ASEN 3112

Spring 2020

Lecture 5

January 28, 2020

5
Torsion of Open Thin Wall (OTW) Sections

Classification of Thin Wall (TW) Cross Sections Under Torque

Closed Thin Wall (CTW) Sections

at least one cell shear flow circuit can be established

Single Cell: just one cell

Multicell: more than one cell

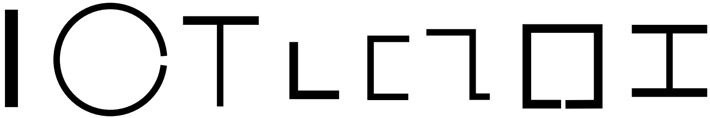
Open Thin Wall (OTW) Sections

no cell shear flow circuit can be established

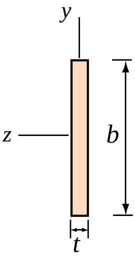
Hybrid Thin Wall (HTW) Sections

contains both OTW and CTW components

Sample OTW Cross Sections



Prototype OTW Cross Section: Solid Rectangle

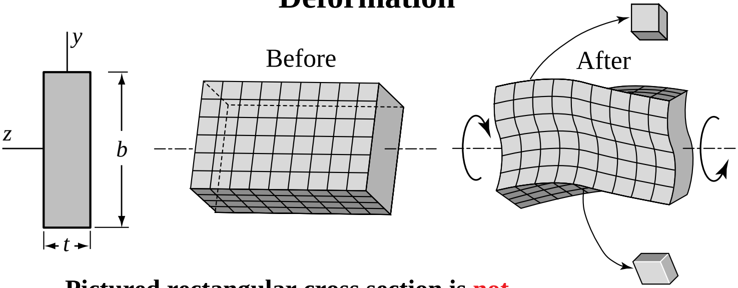


$b >= t$ **is called the long dimension**

t **is the wall thickness or simply thickness**

Rectangle is called thin (or narrow)
if $b >> t$**, usually** $b > 10\,t$

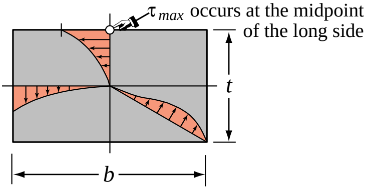
Solid Rectangle Torque Behavior: Deformation

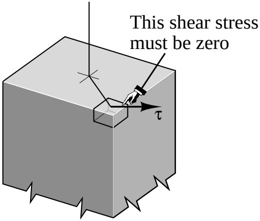


Pictured rectangular cross section is not narrow, to facilitate visualization

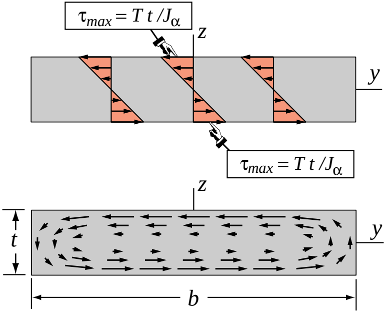
Note that cross sections warp as torque is applied
This happens for all non-circular cross sections
(warping => plane sections do not remain plane)

Solid Rectangle Torque Behavior: Shear Stresses





Solid Rectangle Torque Behavior: Shear Stresses For Thin Rectangle



Basic Formulas for Single Rectangular Cross Section (Not Necessarily Thin)

Let T **= applied torque,** b **= longest rectangle dimension,**
t **= shortest rectangle dimension,** G **= shear modulus.**
Then the stress and twist angle formulas are

$$\tau_{max} = \frac{T\,t}{J_\alpha} \qquad \phi' = \frac{d\phi}{dx} = \frac{T}{G\,J_\beta}$$

in which $J_\alpha = \alpha\, b\, t^3$ **and** $J_\beta = \beta\, b\, t^3$

The dimensionless coefficients α **and** β **are functions of the aspect ratio** b/t**, and are tabulated in the next slide**

These formulas are exact in the sense that they are provided by the Theory of Elasticity. They are obtained by solving a Partial Differential Equation (PDE) of Poisson's type

Coefficients α and β for Single Rectangular Cross Section as Functions of Aspect Ratio

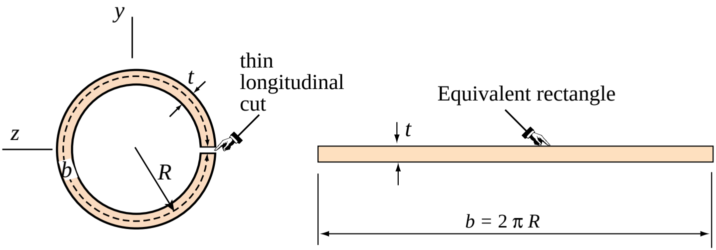
b/t	1.0	1.2	1.5	2.0	2.5	3.0	4.0	5.0	6.0	10.0	∞
α	0.208	0.219	0.231	0.246	0.258	0.267	0.282	0.291	0.299	0.312	1/3
β	0.141	0.166	0.196	0.229	0.249	0.263	0.281	0.291	0.299	0.312	1/3

Interpolation formulas valid for all aspect ratios are given in the Lecture 8 Notes. If $b/t > 3$, $\alpha \sim \beta$ within 1%

If the section is sufficiently thin so that $b/t > 5$ (say) one can take $\alpha = \beta \sim 1/3$, which is easy to remember.

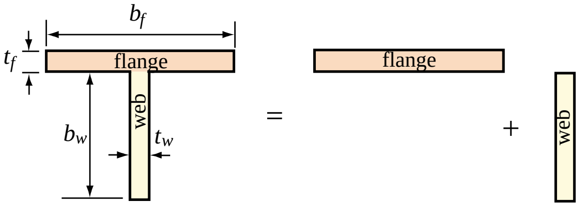
Rectification of Curved TW Profiles

Simple curved TW sections can be "rectified" into an equivalent rectangle. For example the slitted section of Lab 1:

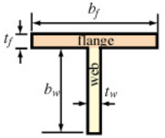


This is valid only if curved wall is truly thin so $b/t > 5$ (say)

Decomposition Into Rectangles: T Section



T Section Torsion Analysis



From statics, decompose the total applied torque T into portions taken by the flange and web:

$$T = T_f + T_w \tag{1}$$

That is one equation for two unknowns. Twist angle rate compatibility provides a second equation

$$\frac{d\phi}{dx} = \frac{d\phi_f}{dx} = \frac{T_f}{G J_{\beta f}} = \frac{d\phi_w}{dx} = \frac{T_w}{G J_{\beta w}} \tag{2}$$

Solving (1) and (2) we get (G drops out)

$$T_f = \frac{J_{\beta f}}{J_{\beta f} + J_{\beta w}} T = \frac{J_{\beta f}}{J_\beta} T, \qquad T_w = \frac{J_{\beta w}}{J_{\beta f} + J_{\beta w}} T = \frac{J_{\beta w}}{J_\beta} T$$

in which $J_\beta = J_{\beta f} + J_{\beta w}$. To obtain the maximum shear stress, apply the stress formula to the flange and web in turn, and pick the largest (note that J_α is now used)

$$\tau_{max\,f} = \frac{T_f t_f}{J_{\alpha f}}, \quad \tau_{max\,w} = \frac{T_w t_w}{J_{\alpha w}} \quad => \quad \tau_{max} = \max(\tau_{max\,f}, \tau_{max\,w})$$

Decomposition Into Rectangles: Z Section