Recitation 10

ASEN 3112 – Spring 2020

Problem 1 Buckling of two-DOF lumped parameter model of a column

The 3-hinged column shown in Figure 2(a) consists of two rigid links (struts) AB and BC, both of length L. The column is pinned at support C, propped at A by an extensional spring of stiffness k, and rotationally stiffened at B by a torsional spring of stiffness $k_T = \beta k L^2$, in which $\beta \geq 0$ is a dimensionless parameter.

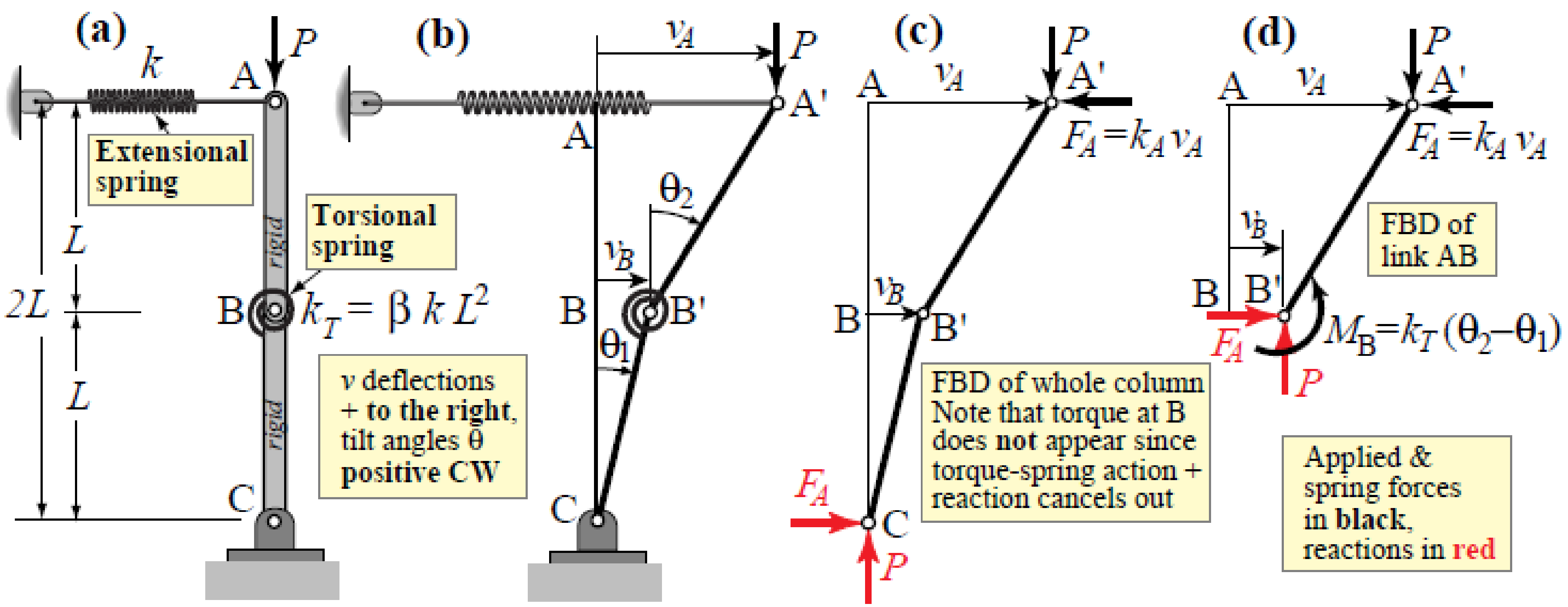

Figure 2: Three-hinged column for Problem 2.

Using the tilt angles θ_1 and θ_2 shown in Figure 2(b) as degrees of freedom (DOF) and linearized stability analysis ($|\theta_1| << 1$ and $|\theta_2| << 1$),

(a) Derive the linearized equilibrium equations in a deflected (tilted) configuration. Two equilibrium

(b) Place the foregoing equations in matrix form

$$\mathbf{A}\theta = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$

where the entries of matrix $\mathbf{A}$ are functions of P, L, k and β.

(c) Find the determinant $\Delta = \det(\mathbf{A})$. This is a quadratic polynomial in P, called the *characteristic polynomial*. The *characteristic equation* is $\Delta = 0$. By solving for its P roots, find the two critical load values. Call them P_{cr1} and P_{cr2}. The smallest of the two is *the* critical load P_{cr}, but which one is the one?

(d) Find the buckling mode shapes.

Problem 2: Cantilever Beam Buckling

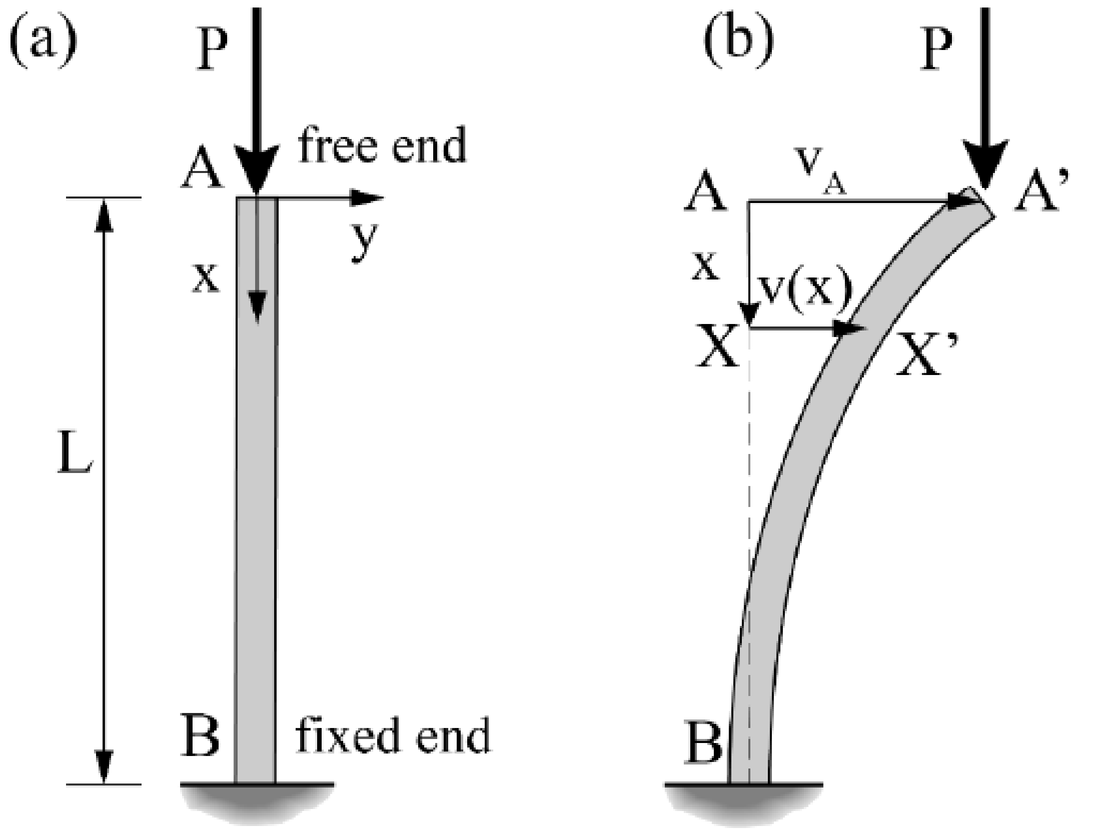

Figure 1. - Column structure for Question I: (a) configuration, (b) sketched buckling

The column of Figure 1(a) has length L, uniform Young's modulus E, and constant second moment of area I. It is fixed at B and free at A. A vertical load P is applied on the free end, causing the beam to buckle as P exceeds a critical value.

(a) Give a FBD of the *entire buckled* column, and calculate the reactions at Point B. Keep v_A as a parameter. Figure 1(b) is *not* a column FBD, only a buckling shape sketch.

(b) Give a FBD of the column cut at distance x from top, to derive the ODE for the lateral deflection $v(x)$. Hint: Take moments with respect to X'.

(c) Verify that the solution has the form: $v(x) = A\cos(\lambda x) + B\sin(\lambda x) + C$, and find the value for λ and C in terms of the parameters E, I, P, and v_A.

(d) Three kinematic boundary conditions are needed in this problem. One boundary condition is obvious from Figure 1(b): $v(0) = v_A$. What are the other two?

(e) Using the boundary conditions, give the characteristic equation for the buckling load. Solve this equation and calculate the critical load, P_{cr}. Hint: As a check, the effective length parameter for this boundary condition is $K = 2$. Note: even if you have the value in the crib-sheets, you still need to calculate P_{cr}.

(f) Write down the final solution for the deflection of the beam, using the results from pars (c), (d), and (e).

(g) Consider now that the beam has indeed buckled, and the displacement at Point A is v_A. Calculate the maximum stress due to bending as a function of the displacement v_A. Assume that the beam has a uniform circular cross section of radius r.