ASEN 3112 - Structures - Spring 2020

Homework 5 Solution

Exercise 1.1.

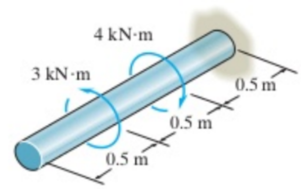


Figure 1.1

Determine the torsional strain energy in the following steel shaft with a radius of 30 mm, assuming a shear modulus of 75GPa.

Solution 1.1.

Givens and unknowns:

$$\boxed{r = 30\,mm, G = 10{,}800\,ksi}$$
$$\boxed{U = ?}$$

$$U = \sum \frac{T^2 L}{2GJ}$$

$$U = \frac{[0^2 * 0.5 + (3 * 10^3)^2 * 0.5 + (1 * 10^3)^2 * 0.5]}{2 * 75 * 10^9 * \frac{\pi}{2} * r^4}$$

$$U = 26.2\,J$$

Exercise 1.2.

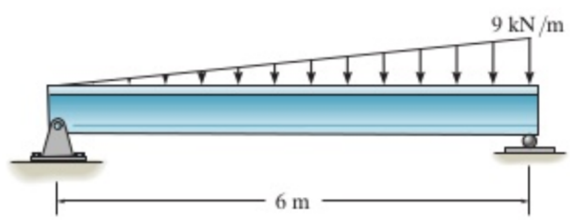


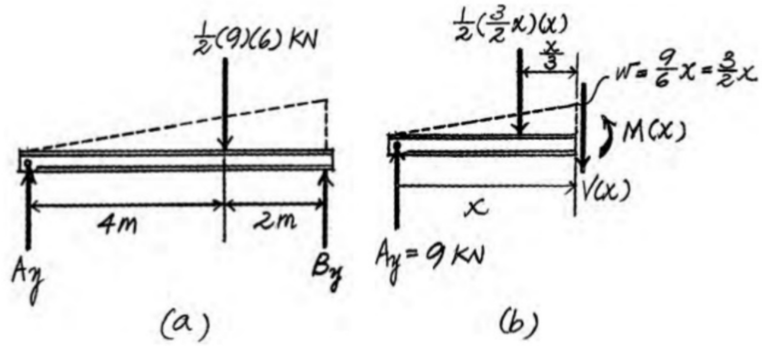
Figure 1.2

Determine the bending strain energy in the steel beam with a moment of inertia of 100 x 10^6 mm^4 and an elastic modulus of 180 GPa.

Solution 1.2.

$$\boxed{w = 9kN/m, \mathrm{I} = 100 \text{ x } 10\text{^}6 \text{ mm^}4, \mathrm{E} = 180 \text{ GPa}}$$

$$\boxed{U = ?}$$



Based on FBD of the whole system shown in Figure (a):

$$\sum \mathrm{M_B} = 0$$

$$\frac{1}{2} * 9 * 6 * 2 - A_y * 6 = 0 \qquad => \qquad A_y = 9\ kN$$

Based on FBD of the cut shown in Figure (b):

$$\sum \mathrm{M_o} = 0 = \mathrm{M(x)} + \frac{x^3}{4} - 9x$$

$$\mathrm{M(x)} = -\frac{x^3}{4} + 9x$$

$$\mathrm{U} = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI} \int_0^L \left(-\frac{x^3}{4} + 9x\right)^2 dx$$

$$U = 37\ J$$

Exercise 1.3.

Determine the horizontal displacement of joint D using the conservation of energy principle. Assume that the product of the cross-sectional area and the elastic modulus (AE) is constant.

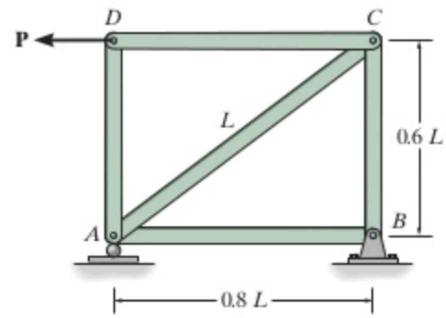


Figure 1.3

Solution 1.3.

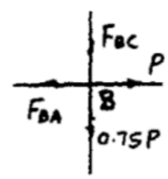
Unknowns:

$$\boxed{\Delta_D = ?}$$

Joint B:

$$\sum F_y = 0 \quad => \quad F_{BD} = \frac{3}{4}P$$

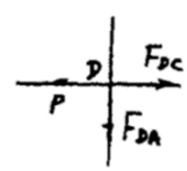
$$\sum F_x = 0 \quad => \quad F_{BA} = P$$



Joint D:

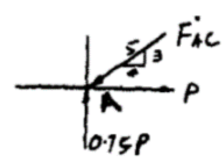
$$\sum F_y = 0 \quad => \quad F_{DA} = 0$$

$$\sum F_x = 0 \quad => \quad F_{DC} = P$$



Joint A:

$$\sum F_y = 0 \quad => \quad \frac{3}{5}F_{AC} - \frac{3}{4}P = 0 \quad => \quad F_{AC} = 1.25P$$



Conservation of Energy:

$U_e = U_i$

$$\frac{1}{2}P\Delta_D = \sum \frac{N^2 L}{2AE} = \frac{1}{2AE}[(0.75P)^2(0.6L) + P^2(0.8L) + 0^2(0.6L) + P^2(0.8L) + (1.25P)^2(L)]$$

$$\Delta_D = \frac{3.50PL}{AE}$$

Exercise 1.4.

Determine the slope at the end B of the steel beam with a moment of inertia of 80 x 10^6 mm^4 and an elastic modulus of 200 GPa.

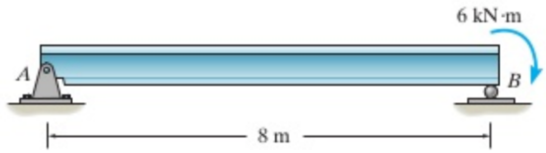
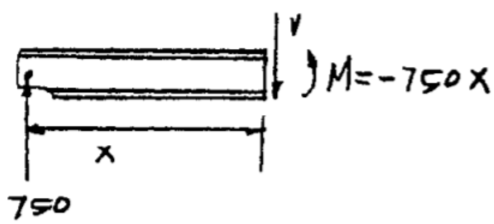


Figure 1.4

Solution 1.4.

Givens and unknowns:

$$\boxed{M = 6\ kNm, L = 8\ m, I = 80\ x\ 10^6\ mm^4, E = 200\ GPa, \theta_B = ?}$$



$$M = -750\ x$$

$$\frac{1}{2}M\theta_B = \int_0^L \frac{M^2 dx}{2EI}$$

$$\theta_B = 1 * 10^{-3}\ rad$$

Exercise 1.5.

Using the conservation of energy principle, determine the displacement of point B on the steel beam with a moment of inertia of 80 x 10^6 mm^4 and an elastic modulus of 180 GPa.

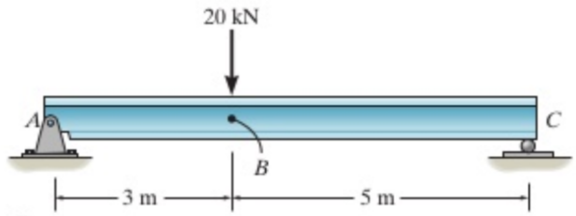
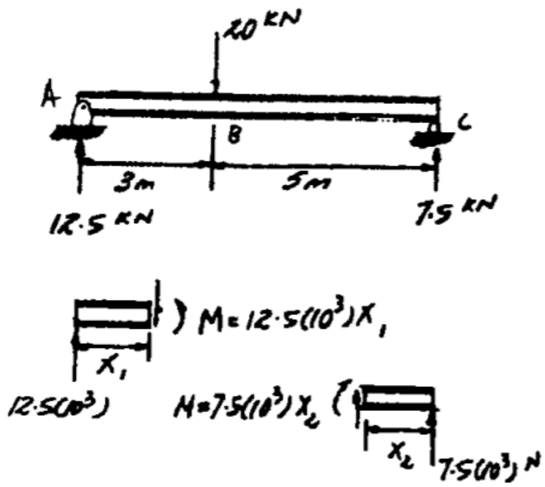


Figure 1.5

Solution 1.5.

Givens and unknowns:

$$\boxed{P = 20\ kN, I = 80\ x\ 10^6\ mm^4, E = 180\ GPa, \Delta_B = ?}$$



$$Ui = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left(\int_0^3 (12.5 * 10^3 * x_1)^2 dx_1 + \int_0^5 (7.5 * 10^3 * x_2)^2 dx_2\right) = \frac{1.875 * 10^9}{EI}$$

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(20 * 10^3 * \Delta_B) = 10 * 10^3 \Delta_B$$

Conservation of energy:

$$U_e = U_i$$

$$10 * 10^3 * \Delta_B = \frac{1.875 * 10^9}{EI}$$

$$\Delta_B = \frac{187500}{EI} = 12.5\ mm$$
