ASEN 3112

Spring 2020

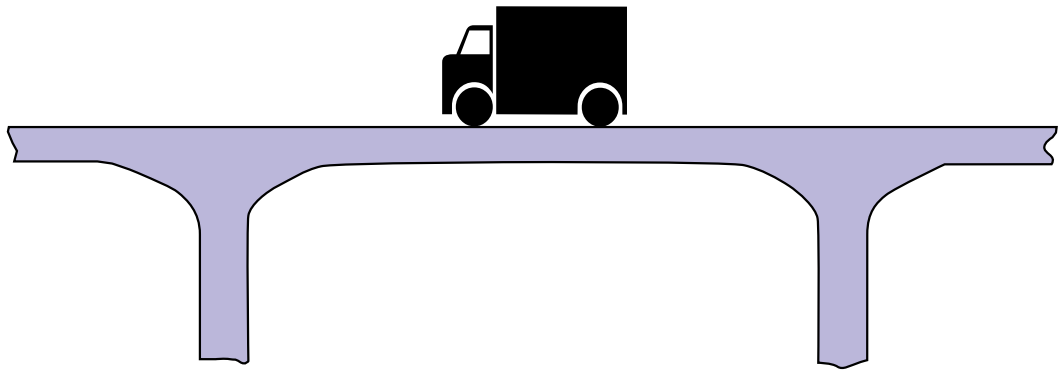
Lecture 7

February 4, 2020

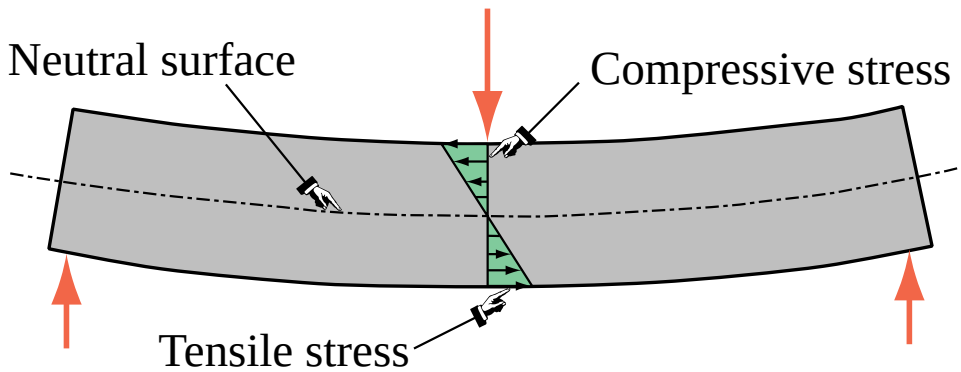
7

Beam Deflections by 2nd Order Method

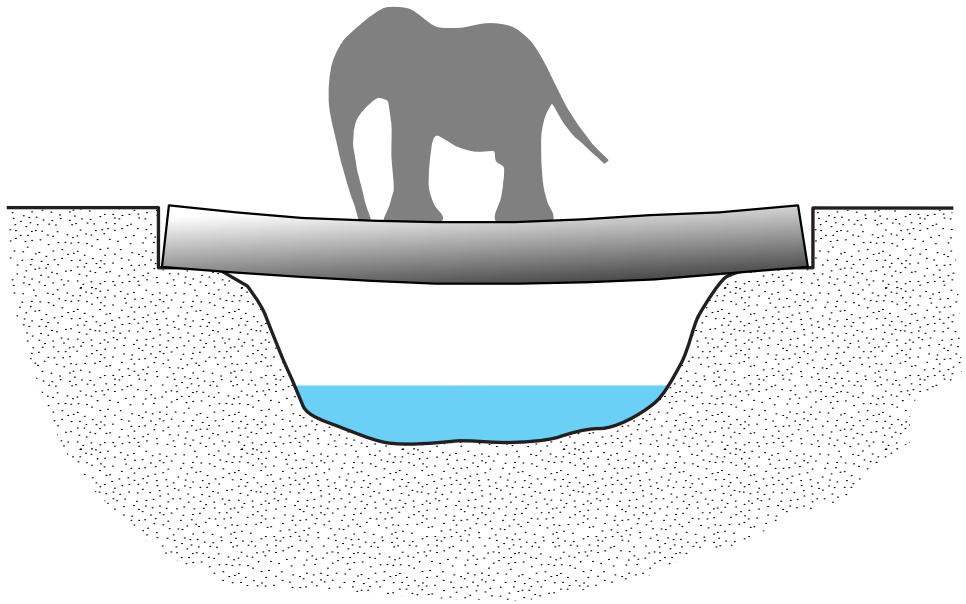
A Beam is a Structural Member Designed to Resist Primarily Transverse Loading



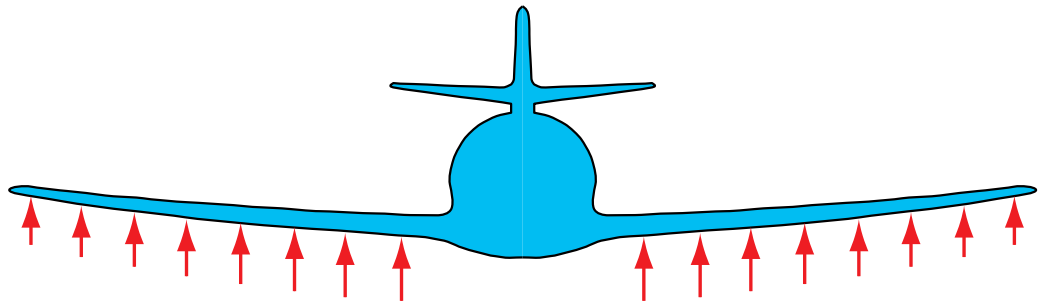
Distribution of Beam Normal Stress Due to Bending Moment



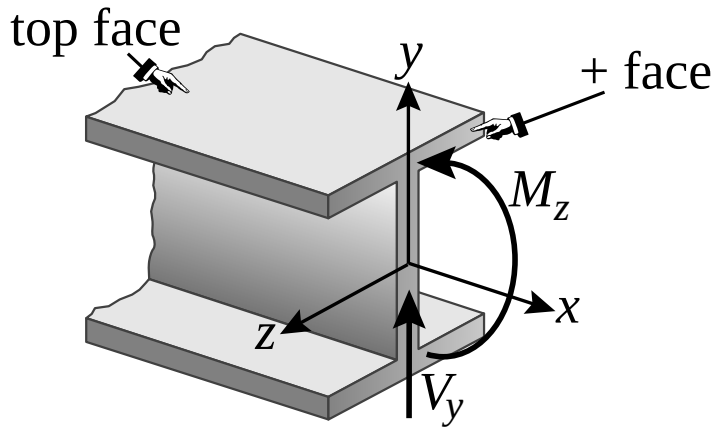
Simply Supported Beam BC Example



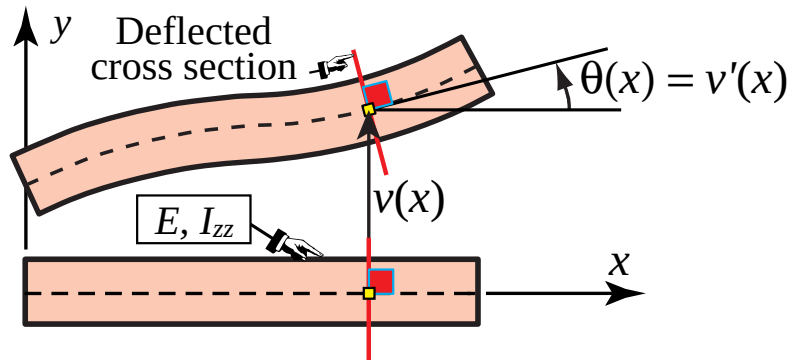
Cantilever Beam BC Example



Transverse Shear and Bending Moment Sign Conventions



Beam Slope and Deflection



Beam Notation & Sign Conventions

Quantity	Symbol	Sign convention(s)
Problem specific load	varies	You pick'em
Generic load for ODE work	$p(x)$	+ if up
Transverse shear force	$V_y(x)$	+ if up on +x face
Bending moment	$M_z(x)$	+ if it produces compression on top face
Slope of deflection curve	$dv(x)/dx = v'(x)$	+ if positive slope, or cross-section rotates CCW
Deflection curve	$v(x)$	+ if beam cross-section moves upward

Note 1: Some textbooks (e.g. Vable and Beer-Johnson-DeWolf) use $V = -V_y$ as alternative transverse shear force symbol. This has the advantage of eliminating the minus sign in two of the ODEs listed on the next slide. V will only be used occsionally in this course.

Note 2. In our beam model, the slope $v'(x) = dv(x)/dx$ is equal to the rotation $\theta(x)$ of the cross section

Beam Differential Equations

Connected quantities	Ordinary Differential Equations (ODEs)
From load to transverse shear force	$\frac{dV_y}{dx} = -p$ or $p = -V_y' = V'$
From transverse shear to bending moment	$\frac{dM_z}{dx} = -V_y$ or $M_z' = -V_y = V$
From bending moment to deflection	$E\,I_{zz}\,v'' = M_z$ or $v'' = \frac{M_z}{E\,I_{zz}}$
From load to moment	$M_z'' = p$
From load to deflection	$E\,I_{zz}\,v^{IV} = p$

Beam Boundary Conditions for Shear, Moment, Slope & Deflection

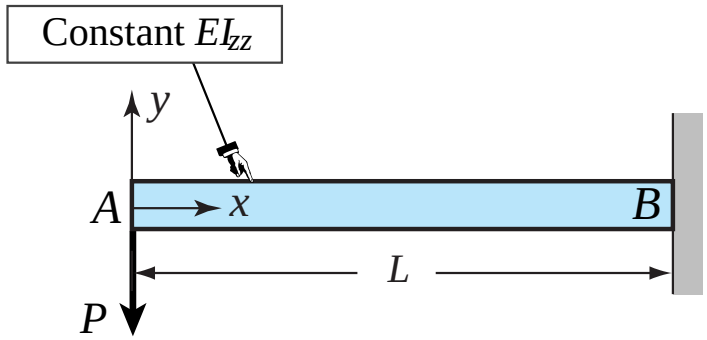
(Part of a Supplementary Crib Sheet for Midterm Exams 2 & 3)

Beam Boundary Conditions for Shear, Moment, Slope & Deflection

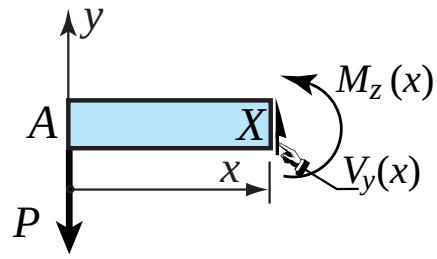
Condition	Shear force $V_y(x)$	Bending moment $M_z(x)$	Slope (= rotation) $v'(x)= \theta(x)$	Deflection $v(x)$
Simple support		0 &		0
Fixed end			0	0
Free end	0 *	0 #		
Symmetry	0		0	
Antisymmetry		0		0

* Unless a point force is applied at the free end
& Unless a point moment is applied at the simple support
\# Unless a point moment is applied at the free end
Blank entry means that value is unknown and has to be determined by solving problem

Example 1: Cantilever Beam Under End Point Load

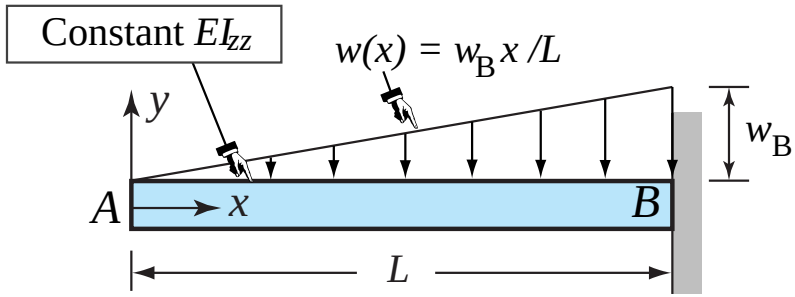


(a) Problem definition

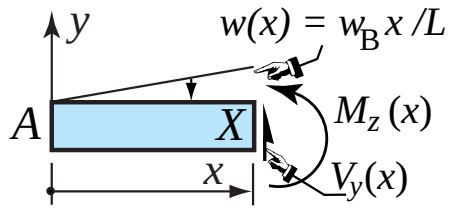


(b) FBD to find $M_z(x)$

Example 2: Cantilever Beam Under Triangular Load

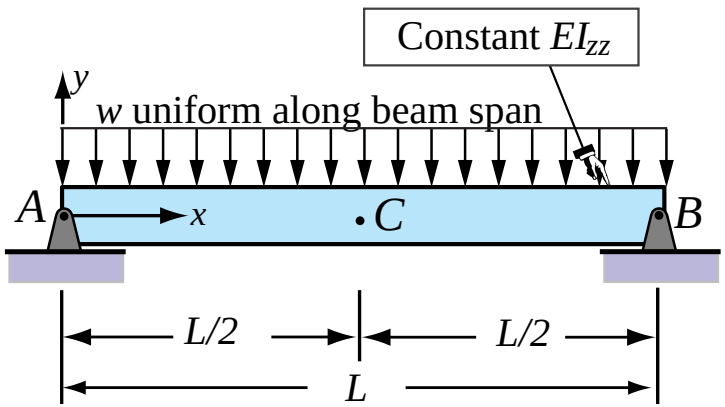


(a) Problem definition

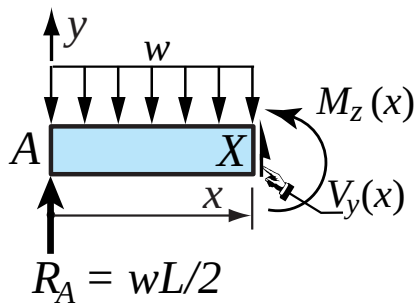


(b) FBD to find $M_z\,(x)$

Example 3: Simply Supported Beam Under Uniform Load



(a) Problem definition



(b) FBD to find $M_z(x)$