Foundations of Audio Signal Processing:

Exercise sheet65

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1. Dezember 2018

Exercise 6.1.

(a).

$$\widehat{f'(w)} = \int_{-\infty}^{+\infty} f'(t)e^{-2\pi iwt}dt = e^{-2\pi iwt}f(t)|_{t=-\infty}^{+\infty} - \int_{-\infty}^{+\infty} -2\pi iwf(t)e^{-2\pi iwt}dt = \tag{1}$$

$$\int_{-\infty}^{+\infty} 2\pi iwf(t)e^{-2\pi iwt}dt = 2\pi iwf(\hat{w}) \tag{2}$$

(b).

$$\widehat{tf(t)} = \int_{-\infty}^{+\infty} e^{-2\pi iwt}tf(t)dt = -2\pi i\frac{d}{dw}\int_{-\infty}^{+\infty} e^{-2\pi iwt}f(t)dt = -2\pi if'(w) \tag{3}$$

(c).

$$\widehat{f'(w)} = \int_{-\infty}^{+\infty} cos(2\pi wt)f(t)dt + i\int_{-\infty}^{+\infty}(-sin(2\pi wt)f(t)dt) \tag{4}$$

$$Re(\widehat{f'(-w)}) = \int_{-\infty}^{+\infty} cos(-2\pi wt)f(t)dt = \int_{-\infty}^{+\infty} cos(2\pi wt)f(t)dt = Re(\widehat{f'(w)} \tag{5}$$

$$Im(\widehat{f'(-w)}) = \int_{-\infty}^{+\infty} -sin(-2\pi wt)f(t)dt = \int_{-\infty}^{+\infty} sin(2\pi wt)f(t)dt = -Im(\widehat{f'(w)} \tag{6}$$

(d).

$$\widehat{f'(w)} = \int_{-\infty}^{+\infty} cos(2\pi wt)f(t)dt - i\int_{-\infty}^{+\infty} sin(2\pi wt)f(t)dt \tag{7}$$

$$\text{Since } f(-t) = f(t), \int_{-\infty}^{+\infty} sin(2\pi wt)f(t)d = 0 \tag{8}$$

$$\text{So, } \widehat{f'(w)} = \int_{-\infty}^{+\infty} cos(2\pi wt)f(t)dt \tag{9}$$

$$f(t) \text{ is real, so } \widehat{f'(w)} \text{ is real} \tag{10}$$

$$f(t) \text{ and } cos(2\pi wt) \text{ are both even, so } \widehat{f'(w)} \text{ is even} \tag{11}$$

Exercise 6.2.

(a).

$$\sqrt{2}\int_0^1 f(t)\cos(2\pi kt)dt = \sqrt{2}(\int_{1/2}^1 f(t)\cos(2\pi kt)dt + \int_0^{1/2} f(t)\cos(2\pi kt)dt) = \tag{12}$$

$$\sqrt{2}(\int_{1/2}^1 \cos(2\pi kt)dt - \int_0^{1/2} \cos(2\pi kt)dt) = \sqrt{(}2)(\frac{\sin(2\pi kt)}{2\pi k}|_{t=1/2}^1 - \frac{\sin(2\pi kt)}{2\pi k}|_{t=0}^{1/2}) = \tag{13}$$

$$\sqrt{2}(\frac{\sin(2\pi k) - \sin(\pi k)}{2\pi k} - \frac{\sin(\pi k) - \sin(0)}{2\pi k}) = 0 \tag{14}$$

$$\sqrt{2}\int_0^1 f(t)\sin(2\pi kt)dt = \sqrt{2}(\int_{1/2}^1 f(t)\sin(2\pi kt)dt + \int_0^{1/2} f(t)\sin(2\pi kt)dt) = \tag{15}$$

$$\sqrt{2}(\int_{1/2}^1 \sin(2\pi kt)dt - \int_0^{1/2} \sin(2\pi kt)dt) = -\sqrt{2}(\frac{\cos(2\pi k) - \cos(\pi k)}{2\pi k} - \frac{\cos(\pi k) - \cos(0)}{2\pi k}) = \tag{16}$$

$$-\sqrt{2}(\frac{4}{2\pi k}) = -\frac{2\sqrt{2}}{\pi k} \tag{17}$$