University of Colorado - Boulder

ASEN 3112: Structures

ASEN 3112 Lab 1

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Torsional rigidity, a product of the Shear Modulus (G) and the Second Moment of Inertia (J), describes a cross section's resistance to torsional deformation. It is an important property in design and construction as structural integrity is dependent on an accurate model of when a cross section will fail due to torsion. The primary purpose of this lab is to examine the different methods of calculating torsional rigidity and their validity. Exact Theory, Closed Thin Wall Theory, and Open Thin Wall Theory are compared to experimental material testing data retrieved from the in house MTS Torsion Machine. The secondary purpose is to consider the the impact of the use of extensometers on the accuracy and precision of experimental data collection.

Contents

Nomenclature

γ	=	Shear Strain [-]
T	=	Torque [lb]
ϕ	=	Twist Angle [rad]
G	=	Shear Modulus [psi]
J	=	Second Moment of Inertia [in^4]
t	=	Thickness [in]
L	=	Length of the Tube [in]
L_{ext}	=	Length of the Extensometer [in]
R_e	=	Exterior Radius [in]
τ	=	Shear Stress [psi]
R_i	=	Inner Radius [in]
CTW	=	Closed Thin Wall
OTW	=	Open Thin Wall

I. Analysis of the Closed Thin Walled Specimen

The main analysis for this section is for the CTW specimen, both experimental and analytical methods are evaluated to predict the behavior of the specimen under torsion. The experimental section consists of a specimen undergoing a pure torsion test procedure, and experimental data is saved as the test is initiated. On the other hand, given basic material properties, the desired variables, such as torque can be analytically calculated, shear stress, etc. For closed thin wall approximations, maximum shear stress can be analytically approximated, J_{CTW} with Close Thin Wall Theory that consists of the following equations:

$$\tau_{max} = T/(2t_{min}A_e) \tag{1}$$

$$J_{CTW} = \frac{4A_e^2}{\oint_s (ds/t)} \tag{2}$$

$$T = GJ_{CTW}(\frac{d\phi}{dx}) \tag{3}$$

$$\phi_{AB} = \frac{TL}{GJ_{CTW}} \tag{4}$$

The experimental data recorded time, torsional angle, shear strain, torque, normal strain. The experimental data was imported into a matlab script which parsed and manipulated the data to provide the desired plots. Hence, a graph of Torque vs Shear Strain is seen at 1.

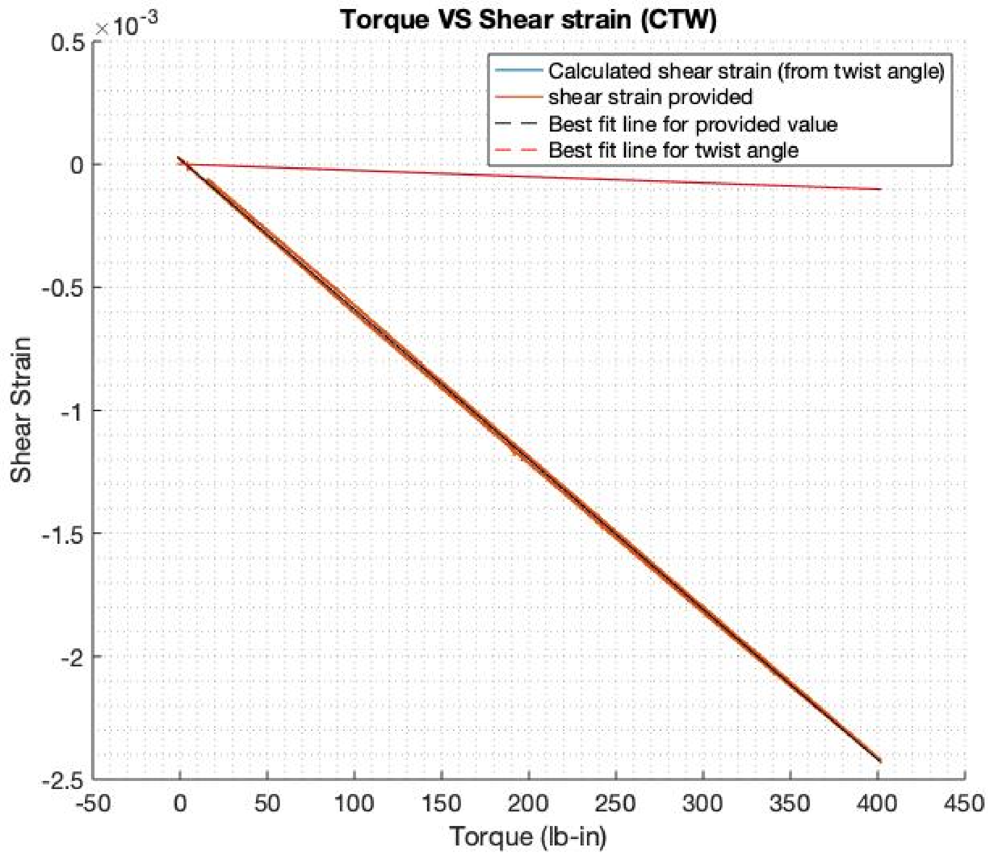

Fig. 1 Torque vs Shear Strain

Furthermore, polyfit matlab built in function was used to find the torsional rigidity GJ for both methods (experimental data provided and calculated based on data). As far for uncertainty, Polyfit, calculates the standard deviation and based on this result, it is quantifiable to calculate the uncertainty of the calculations. The found standard deviation is, $\sigma(x) = 114.6$. Standard error of the mean is used to find uncertainty.

$$SE(\sigma) = \frac{\sigma}{\sqrt{N}} \tag{5}$$

By convention N is the number of data points or samples in the approximation, which was set to be 53301. When solving for standard error, the result found is ± 0.496 to the computed torsional rigidity value.

Using Closed Thin Wall Theory, theoretical analysis can be modeled to predict the specimens shear strain, shear stress, displacement,etc; while undergoing torque, these relations may be calculated by using the equations 1-5 presented above. Hence the Torsional Rigidity was theoretically calculated using exact and CTW theory.

Consequently a comparison table is presented, with the obtained torsional rigidity values. The experimental(provided data) method result is very similar to the value obtained by theoretically approximating GJ with CTW theory. Exact theory seems to be a short overestimate of GJ. The largest offset value is calculating shear strain based on the twist angle which was found to be around 25 times higher, this could be a result of inaccuracy while manipulating data, or inaccuracy of measurements, while the specimens underwent torsion.The other possible scenario is that the calculated values are not in reference with the specimen (not the local shear strain).

Method	Torsional Rigidity (x10^4)$(lb-in^2)$
Experimental(Provided Data)	5.635
Experimental(Calculated)	135.25
Theoretical Exact Theory	6.0311
Theoretical CTW Theory	5.982

II. Analysis of the Open Thin Walled Specimen

The analysis for the OTW specimen is similar to the previous method, but using open thin-walled theory. First, torque is plotted versus shear strain measured by both the torsion machine and an extensometer. The shear strain for the machine data is calculated using the following.

$$\gamma = \phi \frac{t}{L} \tag{6}$$

The plots of torque vs shear strain for both data sets are shown below.

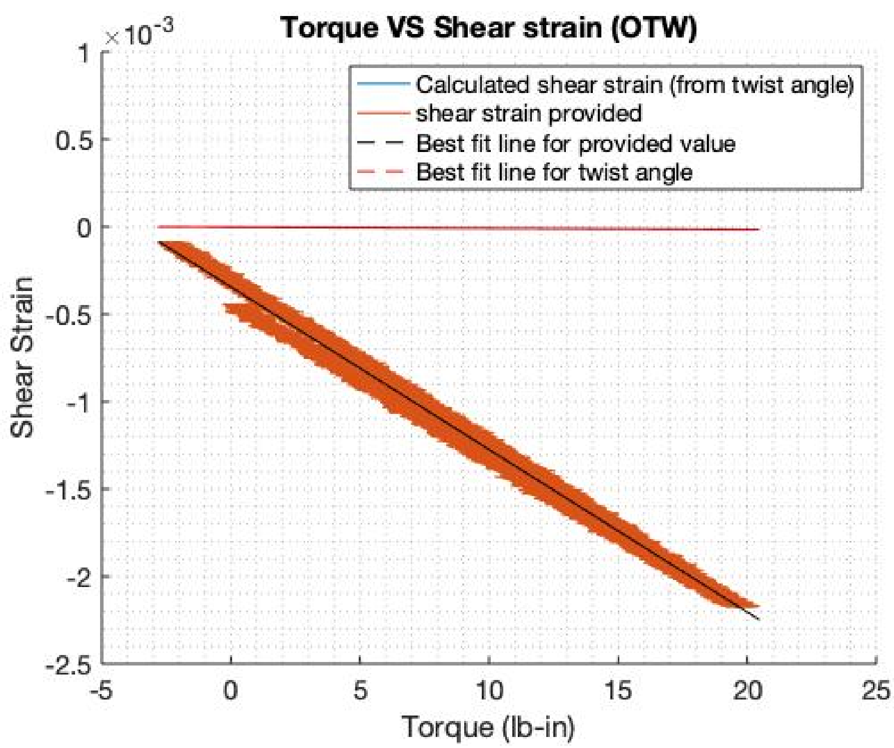

Fig. 2 Torque vs Shear Strain

Next, the torsional rigidity (GJ) is found by using a first order least squares approximation. The equation used to calculate the experimental torsional rigidity is found by substituting γ for $t\frac{d\phi}{dx}$ resulting in the equation below.

$$\gamma = \frac{t}{GJ} T \tag{7}$$

Torsional rigidity is then just thickness divided by the slope of the best fit line of T vs γ. These calculations are done twice using both the strain from the extensometer and from the machine. The polyfit function in Matlab was used for the least squares approximation. Below are the plots of torque vs shear strain for both sets of data, including the lines of best fit.

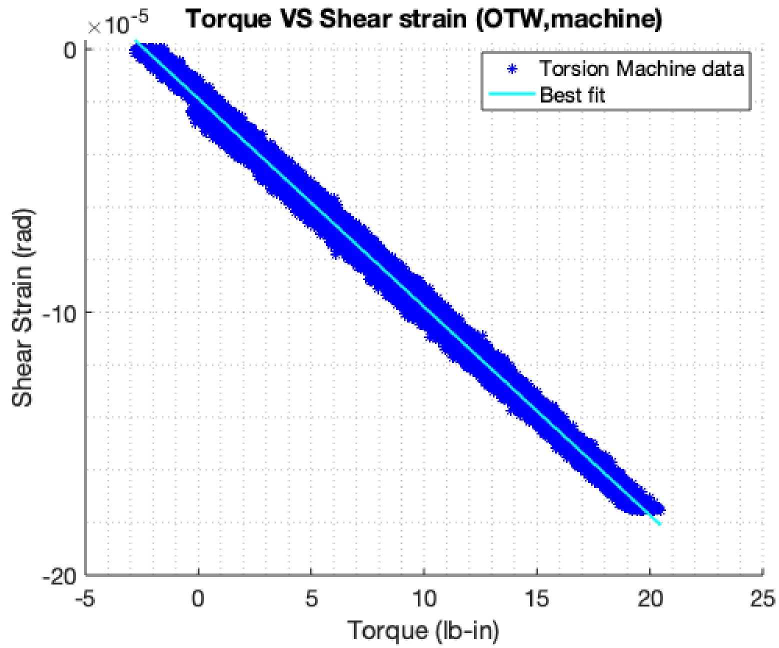

Fig. 3 Machine data

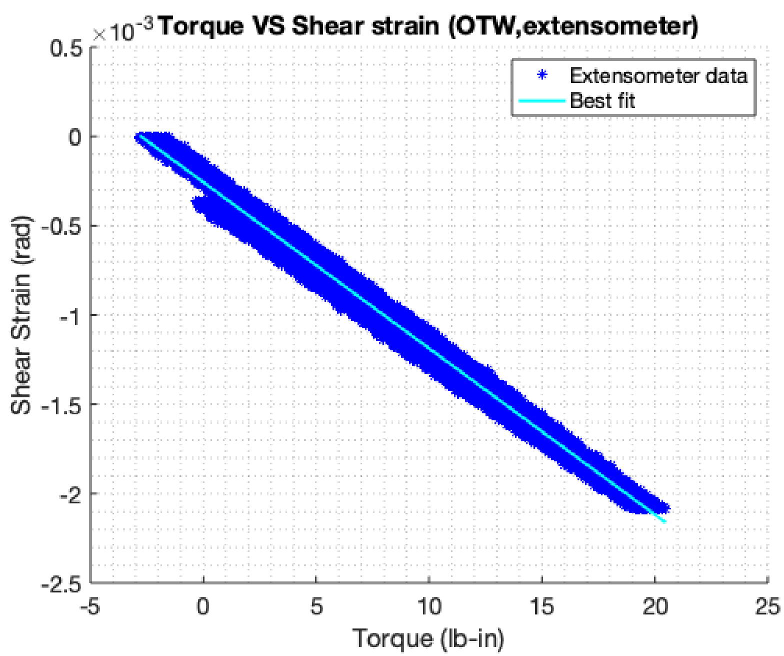

Fig. 4 Extensometer data

The calculated values of GJ for the machine data and the extensometer data were 7892.7 and 672.1 lbin^2. The associated errors were calculated using the following equation.

$$SE(\sigma) = \frac{\sigma}{\sqrt{N}} \tag{8}$$

Where std is the standard deviation of the variable associated with the Y vector used in the polyfit function. In this case, torque was assigned as X and shear strain as Y. Details and code can be seen in the code for section 2.2 attached in the appendix. The resulting errors for the machine and extensometer data were ± 339.5 and 28.2 lbin^2 respectively. The theoretical value of GJ was computed using the following equation.

$$GJ = G\alpha Lt^3 \tag{9}$$

All of these variables were given and yielded an OTW theoretical torsional rigidity of 2746.6 lbin^2. Comparing the three calculated values, it is clear to see the discrepancies between them. The theoretical GJ calculated from the exact theory was 60311 lbin^2.

Method	Torsional Rigidity (lbin^2)
Experimental(Provided Data)	672
Experimental(Calculated)	7893
Theoretical Exact Theory	60311
Theoretical OTW Theory	2747

All GJ values obtained using OTW were at least an order of magnitude under the value associated with the Exact theory. The assumption that the slit has negligible thickness compared to the radius of the specimen must not be a good one when calculating torsional rigidity. Therefore it is safe to assume that the most accurate value of GJ must be the theoretical OTW value.

III. Importance of Extensometer

For both specimens, the extensometer measured strain much more accurately than the calculated data from the torsion machine. For the CTW specimen, this proved to be invaluable as the GJ value calculated from the machine data was way off of the value from either theoretical values. On the other hand, the calculated GJ was much closer to the theoretical for the OTW sample. However, the data from the extensometer was still closer to the theoretical OTW and the associated error when using the least squares approximation was much less. The results clearly show that the extensometer allows for a much more accurate measurement of strain.

The MTS Torsion Module is configured to measure shear strain, total rotation, and torque, however an external extensometer was used to generate more accurate measurements. The decision to use this device was motivated for two reasons. First, the machine uses an angular displacement transducer located in its bottom grip to measure twist angle which leads to complications when shear strain is calculated. This is because this twist angle is measured along a length relative to the top of the upper grip rather than the length of the specimen. Therefore the twist angle measured by the machine is associated with material properties of the grip and machine assembly rather than the pure twist angle of the aluminum specimen.

While an extensometer is a fantastic piece of hardware and is ideal when testing multiple specimens, there are relevant factors to consider when choosing a sensor. While the extensometer used in this lab measures both shear and axial strains, more than one type of extensometer is needed to measure other strains. A singular extensometer can cost thousands of dollars which means the price to completely analyze materials with these devices is certainly a consideration. One cost efficient alternative that engineers might choose is the strain gauge, a small sensor that converts forces and moments into measurable changes in electrical resistance, yielding stress and strain values. With a price tag of roughly five dollars, a strain gauge effectively functions the same as an extensometer without the concern of breaking one. In this lab, two strain gauges could have been used in place of the extensometer to measure shear stress. By placing two strain gauges perpendicular to each other at a 45 degree angle to the longitudinal axis of the specimen, the shear stress and strains could be measured. In fact, strain gauges have a long history of application in aerospace engineering. Specifically, "Strain gauges are bonded directly to structural load bearing components to measure stresses along load paths for wind deflection." Clearly, strain gauges have advantages, however there are some important limitations to consider. Physically applying strain gauges to specimens is a taxing process which involves using special glues that take hours to dry completely, and is a process that must be repeated for multiple measurements on a variety of specimens. The Temperature Coefficent of Gauge Factor (TCGF) measures the change in strain gauge sensitivity over a temperature gradient, a parameter which must be considered when dealing with sizeable temperature changes of specimens since strain gauges make direct contact with the material. Therefore, in the interest of saving time and maximizing productivity, many engineers will rely on extensometers for material and specimen testing.

IV. Plastic Deformation

We know that shear strain is given by the following equation;

$$\gamma = G\tau, \tag{10}$$

where G is the modulus of rigidity and τ is the shear stress. In order to find an equation that relates the shear strain and the geometry of the sample, the open thin wall approximation must be used,

$$\gamma = \frac{GT}{2t_{min}A_e} \tag{11}$$

in this equation T represents the applied torque, t_{min} is the minimum thickness, and A_e is the area enclosed by the mid-line concentric with the tube. Finally, the various geometric parameters of the tube can be plugged in to obtain the desired relationship

$$\gamma = \frac{GT}{2\pi(R_e - Ri)(R_e - \frac{R_e - R_i}{2})^2} \tag{12}$$

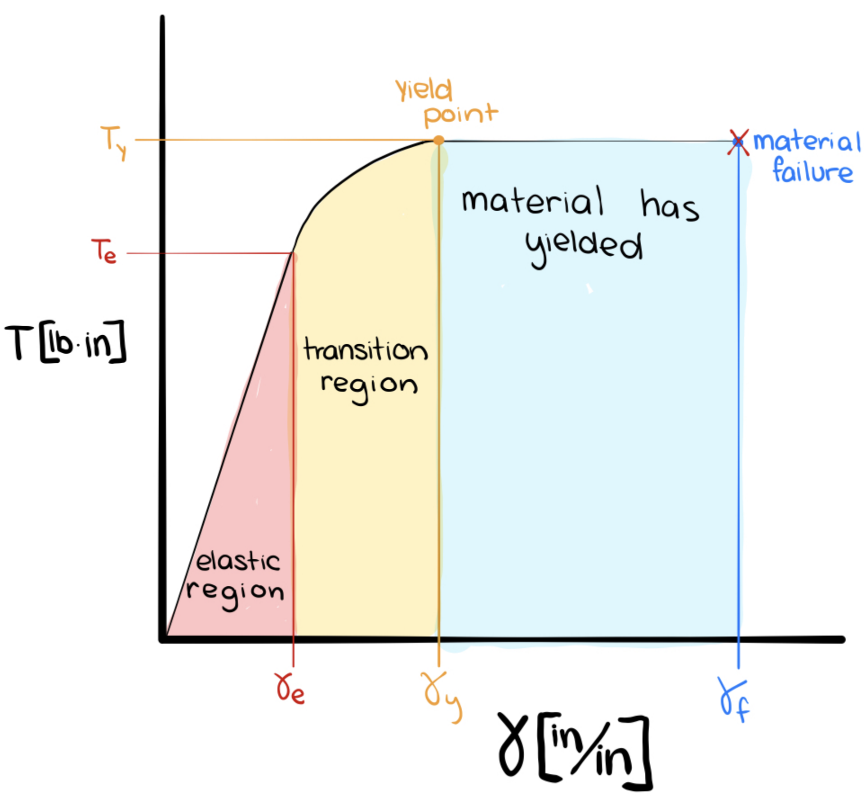

Fig. 5 Expected response of the material with respect to torque.

The elastic region T<T_e, highlighted in red in Figure 5, is the linear region in which the material can be loaded and unloaded while still returning to it's initial dimensions. This is the region this lab focused on because Young's Modulus (E) could be used for calculations. The transitional region T_e<T<T_y is where the material experiences pockets of plastic deformation. After the yielding point, the material has fully yielded and is completely plasticized. While some T-γ graphs may show a slight increase in applied torque between γ_y and γ_f, this graph has been simplified to make the maximum applied torque equal to the torque where yielding occurs.

V. Conclusion

When considering which machines, sensors, and setups are needed for material testing, it is important to understand the options, as discussed in the lab introduction video, Professor Shwartz explains that while measurements for torsional strain could be obtained by placing strain gauges at a 45° angle on the specimen, the extensometer is both cheaper and less labor intensive for this experiment. In addition, the extensometer was able to measure torsional strain much more accurately than the calculated data from the MTS in both the closed and open thin walled specimens. For the CTW analysis, the standard deviation of the experimental data was found to be ±0.496. The Torsional rigidity calculated from experimental data ($5.635*10^4$ lb-in^2) was most similar to the theoretical CTW theory approximation ($5.982*10^4$ lb-in^2). For the OTW analysis, the standard deviation of the experimental data from the machine and extensometer were found to be ±339.5[lb-in^2] and ±28.2[lb-in^2] respectively. The Torsional rigidity calculated from experimental data (672 lb-in^2) was most similar to the theoretical CTW theory approximation (2747 lb-in^2). These values were not nearly as close as the CTW results, implying that the assumptions in this lab were not very accurate for this specimen

Acknowledgements

Special thanks to the following for their help during the challenging portions of this lab.
Dr. Johnson

VI. Appendix

A. Participation Report

Team Member	Experiment	2.1	2.2	2.3	2.4	Report	Total Score
Matt Bridges:	2	1	2	1	1	1	100
Marin Grgas:	2	1	1	1	2	1	100
Fiona McGann:	2	1	1	1	1	2	100
Ventura Morales:	2	2	1	1	1	1	100
Chris Nylund:	2	1	1	2	1	1	100
Panitnan Yuvanondha:	2	2	2	1	1	1	100

Table 1 Participation table is scored as follows: 2 - Lead Section, 1 - Worked On, 0 - Not Responsible For. Total Score is a numerical reflection of team members' contributions out of 100.

B. Code

```
% housekeeping
clc;clear;close all

%% Loading Data and Organizing data

ctwdata= importfile('CTW.txt');
otwdata= importfile('OTW.txt');

% Organzie the data
% Closed thin walled
gammactw= deg2rad(ctwdata(:,2)); % shear strain
phictw= ctwdata(:,3)*pi/180; % total twist angle in rad
Tctw= ctwdata(:,4); % applied torque in-lb

% Open thin walled
gammaotw= otwdata(:,2)*pi/180; % shear strain
phiotw= otwdata(:,3)*pi/180; % total twist angle in rad
Totw= otwdata(:,4); % applied torque in-lb


%% Constant Variables
G=3.75*10^6; % psi
Re= 3/8; % in. for exterior radius
t= 1/16; % in. for thickness
Ri= Re-t; %in. for inner radius
L= 9; % in. length of the tube
Le= 1; % in. length of extensometer
alpha= 1/3;

%% Section 2.1 : CTW

% Section 2.1.1
% Calculating strain by using total rotation
strain = (phictw(:,1)*Re)/L;

% Plot
figure(1)
hold on
xlabel('Torque (lb-in)')
ylabel('Shear Strain')
title('Torque VS Shear strain (CTW)')
plot(Tctw,strain) % by total rotation (Calculation)
grid minor
plot(Tctw,phictw) % by extensomter (provided)


% Section 2.1.2 polyfit (?????

lsgamma= polyfit(Tctw,phictw,1); % least square fitting for shear strain provided
```

```
lsstrain = polyfit(Tctw,strain,1); % least sqaure fitting for shear strain calculated

hold on
plot(Tctw, lsgamma(1)*Tctw+lsgamma(2),'g--')
plot(Tctw, lsstrain(1)*Tctw+ lsstrain(2),'c--')
legend('Calculated shear strain (from twist angle)','shear strain provided'...
    ,'Best fit line for provided value','Best fit line for twist angle')


%Section 2.1.3
% Calculate J CTW (theoretical)
R=(Re+Ri)/2; % midline
Ae= pi*R^2; % Enclosed Area
Jctw= 4*Ae^2/(2*pi*R/t);

% Calculate J exact theory (theoretical)
Jex= pi/2*(Re^4-Ri^4);

% Calculate torsion regidity (theoretical)
TRctw= G*Jctw; % closed thoery
TRex= G* Jex; % exact theory

% Calculate torsion regidity (experimental)
TRctwexp= R/lsgamma(1); % from shear strain provided

% Calculate torsion regidity (experimental)
TRctwexp1= R/lsstrain(1); % from shear strain calculated


%% Section 2.2: OTW

% Section 2.2.1
% Calculating strain by using total rotation
strainotw = (phiotw(:,1)*t)/L;

% Plot
figure(2)
hold on
xlabel('Torque (lb-in)')
ylabel('Shear Strain')
title('Torque VS Shear strain (OTW)')
plot(Totw,strainotw)
grid minor
plot(Totw,phiotw)
ylim([-2.5*10^-3,1*10^-3])


% Section 2.2.2
```

```
lsgammaotw= polyfit(Totw,phiotw,1); % least square fitting for shear strain provided
lsstrainotw = polyfit(Totw,strainotw,1); % least sqaure fitting for shear strain
calculated

hold on
plot(Totw, lsgammaotw(1)*Totw+lsgammaotw(2),'g--')
plot(Totw, lsstrainotw(1)*Totw+ lsstrainotw(2),'c--')
legend('Calculated shear strain (from twist angle)','shear strain provided'...
    ,'Best fit line for provided value','Best fit line for twist angle')


%Section 2.1.3
% Calculate J OTW (theoretical)
Jotw= alpha*2*pi*R*t^3;

% Calculate torsion regidity (theoretical)
TRotw= G*Jotw;

% Calculate torsion regidity (experimental)
TRotwexp= R/lsgammaotw(1); % from shear strain provided
TRotwexp1= R/lsstrainotw(1); % from shear strain calculated
```

```
%% ASEN 3112 Lab 1
% Section 2.2
% Matthew Bridges (105070470)

% Housekeeping
clear;
clc;
close all;

%% Constant Variables
G=3.75*10^6; % psi
Re= 3/8; % in. for exterior radius
t= 1/16; % in. for thickness
Ri= Re-t; % in. for inner radius
Rav = (Ri+Re)/2; % in. for average radius
L= 9; % in. length of the tube
Le= 1; % in. length of extensometer
alpha= 1/3;

%% Loading Data and Organizing data

ctwdata= importfile('CTW.txt');
otwdata= importfile('OTW.txt');

time_otw = otwdata(:,1);
time_ctw = ctwdata(:,1);

% Organzie the data
% Closed thin walled
gammactw= deg2rad(ctwdata(:,2)); % shear strain
phictw= ctwdata(:,3)*pi/180; % total twist angle in rad
Tctw= ctwdata(:,4); % applied torque in-lb

% Open thin walled
% machine twist angle reset for zero at first data point (rad)
phi_otw= (otwdata(:,2)-otwdata(1,2))*pi/180;
% extensometer shear stress (rad)
strain_ext_otw= (otwdata(:,3)-otwdata(1,3))*pi/180;
Totw= otwdata(:,4); % applied torque in-lb
dx = otwdata(:,5);


%% Section 2.2: OTW

% Section 2.2.1
% Calculating twist angle using shear strain
strain_otw = phi_otw*(t/L);
phi_ext_otw = strain_ext_otw*L*(L/t);
```

```
% Plotting torque and shear strain for machine and extensometer data
figure(1)
hold on
xlabel('Torque (lb-in)')
ylabel('Shear Strain (rad)')
title('Torque VS Shear strain (OTW)')
plot(Totw,strain_otw)
grid minor
plot(Totw,strain_ext_otw)
legend('Calculated','Extensometer')

%% Finding torsional rigidty
% Preparing variables for least fit approx
strain_otw = strain_otw(2:end);
Totw = Totw(2:end);
strain_ext_otw = strain_ext_otw(2:end);
% Least squares approxs
p_otw = polyfit(Totw,strain_otw,1);
p_ext_otw = polyfit(Totw,strain_ext_otw,1);

% Plotting Strain and Torque with best fit line
figure(2)
hold on
scatter(Totw,strain_otw,'*b')
plot(Totw,(p_otw(1)*Totw+p_otw(2)),'k')
grid minor
ylabel('Shear Strain (rad)')
xlabel('Torque (lb-in)')
title('Torque VS Shear strain (OTW,machine)')
legend('Torsion Machine data','Best fit')

% Error analysis wtih least squares equations from Taylor
GJ_otw = -t/p_otw(1) % Torsional rigidity is the slope of T/gamma
sigmaY_otw = std(Totw); % standard deviation of T
sigma_GJotw = sigmaY_otw/sqrt(sum(strain_otw.^2)) % error in GJ

% Repeatting for extensometer data
figure(3)
hold on
scatter(Totw,strain_ext_otw,'*b')
plot(Totw,(p_ext_otw(1)*Totw+p_ext_otw(2)),'k')
grid minor
ylabel('Shear Strain (rad)')
xlabel('Torque (lb-in)')
title('Torque VS Shear strain (OTW,extensometer)')
legend('Extensometer data','Best fit')

GJ_ext_otw = -t/p_ext_otw(1)
sigmaY_ext_otw = std(Totw);
```

```
sigma_GJotw_ext = sigmaY_ext_otw/sqrt(sum(strain_ext_otw.^2))

GJ_theo_otw = G*alpha*L*(t^3)
JHollow = ((Re^4)-(Ri^4))*pi/2;
GJ_theo_exact = G*JHollow
```

```
function Data = importfile(filename, dataLines)
%% Input handling

% If dataLines is not specified, define defaults
if nargin < 2
    dataLines = [3, Inf];
end

%% Setup the Import Options and import the data
opts = delimitedTextImportOptions("NumVariables", 5);

% Specify range and delimiter
opts.DataLines = dataLines;
opts.Delimiter = "\t";

% Specify column names and types
opts.VariableNames = ["Time", "TorsionalAngle", "TorsionalEpsilon3550Torsion",
"TorsionalTorque", "AxialEpsilon3550Axial"];
opts.VariableTypes = ["double", "double", "double", "double", "double"];

% Specify file level properties
opts.ExtraColumnsRule = "ignore";
opts.EmptyLineRule = "read";

% Import the data
Data = readtable(filename, opts);

%% Convert to output type
Data = table2array(Data);
end
```

References

[1] Johnson, Aaron "ASEN 3112 Lab 1" published on Jan. 20, 2020, pp.1-8.