Recitation 5

ASEN 3112 - Spring 2020

Problem 1 (20 mins)

Beam AB is clamped at point B, Fig. 2. Beam has constant bending inertia *I* and elastic modulus *E.* Point force *P* applied at point A. Determine the deflection at point A using the **Conservation of Energy Principle**:

a) Without considering the spring.
b) With considering the effect of a spring with stiffness k.
   Hint: Split the problem into a beam problem and a spring problem; apply the Conservation of Energy principle to these subsystems individually.

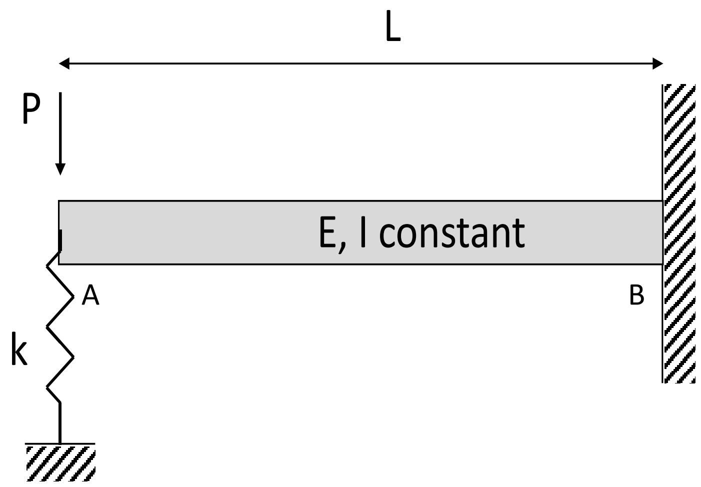


Figure 2: Clamped beam

Solution 1

1. Cutting the beam at some position x:

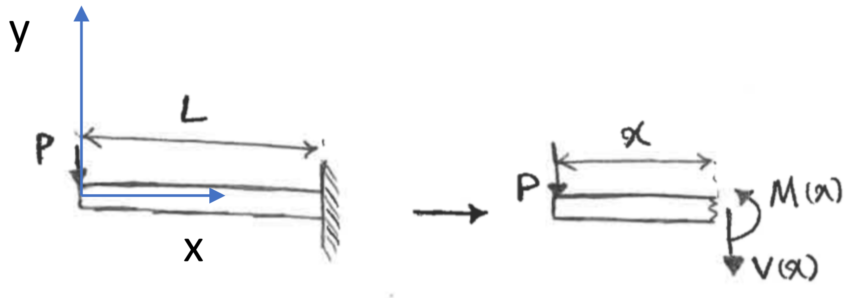
$$M(x) + Px = 0 \rightarrow M(x) = -Px$$

Strain energy of the beam:

$$U_i = \int_0^L \frac{M(x)^2 dx}{2EI} = \frac{1}{2EI}\int_0^L P^2 x^2 dx = \frac{P^2 L^3}{6EI}$$

Using conservation of energy principle:

$$U_i = U_e \rightarrow \frac{P^2 L^3}{6EI} = \frac{1}{2}(-P)v_{(x=0)} \rightarrow \boxed{v_{(x=0)} = \frac{-PL^3}{3EI}}$$



2. Cutting the beam at some position x:

$$M(x) + Px - F_s x = 0 \rightarrow M(x) = (F_s - P)\,x$$

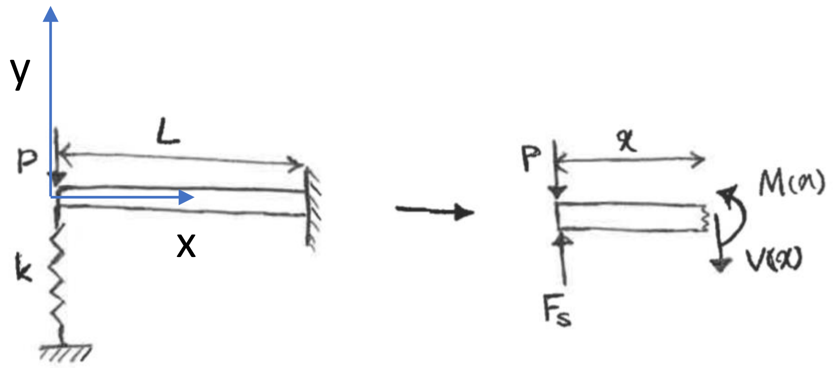
Strain energy of the beam:

$$U_i = \int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI}\int_0^L [(F_S - P)\,x]^2\, dx = \frac{(F_s - P)^2 L^3}{6EI}$$

$$U_i = U_e \rightarrow \frac{(F_s - P)^2 L^3}{6EI} = \frac{1}{2}(F_s - P)v_{(x=0)} \rightarrow v_{(x=0)} = \frac{(F_s - P)L^3}{3EI}$$

We know that $F_s = -kv_{(x=0)}$, so

$$\boxed{v_{(x=0)} = \frac{-PL^3}{3EI\left(\frac{kL^3}{3EI} + 1\right)}}$$



Problem 2a (15 mins)

Consider the two-bar truss shown in the Fig. 3a. Compute the displacement in horizontal direction at joint B due to the external force $P=4.0\text{x}10^3$ N using **Virtual Displacement Method**

Given:

Area of AB = 0.15 m^2

Area of CB = 0.25 m^2

E for both AB and CB = $3\text{x}10^6$ Pa

Hint: For a single bar: $\delta W_{ie,bar} = \frac{E\ A}{L}(\Delta L)\,\delta(\Delta L)$

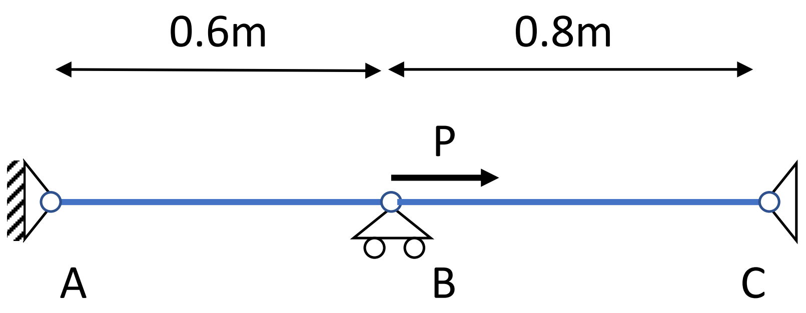


Figure 3a: Two-Bar truss

Solution 2a

$$\Delta L_{AB} = u_B \rightarrow \delta\Delta L_{AB} = \delta u_B,\ \Delta L_{BC} = -u_B \rightarrow \delta\Delta L_{BC} = -\delta u_B$$

Create the virtual work equation for horizontal direction using $\delta W_{ie} = \frac{EA}{L}\Delta L\delta\Delta L$.

$$\delta W_{ie} = \frac{EA_{AB}}{L_{AB}}\Delta L_{AB}\delta\Delta L_{AB} + \frac{EA_{BC}}{L_{BC}}\Delta L_{BC}\delta\Delta L_{BC}$$

$$\delta W_{ie} = \frac{EA_{AB}}{L_{AB}}(u_B)(\delta u_B) + \frac{EA_{BC}}{L_{BC}}(-u_B)(-\delta u_B) = u_B(\frac{EA_{AB}}{L_{AB}} + \frac{EA_{BC}}{L_{BC}})\delta u_B$$

External work:

$$\delta W_e = P\delta u_B$$

Equate the external work to the virtual work yields:

$$\delta W_{ie} = \delta W_e \rightarrow u_B(\frac{EA_{AB}}{L_{AB}} + \frac{EA_{BC}}{L_{BC}})\delta u_B = P\delta u_B \rightarrow \boxed{u_B = \frac{P}{(\frac{EA_{AB}}{L_{AB}} + \frac{EA_{BC}}{L_{BC}})} = 0.0024\text{m}}$$

Problem 2b (25 mins)

Consider the two-bar truss shown in the Fig. 3b. Compute the displacements in vertical and horizontal direction at joint B due to the external force $P = 4.243 \times 10^3$ N inclined at an angle of 45 deg with the horizontal using **Virtual Displacement Method**

Given:

Area of AB = 0.15 m^2

Area of CB = 0.25 m^2

E for both AB and CB = 3×10^6 Pa

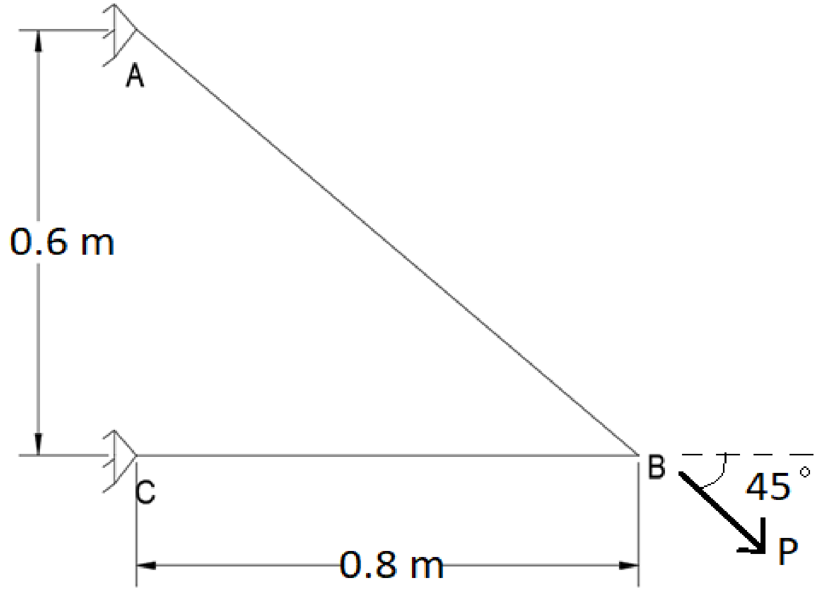
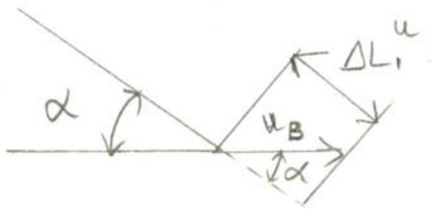


Figure 3b: Two-Bar truss

Solution 2b

Static equilibrium is established by $\delta w_e = \delta w_{ie}$

A and C are fixed, so apply virtual displacements at point B



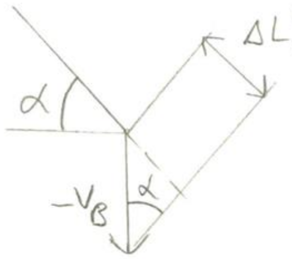
From geometry, we can say L1 = 1m

We know L2 = 0.8m

$P cos(45°) = 3E3\ N$

$P_V = P sin(45°) = 3E3\ N$

$cos(\propto) = \frac{4}{5},\ sin(\propto) = \frac{3}{5}$



Step 1: We first compute the elongations of the bar due to unknown displacements at joint B in horizontal direction

Bar 1 is elongated due to a displacement u_B by:

$$\Delta L_1^u = cos(\propto) u_B = \frac{4}{5} u_B$$

Bar 2 is elongated due to a displacement u_B by:

$$\Delta L_2^u = u_B$$

Now, Bar 1 is elongated due to a vertical displacement v_B by:

$$\Delta L_1 = -sin(\propto) v_B = -\frac{3}{5} v_B$$

Bar 2 elongation due to vertical displacement v_B

$$\Delta L_2^v = 0$$

Using superposition principle:

$$\Delta L_1 = \Delta L_1^u + \Delta L_1^v = \frac{4}{5} u_B - \frac{3}{5} v_B$$

$$\Delta L_2 = \Delta L_2^u + \Delta L_2^v = u_B$$

Using $\delta w_{ie,bar} = \frac{EA}{L} (\Delta L) \delta(\Delta L)$

Here $\delta w_{ie}^u = \frac{EA_1}{L} (\Delta L_1) \delta(\Delta L_1^u) + \frac{EA_2}{L} (\Delta L_2) \delta(\Delta L_2^u)$

Here $\delta(\Delta L_1^u) = \frac{4}{5} \delta u_B;\ \ \delta(\Delta L_1^v) = -\frac{3}{5} \delta v_B$

$$\delta(\Delta L_2^u) = \delta u_B;\ \ \delta(\Delta L_1^v) = 0$$

Now, $\delta w_{ie}^u = \frac{EA_1}{L_1} \left(\frac{4}{5} u_B - \frac{3}{5} v_B\right) \frac{4}{5} \delta u_B + \frac{EA_2}{L_2} (u_B) \delta u_B$

Similarly, $\delta w_{ie}^v = \frac{EA_1}{L} \left(\frac{4}{5} u_B - \frac{3}{5} v_B\right) \left(-\frac{3}{5} \delta v_B\right) + 0$

External works, $\delta w_e^u = P_H \delta u_B\ ; \delta w_e^v = -P_v \delta v_B$

Use, $\delta w_{ie}^u = \delta w_e^u\ \&\ \delta w_{ie}^v = \delta w_e^v$

$$\left(\left(\frac{A_1}{L_1}\frac{16}{25} + \frac{A_2}{L_2}\right) u_B + \left(\frac{A_1}{L_1}\frac{-12}{25} + 0\right) v_B\right) \delta u_B = \frac{P_H}{E} \delta u_B$$

$$\left(\left(-\frac{A_1}{L_1}\frac{12}{25}\right) u_B + \left(\frac{A_1}{L_1}\frac{9}{25}\right) v_B\right) \delta v_B = \frac{-P_v}{E} \delta v_B$$

Solution 2b cont.

$$\left[\left(\left(\frac{A_1}{L_1}\frac{16}{25}+\frac{A_2}{L_2}\right)u_B+\left(\frac{A_1}{L_1}\frac{-12}{25}+0\right)v_B\right)\delta u_B=\frac{P_H}{E}\delta u_B\right.$$

$$\left(\left(-\frac{A_1}{L_1}\frac{12}{25}\right)u_B+\left(\frac{A_1}{L_1}\frac{9}{25}\right)v_B\right)\delta v_B=\frac{-P_v}{E}\delta v_B$$

$$\begin{bmatrix}\left(\frac{A_1}{L_1}\frac{16}{25}+\frac{A_2}{L_2}\right) & \left(\frac{A_1}{L_1}\frac{-12}{25}+0\right)\\ \left(-\frac{A_1}{L_1}\frac{12}{25}\right) & \left(\frac{A_1}{L_1}\frac{9}{25}\right)\end{bmatrix}\begin{bmatrix}u_B\\ v_B\end{bmatrix}=\begin{bmatrix}\frac{P_H}{E}\\ \frac{-P_v}{E}\end{bmatrix}$$

Substitute known values and solve for u_B and v_B

$$\boxed{\begin{aligned}u_B&=-0.0011m\\ v_B&=-0.0199m\end{aligned}}$$