Problems

Refer to the problem sheet posted in the google classroom.

(a) Working with the n-type enhancement mode FET, let L be the length of the channel, source-drain voltage be V_{ds} and source-drain current be I_{ds}. Since conduction occurs through the channel, let the time taken by an electron to move from source to drain be τ given by

$$\tau = \frac{L}{v_d} = \frac{L}{\mu E} = \frac{L^2}{\mu V_{ds}} \tag{1}$$

where v_d is the drift velocity and μ is the mobility of electrons in the channel.

Now, along the channel length, we define a point C at a distance x from the source. Hence,

$$V_{GS} = V_G - V_S = V_G - V_C + V_C - V_S$$

Now, $V_{GC} = V_G - V_C$ and $V(x) = V_C - V_S$. Hence, substituting in the above expression, we get

$$V_{GS} = V_G - V_C + V(x) \implies \boxed{V_{GC} = V_{GS} - V(x)}$$

With the oxide layer acting as a dielectric, the charge (per unit area) accumulated within the small region dx at C,

$$dQ_{inv}^* = C_{ox}(V_{GS} - V(x) - V_{th})$$

where C_{ox} is the capacitance of oxide layer, V_{th} is the threshold voltage.[1]

Hence, the total charge accumulated within this small volume around C is

$$dQ_{inv} = C_{ox}W dx(V_{GS} - V(x) - V_{th})$$

where W is the width of the channel. From the above expression, we now calculate the channel current I_{ds} as

$$I_{ds} = \frac{dQ_{inv}}{d\tau} = \frac{C_{ox}W dx(V_{GS} - V(x) - V_{th})}{(dx)^2/\mu dV}$$
$$= \frac{\mu C_{ox}W(V_{GS} - V(x) - V_{th})dV}{dx}$$

$\implies$

$$I_{ds}dx = C_{ox}W\mu(V_{GS} - V(x) - V_{th})dV$$
$$\implies I_{ds}\int_0^L dx = C_{ox}W\mu\int_0^{V_{ds}} dV(V_{GS} - V(x) - V_{th})$$
$$\implies I_{ds}L = C_{ox}W\mu\left((V_{GS} - V_{th})V_{ds} - \frac{V_{ds}^2}{2}\right)$$
$$I_{ds} = \frac{C_{ox}W\mu}{L}\left((V_{GS} - V_{th})V_{ds} - \frac{V_{ds}^2}{2}\right)$$

Note that this is the expression in the linear regime when V_{ds} is low.

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[1]This is the initial voltage threshold that needs to be overcome in order to have non-zero I_{ds}.

(b) (i) OFET-based biosensor devices have shown potential in the detection of low concentration of biomolecules used in various biochemical reactions. One such study was able to detect a significantly low concentration (10^{-16} M) of dopamine using an electrolyte-gated OFET [1].

In electrolyte-gated OFETs, a large electrical double-layer capacitance (~ 10 mF cm^{-2} forms near the interface between the organic semiconductor layer and electrolyte solution. This has its use in the detection of sub-femtomolar concentration of protein. This was the idea used by Magliulo *et al.* [2] with the introduction of a functional bio-interlayer between the dielectric and organic semiconductor material in order to fabricate highly-efficient OFET-based biosensors.

(ii) Hybrid materials i.e. a combination of organic and inorganic materials, have several advantages of conventional purely organic and purely inorganic materials. Both organic and inorganic materials have their pros and cons (often complimentary to each other), and the idea behind introducing hybrid materials is to leverage the advantages of both the materials into a single material. Some of these advantages are:

- Inorganic materials due to their crystalline nature have a well-defined bandstructure, whereas organic materials being amorphous do not have a bandstructure. In hybrid materials, although employing an organic nature, have a well-define bandstructure allowing for fundamental understanding of the physics in charge transport in these materials.

- The fabrication temperature of organic materials cannot exceed more than 100° C whereas inorganic materials have a significantly high fabrication temperature, often in the order $\sim 1000°$ C, which is energy-wise very inefficient. Hybrid materials have a fabrication temperature

- Mobility of charge carriers is significantly higher in inorganic semiconductors (~ 1000 cm^2/Vs) compared to organic materials (~ 0.5 cm^2/Vs). For practical devices, we require high mobility, leading to high conductivity. Hybrid materials have a mobility between the two materials i.e. $\sim 5 - 10$ cm^2/Vs.

  This is clearly better than organic materials, and also allows for devices to be flexible owing to the organic nature of the hybrid material.

- Speaking of flexibility, inorganic materials are brittle due to their crystalline nature and hence cannot be "bent". On the other hand, organic materials being amorphous are very flexible, a feature which is heavily employed in hybrid-material based devices.

(iii) Organic photovoltaics are solar cells with an organic semiconductor material as the layer where charge-separation occurs. For photovoltaice, organic semiconductors with high conjugation lengths are used, where conductance is via π-electron resonance. In these materials, a finite bandgap exists (akin to inorganice semiconductors) but instead of "conduction" and "valence" bands, the gap exists between the highest occupied molecular orbital (HOMO) and lowest unoccupied molecular orbital (LUMO).

When a photon is incident with energy greater than the bandgap of the organic semiconductor, an electron from HOMO is excited to LUMO and subsequently a hole is formed at HOMO. This gives us an exciton with the electron and hole pair bound via Coulombic interaction with an exciton-binding energy, which is typically in the range of 0.3 - 0.5 eV

for an organic semiconductor. This is already much higher than thermal energy at room temperature which is $\sim$25 meV.

Another organic semiconductor material acting as an acceptor is present in this photovoltaic. At the interface of the original organic semiconductor (acting as donor) and the acceptor material, the difference in potential between LUMO of the latter and former, allows for the electron from the exciton to jump to the LUMO of acceptor material, thus forbidding recombination of this electron with the hole from the exciton.[2] The energy difference between the state of the current electron and the hole left behind is characterized by the potential V_{OC}. This forbidding of recombination allows for better charge separation and a functioning photovoltaic cell, where the electrons and holes move to their respective electrodes, and thus completing the circuit.

(iv) A typical hybrid halide perovskite is $CH_3NH_3PbI_3$. These hybrid materials are crystalline in nature and thus have a clean bandstructure as opposed to purely organic materials which are amorphous and do not have a well defined bandstructure.

When a bias voltage is applied to this perovskite, it dissociates into CH_3NH_3I and PbI_2, which further splits into Pb^{2+} and $2I^-$.

With the application of the gate voltage (positive), negative charges accumulate near the perovskite-oxide interface on the perovskite side and between the source-drain channel region. This makes the ions to move such that Pb^{2+} is near the induced negative charges in the perovskite layer and I^- is away from it. Hence, the effective field is reduced due to this ionic field gradient due to the Pb^{2+} and I^- ions i.e. $E_{\text{eff}} = E_{\text{elec}} - E_{\text{ionic}}$, where the subscripts represent the contribution from respective charge carriers. This reduction in field is seen as a reduction in the drain current as a function of gate voltage compared to inorganic FETs. This difference is often in the order of $\sim 10^3$ mA at a gate voltage of 5 V. This ionic field gradient can be reduced by decreasing ionic mobility in the perovskite layer, which can be done by reducing temperature.

(c) Photo-generated charge carriers exist as bound states in an organic semiconductor as compared to inorganic semiconductors. This can be understood via the expression for the exciton-binding energy,

$$E_{\text{binding}} = \frac{-e^2}{4\pi\epsilon_0\epsilon_r r} \tag{2}$$

In the above equation, for organic and inorganic materials, the only deciding parameter is the relative permittivity ϵ_r. For organic materials, $\epsilon_r \sim 3 - 4$ whereas for inorganic materials (silicon for example), $\epsilon_r^{\text{Si}} \sim 11$.

From this we can infer that for organic materials, the exciton-binding energy is much greater than that of inorganic materials, meaning that the probability of the photo-generated charge carriers to exist in the exciton-bound state is higher than inorganic materials.

Frenkel excitons are those bound-states where the hole is localized at a lattice site in the crystal. Mott-Wannier exciton on the other hand, is not localized at a lattice site position and is free to diffuse across the lattice.

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[2]For this jump to happen, the exciton pair must first move towards the interface of the two organic semiconductors, meaning that this movement length must be less than the diffusion length of the exciton, otherwise the exciton will simply be lost before this electron jump to the adjacent material could occur.

(d) For improving the power operation of a MOSFET, we can employ doing the following:

- **Shorter channel length**: By decreasing the channel length (but only upto a certain limit, otherwise, the depletion regions formed around the source and drain would merge, leading to an always switched on device) we effectively reduce the path for carriers to propagate from source to drain, hence decreasing power consumption.

- **Decreasing dielectric thickness**: The capacitance of the dielectric is inversely proportional to its thickness and hence, decreasing its thickness would increase the capacitance. Hence, for the same V_g, more charges would get induced at the interface, implying higher I_{ds} for less V_g, yielding a low power dissipation. However, we also have to keep in mind that decreasing the thickness would increase the probability of leakage current through the dielectric.

- **High-dielectric constant**: Using a dielectric of higher ϵ_r, would increase its capacitance (since ϵ_r is directly proportional to capacitance), meaning that for a lower gate voltage, we get an increase in charge induction near the interface, leading to higher I_{ds} current, and therefore low power consumption.

- **Using organic semiconductor**: Since organic semiconductors do not have a depletion region, one can decrease the channel length without worrying too much about the merger of depletion regions as seen in inorganic semiconductors.

(e) The device structure of a photo-FET is provided in **??**. A highly p-doped silicon substrate is used, which acts as a gate electrode and is well separated from the semiconductor channel via an oxide layer such as SiO_2. Two metal electrodes are introduced to be in contact with the channel, forming the source and drain terminals.

For the photo-FETs to be efficient, the respective channel has to provide high carrier mobility and hence conductivity. One can tune the gate voltage to modulate conductivity during the dark state[3] via field effect. Switching off the device would mean, setting gate voltage in the negative direction, so as to add positive charge accumulation in the channel, thus yielding no I_{ds} (since channel is n-doped). This defines the I_{off} value. Now, when light of energy equal to or greater than the bandgap of the semiconductor is incident, the photo-generated electrons populate the channel, thus increasing the number density and yielding a non-zero I_{ds}. The gate bias voltage can be further used to modulate I_{ds} with an increase in positive gate voltage hence giving higher conductivity in the channel.

Advantages over conventional photodetectors

- **High sensitivity**: Photo-FETs have a high sensitivity compared to photodetectors.

- **Amplification**: Compared to photodetectors that have a gain restricted to 1, photo-FETs have a much higher gain.

- **Greater absorbance**: The active material used in a photo-FET have higher absorbance compared to inorganic semiconductors used in conventional photodetectors, thus leading to a more efficient device.

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[3]State of the FET when no light is incident.

Responsivity of photo-FET   The responsivity of a device is defined as the ratio between photocurrent and power input given as

$$R = \frac{I_{\text{photo}}}{P_{\text{in}}} = \frac{eG\epsilon_Q}{h\nu\sqrt{1 + (2\pi f\tau)^2}} \tag{3}$$

where ϵ_Q is the quantum efficiency, G is the gain of the device, e is the charge of an electron, f is the modulation frequency, ν is the frequency of incident photon and τ is the time constant.

The gain G is calculated as

$$G = \frac{\tau_{lt}}{L^2} \cdot \mu V_{DS} \tag{4}$$

where τ_{lt} is the lifetime of the charge carrier, L is the length of the channel and μ is the carrier mobility.

(f) A single monolayer arrangement of carbon atoms in a honeycomb-like fashion is what comprises of graphene. Let us define two sub-lattices A and B, with the corresponding primitive vectors of the lattice given by

$$\mathbf{a}_1 = \frac{a}{2}(3, \sqrt{3}) \qquad \mathbf{a}_2 = \frac{a}{2}(3, -\sqrt{3}) \tag{5}$$

with the respective nearest-neighbour vectors being

$$\delta_1 = \frac{a}{2}(1, \sqrt{3}) \qquad \delta_2 = \frac{a}{2}(1, -\sqrt{3}) \qquad \delta_3 = a(-1, 0) \tag{6}$$

The reciprocal lattice vectors in the Brillouin zone is given as

$$\mathbf{b}_1 = \frac{2\pi}{3a}(1, \sqrt{3}) \qquad \mathbf{b}_2 = \frac{2\pi}{3a}(1, -\sqrt{3}) \tag{7}$$

We then define Dirac points, which are corners of the first Brillouin zone of graphene:

$$\mathbf{K} = \frac{2\pi}{3\sqrt{3}a}(\sqrt{3}, 1) \qquad \mathbf{K}' = \frac{2\pi}{3\sqrt{3}a}(\sqrt{3}, -1) \tag{8}$$

Via the tight-binding model, the 2×2 Hamiltonian consisting of a uniform hopping coefficient t, meaning that the charge carriers (electrons) can "hop" from an atom a of sublattice A to an atom b of sublattice B, and vice-versa. The Hamiltonian then becomes

$$\mathcal{H} = -t \sum_{a,b} \left((c_a^A)^\dagger c_b^B + c_a^A (c_b^B)^\dagger \right) \tag{9}$$

Expressing the sublattice B via the sublattice A and 3 vectors δ_i,

$$\mathcal{H} = -t \sum_a \sum_\delta \left((c_a^A)^\dagger c_{a+\delta}^B + c_a^A (c_{a+\delta}^B)^\dagger \right) \tag{10}$$

Via a Fourier transform to momentum space for the sublattice A with $N/2$ atoms, we get (momentum k)

$$(c_a^A)^\dagger = \frac{1}{\sqrt{N/2}} \sum_k e^{i\mathbf{k}\cdot\mathbf{r}} (c_k^A)^\dagger \tag{11}$$

similarly, performing the same transformation for sublattice B to momentum k' and substituting the operators A and B, along with applying the equation $\sum_a e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_a} = \frac{N}{2}\delta_{k,k'}$, the tight-binding Hamiltonian becomes

$$
\mathcal{H} = t\sum_k \left((c_k^A)^\dagger (c_k^B)^\dagger\right) \begin{pmatrix} 0 & \sum_i e^{i\mathbf{k}\cdot\delta_i} \\ \sum_i e^{-i\mathbf{k}\cdot\delta_i} & 0 \end{pmatrix} \begin{pmatrix} c_k^A \\ c_k^B \end{pmatrix} \tag{12}
$$

Defining $\sum_i e^{i\mathbf{k}\cdot\delta_i} = \Delta(k)$

$$
\begin{aligned}
\Delta(k) = \sum_i e^{i\mathbf{k}\cdot\delta_i} &= e^{i\mathbf{k}\cdot\delta_3}(e^{i\mathbf{k}\cdot(\delta_1-\delta_3)} + e^{i\mathbf{k}\cdot(\delta_2-\delta_3)} + 1) \\
&= e^{i\mathbf{k}\cdot\delta_3}\left(e^{3ik_xa/2}e^{\sqrt{3}ik_ya/2} + e^{3ik_xa/2}e^{-\sqrt{3}ik_ya/2} + 1\right) \\
&= e^{-ik_xa}\left(1 + 2e^{3ik_xa/2}\cos\left(\frac{\sqrt{3}k_ya}{2}\right)\right)
\end{aligned} \tag{13}
$$

The eigenvalues of the Hamiltonian is given by

$$
\begin{aligned}
E_\pm(k) = \pm t\sqrt{\Delta(k)\Delta^*(k)} &= \pm t\sqrt{1 + 4\cos\left(\frac{3k_xa}{2}\right)\cos\left(\frac{\sqrt{3}k_ya}{2}\right) + 4\cos^2\left(\frac{\sqrt{3}k_ya}{2}\right)} \\
&= \pm t\sqrt{3 + 4\cos\left(\frac{3k_xa}{2}\right)\cos\left(\frac{\sqrt{3}k_ya}{2}\right) + 2\cos\left(\sqrt{3}k_ya\right)}
\end{aligned} \tag{14}
$$

From the expression for the eigenenergies, a value of 0 is obtained at the Dirac points. This denotes that the two energy bands (conduction and valence bands) touch at Dirac point.

(g) The following is a non-exhaustive list of methods to differentiate between an n-type and p-type semiconductor:

References

[1] Jungkyun Oh et al. "Ultrasensitive and selective organic FET-type nonenzymatic dopamine sensor based on platinum nanoparticles-decorated reduced graphene oxide". In: *ACS applied materials & interfaces* 9.45 (2017), pp. 39526–39533.

[2] Maria Magliulo et al. "Part per trillion label-free electronic bioanalytical detection". In: *Analytical chemistry* 85.8 (2013), pp. 3849–3857.