Foundations of Audio Signal Processing:

Exercise sheet 5

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Exercise 5.1.

(a).

$$||f(t)||_1 = \int_{-\infty}^{+\infty} f(t)dt = \int_0^1 f(t)dt = \int_0^1 t^{-1/2}dt = \lim_{t\to 1} 2t^{1/2} - \lim_{t\to 0} 2t^{1/2} = 2 \quad (1)$$

$$||f(t)||_2 = \int_{-\infty}^{+\infty} f(t)^2dt = \int_0^1 f(t)^2dt = \int_0^1 t^{-1}dt = \lim_{t\to 1} \ln(t) - \lim_{t\to 0} ln(t) = \infty \quad (2)$$

(b).

$$||f(t)||_1 = \int_{-\infty}^{+\infty} f(t)dt = \int_0^{+\infty} f(t)dt = \int_0^{+\infty} t^{-1}dt = \lim_{t\to +\infty} \ln(t) - \lim_{t\to 0} \ln(t) = \infty \quad (3)$$

$$||f(t)||_2 = \int_{-\infty}^{+\infty} f(t)^2dt = \int_0^{+\infty} f(t)^2dt = \int_0^{+\infty} t^{-2}dt = \lim_{t\to +\infty} (-\frac{1}{t}) - \lim_{t\to 0}(-\frac{1}{t}) = 0 + 1 = 1 \quad (4)$$

Exercise 5.2.

(a).

$$||\sum_{n=1}^{+\infty} x(n)||_p = \sum_{n=1}^{+\infty} (e^n)^p = \sum_{n=1}^{+\infty} e^{np} \to +\infty \text{ for } \forall p \in \mathbb{Z} \quad (5)$$

$$\text{So } x(n) \notin l^p(\mathbb{Z}) \forall p \in \mathbb{Z} \quad (6)$$

(b).

$$||\sum_{n=1}^{+\infty} x(n)||_p = \sum_{n=1}^{+\infty} (e^{2\pi i n})^p = \sum_{n=1}^{+\infty} \cos 2\pi p n + i \sum_{n=1}^{+\infty} \sin 2\pi p n = \sum_{n=1}^{+\infty} 1 + i \sum_{n=1}^{+\infty} 0 = \sum_{n=1}^{+\infty} 1 \to +\infty \tag{7}$$

$$\text{So } x(n) \notin l^p(\mathbb{Z}) \forall p \in \mathbb{Z} \tag{8}$$

(c).

$$\text{For even } p = 2k \tag{9}$$

$$||\sum_{n=1}^{+\infty} x(n)||_p = \sum_{n=1}^{+\infty} (\frac{1}{\sqrt{n}})^p = \sum_{n=1}^{+\infty} (\frac{1}{\sqrt{n}})^{2k} = \sum_{n=1}^{+\infty} (\frac{1}{n})^k \tag{10}$$

$$\text{Diverges for } k = 1\text{, converges for } k > 1 \text{ as } \zeta(k) \tag{11}$$

$$\text{So } x(n) \in l^p(\mathbb{Z}) \ \forall \text{ even } p > 2 \tag{12}$$

$$\tag{13}$$

$$\text{For odd } p = 2k + 1 \tag{14}$$

$$||\sum_{n=1}^{+\infty} x(n)||_p = \sum_{n=1}^{+\infty} (\frac{1}{\sqrt{n}})^p = \sum_{n=1}^{+\infty} (\frac{1}{\sqrt{n}})^{2k+1} = \sum_{n=1}^{+\infty} (\frac{1}{\sqrt{n}})^{2k} \frac{1}{\sqrt{n}} = \sum_{n=1}^{+\infty} (\frac{1}{n})^k \frac{1}{\sqrt{n}} \tag{15}$$

$$\text{Diverges because } \frac{1}{\sqrt{n}} \text{ diverges and } \frac{1}{n} \text{ behaves as for even } p \tag{16}$$

$$\text{So } x(n) \notin l^p(\mathbb{Z}) \ \forall \text{ odd } p \tag{17}$$