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Exercise – *A New Hope*

The Galactic Empire is building a heavily armed and armored space station, the *Death Star*, capable of destroying an entire planet. Fortunately, Princess Leia managed to get hold of the construction plans and sent them off to Obi-Wan Kenobi. These plans contain details about the internal organization of the Death Star and Leia hopes that Obi-Wan can use this information to devise a strategy to take the Death Star down.

The Death Star is divided into k *command centers* $C_0, \ldots, C_{k-1}$. At every command center C_i, exactly s stormtroopers $t_{i,0}, \ldots, t_{i,s-1}$ are stationed. One of the most important bits of leaked information concerns the command structure of the stormtroopers: Every stormtrooper supervises some nonnegative number of stormtroopers (possibly zero) from the same command center. In addition, a stormtrooper may also supervise stormtroopers from a different command center, subject to the following three constraints.

(1) Darth Vader puts his most trustworthy minions at C_0. No stormtrooper from C_0 is supervised by a stormtrooper from a different command center.

(2) For every command center C_i, with $i \in \{1, \ldots, k-1\}$, there exists *exactly one* $j \in \{0, \ldots, k-1\} \setminus \{i\}$ such that there exists a stormtrooper from C_j that supervises some stormtrooper(s) from C_i.

(3) For every two distinct $i, j \in \{0, \ldots, k-1\}$, *at most two* stormtroopers from C_j supervise some stormtrooper(s) from C_i.

Finally, it is also known that there is a *chain of supervision* from C_0 to every other command center. More formally, for every command center C_i, with $i \in \{1, \ldots, k-1\}$, there is a sequence $r_0, \ldots, r_\ell$ such that $r_0 = 0$, $r_\ell = i$, and a stormtrooper from C_{r_t} supervises a stormtrooper from $C_{r_{t+1}}$, for every $t \in \{0, \ldots, \ell - 1\}$.

Based on this information Obi-Wan has an idea how to sabotage the Death Star. But first all the stormtroopers must be dealt with. Using a Jedi trick for mind control Obi-Wan can disable any number of stormtroopers. But the trick fails if he tries to control two stormtroopers one of which supervises the other. Your task is to calculate the largest number of stormtroopers Obi-Wan can disable. Help Obi-Wan in his task and give the galaxy a new hope!

Remarks: Obviously, a stormtrooper cannot supervise himself. But any stormtrooper can supervise multiple stormtroopers and any stormtrooper can be supervised by multiple stormtroopers.

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains three integers `k s m`, separated by a space and such that $1 \leq k \leq 2 \cdot 10^4$, $1 \leq s \leq 14$, and $0 \leq m \leq 2k-1$. You may suppose that $k \cdot 2^s \leq 2.5 \cdot 10^5$. Here k denotes the total number of command centers of the Death Star, s denotes the number of stormtroopers per command center, and m denotes the number of different pairs of—not necessarily distinct—command centers C_u and C_v such that there exists a stormtrooper from C_u that supervises a stormtrooper from C_v.

- The following m lines define the command hierarchy among the stormtroopers. Each line defines h pairs of stormtroopers where one supervises the other. More specifically, each line contains $2h + 3$ integers `u v h` x_1 y_1 ... x_h y_h separated by a space and such that $u, v \in \{0, \ldots, k-1\}$, $u \leq v$, and $0 \leq x_i, y_i \leq s-1$, for all $i \in \{1, \ldots, h\}$. This means that stormtrooper t_{u,x_j} supervises stormtrooper t_{v,y_j}, for all $j \in \{1, \ldots, h\}$.

Output For each test case output the size of a largest subset $S \subseteq \{t_{i,j} : 0 \leq i < k \wedge 0 \leq j < s\}$ of stormtroopers such that *a does not supervise b*, for any two $a, b \in S$.

Points There are three groups of test sets, worth 100 points in total.

1. For the first group of test sets, worth 50 points, you may assume $s = 1$, that is, at every command center there is one stormtrooper only.
2. For the second group of test sets, worth 25 points, you may assume $s \leq 2$, that is, at every command center there are at most two stormtroopers.
3. For the third group of test sets, worth 25 points, there are no additional assumptions.

Corresponding sample test sets are contained in `testi.in/out`, for $i \in \{1, 2, 3\}$.

Sample Input

```
2
3 1 2
0 1 1 0 0
0 2 1 0 0
4 3 6
0 0 2 0 1 0 2
2 2 2 0 1 1 2
3 3 1 0 2
0 1 5 0 2 0 0 0 1 2 1 2 2
0 2 2 1 0 1 1
2 3 4 1 0 0 1 0 0 0 2
```

Sample Output

```
2
7
```

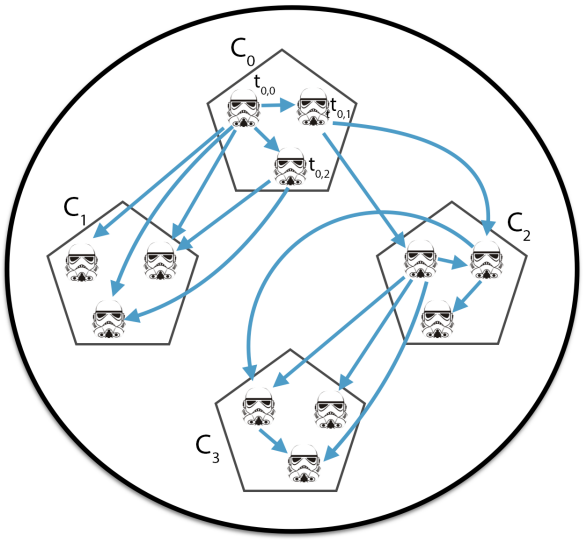

Figure 1: Sample input: test case 2