ASEN 3112

Spring 2020

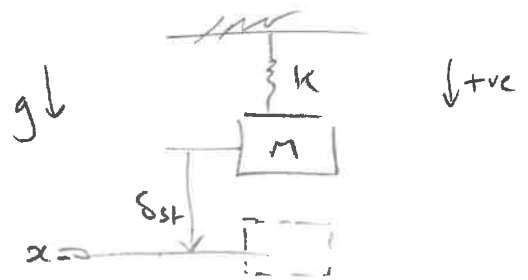
Lecture 17

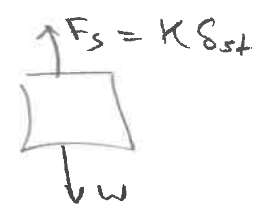
Whiteboard

March 12, 2020

$\frac{L17}{1}$ Single degree of freedom (1DOF)

- Undamped
- Free



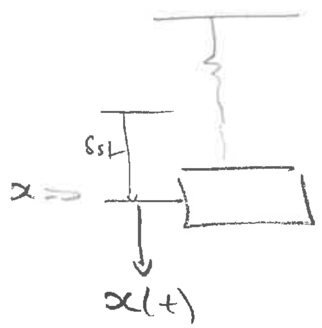


Statics

$\Sigma F_y = 0 \qquad W - F_s = 0$

$W - k\delta_{st} = 0 \Rightarrow \delta_{st} = \frac{W}{k}$

Dynamics



Note: x: DoF
(u: DOF in notes)

$\Sigma F_y = ma$

$W - F_s = m\ddot{x}$

$m\ddot{x} = W - k(\delta_{st} + x)$

$m\ddot{x} = \underbrace{\cancel{W} - \cancel{k\delta_{st}}} - kx$

$m\ddot{x} + kx = 0 \longrightarrow$ EOM (2nd order ODE)

$\ddot{x} + \frac{k}{m}x = 0 \qquad \omega_n^2 = \frac{k}{m} \Rightarrow \omega_n = \sqrt{\frac{k}{m}}$

$\boxed{\ddot{x} + \omega_n^2 x = 0}$

$\hookrightarrow$ natural

$$x(t) = B_1 \cos \omega_n t + B_2 \sin \omega_n t \longleftarrow (1)$$

ω_n: natural frequency $\quad \omega_n = \frac{2\pi}{T}$, T: period.

ω_n: $\frac{rad}{s}$

$$B = \sqrt{B_1^2 + B_2^2}$$

$$\phi = \tan^{-1} \frac{B_1}{B_2}$$

Example $\quad$ consider same system

$x(0) = x_0 \longleftarrow$ given.

$\dot{x}(0) = V_0 \longleftarrow$ given $\qquad V = \dot{x} = \frac{dx}{dt}$

Use EOM (1) + IC' to obtain complete solution

$$\dot{x}(t) = -B_1 \omega_n \sin \omega_n t + B_2 \omega_n \cos \omega_n t \text{ ——— } (2)$$

(1) At $t=0$, $x(0) = B_1 = x_0$

(2) At $t=0$, $\dot{x}(0) = B_2 \omega_n = V_0 \Longrightarrow B_2 = \frac{V_0}{\omega_n}$

$$\boxed{x(t) = x_0 \cos \omega_n t + \frac{V_0}{\omega_n} \sin \omega_n t, \quad \omega_n = \sqrt{\frac{k}{m}}}$$

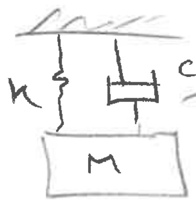
L17
3

1DOF

— Damping
— Free

Restoring force: Kx

Damping force: $c\dot{x}$



viscous damping

K: $\frac{N}{m}$

c: $\frac{N}{m/s}$

$$\underbrace{m\ddot{x}}_{\text{inertial force}} + \underbrace{c\dot{x}}_{\text{damping force}} + \underbrace{kx}_{\text{elastic restoring force}} = 0$$

$$\underbrace{\omega_n = \sqrt{\frac{k}{m}}}_{\text{natural frequency}}, \quad \underbrace{\zeta = \frac{c}{2m\sqrt{\frac{k}{m}}}}_{\text{damping ratio}} = \frac{c}{2\sqrt{km}} = \frac{c}{c_c}$$

$$\zeta: \frac{c}{c_r} : \frac{\text{prescribing damper quantity}}{\text{critical damping quantity}}$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0 \quad \text{— EOM}$$

Assume $x(t) = Ae^{\lambda t}$

$$\dot{x} = \lambda A e^{\lambda t}$$

$$\ddot{x} = \lambda^2 A e^{\lambda t}$$

substitute into EOM:

$$\boxed{\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0} \quad \text{— characteristic equation}$$