Foundations of Audio Signal Processing:

Exercise sheet 2

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2. November 2018

Exercise 2.1.

(a).

$$2e^{\frac{\pi}{2}i}(1+i) \tag{1}$$
$$For\ a+bi = 1+i \tag{2}$$
$$r = \sqrt{1^2 + 1^2} = \sqrt{2} \tag{3}$$
$$1 = \sqrt{2}\cos\phi \tag{4}$$
$$\cos\phi = \frac{1}{\sqrt{2}} \tag{5}$$
$$\phi = \frac{\pi}{4} \tag{6}$$
$$\tag{7}$$
$$2e^{\frac{\pi}{2}i}(1+i) = 2e^{\frac{\pi}{2}i}\sqrt{2}e^{\frac{\pi}{4}i} = 2\sqrt{2}e^{\frac{3\pi}{4}i} \tag{8}$$

(b).

$$z \cdot \bar{z} = |z|^2 \tag{9}$$
$$z = a + bi = re^{i\phi} \tag{10}$$
$$\bar{z} = a - bi = re^{-i\phi} \tag{11}$$
$$z \cdot \bar{z} = re^{i\phi} \cdot re^{-i\phi} = r^2 = a^2 + b^2 = |z|^2 \tag{12}$$

(c).

$$e^{i\alpha} = \cos\alpha + i\sin\alpha \tag{13}$$

$$i\sin\alpha = e^{i\alpha} - \cos\alpha \tag{14}$$

$$e^{-i\alpha} = \cos(-\alpha) + i\sin(-\alpha) = \cos\alpha - i\sin\alpha \tag{15}$$

$$i\sin\alpha = \cos\alpha - e^{-i\alpha} \tag{16}$$

$$Sum\,of\,(14)\,and\,(16)\,gives\,2i\sin\alpha = e^{i\alpha} - e^{-i\alpha} \tag{17}$$

$$sin\alpha = \frac{1}{2i}(e^{i\alpha} - e^{-i\alpha}) \tag{18}$$

Exercise 2.2.

(a).

$$For\,4:\,\{1, i, -1, -i\} \tag{19}$$

$$For\,6:, \{1, \frac{1}{2} + i\frac{\sqrt{3}}{2}, -\frac{1}{2} + i\frac{\sqrt{3}}{2}, -1, -\frac{1}{2} - i\frac{\sqrt{3}}{2}, -\frac{1}{2} + i\frac{\sqrt{3}}{2}\} \tag{20}$$

(b).

$$\sum_{k=0}^{n-1} e^{\frac{2\pi ik}{n}} = 1 + e^{\frac{2\pi i}{n}} + (e^{\frac{2\pi i}{n}})^2 + ... + (e^{\frac{2\pi i}{n}})^{n-1} = \frac{(e^{\frac{2\pi i}{n}})^n - 1}{(e^{\frac{2\pi i}{n}}) - 1} = \frac{1 - 1}{(e^{\frac{2\pi i}{n}}) - 1} = 0 \tag{21}$$