

Exercises 1

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样本空间：

$$S = \{2, 3, 4, \dots\}$$

刚好在 k 次出现两次正面向上，相当于前 $k-1$ 次刚好出现一次证明向上且第 k 次正面向上：

$$P(X = k) = \frac{k-1}{2^{k-1}} \times \frac{1}{2} = \frac{k-1}{2^k}$$

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(a) $\Omega = \{2, 3, 4, \dots\}$

(b) 需要三次投掷的情况对应于事件HHT与TTH

$$P = 2 \times \frac{1}{2}^3 = \frac{1}{4}$$

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(a) 记“至少一个小孩有蓝色眼睛”为事件A，“至少两个小孩有蓝色眼睛”为事件B。

$$P(A) = 1 - \frac{3}{4}^3 = \frac{37}{64}$$
$$P(B) = 3 \times \frac{1}{4}^2 \times \frac{3}{4} + \frac{1}{4}^3 = \frac{5}{32}$$

于是

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(B)}{P(A)} = \frac{10}{37}$$

(b) 该事件等价于“两个小孩中至少一个拥有蓝色眼睛”

$$P = 1 - \frac{3}{4}^2 = \frac{7}{16}$$

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$$\begin{aligned} P(A|BC)P(B|C)P(C) &= P(A|BC) [P(B|C)P(C)] \\ &= P(A|BC)P(BC) \\ &= P(ABC) \end{aligned}$$

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	Macintosh	Windows	Linux
V	0.195	0.41	0.1
V^C	0.105	0.09	0.1

$$P(\text{Windows}|V) = \frac{0.41}{0.195 + 0.41 + 0.1} = 0.58$$

(a)

$$P(H) = \sum_{i=1}^5 P(H|C_i) = \sum_{i=1}^5 \frac{p_i}{5} = \frac{1}{2}$$

$$P(C_i|H) = \frac{P(C_i H)}{P(H)} = \frac{\frac{1}{5}p_i}{\frac{1}{2}} = \frac{2}{5}p_i$$

(b)

$$\begin{aligned} P(H_2|H_1) &= \sum_{i=1}^5 P(H_2 C_i | H_1) \\ &= \sum_{i=1}^5 \frac{P(H_2 H_1 C_i)}{P(H_1)} \\ &= \sum_{i=1}^5 \frac{P(H_2 H_1 | C_i) P(C_i)}{P(H)} \\ &= \frac{2}{5} \sum_{i=1}^5 p_i^2 \\ &= \frac{3}{4} \end{aligned}$$

(c)

$$\begin{aligned} P(C_i|B_4) &= \frac{P(C_i B_4)}{P(B_4)} \\ &= \frac{P(C_i B_4)}{\sum_{j=1}^5 P(C_j B_4)} \end{aligned}$$

其中

$$P(C_i B_4) = P(B_4|C_i)P(C_i) = \frac{1}{5}(1-p_i)^3 p_i$$

于是

$$\sum_{j=1}^5 P(C_j B_4) = \frac{23}{640}$$

$$P(C_i|B_4) = \frac{(1-p_i)^3 p_i}{\sum_{j=1}^5 (1-p_j)^3 p_j} = \frac{128}{23}(1-p_i)^3 p_i$$

$P(C_1 B_4)$	$P(C_2 B_4)$	$P(C_3 B_4)$	$P(C_4 B_4)$	$P(C_5 B_4)$
0	$\frac{27}{46}$	$\frac{8}{23}$	$\frac{3}{46}$	0