

## Exercise 4

### 6.1

$$E(\hat{\lambda}) = n^{-1} \sum_{i=1}^n E(X_i) = n^{-1} n \lambda = \lambda$$

$$Var(\hat{\lambda}) = n^{-2} \sum_{i=1}^n Var(X_i) = n^{-2} n \lambda = n^{-1} \lambda$$

于是

$$\begin{aligned} \text{bias} &= E(\hat{\lambda}) - \lambda = 0 \\ \text{se} &= \sqrt{Var(\hat{\lambda})} = \sqrt{n^{-1} \lambda} \\ \text{MSE} &= n^{-1} \lambda \end{aligned}$$

### 6.2

对于  $0 < \hat{\theta} < \theta$ , 有

$$\begin{aligned} F(\hat{\theta}) &= \left( \frac{\hat{\theta}}{\theta} \right)^n \\ f(\hat{\theta}) &= \frac{n}{\theta^n} \hat{\theta}^{n-1} \\ E(\hat{\theta}) &= \int_0^{\theta} f(\hat{\theta}) \hat{\theta} d\hat{\theta} = \frac{n}{n+1} \theta \\ E(\hat{\theta}^2) &= \int_0^{\theta} f(\hat{\theta}) \hat{\theta}^2 d\hat{\theta} = \frac{n}{n+2} \theta^2 \end{aligned}$$

于是

$$\begin{aligned} \text{bias} &= E(\hat{\theta}) - \theta = -\frac{1}{n+1} \theta \\ \text{se} &= \sqrt{E(\hat{\theta}^2) - E^2(\hat{\theta})} = \sqrt{\frac{n}{(n+1)^2(n+2)}} \theta \\ \text{MSE} &= \frac{2}{(n+1)(n+2)} \theta^2 \end{aligned}$$

### 6.3

$$\begin{aligned} E(\hat{\theta}) &= \theta \\ Var(\hat{\theta}) &= \frac{\theta^2}{3n} \\ \text{bias} &= E(\hat{\theta}) - \theta = 0 \\ \text{se} &= \sqrt{Var(\hat{\theta})} = \sqrt{\frac{1}{3n}} \theta \\ \text{MSE} &= \frac{1}{3n} \theta^2 \end{aligned}$$

## 9.1

$$a_1 = E(X) = \frac{\hat{\alpha}}{\hat{\beta}}$$

$$a_2 = E(X^2) = \frac{\hat{\alpha}}{\hat{\beta}^2} + \left(\frac{\hat{\alpha}}{\hat{\beta}}\right)^2$$

于是

$$\hat{\alpha} = \frac{E^2(X)}{Var(X)}$$

$$\hat{\beta} = \frac{E(X)}{Var(X)}$$