មេរៀនទី ២

រូបមន្តត្រីកោណមាត្រ

1. រូបមន្តផលបូក និងផលដក

1.1. គណនា $\cos(\alpha-\beta)$ និង $\cos(\alpha+\beta)$

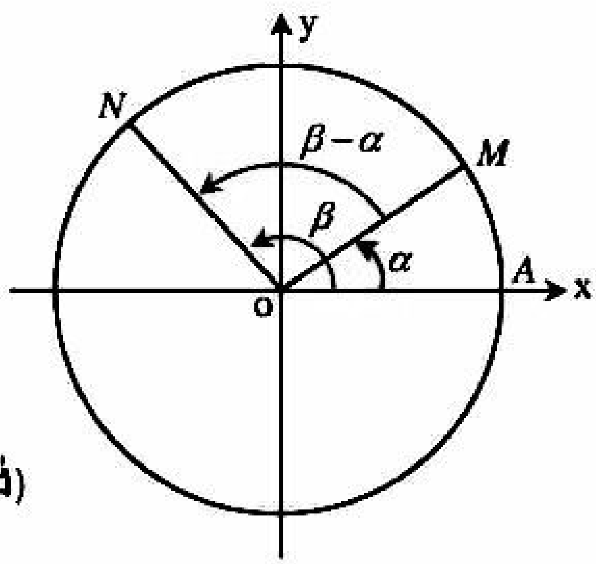
រំលឹក: គេមាន $\vec{u}=(a,b)$ និង $\vec{v}=(a',b')$។

➲ និយមន័យផលគុណស្កាលែ

$\vec{u}\cdot\vec{v}=|\vec{u}|\cdot|\vec{v}|\cos\alpha$ ដែល $\alpha=(\vec{u},\vec{v})$

➲ កន្សោមវិភាគផលគុណស្កាលែ

$\vec{u}\cdot\vec{v}=aa'+bb'$



គេមានរង្វង់ត្រីកោណមាត្រដូចរូប។

គេបាន: $\vec{OM}=(\cos\alpha,\sin\alpha)$

$\vec{ON}=(\cos\beta,\sin\beta)$

$(\vec{OM},\vec{ON})=\beta-\alpha$

$OM=ON=1$ (កាំរង្វង់)

❖ តាមនិយមន័យផលគុណស្កាលែ:

$\vec{OM}\cdot\vec{ON}=OM\times ON\times\cos(\beta-\alpha)$

$=\cos[-(\alpha-\beta)]=\cos(\alpha-\beta)$

❖ ម្យ៉ាងទៀតតាមកន្សោមវិភាគផលគុណស្កាលែ:

$$\overrightarrow{OM}\cdot\overrightarrow{ON}=\cos\alpha\cos\beta \quad \sin\alpha\sin\beta$$

ចំពោះ $\begin{cases}\overrightarrow{OM}\cdot\overrightarrow{ON}=\cos(\alpha \quad \beta) \\ \overrightarrow{OM}\cdot\overrightarrow{ON}=\cos\alpha\cos\beta \quad \sin\alpha\sin\beta\end{cases}$

គេទាញបាន $\cos(\alpha-\beta)\cos\alpha\cos\beta+\sin\alpha\sin\beta$

ដូចនេះ: $\boxed{\cos(\alpha-\beta)=\cos\alpha\cos\beta \quad \sin\alpha\sin\beta}$ (1)

ចំពោះ: $\cos(\alpha+\beta)=\cos[\alpha \quad (\quad \beta)]$

$$=\cos\alpha\cos(\quad \beta) \quad \sin\alpha\sin(\quad \beta)$$

$$=\cos\alpha\cos\beta \quad \sin\alpha\sin\beta$$

ដូចនេះ: $\boxed{\cos(\alpha+\beta)=\cos\alpha\cos\beta \quad \sin\alpha\sin\beta}$ (2)

លំហាត់គំរូ ១ : គណនា $\cos 15^\circ$, $\cos 105^\circ$ និង $\cos\dfrac{5\pi}{12}$ ។

ចម្លើយ

$$\cos 15^\circ=\cos(45^\circ \quad 30^\circ)$$

$$=\cos 45^\circ\cos 30^\circ+\sin 45^\circ\sin 30^\circ$$

$$=\frac{\sqrt{2}}{2}\ \frac{\sqrt{3}}{2} \quad \frac{\sqrt{2}}{2}\ \frac{1}{2} \quad \frac{\sqrt{6}+\sqrt{2}}{4}$$

ដូចនេះ: $\boxed{\cos 15^\circ=\dfrac{\sqrt{6}+\sqrt{2}}{4}}$

$$\cos 105^\circ=\cos(45^\circ \quad 60^\circ)$$

$$=\cos 45^\circ\cos 60^\circ-\sin 45^\circ\sin 60^\circ$$

$$=\frac{\sqrt{2}}{2}\ \frac{1}{2} \quad \frac{\sqrt{2}}{2}\ \frac{\sqrt{3}}{2} \quad \frac{\sqrt{2}-\sqrt{6}}{4}$$

ដូចនេះ: $\boxed{\cos 105^\circ=\dfrac{\sqrt{2}-\sqrt{6}}{4}}$

$$\cos\frac{5\pi}{12}=\cos(\frac{\pi}{4}\quad\frac{\pi}{6})$$
$$=\cos\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\sin\frac{\pi}{6}$$
$$=\frac{\sqrt{2}}{2}\ \frac{\sqrt{3}}{2}\quad\frac{\sqrt{2}}{2}\ \frac{1}{2}\quad=\frac{\sqrt{6}-\sqrt{2}}{4}$$

ដូចនេះ: $\boxed{\cos\frac{5\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}}$

លំហាត់គំរូ ២ : គេមានមុំ α និង β ដែល $\sin\alpha=\frac{4}{5}$, $\cos\alpha\quad\frac{3}{5}$, $\sin\beta=\ \frac{12}{13}$ និង $\cos\beta=\frac{5}{13}$ ។ គណនាតម្លៃ $\cos(\alpha+\beta)$ និង $\cos(\alpha-\beta)$ ។

ចម្លើយ

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta\quad\sin\alpha\sin\beta$$
$$=\frac{3}{5}\ \frac{5}{13}\quad\frac{4}{5}\ \frac{12}{13}\quad\frac{15}{65}\quad\frac{48}{65}\qquad\frac{33}{65}$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta\quad\sin\alpha\sin\beta$$
$$=\frac{3}{5}\cdot\frac{5}{13}+\frac{4}{5}\cdot\frac{12}{13}=\frac{15}{65}+\frac{48}{65}=\frac{63}{65}$$

ដូចនេះ: $\boxed{\cos(\alpha+\beta)=\quad\frac{33}{65}\ ,\ \cos(\alpha\quad\beta)\quad\frac{63}{65}}$

លំហាត់គំរូ ៣ : ស្រាយបញ្ជាក់ថា

$$\frac{\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta)}{2\sin^2\alpha\sin^2\beta}-1=\cot^2\alpha\cot^2\beta\ ។$$

សម្រាយបញ្ជាក់

គេបាន $\frac{\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta)}{2\sin^2\alpha\sin^2\beta}-1$

$$=\frac{(\cos\alpha\cos\beta-\sin\alpha\sin\beta)^2+(\cos\alpha\cos\beta+\sin\alpha\sin\beta)^2}{2\sin^2\alpha\sin^2\beta}-1$$

$$=\frac{\cos^2\alpha\cos^2\beta-2(\cos\alpha\cos\beta)(\sin\alpha\sin\beta)+\sin^2\alpha\sin^2\beta}{2\sin^2\alpha\sin^2\beta}$$

$$+\frac{\cos^2\alpha\cos^2\beta+2(\cos\alpha\cos\beta)(\sin\alpha\sin\beta)+\sin^2\alpha\sin^2\beta}{2\sin^2\alpha\sin^2\beta}-1$$

$$=\frac{2\cos^2\alpha\cos^2\beta+2\sin^2\alpha\sin^2\beta)}{2\sin^2\alpha\sin^2\beta}-1$$

$$=\frac{2(\cos^2\alpha\cos^2\beta+\sin^2\alpha\sin^2\beta)}{2\sin^2\alpha\sin^2\beta}-1$$

$$=\frac{\cos^2\alpha\cos^2\beta}{\sin^2\alpha\sin^2\beta}+\frac{\sin^2\alpha\sin^2\beta}{\sin^2\alpha\sin^2\beta}-1$$

$$=\cot^2\alpha\cot^2\beta+1-1=\cot^2\alpha\cot^2\beta$$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\frac{\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta)}{2\sin^2\alpha\sin^2\beta}-1=\cot^2\alpha\cot^2\beta}$

1.2. គណនា $\sin(\alpha+\beta)$ **និង** $\sin(\alpha-\beta)$

$$\sin(\alpha+\beta)=\cos\left[\frac{\pi}{2}\quad(\alpha-\beta)+\right]\quad\cos\left[(\frac{\pi}{2}\quad\alpha)\quad\beta\right]$$

$$=\cos(\frac{\pi}{2}-\alpha)\cos\beta+\sin(\frac{\pi}{2}-\alpha)\sin\beta$$

$$=\sin\alpha\cos\beta\quad\cos\alpha\sin\beta$$

ដូចនេះ: $\boxed{\sin(\alpha+\beta)=\sin\alpha\cos\beta\quad\cos\alpha\sin\beta}$ (3)

$$\sin(\alpha-\beta)=\sin[\alpha\quad(\quad\beta)]$$

$$=\sin\alpha\cos(\quad\beta)\quad\cos\alpha\sin(\quad\beta)=\sin\alpha\cos\beta\quad\cos\alpha\sin\beta$$

ដូចនេះ: $\boxed{\sin(\alpha-\beta)=\sin\alpha\cos\beta\quad\cos\alpha\sin\beta}$ (4)

លំហាត់គំរូ ១ : គណនា $\sin\frac{\pi}{12}$, $\sin 75^\circ$ និង $\sin 105^\circ$ ។

ចម្លើយ

$$\sin\frac{\pi}{12} = \sin\frac{3\pi - 2\pi}{12} = \sin\left(\frac{\pi}{4} \quad \frac{\pi}{6}\right)$$

$$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} \quad \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2} \ \frac{\sqrt{3}}{2} \quad \frac{\sqrt{2}}{2} \ \frac{1}{2} \quad \frac{\sqrt{6}-\sqrt{2}}{4}$$

ដូចនេះ: $\boxed{\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}}$

$$\sin 75^\circ = \sin(30^\circ \quad 45^\circ)$$

$$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$$

$$= \frac{1}{2} \ \frac{\sqrt{2}}{2} \quad \frac{\sqrt{3}}{2} \ \frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}+\sqrt{6}}{4}$$

ដូចនេះ: $\boxed{\sin 75^\circ = \frac{\sqrt{2}+\sqrt{6}}{4}}$

$$\sin 105^\circ = \sin(60^\circ \quad 45^\circ)$$

$$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \ \frac{\sqrt{2}}{2} \quad \frac{1}{2} \ \frac{\sqrt{2}}{2} \quad \frac{\sqrt{6}+\sqrt{2}}{4}$$

ដូចនេះ: $\boxed{\sin 105^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}}$

លំហាត់គំរូ ២ : ចូរស្រាយបញ្ជាក់ថា $\sin\theta + \cos\theta = \sqrt{2}\sin(\theta \quad 45^\circ)$

រួចទាញថា $-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$ ។

សម្រាយបញ្ជាក់

គេបាន $\sin\theta+\cos\theta=\sqrt{2}(\frac{\sqrt{2}}{2}\sin\theta \quad \frac{\sqrt{2}}{2}\cos\theta)$

$$=\sqrt{2}(\cos 45^\circ \sin\theta+\sin 45^\circ \cos\theta)$$

$$=\sqrt{2}(\sin\theta\cos 45^\circ+\cos\theta\sin 45^\circ)$$

$$=\sqrt{2}\sin(\theta \quad 45^\circ)$$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\sin\theta+\cos\theta=\sqrt{2}\sin(\theta \quad 45^\circ)}$

គេមាន $-1\le\sin(\theta+45^\circ)\le 1$ ចំពោះគ្រប់ θ

$$(-1)\sqrt{2}\le\sin(\theta+45^\circ)\le(1)\sqrt{2}$$

$$-\sqrt{2}\le\sqrt{2}\sin(\theta+45^\circ)\le\sqrt{2}$$

$$-\sqrt{2}\le\sin\theta+\cos\theta\le\sqrt{2}$$

ដូចនេះ: $\boxed{-\sqrt{2}\le\sin\theta+\cos\theta\le\sqrt{2}}$

លំហាត់គំរូ ៣ : បង្ហាញថា $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}+\frac{\sin(\beta-\theta)}{\cos\beta\cos\theta}+\frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha}=0$ ។

សម្រាយបញ្ជាក់

គេបាន $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}+\frac{\sin(\beta-\theta)}{\cos\beta\cos\theta}+\frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha}$

$$=\frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta}+\frac{\sin\beta\cos\theta-\cos\beta\sin\theta}{\cos\beta\cos\theta}$$

$$+\frac{\sin\theta\cos\alpha-\cos\theta\sin\alpha}{\cos\alpha\cos\theta}$$

$$=\left(\frac{\sin\alpha}{\cos\alpha} \quad \frac{\sin\beta}{\cos\beta}\right)+\left(\frac{\sin\beta}{\cos\beta} \quad \frac{\sin\theta}{\cos\theta}\right)+\left(\frac{\sin\theta}{\cos\theta} \quad \frac{\sin\alpha}{\cos\alpha}\right)$$

$$=\tan\alpha-\tan\beta+\tan\beta-\tan\theta+\tan\theta-\tan\alpha=0$$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}+\frac{\sin(\beta-\theta)}{\cos\beta\cos\theta}+\frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha}=0}$

លំហាត់គំរូ ៤ : ស្រាយបញ្ជាក់ថា

$$\frac{\sin^2(\alpha+\beta)+\sin^2(\alpha-\beta)}{2\cos^2\alpha\cos^2\beta}=\tan^2\alpha+\tan^2\beta$$ ។

សម្រាយបញ្ជាក់

គេបាន $\dfrac{\sin^2(\alpha+\beta)+\sin^2(\alpha-\beta)}{2\cos^2\alpha\cos^2\beta}$

$$=\frac{(\sin\alpha\cos\beta+\cos\alpha\sin\beta)^2+(\sin\alpha\cos\beta-\cos\alpha\sin\beta)^2}{2\cos^2\alpha\cos^2\beta}$$

$$=\frac{\sin^2\alpha\cos^2\beta+2(\sin\alpha\cos\beta)(\cos\alpha\sin\beta)+\cos^2\alpha\sin^2\beta}{2\cos^2\alpha\cos^2\beta}$$

$$+\frac{\sin^2\alpha\cos^2\beta-2(\sin\alpha\cos\beta)(\cos\alpha\sin\beta)+\cos^2\alpha\sin^2\beta}{2\cos^2\alpha\cos^2\beta}$$

$$=\frac{2\sin^2\alpha\cos^2\beta+2\cos^2\alpha\sin^2\beta)}{2\cos^2\alpha\cos^2\beta}$$

$$=\frac{2(\sin^2\alpha\cos^2\beta+\cos^2\alpha\sin^2\beta)}{2\cos^2\alpha\cos^2\beta}$$

$$=\frac{\sin^2\alpha\cos^2\beta}{\cos^2\alpha\cos^2\beta}+\frac{\cos^2\alpha\sin^2\beta}{\cos^2\alpha\cos^2\beta}$$

$$=\frac{\sin^2\alpha}{\cos^2\alpha}+\frac{\sin^2\beta}{\cos^2\beta}=\tan^2\alpha+\tan^2\beta$$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\dfrac{\sin^2(\alpha+\beta)+\sin^2(\alpha-\beta)}{2\cos^2\alpha\cos^2\beta}=\tan^2\alpha+\tan^2\beta}$

1.3. គណនា $\tan(\alpha+\beta)$ **និង** $\tan(\alpha-\beta)$

គេមាន $\tan(\alpha+\beta)=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$

គេចែកតួទាំងពីរនៃផលធៀបនឹង $\cos\alpha\cos\beta$ គេបាន:

$$\tan(\alpha+\beta)=\frac{\left(\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}\right)}{\left(\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}\right)}$$

$$=\frac{\left(\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\sin\beta}{\cos\beta}\right)}{\left(1-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}\right)}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

ដូចនេះ: $\boxed{\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}}$ (5)

បើគេជំនួស β ដោយ $(-\beta)$ ក្នុងរូបមន្ត $\tan(\alpha+\beta)$ គេបាន:

$$\tan(\alpha-\beta)=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$$

$$=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

ដូចនេះ: $\boxed{\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}}$ (6)

លំហាត់គំរូ ១ : ដោយប្រើ $15^\circ=45^\circ-30^\circ$ ចូរគណនាតម្លៃនៃ $\tan15^\circ$ ។

ចម្លើយ

គណនាតម្លៃនៃ $\tan15^\circ$

$$\tan15^\circ=\tan(45^\circ-30^\circ)=\frac{\tan45^\circ-\tan30^\circ}{1+\tan45^\circ\tan30^\circ}$$

$$=\frac{1-\dfrac{\sqrt3}{3}}{1+\dfrac{\sqrt3}{3}}=\frac{3-\sqrt3}{3+\sqrt3}=\frac{(3-\sqrt3)^2}{(3+\sqrt3)(3-\sqrt3)}=\frac{9-6\sqrt3+3}{9-3}$$

$$= \frac{12-6\sqrt{3}}{6} = \frac{6(2-\sqrt{3})}{6} = 2-\sqrt{3}$$

ដូចនេះ: $\boxed{\tan 15^\circ = 2-\sqrt{3}}$

លំហាត់គំរូ ២ : ដោយប្រើ $75^\circ = 45^\circ + 30^\circ$ ចូរគណនាតម្លៃនៃ $\tan 75^\circ$ ។

ចម្លើយ

គណនាតម្លៃនៃ $\tan 75^\circ$

$$\tan 75^\circ = \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1-\tan 45^\circ \tan 30^\circ}$$

$$= \frac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})^2}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{9+6\sqrt{3}+3}{9-3}$$

$$= \frac{12+6\sqrt{3}}{6} = \frac{6(2+\sqrt{3})}{6} = 2+\sqrt{3}$$

ដូចនេះ: $\boxed{\tan 75^\circ = 2+\sqrt{3}}$

លំហាត់គំរូ ៣ : ចំពោះ ΔABC ចូរបង្ហាញថា

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$ ។

សម្រាយបញ្ជាក់

បង្ហាញថា $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

ដោយ A, B, C ជាមុំនៃ ΔABC នោះ: $A, B, C > 0^\circ$ និង $A+B+C = 180^\circ$

នាំឲ្យ $C = 180^\circ - (A+B)$ គេបាន:

$$\tan A + \tan B + \tan C = \tan A + \tan B + \tan[180^\circ - (A+B)]$$

$$= \tan A + \tan B - \tan(A+B)$$

$$= \tan A + \tan B - \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$=(\tan A \quad \tan B)\left(-1 \quad \frac{1}{1-\tan A\tan B}\right)$$

$$=(\tan A \quad \tan B)\left(\frac{1-\tan A\tan B-1}{1-\tan A\tan B}\right)$$

$$=(\tan A \quad \tan B)\left(\frac{-\tan A\tan B}{1-\tan A\tan B}\right)$$

$$=(\quad \tan A\tan B)\left(\frac{\tan A+\tan B}{1-\tan A\tan B}\right)$$

$$=(-\tan A\tan B)\tan(A+B)=\tan A\tan B(\quad \tan(A \quad B))$$

$$=\tan A\tan B(\tan(180^\circ-(A+B)))$$

$$=\tan A\tan B(\tan C)=\tan A\tan B\tan C$$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\tan A+\tan B+\tan C=\tan A\tan B\tan C}$

លំហាត់គំរូ k : ចូរសម្រួលកន្សោម $\dfrac{\tan 3\alpha-\tan\alpha}{1+\tan\alpha\tan 3\alpha}+\cot\left(\dfrac{\pi}{2}+2\alpha\right)$។

ចម្លើយ

គេបាន $\dfrac{\tan 3\alpha-\tan\alpha}{1+\tan\alpha\tan 3\alpha}+\cot\left(\dfrac{\pi}{2}+2\alpha\right)$

$$=\tan(3\alpha \quad \alpha)-\tan 2\alpha$$

$$=\tan 2\alpha-\tan 2\alpha=0$$

ដូចនេះ: $\boxed{\dfrac{\tan 3\alpha-\tan\alpha}{1+\tan\alpha\tan 3\alpha}+\cot\left(\dfrac{\pi}{2}+2\alpha\right)=0}$

1.4. គណនា $\cot(\alpha+\beta)$ **និង** $\cot(\alpha-\beta)$

គេមាន $\cot(\alpha+\beta)=\dfrac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}=\dfrac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta}$

គេចែកតួទាំងពីរនៃផលធៀបនឹង $\sin\alpha\sin\beta$ គេបាន:

$$\cot(\alpha+\beta)=\frac{\left(\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}-\dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}\right)}{\left(\dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta}+\dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}\right)}$$

$$=\frac{\left(\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}-1\right)}{\left(\dfrac{\cos\beta}{\sin\beta}+\dfrac{\cos\alpha}{\sin\alpha}\right)}=\frac{\left(\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}-1\right)}{\left(\dfrac{\cos\beta}{\sin\beta}+\dfrac{\cos\alpha}{\sin\alpha}\right)}=\frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha}$$

ដូចនេះ: $\boxed{\cot(\alpha+\beta)=\dfrac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha}}$ (7)

បើគេជំនួស β ដោយ $(-\beta)$ ក្នុងរូបមន្ត $\cot(\alpha+\beta)$ គេបាន:

$$\cot(\alpha-\beta)=\frac{\cot\alpha\cot(-\beta)-1}{\cot(-\beta)+\cot\alpha}$$

$$=\frac{-\cot\alpha\cot\beta-1}{-\cot\beta+\cot\alpha}=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$$

ដូចនេះ: $\boxed{\cot(\alpha-\beta)=\dfrac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}}$ (8)

លំហាត់គំរូ ១ : គេមានមុំ α និង β ដែល $\cot\alpha=\dfrac{3}{4}$ និង $\cot\beta=\dfrac{5}{12}$ ។

គណនាតម្លៃ $\cot(\alpha+\beta)$ និង $\cot(\alpha-\beta)$ ។

ចម្លើយ

គណនាតម្លៃ $\cot(\alpha+\beta)$ និង $\cot(\alpha-\beta)$

$$\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha} \quad \frac{\left(\dfrac{3}{4}\right)\left(\dfrac{5}{12}\right)-1}{\dfrac{5}{12}+\dfrac{3}{4}}=\frac{\left(\dfrac{5}{16}-1\right)}{\left(\dfrac{5+9}{12}\right)}$$

$$=\frac{\left(-\frac{11}{16}\right)}{\left(\frac{14}{12}\right)} \times \frac{11}{16} = \frac{12}{14} \quad \frac{33}{56}$$

ដូចនេះ: $\boxed{\cot(\alpha+\beta) = \quad \frac{33}{56}}$

$$\cot(\alpha-\beta)=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha} \quad \frac{\left(\frac{3}{4}\right)\left(\frac{5}{12}\right)+1}{\frac{5}{12}-\frac{3}{4}} = \frac{\left(\frac{5}{16}+1\right)}{\left(\frac{5-9}{12}\right)}$$

$$=\frac{\left(\frac{21}{16}\right)}{\left(-\frac{4}{12}\right)} - \left(\frac{21}{16}\right)\left(\quad\frac{12}{4}\right) \quad \frac{63}{16}$$

ដូចនេះ: $\boxed{\cot(\alpha+\beta) = \quad \frac{63}{16}}$

លំហាត់គំរូ ២ : ចូរបង្ហាញថា $\frac{\cot(\alpha+\beta)}{\cot\alpha\cot\beta}=\frac{1-\tan\alpha\tan\beta}{\cot\alpha+\cot\beta}$ ។

សម្រាយបញ្ជាក់

គេបាន $\frac{\cot(\alpha+\beta)}{\cot\alpha\cot\beta}=\frac{1}{\cot\alpha\cot\beta} \ \cot(\alpha \quad \beta)$

$$=\frac{1}{\cot\alpha\cot\beta} \ \frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}=\frac{1}{\cot\alpha+\cot\beta}\cdot\frac{\cot\alpha\cot\beta-1}{\cot\alpha\cot\beta}$$

$$=\frac{1}{\cot\alpha+\cot\beta}\left(1 \quad \frac{1}{\cot\alpha\cot\beta}\right)$$

$$=\frac{1}{\cot\alpha+\cot\beta}\ (1 \quad \tan\alpha\tan\beta)=\frac{1-\tan\alpha\tan\beta}{\cot\alpha+\cot\beta}$$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\dfrac{\cot(\alpha+\beta)}{\cot\alpha\cot\beta}=\dfrac{1-\tan\alpha\tan\beta}{\cot\alpha+\cot\beta}}$

លំហាត់គំរូ ៣ : ចូរបង្ហាញថា

$$\cot(\alpha+\beta)\cdot\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}+\cot(\alpha-\beta)(\cot\alpha-\cot\beta)=-2\text{។}$$

សម្រាយបញ្ជាក់

គេបាន $\cot(\alpha+\beta)\cdot\dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}+\cot(\alpha-\beta)(\cot\alpha-\cot\beta)$

$$=\frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}\cdot\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}-\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}(\cot\alpha-\cot\beta)$$

$$=\frac{\cos(\alpha+\beta)}{\sin\alpha\sin\beta}+\frac{(\cot\alpha\cot\beta+1)}{(\cot\alpha-\cot\beta)}(\cot\alpha-\cot\beta)$$

$$=\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\sin\beta}-(\cot\alpha\cot\beta+1)$$

$$=\left(\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}-1\right)-(\cot\alpha\cot\beta+1)$$

$$=(\cot\alpha\cot\beta-1)-(\cot\alpha\cot\beta+1)$$

$$=\cot\alpha\cot\beta-1-\cot\alpha\cot\beta-1=-2$$

ដូចនេះ: $\boxed{\cot(\alpha+\beta)\cdot\dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}+\cot(\alpha-\beta)(\cot\alpha-\cot\beta)=-2}$

2. អនុវត្តន៍រូបមន្តផលបូក

2.1. រូបមន្តមុំឌុប

ក/. គេឲ្យ $\alpha=\beta$ ជំនួស β ដោយ α ក្នុងរូបមន្ត $\sin(\alpha+\beta)$

គេបាន: $\sin 2\alpha=\sin(\alpha+\alpha)$

$$=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha$$

$$= \sin\alpha\cos\alpha + \sin\alpha\cos\alpha$$

$$= 2\sin\alpha\cos\alpha$$

ដូចនេះ: $\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$ (9)

ខ/. បើជំនួស β ដោយ α ទៅក្នុងរូបមន្ត $\cos(\alpha+\beta)$

គេបាន: $\cos 2\alpha = \cos(\alpha + \alpha)$

$$= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$$

$$= \cos^2\alpha - \sin^2\alpha$$

$$= \cos^2\alpha - (1 - \cos^2\alpha) = 2\cos^2\alpha - 1$$

$$= (1 - \sin^2\alpha) - \sin^2\alpha = 1 - 2\sin^2\alpha$$

ដូចនេះ: $$\boxed{\begin{aligned} \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \end{aligned}} \quad (10)$$

❖ ចំពោះ: $\cos 2\alpha = 2\cos^2\alpha - 1$

$$2\cos^2\alpha = 1 + \cos 2\alpha$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

ដូចនេះ: $\boxed{\cos^2\alpha = \frac{1+\cos 2\alpha}{2}}$

❖ ចំពោះ: $\cos 2\alpha = 1 - 2\sin^2\alpha$

$$2\sin^2\alpha = 1 - \cos 2\alpha$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

ដូចនេះ: $\boxed{\sin^2\alpha = \frac{1-\cos 2\alpha}{2}}$

គេទាញបាន $\tan^2\alpha = \dfrac{\sin^2\alpha}{\cos^2\alpha} = \dfrac{\left(\dfrac{1-\cos 2\alpha}{2}\right)}{\left(\dfrac{1+\cos 2\alpha}{2}\right)}$

$$=\left(\frac{1-\cos 2\alpha}{2}\right)\left(\frac{2}{1+\cos 2\alpha}\right)=\frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

ដូចនេះ: $\boxed{\tan^2\alpha = \dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}}$

គ/. បើជំនួស β ដោយ α ទៅក្នុងរូបមន្ត $\tan(\alpha+\beta)$

គេបាន: $\tan 2\alpha = \tan(\alpha+\alpha)$

$$=\frac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

ដូចនេះ: $\boxed{\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}}$ (11)

ឃ/. បើជំនួស β ដោយ α ទៅក្នុងរូបមន្ត $\cot(\alpha+\beta)$

គេបាន: $\cot 2\alpha = \cot(\alpha+\alpha)$

$$=\frac{\cot\alpha\cot\alpha-1}{\cot\alpha+\cot\alpha}=\frac{\cot^2\alpha-1}{2\cot\alpha}$$

ដូចនេះ: $\boxed{\cot 2\alpha = \dfrac{\cot^2\alpha-1}{2\cot\alpha}}$ (12)

ង/. ចំពោះរូបមន្ត $\sin 3\alpha$ ដោយ $\cos 3\alpha$ គេបាន:

$$\sin 3\alpha = \sin(2\alpha+\alpha) = \sin 2\alpha\cos\alpha + \cos 2\alpha\sin\alpha$$

$$=(2\sin\alpha\cos\alpha)\cos\alpha+\cos 2\alpha\sin\alpha$$

$$=2\sin\alpha\cos^2\alpha+\cos 2\alpha\sin\alpha$$

$$=2\sin\alpha(1-\sin^2\alpha)+(1-2\sin^2\alpha)\sin\alpha$$

$$=2\sin\alpha-2\sin^3\alpha+\sin\alpha-2\sin^3\alpha$$

$$= 3\sin\alpha - 4\sin^3\alpha$$

ដូចនេះ: $\boxed{\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha}$ (13)

$$\cos 3\alpha = \cos(2\alpha + \alpha) = \cos 2\alpha\cos\alpha - \sin 2\alpha\sin\alpha$$

$$= (2\cos^2\alpha - 1)\cos\alpha - (2\sin\alpha\cos\alpha)\sin\alpha$$

$$= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha\sin^2\alpha$$

$$= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha(1 - \cos^2\alpha)$$

$$= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha$$

$$= 4\cos^3\alpha - 3\cos\alpha$$

ដូចនេះ: $\boxed{\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha}$ (14)

លំហាត់គំរូ ១ : គណនា $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$ និង $\cot 2\alpha$ បើគេដឹងថា

$$\cos\alpha = -\frac{4}{5} \text{ និង } \frac{\pi}{2} < \alpha < \pi \text{ ។}$$

ចម្លើយ

គណនា $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$ និង $\cot 2\alpha$

តាមរូបមន្ត $\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \sin\alpha = \pm\sqrt{1 - \cos^2\alpha}$

ដោយ $\frac{\pi}{2} < \alpha < \pi$ នោះ $\sin\alpha > 0$ គេបាន:

$$\sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - \left(-\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$$

$$\cos 2\alpha = 2\cos^2\alpha - 1 = 2\left(-\frac{4}{5}\right)^2 - 1 = \frac{32}{25} - 1 = \frac{7}{25}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\left(-\frac{24}{25}\right)}{\left(\frac{7}{25}\right)} = \left(-\frac{24}{25}\right)\left(\frac{25}{7}\right) = -\frac{24}{7}$$

$$\cot 2\alpha = \frac{1}{\tan 2\alpha} = -\frac{7}{24}$$

ដូចនេះ: $\boxed{\sin 2\alpha = -\frac{24}{25}, \cos 2\alpha = \frac{7}{25}, \tan 2\alpha = -\frac{24}{7}, \cot 2\alpha = -\frac{7}{24}}$

លំហាត់ទី ២ : បង្ហាញថា $\frac{4\tan\alpha(1-\tan^2\alpha)}{(1+\tan^2\alpha)^2} = \sin 4\alpha$ ។

សម្រាយបញ្ជាក់

គេបាន $\frac{4\tan\alpha(1-\tan^2\alpha)}{(1+\tan^2\alpha)^2} = \frac{4\cdot\frac{\sin\alpha}{\cos\alpha}\left(1-\frac{\sin^2\alpha}{\cos^2\alpha}\right)}{\left(\frac{1}{\cos^2\alpha}\right)^2}$

$$= \frac{4\cdot\frac{\sin\alpha}{\cos\alpha}\left(\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha}\right)}{\left(\frac{1}{\cos^4\alpha}\right)} = 4\left(\frac{\sin\alpha}{\cos\alpha}\right)\left(\frac{\cos 2\alpha}{\cos^2\alpha}\right)(\cos^4\alpha)$$

$$= 4\left(\frac{\sin\alpha}{\cos\alpha}\right)\left(\frac{\cos 2\alpha}{\cos^2\alpha}\right)(\cos^4\alpha) = 4\sin\alpha\cos 2\alpha\cos\alpha$$

$$= 2(2\sin\alpha\cos\alpha)\cos 2\alpha = 2\sin 2\alpha\cos 2\alpha = \sin 4\alpha$$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\frac{4\tan\alpha(1-\tan^2\alpha)}{(1+\tan^2\alpha)^2} = \sin 4\alpha}$

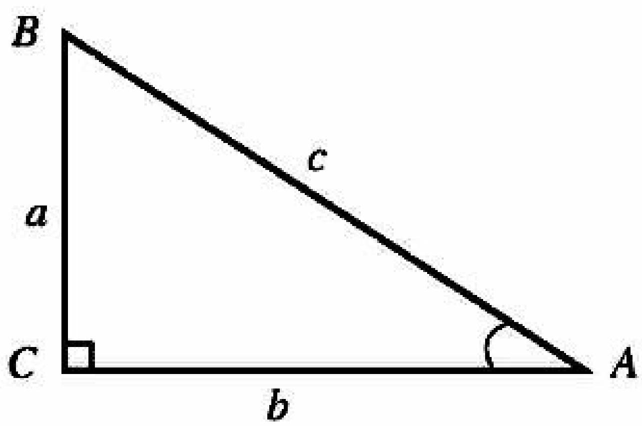
លំហាត់ទី ៣ : គេឲ្យត្រីកោណកែង ΔABC ដែល $C = 90^\circ$ ។ ចូរបង្ហាញថា:

ក/. $\sin 2A = \dfrac{2ab}{c^2}$ ខ/. $\cos 2A = \dfrac{b^2 - a^2}{c^2}$

គ/. $\tan 2A = \dfrac{2ab}{b^2 - a^2}$

ចម្លើយ

គេមានត្រីកោណកែង ΔABC ដែល $C = 90^\circ$



បង្ហាញថា:

ក/. $\sin 2A = \dfrac{2ab}{c^2}$

តាមរូបមន្ត $\sin 2A = 2\sin A \cos A$

ដោយ $\sin A = \dfrac{a}{c}$ និង $\cos A = \dfrac{b}{c}$

គេបាន $\sin 2A = 2 \cdot \dfrac{a}{c} \cdot \dfrac{b}{c} = \dfrac{2ab}{c^2}$

ដូចនេះ បញ្ជាក់ថា $\boxed{\sin 2A = \dfrac{2ab}{c^2}}$

ខ/. $\cos 2A = \dfrac{b^2 - a^2}{c^2}$

តាមរូបមន្ត $\cos 2A = 1 - 2\sin^2 A$

គេបាន $\cos 2A = 1 \quad 2\left(\frac{a}{c}\right)^2 \quad 1 = \frac{2a^2}{c^2} \quad \frac{c^2-2a^2}{c^2}$

តាមច្បាប់ពីតាគ័រ $c^2 = a^2 \quad b^2 +$

នាំឲ្យ $\cos 2A = \frac{a^2+b^2-2a^2}{c^2} = \frac{b^2-a^2}{c^2}$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\cos 2A = \frac{b^2-a^2}{c^2}}$

គ/. $\tan 2A = \frac{2ab}{b^2-a^2}$

$$\tan 2A = \frac{\sin 2A}{\cos 2A} \quad \frac{\left(\frac{2ab}{c^2}\right)}{\left(\frac{b^2-a^2}{c^2}\right)} \quad \left(\frac{2ab}{c^2}\right)\left(\frac{c^2}{b^2-a^2}\right) = \frac{2ab}{b^2-a^2}$$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\tan 2A = \frac{2ab}{b^2-a^2}}$

លំហាត់ទី ៤ : បង្ហាញថា $\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)} = 1$ ។

សម្រាយបញ្ជាក់

គេបាន $\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$

$$= \frac{\cos 2\alpha}{2\cot\left(\frac{\pi}{2}-\frac{\pi}{2}+\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$$

$$=\frac{\cos 2\alpha}{2\cot\left(\frac{\pi}{2}-\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$$

$$=\frac{\cos 2\alpha}{2\cot\left(\frac{\pi}{2}-(\frac{\pi}{4}-\alpha)\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$$

$$=\frac{\cos 2\alpha}{2\tan\left(\frac{\pi}{4}-\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$$

$$=\frac{\cos 2\alpha}{2\cdot\frac{\sin\left(\frac{\pi}{4}-\alpha\right)}{\cos\left(\frac{\pi}{4}-\alpha\right)}\cdot\cos^2\left(\frac{\pi}{4}-\alpha\right)}$$

$$=\frac{\cos 2\alpha}{2\cdot\sin\left(\frac{\pi}{4}-\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)}$$

$$=\frac{\cos 2\alpha}{\sin 2\left(\frac{\pi}{4}-\alpha\right)}=\frac{\cos 2\alpha}{\sin\left(\frac{\pi}{2}-2\alpha\right)}=\frac{\cos 2\alpha}{\cos 2\alpha}\quad 1$$

ដូចនេះ បញ្ជាក់ថា $\boxed{\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}=1}$

2.2. រូបមន្តកន្លះមុំ

គេមានរូបមន្ត $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\tan^2\alpha = \frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

ក/. បើគេជំនួស 2α ដោយ α និង α ដោយ $\frac{\alpha}{2}$ ទៅក្នុងរូបមន្តខាងលើ

គេបាន: $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$\tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

ដូចនេះ:
$$\boxed{\begin{array}{ll} \sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\,, & \sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} \\ \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}\,, & \tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha} \end{array}}$$

➲ រូបមន្តម្យ៉ាងទៀតរបស់ $\tan\frac{\alpha}{2}$ គឺ

$$\tan\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \quad \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \frac{\sin\alpha}{2\cos^2\frac{\alpha}{2}} = \frac{\sin\alpha}{1+\cos\alpha}$$

ដូចនេះ: $\boxed{\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}}$

ខ/. បើគេជំនួស 2α ដោយ α និង α ដោយ $\frac{\alpha}{2}$ ទៅក្នុងរូបមន្ត $(9),(10),(11)$

គេបាន: $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$$\cos\alpha = \cos^2\frac{\alpha}{2} \quad \sin^2\frac{\alpha}{2}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$

ដោយ $\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}=1$ គេបាន:

$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{1} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$$

$$\cos\alpha = \cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2} = \frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{1} = \frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$$

ចែកភាគយក និងភាគបែងនៃកន្សោមទាំងពីរនឹង $\cos^2\frac{\alpha}{2}$ ដែល $\cos\frac{\alpha}{2}\neq 0$

គេបាន $\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ និង $\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$

បើគេឲ្យ $\tan\frac{\alpha}{2}=t$ គេបានរូបមន្ត៖

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \quad \frac{2t}{1+t^2}$$

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-t^2}{1+t^2}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2t}{1-t^2}$$

ដូចនេះ: $\boxed{\sin\alpha = \frac{2t}{1+t^2}\ ,\ \cos\alpha = \frac{1-t^2}{1+t^2}\ ,\ \tan\alpha = \frac{2t}{1-t^2}}$ (ដែល $t = \tan\frac{\alpha}{2}$)

លំហាត់គំរូ ១ : គណនា $\sin 22^\circ30'$, $\sin\frac{\pi}{12}$, $\cos 67^\circ30'$, $\tan\frac{3\pi}{8}$ និង $\tan 112^\circ30'$ ។

ចម្លើយ

គណនា $\sin 22^\circ30'$, $\sin\frac{\pi}{12}$, $\cos 67^\circ30'$, $\tan\frac{3\pi}{8}$ និង $\tan 112^\circ30'$

គេបាន $\sin 22^\circ30' = \sin\frac{45^\circ}{2} = \sqrt{\frac{1-\cos 45^\circ}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

$$\sin\frac{\pi}{12} = \sin\frac{(\frac{\pi}{6})}{2} = \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

ដូចនេះ: $\boxed{\sin 22^\circ30' = \frac{\sqrt{2-\sqrt{2}}}{2}\ ,\ \sin\frac{\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2}}$

$$\cos 67^\circ30' = \cos\frac{135^\circ}{2} = \sqrt{\frac{1+\cos 135^\circ}{2}} = \sqrt{\frac{1+\cos(180^\circ-45^\circ)}{2}}$$

$$=\sqrt{\frac{1-\cos 45^\circ}{2}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2-\sqrt{2}}}{2}$$

ដូចនេះ: $\boxed{\cos 67^\circ 30' = \frac{\sqrt{2-\sqrt{2}}}{2}}$

$$\tan\frac{3\pi}{8}=\tan\frac{(\frac{3\pi}{4})}{2}=\sqrt{\frac{1-\cos\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}}=\sqrt{\frac{1-\cos(\pi-\frac{\pi}{4})}{1+\cos(\pi-\frac{\pi}{4})}}$$

$$=\sqrt{\frac{1+\cos\frac{\pi}{4}}{1-\cos\frac{\pi}{4}}}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}}=\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}=\sqrt{\frac{(2+\sqrt{2})^2}{4-2}}$$

$$=\sqrt{\frac{4+4\sqrt{2}+2}{2}}=\sqrt{3+2\sqrt{2}}=\sqrt{2+2\sqrt{2}+1}$$

$$=\sqrt{(\sqrt{2}+1)^2}=\sqrt{2}+1$$

- ចំពោះ $\tan 112^\circ 30' < 0$ គេបាន:

$$\tan 112^\circ 30' = \tan\frac{225^\circ}{2}=-\sqrt{\frac{1-\cos 225^\circ}{1+\cos 225^\circ}}=-\sqrt{\frac{1-\cos(180^\circ+45^\circ)}{1+\cos(180^\circ+45^\circ)}}$$

$$=-\sqrt{\frac{1+\cos 45^\circ}{1-\cos 45^\circ}}=-\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}}=-\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$$

$$=-\sqrt{\frac{(2+\sqrt{2})^2}{4-2}}=-\sqrt{\frac{4+4\sqrt{2}+2}{2}}=-\sqrt{3+2\sqrt{2}}$$

$$=-\sqrt{2+2\sqrt{2}+1}=-\sqrt{(\sqrt{2}+1)^2}=-\sqrt{2}-1$$

ដូចនេះ: $\boxed{\tan\frac{3\pi}{8}=\sqrt{2}\quad 1\ ,\ \tan 112^\circ 30' - \ \sqrt{2}\quad 1}$

លំហាត់ទី ២ : ផ្ទៀងផ្ទាត់សមភាព $\frac{1-\cos x}{\sin x}=\tan\frac{x}{2}$ ។

សម្រាយបញ្ជាក់

គេបាន $\frac{1-\cos x}{\sin x}=\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\quad \tan\frac{x}{2}$

ដូចនេះ: បញ្ជាក់ថា $\boxed{\frac{1-\cos x}{\sin x}=\tan\frac{x}{2}}$

លំហាត់ទី ៣ : គេមាន $\cos\alpha = -\frac{3}{5}$ និង $\frac{\pi}{2}<\alpha<\pi$ ។

គណនា $\sin\frac{\alpha}{2}$ និង $\cos\frac{\alpha}{2}$ ។

ចម្លើយ

គណនា $\sin\frac{\alpha}{2}$ និង $\cos\frac{\alpha}{2}$

ដោយ $\frac{\pi}{2}<\alpha<\pi$ នោះ: $\frac{\pi}{4}<\frac{\alpha}{2}<\frac{\pi}{2}$ នាំឲ្យ $\begin{cases}\sin\frac{\alpha}{2}>0\\ \cos\frac{\alpha}{2}>0\end{cases}$

$$\sin\frac{\alpha}{2}=\sqrt{\frac{1-\cos\alpha}{2}}\quad\sqrt{\frac{1-(-\frac{3}{5})}{2}}\quad\sqrt{\frac{1+\frac{3}{5}}{2}}\quad\sqrt{\frac{5+3}{2\times 5}}$$

$$=\sqrt{\frac{8}{2\times 5}}\quad\sqrt{\frac{4}{5}}\quad\frac{2}{\sqrt{5}}\quad\frac{2\sqrt{5}}{5}$$

$$\cos\frac{\alpha}{2}=\sqrt{\frac{1+\cos\alpha}{2}}\quad\sqrt{\frac{1+(-\frac{3}{5})}{2}}\quad\sqrt{\frac{1-\frac{3}{5}}{2}}\quad\sqrt{\frac{5-3}{2\times5}}$$

$$=\sqrt{\frac{2}{2\times5}}\quad\sqrt{\frac{1}{5}}\quad\frac{1}{\sqrt{5}}\quad\frac{\sqrt{5}}{5}$$

ដូចនេះ $\boxed{\sin\frac{\alpha}{2}=\frac{2\sqrt{5}}{5}\ ,\ \cos\frac{\alpha}{2}=\frac{\sqrt{5}}{5}}$

លំហាត់គំរូ ៤ : គណនា $\tan\frac{\alpha}{2}$ ដោយស្គាល់ $\tan\alpha=\frac{24}{7}$ និង $180^\circ<\alpha<270^\circ$

ចម្លើយ

គណនា $\tan\frac{\alpha}{2}$

គេមាន $180^\circ<\alpha<270^\circ$ នោះ $90^\circ<\frac{\alpha}{2}<135^\circ$ នាំឲ្យ $\tan\frac{\alpha}{2}<0$

តាមរូបមន្ត $\tan\alpha=\frac{2t}{1-t^2}$ ដែល $t=\tan\frac{\alpha}{2}$

ដោយ $\tan\alpha=\frac{24}{7}$ គេបាន $\frac{24}{7}=\frac{2t}{1-t^2}$

$$24(1-t^2)=2t\ \ 7$$

$$24-24t^2=14t$$

$$24t^2+14t-24=0$$

$$12t^2+7t-12=0$$

តាម $\Delta=7^2\quad4(12)(\ 12)\quad49\quad576+625=25^2$

មានឫស $t=\frac{-7-25}{2(12)}\quad\frac{32}{24}<\ \frac{4}{3}\quad0$ យក

$$t=\frac{-7+25}{2(12)}\quad\frac{18}{24}\quad0$$ មិនយក

ដូចនេះ: $\boxed{t=\tan\frac{\alpha}{2} = \frac{4}{3}}$

លំហាត់ទី ៥ : គេឲ្យ $\tan\frac{\alpha}{2}=\frac{7}{8}$ ។ គណនា $\sin\alpha , \cos\alpha , \tan\alpha$ ។

ចម្លើយ

គណនា $\sin\alpha , \cos\alpha , \tan\alpha$

តាង $t=\tan\frac{\alpha}{2} \quad \frac{7}{8}$

គេបាន $\sin\alpha=\frac{2t}{1+t^2} = \frac{2\left(\frac{7}{8}\right)}{1+\left(\frac{7}{8}\right)^2} \quad \frac{2\left(\frac{7}{8}\right)}{\frac{64+49}{64}} \quad 2\left(\frac{7}{8}\right)\left(\frac{64}{113}\right)=\frac{112}{113}$

$$\cos\alpha=\frac{1-t^2}{1+t^2} = \frac{1-\left(\frac{7}{8}\right)^2}{1+\left(\frac{7}{8}\right)^2} \quad \frac{1-\frac{49}{64}}{1+\frac{49}{64}}$$

$$=\frac{64-49}{64}\times\frac{64}{64+49}=\frac{15}{113}$$

$$\tan\alpha=\frac{\sin\alpha}{\cos\alpha} \quad \frac{\left(\frac{112}{113}\right)}{\left(\frac{15}{113}\right)} = \left(\frac{112}{113}\right)\left(\frac{113}{15}\right)=\frac{112}{15}$$

ដូចនេះ: $\boxed{\sin\alpha=\frac{112}{113} , \cos\alpha \quad \frac{15}{113} , \tan\alpha= \quad \frac{112}{15}}$

3. រូបមន្តបំប្លែង

3.1. បំប្លែងពីផលគុណទៅជាផលបូកនិងផលដក

គេមានរូបមន្ត :

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta \quad (1)$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta \quad (2)$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \quad (3)$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta \quad (4)$$

ក/. បូករូបមន្ត (1) និង (2) គេបាន:

$$\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta$$

$$\Rightarrow \cos\alpha\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

យករូបមន្ត (1) ដក (2) គេបាន:

$$\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta$$

$$\Rightarrow \sin\alpha\sin\beta=-\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$$

ខ/. បូករូបមន្ត (3) និង (4) គេបាន:

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$$

$$\Rightarrow \sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

យករូបមន្ត (3) ដក (4) គេបាន:

$$\sin(\alpha+\beta)-\sin(\alpha-\beta)=2\cos\alpha\sin\beta$$

$$\Rightarrow \cos\alpha\sin\beta=\frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$$

ដូចនេះ: $\boxed{\begin{aligned} \cos\alpha\cos\beta &= \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \\ \sin\alpha\sin\beta &= -\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)] \\ \sin\alpha\cos\beta &= \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)] \\ \cos\alpha\sin\beta &= \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)] \end{aligned}}$

លំហាត់គំរូ ១ : គណនា $\cos 75^\circ \cos 45^\circ$ និង $\sin\frac{5\pi}{12}\sin\frac{\pi}{4}$ ។

ចម្លើយ

គណនា $\cos 75^\circ \cos 45^\circ$

$$\begin{aligned} \cos 75^\circ \cos 45^\circ &= \frac{1}{2}[\cos(75^\circ+45^\circ)+\cos(75^\circ-45^\circ)] \\ &= \frac{1}{2}(\cos 120^\circ+\cos 30^\circ) \\ &= \frac{1}{2}(\cos(180^\circ-60^\circ)+\cos 30^\circ) \\ &= \frac{1}{2}(-\cos 60^\circ+\cos 30^\circ) \\ &= \frac{1}{2}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)=\frac{1}{2}\cdot\frac{\sqrt{3}-1}{2}=\frac{\sqrt{3}-1}{4} \end{aligned}$$

ដូចនេះ: $\boxed{\cos 75^\circ \cos 45^\circ = \frac{\sqrt{3}-1}{4}}$

គណនា $\sin\frac{5\pi}{12}\sin\frac{\pi}{4}$

$$\sin\frac{5\pi}{12}\sin\frac{\pi}{4} = -\frac{1}{2}\left[\cos\left(\frac{5\pi}{12}+\frac{\pi}{4}\right)-\cos\left(\frac{5\pi}{12}-\frac{\pi}{4}\right)\right]$$

$$= -\frac{1}{2}(\cos\frac{5\pi+3\pi}{12} - \cos\frac{5\pi-3\pi}{12})$$

$$= \frac{1}{2}(\cos\frac{8\pi}{12} \quad \cos\frac{2\pi}{12})$$

$$= \frac{1}{2}(\cos\frac{2\pi}{3} \quad \cos\frac{\pi}{6})$$

$$= \frac{1}{2}(\cos(\pi \quad \frac{\pi}{3}) \quad \cos\frac{\pi}{6})$$

$$= \frac{1}{2}(\quad \cos\frac{\pi}{3} \quad \cos\frac{\pi}{6}) \quad \frac{1}{2}\left(-\frac{1}{2} \quad \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2}\left(\quad \frac{\sqrt{3}+1}{2}\right) \quad \frac{\sqrt{3}+1}{4}$$

ដូចនេះ: $\boxed{\sin\frac{5\pi}{12}\sin\frac{\pi}{4} = \frac{\sqrt{3}+1}{4}}$

លំហាត់គំរូ ២ : គណនា $\sin 75^\circ \cos 45^\circ$ និង $\cos 75^\circ \sin 45^\circ$ ។

ចម្លើយ

គណនា $\sin 75^\circ \cos 45^\circ$

$$\sin 75^\circ \cos 45^\circ = \frac{1}{2}[\sin(75^\circ \quad 45^\circ) + \sin(75^\circ \quad 45^\circ)]$$

$$= \frac{1}{2}(\sin 120^\circ \quad \sin 30^\circ)$$

$$= \frac{1}{2}(\sin(180^\circ \quad 60^\circ) - \sin 30^\circ)$$

$$= \frac{1}{2}(\sin 60^\circ \quad \sin 30^\circ) \quad \frac{1}{2}(\frac{\sqrt{3}}{2} \quad \frac{1}{2}) \quad \frac{\sqrt{3}+1}{4}$$

ដូចនេះ: $\boxed{\sin 75^\circ \cos 45^\circ = \frac{\sqrt{3}+1}{4}}$

គណនា $\cos 75^\circ \sin 45^\circ$

$$\cos 75^\circ \sin 45^\circ \frac{1}{2}[\sin(75^\circ + 45^\circ) - \sin(75^\circ - 45^\circ)]$$

$$= \frac{1}{2}(\sin 120^\circ \quad \sin 30^\circ)$$

$$= \frac{1}{2}(\sin(180^\circ \quad 60^\circ) - \sin 30^\circ)$$

$$= \frac{1}{2}(\sin 60^\circ \quad \sin 30^\circ) \quad \frac{1}{2}(\frac{\sqrt{3}}{2} \quad \frac{1}{2}) \quad \frac{\sqrt{3}-1}{4}$$

ដូចនេះ: $\boxed{\cos 75^\circ \sin 45^\circ = \frac{\sqrt{3}-1}{4}}$

3.2. បំប្លែងពីផលបូកទៅជាផលគុណ

គេមាន $\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$

$$\cos(\alpha+\beta) - \cos(\alpha-\beta) = \quad 2\sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$$

គេតាង $\begin{cases} p = \alpha \quad \beta \\ q = \alpha \quad \beta \end{cases}$

គេបាន: $\begin{array}{l} + \begin{cases} p = \alpha \quad \beta \\ q = \alpha \quad \beta \end{cases} \\ \hline p + q = 2\alpha \end{array}$

$$\Rightarrow \alpha = \frac{p+q}{2}$$

$$\Rightarrow \beta = p - \alpha = p - \frac{p+q}{2} = \frac{p-q}{2}$$

ដូចនេះ:

$$\boxed{\begin{aligned} &\cos p+\cos q=2\cos\frac{p+q}{2}\cos\frac{p-q}{2}\\ &\cos p-\cos q=\quad 2\sin\frac{p+q}{2}\sin\frac{p-q}{2}\\ &\sin p+\sin q=2\sin\frac{p+q}{2}\cos\frac{p-q}{2}\\ &\sin p+\sin q=2\cos\frac{p+q}{2}\sin\frac{p-q}{2}\end{aligned}}$$

- ចំពោះ: $\tan p+\tan q=\frac{\sin p}{\cos p}+\frac{\sin q}{\cos q}$

$$=\frac{\sin p\cos q+\cos p\sin q}{\cos p\cos q}=\frac{\sin(p+q)}{\cos p\cos q}$$

- ចំពោះ: $\tan p-\tan q=\frac{\sin p}{\cos p}\quad\frac{\sin q}{\cos q}$

$$=\frac{\sin p\cos q-\cos p\sin q}{\cos p\cos q}=\frac{\sin(p-q)}{\cos p\cos q}$$

- ចំពោះ: $\cot p+\cot q=\frac{\cos p}{\sin p}+\frac{\cos q}{\sin q}$

$$=\frac{\sin p\cos q+\cos p\sin q}{\sin p\sin q}=\frac{\sin(p+q)}{\sin p\sin q}$$

- ចំពោះ: $\cot p-\cot q=\frac{\cos p}{\sin p}\quad\frac{\cos q}{\sin q}$

$$=\frac{\cos p\sin q-\sin p\cos q}{\sin p\sin q}$$

$$=-\frac{\sin p\cos q-\cos p\sin q}{\sin p\sin q}=-\frac{\sin(p-q)}{\sin p\sin q}$$

ដូចនេះ: $$\boxed{\begin{aligned} \tan p+\tan q &= \frac{\sin(p+q)}{\cos p\cos q} \\ \tan p-\tan q &= \frac{\sin(p-q)}{\cos p\cos q} \\ \cot p+\cot q &= \frac{\sin(p+q)}{\sin p\sin q} \\ \cot p-\cot q &= \frac{\sin(p-q)}{\sin p\sin q} \end{aligned}}$$

លំហាត់គំរូ ១ : គណនា $\sin 105^\circ+\sin 15^\circ$ និង $\cos 105^\circ+\cos 15^\circ$ ។

ចម្លើយ

គណនា $\sin 105^\circ+\sin 15^\circ$

$$\sin 105^\circ+\sin 15^\circ = 2\sin\frac{105^\circ+15^\circ}{2}\cos\frac{105^\circ-15^\circ}{2}$$

$$= 2\sin 60^\circ\cos 45^\circ = 2\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{2}$$

ដូចនេះ: $\boxed{\sin 105^\circ+\sin 15^\circ = \frac{\sqrt{6}}{2}}$

គណនា $\cos 105^\circ+\cos 15^\circ$

$$\cos 105^\circ+\cos 15^\circ = 2\cos\frac{105^\circ+15^\circ}{2}\cos\frac{105^\circ-15^\circ}{2}$$

$$= 2\cos 60^\circ\cos 45^\circ = 2\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

ដូចនេះ: $\boxed{\cos 105^\circ+\cos 15^\circ = \frac{\sqrt{2}}{2}}$

លំហាត់គំរូ ២ : គេមាន A,B,C ជាមុំក្នុងត្រីកោណមួយ។ ចូរបង្ហាញថា

$$\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$ ។

សម្រាយបញ្ជាក់

ដោយ A,B,C ជាមុំក្នុងត្រីកោណមួយ នោះ $A+B+C=\pi$

ឬ $A+B=\pi-C\Leftrightarrow\frac{A+B}{2}=\frac{\pi}{2}-\frac{C}{2}$

គេបាន $\sin\frac{A+B}{2}=\sin(\frac{\pi}{2}\quad\frac{C}{2})\quad\cos\frac{C}{2}$

$$\cos\frac{A+B}{2}=\cos(\frac{\pi}{2}\quad\frac{C}{2})\quad\sin\frac{C}{2}$$

គេបាន $\sin A+\sin B+\sin C=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+2\sin\frac{C}{2}\cos\frac{C}{2}$

$$=2\cos\frac{C}{2}\cos\frac{A-B}{2}+2\cos\frac{A+B}{2}\cos\frac{C}{2}$$

$$=2\cos\frac{C}{2}\left(\cos\frac{A+B}{2}+\cos\frac{A-B}{2}\right)$$

$$=2\cos\frac{C}{2}\quad 2\cos\frac{A}{2}\cos\frac{B}{2}$$

$$=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

ដូចនេះ: $\boxed{\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$

លំហាត់គំរូ ៣ : គេមាន A,B,C ជាមុំក្នុងត្រីកោណមួយ។ ចូរបង្ហាញថា

$$\cos A+\cos B+\cos C=1\quad 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$ ។

សម្រាយបញ្ជាក់

ដោយ A,B,C ជាមុំក្នុងត្រីកោណមួយ នោះ $A+B+C=\pi$

ឬ $A+B=\pi-C \Leftrightarrow \frac{A+B}{2}=\frac{\pi}{2}-\frac{C}{2}$

គេបាន $\sin\frac{A+B}{2}=\sin(\frac{\pi}{2} \quad \frac{C}{2}) \quad \cos\frac{C}{2}$

$\cos\frac{A+B}{2}=\cos(\frac{\pi}{2} \quad \frac{C}{2}) \quad \sin\frac{C}{2}$

គេបាន $\cos A+\cos B+\cos C=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}+1-2\sin^2\frac{C}{2}$

$$=1+2\sin\frac{C}{2}\cos\frac{A-B}{2}-2\sin\frac{C}{2}\cos\frac{A+B}{2}$$

$$=1+2\sin\frac{C}{2}\left(\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right)$$

$$=1 \quad 2\sin\frac{C}{2}\left(\quad 2\sin\frac{A}{2}\sin(\quad \frac{B}{2})\right)$$

$$=1 \quad 2\sin\frac{C}{2}\left(2\sin\frac{A}{2}\sin\frac{B}{2}\right)$$

$$=1 \quad 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

ដូចនេះ $\boxed{\cos A+\cos B+\cos C=1 \quad 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$

លំហាត់គំរូ ៤ : គេមាន A,B,C ជាមុំក្នុងត្រីកោណមួយ។ ចូរបង្ហាញថា

$\tan A+\tan B+\tan C=\tan A\tan B\tan C$ ។

សម្រាយបញ្ជាក់

ដោយ A,B,C ជាមុំក្នុងត្រីកោណមួយ នោះ $A+B+C=\pi$

ឬ $C=\pi- \ (A+ \ B)$

គេបាន $\tan C=\tan[\pi \quad (A \quad B)]$

$$-=\tan(A+B)=-\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan C=\frac{\tan A+\tan B}{\tan A\tan B-1}$$

$$\tan A+\tan B=\tan C(\tan A\tan B\quad 1)$$

$$\tan A+\tan B=\tan A\tan B\tan C\quad \tan C$$

$\tan A+\tan B+\tan C=\tan A\tan B\tan C$ ពិត

ដូចនេះ: $\boxed{\tan A+\tan B+\tan C=\tan A\tan B\tan C}$

លំហាត់ទី ៥ : គេមាន A,B,C ជាមុំក្នុងត្រីកោណមួយ។ ចូរបង្ហាញថា

$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$ ។

សម្រាយបញ្ជាក់

ដោយ A,B,C ជាមុំក្នុងត្រីកោណមួយ នោះ: $A+B+C=\pi$

ឬ $C=\pi-(A+B)$ ឬ $\frac{C}{2}=\frac{\pi}{2}\quad\frac{A+B}{2}$

គេបាន $\cot\frac{C}{2}=\cot\left(\frac{\pi}{2}-\frac{A+B}{2}\right)=\tan\frac{A+B}{2}=\tan\left(\frac{A}{2}+\frac{B}{2}\right)$

$$=\frac{1}{\cot\left(\frac{A}{2}+\frac{B}{2}\right)}=\frac{1}{\dfrac{\cot\frac{A}{2}\cot\frac{B}{2}-1}{\cot\frac{A}{2}+\cot\frac{B}{2}}}=\frac{\cot\frac{A}{2}+\cot\frac{B}{2}}{\cot\frac{A}{2}\cot\frac{B}{2}-1}$$

$$\cot\frac{C}{2}=\frac{\cot\frac{A}{2}+\cot\frac{B}{2}}{\cot\frac{A}{2}\cot\frac{B}{2}-1}$$

$$\cot\frac{A}{2}+\cot\frac{B}{2}=\cot\frac{C}{2}\left(\cot\frac{A}{2}\cot\frac{B}{2}-1\right)$$

$$\cot\frac{A}{2}+\cot\frac{B}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}\quad\cot\frac{C}{2}$$

$$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$

ដូចនេះ: $\boxed{\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}}$