ASEN 3112 Spring 2020

Lab 4: Beam Buckling - Description Document

I Summary

In the fourth and last Structures Lab, you will investigate the buckling of beams. The goal of this lab is to predict the buckling load for two beams of different materials and/or cross sections, track the post-buckling behavior of the beam, and compare elastic and plastic post-buckling.

This document provides theoretical background to beam buckling, a description of the experimental procedure, and information on the expected data processing.

II Timetable, Groups and Logistics

II.1 Timetable

This lab spans nearly three weeks, with reports due on Tuesday April 30th, 2020. Under normal circumstance, the experimental procedure demos for this lab are held in the Co-PILOT and students are asked to physically attend and observe the experimental testing. However, due to the current circumstances and campus closure, the entire lab will be conducted remotely via online tools.

This lab will be done in groups. The groups are formed based on students signing up for a group; see Section II.2. Groups should meet online ASAP and get organized. Each student in each group is expected to make a significant contribution to the lab activities, see Section VII.3 for more details.

II.2 Groups

The groups are formed by the students by signing up using SignUpGenius. Each group has a limit of 7 students. As soon as this limit is reached, a student is no longer permitted to sign up for that group. The sign up is available at the Sign-up Genius website. See the file "ASEN 3112 Lab 4 Project Steps.pdf" for the URL links. There is a single sign-up URL link common for both sections, i.e., students can team up with classmates from the same section or the other section.

As in previous labs, each group should select a leader (the Group Leader, or GL) who will have the following responsibilities:

- Divide tasks to be accomplished in writing the report (e.g. writing specific sections, analyzing data and producing necessary plots/figures).
- Compile and edit the final report (ensure consistency between sections and make sure that other member's contributions are satisfactory).
- Provide internal deadlines to group members so that the lab report project stays on schedule.
- Keep a record of delegated tasks, internal deadlines, and confirmations from team members. This record can simply be a thread of emails between the group leader and group members. This will not be turned in but will be used by the TAs to resolve any disputes about participation scores.
- Provide a participation report for your group with a brief summary of each group member's tasks, contributions, and performance as a group member. It is the group leader's responsibility to organize a peer evaluation process to determine each group member's contribution grade; see Section VII.3. Note, the group leader should not assign a participation score without the input of the entire group.

Group leaders do not have to write their own section of the report (though they can if they want to). Group members are responsible for timely communication with the group leader. If a student is assigned a task to complete with a deadline, the student should confirm that she/he will do so. If the student does not agree to the task or has difficulty with it and needs more time or help with the task, this should also be communicated to the group leader (well before the deadline).

II.3 Lab Reports

Each group prepares and submits one word-processed electronic copy of the report, which is due by 5 pm on Thursday April 30th. Instructions for preparing this report are given in Section VII of this document.

III Buckling of beams

The buckling of beams are covered in Lectures 22, 23, and 25, and a detailed description of the problem is presented in Chapter 28 and the corresponding lecture notes. A brief summary, sufficient to conduct the lab, is presented here (please read it carefully to gain the necessary background for the problem).

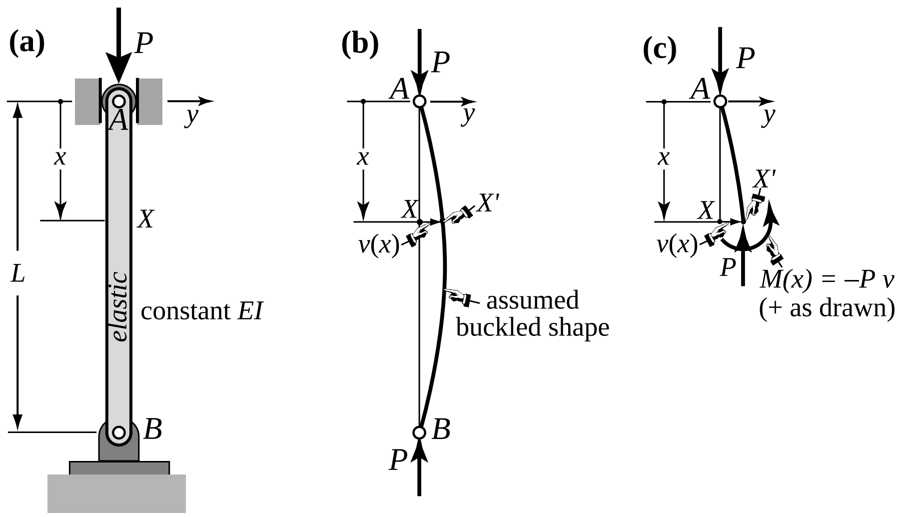

Figure 1: The Euler column: (a) problem definition; (b) FBD of whole column assuming a buckled shape $v(x)$ with its amplitude exaggerated for visibility; (c) FBD at distance x from top.

The simplest case to study the buckling of beams is the so-called Euler column, shown in Figure 1(a). It is a homogeneous, prismatic, elastic column pinned (*i.e.*, hinged, simply supported) at both ends A and B, and subjected to an axial loading P. The typical response, axial loading P versus column shortening (vertical deflection) u, is shown in Figure 2. As the compressive load increases, the beam remains initially straight and it shortens due to compressive strain, and the corresponding $P - u$ slope is very high. As a critical buckling load P_{cr} is reached, the beam buckles and no longer remains straight, see Fig. 1(b-c). If further compression is applied, the response of the now buckled beam changes drastically, with a significant reduction of the slope, as shown in the schematic in Figure 2. This is usually referred to as post-buckling regime. Depending on the slenderness of the beam, plastic yield will take place before buckling, just after buckling, or well into the post-buckling regime. The precise shape of the curve depends on both the geometry and material properties of the beam.

In the case of the Euler buckling of beam, the buckling load can be calculated as:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{1}$$

where E is the Young's modulus, I is the second moment of area of the beam, and L is its length.

The buckled beam deforms according to the corresponding mode shape, which is determined by the geometry and boundary conditions of the problem. It is shown in Lecture 23 that, in the case of pinned-pinned boundary conditions, the mode shape corresponds to a sine, shown in Figure 1(b), of equation:

$$v(x) = \delta \sin \frac{\pi x}{L} \tag{2}$$

where the precise value of the maximum deflection, δ, evolves as additional loading is applied. The relationship between load and lateral deflection is given by an elliptic integral with no exact solution, but it can be approximated as:

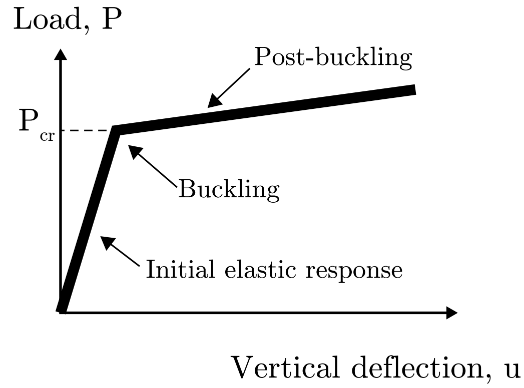

Figure 2: Schematic of the buckling response of an Euler column, showing initial elastic regime, buckling, and post-buckling behavior.

$$P = P_{cr} \left(1 + \frac{\pi^2}{8L^2} \delta^2 \right) \tag{3}$$

where P is the applied loading after the beam has buckled.

This provides a different way to present the buckling response of a beam. In Figure 3, we plot the evolution of the load, as a function of the maximum *horizontal* displacement of the beam, δ. In this case the response before buckling is a vertical line (since the load increases, but there is no lateral deflection, only compression), and then after buckling the horizontal deflection increases rapidly. As with Figure 2, the precise nature of the post-buckling response depends on geometry and material properties.

Due to the geometry of a sine, the vertical displacement at the end where the load is applied (*i.e.*, the *shortening* of the beam) is much smaller than the maximum horizontal deflection, meaning that for the same value of the loading, the number of the x-axis in Fig. 3 will in general be much larger than those in the x-axis of Fig. 2. Measuring the maximum lateral displacement, δ, is therefore much simpler, and it will be the approach followed in this lab.

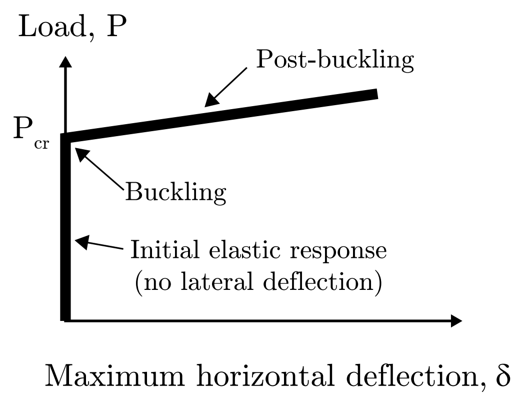

Figure 3: Schematic of the buckling response of an Euler column, showing initial elastic regime, buckling, and post-buckling behavior.

We now look at the strains in the bar, with the goal of predicting yielding. For a sufficiently slender beam, such as those used in this lab, buckling will take place in the elastic regime, and plastic yield will occur during post-buckling, once some region of the beam reaches the yield strain ϵ_y. We can assume that the strain due to net compression of the beam is much smaller than the strain due to the bending following the buckling mode. In that case, the strain at every point of the beam can be calculated by:

$$\epsilon = \kappa(x) y \tag{4}$$

where κ is the curvature of the beam, and y is the distance to the neutral axis. In the case of beams with symmetric cross section, the neutral axis is at the center of the cross section, and the maximum distance, y_{max}, is equal to half the thickness of the beam in the case of rectangular or square cross sections. Please note that the thickness of the beam is NOT the wall thickness in the case of a beam where the cross section is a hollow tube.

As a reminder, the curvature of a beam can be approximated by:

$$\kappa(x) = v(x)'' = \frac{d^2 v}{dx^2} \tag{5}$$

which can easily be calculated from the buckling mode, Equation 2.

Combining Equations 2, 4 and 5 we can predict the lateral displacement δ associated with yielding, δ_y. This will result in a clear transition between elastic and plastic post-buckling in the $P - \delta$ curve shown in Fig. 3.

IV Test articles

You are given the results for test two different samples for two different types of beams (i.e., four data sets in total). All were obtained using the same procedure. Due to the different geometries of the two configurations, one type will reach the plastic regime early in the test, while the other one will remain elastic for much longer.

The two different test configurations are:

- Aluminum square hollow cross section. Outer dimensions 0.25 in x 0.25 in, wall thickness 0.0625 in. Maximum length 12 in, minimum length 10 in. Material is 6061-T6 aluminum, with Young's modulus 10,000,000 psi and yield stress 35,000 psi.
- Aluminum rectangular solid cross section. Outer dimensions 0.125 in x 1 in. Maximum length 12 in, minimum length 10 in. Material is 6061-T6 aluminum, with Young's modulus 10,000,000 psi and yield stress 35,000 psi.

Assume the length of all samples to be 11 and 3/8 inches. For each type, either select the highest quality data set and omit the other, or if both data sets are of good quality, analyse both and for your results report the statistical average. This is your choice. The length of a sample has to be as such that the specimen fits in the test rig, and also such that the buckling load will not exceed the maximum load admissible by the load cell. The length is an important parameter in the calculations.

V Pre-test calculations

The following calculations ideally should be done before the experimental component of the lab, as a function of the specimen length, which can be incorporated in the calculations at the beginning of the test. In all cases, it should be remembered to add 0.4 inches to the length of your test articles, to account for the fixtures used to connect them to the testing rig (more details below).

Since you are provided the lab by video and online data, please just make sure you make these calculations before you start examining the experimental results.

- Calculate the buckling load for both test articles.
- Calculate the lateral deflection at which each test article yields, i.e., the point at which the maximum strain is equal to the yield strain of the corresponding material. That deflection should mark the transition between elastic and plastic post-buckling.

VI Test procedure

This section will provide a brief description of the test goals and procedure. A detailed description of the Setup and Operational Procedure is provided in a different document (which is optional reading).

The goal of the test is to obtain the critical buckling load of each of the test articles, as well as the evolution of the loading as a function of the lateral displacement of the beam. The test articles are connected to the testing frame using custom made fixtures. The fixtures have groves that connect to spikes in the test stand, allowing free rotation at both ends of the test article, which corresponds to pinned-pinned boundary conditions.

During the test, we would slowly turn the knob of the test stand clockwise. This increases the compressive load on the specimen, which is measured by the load cell. Buckling can be identified as the point at which the beam is no longer straight. Due to initial imperfections (*i.e.*, initial waviness of the beam) it is possible that this load is significantly lower than the theoretical prediction, or that it drops suddenly after buckling. It is therefore recommended to record the evolution of the load during the experiment, so that the buckling load can be determined more precisely. It is also important to ensure the maximum operating load of the load cell is not surpassed.

Once buckling occurs, there will be noticeable lateral displacement on the test article. The test stand has a ruler attached, which allows for measuring the maximum lateral displacement, which takes place at the center of the specimen. We would keep turning the knob and record the measured voltage after every 1/8 inch of lateral deflection, until we reach at least two inches of lateral deflection (we may continue loading further if we wish), recording the load at each 1/8 inch increment. This is the data you will use to generate a load vs. lateral deflection curve for each test article, and compare it with Equation 3.

VII Writing A Report

VII.1 Organization

Reports are due April 30th, 2020, by 5 pm. **The report must be electronically processed, e.g. by WORD or LATEX; and submitted via Gradescope**.

It must include:

- **Title Page**: Describes Lab, lists the name of the team members and identifies the group leader.
- **Results:** The results should address the questions in Section VII.2.
- **Appendix - Code.** A printout of all the code used to produce the results.
- **Appendix - Participation report.** More details on how the grade of individual group members is calculated can be found in Section VII.3.

VII.2 Report Content

The report will be graded on both technical content and presentation. Regarding the content, instead of an open-ended report, you should process the experimental data and computational results to address the specific questions posted below. Regarding presentation, make sure that you follow these guidelines:

- All plots should be readable. This includes using different color or line styles and a suitable font size for the axis labels and all other text in the plot. The range of both axes should be chosen to focus on the region of interest (*i.e.*, the data).
- Show your work, including equations used and partial results.
- All results should be presented with appropriate units.
- Be quantitative when comparing results. Use percentage of error or deviation. Refer back to predicted error or variance when applicable.

VII.2.1 Question 1: Buckling Load

- Predicted and experimentally measured buckling load for each of the test articles. Quantify and discuss the errors.

VII.2.2 Question 2: Post-Buckling Behavior

- Predicted lateral deflection corresponding to the initiation of plastic post-buckling.
- Plots of the predicted and experimentally measured curves of applied load vs. lateral deflection for each of the test articles, similar to the schematic shown in Fig. 3. Include a vertical line identifying the predicted initiation of plastic post-buckling.

VII.2.3 Question 3: Theoretical Design Study

This part is independent of the lab data.

Consider now two beams with the same cross section and made of the same material as the two specimens tested, but with different length, L.

- Plot the buckling load for both beams, as a function of the length, L, considering: (a) simply supported - simply supported, and (b) fixed-fixed boundary conditions.
- On the same figure, plot the load that will result in yielding of the beams under compressive loading.

VII.3 Individual Contribution Evaluation and Deductions for No-Show

The group leader submits along with the report a separate participation report for your group with a brief summary of each group member's tasks, contributions, and performance as a group member. **Please make sure to include this as an Appendix in the report**.

The performance of each group member is rated with a "contribution factor" on a scale of 0 to 100%. A score of 100% indicates that the member contributed the expected share to the experiment and to the preparation of the report. The scores are normally assigned by *peer evaluation*, using the same procedures followed in the ASEN 200x sophomore courses. The group leader is responsible for administering the peer evaluation, tabulating and submitting the results of said peer evaluation.

The individual score will be equal to:

$$\text{Individual score} = \text{Group score} \times \frac{100 + \text{Contribution factor}}{200} \tag{6}$$

For example, if the group receives an overall score of 88.75 and the individual received a "contributing factor" of 90%, the individual score is $88.75 \times (100 + 90)/200 = 84.31$.

Appendix. Report Grading

The score assigned to the lab report includes technical content (75%) and presentation (25%). This is a more detailed breakdown of the weights:

Category	Weight	Score	Contribution
Technical content			
Question 1	0.20		
Question 2	0.30		
Question 3	0.25		
Presentation			
Plots	0.10		
Grammar, style & spelling	0.10		
Formatting	0.05		
Total	1.00		(overall score)

The score within each category ranges from 0 to 100%. For example, if the score for 'Question 1' is 80%, it contributes $0.25 \times 80 = 20\%$ to the overall score. The final score of each team member is then calculated following the procedure detailed in Section VII.3.