HW 2

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Question 1:

D=(0.4223, 0.4223)

Question 2:

The convergence process is listed as follows, while the elapsed time is 0.011359 seconds.

iter 1: p(1) = 1.000000, p(2) = 1.000000, norm(f(x)) = 0.25222947
iter 2: p(1) = 1.595733, p(2) = 1.595733, norm(f(x)) = 0.00131883
iter 3: p(1) = 1.598864, p(2) = 1.598864, norm(f(x)) = 0.00003186
iter 4: p(1) = 1.598942, p(2) = 1.598942, norm(f(x)) = 0.00000001

Question 3:

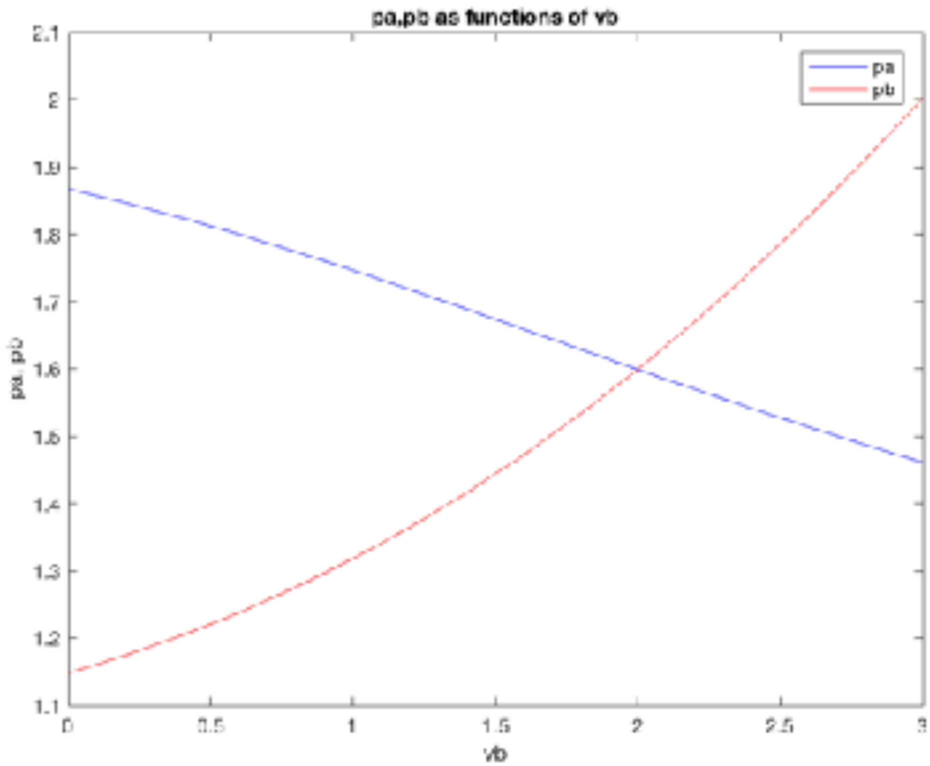
The elapsed time is 0.008439 seconds.

Question 4:

The elapsed time is 0.011076 seconds.

Questions 5:

I used Broyden's method, and the result is the following.



Code:

```
%HW2 Code


%% Question 1
clear;
v=[2,2]; p=[2,2];
[fval,D]=bertrand2(v,p);
display(D);

%% Question 2 Broyden's Method
clear;
p=[1; 1]; %inital guess
v=[2;2];
fVal = bertrand2(v,p); %inital value of FOC
f=@(p) bertrand2(v,p);
iJac = inv(myJac(f, p)); %intial Jacobian of FOC

%Broyden iterations:
tic
```

```
maxit = 100;
tol = 1e-6;
for iter = 1:maxit
   fnorm = norm(fVal);
   fprintf('iter %d: p(1) = %f, p(2) = %f, norm(f(x)) = %.8f\n', iter, p(1), p(2),
norm(fVal));
   if norm(fVal) < tol
      break
   end
   d = - (iJac * fVal);
   p = p+d;
   fOld = fVal;
   fVal = bertrand(p);
   u = iJac*(fVal - fOld);
   iJac = iJac + ( (d - u) * (d'*iJac) )/ (d'*u);
end
toc

%% Question 3 Secant Method
clear;
v=[2;2];

f1= @(p1,p2) p1*(1+exp(v(2)-p2))/(1+exp(v(1)-p1)+exp(v(2)-p2))-1;
f2= @(p1,p2) p2*(1+exp(v(1)-p1))/(1+exp(v(1)-p1)+exp(v(2)-p2))-1;

%Assign initial values
pa=1.2; paold=1;
pb=1.2; pbold=1;

% Secant iterations:
tol = 1e-8;
maxit = 100;

tic
for iter =1:maxit

   f1Val=f1(pa,pb); f2Val=f2(pa,pb);

   if abs(max(f1Val,f2Val)) < tol
      break
```

```
    else

    %given pb, update the guess for pa using FOC1
    f=@(p1) f1(p1, pb);
    fold=f(paold);
    fVal = f(pa);
    paNew = pa - ( (pa - paold) / (fVal - fold) )* fVal;
    paold = pa;
    pa = paNew;

    %given updated pa, update pb using FOC2
    f=@(p2) f2(pa, p2);
    fold=f(pbold);
    fVal = f(pb);
    pbNew = pb - ( (pb - pbold) / (fVal - fold) )* fVal;
    pbold = pb;
    pb = pbNew;
    end

end
toc

%% Question 4
clear;
%initial price
p=[1; 1];
v=[2,2];
fVal = bertrand2(v,p);

%Iterations by rule: p'=1/(1-D(p))
tic
ee=ones(length(p),1);
maxit = 100;
tol = 1e-6;
for iter = 1:maxit
    fnorm = norm(fVal);
    fprintf('iter %d: p(1) = %f, p(2) = %f, norm(f(x)) = %.8f\n', iter, p(1), p(2),
norm(fVal));
    if norm(fVal) < tol
        break
```

```
    end
    [fval,D]=bertrand2(v,p);
    p = ee./(ee-D);
    fVal = bertrand2(v,p);
end
toc

%% Question 5
va=2;
vb=0:0.2:3;
pa=ones(length(vb),1); pb=ones(length(vb),1);

maxit = 100;
tol = 1e-6;

for i=1:length(vb)
    v=[va;vb(i)];

    %initial guess

    p=[1; 1];
    fVal = bertrand2(v,p);
    f=@(p) bertrand2(v,p);
    iJac = inv(myJac(f, p));

    %Broyden iterations:

    for iter = 1:maxit
    fnorm = norm(fVal);
    if norm(fVal) < tol
        break
    end
    d = - (iJac * fVal);
    p = p+d;
    fOld = fVal;
    fVal = bertrand2(v,p);
    u = iJac*(fVal - fOld);
    iJac = iJac + ( (d - u) * (d'*iJac) )/ (d'*u);
   end
```

```
    pa(i)=p(1); pb(i)=p(2);

end


figure
plot(vb, pa, 'b', vb, pb, 'r');
title('pa,pb as functions of vb');
xlabel('vb'); ylabel('pa, pb');
legend('pa', 'pb');
```