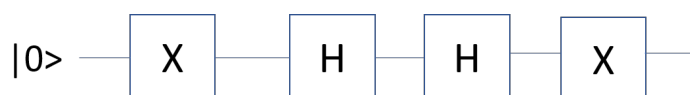
(a)



(b)



(c)



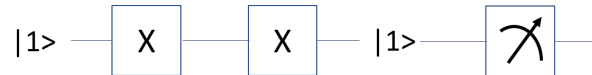
(d)



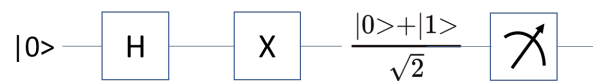
Question 3

What can we expect on measurement?

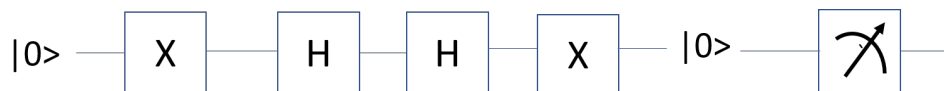
(a)



(b)



(c)



(d)

