NATIONAL INSTITUTE OF SCIENCE EDUCATION AND
RESEARCH

SIXTH SEMESTER PROJECT REPORT

Study on spin current in a NM/HM bilayer

Author:
Ashish PANIGRAHI

Supervisor:
Dr. Kartik SENAPATI

*A report submitted in fulfillment of the requirements
for the course of P398*

in the

Superconducting Spintronics Lab
School of Physical Sciences

June 15, 2021

Declaration of Authorship

I, Ashish PANIGRAHI, declare that this thesis titled, "Study on spin current in a NM/HM bilayer" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature as part of the coursework for sixth semester at this University.

- Where I have consulted the published work of others, this is always clearly attributed.

- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.

- I have acknowledged all main sources of help.

- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Date: June 15, 2021

NATIONAL INSTITUTE OF SCIENCE EDUCATION AND RESEARCH

Abstract

Dr. Kartik Senapati

School of Physical Sciences

P398

Study on spin current in a NM/HM bilayer

by Ashish PANIGRAHI

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Acknowledgement

I would like to take this opportunity to thank my supervisor, Dr. Kartik Senapati, for supervising me for my sixth semester project. His guidance and positivity have helped maintain my morale throughout my work.

I would also like to especially thank my collaborator, Mr. Tapas Ranjan Senapati, who guided me in working with the instruments and made sure that I understood the relevant concepts required for the project.

List of Abbreviations

OHE	**O**rdinary **H**all **E**effect
AHE	**A**nomalous **H**all **E**effect
SHE	**S**pin **H**all **E**effect
ISHE	**I**nverse **S**pin **H**all **E**effect
NM	**N**ormal **M**etal
HM	**H**eavy **M**etal

Contents

Chapter 1

Introduction

1.1

Chapter 2

Primer to Hall effects

2.1 Introduction - Ordinary Hall effect

These effects originally deal with the application of an external magnetic field on a current carrying material and subsequently observing the effect either on the conductor or the electric current itself.

In 1879, Edwin Hall was exploring this interaction and tried to determine the effect of the magnetic field on a current carrying wire, with a suspicion that it either affected the whole length of the wire or only the moving electrons.

He later devised a rather simple experiment based on the argument that "if the current of electricity in a fixed conductor is itself attracted by a magnet, the current should be drawn to one side of the wire, and therefore the resistance experienced should be increased." [5]

Hall couldn't detect this extra resistance (which we now know as magnetoresistance) but concluded that a transverse force in the opposite direction must exist and which appears as a transverse voltage across the width of the conducting material. This is the Hall effect and the transverse voltage is the Hall voltage.

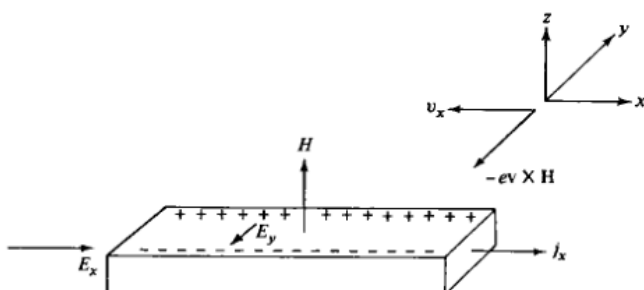The experiment by Hall is shown in the figure below.

Figure 1.3
Schematic view of Hall's experiment.

FIGURE 2.1: Schematic diagram of the Hall effect
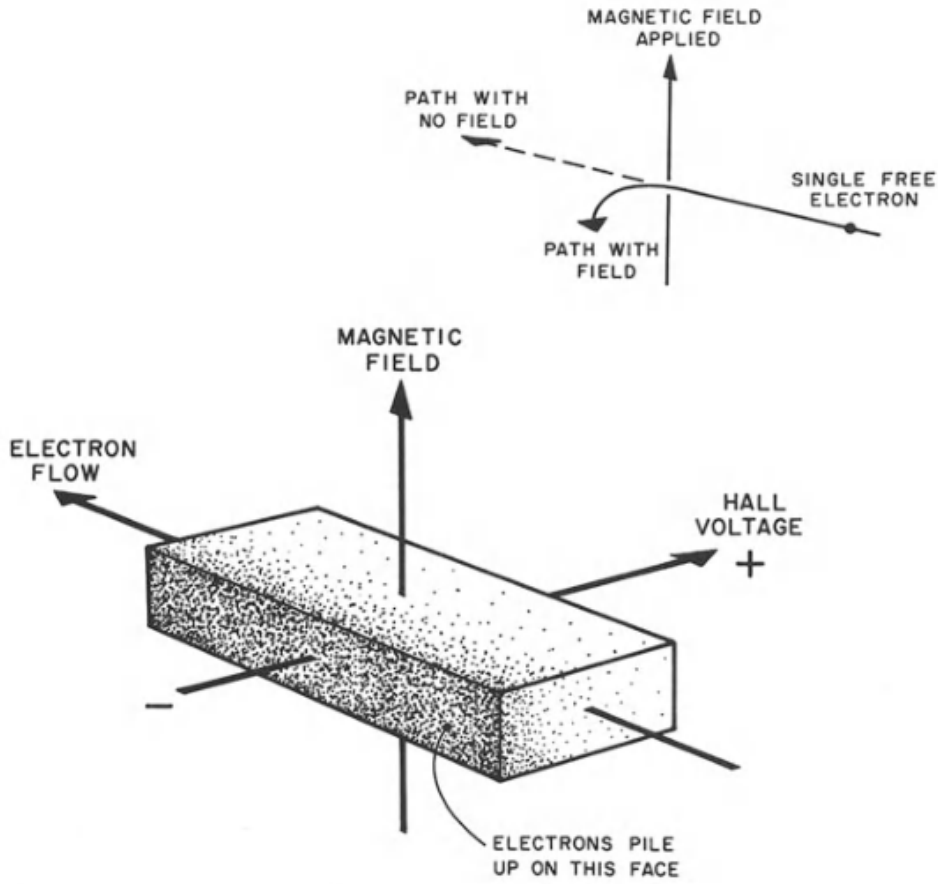*Image credit: Ashcroft & Mermin, Solid State Physics*

FIGURE 2.2: A more "self-explanatory" diagram of the Hall effect
Image credit: C.M Hurd, The Hall effect in metals and alloys

2.1.1 Mechanism of OHE

In the fig. 2.1 , an electric current is passed along the x direction with corresponding current density is j_x. The cause of this current is an external electric field along the same direction, E_x.

An external magnetic field H along the z direction is applied and the Hall effect is observed.

From the Lorentz force equation

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{H}) \tag{2.1}$$

The second term of the eq. (2.1) is responsible for deflecting the trajectory of the electrons in the negative y-direction and accumulating along the sides of the material. As this accumulation takes place, an electric field builds up along the y-direction which opposes the further deflection of electrons towards the sides. This process continues until an equilibrium is reached, at which this transverse field (or **Hall field**) E_y perfectly balances the Lorentz force and the current flows

only along the longitudinal direction.

2.1.2 Hall coefficient

This transverse Hall field E_y can be thought of to be proportional to the external magnetic field H and longitudinal current density j_x. Here, we define the Hall coefficient as

$$R_H = \frac{E_y}{j_x H} \tag{2.2}$$

A rather interesting point to note is that by our construction, j_x and H are along positive x and z directions respectively. E_y however, is along negative y direction, meaning that the resultant sign of the Hall coefficient R_H is negative.

Now, imagine if the charge carriers were positive, this would result in their velocity along x-direction to get reversed (j_x would still be along positive x-direction). The Lorentz force would remain unchanged (as can be seen from eq. (2.1)). Consequentially, the direction of Hall field would be in the opposite direction compared to its direction in the case of negatively charged carriers. This would mean that measuring the Hall coefficient of a material, would help one determine the sign of the charge carriers.

Calculating the Hall coefficient

Let us consider current densities j_x and j_y in the presence of an electric field with components E_x and E_y, and in the presence of an external magnetic field H along the z-axis.

The average force per electron is given by the Lorentz force equation, i.e. $\boldsymbol{F} = -e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{H})$, and hence the average momentum per electron becomes

$$\frac{d\boldsymbol{p}}{dt} = -e\left(\boldsymbol{E} + \frac{\boldsymbol{p}}{m} \times H\right) - \frac{\boldsymbol{p}}{\tau} \tag{2.3}$$

During equilibrium, the current becomes time-independent, and hence p_x and p_y satisfy the equations

$$\begin{aligned}
0 &= -eE_x - \omega_c p_y - \frac{p_x}{\tau} \\
0 &= -eE_y + \omega_c p_x - \frac{p_y}{\tau}
\end{aligned} \tag{2.4}$$

where

$$\omega_c = \frac{eH}{m} \tag{2.5}$$

Solving the above equations, we get

$$E_y = -\left(\frac{\omega_c \tau}{\sigma_0}\right) j_x = -\left(\frac{H}{ne}\right) j_x \tag{2.6}$$

This yields the Hall coefficient (eq. (2.2)) to be

$$\boxed{R_H = -\frac{1}{ne}} \tag{2.7}$$

where n is the number density of the charge carriers.

This is a rather astonishing result, suggesting that the Hall coefficient of a material, depends solely on the density of the carriers.

We then define Hall resistivity ρ_H as the Hall field E_y per unit longitudinal current density j_x, which is given by

$$\boxed{\rho_H = \frac{E_H}{j_x} = R_H H} \tag{2.8}$$

where the symbols have their usual meanings.

We shall stop our investigation of the Ordinary Hall effect in lieu of the main topic of the report.

2.2   Anomalous Hall effect (AHE)

The previous section dealt with OHE where the nature of the current carrying material is immaterial. Now, we deal with specific characteristics of such material, namely, magnetic metals.

It is observed that when observing Hall effect in such magnetic materials, in addition to OHE, certain unusual phenomena are observed.

In low-field conditions as seen from eq. (2.8), the observed Hall resistivity ρ_H varies linearly with external magnetic field H with the slope given by the Hall coefficient R_H. For illustration purposes, the example of Hall resistivity ρ_H has

been used. The trend is the same for Hall field E_H as can be observed from eq. (2.8).

What about magnetic metals? In such materials, a linear but rapid rise in the Hall field is seen with increasing external magnetic field H. This is followed by a secondary linear rise (with a lower gradient compared to the first), which later saturates at large fields and becomes almost independent of the external field. This is depicted in fig. 2.3
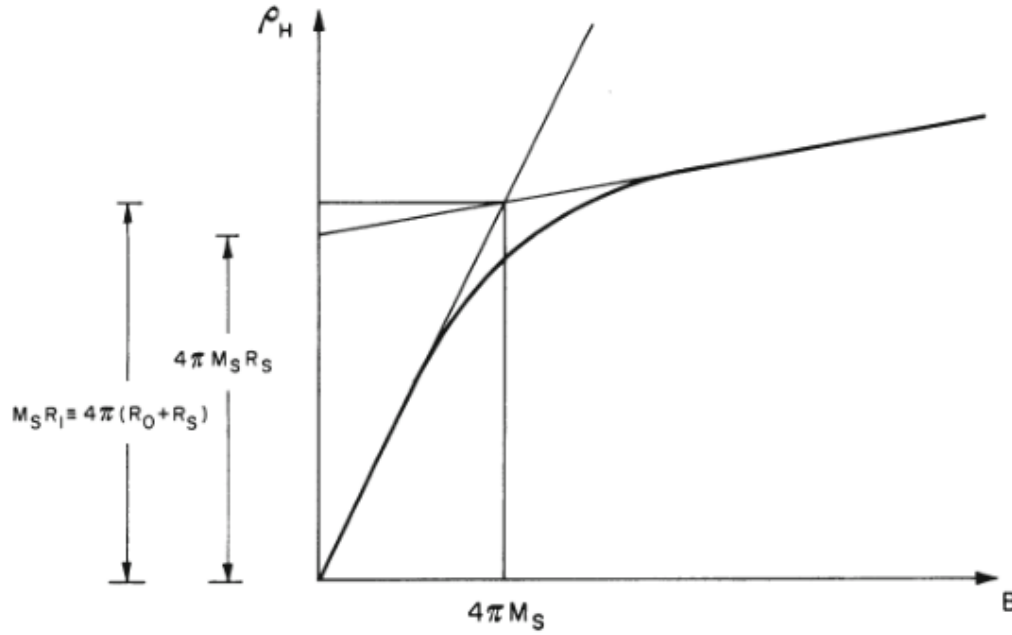

FIGURE 2.3: Schematic behaviour of the Hall resistivity ρ_H as a function of magnetic induction B in a metal showing appreciable magnetization.

Image credit: C.M Hurd, The Hall effect in metals and alloys.

In this case, the Hall effect is not simply from the application of Lorentz force on the charge carriers but rather seen as an *anomaly* and is therefore known as the **anomalous Hall effect.**

2.2.1 Mechanism of AHE

It has been shown empirically that this anomalous behaviour can be explained by a superposition of OHE and a strongly temperature dependent term. The curve shown in fig. 2.3 can be empirically fitted by the following equation (in CGS units):

$$\rho_H = R_0 B + \mu_0 R_s M \tag{2.9}$$

where B is the applied magnetic field, R_0 is the ordinary Hall coefficient, M is the magnetization of the material, R_s is the anomalous Hall coefficient.

The first term in eq. (2.9) accounts for OHE and is characterized by the more familiar eq. (2.8), whereas the second term is a characteristic of magnetic materials. This magnetization M can be present even without the presence of an external magnetic field B, especially in ferromagnetic materials.

Especially in the case of ferromagnets, R_s is experimentally found to be strongly temperature dependent.

2.3   Spin Hall effect (SHE)

In this type of Hall effect, we generally deal with non-magnetic (or weakly magnetic) materials. For example, a paramagnetic material or a ferromagnetic material beyond its Curie temperature. Strictly speaking, an external magnetic field is not *mandatory* to observed SHE.

When a charge current is supplied in a specific direction along a conducting material, an addition parameter of the charge carriers, namely, the spin of the electrons is responsible for a unique phenomena to take place. If these electrons have the directions of their spins along specific directions (in our interest, spins perpendicular to the plane of the conducting material), then such electrons are *spin-polarized* and via scattering mechanisms[1], these electrons get preferentially scattered, giving rise to a spin current [1, 2].

In layman terms, SHE reduces to the following: *Spin accumulation at the lateral boundaries of the material, with the directions of spins being opposite at the opposing sides.*

This phenomena is very similar and analogous to OHE, the only difference being that instead of opposite charge accumulation on lateral sides, electrons with opposite spins get accumulated.

We shall discuss the plausible mechanisms responsible for the spin Hall effect in greater detail in the forthcoming chapters.

2.3.1   Spin Current

A pure spin current can be defined as the flow of electrons of spin ↑ moving in one direction and electrons of the opposite spin i.e. ↓ electrons moving along the opposite direction. This results in no net charge current (since rate of movement of electrons in opposing directions is the same) and a net flow of angular momentum [4].

Mathematical analysis

From the continuity equation,

$$\nabla \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0 \tag{2.10}$$

where $\boldsymbol{J}$ is the current density and ρ is the charge density.

This eq. (2.10) implies charge conservation law and helps us define the charge current [3].

Analogously, we can define the spin current density $\boldsymbol{J}_s$ with reference to conservation of spin angular momentum. Considering that spin angular momentum is conserved, we can then define the corresponding continuity equation as

$$\nabla \boldsymbol{J}_s + \frac{\partial \boldsymbol{M}}{\partial t} = 0 \tag{2.11}$$

where M is the magnetization (magnetic moment per unit volume) of the sample. The eq. (2.11) defines the spin current density j_s [6].
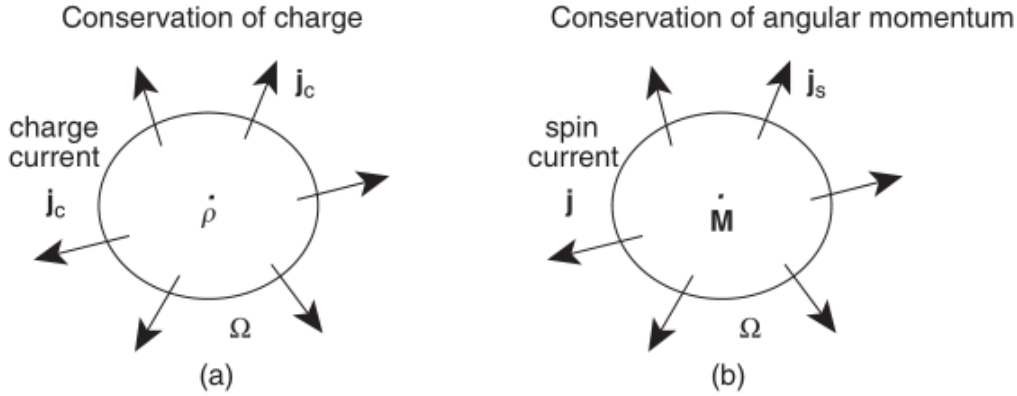

FIGURE 2.4: The total sum of rate of all charge variations across a surface is equal to the total current entering the surface.

Image credit: "Spin Current", Ken-ichi Uchida and Eiji Saitoh

In real solids for most of the cases, the conservation of spin angular momentum is a good approximation. However, in general, the conservation does not hold true due to spin relaxation (due to collisions with impurities in the solid, the spins of the electrons does not remain polazized) [6]. This gives rise to a modified version of eq. (2.11) as:

$$\nabla \boldsymbol{J}_s + \frac{\partial \boldsymbol{M}}{\partial t} = -\boldsymbol{T} \tag{2.12}$$

where T is an indicator of the non-conservation of spin angular momentum (due to spin relaxation and spin generation) [6].

2.4 Inverse Spin Hall effect (ISHE)

Contrary to SHE, the inverse Hall effect is essentially the same but SHE in reverse. When a spin current is supplied into a material (with no charge current), the same scattering mechanisms[1] in case of SHE, allow the spin-polarized electrons to preferentially scatter into opposite directions.

The spin ↑ electrons scatter along one direction and the spin ↓ electrons scatter along another direction. The surprising aspect about this, is that **both the directions are the same!** This leads to a pure charge current.
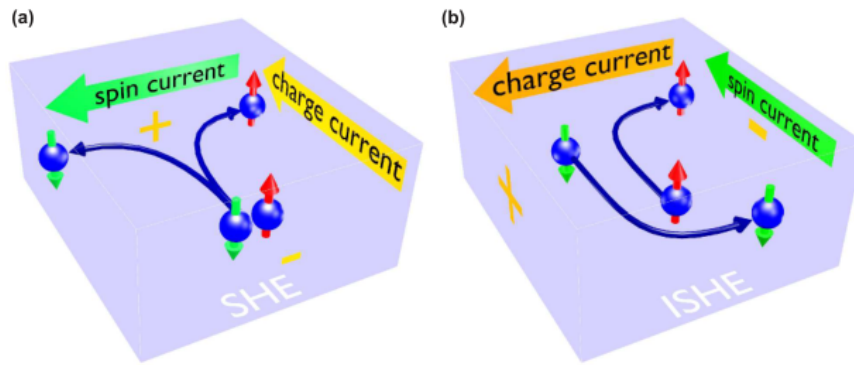
The fig. 2.5 depicts the phenomena:

FIGURE 2.5: Comparison between SHE and ISHE.
Image credit: M.B Jungfleisch, PhD Thesis

---

[1]We shall discuss the plausible mechanisms responsible for the spin Hall effect in greater detail in the forthcoming chapters.

Chapter 3

A semiclassical approach to SHE

In this chapter, we shall look at the semi-classical picture of SHE and try to understand the mechanisms behind SHE, which are mainly classified into intrinsic and extrinsic factors.

3.1 Mechanisms behind the phenomena

Over the past century, the spintronics community has agreed upon three plausible mechanisms responsible for SHE.

Bibliography

[1]   Mikhail I Dyakonov and VI Perel. "Current-induced spin orientation of electrons in semiconductors". In: *Physics Letters A* 35.6 (1971), pp. 459–460.

[2]   JE Hirsch. "Spin hall effect". In: *Physical review letters* 83.9 (1999), p. 1834.

[3]   John David Jackson. *Classical electrodynamics*. 1999.

[4]   Kannan M Krishnan. *Fundamentals and applications of magnetic materials*. Oxford University Press, 2016.

[5]   A. S. "On a new Action of the Magnet on Electric Currents1". In: *Nature* 21.537 (Feb. 1880), pp. 361–361. ISSN: 1476-4687. DOI: 10.1038/021361a0. URL: https://doi.org/10.1038/021361a0.

[6]   Ken-ichi Uchida and Eiji Saitoh. "Spin Current". In: *Spintronics for Next Generation Innovative Devices*. John Wiley & Sons, Ltd, Jan. 2016, pp. 43–76. DOI: 10.1002/9781118751886.ch3. URL: https://doi.org/10.1002%2F9781118751886.ch3.