ASEN 3112

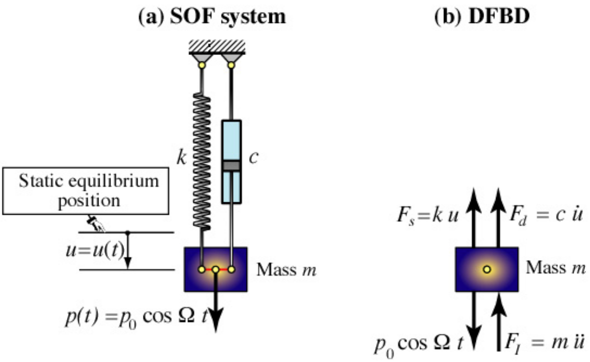
Spring 2020

Lecture 18

March 19, 2020

Harmonically Forced SDOF Oscillator

Harmonically Force Driven SDOF Oscillator



Equation of Motion

From the Dynamic Free Body Diagram (DFBD) of previous slide, we get the EOM:

$$\boxed{m\,\ddot{u} + c\,\dot{u} + k\,u = p_0 \cos \Omega t}$$

Damping, modeled by the $c\,u$ term, is include from the start since it is important in finding the maximum amplication at resonance.

The EOM is **linear and second order ODE**, as in the previous Lecture, but now this ODE is **non-homogeneous**. According to the theory of such equations, the solution $u(t)$, which is called the **displacement response** is the sum of **two components**

Response Decomposition

As remarked in the last slide, the **total response** u(t) can be expressed as the sum of two components: called **homogeneous** and **particular** in applied math textbooks:

$$\boxed{u(t) = u_{\mathrm{H}}(t) + u_{\mathrm{P}}(t)}$$

Engineers use a terminology with closer connection to physics:

homogeneous solution => **transient response**
particular solution => **steady-state response**

Sourse and Significance of Transient Vs. Steady Response

The transient (=homogeneous) response is the solution under zero force. It is primarily determined by initial conditions

The steady-state (=particular) response is produced by the applied force

If there is at least a tiny amount of damping, the transient solution decays at time t grows, and eventually only the steady-state component survives. This explains the "transient" qualifier

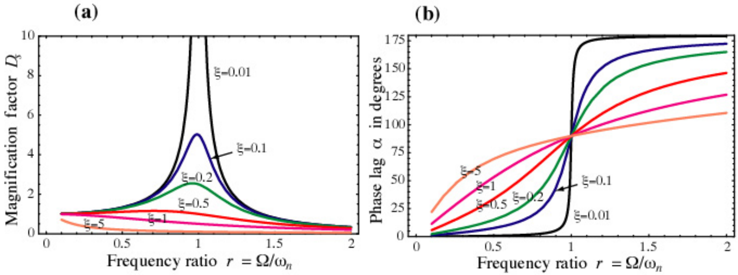
Steady Response Expression

See Lecture Notes

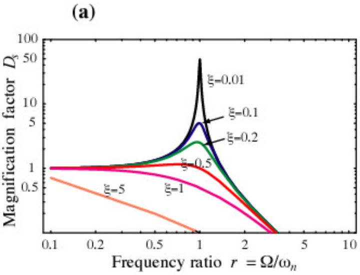
Steady Response Expression

See Lecture Notes

Magnification Factor and Phase Lag as Function of Frequency Ratio



Magnification Factor and Phase Lag as Function of Frequency Ratio - Log-Log Plots



SDOF Oscillator Excited by Harmonic Base Motion