Homework 8

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1. 用投影梯度法求解下列问题

(1)
$$\begin{aligned} \min \quad & x_1^2 + x_2^2 + 2x_2 + 5 \\ \text{s.t.} \quad & x_1 - 2x_2 \geq 0, \\ & x_1, x_2 \geq 0. \end{aligned}$$

取初始点 $x^{(1)} = (2, 0)^T$.

(2)
$$\begin{aligned} \min \quad & x_1^2 + x_1x_2 + 2x_2^2 - 6x_1 - 2x_2 - 12x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 2, \\ & x_1 - 2x_2 \geq -3, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

取初始点 $x^{(1)} = (1, 0, 1)^T$.

2. 考虑约束 $Ax \leq b$，令 $P = I - A_1^T(A_1A_1^T)^{-1}A_1$，其中 A_1 的每一行是在已知点 $\hat{x}$ 处的紧约束的梯度, 试解释下列各式的几何意义:

(1) $P\nabla f(\hat{x}) = 0$;

(2) $P\nabla f(\hat{x}) = \nabla f(\hat{x})$;

(3) $P\nabla f(\hat{x}) \neq 0$.

3. 考虑问题

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \geq 0, \quad i = 1, \cdots, m, \\ & h_j(x) = 0, \quad j = 1, \cdots, l. \end{aligned}$$

设 $\hat{x}$ 是可行点, $I = \{i \mid g_i(x) = 0\}$. 证明 $\hat{x}$ 为 K-T 点的充分必要条件是下列问题的目标函数的最优值为零:

$$\begin{aligned} \min \quad & \nabla f(\hat{x})^T d \\ \text{s.t.} \quad & \nabla g_i(\hat{x})^T d \geq 0, \quad i \in I, \\ & \nabla h_j(\hat{x})^T d = 0, \quad j = 1, \cdots, l, \\ & -1 \leq d_j \leq 1, \quad j = 1, \cdots, n. \end{aligned}$$