Simple DC Power Flow

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1 Simple DC Power Flow

DC power flow is one of power flow methods that is usually used for a transmission system. It defines several assumptions:

- Only real powers take part in the calculation, ignoring the reactive powers.
- The amount of power flowing on the line is calculated based on the difference of phase angle between two buses.
- The voltage magnitudes are undefined, unless being explicitly stated in the constraints or computed afterwards.
- The system has zero losses.

1.1 Notations

1.1.1 Variables

$$
\begin{aligned}
\texttt{p\_g} \rightarrow p_i^{\text{g}} \quad &: \text{Real power of generator at bus } i. \\
\texttt{p\_inj} \rightarrow p_i^{\text{inj}} \quad &: \text{Real power injection of bus } i. \\
\texttt{theta} \rightarrow \theta_i \quad &: \text{Phase angle of bus } i. \\
\texttt{p\_ij} \rightarrow p_{ij}^{\text{line}} \quad &: \text{Real power flowing from bus } i \text{ to } j.
\end{aligned}
$$

*Note: In this study, please ignore the index t in the code.

1.1.2 Parameters

$$
\begin{aligned}
\texttt{Pd} \rightarrow \widehat{p}_i^{\text{load}} \quad &: \text{Load demand at bus } i. \\
\texttt{perB} \rightarrow \widehat{X}_{ij} \quad &: \text{Reactance of line between bus } i \text{ and } j.
\end{aligned}
$$

1.2 Constraints

$$
\begin{aligned}
p_i^{\text{inj}} &= \sum_{h \in \mathcal{N}^+} p_{hi}^{\text{line}} - \sum_{j \in \mathcal{N}^-} p_{ij}^{\text{line}}, && \forall i \in \mathcal{N} && (1)\\
p_i^{\text{inj}} &= p_i^{\text{g}} - \widehat{p}_i^{\text{load}}, && \forall i \in \mathcal{N} && (2)\\
p_{ij}^{\text{line}} &= \frac{1}{\widehat{X}_{ij}}(\theta_i - \theta_j), && \forall ij \in \mathcal{L} && (3)\\
\theta_i &= 0, && \forall i \in \mathcal{N}^{\text{slack}} && (4)
\end{aligned}
$$

where $\mathcal{N}$: the set of buses; $\mathcal{N}^+$ and $\mathcal{N}^-$: the set of preceding and succeeding buses, respectively; $\mathcal{N}^{\text{slack}}$: the set of slack buses (usually only contain one bus); and $\mathcal{L}$: the set of lines.

Constraint (1) is the bus power balance. Constraint (2) manages the bus injection. Constraint (3) determines the line power flow through the phase angle differences between two bus, where the initial slack phase angle is defined in Constraint (4).

Some of the bounding constraints can be expressed directly in the variable declarations, in which bounds are taken from the `net` dataset or other known parameters. The bounds managed in this manner are:

$$
\begin{aligned}
\underline{p}_i^{\text{gen}} \leq p_i^{\text{gen}} \leq \overline{p}_i^{\text{gen}} && (5)\\
-2\pi \leq \theta_i \leq 2\pi && (6)\\
\underline{p}_{ij}^{\text{line}} \leq p_i^{\text{gen}} \leq \overline{p}_{ij}^{\text{line}} && (7)
\end{aligned}
$$

where Constraints (5)–(7) limit the operating conditions for the generator real power dispatch, phase angle, and real power flowing in the conductor.

1.3 Objective

The objective is to minimize the generators' real powers according to their generation coefficients.

$$a(p_i^{\text{g}})^2 + b(p_i^{\text{g}}) + c \tag{8}$$

where $a, b,$ and c are the quadratic generator coefficients, or can be treated as generation costs in the unit of US$.

1.4 Result

The result of the above optimization program implemented for 9-bus transmission system can be seen in Fig. 1.

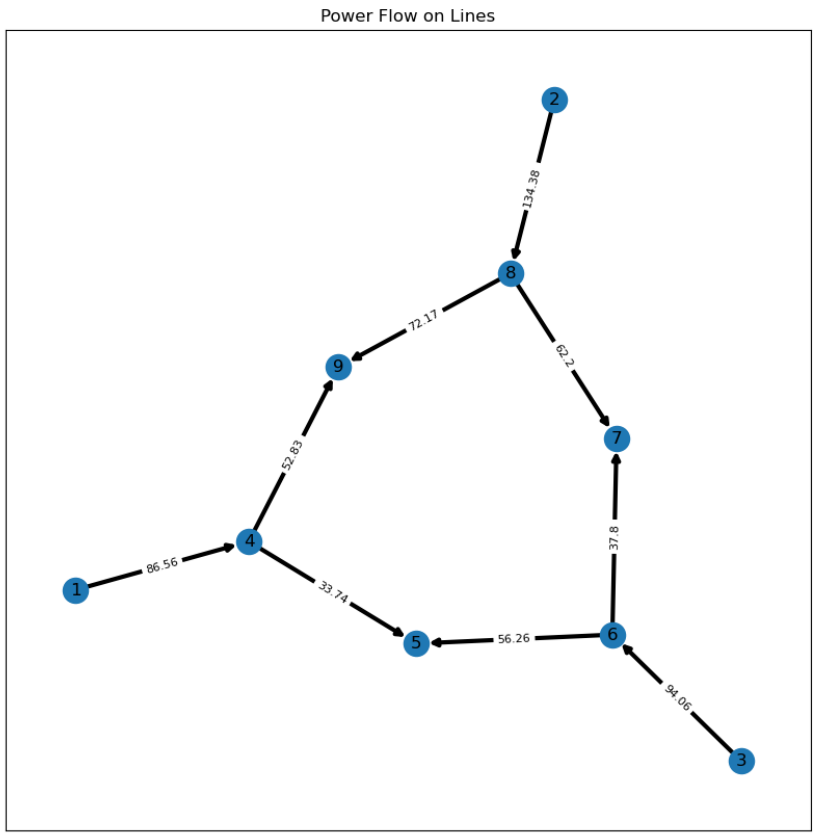

Figure 1: Expected result.