Simulation result

1) Centralized optimization

Complete formulation :

$$\min \sum_{t=1}^{5} [5x^t + 3y^t + 4w_d^t + 50(\Delta n^t)^2 + 100a^t + 200(b^t)^2] \quad (1)$$

Subject to:

$$x^t + y^t + 5n^t + w_d^t = \hat{l} + w_c^t \quad (2)$$
$$\Delta n^t = n^t - n^{t-1};\ n^0 = 0 \quad (3)$$
$$q^t = q^{t-1} + 0.05(w_c^t - 5w_d^t);\ q^0 = 0.8 \quad (4)$$
remove this 5
$$0 \le w_c^t \le 10\beta_c^t \quad (5)$$
$$0 \le w_d^t \le 10\beta_d^t \quad (6)$$
$$\beta_c^t + \beta_d^t \le 1.5;\ \beta_c^t, \beta_d^t \in \{0,1\} \quad (7)$$
$$a^t = \hat{y} - y^t \quad (8)$$
$$b^t = 0.8 - q^t \quad (9)$$
$$0 \le y^t \le \hat{y} \quad (10)$$
$$0 \le n^t \le 8 \quad (11)$$
$$x^t \ge 0 \quad (12)$$
$$0.4 \le q^t \le 1 \quad (13)$$

Where $\hat{y}$ and $\hat{l}$ are parameter vector with values of $\hat{y} = [2, 4, 8, 7, 10]$ and $\hat{l} = [25, 40, 35, 50, 55]$.

a. Relaxed integer

The obtained objective value is $\mathbf{784.705}$ and the result can be seen as follows :

Variables	t				
	1	**2**	**3**	**4**	**5**
x	10.35	24.75	12	25.5	26.21
y	2	4	8	7	10
a	0	0	0	0	0
w_c	0	0	0	0	0
w_d	6.4	0	0	0	0.04
q	0.8	0.8	0.8	0.8	0.81
qq	0	0	0	0	-0.01
n	1.25	2.25	3	3.5	3.75
Δn	1.25	1	0.75	0.5	0.25
β_c	0	0	0	0	0
β_d	0.64	0	0	0	0.004

b. With integer

The obtained objective value is $\mathbf{806.58}$ and the result can be seen as follows :

Variables	t				
	1	**2**	**3**	**4**	**5**
x	11.6	26	12	28	29.96
y	2	4	8	7	10
a	0	0	0	0	0

w_c	0	0	0	0	0
w_d	6.4	0	0	0	0.04
q	0.8	0.8	0.8	0.8	0.81
qq	0	0	0	0	-0.01
n	1	2	3	3	3
Δn	1	1	1	0	0
β_c	0	0	0	0	0
β_d	1	1	1	1	1

2) Distributed optimization

The data exchange between entity :

Variable	Between entity :
y	Entity 1 and 2
w_c, w_d	Entity 1 and 3

With formulation for the three entities is:

a. First entity. In this part, it has information about variables : $x, y, n, \Delta n, w_c, w_d, \beta_c$, and β_d

$$\min \sum_{t=1}^{5} [5x^t + 3y^t + 4w_d + 50(\Delta n^t)^2]$$

Subject to:

$$x^t + y^t + 5n^t + w_d^t = \hat{l} + w_c^t \quad (14)$$
$$\Delta n^t = n^t - n^{t-1};\ \ n^0 = 0 \quad (15)$$
$$0 \le w_c^t \le 10\beta_c^t \quad (16)$$
$$0 \le w_d^t \le 10\beta_d^t \quad (17)$$
$$\beta_c^t + \beta_d^t \le 1.5;\ \ \beta_c^t, \beta_d^t \in \{0,1\} \quad (18)$$
$$0 \le y^t \le \hat{y} \quad (19)$$
$$0 \le n^t \le 8 \quad (20)$$
$$x^t \ge 0 \quad (21)$$

eqn(16-19) is unknown to first entity

eqn(16-18) is known only to third entity

eqn(19) is known only to second entity

b. Second entity. This part has information about y only, which is replaced with auxiliary variable z

$$\min \sum_{t=1}^{5} [100a^t]$$

Subject to:

$$a^t = \hat{y} - z^t \quad (22)$$
$$0 \le z^t \le \hat{y} \quad (23)$$

a. Third entity. This part has information about q, w_c, and w_d, which w_c and w_d are replaced with auxiliary variables u_c and u_d. It does not know the upper bounds of w_c and w_d.

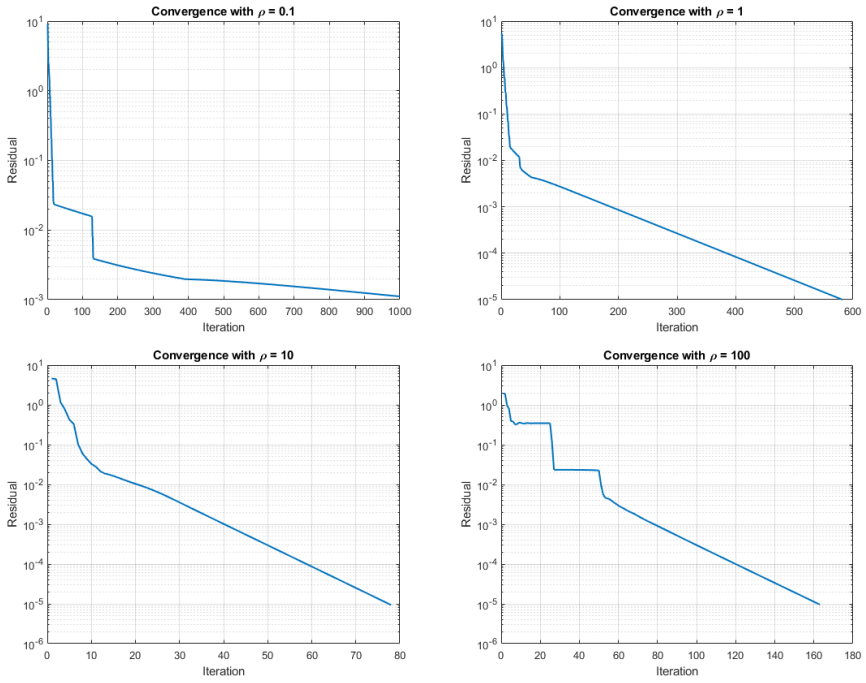
$$\min \sum_{t=1}^{5} [200(b^t)^2]$$

Subject to:

$$q^t = q^{t-1} + 0.05(u_c^t - 5u_d^t); \quad q^0 = 0.8 \tag{24}$$
$$0 \leq u_c^t \tag{25}$$
$$0 \leq u_d^t \tag{26}$$
$$b^t = 0.8 - q^t \tag{27}$$
$$0.4 \leq q^t \leq 1 \tag{28}$$

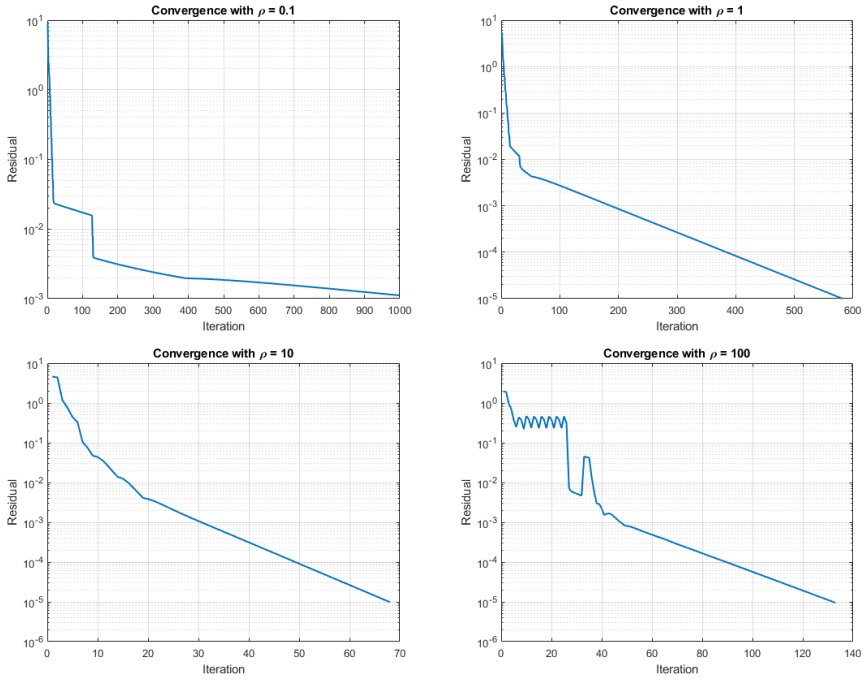
There are four cases that differs the penalty parameter in each simulation of the ADMM $\rho = [0.1, 1, 10, 100]$. The tolerance for the convergence is 10^{-5}. The convergence can be seen below.

a. Relaxed integer



The simulation with the $\rho = 1, 10$, and 100 converges, but $\rho = 0.1$ has not reached convergence in 1000 iterations. The sum of objective values of all entities with penalty $\rho = 10$ produce objective value of $\mathbf{784.705048}$

b. With integer



The simulation with the $\rho = 1, 10$, and 100 converges, but $\rho = 0.1$ has not reached convergence in 1000 iterations. The sum of objective values of all entities with penalty $\rho = 10$ produce objective value of $\mathbf{806.580048}$

Summary:

Difference of objective values between centralized and distributed :

Integer type	Centralized	Distributed	Approx. Error (%)
Relaxed	$\mathbf{784.705}$	$\mathbf{784.705048}$	$\mathbf{0.00000613}$
Integer	$\mathbf{806.58}$	$\mathbf{806.580048}$	$\mathbf{0.00000596}$