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Algorithms Lab

Exercise – San Francisco

After traveling half the globe for more than sixty days, Phileas arrives in San Francisco together with Passepartout and Fix. Their plan is to use the *Pacific Railroad* for their journey to New York. From ocean to ocean—as the Americans say; these four words being a synonym for the 'great trunk line' which crosses the entire width of the United States.

However, unforeseen events start to unroll. The train is ambushed by a band of Sioux warriors and Passepartout is captured. Phileas, a true gentleman, cannot leave a man behind. He pursues the band across the vast white plains and finally manages to catch up with them.

After a lengthy brawl, the chief of the tribe, being a reasonable man, agrees with Phileas upon the following. The two will play an ancient Sioux board game: if Phileas manages to score at least as many points as the chief, he may take Passepartout and they are free to leave; otherwise, they both become slaves for an unspecified amount of time—at the current stage of their journey clearly a suboptimal event.

The game is a single player game, played with a single marble on a wooden board with n carved holes and m carved canals between these holes. An arrow is engraved in each canal to indicate the direction in which the marble may be moved through the canal. Furthermore, each canal carries a nonnegative number of *points*, which the player *scores* whenever rolling the marble through the canal (every canal can be used and scored multiple times throughout a game). Given only a limited number of moves, the goal of the game is to maximise the score, of course.

The chief explains the rules of the game to Phileas. There is a unique starting hole, called *Angvariationu-toke* (a Sioux word for 'another day'). The marble, called *Canowicakte* (a Sioux word for 'forest hunter'), starts at Angvariationu-toke. In each move, the player rolls the marble from the current hole to a neighboring hole through one of the incident canals, while respecting the direction of the engraved arrow. Doing so, (s)he scores as many points as the canal carries. A hole with no outgoing canal is called *Weayaya* (a Sioux word for 'setting sun') and from such a hole the player may take the marble back to Angvariationu-toke as a *free action*. Such a free action does not count as a move and it yields no score.

The chief makes the bold claim that he can achieve a score of x in k moves. Phileas' goal is to beat the chief dramatically: either find the minimum number of moves in order to score at least as much as the chief, or prove that it is impossible to achieve the score of x in k moves. It may be noted that the same canal can be scored more than once.

Input The first line of the input contains the number $t \le 30$ of test cases. Each of the t test cases is described as follows.

- The first line contains four integers n m x k, separated by a space. They denote
 - n, the number of holes in the game board ($2 \le n \le 10^3$);
 - m, the number of canals between the holes $(1 \le m \le 4 \cdot 10^3)$;
 - x, the claimed score of the chief $(1 \le x \le 10^{14})$;

- k, the maximum number of moves allowed $(1 \le k \le 4 \cdot 10^3)$.

Hole 0 always corresponds to Angvariationu-toke.

• The following m lines define the canals. Each line consists of three integers u v p, separated by a space, and such that $0 \le u, v \le n-1$ and $0 \le p < 2^{31}$. This means that the arrow engraved in the canal points from u to v. The player can roll the marble from hole u to hole v, thereby scoring p points. Note that (1) there can be more than one canal from hole v to hole v and (2) possibly v = v.

Output For each test case output one line containing a single integer that denotes the minimum number of moves to get at least x points. If it is not possible to score at least x points in k moves, output 'Impossible'.

Points There are three groups of test sets, worth 100 points in total.

- 1. For the first group of test sets, worth 20 points, you may assume $n \leq 40$ and $k \leq 20$. Furthermore, you may assume that all routes from Angvariationu-toke to a Weayaya hole use exactly k canals.
- 2. For the second group of test sets, worth 30 points, you may assume that all routes from Angvariationu-toke to a Weayaya hole use exactly k canals.
- 3. For the third group of test sets, worth 50 points, there are no additional assumptions.

Corresponding sample test sets are contained in test i.in/out, for $i \in \{1, 2, 3\}$.

Sample Input	Sample Output
3	3
6 6 7 3	5
0 1 1	Impossible
0 2 1	
1 4 2	
2 3 1	
3 5 5	
4 5 2	
6 8 7 5	
0 1 0	
0 2 2	
0 2 1	
0 5 1	
1 3 0	
2 4 0	
3 5 4	
4 5 0	
4 4 1 100	
0 1 0	
1 2 0	
2 3 0	

3 1 0