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CONSTRUCTIONS OF BLOCK-ENCLODING AND THEIR EFFECTS ON QUANTUM SPEEDUPS†

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ABSTRACT. Abstract is here, not exceeding *160* words. It must contain *Main Facts.*

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1. Introcution

On claimed exponential speedups of quantum machine learning algorithms subsequent to HHL algorithm for solving linear system of equations, Tang presented classical counterparts for fair amount of QML algorithms by exploiting basic linear algebraic properties underlying data structures used during accessing matrices in a quantum-advantable way, and corresponding them to classical randomied numerical linear algebra methods. Such process is termed 'dequantization'.

All known linear algebraic QML techinques are captured by Quantum singluar vlaue transformation (QSVT), a unifying framework of quantum algorithms. QSVT can be classified by their input model assumptions, whether if inputs for QML are sparse or low-rank. Since sparse-access input models are known to give expnential speedup, dequantization attacks low-rank input models, where classical data without strong restrictions are appliable.

For classical data, QML algorithms must efficiently prepair them as qunatum states. So we assume the existence of quantum random access memory (QRAM), a qunatum device corresponding to classical RAM. QRAM stores n bits of data

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and query those data in superposition by a polylog(n) time. Dequantization is essentially the process of providing psuedo-QRAM to classical computers, by assumming a input model of sampeling and query access to a vector, which would lead to a fair comparison between quantum and classical machine learning.

Results of dequantization draw a border line for our understanding of QML algorithms and their limitations. Hence, one of the open problems of QML is whether there exist other ways to construct data structures that prevent dequantizaition. We focus on this matter by its basic unit, termed 'Block-encoding'. Our goal is to formally define two alternative data structure implicitly stated by Kerenidis and Prakash, and CHakraborty, Gilyén, and Jeffery, generelizing sparse-input model to QRAM-input model.

2. Nomenclature

3. Block-encoding

The notion of block-encoding was devised as a solution to the problem of Hamiltonian simulation. Hamiltonian simulation, one of the original motivations for designing practical quantum computers, may stated as follows: For time evolution of the wave function $|\psi(t)\rangle$ governed by the Schrödinger equation, that is,

$$i\hbar\frac{\mathrm{d}}{\mathrm{dt}}\left|\psi(t)\right\rangle = H(t)\left|\psi(t)\right\rangle$$

the Hamiltonian, an operator with units of energy, is $H(t)$. Hamiltionian simulation is a problem of designing quantum circuit or unitary matrix U consisting of poly$(n, t, 1/\epsilon)$ gates such that $\|U - e^{iHt}\| \leq \epsilon$. The cost of Hamiltonian simulation depends on the number of qubits n, evolution time t, target error ϵ, and access models of Hamiltonian H. While acheiving optimal Hamiltonian simulation by the process called 'Qubitization', Low and Chuang (2019) defined a *standard-form encoding*, a primitive statement of *block-encoding*. Basically, qubitization is a technique of representing Hermitian or subnormalized matrix as the top-left block of a unitary matrix, that is;

$$U = \begin{bmatrix} A/\alpha & \cdot \\ & \cdot & . \end{bmatrix}$$

where $\cdot$ denotes arbitrary elements of U.

Definition 3.1 (Block-encoding)**.** For $A \in \mathbb{C}^{n\times m}, \alpha, \epsilon \in \mathbb{R}_+$ and $a \in \mathbb{N}$, $(s+a)$-qubit unitary U is an (α, a, ϵ)-block-encoding of A if

$$\|A - \alpha(\langle 0|^{\otimes a} \otimes I)U(|0\rangle^{\otimes a} \otimes I)\| \leq \epsilon.$$

For $n, m \leq 2^s$ we may define an embedding matrix $A_e \in \mathbb{C}^{2^s\times 2^s}$ such that the top-left block of A_e is A and all other entries are 0.

Introduction is here

4. Main results

Main results are here

Theorem 4.1 (Pan and Zhang [1, p. 682 (1.5)])**.** *Theorem is here*

Proof. Proof is here [5, Proposition 1.4] ☐

Lemma 4.2 (Yun [7])**.** *Lemma is here*

Proof. Proof is here ☐

Corollary 4.3 ([7])**.** *Corollary is here*

Definition 4.4. Definition is here

Remark 4.1. Remark is here

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Data availability : In this section, please provide details regarding where data supporting reported results can be found, including links to publicly archived datasets analyzed or generated during the study. If the study did not report any data, you might add "Not applicable" here.

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