VI. Mathematical problem to be solved using ADMM and verify its solution with centralized optimization

Consider a following optimization problem:

$$\mathbf{min} \sum_{t=1}^{5} \Big[5x^t + 3y^t + 50(\Delta n^t)^2 + 100(\hat{y} - y^t) \Big] \tag{43}$$

subject to :

$$x^t \geq 0 \tag{44}$$

$$x^t + y^t + 5n^t = \hat{l} \tag{45}$$

$$0 \leq y^t \leq \hat{y} \tag{46}$$

$$\Delta n^t = n^t - n^{t-1}; \quad n^0 = 0 \tag{47}$$

$$0 \leq n \leq 8, \quad \text{(n is an integer variable)} \tag{48}$$

$$\hat{y}[1:5] = [2, 4, 8, 7, 10] \tag{49}$$

$$\hat{l}[1:5] = [25, 40, 35, 50, 55] \tag{50}$$

We will separate the objective function (43) in two parts, such as:

$$\mathbf{min} \sum_{t=0}^{5} \Big[5x^t + 3y^t + 50(\Delta n^t)^2 \Big] + \mathbf{min} \quad \sum_{t=0}^{5} 100(\hat{y} - y^t) \tag{51}$$

For applying ADMM, we shall separate the common variable y from both objective by introducing an auxiliary variable z, such as:

$$\mathbf{min} \sum_{t=0}^{5} \Big[5x^t + 3y^t + 50(\Delta n^t)^2 \Big] + \mathbf{min} \sum_{t=0}^{5} 100(\hat{y} - z^t) \tag{52}$$

subject to:

$$y^t - z^t = 0 \tag{53}$$

VII. Things to perform

1) First solve the above optimization with centralized optimization.
2) Solve the same problem with integer relaxation using ADMM.
3) Solve the same problem with Branch and Bound integrated ADMM.