ASEN 3112:
Spring 2020
Final Exam

Date: May 2, 2020
Time: 1:30 – 3:30 pm MDT

Name: ____

Student ID: ____________________________________

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this exam.

Name: ____

Signature: ____

Date: ____

Please label your work with the part of the problem you are working on and circle your final answer for each part.

You can ask any questions of Dr. Johnson (Questions 1 and 2) and Prof. Hussein (Questions 3 and 4) in a private chat at this Zoom meeting: https://cuboulder.zoom.us/j/4179468494

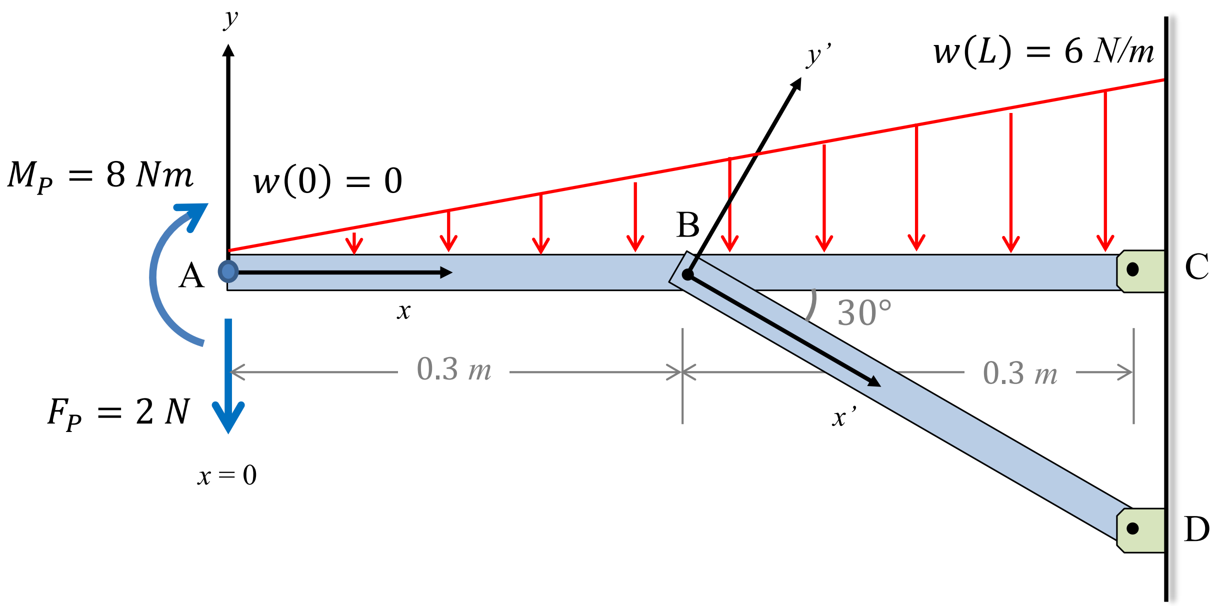
1. This exam is open-book, open-notes.
2. Solve all four problems.
3. Total time for the exam is 2 hours.
4. Do not redefine the problem; carefully read the problem statement and answer the questions that are asked.
5. Make sure that one can follow your analysis; describe briefly what you are doing.
6. You must show how you got to your solution. Simply showing the final results will lead to point reduction.
7. You must cross out all work you do not want graded. Any work that is not crossed out is fair game for grading.
8. Include units on final answers whenever applicable.
9. Write your solution on the exam sheets using the space provided for each question.
10. You can either print the exam and write the answers in the empty parts (and add additional sheets as needed), or you can write all your answers in empty sheets and number them and order them properly.
11. One you finish your exam, scan it, please scan it and upload a single pdf file of your complete exam to Gradescope. Make sure all pages are numbered and included in the proper order.

This page to be used for additional work.

Please write the problem number here: ____________

Question 1. 25 points

You are a member of a senior projects team who is designing a blimp-mounted telescope. As the Structures Lead, you are analyzing the system that holds one of the propellers. A free-body diagram of the system can be seen below:



There are two parts to this system: member ABC and member BD. Both members have an elastic modulus $E = 70\ GPa$, a cross-sectional area $A = 0.25\ m^2$ a moment of inertia $I = 5.2 \times 10^{-7}\ m^4$, and a Poisson's ratio $\nu = 0.3$. All joints at B, C, and D are pinned joints.

The following external forces and moments act on member ABC:

- The distributed load *w(x)*, which represents the member's weight
- The point force $F_P = 2\ N$ at the left end ($x = 0$), which represents the propeller's weight.
- The clockwise point moment $M_P = 8\ Nm$ at the left end, which is caused by the spinning propeller.

Ignore the weight of the spar BD.

(a) Draw a global free-body diagram of the entire two-member system.

The following questions all relate to member BD. In these questions, I refer to the $x' - y'$ coordinate system centered at B, where the x' axis goes from B to D.

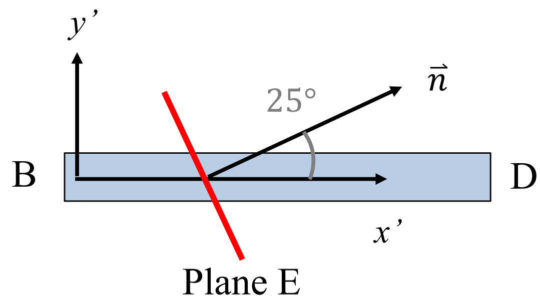
(b) Calculate the normal axial stress $\sigma_{x'}$ in member BD. Is member BD in tension or compression?

(c) Calculate the normal axial strain $\epsilon_{x'}$ in member BD.

Problem 1 continues on the next page

(d) What is the magnitude and sign of the normal stress σ_E on plane E passing through member BD? Plane E is shown in red in the figure below, and the surface normal $\vec{n}$ of plane E is at an angle of $25°$ from the x' axis.

Hint: There is both normal stress and shear stress on plane E. (However, this problem only asks you to calculate the normal stress.)



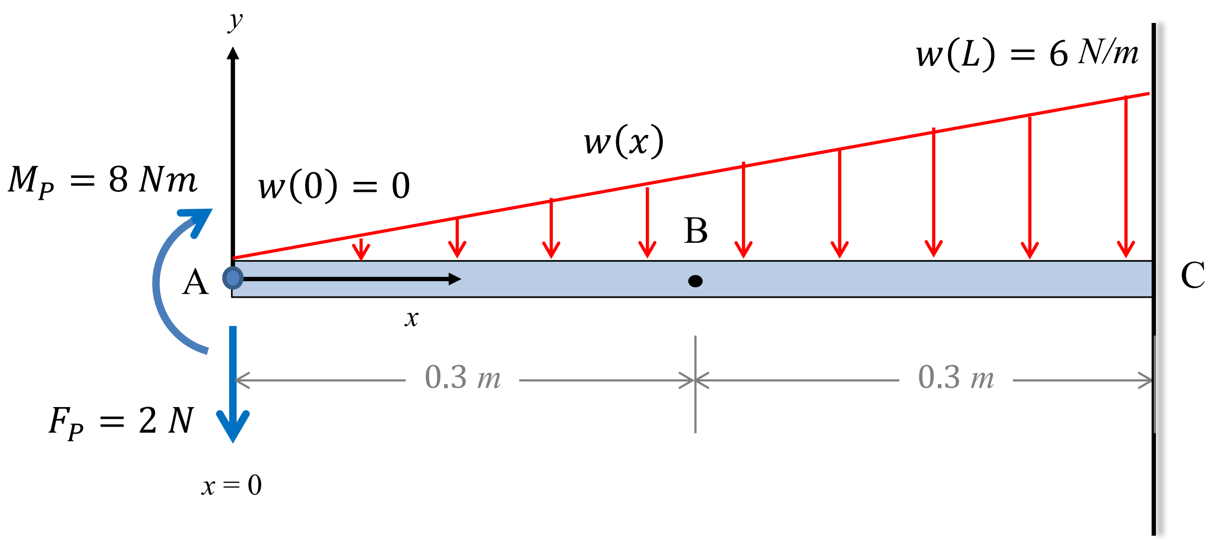
Write work for Problem 1 here:

Continue work for Problem 1 here:

Question 2.   25 points
Note there is both Part I and Part II

Part I.

Later in the design phase, your senior projects team decides to change the design of your system to remove member BD. Member ABC is now clamped (welded) at C. This is seen in the following diagram:



Member ABC has an elastic modulus $E = 70\ GPa$, a cross-sectional area $A = 0.25\ m^2$ a moment of inertia $I = 5.2 \times 10^{-7}\ m^4$, and a Poisson's ratio $\nu = 0.3$.

The following external forces and moments still act on member ABC:

- The distributed load *w(x)*.
- The point force $F_P = 2\ N$ at the left end ($x = 0$).
- The clockwise point moment $M_P = 8\ Nm$ at the left end.

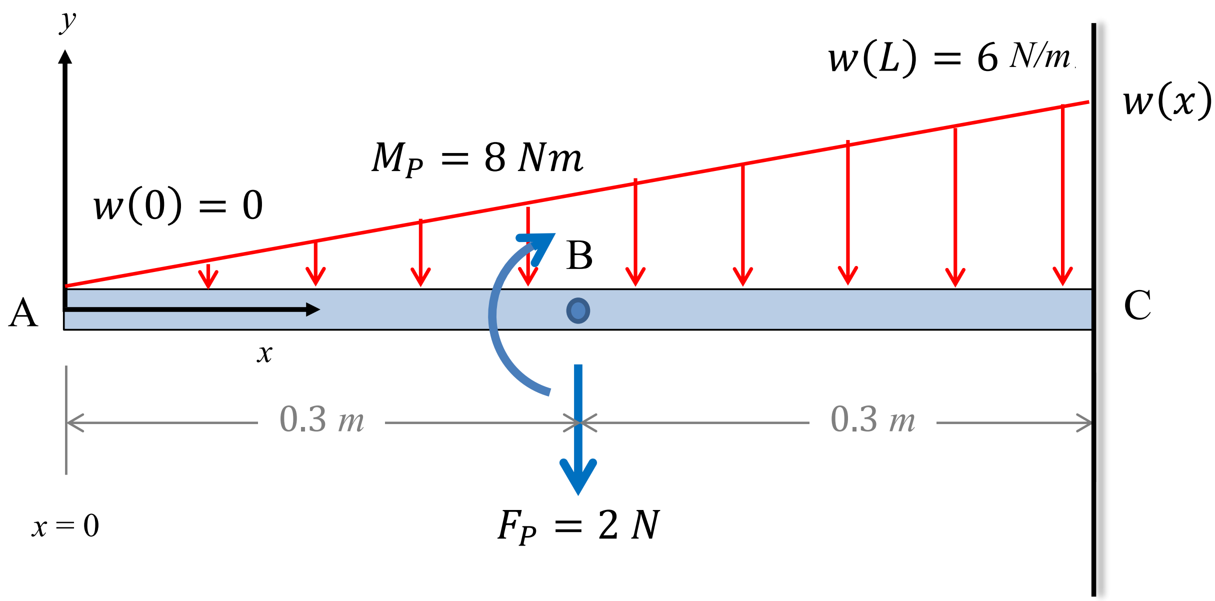
(a) Determine the equation for the bending moment $M(x)$ throughout the entire length of member ABC (i.e. from $x = 0$ to $x = L$).

(b) Is the top of member ABC (in the $+y$ direction) in tension or compression at B ($x = 0.3\ m$)? Justify your answer.

Write work for Problem 2, Part I here:

Part II.

There has been yet another change to your system (this is literally how senior projects goes), and your team decides to move the propeller to the middle of member ABC (at B, $x = 0.3\ m$). The only thing that has changed from Part II is the location of F_P and M_P:



(c) With this new system, is the equation for $M(x)$ a piecewise function? If so, give the x-limits of each piece. **Note: You should NOT calculate the new $M(x)$ equation.**

(d) Write all of the boundary conditions and matching conditions needed to fully solve for all the constants that arise when finding the deflection $v(x)$ by integrating the fourth-order ordinary differential equation $EIv''''(x) = w(x)$. **Note: You should NOT do any integration for this part.**

Write work for Problem 2, Part II here:

Continue work for Problem 2, Part II here:

Question 3. 20 points

Part I

The roots of the characteristic equation of a SDOF system is provided in parts (a) through (d). For each case, indicate in the table the type of damping (undamped, underdamped, critically damped, or overdamped) and the value of the damped natural frequency of oscillation (or natural frequency if the system is undamped). If the system does not exhibit oscillation, then the notion of frequency does not apply. Use the space provided under the table <u>to write the formula that you have used for each of these four cases</u>. You will not get full credit if you do not write the formulas you use.

	Roots of Characteristic Equation [rad/s]	Type of Damping: Undamped, Underdamped, Critically damped, or Overdamped	Damped Natural Frequency (Or Natural Frequency if Undamped) [rad/s] *Write a value or "N/A"*
(a)	$\lambda = \pm 2\mathbb{i}$		
(b)	$\lambda = -2, -2$		
(c)	$\lambda = -1, -3$		
(d)	$\lambda = -1 \pm 2\mathbb{i}$		

<u>**Key**</u>

$\mathbb{i} = \sqrt{-1}$

N/A: Not apply.

Write work for Problem 3, Part I here:

Part II

Consider a SDOF spring-mass-damper system with k = 1000 N/m, m = 10 Kg, c = 10 N-s/m.

(a) What should the damped natural frequency of this system? Write the formula and evaluate it and give the numerical value in the correct units.

(b) Find the transient response (i.e., free vibration response) of the system when subjected to the initial conditions $x_0 = 0.1$ m and $\dot{x}_0 = 1$ m/s. Show the expression for your solution in symbolic form first, then substitute in the numbers.

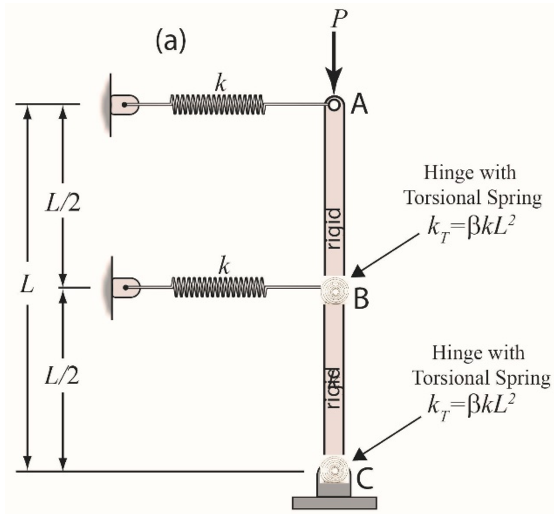
(c) For this system to be critically damped, the dashpot parameter c should be multiplied by a factor of R. What is R? Show how you obtained this answer.

Write work for Problem 3, Part II here:

Continue work for Problem 3, Part II here:

Question 4.   30 points

The 3-hinged column below consists of two rigid links (struts) AB and BC, each of length *L/2*. Each of Points B and C represent a pinned joint with a torsional spring as shown. A lateral spring is also present at each of Points A and B as shown. The torsional stiffness of each torsional spring is defined in terms of $\beta \geq 0$ which is a nondimensional parameter. This system is susceptible to buckling. Under a linear state of buckling, assume all rotational angels to be very small, that is much less than 1.



(a) This system has two degrees of freedom, $\theta_1 = \theta_C$ and $\theta_2 = \theta_B$, each defined about a vertical axis at Point C and B, respectively. Draw two free-body diagrams: one for the entire system, and one for link AB. Show the two degrees of freedom and show all forces and moments in your free-body diagrams.
(b) Derive the linearized equilibrium equations of motion for the deflected (titled) configuration.
(c) Write the two resulting equations in matrix form where the entries of the matrix equation are in terms of *P*, *L*, *k*, and β.

For the following parts [i.e., Parts (d) and (e)], assume $L = k = 1$ and $\beta = 2$.
(d) Find the determinant of matrix **A** and set it equal to zero. This gives you the characteristic equation. By solving this equation for its *P* roots, find the two critical load values. Call them P_{cr1} and P_{cr2}. Among these two values, specify which is *the* critical load P_{cr}.
(e) Find the buckling mode shape corresponding to *the* critical load P_{cr}.

Write work for Problem 4 here:

Continue work for Problem 4 here:

Continue work for Problem 4 here: