ASEN 3112

Spring 2020

Lecture 1

January 14, 2020

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Stress in 3D

Mechanical Stress in 3D: Concept

Mechanical stress measures ***intensity level*** of ***internal forces*** in a material (solid or fluid body) idealized as a mathematical ***continuum***. The physical measure of stress is

Force per unit area e.g. N/mm^2 (MPa) or lbs/sq-in (psi)

This measure is convenient to assess the resistance of a material to permanent deformation (yield, creep, slip) and rupture (fracture, cracking). Comparing working and failure stress levels allows engineers to establish ***strength safety factors*** for structures.

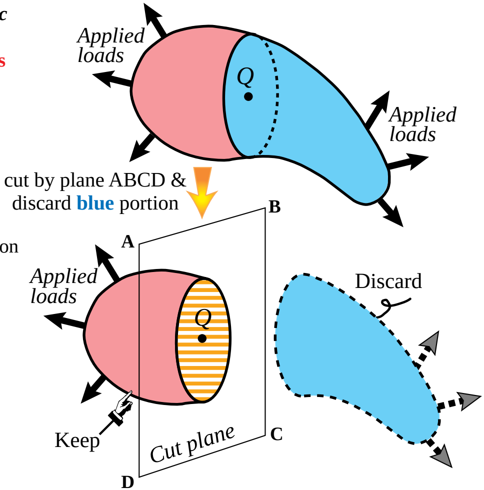
Stresses may vary from point to point. We next consider a solid body (could be a structure or part of one) in 3D.

Cutting a 3D Body

Consider a 3D solid body in ***static equilibrium*** under applied loads. We want to find the **state of stress** at an arbitrary point Q, which generally will be inside the body.

Cut the body by a ***plane*** ABCD that passes through Q as shown (How to ***orient*** the plane is discussed later.) The body is divided into two. Retain one portion (**red** in figure) and discard the other (**blue** in figure)

To ***restore equilibrium***, however, we must replace the discarded portion by the ***internal forces*** it had exerted on the kept portion.



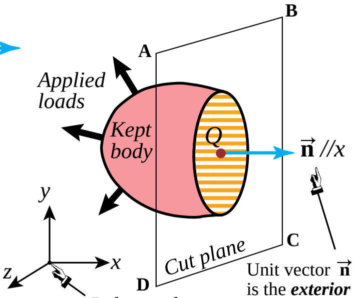
Orienting the Cut Plane

The cut plane ABCD is oriented by its unit normal direction vector **n**, or ***normal*** for short. By convention we will draw **n** as emerging from Q and pointing ***outward*** from the kept body. This direction identifies the ***exterior normal***.

With respect to the RCC system $\{x,y,z\}$, the normal vector has components

$$\mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

where $\{n_x, n_y, n_z\}$ are the direction cosines of **n** with respect to $\{x,y,z\}$. These satisfy $n_x^2 + n_y^2 + n_z^2 = 1$.



Reference frame is a right-handed, RCC system

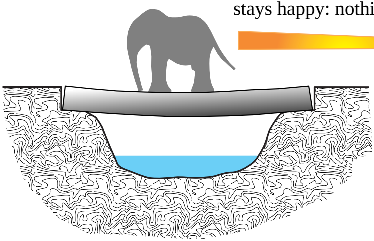
Unit vector $\vec{\mathbf{n}}$ is the ***exterior normal*** at Q, meaning it points ***outward*** from the kept body

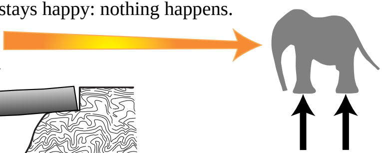
In the figure, the cut plane ABCD has been chosen with its exterior normal ***parallel*** to the $+x$ axis. Consequently $\mathbf{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

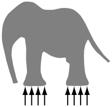
Digression: Action and Reaction

(Newton's 3rd Law, from Physics I)

Remove the "bridge" (log) and replace its effect on the elephant by ***reaction forces*** on the legs. The elephant stays happy: nothing happens.







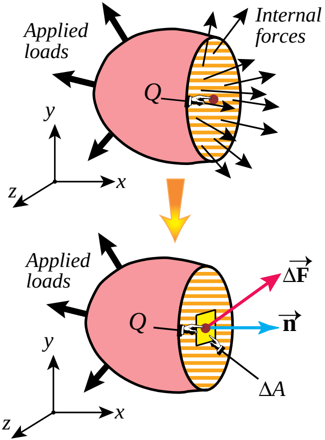
Strictly speaking, reaction forces are ***distributed*** over the elephant leg contact areas. They are replaced above by equivalent point forces, a.k.a. ***resultants***, for visualization convenience

Internal Forces on Cut Plane

Those internal forces generally will form a system of ***distributed forces per unit of area***, which, being vectors, generally will vary in magnitude and direction as we move from point to point of the cut plane, as pictured.

Next, we focus our attention on point Q. Pick an ***elemental area*** ΔA around Q that lies on the cut plane. Call $\Delta\mathbf{F}$ the ***resultant*** of the internal forces that act on ΔA. Draw that vector with origin at Q, as pictured. Don't forget the normal

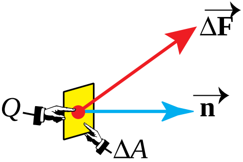
The use of the increment symbol Δ suggests a pass to the limit. This will be done later to define the stresses at Q



Arrows are placed over $\Delta\mathbf{F}$ and $\mathbf{n}$ to remind you that they are ***vectors***

Internal Force Components

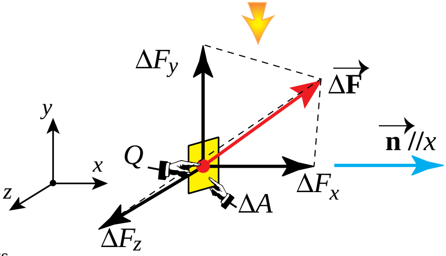
Zoom on the elemental area about Q, omitting the kept-body and applied loads for clarity:



Project vector $\Delta\mathbf{F}$ on axes x, y and z to get its components ΔF_x, ΔF_y and ΔF_z, respectively. See bottom figure.

Note that component ΔF_x is aligned with the cut-plane normal, because $\mathbf{n}$ is parallel to x. It is called the ***normal internal force*** component.

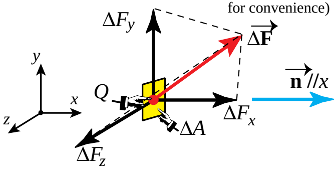
Components ΔF_y and ΔF_z lie on the cut plane. They are called ***tangential internal force*** components.



We are now ready to define stresses.

Stress Components at a Point: x Cut

(Reproduced from previous slide for convenience)



Define the ***x-stress components*** at point Q by taking the limits of internal-force-over-area ratios as the elemental area shrinks to zero:

$$\sigma_{xx} \overset{\text{def}}{=} \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A} \qquad \tau_{xy} \overset{\text{def}}{=} \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} \qquad \tau_{xz} \overset{\text{def}}{=} \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

σ_{xx} is called a ***normal stress,*** whereas τ_{xy} and τ_{xz} are ***shear stresses***.

Stress Components at a Point: y and z Cuts

It turns out we need ***nine*** stress components in 3D to fully characterize the stress state at a point. So far we got only three. Six more are obtained by repeating the same take-the-limit procedure with ***two other cut planes.*** The obvious choice is to pick planes normal to the other two axes: y and z.

Taking **n**//y we get three more components, one normal and two shear:

$$\boxed{\sigma_{yy} \quad \tau_{yx} \quad \tau_{yz}}$$

These are called ***y-stress components***.

Taking **n**//z we get three more components, one normal and two shear:

$$\boxed{\sigma_{zz} \quad \tau_{zx} \quad \tau_{zy}}$$

These are called ***z-stress components***.

Together with the three ***y-stress components*** found before, this makes up a total of nine, as required.