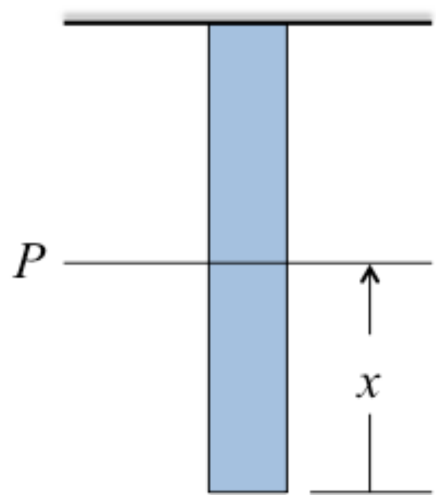
ASEN 3112 - Structures - Fall 2019

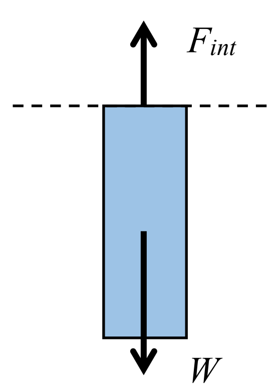
Homework 1

Due at Start of Class on **Wednesday September 4, 2019**
Make sure to follow the **homework guidelines** on the **last page** of this document!

Exercise 1.1.
The prismatic bar shown has a solid circular cross section with 30-mm radius. It is suspended from one end and is loaded only by its own weight. The mass density of the homogeneous material is 2800 kg/m^3. Determine the average normal stress as a function of x at any plane P, where x is the distance from the bottom of the bar in meters. (*Strategy hint*: Draw a free-body diagram of the part of the bar below the plane P.)



Solution 1.1.
Draw a FBD of the part of the bar below plane P, where F_{int} is the internal force in the bar at plane P and W is the weight of the bar below plane P.



The weight of the bar below P can be found by multiplying together the mass density, ρ, the acceleration due to gravity, g, and the volume of the bar below P, Ax, where A is the cross-sectional area and x is the height of the piece of the bar.

$$W = \rho g A x$$

Then, taking the sum of the forces in the vertical direction:

$$\Sigma F_y = 0 = F_{int} - W$$
$$F_{int} = W = \rho g A x$$

Applying the definition of average normal stress, we have:

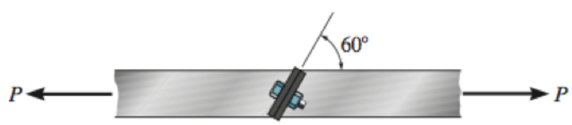
$$\sigma = \frac{F_{int}}{A} = \frac{\rho g A x}{A} = \rho g x$$

Plugging in numbers:

$$\sigma = \rho g x = 2800 \frac{kg}{m^3} \times 9.8 \frac{m}{s^2} \times x = 27.44x\ kPa$$

Exercise 1.2.

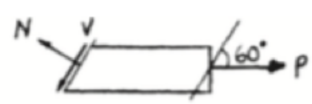
The tension member is fastened together using *two* bolts, one on each side of the member as shown. Each bolt has a diameter of 0.5625 in. Determine the maximum load P that can be applied to the member if the allowable shear stress for the bolts is $\tau_{\text{allow}} = 17$ ksi and the allowable average normal stress is $\sigma_{\text{allow}} = 26$ ksi.



Solution 1.2.

$\nwarrow + \Sigma F_y = 0$; $N - P \sin 60° = 0$

$$P = 1.1547\,N \qquad \textbf{(1)}$$



$\swarrow + \Sigma F_x = 0$; $V - P\cos 60° = 0$

$$P = 2V \qquad \textbf{(2)}$$

Assume failure due to shear:

$$\tau_{\text{allow}} = 17 = \frac{V}{2\frac{\pi}{4}(0.5625)^2}$$

$V = 8.45$ kip

From Eq. (2),

P = 16.90 kip

Assume failure due to normal force:

$$\sigma_{\text{allow}} = 26 = \frac{N}{2\frac{\pi}{4}(0.5625)^2}$$

N = 12.92 kip

From Eq. (1),

P = 14.92 kip (limiting case)

***Note:** The lowest value is the maximum allowable value.

Exercise 1.3.
Two planks, each 25 mm thick and 225 mm wide, are joined by the mortise joint shown. Determine the load P_{ult} that will cause the joint to fail if the ultimate shear stress is 10 MPa and the ultimate normal stress is 25 MPa (along the grain direction parallel to the tensile axis).
Strategy: consider the different ways in which the joint could fail
Hint: because the structure is periodic you only need to consider a portion of it

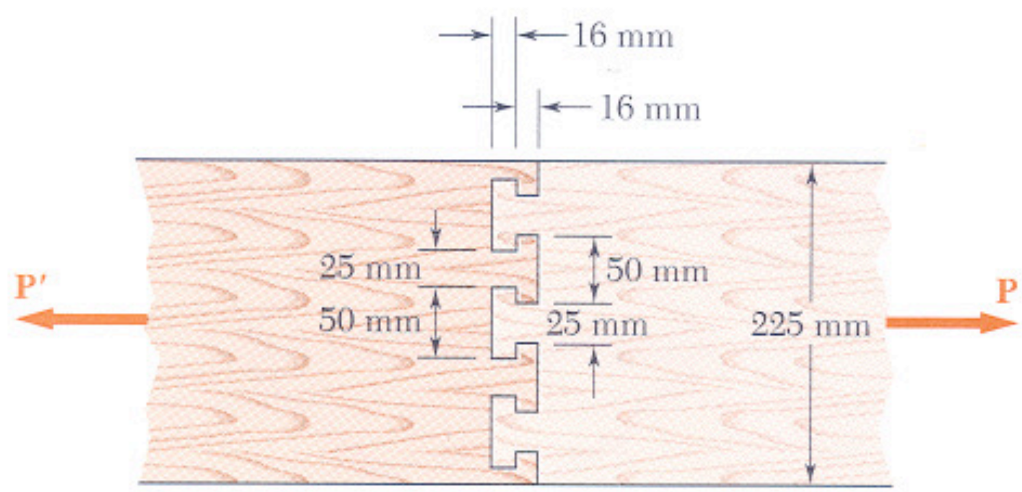
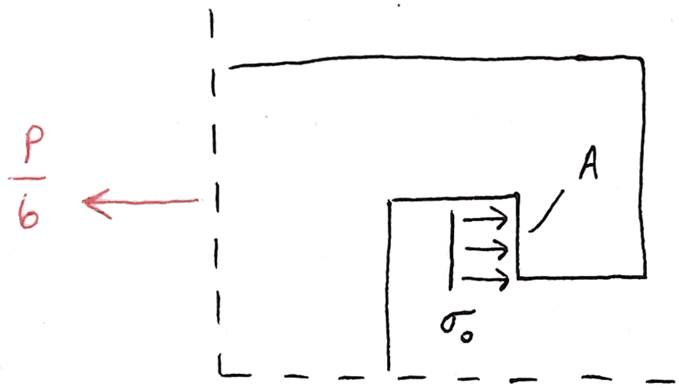
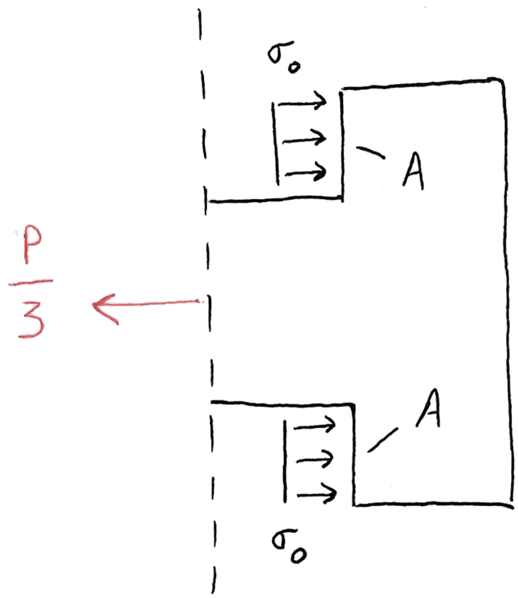


Figure 1.3

Solution 1.3.

To analyze the ways this mortise joint might fail, we can look at the left board. Furthermore, because the structure is periodic, we only need to consider a portion of it in our analyses. When considering a portion of the board, we also consider the portion of the external force P that acts on the portion of the board. Two options for the portion of the board you can look at are:





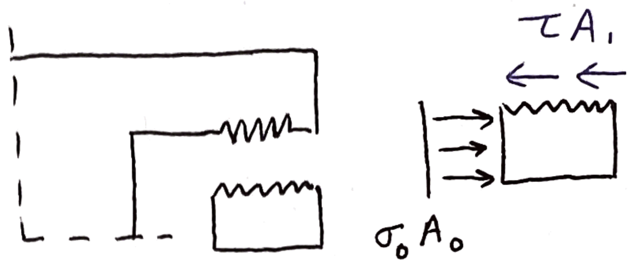
The stress σ_0 is caused by the contact of the two boards. Doing the sum of the forces for both figures will give the same result for σ_0. For the top board, we have:

$$\Sigma F = 0 = -\frac{P}{6} + \sigma_0 A_0$$

$$\sigma_0 A_0 = \frac{P}{6}$$

Where A_0 is the contact surface area between the boards, which is the thickness, 25 mm, multiplied by the vertical distance of $\frac{50-25}{2} = 12.5\ mm$. Therefore, $A_0 = 312.5\ \text{mm}^2$.

To determine the load P_{ult} we must analyze how the board will fail in shear, in tension, and in compression. The board will fail in shear in the following manner:



Doing the sum of the forces for this free body diagram gives:

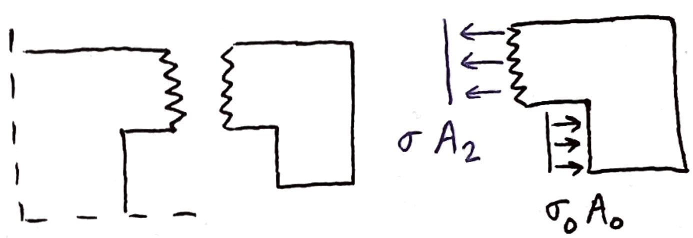
$$\Sigma F = 0 = \sigma_0 A_0 - \tau A_1$$

$$\tau A_1 = \frac{P}{6}$$

Where $A_1 = 16 \times 25 = 400\ mm^2$. **(Be sure to include the thickness of the board when calculating the area.)** Plugging in $\tau_{fail} = 10\ MPa$ gives a value of P_{ult}:

$$P_{ult} = 6\tau_{fail}A_1 = 6(10 \times 10^6)(400 \times 10^{-6}) = 24\ kN$$

The board will fail in tension in the following manner:



Doing the sum of the forces for this free body diagram gives:

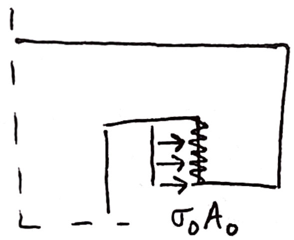
$$\Sigma F = 0 = \sigma_0 A_0 - \sigma A_2$$

$$\sigma A_2 = \frac{P}{6}$$

Where $A_2 = \frac{25}{2} \times 25 = 312.5\ mm^2$. Plugging in $\sigma_{fail} = 25\ MPa$ gives a value of P_{ult}:

$$P_{ult} = 6\sigma_{fail}A_2 = 6(25 \times 10^6)(312.5 \times 10^{-6}) = 46.88\ kN$$

The board will fail in compression in the following manner:



The stress due to failure in compression is σ_0, which gives a P_{ult} of:

$$P_{ult} = 6\sigma_{fail}A_0 = 6(25 \times 10^6)(312.5 \times 10^{-6}) = 46.88\ kN$$

We see that the limiting case is failure in shear. Therefore, P_{ult} = 24 kN

Exercise 1.4.

The pin at C in Figure 1.3 has a diameter of 0.75 in. and is in double shear. The cross section areas of members AB and BC are 4 in^2 and 4.5 in^2, respectively. Determine the average axial stress in member AB (away from pin) and the average shear stress in pin C, both in psi.

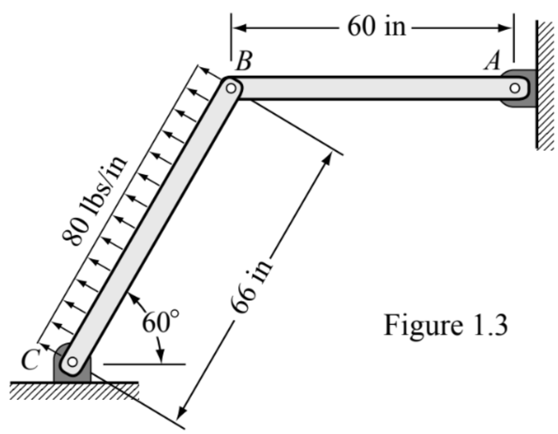


Figure 1.3

Solution 1.4.

Givens and unknowns:

$$\boxed{d_c = 0.75in \quad A_{AB} = 4in^2 \quad A_{BC} = 4.5in^2 \quad \sigma_{AB} =? \quad \tau_C =?}$$

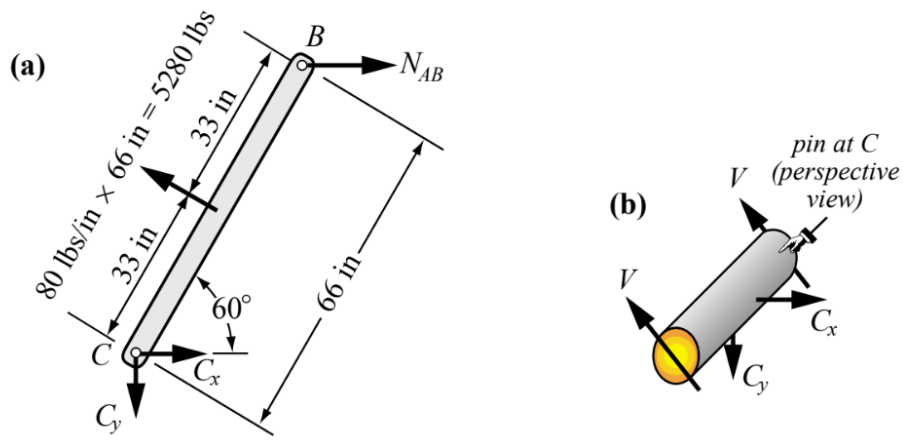


Figure S1.3. FBDs used to solve Exercise 1.3.

The free body diagram (a) can be redrawn with the distributed force replaced by an equivalent force resultant of $80\frac{lbs}{in} * 66in = 5280\ lbs$. By moment equilibrium about C we obtain

$$33in * 5280lbs - 66in * N_{AB} * \sin 60° = 0$$

$$=> N_{AB} = 3048.4\ lbs$$

The axial stress in bar AB away from the pins is

$$\boxed{\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{3048.4\ lbs}{4\ in^2} = 762.1\ psi}$$

By force equilibrium in the x-direction we get $C_x + N_{AB} - 5280 * \cos 30° = 0$ which gives $C_x = 1524.2\ lbs$. By force equilibrium in the y-direction we get $-C_y + 5280 * \sin 30° = 0$ which gives $C_y = 2640\ lbs$.

The FBD in Figure S1.3 (b) shows pin C in double shear. By force equilibrium we get $2V = \sqrt{{C_x}^2 + {C_y}^2} = \sqrt{1524.2^2 + 2640^2} = 3048.4\ lbs$ and $V = 1524.2\ lbs$, where V is the resultant force acting on each pin area. This area is the same as the pin cross-section: $A_s = A_c = \frac{1}{4}\pi {d_c}^2 = 0.4417\ in^2$. The average shear stress in pin C is

$$\boxed{\tau_c = \frac{V}{A_c} = \frac{1524.2\ lbs}{0.4417\ in^2} = 3450.76\ psi}$$

Note: It is also legitimate to divide the total axial force transmitted at the pin, which is $2V = N_{AB}$ by the double shear area. As may be expected, this gives the same τ_c.

Exercise 1.5

A cylindrical pressure tank 20-ft long and 48-in in mean diameter is to be fabricated from a 1/2 in thick steel. A 20 ft long, 12-in wide and 1/2-in thick plate is bonded into the tank to seal the gap, as shown in Figure 1.5(a). What is the average shear stress in the adhesive when the internal pressure in the tank is 725 psi? Assume that the shear stress over the entire inner surface of the attaching plate is uniform. The appropriate FBD is pictured in Figure 1.5(b).

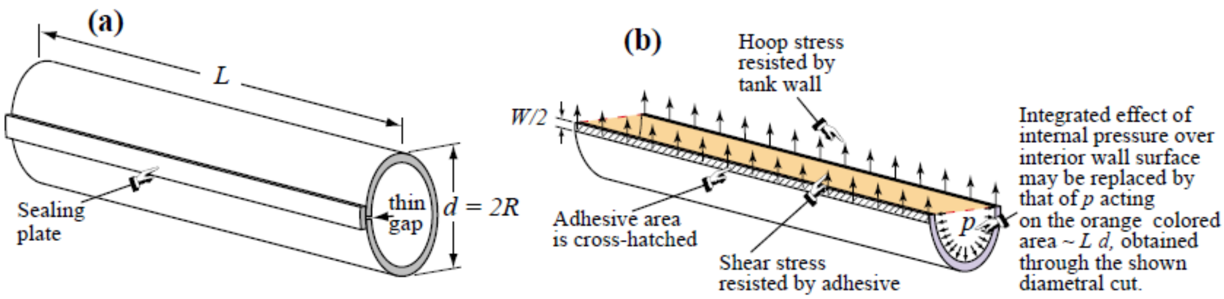


Figure 1.5

Solution 1.5. Givens and unknowns:

$$\boxed{L = 20 \text{ ft}, \ d = 2R = 48 \text{ in}, \ t = 1/2 \text{ in}, \ w = 12 \text{ in}, \ \tau_{av} = ?}$$

Here L is the tank length, $d = 2R$ the diameter, t denotes both the tank wall and sealing plate thicknesses, w the width of the sealing plate, and τ_{av} denotes the average shear stress in the glue.

The hoop stress in a pressurized cylinder of radius R, derived in the Lecture 3 Notes, is

$$\boxed{\sigma_{\theta\theta} = \frac{pR}{t} = \frac{725 \ psi \times 24 \ in}{1/2 in} = 34800 \ psi} \#(1)$$

Make two surface cuts, one through the adhesive, and another one diametrically through the tank to produce the FBD shown in Figure 2.1(b). Force equilibrium in the circumferential direction gives

$$\boxed{\tau_{av}\left(\frac{1}{2}w\right)L + \sigma_{\theta\theta} L t - pLd = 0} \#(2)$$

where t is the wall thickness and w the width of the sealing plate. The last term in the LHS of (2) is the pressure force acting on the orange colored area of Figure 1.5(b). Solving for τ_{av} we get

$$\boxed{\tau_{av} = \frac{pd - \sigma_{\theta\theta} t}{w/2} = \frac{725 psi \times 48 in - 34800 psi \times 1/2 in}{12 in/2} = 2900 psi} \#(3)$$