VIII. Mathematical problem to be solved using ADMM and verify its solution with centralized optimization

Consider a following optimization problem:

$$\textbf{min} \sum_{t=1}^{5} \Big[5x^t + 3y^t + 4w_d^t + 50(\Delta n^t)^2 + 100(\hat{y} - y^t) + 200(0.8 - q^t)^2\Big] \tag{54}$$

subject to :

$$x^t \geq 0 \tag{55}$$

$$x^t + y^t + 5n^t + w_d^t = \hat{l} + w_c^t \tag{56}$$

$$0 \leq y^t \leq \hat{y} \tag{57}$$

$$\Delta n^t = n^t - n^{t-1}; \quad n^0 = 0 \tag{58}$$

$$0 \leq n \leq 8, \quad \text{(n is an integer variable)} \tag{59}$$

$$\hat{y}[1:5] = [2, 4, 8, 7, 10] \tag{60}$$

$$\hat{l}[1:5] = [25, 40, 35, 50, 55] \tag{61}$$

$$q^t = q^{t-1} + 0.05(w_c^t - 5w_d^t); \quad q^0 = 0.8 \tag{62}$$

$$0 \leq w_c^t \leq 10\beta_c^t \tag{63}$$

$$0 \leq w_d^t \leq 10\beta_d^t \tag{64}$$

$$\beta_c^t + \beta_d^t \leq 1.5; \quad \beta_c \text{ and } \beta_d \text{ are binary variables} \tag{65}$$

$$0.4 \leq q^t \leq 1 \tag{66}$$

We will separate the objective function (54) in three parts, such as:

$$\textbf{min} \sum_{t=0}^{5} \Big[5x^t + 3y^t + 4w_d^t + 50(\Delta n^t)^2\Big] + \textbf{min} \quad \sum_{t=0}^{5} 100(\hat{y} - y^t) + \textbf{min} \sum_{t=0}^{5} 200(0.8 - q^t)^2 \tag{67}$$

For applying ADMM, we shall separate the common variable y between first and second objectives, also w_d and w_c between first and third objectives. We introduce auxiliary variables z, u_d, and u_c for separation of variables, such as:

$$\textbf{min} \sum_{t=0}^{5} \Big[5x^t + 3y^t + 4w_d^t + 50(\Delta n^t)^2\Big] + \textbf{min} \sum_{t=0}^{5} 100(\hat{y} - z^t) + \textbf{min} \sum_{t=0}^{5} 200(0.8 - q^t)^2 \tag{68}$$

subject to:

$$y^t - z^t = 0 \tag{69}$$

$$w_d^t - u_d^t = 0 \tag{70}$$

$$w_c^t - u_c^t = 0 \tag{71}$$

IX. Things to perform

1) First solve the above optimization with centralized optimization.
2) Solve the same problem with integer relaxation using ADMM.
3) Solve the same problem with Branch and Bound integrated ADMM.