

The Hassle with Monads

Panicz Maciej Godek

@PaniczGodek

`https://github.com/panicz/writings/tree/
master/talks/datamass`

datamass.io summit, 28.09.2018

Before we begin - questions to the audience:

- do you program?
- do you not program?

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- Java/C#
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- have you heard about monads?
- do you understand monads?
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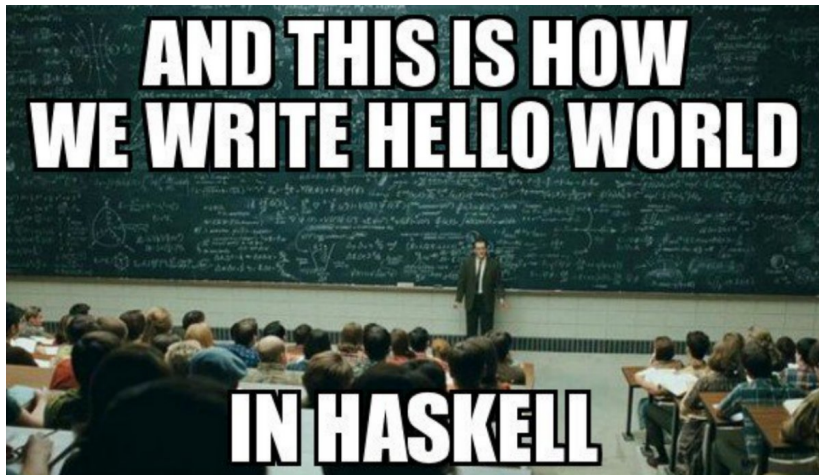
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Why learn monads?

Yes, monads seem to be a form of AspectOrientedProgramming, since they serve to isolate a generalized computational strategy from the specifics of an algorithm. For example, in HaskellLanguage you can write a graph-searching procedure that can either do a depth-first search and return the first result, or do a breadth-first search and return a list of results, merely by running it in a different monad.

<http://wiki.c2.com/?AspectOrientedProgramming>

Why learn monads?



Why (lazy) functional programming?



@selfsame@tiny.tilde.website

@jplur_

Following



The recursive centaur: half horse, half recursive centaur



Why (lazy) functional programming?

```
numbersFrom n = n:numbersFrom(n+1)
numbersFrom 0 = [0,1,2,3,4,5,6,7,8,9,10,...]

sieve (h:t) = h:(sieve [x|x<-t,x`mod`h/=0])
primes = sieve (numbersFrom 2)
primes = [2,3,5,7,11,13,17,19,23,29,31,37,...]
```

Jerzy Karczmarczuk, *Generating Power of Lazy Semantics*
John Hughes, *Why Functional Programming Matters*

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The essence of laziness

```
square x = x * x
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Applicative order (evaluate arguments before expansion):

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

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Lambda the Ultimate

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square x = x * x
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```
square =  $\lambda x \rightarrow x * x$ 
```

```
square = function(x) { return x * x; }
```

```
distance x y = abs(x-y)
```

```
distance =  $\lambda x \rightarrow \lambda y \rightarrow \text{abs}(x-y)$  (currying)
```

```
let name = value in expression
```

```
( $\lambda$  name  $\rightarrow$  expression) value
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The problem with I/O

$1 * 3 + 2 * 0$

`readNumber() * 3 + 2 * readNumber()`

`< 1`

`< 0`

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Attempted solution

```
let a      = readNumber( ) in
  let b      = readNumber( ) in
    a*2 + 3*b
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let  a      = readNumber( ) in
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```

Actual solution

```
let (a,w1) = readNumber(w0) in
  let (b,w2) = readNumber(w1) in
    a*2 + 3*b
```

Better (composable) solution

```
let (a,w1) = readNumber(w0) in
  let (b,w2) = readNumber(w1) in
    (a*2 + 3*b, w2)
```

A function

```
myOperation w0 =  
  let (a,w1) = readNumber(w0) in  
    let (b,w2) = readNumber(w1) in  
      (a*2 + 3*b, w2)
```


A function

```
myOperation :: RealWorld -> (Int, RealWorld)
myOperation w0 =
  let (a,w1) = readNumber(w0) in
    let (b,w2) = readNumber(w1) in
      (a*2 + 3*b, w2)
```

https://wiki.haskell.org/IO_inside

Downsides

- need to pass additional parameter
- prone to errors (e.g. w_0 instead of w_1)
- nesting level increases

Could we do something to make w_0 passed implicitly?

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  (λ a -> pass readNumber
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return value world = (value, world)

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Does this really work?

```
pass readNumber (λ a -> λ w1 ->
  let (y, w2) = readNumber(w1) in
    (λ b -> λ w -> (a*2 + 3*b, w)) y w2)
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  (λ a -> λ w1 -> let (y, w2) = readNumber(w1)  
    in (a*2 + 3*y, w2)) x w3
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It works!

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pass readNumber
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But typing λ and the increased indentation level is annoying!

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Introduce new syntax (do-notation):

```
do result <- action  
  actions ...
```

can be interpreted as:

```
pass action (\ result -> do actions ...)
```

Note: In Haskell, `pass` function is spelled `>=` and pronounced “bind”.

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Emperor's new clothes

Now we can write our program as:

```
do a <- readNumber
   b <- readNumber
   return a*2 + 3*b
```

The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

The monad laws:

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

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In Haskell

```
class Monad m where  
  return :: a -> m a  
  (»=) :: m a -> (a -> m b) -> m b
```

Monads with the `do` notation provide a general and systematic solution to the common anti-pattern known as *the Pyramid of Doom*.

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The Pyramid of Doom

```
function register()
{
    if (!empty($_POST)) {
        $msg = '';
        if ($_POST['user_name']) {
            if ($_POST['user_password_new']) {
                if ($_POST['user_password_new'] === $_POST['user_password_repeat']) {
                    if (strlen($_POST['user_password_new']) > 5) {
                        if (strlen($_POST['user_name']) < 65 && strlen($_POST['user_name']) > 1) {
                            if (preg_match('/^[a-z\d]{2,64}$/i', $_POST['user_name'])) {
                                $user = read_user($_POST['user_name']);
                                if (!isset($user['user_name'])) {
                                    if ($_POST['user_email']) {
                                        if (strlen($_POST['user_email']) < 65) {
                                            if (filter_var($_POST['user_email'], FILTER_VALIDATE_EMAIL)) {
                                                create_user();
                                                $_SESSION['msg'] = 'You are now registered so please login';
                                                header('Location: ' . $_SERVER['PHP_SELF']);
                                                exit();
                                            } else $msg = 'You must provide a valid email address';
                                        } else $msg = 'Email must be less than 64 characters';
                                    } else $msg = 'Email cannot be empty';
                                } else $msg = 'Username already exists';
                            } else $msg = 'Username must be only a-z, A-Z, 0-9';
                        } else $msg = 'Username must be between 2 and 64 characters';
                    } else $msg = 'Password must be at least 6 characters';
                } else $msg = 'Passwords do not match';
            } else $msg = 'Empty Password';
        } else $msg = 'Empty Username';
        $_SESSION['msg'] = $msg;
    }
    return register_form();
}
```



The Pyramid of Doom

*In computer programming, the **pyramid of doom** is a common problem that arises when a program uses many levels of nested indentation to control access to a function. It is commonly seen when checking for null pointers or handling callbacks.*

Wikipedia/Pyramid_of_doom_(programming)

The Pyramid of Doom – example

```
theWidth = windows("Main").views(5).size().width();

if windows.contains("Main") {
    if windows("Main").views.contains(5) {
        theWidth = windows("Main").views(5).size().width();
        //more code that works with theWidth
    }
}
```

With “optional chaining”/“null-conditional”/“safe navigation” operator:

```
theWidth = windows("Main")?.views(5)?.size.width;
```

The Pyramid of Doom – example

```
theWidth = windows("Main").views(5).size().width();

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Dealing with “no values” in Haskell

```
data Maybe a = Nothing | Just a

let theWidth = do window <- windows("Main")
                  view <- views 5 window
                  return width (size view)

instance Monad Maybe where
  (Nothing >= f) = Nothing
  (Just a >= f)  = (f a)
  return a      = Just a
```


Dealing with “no values” in Haskell

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data Maybe a = Nothing | Just a
```

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let theWidth = do window <- windows("Main")  
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The List Monad

```
instance Monad List where
  (x >=> f) = concatMap f x
  return a = [a]
```

```
>>> concatMap (\n -> [1..n]) [1,2,3]
[1, 1,2, 1,2,3]
```

```
>>> do a <- [1,2,3]
     b <- [4,5]
     return (a, b)
```

```
[(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)]
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```

The Amb Monad

```
import Control.Monad
import Control.Monad.Amb

pyTriple n = do a <- anIntegerBetween 1 n
                b <- anIntegerBetween (a+1) n
                c <- anIntegerBetween (b+1) n
                when (a*a + b*b /= c*c) empty
                return (a,b,c)
```

```
>>> oneValue (pyTriple 20)
(3,4,5)
```

```
>>> allValues (pyTriple 20)
[(3,4,5), (5,12,13), (6,8,10), (8,15,17),
 (9,12,15), (12,16,20)]
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Other instances of monads

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<https://www.youtube.com/watch?v=yjmKMhJOJos>

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Monad Laws revisited

Function composition:

$$(f \cdot g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \cdot (g \cdot h) = (f \cdot g) \cdot h - \textit{associativity of } (.)$$

$$\begin{aligned}(f \cdot (g \cdot h)) \ x &= f \ ((g \cdot h) \ x) = f \ (g \ (h \ x)) \\ ((f \cdot g) \cdot h) \ x &= (f \cdot g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

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```

$$f \ . \ (g \ . \ h) = (f \ . \ g) \ . \ h - \textit{associativity of (.)}$$

$$(f \ . \ (g \ . \ h)) \ x = f \ ((g \ . \ h) \ x) = f \ (g \ (h \ x))$$

$$(((f \ . \ g) \ h)) \ x = (f \ . \ g) \ (h \ x) = f \ (g \ (h \ x))$$

Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
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Monad Laws revisited

associativity + identity = monoid (semi-group with neutral element)

(\circ, id) is a monoid. Other examples:

- $(+, 0)$
- $(*, 1)$
- $(min, +\infty)$
- $(max, -\infty)$

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Kleisli composition:

```
(f >=> g) x = do y <- f x  
              g y
```

```
(f >=> g) x = (f x) >=> g
```

```
(return >=> f) = f
```

```
(f >=> return) = f
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((f >=> g) >=> h) = (f >=> (g >=> h))
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Monad Laws revisited

Kleisli composition:

$$(f \ggg g) \ x = \text{do } y \leftarrow f \ x \\ \qquad \qquad \qquad g \ y$$
$$(f \ggg g) \ x = (f \ x) \ggg g$$
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Functors, Applicatives, Monads

```
class Functor f where  
  fmap :: (a -> b) -> f a -> f b
```

```
class Functor f => Applicative f where  
  pure :: a -> f a  
  (<*>) :: f (a -> b) -> f a -> f b
```

```
class Applicative m => Monad m where  
  (»=) :: m a -> (a -> m b) -> m b  
  return :: a -> m a  
  return = pure
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Free monads

```
data IO a =  
  PutStrLn String (IO a)  
  | GetStrLn (String -> IO a)  
  | Sleep Int (IO a)  
  | DeleteFile String (IO a)  
  | LaunchTheMissles (IO a)  
  ...  
  | forall a0. Chain (IO a0) (a0 -> IO a)  
  | Return a
```

Problems:

- IO can do really anything
- we may want to restrict it to a smaller number of capabilities
- we may want to simulate (mock) the behavior

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Monad transformers

Problem: different monads do not stack together well.

For example, `Future (Maybe a)` cannot be composed with the single `>>=` operator.

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Thank you

Questions?

@PaniczGodek

`https://www.quora.com/profile/Panicz-Godek`

`https://github.com/panicz/writings/tree/
master/talks/datamass`