ASEN 3112 STRUCTURES

Lab 3 DESCRIPTION

Spring 2020

Version: April 2, 2020

I Summary

In the third Structures Lab, you will investigate frequency resonances of a highly simplified, scaled model of an airplane, vibrating under prescribed harmonic base excitation produced by a shaker device. The objectives of the lab are:

- To expose you to the phenomenon of vibration resonances of a flexible continuous structure subject to harmonic excitation applied through a shaker.
- To give you a hands-on first experience of an experimental technique (frequency sweep) widely used in industry to locate natural frequencies and thus reveal possibly dangerous frequency ranges exhibiting resonance response.
- To compare a subset of the observed resonant frequencies with natural frequencies predicted by two very coarse Bernoulli-Euler beam finite-element models.

This document provides a description of the experimental setup, measurement procedure, and provides detailed information on the two FEM models.

Before you continue, please read the following file which is available on Canvas: ASEN 3112 Lab 3 Project Steps.pdf

This file gives you a summary of all the main steps you need to take to carry out this lab, including downloading and the reading the present document.

II Timetable, Groups and Logistics

II.1 Timetable

This lab spans nearly three weeks, with reports due on Tuesday April 21st, 2020. Under normal circumstance, the experimental procedure demos for this lab are held in the Co-PILOT and students are asked to physically attend and observe the experimental testing. However, due to the current circumstances and campus closure, the entire lab will be conducted remotely via online tools.

This lab will be done in groups. The groups are formed based on students signing up for a group; see Section II.2. Groups should meet online ASAP and get organized. Each student in each group is expected to make a significant contribution to the lab activities, see Section VI.3 for more details.

II.2 Groups

The groups are formed by the students by signing up using SignUpGenius. Each group has a limit of 7 students. As soon as this limit is reached, a student is no longer permitted to sign up for that group. The sign up is available at the Sign-up Genius website. See the file "ASEN 3112 Lab 3 Project Steps.pdf" for the URL links. There is a sign-up URL link for each section; one for Section 011 and one for Section 012.

As in previous labs, each group should select a leader (the Group Leader, or GL) who will have the following responsibilities:

- Divide tasks to be accomplished in writing the report (e.g. writing specific sections, analyzing data and producing necessary plots/figures).

- Compile and edit the final report (ensure consistency between sections and make sure that other member's contributions are satisfactory).
- Provide internal deadlines to group members so that the lab report project stays on schedule.
- Keep a record of delegated tasks, internal deadlines, and confirmations from team members. This record can simply be a thread of emails between the group leader and group members. This will not be turned in but will be used by the TAs to resolve any disputes about participation scores.
- Provide a participation report for your group with a brief summary of each group member's tasks, contributions, and performance as a group member. It is the group leader's responsibility to organize a peer evaluation process to determine each group member's contribution grade; see Section VI.3. Note, the group leader should not assign a participation score without the input of the entire group.

Group leaders do not have to write their own section of the report (though they can if they want to). Group members are responsible for timely communication with the group leader. If a student is assigned a task to complete with a deadline, the student should confirm that she/he will do so. If the student does not agree to the task or has difficulty with it and needs more time or help with the task, this should also be communicated to the group leader (well before the deadline).

II.3 Lab Reports

Each group prepares and submits one electronic copy of the report through Gradescope, which is due by 5 pm on Tuesday, April 21st, 2020. Instructions for preparing this report are given in Section VI of this document.

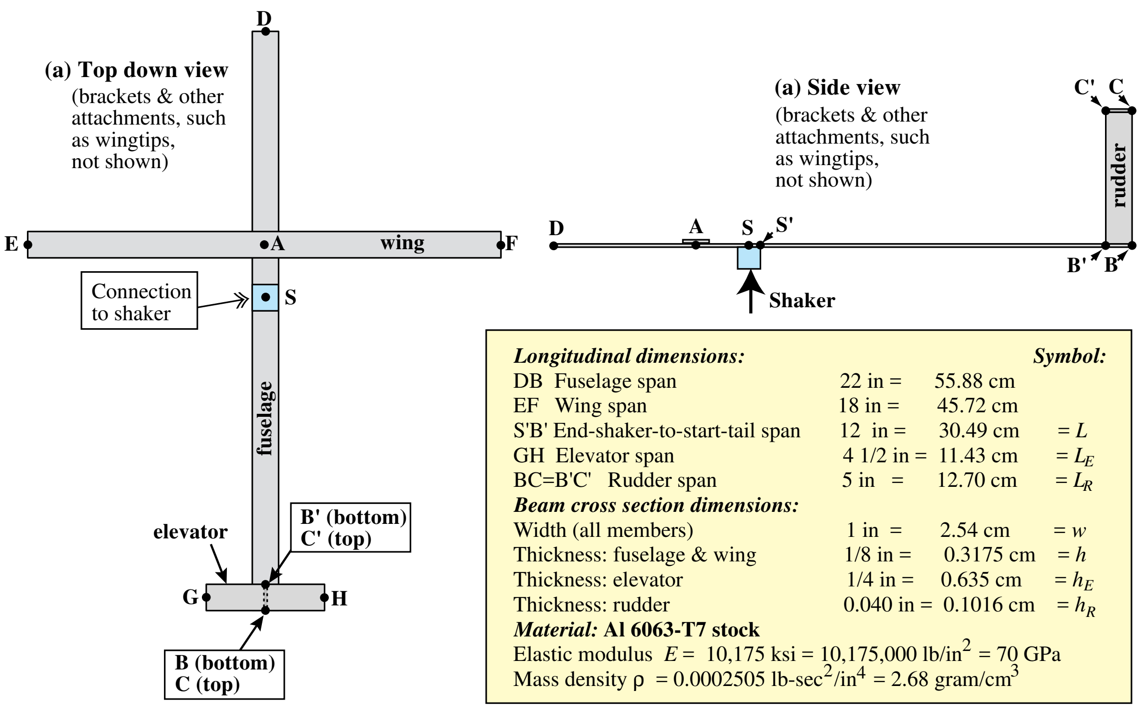

Figure 1: Data for test article of Airplane Shaker demo. Values with specific names such as E, ρ, L, L_E, etc., are those used in the FEM analysis to predict a natural frequency subset. They are also restated in Table 1, along with other information.

III Experimental Procedure Summary

In the experiments you will characterize the dynamical (vibrational) response of a simplified scaled model of an airplane, shown in Figure 1). It consists of nose (section AD), wings (section EAF) and tail (section AB). The experimental procedure is detailed in a separate document (which is provided on Canvas as a reference) as well as a demo video (see ”ASEN 3112 Lab 3 Project Steps.pdf” for information on how to access this video); we only present a brief summary here.

The structure is instrumented with four accelerometers, one of which will always be used to measure the input acceleration provided by the shaker. A laser vibrometer is used to monitor the horizontal displacement of the tail, since the weight of an accelerometer will be enough to alter its dynamics.

First, a frequency sweep is performed, in which the excitation frequency is increased slowly, in order to excite all modes. All sensors (laser vibrometer and accelerators) are placed in ”standard” locations, so that the vibration of all elements (tail, nose, and wings) are monitored.

Then, you will explore the two modes that can be predicted with the finite-element model described in Section IV. These are the modes that exhibit motion mainly in the tail of the model airplane. If this is a research project (e.g., in senior projects), one would need to investigate where to optimally relocate the accelerometers to obtain the best data possible to characterize the mode shape. In this lab, we will use the current locations of the accelerometers and assume they are reasonably well placed.

We have conducted the experimental tests and are providing the raw data to you in the file ”5mincenterrenamed.txt” whic is available on Canvas.

The key task for you that follows is to write a Matlab code to read this data and analyse it.

IV Finite-Element Analysis

The objective of the computer portion of this Lab is to predict, using very simple FEM models, two of the five natural frequencies observed experimentally. These two involve *cantilever modes* of the aft-fuselage (the portion of the fuselage behind the shaker plus the tail assembly, see Figure 1). They appear as modes #2 and #5 in the shaker frequency sweep and are identified as **Horizontal tail vertical + “T” section sideways**, and **Horizontal tail vertical second mode vibrations**, respectively.

A diagram of the test article for use in the FEM model is shown in Figure 2. That figure provides geometric dimensions and material properties. It also defines the symbols used in this Lab. Values are listed in both English and metric units. For this Lab, we will use the following *English units*: length in inches (in), force in pounds (lb), and time in seconds (sec). The mass density ρ is a derived unit: lb-sec^2/in^4.

Note: the element stiffness and mass matrices supplied below for the FEM analysis are given as recipe. Their derivation requires the Method of Virtual Displacements and the Lagrange equations applied to beams; the latter topic is beyond the scope of this course.

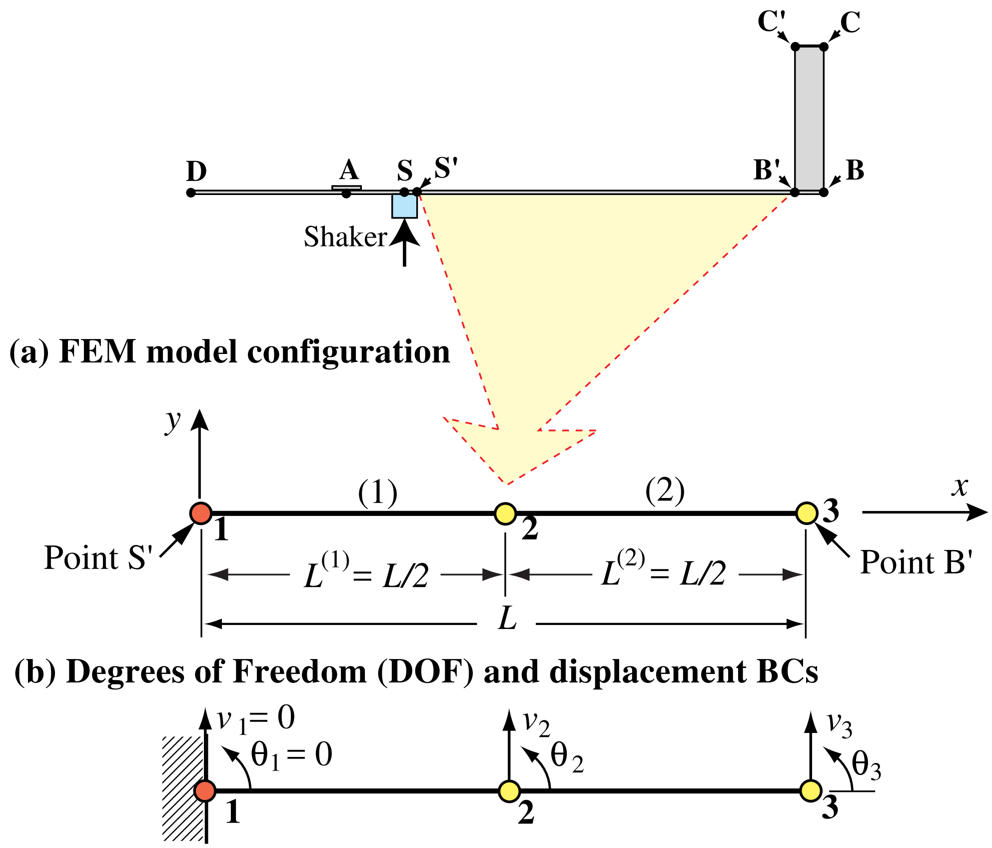

Figure 2: Two-element FEM model of aft-fuselage plus "lumped tail" assembly.

IV.1 Two-Element Model

A very coarse FEM model is shown in Figure 2. The aft-fuselage span S'B' between the end of the shaker and the start of the tail assembly is modeled with two Bernoulli-Euler plane beam elements as shown. The tail assembly is modeled as a combination of point-mass, point-first-mass-moment-of-inertia and point-second-mass-moment-of-inertia. Those three quantities were evaluated using a symbolic computation program, e.g., *Mathematica*, to carry out straightforward but error-prone integrations. They are lumped to node 3 (located at B').

Since the FEM model is two-dimensional, *it can only account for aft-shake vertical cantilever modes that occur in the fuselage-tail plane*. As such, it misses three of the five observed natural frequencies, because their associated mode shapes do not fit the stated 2D motion restrictions.

Before applying displacement boundary conditions this FEM model has six DOF collected in the vector

$$\mathbf{u} = \begin{bmatrix} v_1 & \theta_1 & v_2 & \theta_2 & v_3 & \theta_3 . \end{bmatrix}^T \tag{1}$$

Here v_i denotes the displacement of node i normal to the beam while θ_i is the rotation of the beam cross section about z, as sketched in Figure 3. The master mass and stiffness matrices for this model are

$$\mathbf{M}_2 = c_{M2}\begin{bmatrix} 19272 & 1458L & 5928 & -642L & 0 & 0 \\ 1458L & 172L^2 & 642L & -73L^2 & 0 & 0 \\ 5928 & 642L & 38544 & 0 & 5928 & -642L \\ -642L & -73L^2 & 0 & 344L^2 & 642L & -73L^2 \\ 0 & 0 & 5928 & 642L & 19272 & -1458L \\ 0 & 0 & -642L & -73L^2 & -1458L & 172L^2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_T & S_T \\ 0 & 0 & 0 & 0 & S_T & I_T \end{bmatrix},$$

$$\mathbf{K}_2 = c_{K2}\begin{bmatrix} 24 & 6L & -24 & 6L & 0 & 0 \\ 6L & 2L^2 & -6L & L^2 & 0 & 0 \\ -24 & -6L & 48 & 0 & -24 & 6L \\ 6L & L^2 & 0 & 4L^2 & -6L & L^2 \\ 0 & 0 & -24 & -6L & 24 & -6L \\ 0 & 0 & 6L & L^2 & -6L & 2L^2 \end{bmatrix},$$

$$\text{in which} \quad c_{M2} = \rho A\, L/100800, \quad c_{K2} = 4EI_{zz}/L^3, \quad A = wh, \quad I_{zz} = wh^3/12. \tag{2}$$

Symbols E, ρ, L, w, h, M_T, S_T and I_T, which appear above, are defined in Table 1 on page 6 of this document. (For reference convenience that Table also lists tail assembly member dimensions; these were used to compute the lumped values M_T, S_T and I_T, but do not appear in the matrices above.)

Denoted by $\hat{\mathbf{M}}_2$ and $\hat{\mathbf{K}}_2$, the reduced mass and stiffness matrices are obtained by removing rows 1,2 and columns 1,2 from $\mathbf{M}_2$ and $\mathbf{K}_2$, respectively, on account of the fixed-end displacement BC at node 1. Those reduced matrices are of order 4×4. The vibration eigen problem for this model is

$$\boxed{\hat{\mathbf{K}}_2 \mathbf{U} = \omega^2\, \hat{\mathbf{M}}_2\, \mathbf{U}.} \tag{3}$$

You are required to either build your own code or use a provided code to solve the Eigenvalue problem in Equation 3 and report only the smallest three frequencies obtained, i.e., $f_i = \omega_i/(2\pi)$, $i = 1, 2, 3$ in Hz. Compare the mode shapes to the partial reference results listed in Figure 4. Then compare f_1 and f_2 to frequencies #2 and #5 observed in the shaker demo.

To plot the eigenvectors (mode shapes), you can use the procedure described in Section V. Those are shown only for informational purposes; they need not be compared with experimental results. The FEM code provided automatically plots the mode shapes.

The provided FEM code is in the form of a Matlab file called ”FEMcodeLab3.m” and is available on Canvas.

IV.2 Four-Element Model

A more refined FEM model is shown in Figure 3. Here the fuselage span S'B' is modeled with four Bernoulli-Euler plane beam elements as shown. The tail assembly is modeled exactly as done for the two-element model, with lumped values assigned to node 5. Before applying displacement boundary conditions, the model has ten DOF collected in the vector

$$\mathbf{u} = \begin{bmatrix} v_1 & \theta_1 & v_2 & \theta_2 & v_3 & \theta_3 & v_4 & \theta_4 & v_5 & \theta_5 \end{bmatrix}^T. \tag{4}$$

The master mass and stiffness matrices for this model are

$$
\mathbf{M}_4 = c_{M4} \begin{bmatrix}
77088 & 2916L & 23712 & -1284L & 0 & 0 & 0 & 0 & 0 & 0 \\
2916L & 172L^2 & 1284L & -73L^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
23712 & 1284L & 154176 & 0 & 23712 & -1284L & 0 & 0 & 0 & 0 \\
-1284L & -73L^2 & 0 & 344L^2 & 1284L & -73L^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 23712 & 1284L & 154176 & 0 & 23712 & -1284L & 0 & 0 \\
0 & 0 & -1284L & -73L^2 & 0 & 344L^2 & 1284L & -73L^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 23712 & 1284L & 154176 & 0 & 23712 & -1284L \\
0 & 0 & 0 & 0 & -1284L & -73L^2 & 0 & 344L^2 & 1284L & -73L^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 23712 & 1284L & 77088 & -2916L \\
0 & 0 & 0 & 0 & 0 & 0 & -1284L & -73L^2 & -2916L & 172L^2
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_T & S_T \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_T & I_T
\end{bmatrix},
$$

$$
\mathbf{K}_4 = c_{K4} \begin{bmatrix}
96 & 12L & -96 & 12L & 0 & 0 & 0 & 0 & 0 & 0 \\
12L & 2L^2 & -12L & L^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
-96 & -12L & 192 & 0 & -96 & 12L & 0 & 0 & 0 & 0 \\
12L & L^2 & 0 & 4L^2 & -12L & L^2 & 0 & 0 & 0 & 0 \\
0 & 0 & -96 & -12L & 192 & 0 & -96 & 12L & 0 & 0 \\
0 & 0 & 12L & L^2 & 0 & 4L^2 & -12L & L^2 & 0 & 0 \\
0 & 0 & 0 & 0 & -96 & -12L & 192 & 0 & -96 & 12L \\
0 & 0 & 0 & 0 & 12L & L^2 & 0 & 4L^2 & -12L & L^2 \\
0 & 0 & 0 & 0 & 0 & 0 & -96 & -12L & 96 & -12L \\
0 & 0 & 0 & 0 & 0 & 0 & 12L & L^2 & -12L & 2L^2
\end{bmatrix},
$$

$$
\text{in which} \quad c_{M4} = \rho A L/806400, \quad c_{K4} = 8 E I_{zz}/L^3, \quad A = wh, \quad I_{zz} = wh^3/12. \tag{5}
$$

Here E, ρ, L, w, h, M_T, S_T and I_T, which appear above, are the same as for the two-element model. Their values may be retrieved from Table 1.

Denoted by $\hat{\mathbf{M}}_4$ and $\hat{\mathbf{K}}_4$, the reduced mass and stiffness matrices are obtained by removing rows 1,2 and columns 1,2 from $\mathbf{M}_4$ and $\mathbf{K}_4$, respectively. Those reduced matrices are of order 8×8. The vibration eigenproblem for this model is

$$
\boxed{\hat{\mathbf{K}}_4 \, \mathbf{U} = \omega^2 \, \hat{\mathbf{M}}_4 \, \mathbf{U}}. \tag{6}
$$

You are required to either build your own code or use a provided code to solve the Eigenvalue problem in Equation 6 and report only the smallest three frequencies obtained, that is, $f_i = \omega_i/(2\pi)$, $i = 1, 2, 3$ in Hz. Compare them to the first three frequencies from the two-element eigenproblem Equation 3. Then compare f_1 and f_2 to frequencies #2 and #5 observed in the shaker demo.

To plot the eigenvectors (mode shapes), you can use the procedure described in Section V. Those are shown only for informational purposes; they need not be compared with experimental results. The FEM code provided automatically plots the mode shapes for this model as well.

The provided FEM Matlab code "FEMcodeLab3.m" also implements this four-element FEM model.

To provide a beacon for the computational work, Figure 4 shows partial results obtained using *Mathematica* for the two-finite-element model.

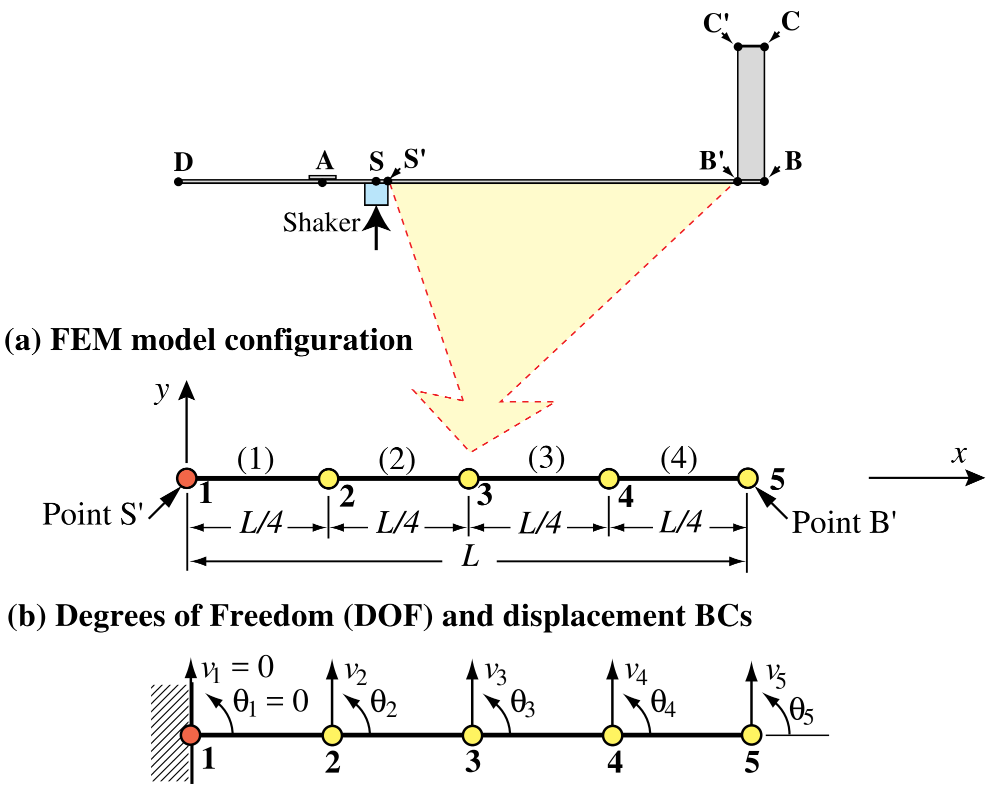

Figure 3: Four-element FEM model of aft-fuselage plus "lumped tail" assembly.

Description	Symbol	Value in English units
Cantilever span S'B'	L	12 in
Elevator span	L_E	4.5 in
Rudder span	L_R	5 in
Width of all members	w	1 in
Thickness of fuselage member	h	1/8 in
Thickness of elevator member	h_E	1/4 in
Thickness of rudder member	h_R	0.040 in
Material elastic modulus	E	10,175 ksi = 10,175,000 psi
Material mass density	ρ	0.0002505 lb-sec^2/in^4
Mass of tail assembly	M_T	(1.131 in^3) ρ
First mass-moment of tail assembly wrt B'	S_T	(0.5655 in^4) ρ
Second mass-moment of tail assembly wrt B'	I_T	(23.124 in^5) ρ

Note 1. Values listed for M_T, S_T and I_T must be scaled by the mass density ρ, as shown.
Note 2. Calculations may be done either using English units: in, lb and sec, or metric units: cm, N and sec. Use of English units, as given above, is recommended.

Table 1: Numerical values to be used in FEM models of aft-fuselage.

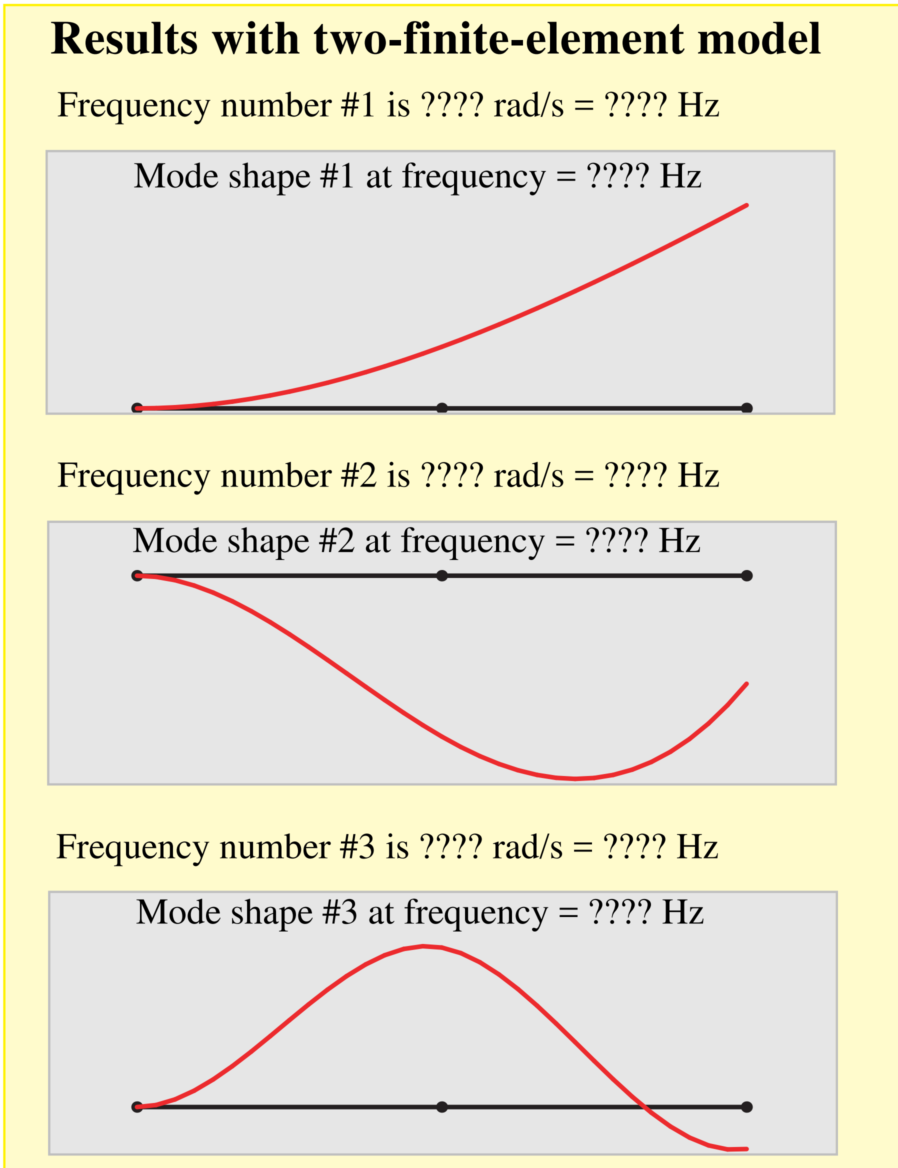

Figure 4: First three vibration frequencies — to be determined by groups — and associated mode shapes predicted by the FEM two-element model. Note: the FEM frequency associated with mode shape #3 is likely out of range of the frequency sweep range in group experiments. It is pictured only for completeness.

V Plotting Mode Shapes (Optional Reading)

The mode shapes are the first 3 eigenvectors associated with the 3 lowest computed natural frequencies. To show pictures of those modes, the C-like pseudocode listed in Figure 5 may be used. This code can be readily translated to a higher order language such as *Matlab* or *Mathematica*, which provide built-in graphics. Note, however, that the actual plotting statement will be very language dependent.

The procedure is invoked as

```
ploteigenvector(L,ev,ne,nsub,scale)
```

The arguments are:

`L`	The length of the FEM mode; see Figure 2 or Figure 3.
`ev`	The eigenvector to be plotted; see below for more details.
`ne`	The number of beam finite elements in the model (2 or 4)
`nsub`	The number of subdivisions of each element for plotting the eigenvector cubic curve as sequence of line segments. For a visually smooth plot 10 or more are recommended.
`scale`	A scaling factor for the eigenvector values. Initially should be set to 1. If the plot aspect ratio is unpleasant (for example, too flat), this argument may be adjusted.

An additional text argument: `title`, could be optionally appended to the foregoing list to write a heading over the plot, for example `"Mode shape #x, for frequency xx.xx Hz"`. But it is just as easy to write the heading before the procedure is called.

Argument `ev` requires some explanation. It is a vector containing `2*ne+2` entries. If the model contains 2 elements (`ne=2`), `ev` has the configuration in Equation 1 in which $v_1 = \theta_1 = 0$. If the reduced eigenproblem Equation 3 is solved by, say, *Matlab*, the program will return the last 4 eigenvector components, and the two zero ones will have to be inserted (prepended) before submitting to the plotting procedure. Similarly, if the model contains 4 beam elements (`ne=4`) , then `ev` has the configuration in Equation 4, with $v_1 = \theta_1 = 0$. Again if the reduced eigenproblem in Equation 6 is solved by *Matlab*, the program will return the last 8 components, and the two zero entries will have to be prepended for plotting.

```
procedure ploteigenvector (L,ev,ne,nsub,scale);
// declare local variables here if required by language
nv=ne*nsub+1; Le=L/ne; dx=Le/nsub; k=1;
x=v=zeroarray(nv); // declare and set to zero plot arrays
for (e=1,e<=ne,e++)  // loop over elements
    xi=Le*(e-1); vi=ev(2*e-1); qi=ev(2*e); vj=ev(2*e+1); qj=ev(2*e+2);
    for (n=1,n<=nsub,n++) // loop over subdivisions
        xk=xi+dx*n; x=float(2*n-nsub)/nsub; // isoP coordinate
        vk=scale*(0.125*(4*(vi+vj)+2*(vi-vj)*(x^2-3)*x+
            Le*(x^2-1)*(qj-qi+(qi+qj)*x))) // Hermitian interpolant
        k = k+1; x(k)=xk; v(k)=vk; // build plot functions
    endfor  // end n loop
endfor // end e loop
// plot v (vertical) vs x (horizontal) -- language dependent
endprocedure
```

Figure 5: Pseudo-code for plotting mode shapes.

VI Writing A Report

VI.1 Organization

Reports are due April 21st, 2020, before class time. **The report must be electronically processed, e.g. by WORD or LATEX; pdf copies only are to be turned in via electronic submission through Gradescope**.

Each group produces a single report. It must include:

- **Title Page**: Describes Lab, lists the name of the team members and identifies the group leader.

- **Results:** The results should address the questions in Section VI.2.
- **Appendix - Code.** A printout of all the code used to produce the results.
- **Appendix - Participation report.** More details on how the grade of individual group members is calculated can be found in Section VI.3.

VI.2 Report Content

The report will be graded on both technical content and presentation. Regarding the content, instead of an open-ended report, you should process the experimental data and computational results to address the specific questions posted below. Regarding presentation, make sure that you follow these guidelines:

- All plots should be readable. This includes using different color or line styles and a suitable font size for the axis labels and all other text in the plot. The range of both axes should be chosen to focus on the region of interest (*i.e.*, the data).
- Show your work, including equations used and partial results.
- All results should be presented with appropriate units.
- Be quantitative when comparing results. Use percentage of error or deviation. Refer back to predicted error or variance when applicable.

VI.2.1 Question 1: Experimental Results

- Provide plots of the response of the system as a function of the excitation frequency. Make sure that you factor out possible changes in the input excitation (*e.g.*, by using the magnification factor instead of just the output of the accelerometers). Provide a detailed explanation of the method used to process the data.
- Use your data to identify the resonant frequency of all five modes.
- For each mode, identify which sensors capture resonance, and use them to describe the shape of the mode.

VI.2.2 Question 2: FEM Results - Resonant frequencies

- Compute the first three resonant frequencies using both finite-element models.
- Compare the frequencies, when possible, with the experimental results. Quantify and discuss the errors.

VI.2.3 Question 3: FEM Results - Mode shapes

- Plot the shapes of the first three modes provided by each of the FEM codes.
- Compare, when possible, with the experimental results of the corresponding modes. Plot the experimental results on the same figure as the predictions. Normalize the shapes appropriately.

VI.3 Individual Contribution Evaluation and Deductions for No-Show

The group leader submits along with the report a separate participation report for your group with a brief summary of each group member's tasks, contributions, and performance as a group member. **Please make sure to include this as an Appendix in the report**.

The performance of each group member is rated with a "contribution factor" on a scale of 0 to 100%. A score of 100% indicates that the member contributed the expected share to the experiment and to the preparation of the report. The scores are normally assigned by *peer evaluation*, using the same procedures followed in the ASEN 200x sophomore courses. The group leader is responsible for administering the peer evaluation, tabulating and submitting the results of said peer evaluation.

The individual score will be equal to:

$$\text{Individual score} = \text{Group score} \times \frac{100 + \text{Contribution factor}}{200} \tag{7}$$

For example, if the group receives an overall score of 88.75 and the individual received a "contributing factor" of 90%, the individual score is $88.75 \times (100 + 90)/200 = 84.31$.

Addendum I. Report Grading

The score assigned to the lab report includes technical content (75%) and presentation (25%). This is a more detailed breakdown of the weights:

Category	Weight	Score	Contribution
Technical content			
Question 1	0.3		
Question 2	0.15		
Question 3	0.3		
Presentation			
Plots	0.10		
Grammar, style & spelling	0.10		
Formatting	0.05		
Total	1.00		(overall score)

The score within each category ranges from 0 to 100%. For example, if the score for 'Question 1' is 80%, it contributes $0.25 \times 80 = 20\%$ to the overall score. The final score of each team member is then calculated following the procedure detailed in Section VI.3.