Supplemental material: Multi-objective optimization of transport processes on complex networks

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In this supplemental material, we validate the effectiveness of the proposed edge centrality metric by comparing it with representative alternatives. In our optimization problem, defined in Eq. (15) in the manuscript, network capacity is a function of the largest node betweenness centrality, while average number of hops is dependent on the average node betweenness centrality. Since both of the two objectives have strong correlation with node betweenness, we define the centrality of an edge as the arithmetic mean of the betweenness of its two end nodes with normalization, as shown in Eq. (18) of the manuscript. We name it by node-betweenness based edge centrality (NBEC).

There already exist several edge centrality metrics in network science, and we consider two most representatives here. One is edge betweenness centrality (EBC) [1] and the other is node-degree based edge centrality (NDEC) [2], [3]. Let us assume a graph $G(V,E)$, where V and E are respectively the node and edge sets. The betweenness centrality of an edge $e \in E$ is defined as

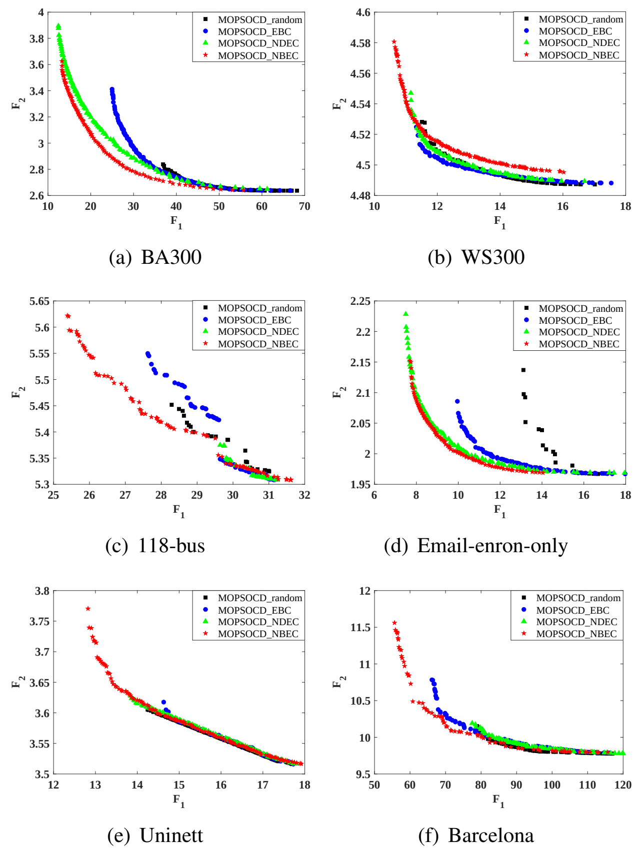
$$C_1(e) = \frac{1}{|V||V-1|} \sum_{s \neq t \in V} \frac{\delta_{st}(e)}{\delta_{st}}, \tag{1}$$

where δ_{st} is the total number of shortest paths connecting s to t, and $\delta_{st}(e)$ is the number of those shortest paths connecting s to t and passing through edge e. According to this definition, we can infer that the edges with large betweenness centrality will bear more traffic load than those with small betweenness centrality. The NDEC quantifies the centrality of edge $e(i,j)$ with the product of the degree of its two end nodes, i and j,

$$C_2(e) = k_i \times k_j, \tag{2}$$

where k_i is the degree of node i. This definition ensures that an edge connected with a large number of other edges has a large centrality value in a network.

All the aforementioned edge centrality metrics can be used in the initialization of NC-MOPSO to redistribute the element values of a solution so that an edge of larger centrality has a larger weight. We present in Fig. 1 the nondominated solutions (PFs) of network models and real networks obtained by MOPSOCD with four different initialization mechanisms, i.e., random, EBC, NDBC, and NBEC. Based on the corresponding distributions of nondominated solutions, we can see that MOPSOCD with heuristic initialization performs always better than with random initialization. Furthermore, the heuristic initialization based on our edge centrality metric, NBEC, is superior to those with other edge centrality metrics. These results validate the effectiveness of our definition of edge centrality metric and thus the efficiency of NC-MOPSO.



(a) BA300 (b) WS300 (c) 118-bus (d) Email-enron-only (e) Uninett (f) Barcelona

Fig. 1. Nondominated solutions (PFs) of heuristic initialization with different edge centrality metrics on all instance.

References

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