ASEN 3112

Spring 2020

Lecture 6

January 30, 2020

6
Torsion of Closed Thin Wall (CTW) Sections

Recall the Classification of Thin Wall (TW) Cross Sections Under Torque

Closed Thin Wall (CTW) Sections

at least one cell shear flow circuit can be established

Single Cell: just one cell

Multicell: more than one cell

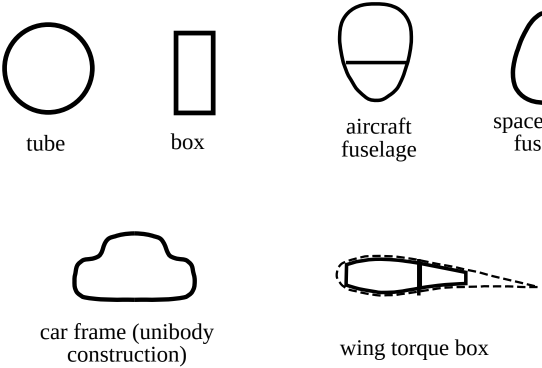
Open Thin Wall (OTW) Sections

no cell shear flow circuit can be established

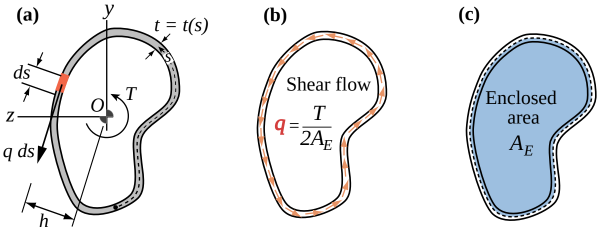
Hybrid Thin Wall (HTW) Sections

contains both OTW and CTW components

Sample CTW Cross Sections



Single-Cell CTW Section



Shear Stress Formula

Equilibrium of internal torque T with the assumed shear stress distribution (constant across wall thickness, directed along tangent to midline) gives

$$T = \oint dT = \oint q.h\,ds = q\oint h\,ds = q\oint 2\,dA_E = q.2A_E = 2q\,A_E$$

in which A_E is the *enclosed area* defined by the wall midline (which should NOT be confused with the *cross section area* or material area)
Solving for q gives the shear flow and wall stress formula

$$\boxed{q = \frac{T}{2A_E} \qquad \tau = \frac{q}{t} = \frac{T}{2t\,A_E}}$$

Since q is constant along the contour, plainly the maximum shear stress occurs where the wall thickness t is minimum:

$$\boxed{\tau_{max} = \frac{q}{t_{min}} = \frac{T}{2t_{min}A_E}}$$

Twist Angle Formula

It can be shown that the twist-angle rate is given by

$$\boxed{\phi' = \frac{d\phi}{dx} = \frac{q}{2G\,A_E}\oint\frac{ds}{t} = \frac{T}{4G\,A_E^2}\oint\frac{ds}{t} = \frac{T}{GJ} \quad \text{in which} \quad J = \frac{4A_E^2}{\oint\frac{ds}{t}}}$$

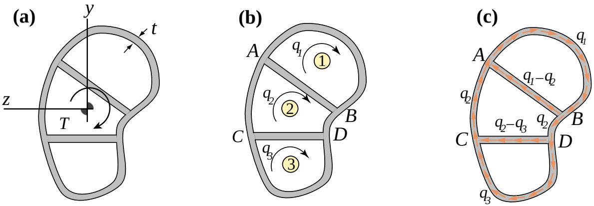
Here G is the shear modulus. Derivation of this formula requires the use of energy methods in mechanics, which is a graduate-level topic.

If the wall thickness t is constant along the cell contour, the foregoing formula simplifies to

$$\boxed{\phi' = \frac{d\phi}{dx} = \frac{q\,p}{2A_E\,t} = \frac{T}{GJ} \qquad \text{in which} \qquad J = \frac{4A_E^2\,t}{p}}$$

where $p = \oint\, ds$ is the midline perimeter length, or simply the *perimeter*.

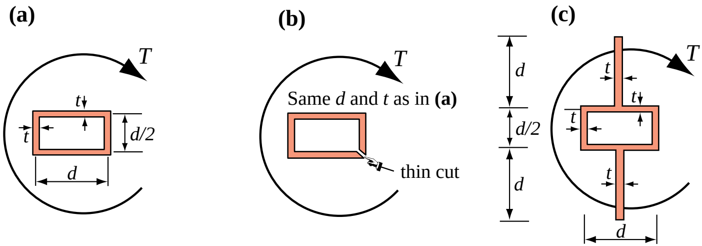
Multicell CTW Sections Are Not Treated in This Course



Cell flows

Wall flows

Worked Out Example Cross Sections



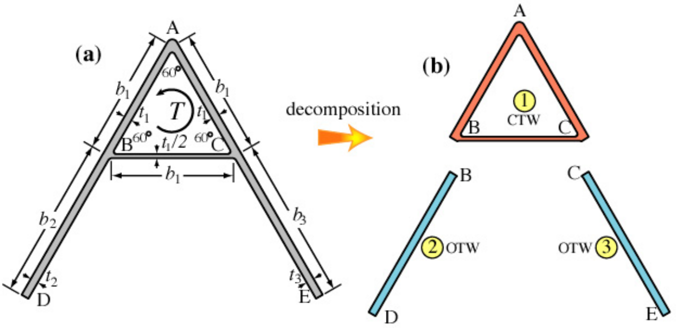
CTW section:
rectangular tube

OTW section:
slitted rectangular tube

Hybrid section:
CTW + OTW (2)

All d's are *midline* dimensions

Torqued Hybrid TW (HTW) "A" Section For Recitation



$b_1 = 200$ mm, $b_2 = b_3 = 400$ mm, $t_1 = t_2 = t_3 = 6$ mm

$G = 80$ GPa (steel), $T = 604\ 10^3$ mm-N, $L = 2514$ mm