ASEN 3112 Structures – Spring 2020: Homework 2
Solutions

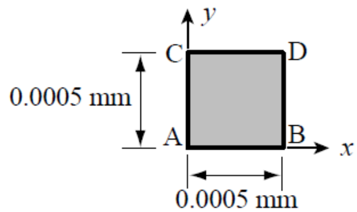

Figure 2.3 for Exercise 2.3

Exercise 2.1. Displacements *u* and *v* in the *x* and *y* directions, respectively, were measured by the Moiré interferometry method at several points on a body surface. The displacements at the 4 points labeled as ABCD in Figure 2.3, are

$$u_A = 1.6000 \cdot 10^{-6}\text{ mm},\ v_A = -0.6000 \cdot 10^{-6}\text{ mm},$$
$$u_B = 0.8000 \cdot 10^{-6}\text{ mm},\ v_B = -0.8000 \cdot 10^{-6}\text{ mm},$$
$$u_C = 0.9876 \cdot 10^{-6}\text{ mm},\ v_C = 0.3500 \cdot 10^{-6}\text{ mm},$$
$$u_D = 1.4000 \cdot 10^{-6}\text{ mm},\ v_D = -1.7000 \cdot 10^{-6}\text{ mm},$$

Find the average values of the strain components ϵ_{xx} , ϵ_{yy} and γ_{xy} with respect to point A.

Solution. Givens and unknowns:

$$\Delta x = \Delta y = 0.0005\ mm = 500 \cdot 10^{-6} mm, u_A\ through\ v_D\ listed\ in\ above,$$

$$\epsilon_{xx} = ?, \epsilon_{yy} = ?, \gamma_{xy} = ?\ \#$$

Here Δx and Δy denote the dimensions of the square "gage region" shown in Figure 2.3 along the *x* and *y* directions, respectively. Those are known as the *gage lengths* or *gage dimensions*.
To reduce clutter the average strains $\epsilon_{yy,av}$, $\epsilon_{yy,av}$ and $\gamma_{xy,av}$ are simply denoted by ϵ_{xx} , ϵ_{yy} and γ_{xy}, respectively.
The average strains are computed using the appropriate displacement increments divided by the pertinent gage lengths:

$$\epsilon_{xx} = \frac{\Delta u_{BA}}{\Delta x} = \frac{u_B - u_A}{x_B - x_A} = \frac{(0.8000 - 1.6000)\ \cdot 10^{-6} mm}{500 \cdot 10^{-6} mm} = -0.1600\%$$
$$= -1600 \cdot 10^{-6},$$

$$\epsilon_{yy} = \frac{\Delta v_{CA}}{\Delta y} = \frac{v_C - v_A}{y_C - y_A} = \frac{(0.3500 + 0.6000) \cdot 10^{-6} mm}{500 \cdot 10^{-6} mm} = 0.1900\%$$
$$= 1900 \cdot 10^{-6},$$

$$\gamma_{xy} = \frac{\Delta u_{CA}}{\Delta y} + \frac{\Delta v_{BA}}{\Delta x} = \frac{u_C - u_A}{y_C - y_A} + \frac{v_B - v_A}{x_B - x_A}$$
$$= \frac{(0.9876 - 1.6000) \cdot 10^{-6} mm}{500 \cdot 10^{-6} mm} + \frac{(-0.8000 - (-0.6000)) \cdot 10^{-6} mm}{500 \cdot 10^{-6} mm}$$
$$= -0.1625\% = -1625\ \cdot 10^{-6}.$$

Note that the displacement of point *D* does not appear in these computations.

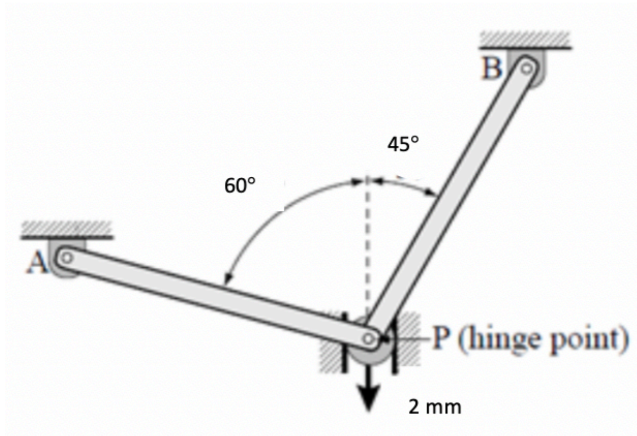

Figure 2.2 for Exercise 2.2

Exercise 2.2. Determine the elongations in mm of the bars AP and BP shown in Figure 2.2, if hinge point P moves = 5 mm downward along the vertical guide.

Solution. Givens and unknowns:

$$\delta_P = 5mm, \qquad \delta_{AP} = ? \quad \delta_{BP} = ? \#(4)$$

plus the bar orientation angles shown in Figure 2.2.

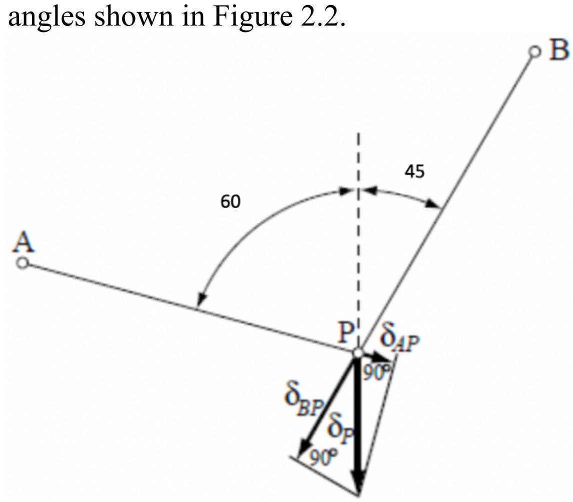

Figure S2.2 for Exercise 2.2

Figure S2.1 zooms on point *P* to clearly display the (as yet unknown) elongations δ_{AP} and δ_{BP} of the bars along their original directions. In this figure *P'* denotes the deformed position of *P*. As per the small deformation approximation, we need the *projections* of *PP'* on the original bar directions. From examination of Figure S2.1, these are easily calculated as

$$\delta_{AP} = \delta_P cos60° = 2.5\ mm, \quad \delta_{BP} = \delta_P\ cos45° = 4.330\ \ mm.\ \#(5)$$

The Exercise does not ask for the axial strains in the bars. Should these be required, the Lagrangian strains result on dividing the elongations found in (5) by the original bar lengths. These lengths would then have to be given as part of the data (they were not).

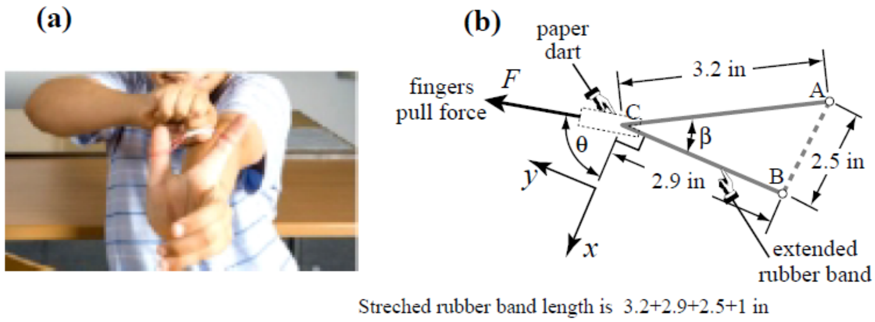

Figure 2.3 for Exercise 2.3.

Exercise 2.3. As pictured in Figure 2.3(a), a boy is shooting paper darts using a rubber band that has an unstretched length of 7.5 in. The piece of rubber band between points A and B is pulled to form the two sides of a triangle as shown in Figure 2.3(b). Assume the same average normal strain in AC and CB (use the Lagrangian strain measure). The triangle ABC as shown in the figure has sides of 3.2 in. (AC), 2.9 in. (BC), and 2.5 in. (AB). Assume that the total length of the stretched rubber band is 9.6 inches, because the length of band around the thumb (point B) and forefinger (point A) is 1 in. This extra 1 in. of stretched band length is not shown in triangle ABC. The band cross-sectional area is 1/120 in^2. Assume rubber has a modulus of elasticity of $E \approx 150$ psi. (This is a rough average, since material behavior of rubber is highly nonlinear.) Determine the approximate pull force F and the angle θ at which the paper leaves the boy's hand.

Solution. Givens and unknowns:

$$E = 150\,psi, \qquad A = \frac{1}{120} sqin, \qquad L_0 = 7.5in, \qquad \theta = ?, \qquad F = ? \#(9)$$

Other geometric data is defined in Figure 2.4.
Begin by calculating the *average strain* in the deformed rubber band. The original length is $L_0 = 7.5\,in$, and the final (deformed, stretched) length is $L_f = 3.2 + 2.9 + 2.5 + 1 = 9.6\,in$. Using the Lagrangian strain measure:

$$\epsilon_{av}^{L} = \frac{L_f - L_0}{L_0} = \frac{(9.6 - 7.5)\,in}{7.5\,in} = 0.28 = 28\%\,(extension)\,\#(10)$$

The stress is computed by Hooke's law (as it says in the Exercise statement this assumption is a very rough approximation for rubber, but the use of finite strains and hyperelastic nonlinear constitutive equations is beyond the scope of the course):

$$\sigma = E\epsilon_{av}^{L} = 150\,psi \times 0.28 = 42\,psi\,(Tension)\,\#(11)$$

The internal force in the stretched band is

$$N = A\,\sigma = \frac{1}{120} in^2 \times 42\ psi = 0.35\ lbs\ (Tension) \#(12)$$

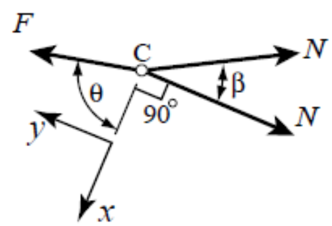

Figure S2.3. FBD for Exercise 2.3.

The FBD to do force calculations using static equilibrium conditions is shown in Figure S2.2. First we find the value of the subtended angle identified as β in Figure 2.4. Applying the cosine law over triangle ABC,

$$cos\beta = \frac{3.2^2 + 2.9^2 - 2.5^2}{2 \times 3.2\ in \times 2.9\ in} = 0.668104$$

Equilibrium of forces in the x direction: $F \cos\theta - N \sin\beta = 0$, gives

$$Fcos\theta = Nsin\beta = 0.35 \times sin48.08° = 0.260\ lbs \#(14)$$

Equilibrium of forces in the y direction: $F \sin\theta - N - N \cos\beta = 0$, gives

$$Fsin\theta = N(1 + cos\beta) = 0.35\ lbs \times (1 + cos48.08°) = 0.5838\ lbs. \#(15)$$

On dividing (15) by (14) we get $\tan\theta = 2.2454$, where

$$\theta = 65.99°, \#(16)$$

and finally using (14) yields the pull force F:

$$F = \frac{Fcos\theta}{cos\theta} = \frac{0.260\ lbs}{0.4069} = 0.6389 lbs. \#(17)$$

Exercise 2.4. A De Havilland Dash 8 wing is assumed to be in a state of plane stress. It is made from aluminum with a modulus of elasticity E = 73.1 GPa and a shear modulus G = 27 GPa.

Three strain gauges are mounted on the wing at point *P*, and from these gauges three strains are calculated:

$\epsilon_x = 2.8 \times 10^{-3}\ mm/mm$
$\epsilon_y = 1.3 \times 10^{-3}\ mm/mm$
$\gamma_{xy} = 6.2 \times 10^{-4}\ mm/mm$

It is important to note that $\epsilon_z \neq 0$, but it cannot be measured with strain gauges. Given these three strain measurements, calculate:

1. The Poisson's ratio of the material.
2. The components of plane stress at point *P*.
3. The principal stresses at point *P*.
4. The maximum in-plane shear stress at point *P*.

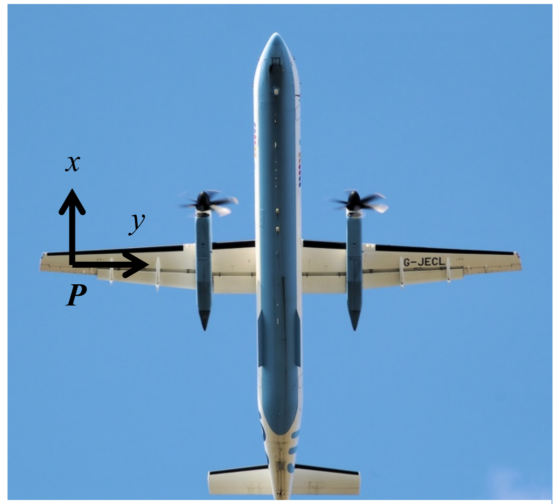

Figure 2.4 for Exercise 2.4

Solution. Given and unknowns:

$$\epsilon_x = 2.8 \times 10^{-3}\ mm/mm,\ \ \epsilon_y = 1.3 \times 10^{-3}\ mm/mm, \gamma_{xy} = 6.2 \times 10^{-4}\ mm/mm$$

$$\nu = ?\,, \sigma_x = ?\,, \sigma_y = ?\,, \sigma_1 = ?\,, \sigma_2 = ?\,,\ \tau_{max} = ?$$

1. We have two of the elastic parameters, E and G. By means of the relations in Ch. 5 of the textbook it is easy to obtain to the Poisson's ratio.
$$G = \frac{E}{2(1+\upsilon)}, \upsilon = \frac{E}{2G} - 1 = 0.35$$
2. Using Hook's law as a constitutive model, we will have the following set of equations.
$$\begin{cases} \epsilon_x = \dfrac{1}{E}(\sigma_x - \nu\sigma_y) \\ \epsilon_y = \dfrac{1}{E}(\sigma_y - \nu\sigma_x) \end{cases}$$
This is a system of linear equations with 2 unknowns and 2 equations. So once can solve for unknowns(σ_x, σ_y) in terms of knowns(ϵ_x, ϵ_y) .
Solving this system for σ_x and σ_y along with the constitutive equation for shear stress, will give the following results:
$$\begin{cases} \sigma_x = E\dfrac{\epsilon_x + \nu\epsilon_y}{1-\nu^2} = 272.4\ MPa \\ \sigma_y = E\dfrac{\epsilon_y + \nu\epsilon_x}{1-\nu^2} = 191.4\ MPa \\ \tau_{xy} = G\gamma_{xy} = 16.8\ MPa \end{cases}$$
3. To find the maximum and minimum stress, we must calculate the center and radius of the Mohr's circle as follow:

$$\begin{cases} C = \dfrac{\sigma_x + \sigma_y}{2} = 231.9\ MPa \\ R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 43.8\ MPa \end{cases}$$

Now we can find the σ_1 and σ_2 based on the following relations.

$$\begin{cases} \sigma_2 = C - R = 188.0\ MPa \\ \sigma_1 = C + R = 275.7\ MPa \end{cases}$$

4. From the Mohr's circle one can easily deduce that:

$$\tau_{max} = R = 43.8\ MPa$$