ASEN 3112:
Spring 2020
Exam 1

Date: February 11, 2020

Name: ______________________________________

Student ID: __________________________________

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this exam.

Name: ____

Signature: ____

Date: ____

Please label your work with the part of the problem you are working on and circle your final answer for each part.

1. This exam is closed-book. Up to 2 sheets (equivalent to 4 pages) of crib sheet are allowed.

2. Solve all four problems.

3. Total time for the exam is 1 hour and 15 minutes.

4. Do not redefine the problem; carefully read the problem statement and answer the questions that are asked.

5. Make sure that one can follow your analysis; describe briefly what you are doing.

6. You must show how you got to your solution. Simply showing the final results will lead to point reduction.

7. You must cross out all work you do not want graded. Any work that is not crossed out is fair game for grading.

8. Include units on final answers whenever applicable

9. Write your solution on the exam sheets using the space provided for each question.

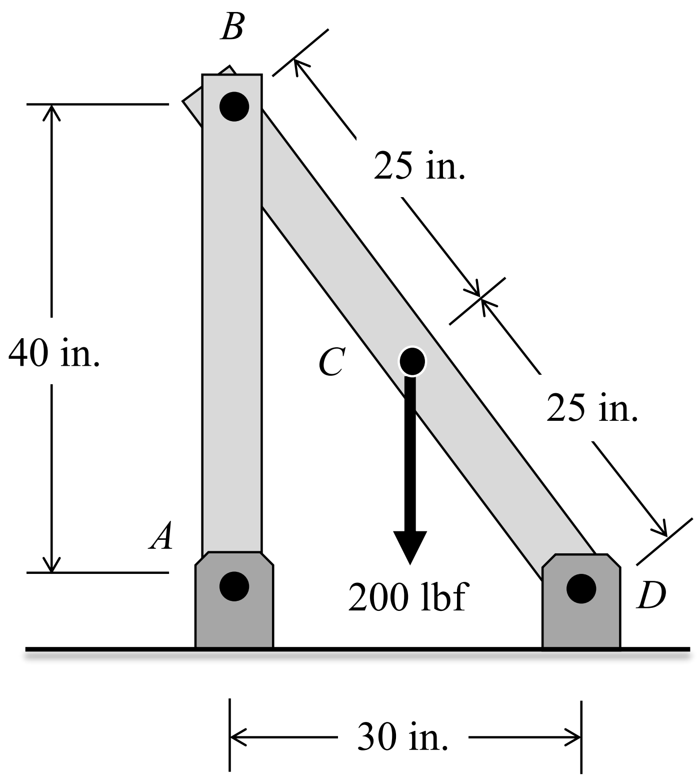
10. If you take your exam apart to work on it, please turn it in stapled in the exact same order in which you received it.

Question 1.   25 points

Dr. Johnson is climbing the ladder shown in the figure. The frame consists of two members: AB (length = 40 in.) and BCD (length = 50 in.). (Note that ABD makes a 3-4-5 triangle.) Both members are made from aluminum with a cross-sectional area $A_{\text{bar}} = 1.25\ \text{in}^2$. All joints (at A, B, and D) are pinned joints with pins constructed of aluminum with cross-sectional area $A_{\text{pin}} = 0.5\ \text{in}^2$. Pins A and D are in double shear, while pin B is in single shear. All aluminum parts (members and pins) in this problem have the following properties:

- Normal failure stress (in tension and compression) $\sigma_{fail} = 60 \times 10^3$ psi
- Shear failure stress $\tau_{fail} = 25 \times 10^3$ psi

Member BCD is loaded by Dr. Johnson's weight, represented by a vertical external force at point C with magnitude 200 lbf. Note that point C is at the center of the length of member BCD. Neglect the weight of members AB and BCD.



a) Draw a free-body diagram of the entire system.

b) Calculate the normal stress in member AB. Is member AB in tension or compression?

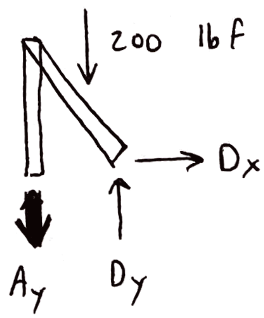
c) Calculate the shear stress in pin A.

d) Calculate the shear stress in pin B.

e) Calculate the safety factor of the part of the system – either member AB, pin A, or pin B – that will fail first (i.e. has the lowest safety factor). Should Dr. Johnson be worried about the ladder failing as he climbs it?

Problem 1

a.



b. $\Sigma M_D = 0 = 200(15) + A_y(30)$

$$A_y = \frac{-200(15)}{30} = -100 \text{ lbf}$$

$N_{AB} = A_y = -100$ lbf. AB is in compression.

$\uparrow N_{AB}$

$\downarrow A_y$

$$\sigma_{AB} = \frac{-100}{1.25} = \boxed{-80 \text{ psi} = \sigma_{AB}}$$

c. A is in double shear, so $|V_A| = \frac{100}{2} = 50$

$$\tau_A = \frac{50}{0.5} = \boxed{100 \text{ psi} = \tau_A}$$

d. B is in single shear, so $|V_B| = 100$

$$\tau_B = \frac{100}{0.5} = \boxed{200 \text{ psi} = \tau_B}$$

e. Don't need to check pin A, b/c pin B will fail before it

$$SF_{AB} = \frac{60 \times 10^3}{80} = 750$$

$$SF_B = \frac{25 \times 10^3}{200} = \boxed{125 = SF_{min}}$$

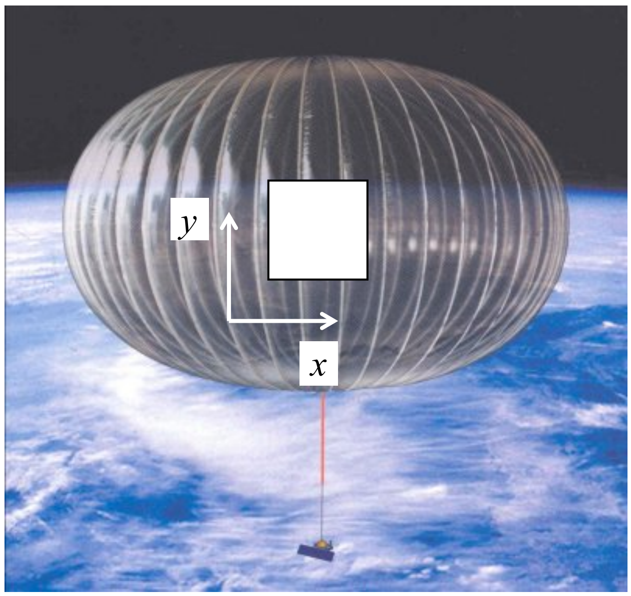
← Fails first

Very high SF. Dr. Johnson shouldn't worry

Question 2. 20 points

A high-altitude science balloon, which is approximated as a **spherical pressure vessel**, is filled with gas on the ground then inflates as it ascends. The skin is thin and can be considered to be in a state of **plane stress**. The balloon skin has a thickness 5 mm, an elastic modulus E = 4.8 GPa, and a Poisson's ratio v = 0.4.

At launch, the pressure inside the balloon is 120 kPa and the radius of the balloon is 25 m.



NASA is interested in the states of stress and strain on the square element shown in the figure above. This element is aligned with the x and y axes as shown, and lies in the plane of the balloon skin (the x-y plane).

a) Calculate the shear modulus G of the balloon skin material.
b) Calculate and write the full 3 × 3 **stress** matrix for the square element of balloon skin oriented as shown in the figure above.
c) What is the maximum **in-plane** shear stress experienced in any orientation of the square element?
d) What is the maximum **out-of-plane** shear stress experienced in any orientation of the square element?
e) Calculate and write the full 3 × 3 **strain** matrix for the square element of balloon skin oriented as shown in the figure above.

If you cannot calculate G in part a, use a value of G = 1 GPa for subsequent parts. Note that this is not the same value you get if you correctly answer part a

Problem 2

a. $G = \frac{E}{2(1+\nu)} = \frac{4.8}{2(1+0.4)} = \boxed{1.71 \text{ GPa} = G}$

b. $\sigma = \frac{pR}{2t} = \frac{120\times10^3(25)}{2(0.005)} = 300 \text{ MPa}$

$$\boxed{\sigma = \begin{bmatrix} 300 & 0 & 0 \\ 0 & 300 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}}$$

c. No τ w/ any rotation of element in plane. $\boxed{\tau_{max,in\ plane} = 0}$

e. $\epsilon_{xx} = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{4.8\times10^9}(300\times10^6 - 0.4(300\times10^6))$

$\epsilon_{xx} = 0.0375$

$\epsilon_{yy} = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0.0375$

$\epsilon_{zz} = -2\frac{\nu}{E}\sigma = -2\frac{0.4}{4.8\times10^9}(300\times10^6) = -0.05$

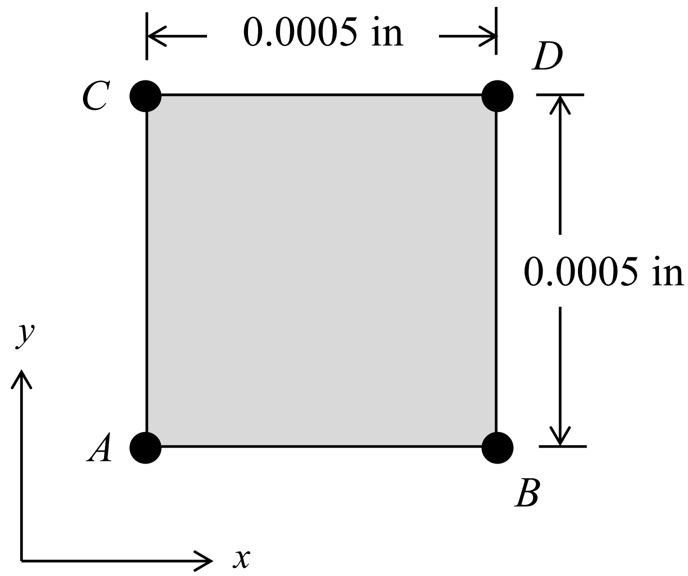
$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0$

$$\boxed{\epsilon = \begin{bmatrix} 0.0375 & 0 & 0 \\ 0 & 0.0375 & 0 \\ 0 & 0 & -0.05 \end{bmatrix}}$$

d. τ_{max} in any plane $= \frac{\sigma_1 - \sigma_3}{2} = \frac{300}{2} = \boxed{150 \text{ MPa} = \tau_{max,out\ of\ plane}}$

Question 3.   15 points

An extensometer is used to measure the strain in a Boeing Starliner spacecraft during testing. The extensometer measures the horizontal and vertical displacement of four close points on the surface of the spacecraft. These four points – A, B, C, and D – are shown in their undeformed configuration below.



The horizontal and vertical displacement of each point is measured as follows (all measurements in inches):

Point	Horizontal displacement u	Vertical displacement v
A	2.3×10^{-6}	-2.2×10^{-6}
B	-1.2×10^{-6}	-1.4×10^{-6}
C	-6.0×10^{-6}	5.2×10^{-6}
D	3.3×10^{-6}	2.8×10^{-6}

Calculate the following quantities with respect to **point D**:

a) The extensional strain in the x-direction, ϵ_{xx}

b) The extension strain in the y-direction, ϵ_{yy}

c) The shear strain, γ_{xy}

Problem 3

a. $\epsilon_{xx} = \frac{\Delta u_{CD}}{\Delta x} = \frac{3.3\times10^{-6} + 6.0\times10^{-6}}{0.0005} = \boxed{0.0186 = \epsilon_{xx}}$

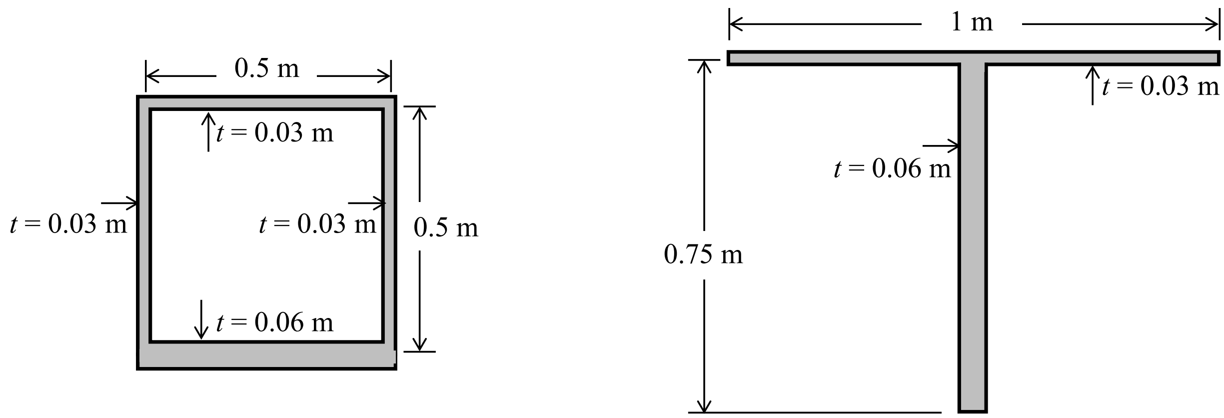
b. $\epsilon_{yy} = \frac{\Delta u_{BD}}{\Delta y} = \frac{2.8\times10^{-6} + 1.4\times10^{-6}}{0.0005} = \boxed{0.0084 = \epsilon_{yy}}$

c. $\gamma_{xy} = \frac{\Delta v_{CD}}{\Delta x} + \frac{\Delta u_{BD}}{\Delta y} = \underbrace{\frac{(2.8\times10^{-6} - 5.2\times10^{-6})}{0.0005}}_{-4.8\times10^{-3}} + \underbrace{\frac{(3.3\times10^{-6} + 1.2\times10^{-6})}{0.0005}}_{+9.0\times10^{-3}}$

$\boxed{\gamma_{xy} = 0.0042 \text{ radians}}$

Question 4. 40 points

You work at a German engineering firm designing a bridge. Your job is to analyze the performance of the bridge's main spar in torsion. In the current design, the spar has the square cross-section shown in the left figure. However, another engineer has proposed to redesign the spar to have an T cross-section, as shown in the right figure. Both cross-sections have the same material area, so the beams cost the same to manufacture. Both cross-sections are made of steel, which has a shear failure stress $\tau_{fail} = 200\ MPa$. You estimate the torque experienced by the spar during normal loading is $T = 1.9 \times 10^5\ N \cdot m$.



All dimensions given are midline dimensions.

Complete the following calculations using the open and/or closed thin-wall approximation. Also, if using open thin-wall theory, assume $\alpha = \beta = \frac{1}{3}$.

a) Calculate the maximum shear stress τ_{max} in the square cross-section (left figure). Where does this τ_{max} occur in the cross-section?

b) If you rotated the square cross-section so that the thickest wall was on the left, would it change your calculation for part a? Justify your answer.

c) Calculate the maximum shear stress τ_{max} in the T cross-section (right figure). Where does this τ_{max} occur in the cross-section?

d) Was it reasonable to use the thin-walled approximation for both sections? If the answer is no, there is no need to redo calculations.

e) As the torsion analyst for this bridge project, do you recommend switching to the T cross-section? Justify your answer numerically.

Problem 4

a. $\tau_{max} = \dfrac{T}{2 t_{min} A_E} = \dfrac{1.9 \times 10^5}{2(0.03)(0.5)^2} = \boxed{12.7 \text{ MPa} : \tau_{max}}$

occurs in all of the walls where $t = 0.03$ m

$A_E = 0.5^2$

b. No, it wouldn't matter. The formula for τ_{max} has nothing to do with orientation. τ_{max} still occurs in all walls with $t = 0.03$ m.

c. Find τ_{max}

$J_{flange} = \frac{1}{3}(1)(0.03)^3 = 9 \times 10^{-6}$ m^4

$J_{web} = \frac{1}{3}(0.75)(0.06)^3 = 5.4 \times 10^{-5}$ m^4

$J_T = J_f + J_w = 6.3 \times 10^{-5}$

$T_f = \dfrac{9 \times 10^{-6}}{6.3 \times 10^{-5}}(1.9 \times 10^5) = 2.71 \times 10^4$ N·m

$T_w = 1.9 \times 10^5 - T_f = 1.9 \times 10^5 - 2.71 \times 10^4 = 1.63 \times 10^5$ N·m

$\tau_{max} = \dfrac{Tt}{J}$

$\tau_f = \dfrac{2.71 \times 10^4 (0.03)}{9 \times 10^{-6}} = 90.3$ MPa

$\tau_w = \dfrac{1.63 \times 10^5 (0.06)}{5.4 \times 10^{-5}} = \boxed{181 \text{ MPa} = \tau_{max}}$ occurs in the web

d. Check $\dfrac{b_{min}}{t_{max}}$ for each

Square: $\dfrac{0.5}{0.06} = 8.3$ not too bad, close to 10. Thin-walled approx ok.

T: $\dfrac{0.75}{0.06} = 12.5$ > 10, so thin-walled approx good.

e. Check SF

$\tau_{fail} = 200 \text{ MPa}$

$SF_{square} = \frac{200}{12.7} = 15.75$ ✓ good

$SF_T = \frac{200}{181} = 1.1$ X not good! Too close to 1. and, a loading of $T = 1.9 \times 10^5$ N·m is normal loading, not even the worst case scenario.

DO NOT SWITCH!