Foundations of Audio Signal Processing:

Exercise sheet 4

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Exercise 4.1.

$$||\sum_{j=1}^{n} x_j|| = (\sum_{j=1}^{n} x_j, \sum_{i=1}^{n} x_i) = \sum_{j=1}^{n}(x_j, \sum_{i=1}^{n} x_i) = \sum_{j=1}^{n} x_j \cdot \sum_{i=1}^{n} x_i = \sum_{j=1}^{n}\sum_{i=1}^{n} x_j \cdot x_i = \sum_{j=1}^{n}\sum_{i=1}^{n}(x_i, x_j) \tag{1}$$

$$\text{Because of orthogonality of } x,\ (x_i, x_j) = 0 \text{ for } i \neq j \tag{2}$$

$$\text{So } (1) = \sum_{j=1}^{n}(x_j, x_j) = \sum_{j=1}^{n} ||x_j||^2 \tag{3}$$

Exercise 4.2.

(a).

$$d(x,y) = |x-y| = \sqrt{(Re(x)-Re(y))^2+(Im(x)-Im(y))^2} \tag{4}$$

$$\text{property 1} \tag{5}$$

$$(Re(x)-Re(y))^2 \geq 0, (Im(x)-Im(y))^2 \geq 0 \Rightarrow \tag{6}$$

$$\sqrt{(Re(x)-Re(y))^2+(Im(x)-Im(y))^2} \geq 0 \tag{7}$$

$$\text{property 2} \tag{8}$$

$$\Longrightarrow \tag{9}$$

$$\sqrt{(Re(x)-Re(y))^2+(Im(x)-Im(y))^2} = 0 \Rightarrow (Re(x)-Re(y))^2+(Im(x)-Im(y))^2 = 0 \tag{10}$$

$$\Rightarrow Re(x) = Re(y), Im(x) = Im(y) \Rightarrow x = y \tag{11}$$

$$\Longleftarrow \tag{12}$$

$$x = y \Rightarrow Re(x) = Re(y), Im(x) = Im(y) \Rightarrow \sqrt{(Re(x)-Re(y))^2+(Im(x)-Im(y))^2} = 0 \tag{13}$$

$$\text{So, } d(x,y) = 0 \Leftrightarrow x = y \tag{14}$$

$$\text{property 3} \tag{15}$$

$$d(x,y) = \sqrt{(Re(x)-Re(y))^2+(Im(x)-Im(y))^2} = \tag{16}$$

$$\sqrt{((-1)\cdot(Re(x)-Re(y)))^2+((-1)\cdot(Im(x)-Im(y)))^2} = \tag{17}$$

$$\sqrt{(Re(y)-Re(x))^2+(Im(y)-Im(x))}^2 = d(y,x) \tag{18}$$

$$\text{property} \quad 4 \tag{19}$$

$$\forall x,\ y,\ z\ \in \mathbb{C}\, |x-z| \leq |x-y| + |y-z| \tag{20}$$

$$\text{because they can be treated as points on the Eucleadean plane with coordinates Re, Im} \tag{21}$$

$$\text{All 4 properties hold, so it is a metric} \tag{22}$$

(b).

$$d(x, y) = |x| \cdot |y| \tag{23}$$

$$\text{Let x=y. Then } d(x, x) = |x|^2 = Re(x) + Im(x) \text{ not always equals 0.} \tag{24}$$

$$\text{Property 3 doesn't hold, so it is not a metric} \tag{25}$$

(c).

$$d(x,y) = \begin{cases} 1 & \text{if y} \neq \text{x} \\ 0 & \text{else} \end{cases} \tag{26}$$

$$\text{Property 1} \tag{27}$$

$$\text{By definition, d(x, y) is always} \geq 0 \tag{28}$$

$$\text{Property 2} \tag{29}$$

$$\text{By definition, d(x, x)} \Leftrightarrow 0 \tag{30}$$

$$\text{Property 3} \tag{31}$$

$$d(x,y) = \begin{cases} 1 & \text{if y} \neq \text{x} \\ 0 & \text{else} \end{cases} \tag{32}$$

$$d(y,x) = \begin{cases} 1 & \text{if x} \neq \text{y} \\ 0 & \text{else} \end{cases} \tag{33}$$

$$d(x,y) = d(y,x) \tag{34}$$

$$\text{Property 4} \tag{35}$$

$$\text{d(x, z)} \in \{0, 1\}, \text{d(x, y)} + \text{d(y, z)} \in \{0, 1, 2\} \tag{36}$$

$$\text{The only case, where } d(x,z) > d(x,y) + d(y,z) \text{ is } d(x,z) = 1 \text{ and } d(x,y) + d(y,z) = 0 \tag{37}$$

$$\text{In this case } x = y = z\text{, which means } d(x,z) = 0\text{, which contracicts with } d(x,z) = 1 \tag{38}$$

$$\text{So, this case never happens.} \tag{39}$$

$$\text{All 4 properties hold, so it is a metric} \tag{40}$$