

# The Hassle with Monads

Panicz Maciej Godek

@PaniczGodek

`https://github.com/panicz/writings/tree/  
master/talks/datamass`

**datamass.io summit, 28.09.2018**

# Before we begin - questions to the audience:

- do you program?
- do you not program?

# Before we begin - questions to the audience:

- do you program?
- do you not program?

# Before we begin - questions to the audience:

- do you program?
- do you not program?

# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# Which languages do you know?

- **Java/C#**
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?



# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# Which languages do you know?

- Java/C#
- Python
- JavaScript
- C/C++
- Scala
- Erlang/Elixir
- OCaml/F#
- Haskell
- some other?

# On to the topic

- do you know functional programming?
- have you heard about monads?
- do you understand monads?
- did you try to understand monads and failed?

# On to the topic

- do you know functional programming?
- have you heard about monads?
- do you understand monads?
- did you try to understand monads and failed?



# On to the topic

- do you know functional programming?
- have you heard about monads?
- do you understand monads?
- did you try to understand monads and failed?

# On to the topic

- do you know functional programming?
- have you heard about monads?
- do you understand monads?
- did you try to understand monads and failed?

# On to the topic

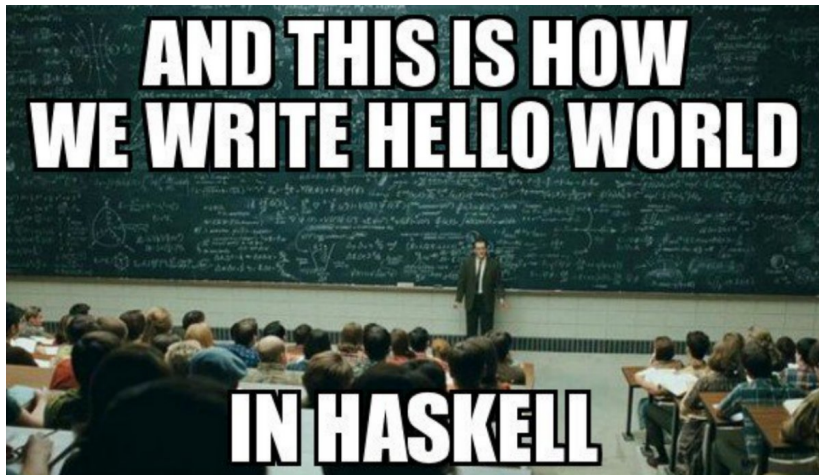
- do you know functional programming?
- have you heard about monads?
- do you understand monads?
- did you try to understand monads and failed?

# Why learn monads?

*Yes, monads seem to be a form of AspectOrientedProgramming, since they serve to isolate a generalized computational strategy from the specifics of an algorithm. For example, in HaskellLanguage you can write a graph-searching procedure that can either do a depth-first search and return the first result, or do a breadth-first search and return a list of results, merely by running it in a different monad.*

<http://wiki.c2.com/?AspectOrientedProgramming>

# Why learn monads?



# Why (lazy) functional programming?



@selfsame@tiny.tilde.website

@jplur\_

Following



The recursive centaur: half horse, half recursive centaur



# Why (lazy) functional programming?

```
numbersFrom n = n:numbersFrom(n+1)
numbersFrom 0 = [0,1,2,3,4,5,6,7,8,9,10,...]

sieve (h:t) = h:(sieve [x|x<-t,x`mod`h/=0])
primes = sieve (numbersFrom 2)
primes = [2,3,5,7,11,13,17,19,23,29,31,37,...]
```

Jerzy Karczmarczuk, *Generating Power of Lazy Semantics*  
John Hughes, *Why Functional Programming Matters*

# Why (lazy) functional programming?

```
numbersFrom n = n:numbersFrom(n+1)  
numbersFrom 0 = [0,1,2,3,4,5,6,7,8,9,10,...]
```

```
sieve (h:t) = h:(sieve [x|x<-t,x`mod`h/=0])  
primes = sieve (numbersFrom 2)  
primes = [2,3,5,7,11,13,17,19,23,29,31,37,...]
```

Jerzy Karczmarczuk, *Generating Power of Lazy Semantics*  
John Hughes, *Why Functional Programming Matters*



# Why (lazy) functional programming?

```
numbersFrom n = n:numbersFrom(n+1)
numbersFrom 0 = [0,1,2,3,4,5,6,7,8,9,10,...]

sieve (h:t) = h:(sieve [x|x<-t,x`mod`h/=0])
primes = sieve (numbersFrom 2)
primes = [2,3,5,7,11,13,17,19,23,29,31,37,...]
```

Jerzy Karczmarczuk, *Generating Power of Lazy Semantics*  
John Hughes, *Why Functional Programming Matters*

# Why (lazy) functional programming?

```
numbersFrom n = n:numbersFrom(n+1)
numbersFrom 0 = [0,1,2,3,4,5,6,7,8,9,10,...]

sieve (h:t) = h:(sieve [x|x<-t,x`mod`h/=0])
primes = sieve (numbersFrom 2)
primes = [2,3,5,7,11,13,17,19,23,29,31,37,...]
```

Jerzy Karczmarczuk, *Generating Power of Lazy Semantics*  
John Hughes, *Why Functional Programming Matters*

# Why (lazy) functional programming?

```
numbersFrom n = n:numbersFrom(n+1)
numbersFrom 0 = [0,1,2,3,4,5,6,7,8,9,10,...]

sieve (h:t) = h:(sieve [x|x<-t,x`mod`h/=0])
primes = sieve (numbersFrom 2)
primes = [2,3,5,7,11,13,17,19,23,29,31,37,...]
```

Jerzy Karczmarczuk, *Generating Power of Lazy Semantics*  
John Hughes, *Why Functional Programming Matters*

# Why (lazy) functional programming?

```
numbersFrom n = n:numbersFrom(n+1)
numbersFrom 0 = [0,1,2,3,4,5,6,7,8,9,10,...]

sieve (h:t) = h:(sieve [x|x<-t,x`mod`h/=0])
primes = sieve (numbersFrom 2)
primes = [2,3,5,7,11,13,17,19,23,29,31,37,...]
```

Jerzy Karczmarczuk, *Generating Power of Lazy Semantics*

John Hughes, *Why Functional Programming Matters*

# Why (lazy) functional programming?

```
numbersFrom n = n:numbersFrom(n+1)
numbersFrom 0 = [0,1,2,3,4,5,6,7,8,9,10,...]

sieve (h:t) = h:(sieve [x|x<-t,x`mod`h/=0])
primes = sieve (numbersFrom 2)
primes = [2,3,5,7,11,13,17,19,23,29,31,37,...]
```

Jerzy Karczmarczuk, *Generating Power of Lazy Semantics*  
John Hughes, *Why Functional Programming Matters*

# The essence of laziness

```
square x = x * x
```

Applicative order (evaluate arguments before expansion):

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

**Applicative order (evaluate arguments before expansion):**

```
square (2*3) = square 6 =def 6 * 6 = 36
```

**Normal order (evaluate arguments after expansion):**

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

**Applicative order (evaluate arguments before expansion):**

```
square (2*3) = square 6 =def 6 * 6 = 36
```

**Normal order (evaluate arguments after expansion):**

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```



# The essence of laziness

```
square x = x * x
```

**Applicative order (evaluate arguments before expansion):**

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

**Applicative order (evaluate arguments before expansion):**

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

**Applicative order (evaluate arguments before expansion):**

```
square (2*3) = square 6 =def 6 * 6 = 36
```

**Normal order (evaluate arguments after expansion):**

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

Applicative order (evaluate arguments before expansion):

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

Applicative order (evaluate arguments before expansion):

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

Applicative order (evaluate arguments before expansion):

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

Applicative order (evaluate arguments before expansion):

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```

# The essence of laziness

```
square x = x * x
```

Applicative order (evaluate arguments before expansion):

```
square (2*3) = square 6 =def 6 * 6 = 36
```

Normal order (evaluate arguments after expansion):

```
square (2*3) =def (2*3) * (2*3) = 6 * 6 = 36
```



# Lambda the Ultimate

```
square x = x * x
```

```
square =  $\lambda$  x -> x * x
```

```
square = function(x) { return x * x; }
```

```
distance x y = abs(x-y)
```

```
distance =  $\lambda$  x ->  $\lambda$  y -> abs(x-y) (currying)
```

```
let name = value in expression
```

```
( $\lambda$  name -> expression) value
```

# Lambda the Ultimate

```
square x = x * x  
square =  $\lambda$  x -> x * x  
square = function(x) { return x * x; }
```

```
distance x y = abs(x-y)  
distance =  $\lambda$  x ->  $\lambda$  y -> abs(x-y) (currying)
```

```
let name = value in expression  
( $\lambda$  name -> expression) value
```

# Lambda the Ultimate

```
square x = x * x  
square =  $\lambda$  x -> x * x  
square = function(x) { return x * x; }
```

```
distance x y = abs(x-y)  
distance =  $\lambda$  x ->  $\lambda$  y -> abs(x-y) (currying)
```

```
let name = value in expression  
( $\lambda$  name -> expression) value
```

# Lambda the Ultimate

```
square x = x * x  
square =  $\lambda$  x -> x * x  
square = function(x) { return x * x; }
```

```
distance x y = abs(x-y)  
distance =  $\lambda$  x ->  $\lambda$  y -> abs(x-y) (currying)
```

```
let name = value in expression  
( $\lambda$  name -> expression) value
```

# Lambda the Ultimate

```
square x = x * x  
square =  $\lambda$  x -> x * x  
square = function(x) { return x * x; }
```

```
distance x y = abs(x-y)  
distance =  $\lambda$  x ->  $\lambda$  y -> abs(x-y) (currying)
```

```
let name = value in expression  
( $\lambda$  name -> expression) value
```

# Lambda the Ultimate

```
square x = x * x  
square =  $\lambda$  x -> x * x  
square = function(x) { return x * x; }
```

```
distance x y = abs(x-y)  
distance =  $\lambda$  x ->  $\lambda$  y -> abs(x-y) (currying)
```

```
let name = value in expression  
( $\lambda$  name -> expression) value
```

# Lambda the Ultimate

```
square x = x * x  
square =  $\lambda$  x -> x * x  
square = function(x) { return x * x; }
```

```
distance x y = abs(x-y)  
distance =  $\lambda$  x ->  $\lambda$  y -> abs(x-y) (currying)
```

```
let name = value in expression  
( $\lambda$  name -> expression) value
```

# The problem with I/O

$1 * 3 + 2 * 0$

`readNumber() * 3 + 2 * readNumber()`

`< 1`

`< 0`



# The problem with I/O

$1 * 3 + 2 * 0$

`readNumber() * 3 + 2 * readNumber()`

`< 1`

`< 0`

# The problem with I/O

$1 * 3 + 2 * 0$

`readNumber() * 3 + 2 * readNumber()`

`< 1`

`< 0`

# Attempted solution

```
let a      = readNumber( ) in
  let b      = readNumber( ) in
    a*2 + 3*b
```

# Attempted solution

```
let a      = readNumber( ) in
  let b      = readNumber( ) in
    a*2 + 3*b
```

# Attempted solution

```
let  a      = readNumber( ) in
  let  b      = readNumber( ) in
    a*2 + 3*b
```

# Actual solution

```
let (a,w1) = readNumber(w0) in  
  let (b,w2) = readNumber(w1) in  
    a*2 + 3*b
```

# Better (composable) solution

```
let (a,w1) = readNumber(w0) in
  let (b,w2) = readNumber(w1) in
    (a*2 + 3*b, w2)
```

# A function

```
myOperation w0 =  
  let (a,w1) = readNumber(w0) in  
    let (b,w2) = readNumber(w1) in  
      (a*2 + 3*b, w2)
```



# A function

```
myOperation :: RealWorld -> (Int, RealWorld)
myOperation w0 =
  let (a,w1) = readNumber(w0) in
    let (b,w2) = readNumber(w1) in
      (a*2 + 3*b, w2)
```

[https://wiki.haskell.org/IO\\_inside](https://wiki.haskell.org/IO_inside)

# Downsides

- need to pass additional parameter
- prone to errors (e.g.  $w_0$  instead of  $w_1$ )
- nesting level increases

Could we do something to make  $w_0$  passed implicitly?

# Downsides

- need to pass additional parameter
- prone to errors (e.g.  $w_0$  instead of  $w_1$ )
- nesting level increases

Could we do something to make  $w_0$  passed implicitly?

# Downsides

- need to pass additional parameter
- prone to errors (e.g.  $w_0$  instead of  $w_1$ )
- nesting level increases

Could we do something to make  $w_0$  passed implicitly?

# Downsides

- need to pass additional parameter
- prone to errors (e.g.  $w_0$  instead of  $w_1$ )
- nesting level increases

Could we do something to make  $w_0$  passed implicitly?

# Downsides

- need to pass additional parameter
- prone to errors (e.g.  $w_0$  instead of  $w_1$ )
- nesting level increases

Could we do something to make  $w_0$  passed implicitly?

# How could this look like?

```
pass readNumber
  (λ a -> pass readNumber
           (λ b -> return a*2 + 3*b))

return value world = (value, world)

pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```

# How could this look like?

```
pass readNumber
  (λ a -> pass readNumber
    (λ b -> return a*2 + 3*b))

return value world = (value, world)

pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```



# How could this look like?

```
pass readNumber
  (λ a -> pass readNumber
    (λ b -> return a*2 + 3*b))
```

```
return value world = (value, world)
```

```
pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```

# How could this look like?

```
pass readNumber
  (λ a -> pass readNumber
    (λ b -> return a*2 + 3*b))
```

```
return value world = (value, world)
```

```
pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```

# How could this look like?

```
pass readNumber
  (λ a -> pass readNumber
    (λ b -> return a*2 + 3*b))

return value world = (value, world)

pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```

# Does this really work?

```
pass readNumber  
  (λ a -> pass readNumber  
           (λ b -> return a*2 + 3*b) )
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```

# Does this really work?

```
pass readNumber  
  (λ a -> pass readNumber  
           (λ b -> return a*2 + 3*b))
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```

# Does this really work?

```
pass readNumber  
  (λ a -> pass readNumber  
           (λ b -> return a*2 + 3*b))
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```

# Does this really work?

```
pass readNumber  
  (λ a -> pass readNumber  
    (λ b -> λ w -> (a*2 + 3*b, w)))
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```

# Does this really work?

```
pass readNumber
  (λ a -> pass readNumber
    (λ b -> λ w -> (a*2 + 3*b, w)))

return value world = (value, world)

pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```



# Does this really work?

```
pass readNumber (λ a -> λ w1 ->
  let (y, w2) = readNumber(w1) in
    (λ b -> λ w -> (a*2 + 3*b, w)) y w2)
```

```
return value world = (value, world)
```

```
pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```

# Does this really work?

```
pass readNumber ( $\lambda$  a ->  $\lambda$  w1 ->  
  let (y, w2) = readNumber(w1) in  
    ( $\lambda$  b ->  $\lambda$  w -> (a*2 + 3*b, w)) y w2)
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```

# Does this really work?

```
pass readNumber (λ a -> λ w1 ->
  let (y, w2) = readNumber(w1) in
    (λ w -> (a*2 + 3*y, w)) w2)

return value world = (value, world)

pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```

# Does this really work?

```
pass readNumber (λ a -> λ w1 ->
  let (y, w2) = readNumber(w1) in
    (a*2 + 3*y, w2))

return value world = (value, world)

pass value continuation w0 =
  let (result, w1) = value w0 in
    continuation result w1
```

# Does this really work?

```
pass readNumber ( $\lambda$  a ->  $\lambda$  w1 ->  
  let (y, w2) = readNumber(w1) in  
    (a*2 + 3*y, w2))
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```

# Does this really work?

```
λ w0 -> let (x,w3) = readNumber(w0) in  
  (λ a -> λ w1 -> let (y, w2) = readNumber(w1)  
    in (a*2 + 3*y, w2)) x w3
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```

# Does this really work?

```
λ w0 -> let (x,w3) = readNumber(w0) in  
  (λ a -> λ w1 -> let (y, w2) = readNumber(w1)  
    in (a*2 + 3*y, w2)) x w3
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```

# Does this really work?

```
λ w0 -> let (x,w3) = readNumber(w0) in  
  (λ w1 -> let (y, w2) = readNumber(w1) in  
    (x*2 + 3*y, w2)) w3
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
  let (result, w1) = value w0 in  
    continuation result w1
```



# Does this really work?

```
λ w0 -> let (x,w3) = readNumber(w0) in  
    let (y, w2) = readNumber(w3) in  
        (x*2 + 3*y, w2)
```

```
return value world = (value, world)
```

```
pass value continuation w0 =  
    let (result, w1) = value w0 in  
        continuation result w1
```

# Does this really work?

```
λ w0 -> let (x,w3) = readNumber(w0) in  
    let (y, w2) = readNumber(w3) in  
        (x*2 + 3*y, w2)
```

```
let (a,w1) = readNumber(w0) in  
    let (b,w2) = readNumber(w1) in  
        (a*2 + 3*b, w2)
```

# It works!

```
pass readNumber
  (λ a -> pass readNumber
    (λ b -> return a*2 + 3*b))
```

But typing  $\lambda$  and the increased indentation level is annoying!

```
pass readNumber (λ a
-> pass readNumber (λ b
-> return a*2 + 3*b))
```

# It works!

```
pass readNumber
  (λ a -> pass readNumber
    (λ b -> return a*2 + 3*b))
```

But typing  $\lambda$  and the increased indentation level is annoying!

```
pass readNumber (λ a
-> pass readNumber (λ b
-> return a*2 + 3*b))
```

# It works!

```
pass readNumber
    (λ a -> pass readNumber
        (λ b -> return a*2 + 3*b))
```

But typing  $\lambda$  and the increased indentation level is annoying!

```
pass readNumber (λ a
-> pass readNumber (λ b
-> return a*2 + 3*b))
```

## Introduce new syntax (do-notation):

```
do result <- action  
  actions ...
```

can be interpreted as:

```
pass action (\ result -> do actions ...)
```

Note: In Haskell, `pass` function is spelled `>=` and pronounced “bind”.

**Introduce new syntax (do-notation):**

```
do result <- action
  actions ...
```

**can be interpreted as:**

```
pass action (\ result -> do actions ...)
```

Note: In Haskell, `pass` function is spelled `>=` and pronounced “bind”.

Introduce new syntax (do-notation):

```
do result <- action
  actions ...
```

can be interpreted as:

```
pass action ( $\lambda$  result -> do actions ...)
```

Note: In Haskell, `pass` function is spelled `>=` and pronounced “bind”.



Introduce new syntax (do-notation):

```
do result <- action
  actions ...
```

can be interpreted as:

```
pass action ( $\lambda$  result -> do actions ...)
```

Note: In Haskell, `pass` function is spelled `>=` and pronounced “bind”.

# Emperor's new clothes

Now we can write our program as:

```
do a <- readNumber
    b <- readNumber
    return a*2 + 3*b
```

# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

## The monad laws:

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

**The monad laws:**

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

**The monad laws:**

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

**The monad laws:**

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

## The monad laws:

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

## The monad laws:

`((return v) >>= f) = (f v)` – *left identity*

`(m >>= return) = m` – *right identity*



# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

## The monad laws:

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

## The monad laws:

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

# The sequencing pattern

A monad (sequencing pattern) consists of:

- 1 a `>>=` (bind, pass, chain) function that takes some (decorated) value and a function (*continuation*) and passes that value to the function
- 2 a `return` function that takes some (raw) value and lifts (decorates) it, so that it can be chained using the `>>=` operator

## The monad laws:

$((\text{return } v) \gg= f) = (f \ v) - \textit{left identity}$

$(m \gg= \text{return}) = m - \textit{right identity}$

```
class Monad m where
  return :: a -> m a
  (»=)   :: m a -> (a -> m b) -> m b
```

Monads with the `do` notation provide a general and systematic solution to the common anti-pattern known as *the Pyramid of Doom*.

```
class Monad m where  
  return :: a -> m a  
  (»=)   :: m a -> (a -> m b) -> m b
```

Monads with the `do` notation provide a general and systematic solution to the common anti-pattern known as *the Pyramid of Doom*.

```
class Monad m where  
  return :: a -> m a  
  (»=)   :: m a -> (a -> m b) -> m b
```

Monads with the `do` notation provide a general and systematic solution to the common anti-pattern known as *the Pyramid of Doom*.

```
class Monad m where  
  return :: a -> m a  
  (»=) :: m a -> (a -> m b) -> m b
```

Monads with the `do` notation provide a general and systematic solution to the common anti-pattern known as *the Pyramid of Doom*.



```
class Monad m where  
  return :: a -> m a  
  (»=) :: m a -> (a -> m b) -> m b
```

Monads with the `do` notation provide a general and systematic solution to the common anti-pattern known as *the Pyramid of Doom*.



# The Pyramid of Doom

```
function register()
{
    if (!empty($_POST)) {
        $msg = '';
        if ($_POST['user_name']) {
            if ($_POST['user_password_new']) {
                if ($_POST['user_password_new'] == $_POST['user_password_repeat']) {
                    if (strlen($_POST['user_password_new']) > 5) {
                        if (strlen($_POST['user_name']) < 65 && strlen($_POST['user_name']) > 1) {
                            if (preg_match('/^[a-z\d]{2,64}$/i', $_POST['user_name'])) {
                                $user = read_user($_POST['user_name']);
                                if (!isset($user['user_name'])) {
                                    if ($_POST['user_email']) {
                                        if (strlen($_POST['user_email']) < 65) {
                                            if (filter_var($_POST['user_email'], FILTER_VALIDATE_EMAIL)) {
                                                create_user();
                                                $_SESSION['msg'] = 'You are now registered so please login';
                                                header('Location: ' . $_SERVER['PHP_SELF']);
                                                exit();
                                            } else $msg = 'You must provide a valid email address';
                                        } else $msg = 'Email must be less than 64 characters';
                                    } else $msg = 'Email cannot be empty';
                                } else $msg = 'Username already exists';
                            } else $msg = 'Username must be only a-z, A-Z, 0-9';
                        } else $msg = 'Username must be between 2 and 64 characters';
                    } else $msg = 'Password must be at least 6 characters';
                } else $msg = 'Passwords do not match';
            } else $msg = 'Empty Password';
        } else $msg = 'Empty Username';
        $_SESSION['msg'] = $msg;
    }
    return register_form();
}
```



# The Pyramid of Doom

*In computer programming, the **pyramid of doom** is a common problem that arises when a program uses many levels of nested indentation to control access to a function. It is commonly seen when checking for null pointers or handling callbacks.*

Wikipedia/Pyramid\_of\_doom\_(programming)

# The Pyramid of Doom – example

```
theWidth = windows("Main").views(5).size().width();

if windows.contains("Main") {
    if windows("Main").views.contains(5) {
        theWidth = windows("Main").views(5).size().width();
        //more code that works with theWidth
    }
}
```

With “optional chaining”/“null-conditional”/“safe navigation” operator:

```
theWidth = windows("Main")?.views(5)?.size.width;
```

# The Pyramid of Doom – example

```
theWidth = windows("Main").views(5).size().width();

if windows.contains("Main") {
    if windows("Main").views.contains(5) {
        theWidth = windows("Main").views(5).size().width();
        //more code that works with theWidth
    }
}
```

With “optional chaining”/“null-conditional”/“safe navigation” operator:

```
theWidth = windows("Main")?.views(5)?.size.width;
```

# The Pyramid of Doom – example

```
theWidth = windows("Main").views(5).size().width();

if windows.contains("Main") {
    if windows("Main").views.contains(5) {
        theWidth = windows("Main").views(5).size().width();
        //more code that works with theWidth
    }
}
```

With “optional chaining”/“null-conditional”/“safe navigation” operator:

```
theWidth = windows("Main")?.views(5)?.size.width;
```

# The Pyramid of Doom – example

```
theWidth = windows("Main").views(5).size().width();

if windows.contains("Main") {
    if windows("Main").views.contains(5) {
        theWidth = windows("Main").views(5).size().width();
        //more code that works with theWidth
    }
}
```

With “optional chaining”/“null-conditional”/“safe navigation” operator:

```
theWidth = windows("Main")?.views(5)?.size.width;
```

# The Pyramid of Doom – example

```
theWidth = windows("Main").views(5).size().width();

if windows.contains("Main") {
    if windows("Main").views.contains(5) {
        theWidth = windows("Main").views(5).size().width();
        //more code that works with theWidth
    }
}
```

With “optional chaining”/“null-conditional”/“safe navigation” operator:

```
theWidth = windows("Main")?.views(5)?.size.width;
```

# Dealing with “no values” in Haskell

```
data Maybe a = Nothing | Just a

let theWidth = do window <- windows("Main")
                  view <- views 5 window
                  return width (size view)

instance Monad Maybe where
  (Nothing >= f) = Nothing
  (Just a >= f)  = (f a)
  return a      = Just a
```



# Dealing with “no values” in Haskell

```
data Maybe a = Nothing | Just a
```

```
let theWidth = do window <- windows("Main")  
                  view <- views 5 window  
                  return width (size view)
```

```
instance Monad Maybe where  
  (Nothing >= f) = Nothing  
  (Just a >= f) = (f a)  
  return a = Just a
```

# Dealing with “no values” in Haskell

```
data Maybe a = Nothing | Just a

let theWidth = do window <- windows("Main")
                  view <- views 5 window
                  return width (size view)

instance Monad Maybe where
  (Nothing >= f) = Nothing
  (Just a >= f)  = (f a)
  return a      = Just a
```

# Dealing with “no values” in Haskell

```
data Maybe a = Nothing | Just a

let theWidth = do window <- windows("Main")
                  view <- views 5 window
                  return width (size view)

instance Monad Maybe where
  (Nothing >= f) = Nothing
  (Just a >= f) = (f a)
  return a = Just a
```

# Dealing with “no values” in Haskell

```
data Maybe a = Nothing | Just a

let theWidth = do window <- windows("Main")
                  view <- views 5 window
                  return width (size view)

instance Monad Maybe where
  (Nothing >= f) = Nothing
  (Just a >= f) = (f a)
  return a = Just a
```

# Dealing with “no values” in Haskell

```
data Maybe a = Nothing | Just a

let theWidth = do window <- windows("Main")
                  view <- views 5 window
                  return width (size view)

instance Monad Maybe where
  (Nothing >= f) = Nothing
  (Just a >= f)  = (f a)
  return a      = Just a
```

# Dealing with “no values” in Haskell

```
data Maybe a = Nothing | Just a

let theWidth = do window <- windows("Main")
                  view <- views 5 window
                  return width (size view)

instance Monad Maybe where
  (Nothing >= f) = Nothing
  (Just a >= f) = (f a)
  return a = Just a
```

# The List Monad

```
instance Monad List where
  (x >=> f) = concatMap f x
  return a = [a]
```

```
>>> concatMap (\n -> [1..n]) [1,2,3]
[1, 1,2, 1,2,3]
```

```
>>> do a <- [1,2,3]
     b <- [4,5]
     return (a, b)
```

```
[(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)]
```

# The List Monad

```
instance Monad List where
  (x >>= f) = concatMap f x
  return a = [a]
```

```
>>> concatMap (\n -> [1..n]) [1,2,3]
[1, 1,2, 1,2,3]
```

```
>>> do a <- [1,2,3]
     b <- [4,5]
     return (a, b)
```

```
[(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)]
```



# The List Monad

```
instance Monad List where
  (x >>= f) = concatMap f x
  return a = [a]
```

```
>>> concatMap (\n -> [1..n]) [1,2,3]
[1, 1,2, 1,2,3]
```

```
>>> do a <- [1,2,3]
     b <- [4,5]
     return (a, b)
```

```
[(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)]
```

# The List Monad

```
instance Monad List where
  (x >=> f) = concatMap f x
  return a = [a]
```

```
>>> concatMap (\n -> [1..n]) [1,2,3]
[1, 1,2, 1,2,3]
```

```
>>> do a <- [1,2,3]
     b <- [4,5]
     return (a, b)
```

```
[(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)]
```

# The List Monad

```
instance Monad List where
  (x >=> f) = concatMap f x
  return a = [a]
```

```
>>> concatMap (\n -> [1..n]) [1,2,3]
[1, 1,2, 1,2,3]
```

```
>>> do a <- [1,2,3]
     b <- [4,5]
     return (a, b)
```

```
[(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)]
```

# The List Monad

```
instance Monad List where
  (x >=> f) = concatMap f x
  return a = [a]
```

```
>>> concatMap (\n -> [1..n]) [1,2,3]
[1, 1,2, 1,2,3]
```

```
>>> do a <- [1,2,3]
     b <- [4,5]
     return (a, b)
```

```
[(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)]
```

# The List Monad

```
instance Monad List where
  (x >=> f) = concatMap f x
  return a = [a]
```

```
>>> concatMap (\n -> [1..n]) [1,2,3]
[1, 1,2, 1,2,3]
```

```
>>> do a <- [1,2,3]
      b <- [4,5]
      return (a, b)
```

```
[(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)]
```

# The Amb Monad

```
import Control.Monad
import Control.Monad.Amb

pyTriple n = do a <- anIntegerBetween 1 n
                b <- anIntegerBetween (a+1) n
                c <- anIntegerBetween (b+1) n
                when (a*a + b*b /= c*c) empty
                return (a,b,c)
```

```
>>> oneValue (pyTriple 20)
(3,4,5)
```

```
>>> allValues (pyTriple 20)
[(3,4,5), (5,12,13), (6,8,10), (8,15,17),
 (9,12,15), (12,16,20)]
```

# The Amb Monad

```
import Control.Monad
import Control.Monad.Amb

pyTriple n = do a <- anIntegerBetween 1 n
                b <- anIntegerBetween (a+1) n
                c <- anIntegerBetween (b+1) n
                when (a*a + b*b /= c*c) empty
                return (a,b,c)
```

```
>>> oneValue (pyTriple 20)
(3,4,5)
```

```
>>> allValues (pyTriple 20)
[(3,4,5), (5,12,13), (6,8,10), (8,15,17),
 (9,12,15), (12,16,20)]
```

# The Amb Monad

```
import Control.Monad
import Control.Monad.Amb

pyTriple n = do a <- anIntegerBetween 1 n
                b <- anIntegerBetween (a+1) n
                c <- anIntegerBetween (b+1) n
                when (a*a + b*b /= c*c) empty
                return (a,b,c)
```

```
>>> oneValue (pyTriple 20)
(3,4,5)
```

```
>>> allValues (pyTriple 20)
[(3,4,5), (5,12,13), (6,8,10), (8,15,17),
 (9,12,15), (12,16,20)]
```



# The Amb Monad

```
import Control.Monad
import Control.Monad.Amb

pyTriple n = do a <- anIntegerBetween 1 n
                b <- anIntegerBetween (a+1) n
                c <- anIntegerBetween (b+1) n
                when (a*a + b*b /= c*c) empty
                return (a,b,c)
```

```
>>> oneValue (pyTriple 20)
(3,4,5)
```

```
>>> allValues (pyTriple 20)
[(3,4,5), (5,12,13), (6,8,10), (8,15,17),
 (9,12,15), (12,16,20)]
```

# The Amb Monad

```
import Control.Monad
import Control.Monad.Amb

pyTriple n = do a <- anIntegerBetween 1 n
                b <- anIntegerBetween (a+1) n
                c <- anIntegerBetween (b+1) n
                when (a*a + b*b /= c*c) empty
                return (a,b,c)
```

```
>>> oneValue (pyTriple 20)
(3,4,5)
```

```
>>> allValues (pyTriple 20)
[(3,4,5), (5,12,13), (6,8,10), (8,15,17),
 (9,12,15), (12,16,20)]
```

# The Amb Monad

```
import Control.Monad
import Control.Monad.Amb

pyTriple n = do a <- anIntegerBetween 1 n
                b <- anIntegerBetween (a+1) n
                c <- anIntegerBetween (b+1) n
                when (a*a + b*b /= c*c) empty
                return (a,b,c)
```

```
>>> oneValue (pyTriple 20)
(3,4,5)
```

```
>>> allValues (pyTriple 20)
[(3,4,5), (5,12,13), (6,8,10), (8,15,17),
 (9,12,15), (12,16,20)]
```

# Other instances of monads

Phil Wadler, *The First Monad Tutorial*

<https://www.youtube.com/watch?v=yjmKMhJOJos>

Rob Norris, *Functional Programming with Effects*

<https://www.youtube.com/watch?v=po3wmq4S15A>

# Other instances of monads

Phil Wadler, *The First Monad Tutorial*

<https://www.youtube.com/watch?v=yjmKMhJOJos>

Rob Norris, *Functional Programming with Effects*

<https://www.youtube.com/watch?v=po3wmq4S15A>

# Monad Laws revisited

## Function composition:

$$(f \cdot g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \cdot (g \cdot h) = (f \cdot g) \cdot h - \textit{associativity of } (.)$$

$$\begin{aligned}(f \cdot (g \cdot h)) \ x &= f \ ((g \cdot h) \ x) = f \ (g \ (h \ x)) \\ ((f \cdot g) \cdot h) \ x &= (f \cdot g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \cdot g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \cdot (g \cdot h) = (f \cdot g) \cdot h - \textit{associativity of } (.)$$

$$\begin{aligned}(f \cdot (g \cdot h)) \ x &= f \ ((g \cdot h) \ x) = f \ (g \ (h \ x)) \\ ((f \cdot g) \cdot h) \ x &= (f \cdot g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \cdot g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \cdot (g \cdot h) = (f \cdot g) \cdot h - \textit{associativity of } (.)$$

$$\begin{aligned}(f \cdot (g \cdot h)) \ x &= f \ ((g \cdot h) \ x) = f \ (g \ (h \ x)) \\ ((f \cdot g) \cdot h) \ x &= (f \cdot g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$



# Monad Laws revisited

## Function composition:

$$(f \cdot g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \cdot (g \cdot h) = (f \cdot g) \cdot h - \textit{associativity of } (.)$$

$$\begin{aligned}(f \cdot (g \cdot h)) \ x &= f \ ((g \cdot h) \ x) = f \ (g \ (h \ x)) \\ ((f \cdot g) \cdot h) \ x &= (f \cdot g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \cdot g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \cdot (g \cdot h) = (f \cdot g) \cdot h - \textit{associativity of } (.)$$

$$\begin{aligned}(f \cdot (g \cdot h)) \ x &= f \ ((g \cdot h) \ x) = f \ (g \ (h \ x)) \\ ((f \cdot g) \cdot h) \ x &= (f \cdot g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \cdot g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \cdot (g \cdot h) = (f \cdot g) \cdot h - \textit{associativity of } (.)$$

$$\begin{aligned}(f \cdot (g \cdot h)) \ x &= f \ ((g \cdot h) \ x) = f \ (g \ (h \ x)) \\ ((f \cdot g) \cdot h) \ x &= (f \cdot g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \cdot g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \cdot (g \cdot h) = (f \cdot g) \cdot h - \textit{associativity of } (.)$$

$$\begin{aligned}(f \cdot (g \cdot h)) \ x &= f \ ((g \cdot h) \ x) = f \ (g \ (h \ x)) \\ ((f \cdot g) \cdot h) \ x &= (f \cdot g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \ . \ g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \ . \ (g \ . \ h) = (f \ . \ g) \ . \ h - \textit{associativity of (.)}$$

$$\begin{aligned}(f \ . \ (g \ . \ h)) \ x &= f \ ((g \ . \ h) \ x) = f \ (g \ (h \ x)) \\ ((f \ . \ g) \ . \ h) \ x &= (f \ . \ g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \ . \ g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \ . \ (g \ . \ h) = (f \ . \ g) \ . \ h - \textit{associativity of (.)}$$

$$\begin{aligned}(f \ . \ (g \ . \ h)) \ x &= f \ ((g \ . \ h) \ x) = f \ (g \ (h \ x)) \\ ((f \ . \ g) \ . \ h) \ x &= (f \ . \ g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \ . \ g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \ . \ (g \ . \ h) = (f \ . \ g) \ . \ h - \textit{associativity of (.)}$$

$$\begin{aligned}(f \ . \ (g \ . \ h)) \ x &= f \ ((g \ . \ h) \ x) = f \ (g \ (h \ x)) \\ ((f \ . \ g) \ . \ h) \ x &= (f \ . \ g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$

# Monad Laws revisited

## Function composition:

$$(f \ . \ g) \ x = f \ (g \ x)$$

```
function compose(f, g) {  
  return function(x) {  
    return f(g(x));  
  };  
}
```

$$f \ . \ (g \ . \ h) = (f \ . \ g) \ . \ h - \textit{associativity of (.)}$$

$$\begin{aligned}(f \ . \ (g \ . \ h)) \ x &= f \ ((g \ . \ h) \ x) = f \ (g \ (h \ x)) \\ ((f \ . \ g) \ . \ h) \ x &= (f \ . \ g) \ (h \ x) = f \ (g \ (h \ x))\end{aligned}$$



# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```



# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

$\text{id} \cdot f = f$  — *left identity*

$(\text{id} \cdot f) \ x = \text{id} \ (f \ x) = f \ x$

$f \cdot \text{id} = f$  — *right identity*

$(f \cdot \text{id}) \ x = f \ (\text{id} \ x) = f \ x$

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

```
id . f = f — left identity
```

```
(id . f) x = id (f x) = f x
```

```
f . id = f — right identity
```

```
(f . id) x = f (id x) = f x
```

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

$\text{id} \cdot f = f$  – *left identity*

$(\text{id} \cdot f) \ x = \text{id} \ (f \ x) = f \ x$

$f \cdot \text{id} = f$  – *right identity*

$(f \cdot \text{id}) \ x = f \ (\text{id} \ x) = f \ x$

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

$\text{id} \cdot f = f$  – *left identity*

$(\text{id} \cdot f) \ x = \text{id} \ (f \ x) = f \ x$

$f \cdot \text{id} = f$  – *right identity*

$(f \cdot \text{id}) \ x = f \ (\text{id} \ x) = f \ x$

# Monad Laws revisited

Identity function:

```
id x = x
```

```
function identity(x) { return x; }
```

$\text{id} \cdot f = f$  – *left identity*

$(\text{id} \cdot f) \ x = \text{id} \ (f \ x) = f \ x$

$f \cdot \text{id} = f$  – *right identity*

$(f \cdot \text{id}) \ x = f \ (\text{id} \ x) = f \ x$

# Monad Laws revisited

associativity + identity = monoid (semi-group with neutral element)

$(\circ, id)$  is a monoid. Other examples:

- $(+, 0)$
- $(*, 1)$
- $(min, +\infty)$
- $(max, -\infty)$

# Monad Laws revisited

associativity + identity = monoid (semi-group with neutral element)

$(\circ, id)$  is a monoid. Other examples:

- $(+, 0)$
- $(*, 1)$
- $(min, +\infty)$
- $(max, -\infty)$

# Monad Laws revisited

associativity + identity = monoid (semi-group with neutral element)

$(\circ, id)$  is a monoid. Other examples:

- $(+, 0)$
- $(*, 1)$
- $(min, +\infty)$
- $(max, -\infty)$



# Monad Laws revisited

associativity + identity = monoid (semi-group with neutral element)

$(\circ, id)$  is a monoid. Other examples:

- $(+, 0)$
- $(*, 1)$
- $(min, +\infty)$
- $(max, -\infty)$

# Monad Laws revisited

associativity + identity = monoid (semi-group with neutral element)

$(\circ, id)$  is a monoid. Other examples:

- $(+, 0)$
- $(*, 1)$
- $(min, +\infty)$
- $(max, -\infty)$

# Monad Laws revisited

associativity + identity = monoid (semi-group with neutral element)

$(\circ, id)$  is a monoid. Other examples:

- $(+, 0)$
- $(*, 1)$
- $(\min, +\infty)$
- $(\max, -\infty)$

# Monad Laws revisited

## Kleisli composition:

```
(f >=> g) x = do y <- f x  
               g y
```

```
(f >=> g) x = (f x) >=> g
```

```
(return >=> f) = f
```

```
(f >=> return) = f
```

```
((f >=> g) >=> h) = (f >=> (g >=> h))
```

# Monad Laws revisited

Kleisli composition:

```
(f >=> g) x = do y <- f x  
               g y
```

```
(f >=> g) x = (f x) >=> g
```

```
(return >=> f) = f
```

```
(f >=> return) = f
```

```
((f >=> g) >=> h) = (f >=> (g >=> h))
```

# Monad Laws revisited

Kleisli composition:

```
(f >=> g) x = do y <- f x  
               g y
```

```
(f >=> g) x = (f x) »= g
```

```
(return >=> f) = f
```

```
(f >=> return) = f
```

```
((f >=> g) >=> h) = (f >=> (g >=> h))
```

# Monad Laws revisited

Kleisli composition:

```
(f >=> g) x = do y <- f x  
               g y
```

```
(f >=> g) x = (f x) >>= g
```

```
(return >=> f) = f
```

```
(f >=> return) = f
```

```
((f >=> g) >=> h) = (f >=> (g >=> h))
```

# Monad Laws revisited

Kleisli composition:

```
(f >=> g) x = do y <- f x  
               g y
```

```
(f >=> g) x = (f x) >>= g
```

```
(return >=> f) = f
```

```
(f >=> return) = f
```

```
((f >=> g) >=> h) = (f >=> (g >=> h))
```



# Monad Laws revisited

Kleisli composition:

$$(f \gg g) \ x = \text{do } y \leftarrow f \ x \\ \qquad \qquad \qquad g \ y$$
$$(f \gg g) \ x = (f \ x) \gg g$$
$$(\text{return} \gg f) = f$$
$$(f \gg \text{return}) = f$$
$$((f \gg g) \gg h) = (f \gg (g \gg h))$$



# Functors, Applicatives, Monads

```
class Applicative m => Monad m where
  (»=) :: m a -> (a -> m b) -> m b
  return :: a -> m a
  return = pure
```

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

# Functors, Applicatives, Monads

```
class Applicative m => Monad m where
  (»=) :: m a -> (a -> m b) -> m b
  return :: a -> m a
  return = pure
```

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

# Functors, Applicatives, Monads

```
class Applicative m => Monad m where
  (»=) :: m a -> (a -> m b) -> m b
  return :: a -> m a
  return = pure
```

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

# Functors, Applicatives, Monads

```
class Applicative m => Monad m where
  (»=) :: m a -> (a -> m b) -> m b
  return :: a -> m a
  return = pure
```

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

# Free monads

```
data IO a =  
  PutStrLn String (IO a)  
  | GetStrLn (String -> IO a)  
  | Sleep Int (IO a)  
  | DeleteFile String (IO a)  
  | LaunchTheMissles (IO a)  
  ...  
  | forall a0. Chain (IO a0) (a0 -> IO a)  
  | Return a
```

## Problems:

- IO can do really anything
- we may want to restrict it to a smaller number of capabilities
- we may want to simulate (mock) the behavior

# Free monads

```
data IO a =  
  PutStrLn String (IO a)  
  | GetStrLn (String -> IO a)  
  | Sleep Int (IO a)  
  | DeleteFile String (IO a)  
  | LaunchTheMissles (IO a)  
  ...  
  | forall a0. Chain (IO a0) (a0 -> IO a)  
  | Return a
```

## Problems:

- IO can do really anything
- we may want to restrict it to a smaller number of capabilities
- we may want to simulate (mock) the behavior



# Free monads

```
data IO a =  
  PutStrLn String (IO a)  
  | GetStrLn (String -> IO a)  
  | Sleep Int (IO a)  
  | DeleteFile String (IO a)  
  | LaunchTheMissles (IO a)  
  ...  
  | forall a0. Chain (IO a0) (a0 -> IO a)  
  | Return a
```

## Problems:

- IO can do really anything
- we may want to restrict it to a smaller number of capabilities
- we may want to simulate (mock) the behavior

# Free monads

```
data IO a =  
  PutStrLn String (IO a)  
  | GetStrLn (String -> IO a)  
  | Sleep Int (IO a)  
  | DeleteFile String (IO a)  
  | LaunchTheMissles (IO a)  
  ...  
  | forall a0. Chain (IO a0) (a0 -> IO a)  
  | Return a
```

## Problems:

- IO can do really anything
- we may want to restrict it to a smaller number of capabilities
- we may want to simulate (mock) the behavior

# Free monads

```
data IO a =  
  PutStrLn String (IO a)  
  | GetStrLn (String -> IO a)  
  | Sleep Int (IO a)  
  | DeleteFile String (IO a)  
  | LaunchTheMissles (IO a)  
  ...  
  | forall a0. Chain (IO a0) (a0 -> IO a)  
  | Return a
```

## Problems:

- IO can do really anything
- we may want to restrict it to a smaller number of capabilities
- we may want to simulate (mock) the behavior

# Free monads

```
data IO a =  
  PutStrLn String (IO a)  
  | GetStrLn (String -> IO a)  
  | Sleep Int (IO a)  
  | DeleteFile String (IO a)  
  | LaunchTheMissles (IO a)  
  ...  
  | forall a0. Chain (IO a0) (a0 -> IO a)  
  | Return a
```

## Problems:

- IO can do really anything
- we may want to restrict it to a smaller number of capabilities
- we may want to simulate (mock) the behavior

John De Goes, *FP to the Max*

<https://www.youtube.com/watch?v=sxudIMiOo68>

Igal Tabachnik, *Journey to Functional Programming*

<https://www.youtube.com/watch?v=g1EvM4CbUvM>

Kelley Robinson, *Why the Free Monad isn't Free*

<https://www.youtube.com/watch?v=U0lK0hnb4U>

John De Goes, *FP to the Max*

<https://www.youtube.com/watch?v=sxudIMiOo68>

Igal Tabachnik, *Journey to Functional Programming*

<https://www.youtube.com/watch?v=g1EvM4CbUvM>

Kelley Robinson, *Why the Free Monad isn't Free*

<https://www.youtube.com/watch?v=U0lK0hnb4U>

John De Goes, *FP to the Max*

<https://www.youtube.com/watch?v=sxudIMiOo68>

Igal Tabachnik, *Journey to Functional Programming*

<https://www.youtube.com/watch?v=g1EvM4CbUvM>

Kelley Robinson, *Why the Free Monad isn't Free*

<https://www.youtube.com/watch?v=U0lK0hnb4U>

# Monad transformers

Problem: different monads do not stack together well.

For example, `Future (Maybe a)` cannot be composed with the single `>>=` operator.

Gabriele Petronella, *Monad transformers down to earth*

<https://www.youtube.com/watch?v=jd5e71nFEZM>

Oleg Kiselyov, Hiromi Ishii, *Freer Monads, More Extensible Effects*

<http://okmij.org/ftp/Haskell/extensible/more.pdf>

Daniel Spiewak, *Emm: A Sane Alternative to Monad Transformers in Scala*

<https://www.youtube.com/watch?v=E5Tri3Yow0U>



# Monad transformers

Problem: different monads do not stack together well.

For example, `Future (Maybe a)` cannot be composed with the single `>>=` operator.

Gabriele Petronella, *Monad transformers down to earth*

<https://www.youtube.com/watch?v=jd5e71nFEZM>

Oleg Kiselyov, Hiromi Ishii, *Freer Monads, More Extensible Effects*

<http://okmij.org/ftp/Haskell/extensible/more.pdf>

Daniel Spiewak, *Emm: A Sane Alternative to Monad Transformers in Scala*

<https://www.youtube.com/watch?v=E5Tri3Yow0U>

# Monad transformers

Problem: different monads do not stack together well.

For example, `Future (Maybe a)` cannot be composed with the single `>>=` operator.

Gabriele Petronella, *Monad transformers down to earth*

<https://www.youtube.com/watch?v=jd5e71nFEZM>

Oleg Kiselyov, Hiromi Ishii, *Freer Monads, More Extensible Effects*

<http://okmij.org/ftp/Haskell/extensible/more.pdf>

Daniel Spiewak, *Emm: A Sane Alternative to Monad Transformers in Scala*

<https://www.youtube.com/watch?v=E5Tri3Yow0U>

# Monad transformers

Problem: different monads do not stack together well.

For example, `Future (Maybe a)` cannot be composed with the single `>>=` operator.

Gabriele Petronella, *Monad transformers down to earth*

<https://www.youtube.com/watch?v=jd5e71nFEZM>

Oleg Kiselyov, Hiromi Ishii, *Freer Monads, More Extensible Effects*

<http://okmij.org/ftp/Haskell/extensible/more.pdf>

Daniel Spiewak, *Emm: A Sane Alternative to Monad Transformers in Scala*

<https://www.youtube.com/watch?v=E5Tri3Yow0U>

# Monad transformers

Problem: different monads do not stack together well.

For example, `Future (Maybe a)` cannot be composed with the single `>>=` operator.

Gabriele Petronella, *Monad transformers down to earth*

<https://www.youtube.com/watch?v=jd5e71nFEZM>

Oleg Kiselyov, Hiromi Ishii, *Freer Monads, More Extensible Effects*

<http://okmij.org/ftp/Haskell/extensible/more.pdf>

Daniel Spiewak, *Emm: A Sane Alternative to Monad Transformers in Scala*

<https://www.youtube.com/watch?v=E5Tri3Yow0U>

Using the IO monad is emulating imperative programming.

Ron Pressler, *Please Stop Polluting our Imperative Languages with Pure Concepts*

<https://www.youtube.com/watch?v=449j7oKQVkc>

Conal Elliott, *The C language is purely functional*

[http://conal.net/blog/posts/  
the-c-language-is-purely-functional](http://conal.net/blog/posts/the-c-language-is-purely-functional)

Using the IO monad is emulating imperative programming.

Ron Pressler, *Please Stop Polluting our Imperative Languages with Pure Concepts*

<https://www.youtube.com/watch?v=449j7oKQVkc>

Conal Elliott, *The C language is purely functional*

[http://conal.net/blog/posts/  
the-c-language-is-purely-functional](http://conal.net/blog/posts/the-c-language-is-purely-functional)

Using the IO monad is emulating imperative programming.

Ron Pressler, *Please Stop Polluting our Imperative Languages with Pure Concepts*

<https://www.youtube.com/watch?v=449j7oKQVkc>

Conal Elliott, *The C language is purely functional*

[http://conal.net/blog/posts/  
the-c-language-is-purely-functional](http://conal.net/blog/posts/the-c-language-is-purely-functional)

Thank you

# Questions?

**@PaniczGodek**

`https://www.quora.com/profile/Panicz-Godek`

`https://github.com/panicz/writings/tree/  
master/talks/datamass`