Foundations of Audio Signal Processing:

Exercise sheet 9

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Exercise 9.1

(a)

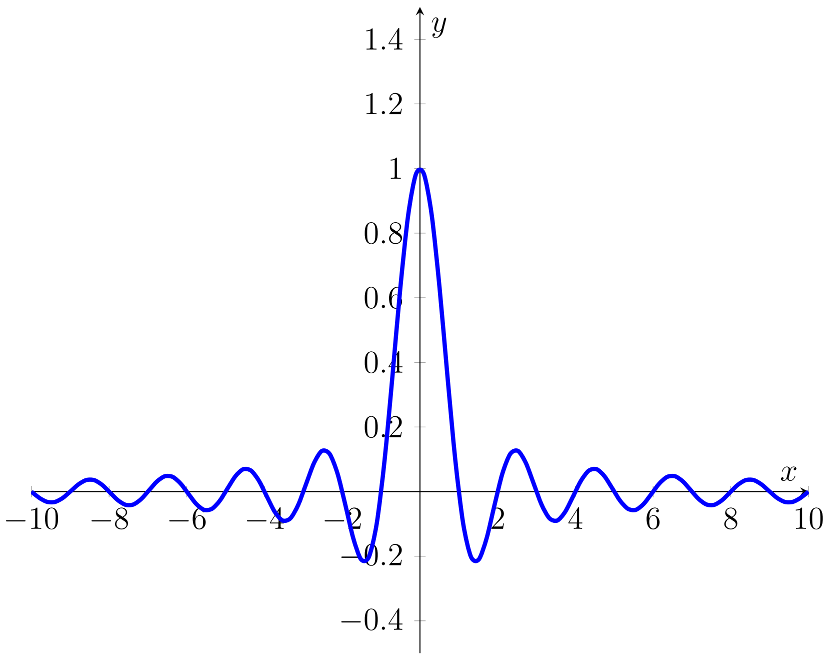

Figure 1: Plot of sinc(x).

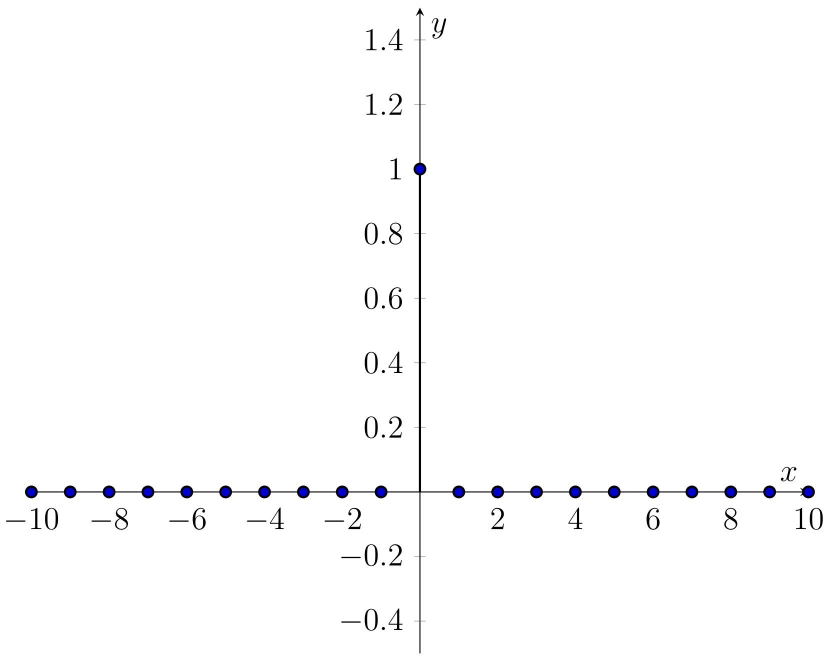

Figure 2: Plot of sampled sinc(x) with T=1.

Sampling rate is $\frac{1}{T} = \frac{1}{1} = 1\text{Hz}$

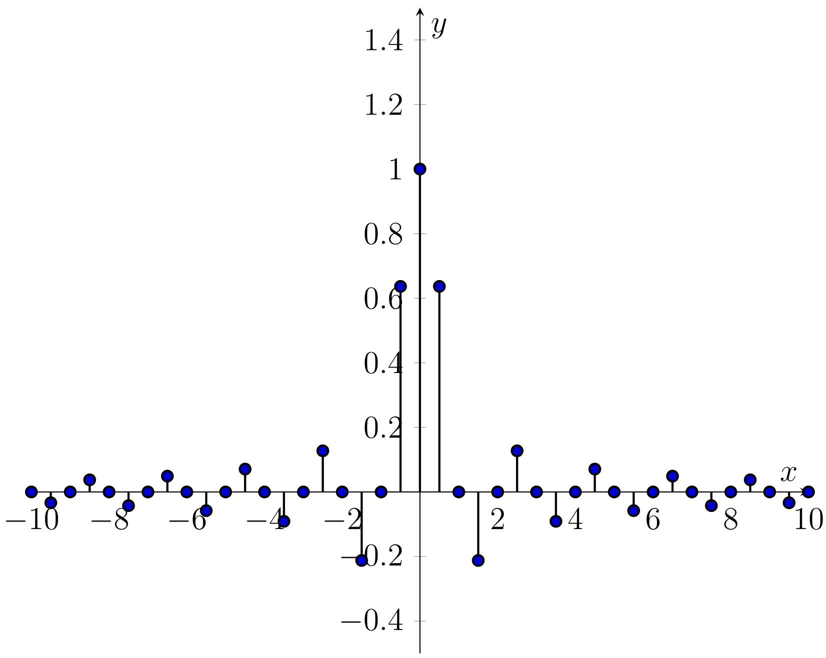

Figure 3: Plot of sampled sinc(x) with T=0.5.

Sampling rate is $\frac{1}{T} = \frac{1}{0.5} = 2\text{Hz}$

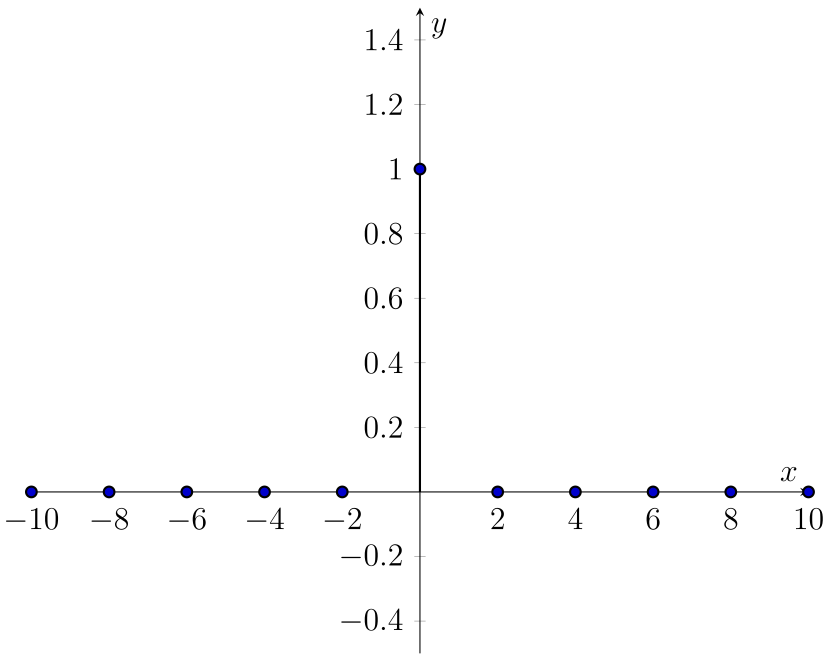

Figure 4: Plot of sampled sinc(x) with T=2.

Sampling rate is $\frac{1}{T} = \frac{1}{2} = 0.5$Hz

(b)

Fourier transform for $f(t) = sinc(t)$ is $\widehat{f}(w) = rect(w)$

$$rect(w) = \begin{cases} 0, & \text{if } |w| > \frac{1}{2} \\ 1/2, & \text{if } |w| = \frac{1}{2} \\ 1, & \text{if } |w| < \frac{1}{2} \end{cases} \tag{1}$$

By definition, sinc(t) is 0.5-bandlimited, and 0.5 is such smallest Ω

(c)

By Shannon's theorem, an Ω-bandlimited signal needs to be sampled with rate $> 2\Omega$ in order to be fully reconstructible. In case of *sinc*, sampling rate should be $> 2 * 0.5 = 1$ Hz
For $T = 1$ sampling rate $\Omega = \frac{1}{T} = 1$Hz is not > 1Hz. Thus, signal is not fully reconstructible
For $T = \frac{1}{2}$ sampling rate $\Omega = \frac{1}{T} = 2$Hz > 1Hz. Thus, signal is fully reconstructible
For $T = 2$ sampling rate $\Omega = \frac{1}{T} = 0.5$Hz < 1Hz. Thus, signal is not fully reconstructible

Exercise 9.2

(a)

$$(\uparrow M \circ T^k)[x](n) = \begin{cases} x(\frac{n}{M} - k), & \text{if } M|n \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$(T^k \circ \uparrow M)[x](n) = \begin{cases} x(\frac{n-k}{M}), & \text{if } M|n-k \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Since $(\uparrow M \circ T^k)[x](n) \neq (T^k \circ \uparrow M)[x](n)$, upsampling operator is not time invariant

(b)

$$(T^k \circ E_\omega)[x](n) = e^{-2\pi i w n} x(n-k) \tag{4}$$

$$(E_\omega \circ T^k)[x](n) = e^{-2\pi i w (n-k)} x(n-k) \tag{5}$$

Since $\forall w \neq 0$, $(\uparrow M \circ T^k)[x](n) \neq (T^k \circ \uparrow M)[x](n)$, frequency shift operator is not time invariant