Mathematics: A Discrete Introduction
$\sharp$10: Quantifiers

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June 24, 2022

10.1

a. $\forall x \in \mathbb{Z}, x$ is prime.

b. $\exists x \in \mathbb{Z}, \neg(x$ is prime or composite).

c. $\exists x \in \mathbb{Z}, x^2 = 2$.

d. $\forall x \in \mathbb{Z}, 5|x$.

e. $\exists x \in \mathbb{Z}, 7|x$.

f. $\forall x \in \mathbb{Z}, x^2 > 0$.

g. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy = 1$.

h. $\exists x, y \in \mathbb{Z}, x/y = 10$.

i. $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, xy = 0$.

j. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x < y$.

10.2

a. There is an integer that is not prime.
$\exists x \in \mathbb{Z}, x$ is not prime.

b. Every integer is prime or composite.
$\forall x \in \mathbb{Z}, x$ is prime or composite.

c. Every integer is not an integer whose square is 2.
$\forall x \in \mathbb{Z}, \neg(x^2 = 2)$.

d. There is an integer that is not divisible by 5.
$\exists x \in \mathbb{Z}, \neg(5|x)$.

e. All integers are not divisible by 7.
$\forall x \in \mathbb{Z}, \neg(7|x)$.

f. There is an integer whose square is negative.
$\exists x \in \mathbb{Z}, x^2 < 0$.

g. There is an integer x for any integer y such that $xy \neq 1$.
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, xy \neq 1$.

h. For every integer x, y, $x/y \neq= 10$.
$\forall x, y \in \mathbb{Z}, x/y \neq 10$.

i. For every integer x there is an integer y such that $xy \neq 0$.
$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy \neq 0$.

j. There is an integer x which is equal or greater than any integer.
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \geq y$.

10.3

No one is invited to the party.

10.4

a. False

b. True

c. True

d. True

e. False

f. True

g. True

h. True

10.5

a. $\exists x \in \mathbb{Z}, x \geq 0$.
There is an integer that is greater than or equal to 0.

b. $\forall x \in \mathbb{Z}, x \neq x + 1$.
Every integer x is not equal to $x + 1$.

c. $\forall x \in \mathbb{Z}, x \leq 10$.
All integers are less or equal to 10.

d. $\exists x \in \mathbb{Z}, x + x \neq 2x$.
   There is an integer x such that $x + x \neq 2x$.

e. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x \leq y$.
   For every integer x there is an integer y such that $x \leq y$.

f. $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x \neq y$.
   There is an integer x such that there is an integer y which satisfies $x \neq y$.

g. $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x + y \neq 0$.
   There is an integer x for any integer y such that $x + y \neq 0$.

10.6

$$\forall x, \forall y, \text{assertions about } x \text{ and } y$$
$$\forall y, \forall x, \text{assertions about } x \text{ and } y$$

These two statements mean the same thing, since we have "uninterrupted subsequences of quantifiers of the same type". Likewise, the following statements mean the same thing.

$$\exists x, \exists y, \text{assertions about } x \text{ and } y$$
$$\exists y, \exists x, \text{assertions about } x \text{ and } y$$

"Being able to swap the use of the variables is of course due to the fact that variables are just 'dummies', and can therefore be replaced by other variables." Check: https://math.stackexchange.com/q/3515811.