Exercise 4

6.1

$$E(\hat{\lambda}) = n^{-1}\sum_{i=1}^{n} E(X_i) = n^{-1}n\lambda = \lambda$$

$$Var(\hat{\lambda}) = n^{-2}\sum_{i=1}^{n} Var(X_i) = n^{-2}n\lambda = n^{-1}\lambda$$

于是

$$\text{bias} = E(\hat{\lambda}) - \lambda = 0$$
$$\text{se} = \sqrt{Var(\hat{\lambda})} = \sqrt{n^{-1}\lambda}$$
$$\text{MSE} = n^{-1}\lambda$$

6.2

对于$0 < \hat{\theta} < \theta$，有

$$F(\hat{\theta}) = \left(\frac{\hat{\theta}}{\theta}\right)^n$$
$$f(\hat{\theta}) = \frac{n}{\theta^n}\hat{\theta}^{n-1}$$

$$E(\hat{\theta}) = \int_o^\theta f(\hat{\theta})\hat{\theta}d\hat{\theta} = \frac{n}{n+1}\theta$$
$$E(\hat{\theta}^2) = \int_o^\theta f(\hat{\theta})\hat{\theta}^2 d\hat{\theta} = \frac{n}{n+2}\theta^2$$

于是

$$\text{bias} = E(\hat{\theta}) - \theta = -\frac{1}{n+1}\theta$$
$$\text{se} = \sqrt{E(\hat{\theta}^2) - E^2(\hat{\theta})} = \sqrt{\frac{n}{(n+1)^2(n+2)}}\theta$$
$$\text{MSE} = \frac{2}{(n+1)(n+2)}\theta^2$$

6.3

$$E(\hat{\theta}) = \theta$$
$$Var(\hat{\theta}) = \frac{\theta^2}{3n}$$

$$\text{bias} = E(\hat{\theta}) - \theta = 0$$
$$\text{se} = \sqrt{Var(\hat{\theta})} = \sqrt{\frac{1}{3n}}\theta$$
$$\text{MSE} = \frac{1}{3n}\theta^2$$

9.1

$$a_1 = E(X) = \frac{\hat{\alpha}}{\hat{\beta}}$$

$$a_2 = E(X^2) = \frac{\hat{\alpha}}{\hat{\beta}^2} + \left(\frac{\hat{\alpha}}{\hat{\beta}}\right)^2$$

于是

$$\hat{\alpha} = \frac{E^2(X)}{Var(X)}$$

$$\hat{\beta} = \frac{E(X)}{Var(X)}$$