图像处理与机器学习

Digital Image Processing and Machine Learning

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第五章　贝叶斯决策

- 基本概念
- 贝叶斯决策
- 判别函数
- 概率密度估计

贝叶斯决策

- 最小错误率贝叶斯决策 (Minimum error Bayes Decision)

- 最小风险贝叶斯决策 (Minimum Risk Bayes decision)

医疗诊断

正常归为异常

异常归为正常

贝叶斯决策

- 最小风险贝叶斯决策 (Minimum Risk Bayes decision)

✓ 分类错误代价 (Loss) λ

True class ω_j decide as α_i , then $\lambda_{ij} = \lambda(\alpha_i | \omega_j)$

✓ 条件风险 (Conditional risk)

$$\boxed{R(\alpha_i|\vec{x}) = \sum_{j=1}^{c} \lambda(\alpha_i|\omega_j)\, p(\omega_j|\vec{x})} \longrightarrow \underset{i}{minimize}\, R(\alpha_i|\vec{x})$$

贝叶斯决策

◆ 最小风险贝叶斯决策 (Minimum Risk Bayes decision)

$$R(\alpha_i|\vec{x}) = \sum_{j=1}^{c} \lambda(\alpha_i|\omega_j)p(\omega_j|\vec{x}) \quad \lambda_{ij} = \lambda(\alpha_i|\omega_j)$$

✓ 两类问题：ω_1 类 与ω_2 类

$$R(\alpha_1|\vec{x}) = \lambda(\alpha_1|\omega_1)p(\omega_1|\vec{x}) + \lambda(\alpha_1|\omega_2)p(\omega_2|\vec{x})$$

$$R(\alpha_2|\vec{x}) = \lambda(\alpha_2|\omega_1)p(\omega_1|\vec{x}) + \lambda(\alpha_2|\omega_2)p(\omega_2|\vec{x})$$

$$R(\alpha_1|\vec{x}) = \lambda_{11}p(\omega_1|\vec{x}) + \lambda_{12}p(\omega_2|\vec{x}) \quad R(\alpha_2|\vec{x}) = \lambda_{21}p(\omega_1|\vec{x}) + \lambda_{22}p(\omega_2|\vec{x})$$

贝叶斯决策

- 最小风险贝叶斯决策 (Minimum Risk Bayes decision)

$$R(\alpha_1|\vec{x}) = \lambda_{11} p(\omega_1|\vec{x}) + \lambda_{12} p(\omega_2|\vec{x})$$

$$R(\alpha_2|\vec{x}) = \lambda_{21} p(\omega_1|\vec{x}) + \lambda_{22} p(\omega_2|\vec{x})$$

✓ 决策规则 （**Decision rule**） $R(\alpha_1|\vec{x}) < R(\alpha_2|\vec{x}), decide \quad \omega_1$

$$\{[\lambda_{21} p(\omega_1|\vec{x}) + \lambda_{22} p(\omega_2|\vec{x})] - [\lambda_{11} p(\omega_1|\vec{x}) + \lambda_{12} p(\omega_2|\vec{x})]\} > 0$$

$$(\lambda_{21} - \lambda_{11}) p(\omega_1|\vec{x}) + (\lambda_{22} - \lambda_{12}) p(\omega_2|\vec{x}) > 0$$

贝叶斯决策

- **最小风险 贝叶斯决策**

✓ 决策规则 （Decision rule）

$$R(\alpha_1|\vec{x}) < R(\alpha_2|\vec{x}), decide \quad \omega_1$$

$$(\lambda_{21}-\lambda_{11})p(\omega_1|\vec{x})+(\lambda_{22}-\lambda_{12})p(\omega_2|\vec{x})>0$$

$$(\lambda_{21}-\lambda_{11})p(\vec{x}|\omega_1)p(\omega_1)>(\lambda_{12}-\lambda_{22})p(\vec{x}|\omega_2)p(\omega_2)$$

$$\boxed{\frac{p(\vec{x}|\omega_1)}{p(\vec{x}|\omega_2)}>\frac{(\lambda_{12}-\lambda_{22})p(\omega_2)}{(\lambda_{21}-\lambda_{11})p(\omega_1)}} \quad decide \quad \omega_1$$

贝叶斯决策

- 假设在某个局部地区细胞识别中正常 ω_1 和异常ω_2 ；

  两类的先验概率分别为 正常：$p(\omega_1)=0.9$ 异常：$p(\omega_2)=0.1$

  现有一待识别的细胞，其观察值为 x ，从类条件概率密度曲线上查得

$$p(x|\omega_1)=0.2 \qquad p(x|\omega_2)=0.4$$

  已知风险参数如下： $\lambda_{11}=0; \lambda_{12}=6; \lambda_{21}=1; \lambda_{22}=0;$

  试分别以最小错误率和最小风险贝叶斯决策对该细胞分类。

贝叶斯决策

$$p(\omega_1)=0.9 \qquad p(x\,|\,\omega_1)=0.2 \qquad p(\omega_2)=0.1 \qquad p(x\,|\,\omega_2)=0.4$$

$$\lambda_{11}=0;\lambda_{12}=6;\lambda_{21}=1;\lambda_{22}=0;$$

最小错误率贝叶斯决策： $\boxed{p(\omega_1|\vec{x})>p(\omega_2|\vec{x})}$ *decide* ω_1

$$\boxed{p(\vec{x}|\omega_1)p(\omega_1)>p(\vec{x}|\omega_2)p(\omega_2)}$$

$$p(\vec{x}|\omega_1)p(\omega_1)=0.2\times0.9=0.18 \qquad p(\vec{x}|\omega_2)p(\omega_2)=0.4\times0.1=0.04 \qquad \rightarrow \quad \omega_1$$

将其归为 ω_1 ， 正常

贝叶斯决策

$$p(\omega_1)=0.9 \quad p(x|\omega_1)=0.2 \quad p(\omega_2)=0.1 \quad p(x|\omega_2)=0.4$$

$$\lambda_{11}=0;\lambda_{12}=6;\lambda_{21}=1;\lambda_{22}=0;$$

最小风险贝叶斯决策： $\boxed{\dfrac{p(\vec{x}|\omega_1)}{p(\vec{x}|\omega_2)} > \dfrac{(\lambda_{12}-\lambda_{22})p(\omega_2)}{(\lambda_{21}-\lambda_{11})p(\omega_1)}}$ $decide \quad \omega_1$

$$\frac{p(\vec{x}|\omega_1)}{p(\vec{x}|\omega_2)}=\frac{0.2}{0.4}=0.5 \qquad \frac{(\lambda_{12}-\lambda_{22})p(\omega_2)}{(\lambda_{21}-\lambda_{11})p(\omega_1)}=\frac{6\times0.1}{1\times0.9}=0.67 \rightarrow \omega_2$$

将其归为 ω_2 ， 异常

贝叶斯决策

最小错误率决策 VS 最小风险决策

- 分类错误代价 (Loss)　$\lambda_{ij} = \lambda(\alpha_i | \omega_j)$

- **0-1损失函数（Zero-one loss function）**

$$\lambda_{ij} = \lambda(\alpha_i | \omega_j) = \begin{cases} 0, i = j \\ 1, i \neq j \end{cases}$$

贝叶斯决策

◆ 0-1损失函数 （Zero-one loss function）

$$R(\alpha_i|x) = \sum_{j=1}^{c} \lambda(\alpha_i|\omega_j) p(\omega_j|x) \qquad \lambda_{ij} = \lambda(\alpha_i|\omega_j) = \begin{cases} 0, i = j \\ 1, i \neq j \end{cases}$$

$$= \sum_{j \neq i} p(\omega_j|x)$$

$$= 1 - p(\omega_i|x) \qquad p(\omega_i|\vec{x})$$ 越大， 风险越小

$$\boxed{Decise \quad \omega_i \quad if \quad p(\omega_i|x) > p(\omega_j|x) \; for \quad all \quad j \neq i}$$

最小错误率决策：0-1 损失函数下的最小风险决策

贝叶斯决策

- **0-1损失函数 （Zero-one loss function）**

$$\lambda_{ij} = \lambda(\alpha_i|\omega_j) = \begin{cases} 0, i = j \\ 1, i \neq j \end{cases}$$

最小风险决策:

$$\boxed{\frac{p(\vec{x}|\omega_1)}{p(\vec{x}|\omega_2)} > \frac{(\lambda_{12} - \lambda_{22})p(\omega_2)}{(\lambda_{21} - \lambda_{11})p(\omega_1)}} \quad decide \quad \omega_1$$

$$\boxed{\frac{p(\vec{x}|\omega_1)}{p(\vec{x}|\omega_2)} > \frac{p(\omega_2)}{p(\omega_1)}} \quad decide \quad \omega_1$$

Likelihood ratio

谢 谢

本课程所引用的一些素材为主讲老师多年的教学积累，来源于多种媒体及同事和同行的交流，难以一一注明出处，特此说明并表示感谢！