ASEN 3112

Spring 2020

Lecture 10

February 18, 2020

Elastic Strain Energy of Bar

General:

$$U_{bar} = \frac{1}{2}\int_L \sigma_{xx}\ \varepsilon_{xx}\ A\, dx$$

Hooke's law:

$$U_{bar} = \frac{1}{2}\int_L E\ A\, \varepsilon_{xx}^2\ dx \quad \text{or} \quad U_{bar} = \frac{1}{2}\int_L \frac{A}{E}\, \sigma_{xx}^2\ dx$$

$$U_{bar} = \frac{1}{2}\int_L \frac{E\ A\, \Delta L^2}{L^2}\, dx \quad \text{or} \quad U_{bar} = \frac{1}{2}\int_L \frac{N^2}{E\ A}\, dx$$

Constant cross section and homogenous material:

$$U_{bar} = \frac{1}{2}\frac{E\ A\, \Delta L^2}{L} \quad \text{or} \quad U_{bar} = \frac{1}{2}\frac{N^2\ L}{E\ A}$$

Elastic Strain Energy of Euler-Bernoulli Beam

General:

$$U_{beam} = \frac{1}{2}\int_L \sigma_{xx}\ \varepsilon_{xx}\ A\, dx$$

Hooke's law:

$$U_{beam} = \frac{1}{2}\int_L E\ A\, \varepsilon_{xx}^2\ dx \quad \text{or} \quad U_{beam} = \frac{1}{2}\int_L \frac{A}{E}\, \sigma_{xx}^2\ dx$$

$$U_{beam} = \frac{1}{2}\int_L E\ I_{zz}\ \kappa^2\ dx \quad \text{or} \quad U_{beam} = \frac{1}{2}\int_L \frac{M^2}{E\ I}\, dx$$

Elastic Strain Energy of Shaft

General:

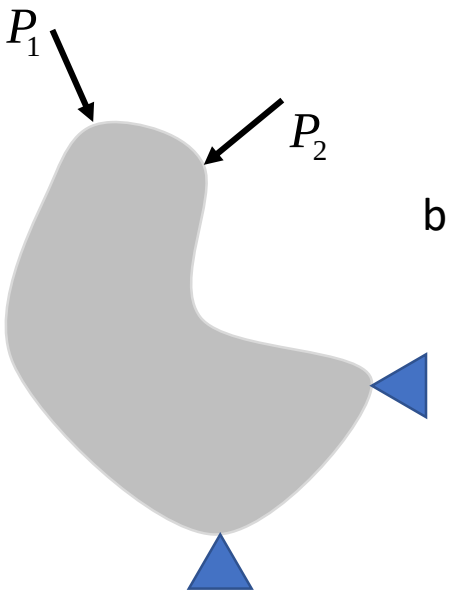
$$U_{shaft} = \frac{1}{2} \int\int\int_V \tau_{x\theta} \, \gamma_{x\theta} \, dx \, dy \, dz$$

Hooke's law:

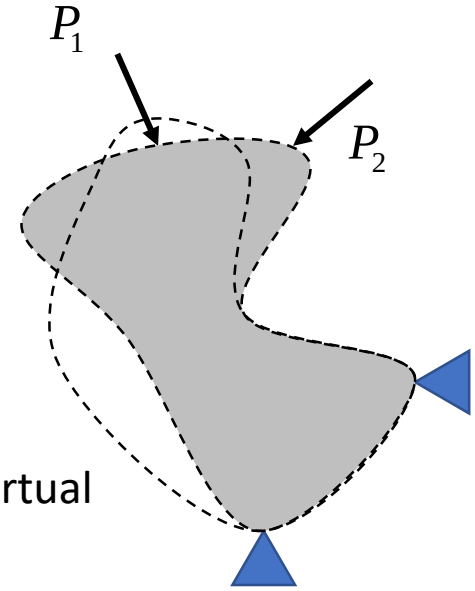
$$U_{shaft} = \frac{1}{2} \int_L G \, J \left(\frac{d\phi}{dx} \right)^2 dx$$

$$U_{shaft} = \frac{1}{2} \int_L \frac{T^2}{G \, J} dx$$

Virtual Work Methods



body in equilibrium



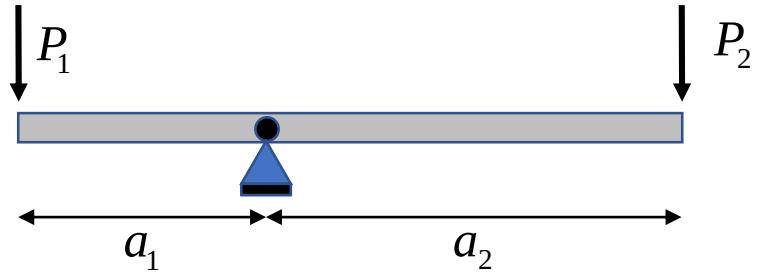
body due to virtual forces or virtual prescribed displacement

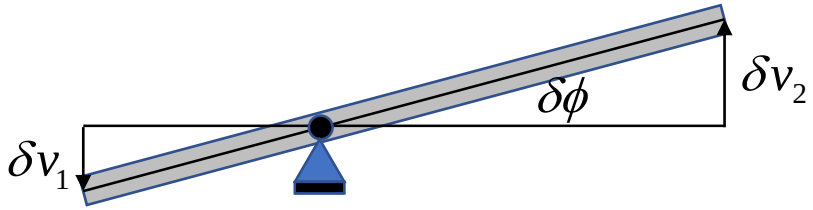
External virtual work + Internal virtual work = 0

$$\delta W = \delta W_e + \delta W_i = 0$$

Virtual Displacement Method

Rigid lever





$$\delta v_1 = a_1\ \delta\phi \qquad \delta v_2 = a_2\ \delta\phi$$

The external virtual work done by **real forces** on **virtual displacements**:

$$\delta W_e = \delta v_1\ P_1 + (-\delta v_2)\ P_2$$

In the absence of internal virtual work (lever is rigid):

$$\delta W_e = \left(a_1\ P_1 - a_2\ P_2\right)\delta\phi = 0$$