图像处理与机器学习

Digital Image Processing and Machine Learning

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判别函数

- 高斯密度下的判别函数

$$y_k(\vec{x}) = -\frac{1}{2}(\vec{x}-\vec{\mu}_k)^t \Sigma_k^{-1}(\vec{x}-\vec{\mu}_k) - \frac{1}{2}\ln|\Sigma_k| + \ln p(\omega_k)$$

- Case 1 $\boxed{\Sigma_k = \sigma^2 I}$
- Case 2 $\boxed{\Sigma_k = \Sigma}$
- Case 3 $\boxed{\Sigma_k = arbitrary}$

判别函数

- 高斯概率密度下的判别函数

➢ Case 2 $\boxed{\Sigma_k = \Sigma}$ $y_k(\vec{x}) = -\frac{1}{2}(\vec{x}-\vec{\mu}_k)^t \Sigma_k^{-1}(\vec{x}-\vec{\mu}_k) + \ln p(\omega_k)$

$$y_k(\vec{x}) = -\frac{1}{2}(\vec{x}-\vec{\mu}_k)^t \Sigma^{-1}(\vec{x}-\vec{\mu}_k) + \ln p(\omega_k)$$

$$y_k(\vec{x}) = \vec{w}_k{}^t \vec{x} + w_{k0}$$

$$\vec{w}_k = \Sigma^{-1}\vec{\mu}_k, \qquad w_{k0} = -\frac{1}{2}\vec{\mu}_k{}^t \Sigma^{-1}\vec{\mu}_k + \ln p(\omega_k)$$

✓ 分类函数为线性函数，线性分类器

Linear Discriminant Function(LDF)

判别函数

- Case 2 $\boxed{\Sigma_k = \Sigma}$

✓ 决策面（判别函数相等的点构成） $y_k(\vec{x}) = y_i(\vec{x})$， $y_k(\vec{x}) - y_i(\vec{x}) = 0$

$$\vec{w}^t(\vec{x} - \vec{x}_0) = 0 \qquad \vec{w} = \Sigma^{-1}(\vec{\mu}_k - \vec{\mu}_i)$$

$$\vec{x}_0 = \frac{1}{2}(\vec{\mu}_k + \vec{\mu}_i) - \frac{1}{(\vec{\mu}_k - \vec{\mu}_i)^t \Sigma^{-1}(\vec{\mu}_k - \vec{\mu}_i)} \ln \frac{p(\omega_k)}{p(\omega_i)} (\vec{\mu}_k - \vec{\mu}_i)$$

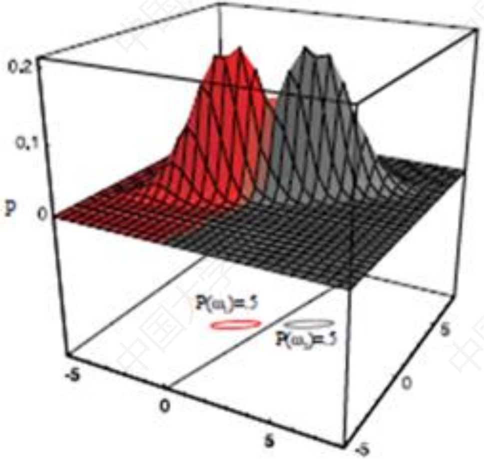
✓ 注意跟μ_1-μ_2的关系，决策面不一定与之垂直

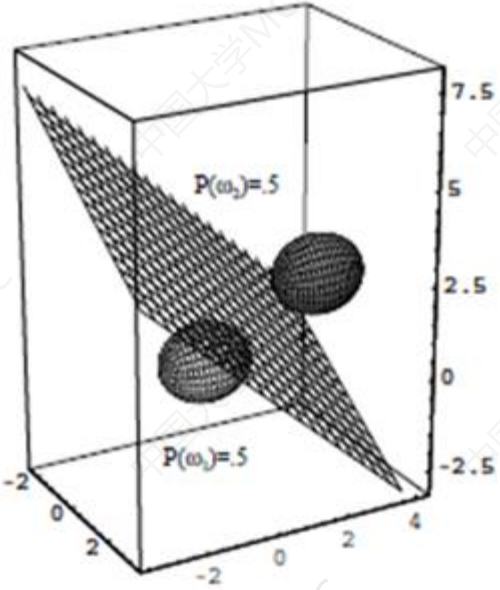
✓ $p(\omega_1)=p(\omega_2)$，决策面经过 $(\mu_1+\mu_2)/2$

判别函数

- Case 2 $\boxed{\Sigma_k = \Sigma}$

$p(\omega_1)=p(\omega_2)$

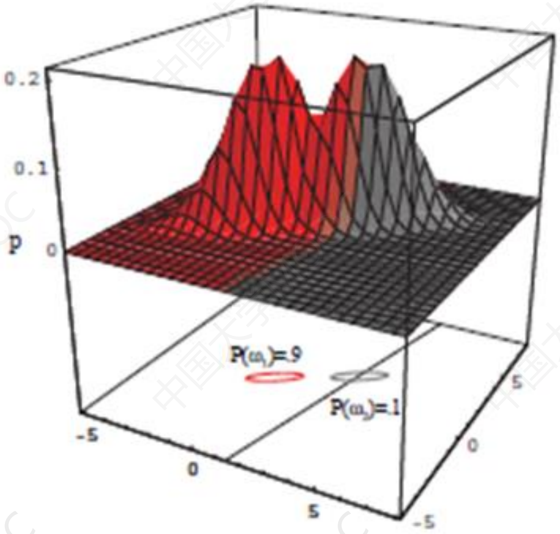


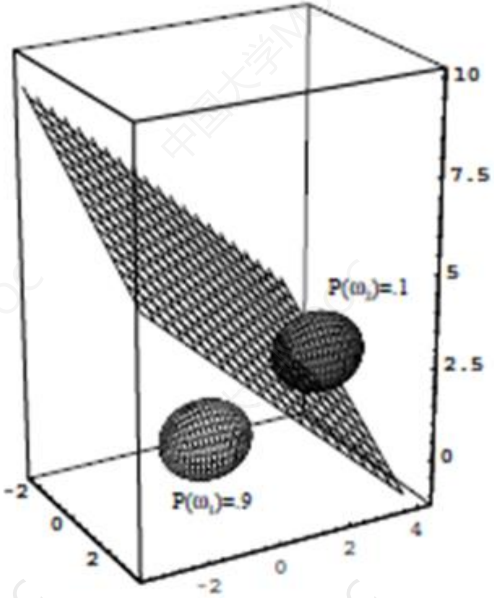


判别函数

- Case 2 $\boxed{\Sigma_k = \Sigma}$

$$p(\omega_1) \neq p(\omega_2)$$





判别函数

- Case 3 $\boxed{\Sigma_k = arbitrary}$

$$y_k(\vec{x}) = -\frac{1}{2}(\vec{x}-\vec{\mu}_k)^t \Sigma_k^{-1}(\vec{x}-\vec{\mu}_k) - \frac{1}{2}\ln|\Sigma_k| + \ln p(\omega_k)$$

$$y_k(\vec{x}) = \vec{x}^t \vec{W}_k \vec{x} + \vec{w}_k{}^t \vec{x} + w_{k0} \qquad \vec{W}_k = -\frac{1}{2}\Sigma_k^{-1} \qquad \vec{w}_k = \Sigma_k^{1} \vec{\mu}_k$$

$$w_{k0} = -\frac{1}{2}\vec{\mu}_k{}^t \Sigma^{-1} \vec{\mu}_k - \frac{1}{2}\ln|\Sigma_k| + \ln p(\omega_k)$$

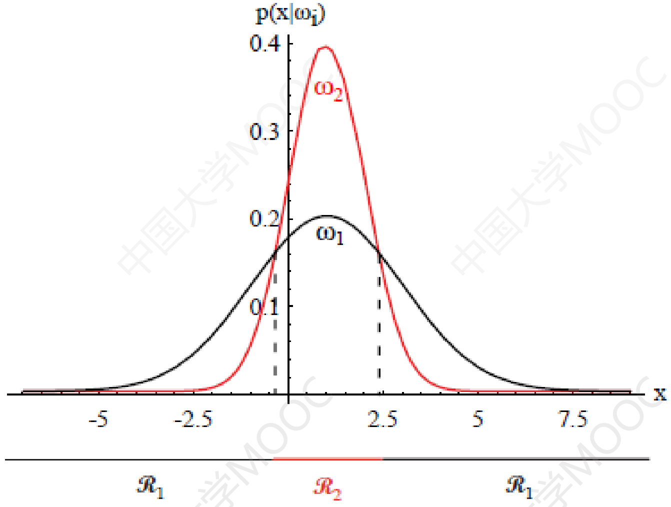
✓ 分类函数为非线性函数，非线性分类器

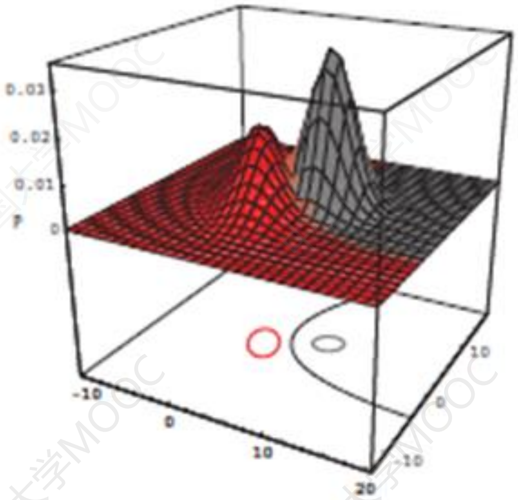
Quadratic Discriminant Function(QDF)

判别函数

- Case 3 $\boxed{\Sigma_k = arbitrary}$

✓ 决策面（判别函数相等的点构成） $y_k(\vec{x}) = y_i(\vec{x})$， $y_k(\vec{x}) - y_i(\vec{x}) = 0$



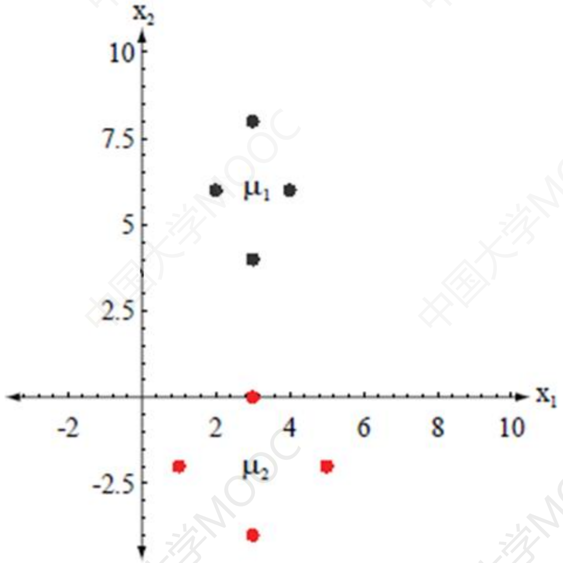


判别函数

- 例: 试确定后验概率为高斯分布的两类样本的决策面，设先验概率相等。

ω_1类$:(2,6)(3,4)(3,8)(4,6)$ $\quad \omega_2$类$:(1,-2)(3,0)(5,-2)(3,-4)$

$$\mu_1=\begin{bmatrix}3\\6\end{bmatrix};\Sigma_1=\begin{pmatrix}1/2 & 0\\0 & 2\end{pmatrix} \qquad \mu_2=\begin{bmatrix}3\\-2\end{bmatrix};\Sigma_2=\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$$



$$\Sigma_k=arbitrary \qquad y_k(\vec{x})=\vec{x}^t\vec{W}_k\vec{x}+\vec{w}_k{}^t\vec{x}+w_{k0}$$

$$\vec{W}_k=-\frac{1}{2}\Sigma_k{}^{-1}, \quad \vec{w}_k=\Sigma_k{}^{-1}\vec{\mu}_k$$

$$w_{k0}=-\frac{1}{2}\vec{\mu}_k{}^t\Sigma^{-1}\vec{\mu}_k-\frac{1}{2}\ln|\Sigma_k|+\ln p(\omega_k)$$

判别函数

$$\mu_1=\begin{bmatrix}3\\6\end{bmatrix};\Sigma_1=\begin{pmatrix}1/2 & 0\\0 & 2\end{pmatrix};\Sigma_1^{-1}=\begin{pmatrix}2 & 0\\0 & 1/2\end{pmatrix} \qquad \mu_2=\begin{bmatrix}3\\-2\end{bmatrix};\Sigma_2=\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix};\Sigma_2^{-1}=\begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}$$

$$\vec{W}_k=-\frac{1}{2}\Sigma_k^{-1},\quad \vec{w}_k=\Sigma_k^{-1}\vec{\mu}_k$$

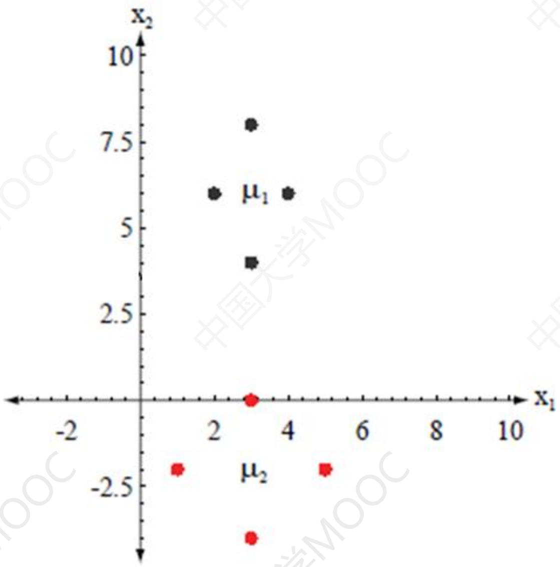
$$w_{k0}=-\frac{1}{2}\vec{\mu}_k{}^t\Sigma^{-1}\vec{\mu}_k-\frac{1}{2}\ln|\Sigma_k|+\ln p(\omega_k)$$

$$y_k(\vec{x})=\vec{x}^t\vec{W}_k\vec{x}+\vec{w}_k{}^t\vec{x}+w_{k0}$$

$$y_1(\vec{x})-y_2(\vec{x})=0$$

$$\Downarrow$$

$$x_2=3.514-1.125x_1+0.1875x_1^2$$



判别函数

$$\mu_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}; \Sigma_1 = \begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix}; \Sigma_1^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix} \qquad \mu_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}; \Sigma_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}; \Sigma_2^{-1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\vec{W}_k = -\frac{1}{2}\Sigma_k^{-1}, \quad \vec{w}_k = \Sigma_k^{-1}\vec{\mu}_k$$

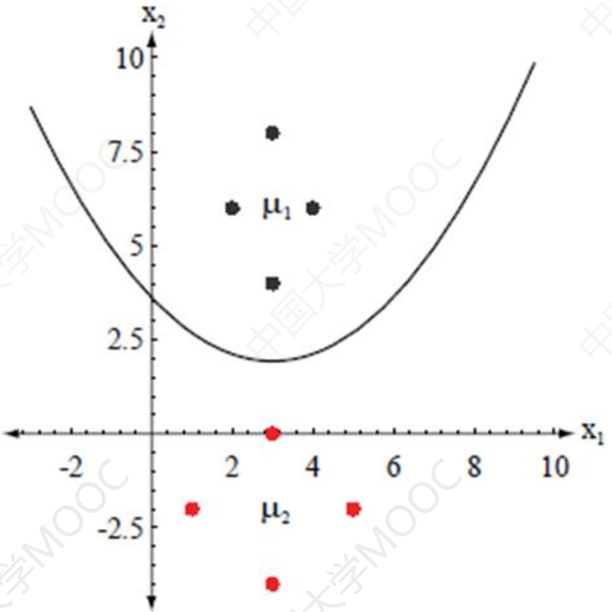
$$w_{k0} = -\frac{1}{2}\vec{\mu}_k^{\,t}\Sigma^{-1}\vec{\mu}_k - \frac{1}{2}\ln|\Sigma_k| + \ln p(\omega_k)$$

$$y_k(\vec{x}) = \vec{x}^t \vec{W}_k \vec{x} + \vec{w}_k^{\,t}\vec{x} + w_{k0}$$

$$y_1(\vec{x}) - y_2(\vec{x}) = 0$$

$$\downarrow$$

$$x_2 = 3.514 - 1.125x_1 + 0.1875x_1^2$$



判别函数

◆ 高斯概率密度下的判别函数

$$p(\vec{x}|\omega_i) = \frac{1}{(2\pi)^{d/2}|\Sigma_i|^{1/2}} \exp\left[-\frac{1}{2}(\vec{x}-\mu_i)\Sigma_i^{-1}(\vec{x}-\mu_i)\right]$$

$$\boxed{\Sigma_k = \sigma^2 I}$$

$$\boxed{\Sigma_k = \Sigma}$$

$$y_k(\vec{x}) = \vec{w}_k{}^t\vec{x} + w_{k0}$$

Linear Discriminant Function

(LDF)

$$\boxed{\Sigma_k = arbitrary}$$

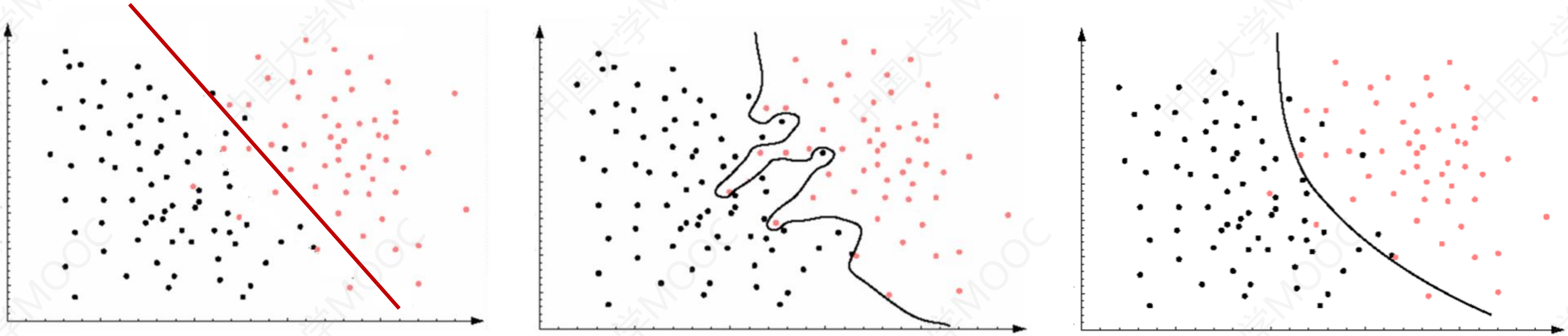
$$y_k(\vec{x}) = \vec{x}^t\vec{W}_k\vec{x} + \vec{w}_k{}^t\vec{x} + w_{k0}$$

Quadratic Discriminant Function

(QDF)

判别函数

如何确定最优的分类界面（函数）？



后验概率密度

谢 谢

本课程所引用的一些素材为主讲老师多年的教学积累，来源于多种媒体及同事和同行的交流，难以一一注明出处，特此说明并表示感谢！