University of Colorado - Boulder

Airplane Shaker

Submited December 2, 2019

Lab 3 - Airplane Shaker

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The objective of this lab is to predict two of the five natural frequencies by using simple FEM models. We approximate the motion of our aircraft as a cantilevered system. In addition to the involvement of the cantilever modes of the aft-fuselage, modes 2 and 5 are identified as horizontal tail vertical + T section sideways and horizontal tail vertical second mode vibrations, respectively. There are three quantities of the tail assembly models that needs to be considered in this lab: combination of point-mass, point-first-mass-moment-of-inertia, and point-second-mass-moment-of inertia. To minimize error-prone integration, these three quantities are evaluated in a computational program symbolically. It should be noted that the FEM model is two-dimensional, therefore, it can only account for aft-shake vertical cantilever modes that occur in the fuselage-rudder plane.

I. Results

A. Experimental Results

Experimental data from the linear sweep experiment was used to analyze the system modes. In order to compare values irrespective of the base displacement, the raw displacements were converted into magnification factors. To facilitate this, the displacement data was taken as a step-wise function with six phases, each phase being distinguished by the peak-to-peak displacement being steady for some amount of time. For a given phase, the average max displacement of the base was determined. Then, for each of the 3 accelerometers, the displacement was scaled by the max base displacement for that frequency span.

The six phases can be observed in the base displacement graph here:

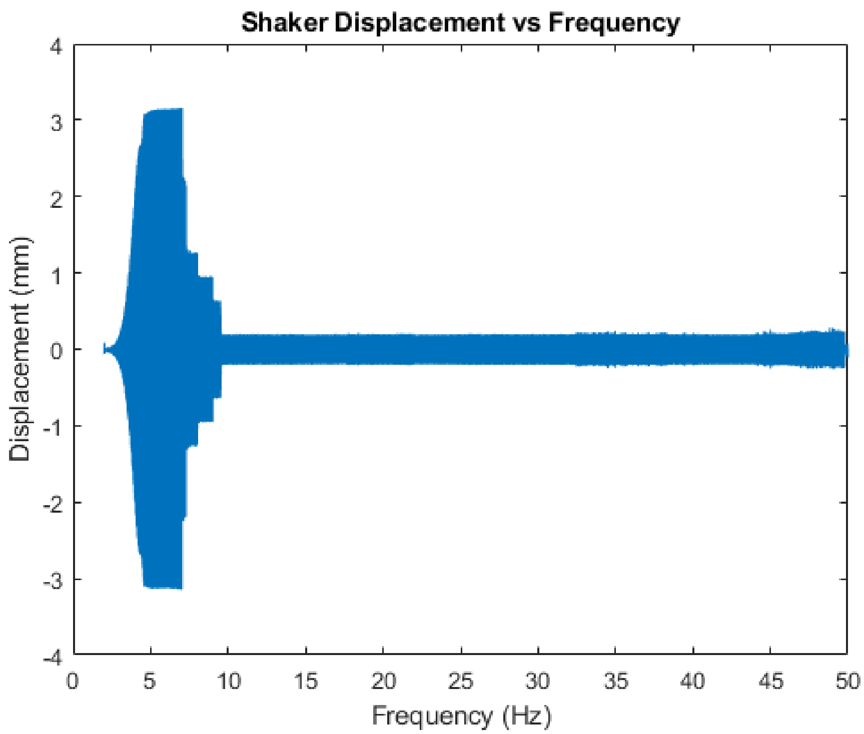

Fig. 1 Displacement of the base during the linear sweep

And the frequency graphs resulting from the above process are as follows:

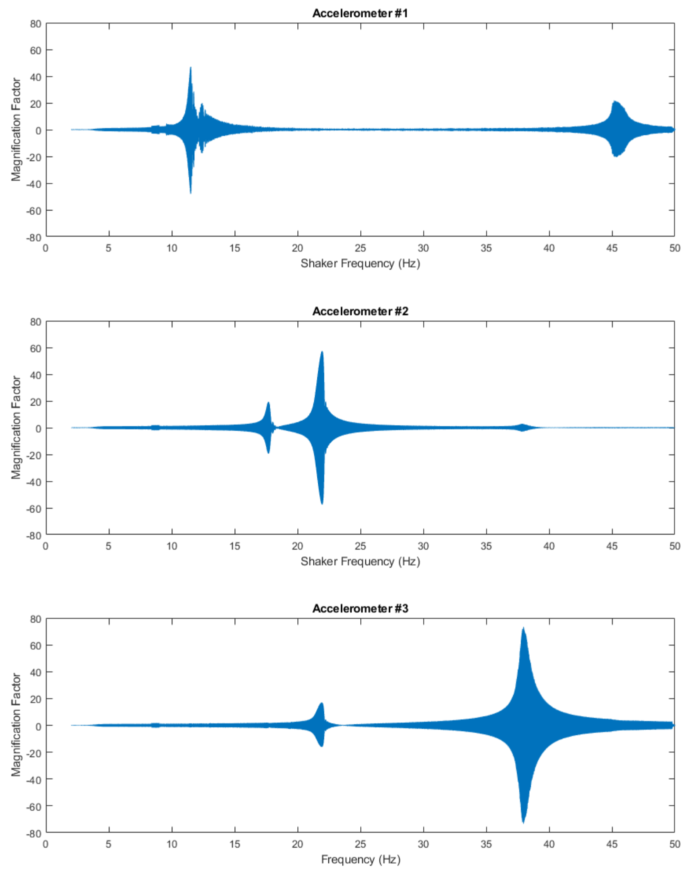

Fig. 2 Magnification Factors for each Accelerometer

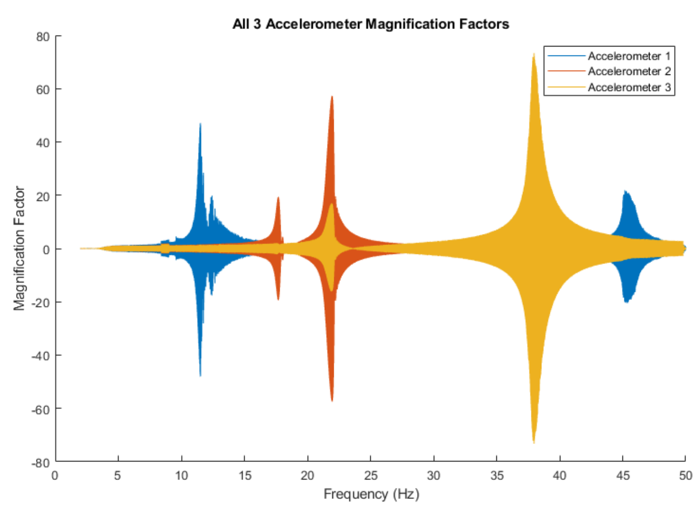

Fig. 3 All 3 Accelerometer graphs superimposed on each other

From the data above, it can be seen that the experimentally derived frequency modes are approximately: 12 Hz, 18 Hz, 22 Hz, 38 Hz, and 45 Hz. By reviewing the system response and correlating it with the accelerometer locations, we can deduce the shape of the modes of the system. Accelerometers 1, 2, and 3 were located on the tail, nose, and wingtip, respectively. Correlating that with figure 6, we describe the mode shapes. Mode 1 is dominated by high tail vibration. Mode 2 is dominated by nose vibrations. Mode 3, the second most energetic mode, is characterized by high nose shake and moderate wingtip shake. Mode 4 is extreme wingtip shake, though a small amount of nose shake was also recorded. Mode 5, the last one experimentally found, is another tail shake mode.

B. FEM Results - Resonant Frequencies

Resonant Frequencies from FEM Simulations		
	2 Element(Hz)	4 Element(Hz)
1st Frequency	12.03	12.03
2nd Frequency	51.03	51.08
3rd Frequency	202.5	203.3

The above table are the results from the FEM simulations for the 2 and 4 element versions. The 1st and 2nd frequencies are calculated values of the 2nd and 5th resonant frequencies from the experimental data. The 3rd resonant frequency in the table above is an additional predicted tail resonant frequency.

Percent Difference between Experimental and FEM		
	2 Element FEM	4 Element FEM
2nd Mode from Exp	38.2%	38.2%
5th Mode from Exp	12.14%	12.26%

The percent differences in the above table are comparing only the first two frequencies from the FEM simulation to the second and fifth frequencies of the experimental data. These comparisons show the lower resonant frequencies show a much greater difference than the higher resonant frequencies. These errors could be due to the fact that the simulations assume that motion of the plane is strictly 2 dimensional and cuts out motion about the nose of the plane and any roll that the entire plane experiences.

C. FEM Results - Mode Shapes

The data gathered is shown in figure 6, as the non-dimensionalized Y displacement is measured against the x position, further examining the planes reaction to resonance at various lengths along the plane. The circles represent the non-dimensionalized acceleration measured by an accelerometer at that x location. For mode 2, we see the largest displacement at the tail, or 12 inches from the nose of the craft which matches the 2 element FEM model nicely. For mode 5, the displacements are more scattered, localizing around 9 inches from the nose of the plane, but causing displacements all along the body or fuselage, matching with the FEM results again. Although there seem to be slight discrepancies in the Y displacements at various lengths, those can be chalked up to uncertainty in the measurements of the accelerometer placements as well as the shaking of outside bodies causing disruptions in the aircraft resonance.

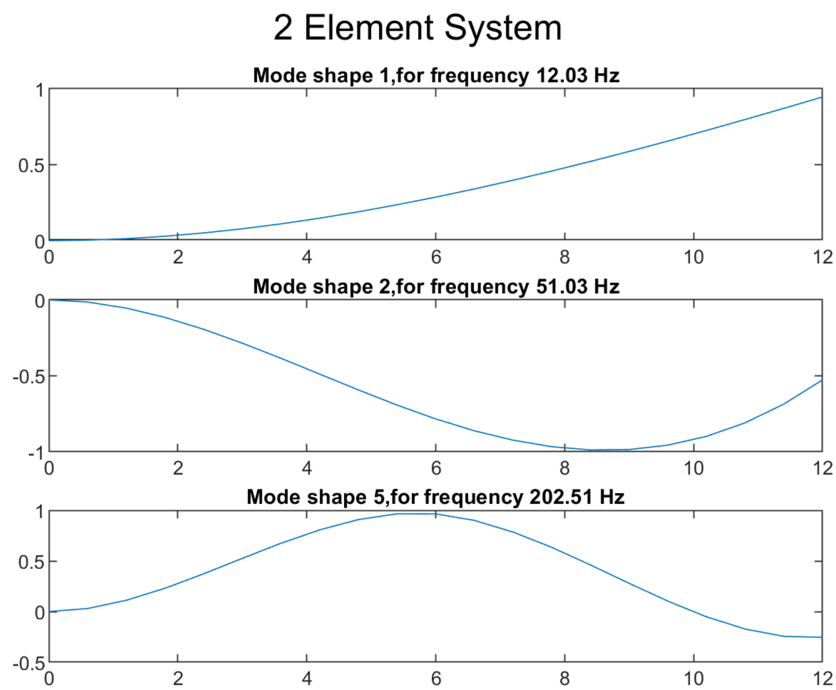

Fig. 4 2 Element FEM Simulation

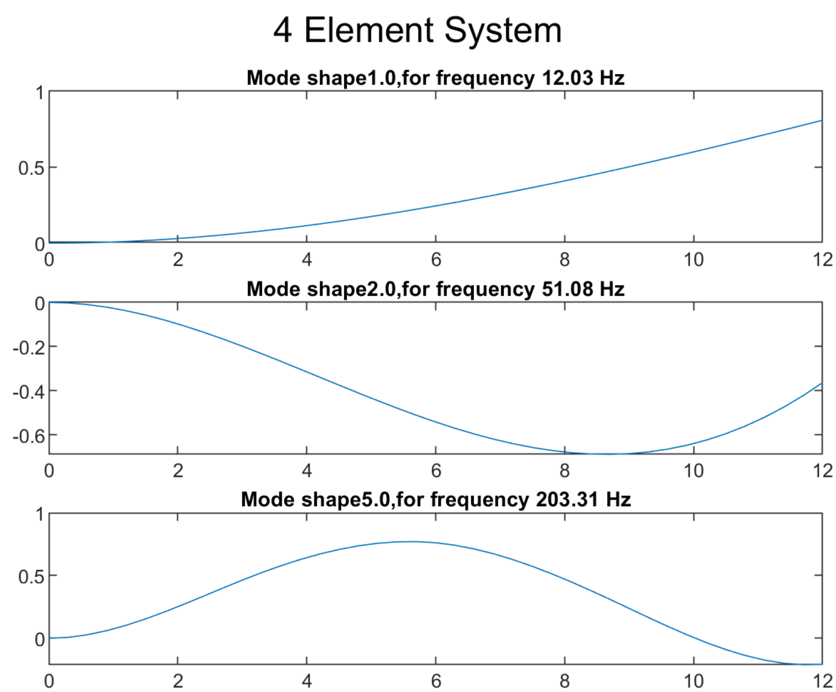

Fig. 5 4 Element FEM Simulation

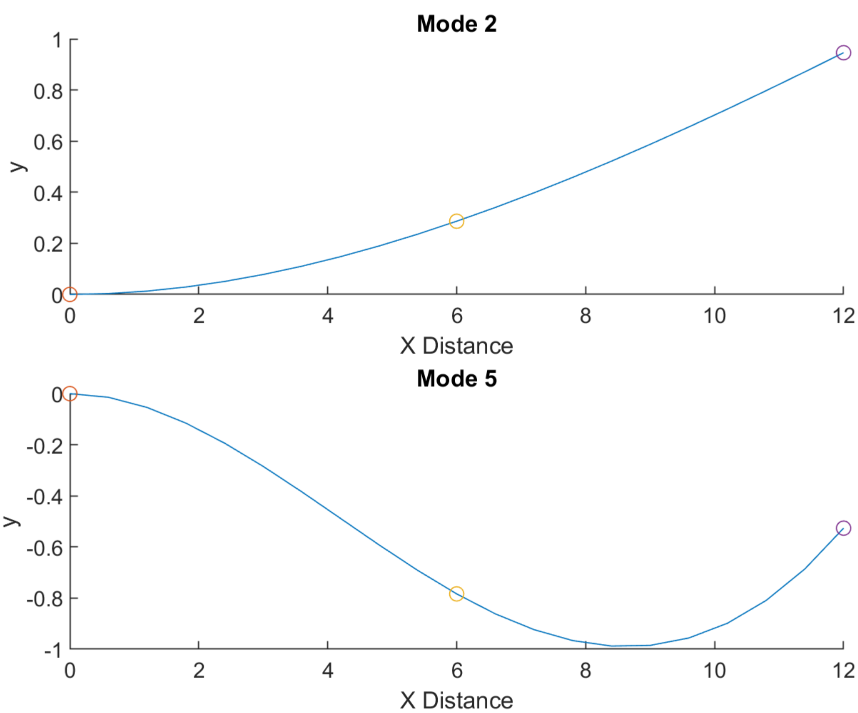

Fig. 6 Y Displacements at Various X Positions Tested for Modes 2 and 5

II. Appendix - Participation Report

Name	Plan	Experiment	Results	Report	Code	Total
Aufa Amirullah	1	1	1	2	0	100%
Tim Breda	1	2	1	1	0	100%
Axel Haugland	1	1	0	1	1	100%
Corey LePine	1	1	0	1	0	100%
Logan Vangyia	1	0	2	1	2	100%
Dean W	1	0	2	2	2	100%
Chenshuo Yang	1	0	1	1	1	100%

III. Appendix Code

```
clear
clc
%Creator:Logan Vangyia
%Lab 3: Plane Resonant Frequency

%% Constants from table 1:
%  Material info: AL 6063-T7 Stock

E = 10175000; %PSI
roh = 0.0002505; %lb-sec^2/in^4


% ALL english units(inches)
L = 12;
L_E = 4.5;
L_R = 5;
w = 1;
h = 1/8;
h_E = 1/4;
h_R = 0.040;
M_T = 1.131*roh;
S_T = 0.5655*roh;
I_T = 23.124*roh;
A = w*h;
Izz = (w*h^3)/12;



%% 2 Element Model
C_M2 = (roh*A*L) / 100800;
C_K2 = (4*E*Izz) / L^3 ;

%building master stiffness matrix by making seperate rows than
%concatenating them all together
row1=[19272, 1458*L, 5928,-642*L,0,0];
row2=[1458*L,172*L^2,642*L,-73*L^2,0,0];
row3=[5928,642*L,38544,0,5928,-642*L];
row4=[-642*L,-73*L^2,0,344*L^2,642*L,-73*L^2];
row5=[0,0,5928,642*L,19272,-1458*L];
row6=[0,0,-642*L,-73*L^2,-1458*L,172*L^2];
```

```

M_2=C_M2*cat(1,row1,row2,row3,row4,row5,row6);
M_2(5,5)=M_2(5,5)+M_T;
M_2(5,6)=M_2(5,6)+S_T;
M_2(6,5)=M_2(6,5)+S_T;
M_2(6,6)=M_2(6,6)+I_T;

row1=[24,6*L,-24,6*L,0,0];
row2=[6*L,2*L^2,-6*L,L^2,0,0];
row3=[-24,-6*L,48,0,-24,6*L];
row4=[6*L,L^2,0,4*L^2,-6*L,L^2];
row5=[0,0,-24,-6*L,24,-6*L];
row6=[0,0,6*L,L^2,-6*L,2*L^2];

K_2=C_K2*cat(1,row1,row2,row3,row4,row5,row6);

%finding eigenvalues of matrix
[vec,val]=eig(M_2(3:end,3:end)^(-1) * K_2(3:end,3:end));
%determining frequencies from eigenvalues
eig_2=sqrt(diag(val));
eig_2=eig_2/(2*pi);
%assigning desired eigenvalues to outputing vector
f2_1=eig_2(4);
f2_2=eig_2(3);
f2_3=eig_2(2);
f=[1,2,5];
f2=[eig_2(4),eig_2(3),eig_2(2)];
fprintf('Lowest 3 Frequencies from the 2-Element Model are %f Hz, %f Hz, and %f Hz
    \n',f2_1,f2_2,f2_3)
figure(1)
for i=1:3%creating subplots of eigenvector mode shapes
   subplot(3,2,i*2-1:2*i)
ev=[0;0;vec(:,5-i)];
ne=2;
nsub=10;
scale=1;
ploteigenvector(L,ev,ne,nsub,scale)
str=sprintf('Mode shape%2.f,for frequency %4.2f Hz',f(i),f2(i));
title(str);
end
sgtitle('2 Element System')
%% 4 Element Model
C_M4 = (roh*A*L) / 806400 ;
C_K4 = (8*E*Izz) / L^3 ;

%building master stiffness matrix by making seperate rows than
%concatenating them all together
row1=[77088,2916*L,23712,-1284*L,0,0,0,0,0,0];
row2=[2916*L,172*L^2,1284*L,-73*L^2,0,0,0,0,0,0];
row3=[23712,1284*L,154176,0,23712,-1284*L,0,0,0,0];
row4=[-1284*L,-73*L^2,0,344*L^2,1284*L,-73*L^2,0,0,0,0];
row5=[0,0,23712,1284*L,154176,0,23712,-1284*L,0,0];
row6=[0,0,-1284*L,-73*L^2,0,344*L^2,1284*L,-73*L,0,0];
row7=[0,0,0,0,23712,1284*L,154176,0,23712,-1284*L];
row8=[0,0,0,0,-1284*L,-73*L^2,0,344*L^2,1284*L,-73*L^2];
row9=[0,0,0,0,0,0,23712,1284*L,77088,-2916*L];
row10=[0,0,0,0,0,0,-1284*L,-73*L^2,-2916*L,172*L^2];

M_4=C_M4*cat(1,row1,row2,row3,row4,row5,row6,row7,row8,row9,row10);
```

```
M_4(9,9)=M_4(9,9)+M_T;
M_4(9,10)=M_4(9,10)+S_T;
M_4(10,9)=M_4(10,9)+S_T;
M_4(10,10)=M_4(10,10)+I_T;

%building master stiffness matrix by making seperate rows than
%concatenating them all together
row1=[96,12*L,-96,12*L,0,0,0,0,0,0];
row2=[12*L,2*L^2,-12*L,L^2,0,0,0,0,0,0];
row3=[-96,-12*L,192,0,-96,12*L,0,0,0,0];
row4=[12*L,L^2,0,4*L^2,-12*L,L^2,0,0,0,0];
row5=[0,0,-96,-12*L,192,0,-96,12*L,0,0];
row6=[0,0,12*L,L^2,0,4*L^2,-12*L,L^2,0,0];
row7=[0,0,0,0,-96,-12*L,192,0,-96,12*L];
row8=[0,0,0,0,12*L,L^2,0,4*L^2,-12*L,L^2];
row9=[0,0,0,0,0,0,-96,-12*L,96,-12*L];
row10=[0,0,0,0,0,0,12*L,L^2,-12*L,2*L^2];

K_4=C_K4*cat(1,row1,row2,row3,row4,row5,row6,row7,row8,row9,row10);
%finding eigenvalues of matrix
[vec,val]=eig(M_4(3:end,3:end)^(-1) * K_4(3:end,3:end));
%finding frequencies from eigenvalues
eig_4=sqrt(diag(val));
eig_4=eig_4/(2*pi);

f4_1=eig_4(8);
f4_2=eig_4(7);
f4_3=eig_4(6);

%outputting values
fprintf('Lowest 3 Frequencies from the 4-Element Model are %f Hz, %f Hz, and %f Hz
    \n',f4_1,f4_2,f4_3)

f=[1,2,5];
f4=[eig_4(8),eig_4(7),eig_4(6)];
figure(2)
for i=1:3%plotting eigenvector modes
    subplot(3,2,i*2-1:2*i)
ev=[0;0;vec(:,5-i)];
ne=4;
nsub=10;
scale=1;
ploteigenvector(L,ev,ne,nsub,scale)
str=sprintf('Mode shape%2.1f,for frequency %4.2f Hz',f(i),f4(i));
title(str);
end
sgtitle('4 Element System')
```

```
function ploteigenvector(L,ev,ne,nsub,scale)
%function to plot eigen modes from the eigenvectors of a matrix
nv=ne*nsub+1;
Le=L/ne;
dx=Le/nsub;
k=1;
x=zeros(nv,1);
v=zeros(nv,1);
for e=1:ne
```

```
xi=Le*(e-1);
vi=ev(2*e-1);
qi=ev(2*e);
vj=ev(2*e+1);
qj=ev(2*e+2);
for n=1:nsub
    xk=xi+dx*n;
    z=(2*n-nsub)/nsub;
    vk=scale*(0.125*(4*(vi+vj)+2*(vi-vj)*(z^2-3)*z+Le*(z^2-1)*(qj-qi+(qi+qj)*z)));
    k=k+1;
    x(k)=xk;
    v(k)=vk;
end


end
plot(x,v)

end
```

```
    close all
    clear all
    clc

    load('data.mat')
    freq = linspace(2,50,length(time));
    freq = freq';

%% Phase 1
% These values were hard coded by using the data picking tool
% We approximate the changing displacement as a step function until it
% reaches the steady displacement in phase 6.
    peak_p1 = 3.162;
    end_p1 = 135834; %index at which the phase ends
    t1 = 201738;

%% Phase 2
    peak_p2 = 2.254;
    end_p2 = 142376;

%% Phase 3
    peak_p3 = 1.313;
    end_p3 = 162137;

%% Phase 4
    peak_p4 = 0.9572;
    end_p4 = 171977;

%% Phase 5
    peak_p5 = 0.6495;
    end_p5 = 202168;
%% Phase 6
    peak_p6 = 0.1966;
    %end is EOF

%% Compute and Conjoin the Data

    disp_1_r = disp_1;
```

```
    disp_2_r = disp_2;
    disp_3_r = disp_3;

    % Normalize into magnification factor
    for i = 1:length(disp_1)
        if i <= end_p1
            disp_1_r(i) = disp_1(i)/peak_p1;
            disp_2_r(i) = disp_2(i)/peak_p1;
            disp_3_r(i) = disp_3(i)/peak_p1;
        elseif i <= end_p2
            disp_1_r(i) = disp_1(i)/peak_p2;
            disp_2_r(i) = disp_2(i)/peak_p2;
            disp_3_r(i) = disp_3(i)/peak_p2;
        elseif i <= end_p3
            disp_1_r(i) = disp_1(i)/peak_p3;
            disp_2_r(i) = disp_2(i)/peak_p3;
            disp_3_r(i) = disp_3(i)/peak_p3;
        elseif i <= end_p4
            disp_1_r(i) = disp_1(i)/peak_p4;
            disp_2_r(i) = disp_2(i)/peak_p4;
            disp_3_r(i) = disp_3(i)/peak_p4;
        elseif i <= end_p5
            disp_1_r(i) = disp_1(i)/peak_p5;
            disp_2_r(i) = disp_2(i)/peak_p5;
            disp_3_r(i) = disp_3(i)/peak_p5;
        else
            disp_1_r(i) = disp_1(i)/peak_p6;
            disp_2_r(i) = disp_2(i)/peak_p6;
            disp_3_r(i) = disp_3(i)/peak_p6;
        end
    end

    figure(1)
    hold on
    subplot(3,1,1)
    plot(freq,disp_1_r)
    ylim([-80,80])
    title('Accelerometer #1')
    xlabel('Shaker Frequency (Hz)')
    ylabel('Magnification Factor')
    subplot(3,1,2)
    plot(freq,disp_2_r)
    xlabel('Shaker Frequency (Hz)')
    ylabel('Magnification Factor')
    title('Accelerometer #2')
    ylim([-80,80])
    subplot(3,1,3)
    plot(freq,disp_3_r)
    xlabel('Shaker Frequency (Hz)')
    ylabel('Displacement Factor')
    title('Accelerometer #3')
    xlabel('Frequency (Hz)')
    ylabel('Magnification Factor')
    ylim([-80,80])
    hold off

%% Extrapolation to FEM Modes
% These are the data for modes 2 and 5. The average accelerometers peaks
% were taken and used to normalize such that they will apply to the FEM
```

```
% curves.
%
%    acc_1_mode_2 = 3.5530*10^4;
%    acc_2_mode_2 = 1576;
%    acc_3_mode_2 = 1.0830*10^4;
%
%    acc_1_mode_5 = 3.916*10^5;
%    acc_2_mode_5 = 3423;
%    acc_3_mode_5 = 5.143*10^5;

    acc_1_mode_2 = 1;
    acc_2_mode_2 = 0.0444;
    acc_3_mode_2 = 0.3048;

    acc_1_mode_5 = 0.7614;
    acc_2_mode_5 = 0.0067;
    acc_3_mode_5 = 1;
```