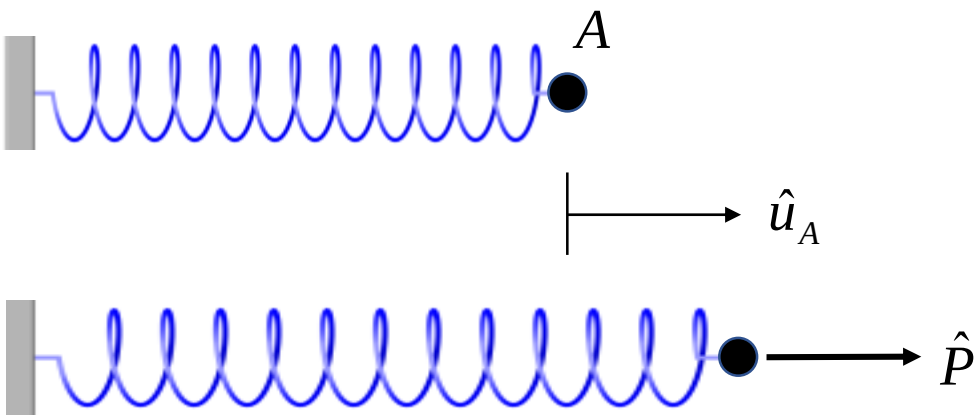
ASEN 3112

Spring 2020

Lecture 9

February 13, 2020

External Work (1)



The displacement at point A increases with the load:

$$k\,u_A = P \quad \text{or} \quad k\,du_A = dP$$

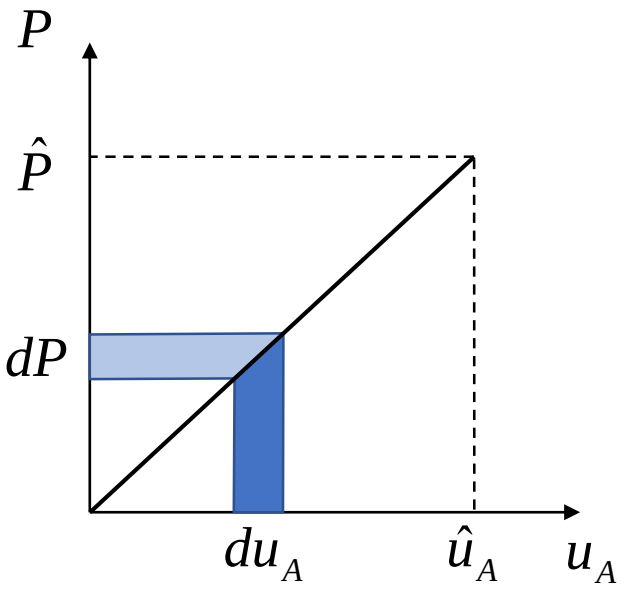
The external work done for by load increment *dP* on displacement u_A

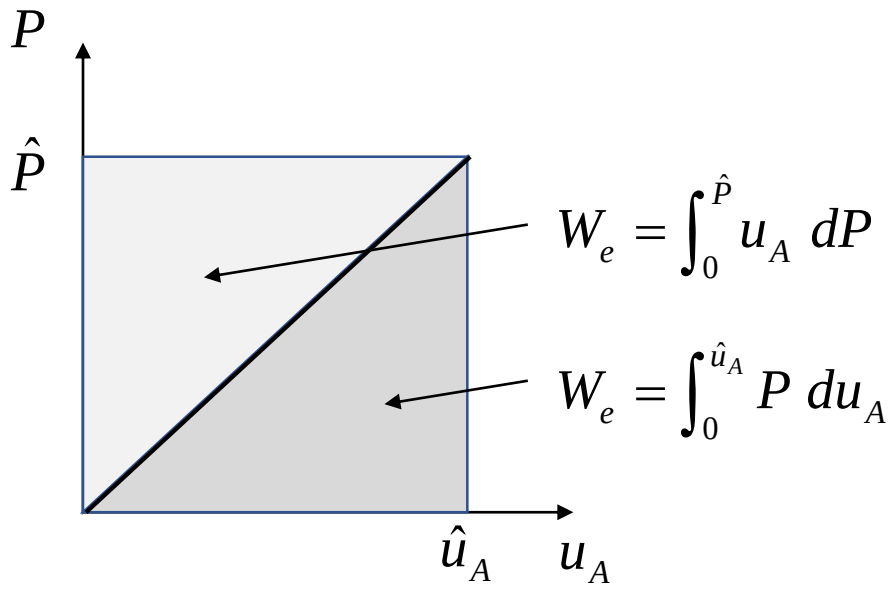
$$\int dW_e = \int_0^{\hat{P}} u_A\,dP \quad \rightarrow \quad W_e = \int_0^{\hat{P}} \frac{1}{k}\,P\,dP = \frac{1}{2}\,\frac{1}{k}\,\hat{P}^2$$

The external work done for by displacement increment du_A on load P

$$\int dW_e = \int_0^{\hat{u}_A} P\,du_A \quad \rightarrow \quad W_e = \int_0^{\hat{u}_A} k\,u_A\,du_A = \frac{1}{2}\,k\,\hat{u}_A^2$$

External Work (2)





The external work done on the spring by the end of deformation process:

$$W_e = \frac{1}{2} \hat{u}_A \, \hat{P}$$

Internal Work (1)

The work done on a body equals the change in energy stored in the body :

$$W_e = U \qquad U: \text{ elastic strain energy}$$

Example: elastic strain energy in spring $U_{spring} = \frac{1}{2} k \hat{u}_A^2$

Conservation of energy: The sum of external and internal work is zero:

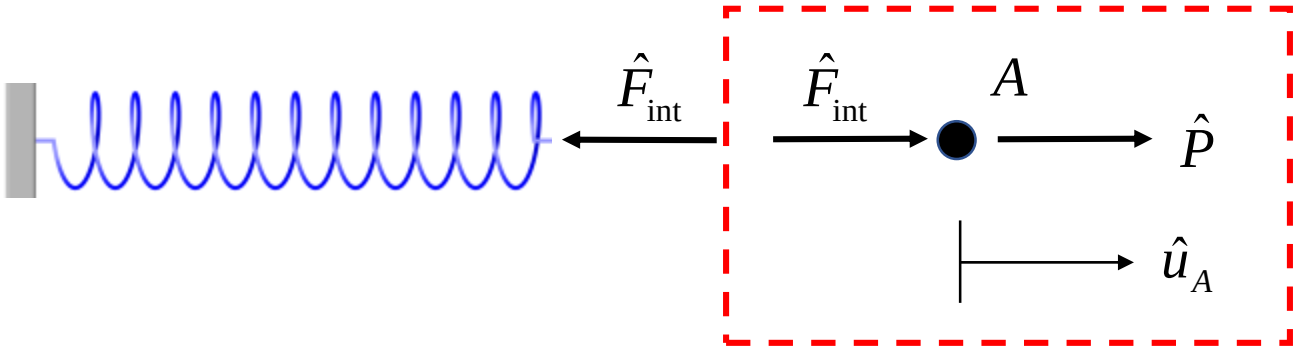
$$W_e + W_i = 0 \qquad W_i: \text{ internal work}$$

$$W_i = -U$$

Internal Work (2)

External work: done by forces, e.g. $\hat{P}$

Internal work: done by internal forces, e.g. $\hat{F}_{int}$



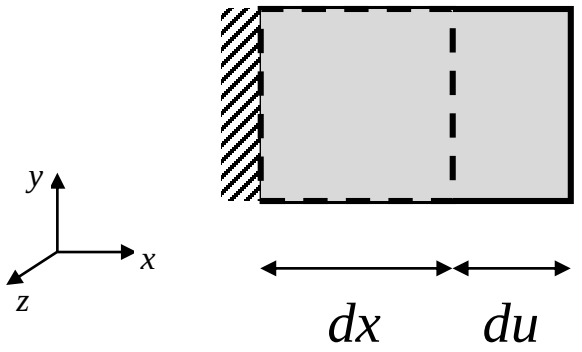
Example: internal work of in spring $W_{\text{int}} = \frac{1}{2}\ \hat{F}_{\text{int}}\ \hat{u}_A$

$$W_e + W_i = \frac{1}{2}\hat{P}\,\hat{u}_A + \frac{1}{2}F_{int}\,\hat{u}_A = 0 \quad \rightarrow \quad F_{int} = -\hat{P}$$

$$W_e - U = \frac{1}{2}\hat{P}\,\hat{u}_A - \frac{1}{2}k\left(\hat{u}_A\right)^2 = 0 \quad \rightarrow \quad \hat{u}_A = \frac{1}{k}\hat{P}$$

Elastic Strain Energy

1D Example



$$dF = \sigma_{xx}\, dy\, dz \qquad dW = \frac{1}{2} dF\, du$$

$$du = \varepsilon_{xx}\, dx \qquad dU = \frac{1}{2}\sigma_{xx}\, \varepsilon_{xx}\, dx\, dy\, dz$$

Elastic Strain Energy Density U_0

$$U_0 = \frac{1}{2}\left(\sigma_{xx}\, \varepsilon_{xx} + \sigma_{yy}\, \varepsilon_{yy} + \sigma_{zz}\, \varepsilon_{zz} + \tau_{xy}\, \gamma_{xy} + \tau_{xy}\, \gamma_{xy} + \tau_{xy}\, \gamma_{xy}\right)$$

Elastic Strain Energy: $$U = \int\int\int_V U_0\, dx\, dy\, dz$$