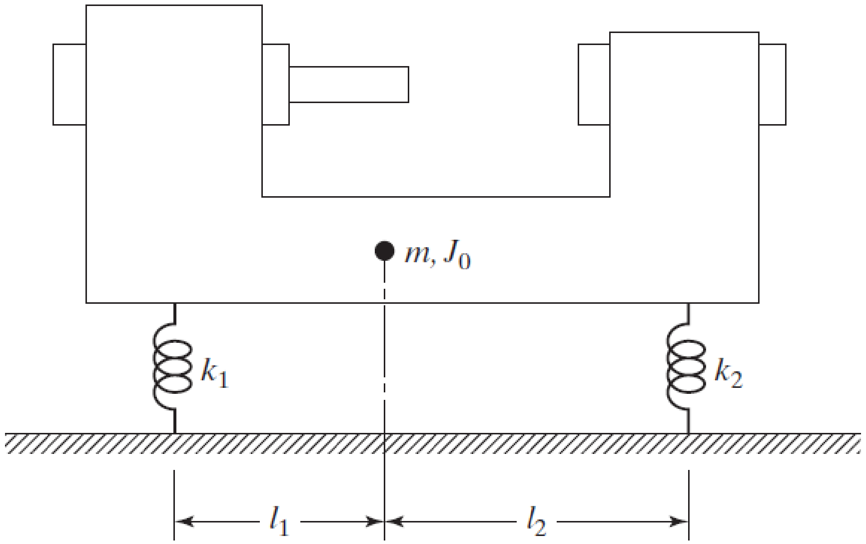
Recitation 8

ASEN 3112 – Spring 2020

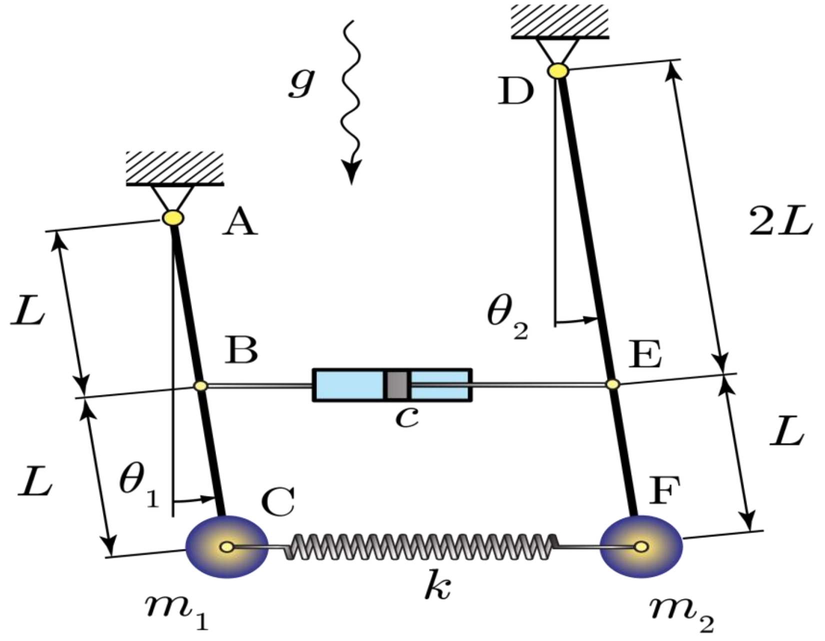
Problem 1: Vibrations of Multi Degree of Freedom System (rigid beam)



A machine tool, having a mass of m = 1000 kg and a mass moment of inertia of J_0 = 300 kg-m^2, is supported on elastic supports, as shown in the figure. If the stiffnesses of the supports are given by k_1 = 3000 N/mm and k_2 = 2000 N/mm, and the supports are located at l_1 = 0.5 m and l_2 = 0.8 m, find the natural frequencies and mode shapes of the machine tool.

Hint: The machine can be modeled as a rigid beam that experiences two independent degrees of freedom of motion at its center of gravity: vertical displacement and rotation.

Problem 2: Vibrations of Multi Degree of Freedom System (two pendulums)



Consider the Multi Degree of Freedom linear dynamic system in the figure. It consists of two pendulums of masses m_1 and m_2, and lengths 2L and 3L, respectively, under gravity g. Both masses are connected by a spring of stiffness k. The pendulums are also connected by a dashpot of coefficient c, at points B and E. Assume that the beams AC and DF are massless, and that both pendulums rotate by angles θ_1 and θ_2, which can be both assumed to be very small, i.e. $\theta_i << 1$, so that $\cos(\theta_i) \sim 1$ and $\sin(\theta_i) \sim \theta_i$. This assumption also implies that the spring and the dashpot always remain horizontal.

a) Consider the numerical values $m_1 = 2$, $m_2 = 2$, $L = 1$, $k = 2$, $c = 0$, $g = 5$ (do not worry about physical units). The mass and stiffness matrices are given below. Set up the vibration eigenproblem, i.e. write the associated matrix equation. $M = \begin{bmatrix} 8 & 0 \\ 0 & 18 \end{bmatrix}$ and $K = \begin{bmatrix} 28 & -12 \\ -12 & 48 \end{bmatrix}$

b) Write the characteristic equation and verify that $\omega^2_1 = 2$ and $\omega^2_2 = 25/6$ are the natural frequencies of the vibrating system.

c) Compute the eigen vector U_1 associated with ω_1. First normalize it so that the largest entry is one. Then normalize it into ϕ_1 so that the generalized mass $M_1 = \phi_1^T \mathbf{M}\, \phi_1 = 1$.

d) The second mass now moves with a prescribed horizontal motion $u(t) = A \cos(\Omega t)$. Which value of Ω would result in resonance in the system?
Hint: The system is now a single DOF system since mass 2 is no longer free, but rather is vibrating in a prescribed manner. The resulting EOM is:

$$4L^2 m_1 \ddot{\theta}_1 + \left(2m_1 g l + 4L^2 k\right)\theta_1 = 2LKA\cos\Omega t$$