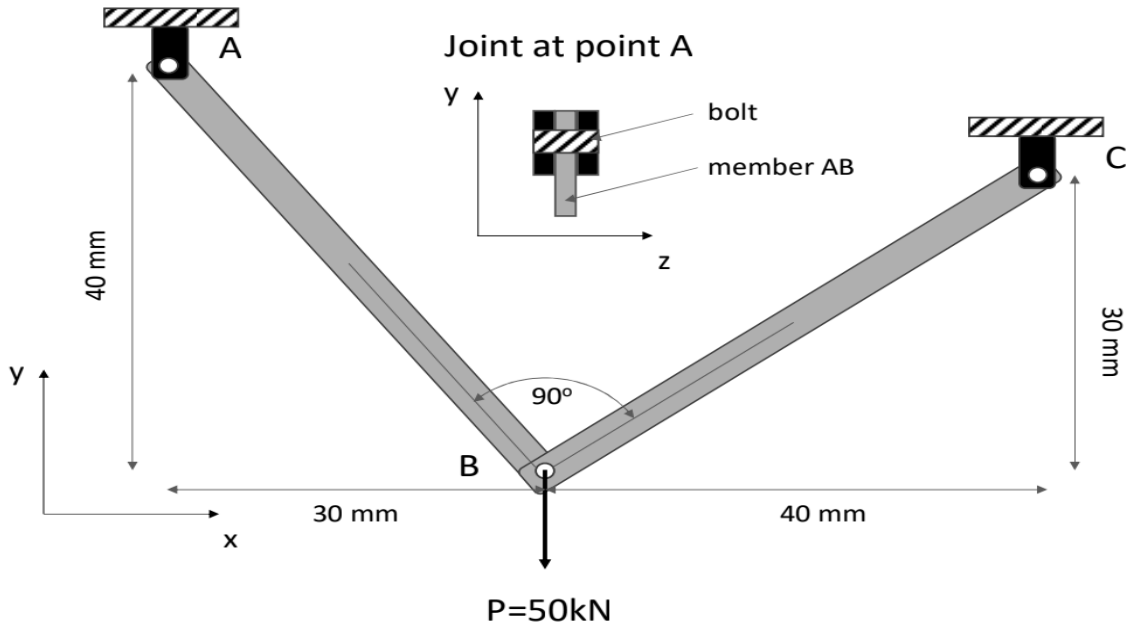
ASEN 3112

Recitation 2

Problem 1: About 30 minutes



The figure shows a two-bar truss to a vertical load $P = 50\ kN$ at joint B. Both bars have a rectangular cross-section and are made of an isotropic material with Young's modulus of $E = 2.1\ Gpa$ and maximum tensile strength of *T.S.* = 150 *Mpa.*

a. Compute the forces in the bars. Use FBD.
b. Compute the smallest possible diameter of the cylindrical bolt joint A such that the shear in the bolt does not exceed $\tau = 100\ MPa$. Use a proper FBD.
c. Determine the cross-section area of each bar such that the factor of safety against failure in normal stress in the bar is $S_f = 4.0$.

Problem 2: About 20 minutes

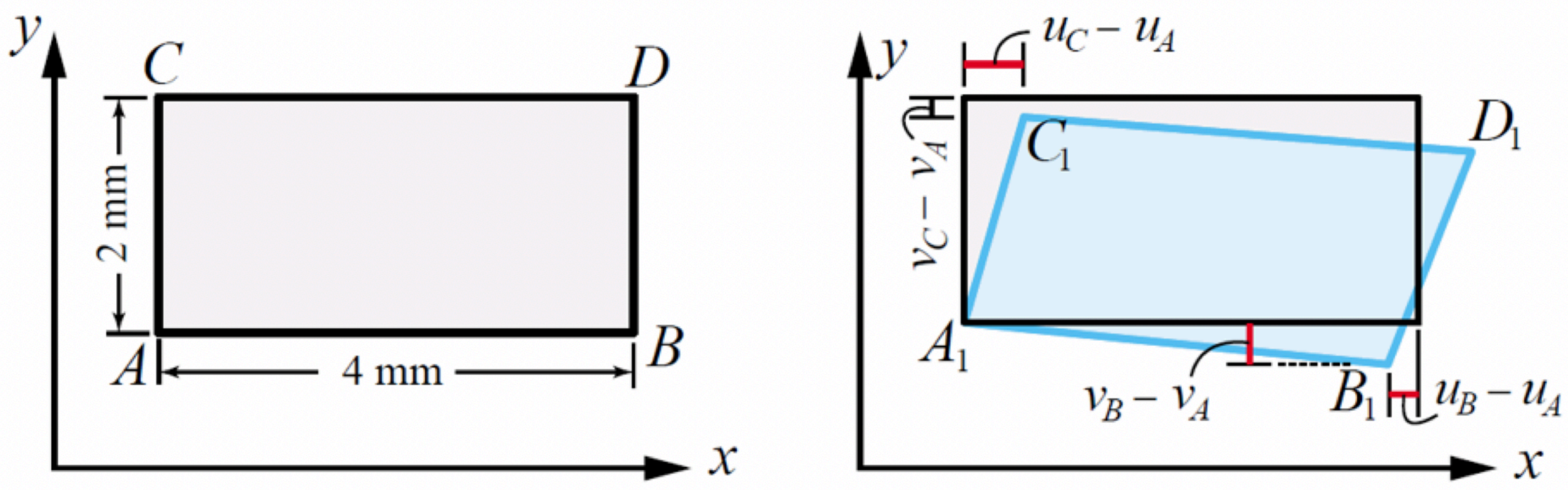


Figure 1: Shape to be analyzed for problem 1.

Displacements *u* and *v* in the *x* and *y* dimensions, respectively, were measured at points *A, B, C,* and *D* of the two-dimensional rectangular body shown above. The gage lengths along *x* and *y* are 4 mm and 2 mm, respectively. Upon deformation, the displacement data is $u_a = -0.0100\ mm$, $v_a = 0.0100\ mm$, $u_b = -0.0050\ mm$, $v_b = -0.0112\ mm$,$u_c = 0.0050\ mm$, $v_c = 0.0068\ mm$, $u_d = 0.0100\ mm$, and $v_d = 0.0080\ mm$. [The displacements of *D* are irrelevant for the computation of the strains at *A*.]

Find the Lagrangian small strains ε_{xx}, ε_{yy}, and γ_{xy} at point A using displacement differences. All answers should be in micros (μ); [one micro is $0.000001 = 0.0001\% = 10^{-6}$].

Problem 3: About 30 minutes

Figure 2: Rubber balloon to be analyzed for problem 2.

A rubber party balloon (see Figure above) is inflated from an initial diameter $D0$ = 50 mm (the reference state) to a final diameter Df = 150 mm (the deformed or final state). Two easy items and a tougher one:

a) Find the average circumferential strain in the deformed state as given by the Lagrangian measure ϵ^L_{circ} and the Eulerian measure ϵ^E_{circ}. Express both in %. Are these values close or far apart? Justify your answer (why is it close or far apart).

b) Initial wall thickness is t_0 = 0.18 mm. The rubber is incompressible (When rubber deforms, it maintains its volume). Find the thickness t_f in the deformed state.

c) (Tougher) For simplicity assume: (I) rubber is linearly elastic, (II) wall is in plane stress, (III) average Young's modulus for the Eulerian strain measure is $E_{rubber} \approx 1.9$ GPa, (IV) Poisson's ratio is $\nu = 1/2$. Under those assumptions, find the inflation pressure p.

Problem 4: About 30 minutes

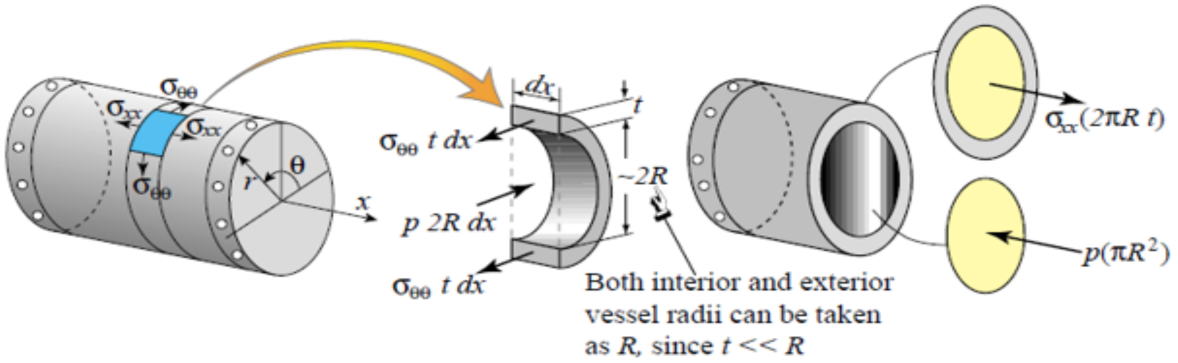


Figure 3: Vessel to be analyzed for problem 3.

$$\sigma_{xx} = \frac{pR}{2t} \quad \text{(axial stress)}, \qquad \sigma_{\theta\theta} = \frac{pR}{t} \quad \text{(hoop stress, also called circumferential stress)}. \qquad (1)$$

(a) For p = 3 psi, R = 10 in, t = 0.05 in, and compute σ_{xx} and $\sigma_{\theta\theta}$

(b) For the plane stress state obtained for the cylindrical pressure vessel above, and using the numerical values determined in part (a), determine the in-plane stresses. Find:

- The principal stresses σ_1 and σ_2, in which $\sigma_1 \geq \sigma_2$.
- The principal planes angles ϕ_1 and ϕ_2 (φ is measured from the x axis, positive CCW). Note: symbol ϕ is used in lieu of θ, as in the Notes, because ϑ is used here to denote the circumferential direction.
- The maximum in-plane shear stress τ_p

(c) Consider next the pressure vessel wall as a *three-dimensional body*. Use the fact that the radial (normal-to-the-wall) stress $\sigma_{rr} \approx 0$ is a zero principal stress, and that σ_{xx} and $\sigma_{\theta\theta}$ are principal stresses. Find the *overall* maximum shear stress $\tau_{max} = max(\frac{1}{2}|\sigma_{xx} - \sigma_{\theta\theta}|, \frac{1}{2}|\sigma_{xx}|, \frac{1}{2}|\sigma_{\theta\theta}|)$, which follows from eqn (6.18) of text.

(d) Compare τ_{max} to the maximum in-plane shear stress τ_p found in (b) above. Are they the same? What can you conclude about failure of the vessel if it is fabricated of a ductile material such a steel or aluminum?