ASEN3112

Recitation 1

Problem 1: (About 10 minutes)

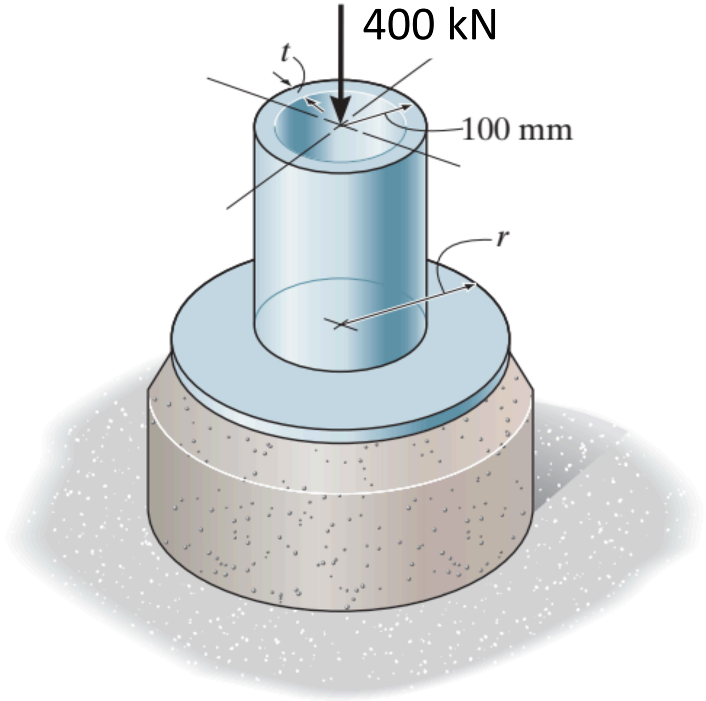


Figure 1: Figure to be analyzed for problem 1.

The steel pipe is supported on the circular base plate and concrete pedestal. If the thickness of the pipe is $t = 4$ mm and the base plate has a radius of 125 mm, determine the factors of safety against failure of the steel and concrete. The applied force is 400 kN, and the normal failure stresses for steel and concrete are $(\sigma_{\text{fail}})_{\text{st}} = 350$ MPa and $(\sigma_{\text{fail}})_{\text{con}} = 25$ MPa, respectively.

Problem 1 Solution

Step 1: Find effective areas

$$Area_{steel} = A_{st} = \pi(0.1^2 - 0.096^2) = 7.84(10^{-4})\pi$$
$$Area_{concrete} = A_{con} = \pi(0.125^2) = 0.015625(\pi)$$

Step 2: Find normal stresses

$$\sigma_{st} = \frac{P}{A_{st}} = \frac{400(10^3)}{7.84(10^{-4})(\pi)} = 162.4\ MPa$$

$$\sigma_{con} = \frac{P}{A_{con}} = \frac{400(10^3)}{0.015625(\pi)} = 8.1487\ MPa$$

Step 3: Calculate safety factors

$$SF_{st} = \frac{(\sigma_{fail})steel}{\sigma_{st}} = \frac{350}{162.4} = 2.155$$

$$SF_{con} = \frac{(\sigma_{fail})con}{\sigma_{con}} = \frac{25}{8.1487} = 3.068$$

Problem 2a: (About 10 minutes)

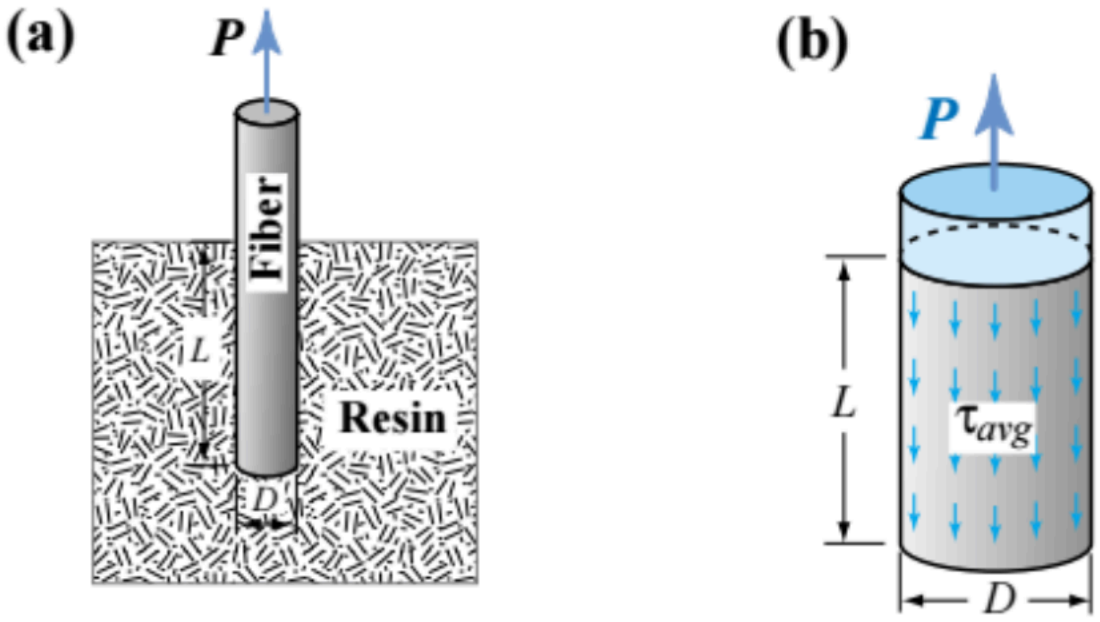


Figure 2: Figure to be analyzed for problem 2a.

A fiber pull-out test is to be conducted to determine the shear strength of the interface between the fiber and the resin matrix in a composite material. See Figure 1(a). Assuming that a uniform shear stress   acts over the fiber-resin interface, derive a formula for the shear stress in terms of the applied force P, the fiber length L, and the fiber diameter D. Use the FBD sketched in Figure 1(b).

Problem 2a Solution

The shear area over which averaging takes place is $(\pi D) * L$

Force equilibrium along the fiber gives P = $\tau_{avg}(\pi DL)$

Resultant shear force is $\tau_{avg} = \frac{P}{\pi DL}$

Problem 2b: (About 10 minutes)

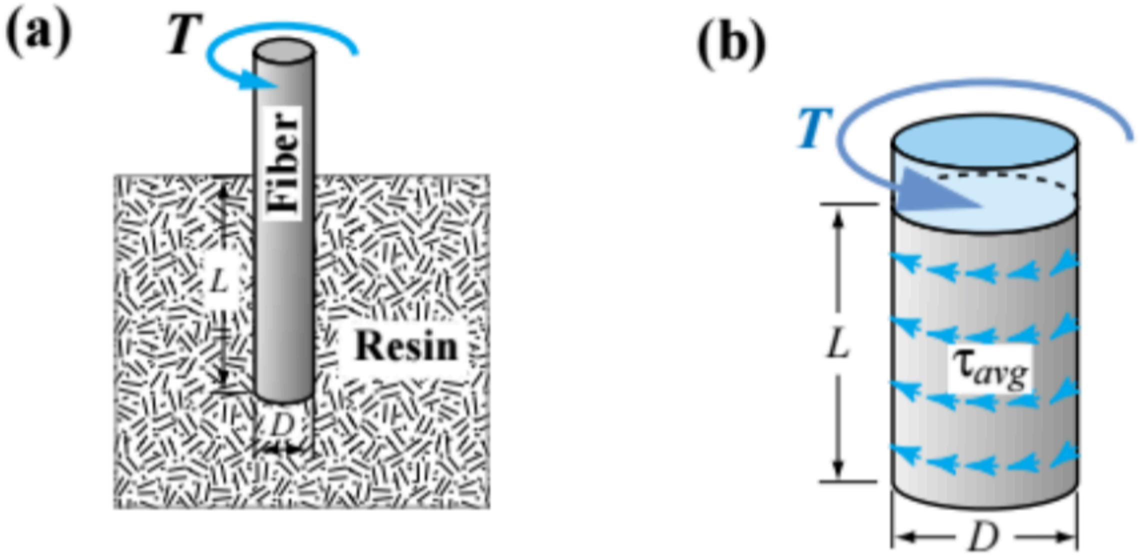


Figure 3: Figure to be analyzed for problem 2b.

A second interface shear strength test applies a torque T to the fiber as shown in Figure 2(a). Assuming a uniform shear stress at the interface, derive a formula for the shear stress in terms of the torque T, the fiber length L, and the fiber diameter D. Use the FBD sketched in Figure 2(b).

Problem 2b Solution

The radius is $R = \frac{D}{2}$

Torque = $T = \left(\frac{D}{2}\right)(\tau_{avg}(\pi DL))$

Solve for shear stress gives $\tau_{avg} = \frac{2T}{LD^2(\pi)}$

Problem 3a: (About 10-15 minutes)

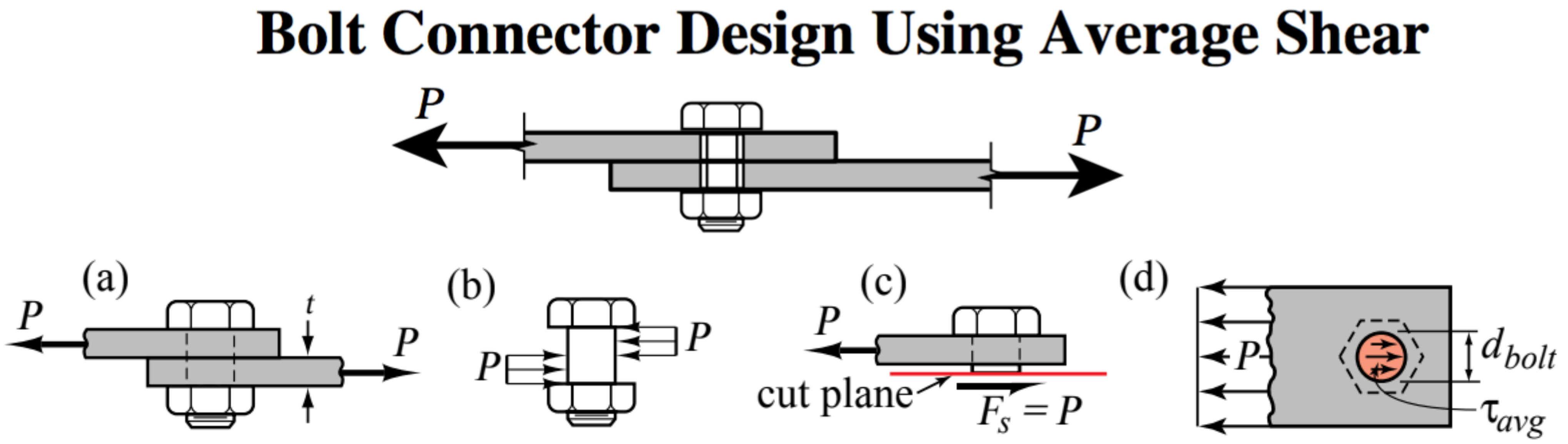


Figure 4: Figure to be analyzed for problem 3a.

A bolt passes through two plates, used to connect two truss members. Given an axial load of 40kN and a safety factor of 4, the bolt fails at a maximum shear of 296MPa. Find the diameter of the bolt using the average shear stress approach.

**Hint: assume the shear stress over the shear area is uniform.

Problem 3a Solution

Step 1: Find effective areas

$$Area_{shear} = A_{bolt} = \pi\left(\frac{d_{bolt}^2}{2}\right)^2 = \frac{\pi d_{bolt}^2}{4}$$

Step 2: Find maximum shear

$$\tau_{avg} = \frac{F}{A} = \frac{P}{\frac{\pi d_{bolt}^2}{4}} = \frac{4P}{\pi d_{bolt}^2}$$

Step 3: The design condition specifies $\tau_{avg} \leq \frac{\tau_{fail}}{SF}$

$$\frac{\tau_{fail}}{SF} = \frac{4P}{\pi d_{bolt}^2}$$

$$d_{bolt}^2 = \frac{4P(SF)}{\pi\tau_{fail}} \quad , \quad d_{bolt} \geq \pm\sqrt{\frac{4P(SF)}{\pi\tau_{fail}}} = +\sqrt{\frac{4(40(10^3))(4)}{\pi(296(10^6))}} = 26.234mm$$

Problem 3b: (About 5-10 minutes)

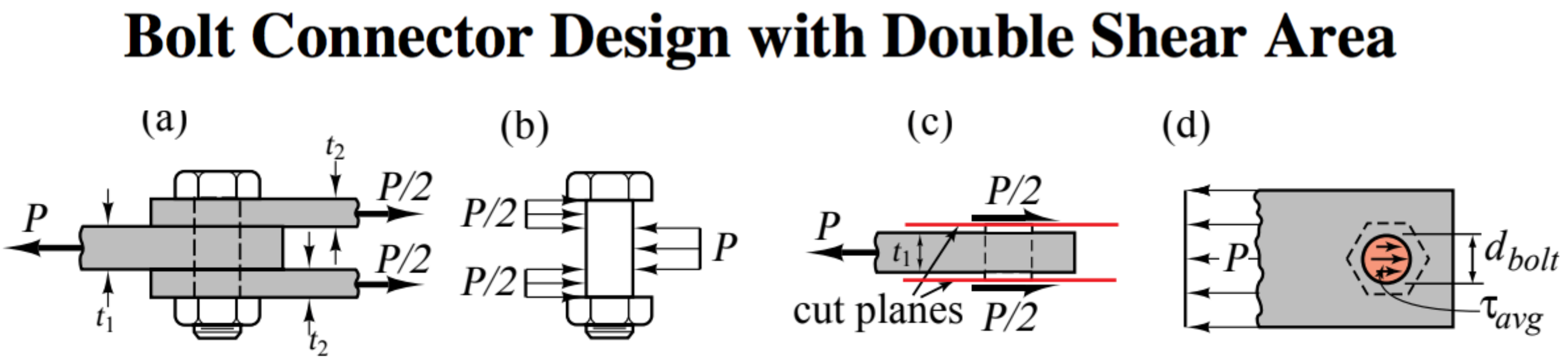


Figure 5: Figure to be analyzed for problem 3b.

A bolt passes through three plates, used to connect three truss members. Given an axial load of 40kN and a safety factor of 4, the bolt fails at a maximum shear of 296MPa. Find the diameter of the bolt using the average shear stress approach.

**Hint: The effective shear area doubles because we make two cut planes for double shear instead of just one.

Problem 3b Solution

Step 1: Find effective areas

$$Area_{shear} = A_{bolt} = 2\,\pi \left(\frac{d_{bolt}^2}{2}\right)^2 = \frac{\pi d_{bolt}^2}{2}$$

Step 2: Find maximum shear

$$\tau_{avg} = \frac{F}{A} = \frac{P}{\frac{\pi d_{bolt}^2}{2}} = \frac{2P}{\pi d_{bolt}^2}$$

Step 3: The design condition specifies $\tau_{avg} \leq \frac{\tau_{fail}}{SF}$

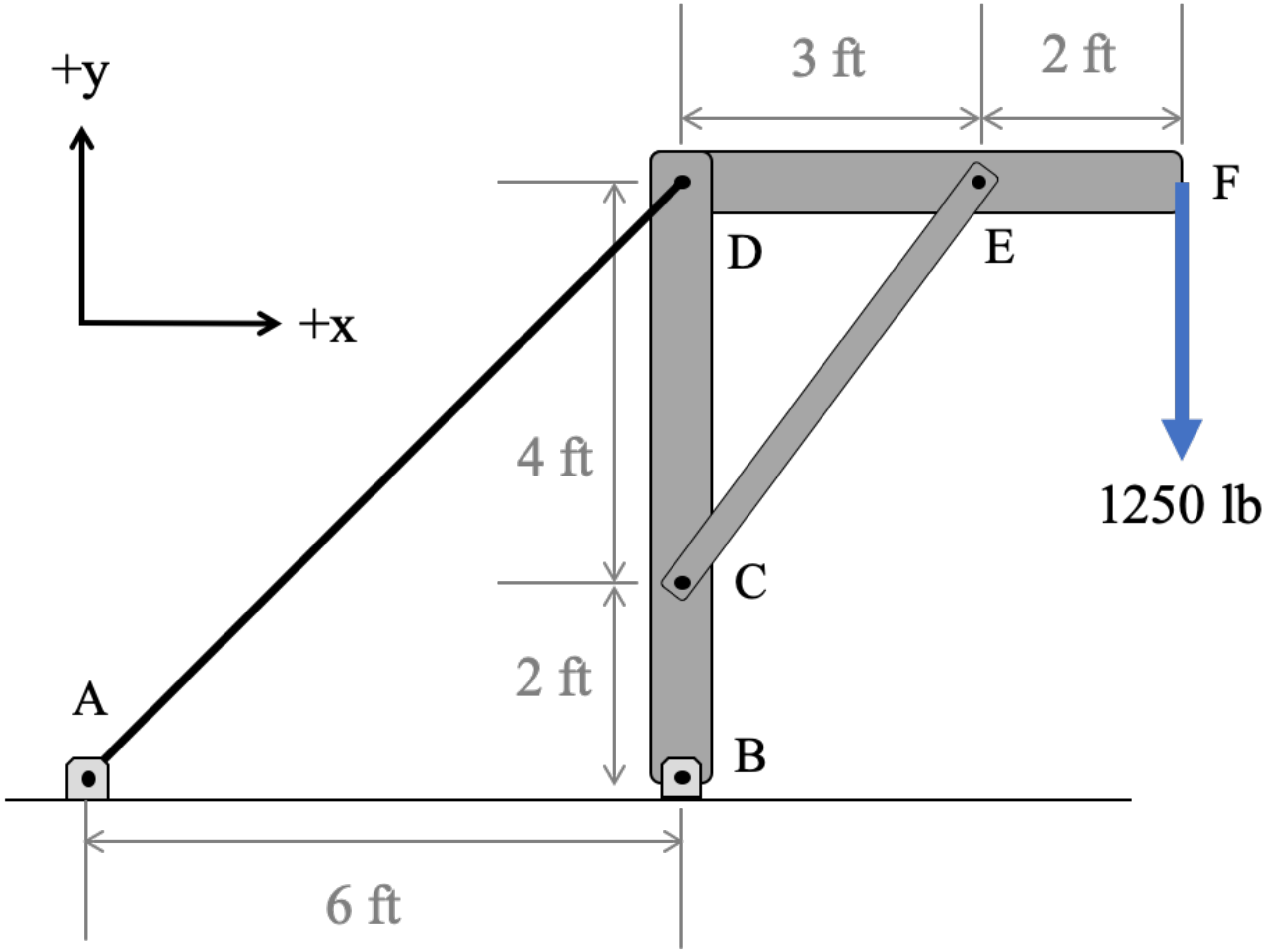
$$\frac{\tau_{fail}}{SF} = \frac{2P}{\pi d_{bolt}^2}$$

$$d_{bolt}^2 = \frac{2P(SF)}{\pi \tau_{fail}} \quad , \quad d_{bolt} \geq \pm\sqrt{\frac{2P(SF)}{\pi \tau_{fail}}} = +\sqrt{\frac{2(40(10^3))(4)}{\pi(296(10^6))}} = 18.55mm$$

Problem 4:
(About 20 minutes)

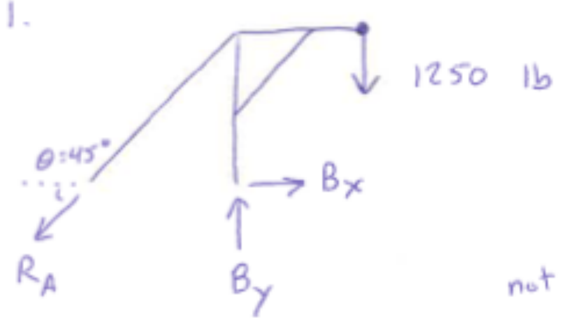
The Ghostbusters have designed a structure to help them capture ghosts. A diagram of the structure is below. The structure consists of three members—*BCD*, *CE*, and *DEF*—and one wire—*AD*. The cross-sectional area of each member (*BCD*, *CE*, and *DEF*) is 4 in^2 and the cross-sectional area of wire AD is 0.8 in^2. All joints are pinned joints. All components (the three members, the wire, and the pins at each joint) are made of aluminum with $E = 1.06 \times 10^7$ psi, $\sigma_{fail} = 60 \times 10^3$ psi, and $\tau_{fail} = 25 \times 10^3$ psi.

1. Draw the global free-body diagram of the entire system and calculate the reaction at A.
2. Determine the internal force(s) and moment(s) in the exact middle of member *CE*.
3. Determine the average normal stress in member *CE*. Is member *CE* in tension or compression?
4. Determine the *change in length* of member *CE*. Does member *CE* get longer or shorter?
5. If the pin at A is in *double shear*, calculate the diameter of the pin required to give a factor of safety of 3.



Problem 4:
(About 20 minutes)

1.



$\Sigma M_B = 0 = -1250(5) + R_A \sin 45 (6)$

$R_A = \frac{1250(5)}{6 \sin 45} = \boxed{1473.1 \text{ lb} = R_A}$

not necessary

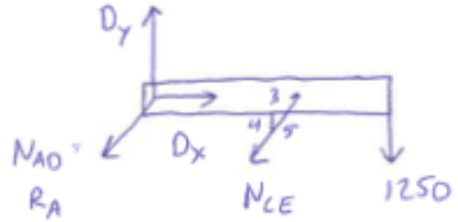
$\Sigma F_x = 0 = B_x - R_A \cos 45$

$B_x = 1041.7$ lb

$\Sigma F_y = 0 = B_y - 1250 - R_A \sin 45$

$B_y = 2291.7$ lb

2. FBD of DEF



$\Sigma M_D = 0 = -N_{CE}\left(\frac{4}{5}\right)(3) - 1250(5)$

$N_{CE} = \frac{-1250(5)}{\left(\frac{4}{5}\right)(3)} =$

$N_{CE} = -2604.2$ lb

$V_{CE} = 0$ lb

$M_{CE} = 0$ lb

known b/c N_{CE} is a two-force member

3. $\sigma_{CE} = \frac{N_{CE}}{A_{CE}} = \frac{-2604.2}{4}$

$\sigma_{CE} = -651.0$ psi compression

4. $\sigma_{CE} = E \epsilon_{CE} \Rightarrow \epsilon_{CE} = \frac{\sigma_{CE}}{E} = \frac{-651.0}{1.06 \times 10^7}$

$\epsilon_{CE} = -6.14 \times 10^{-5} = \frac{\delta_{CE}}{L_{CE}}$

$L_{CE} = 5$ ft.

$\delta_{CE} = \epsilon_{CE} L_{CE} = -6.14 \times 10^{-5}(5)$

$\boxed{\delta_{CE} = -3.07 \times 10^{-4} \text{ ft}} = \boxed{-3.69 \times 10^{-3} \text{ in} = \delta_{CE}}$