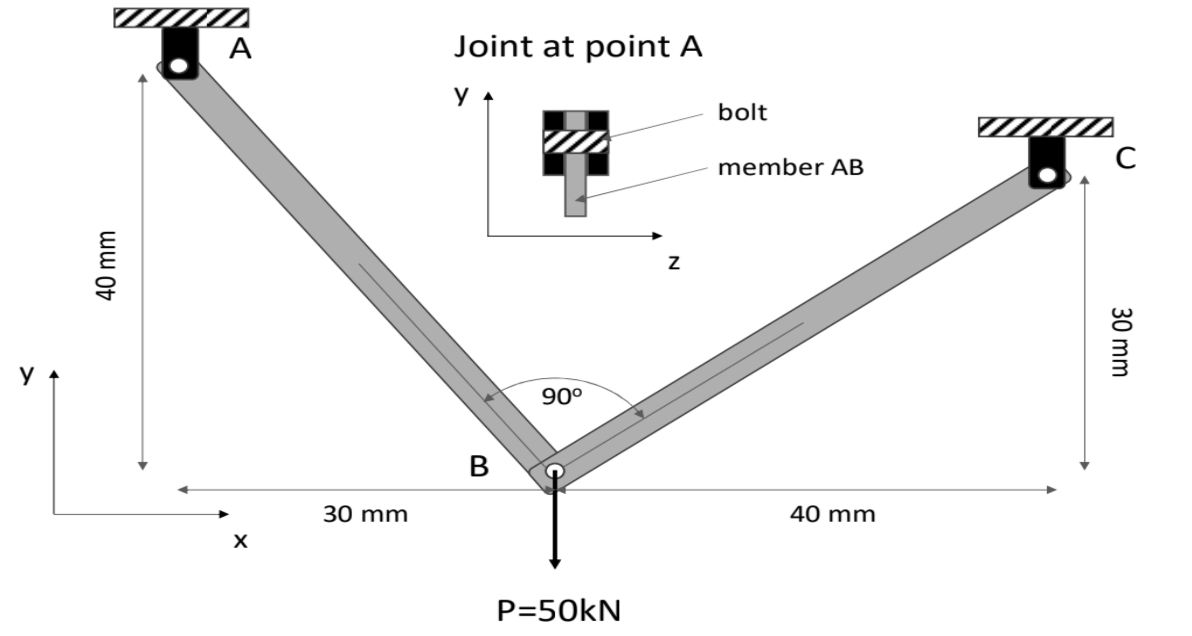
ASEN 3112

Recitation 2

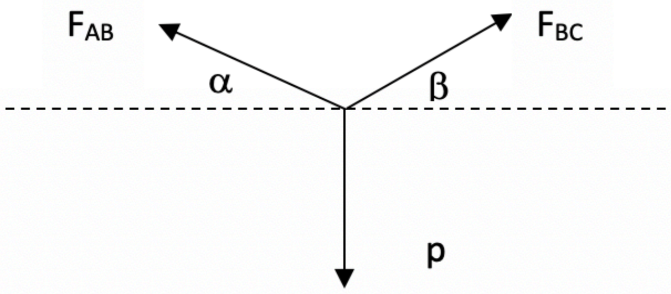
Problem 1: About 30 minutes



The figure shows a two-bar truss to a vertical load *P* = 50 *kN* at joint B. Both bars have a rectangular cross-section and are made of an isotropic material with Young's modulus of *E* = 2.1 *Gpa* and maximum tensile strength of *T.S.* = 150 *Mpa.*

a. Compute the forces in the bars. Use FBD.
b. Compute the smallest possible diameter of the cylindrical bolt joint A such that the shear in the bolt does not exceed $\tau = 100\ MPa$. Use a proper FBD.
c. Determine the cross-section area of each bar such that the factor of safety against failure in normal stress in the bar is $S_f = 4.0$.

Solution 1



a. FBD

$$\tan(\alpha) = 4/3 \;\; \tan(\beta) = 3/4$$

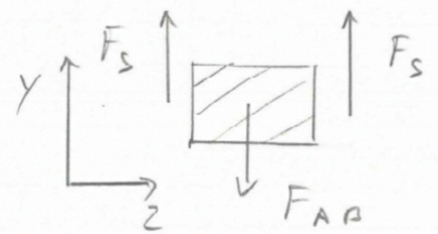
$$\Sigma Fx = 0 \rightarrow -F_{AB}\cos(\alpha) + F_{BC}\cos(\beta) = 0$$

$$\Sigma Fy = 0 \rightarrow F_{AB}\sin(\alpha) + F_{BC}\sin(\beta) - p = 0$$

$$F_{AB} = F_{BC}\frac{\cos(\beta)}{\cos(\alpha)}$$

$$F_{BC}\left(\cos(\beta)\frac{\sin(\alpha)}{\cos(\alpha)} + \sin(\beta)\right) = p,\; F_{BC} = \frac{p}{\sin(\beta)+\cos(\beta)4/3} = 30\;kN,\; F_{AB} = 40\;kN$$

b.



$$\Sigma Fy = 0 \rightarrow F_s = \frac{F_{AB}}{2} = 20\;kN$$

$$\tau = \tau_{max} = \frac{F_s}{A_s},\; A_s = \frac{\pi}{4}d^2 \rightarrow d = \sqrt{\frac{4F_s}{\pi\tau_{max}}} = 0.01596\;m = 15.96\;mm$$

c. $\frac{T.S.}{\sigma_{axial}} = s_f, \quad \sigma_{axial}^{AB} = \frac{F_{AB}}{A_{AB}}, \qquad \sigma_{axial}^{BC} = \frac{F_{BC}}{A_{BC}}$

$$A_{AB} = \frac{s_f F_{AB}}{T.S.} = 0.001066\;m^2 = 1066\;mm^2, \quad A_{BC} = 0.0008\;m^2 = 800\;mm^2$$

Problem 2: About 20 minutes

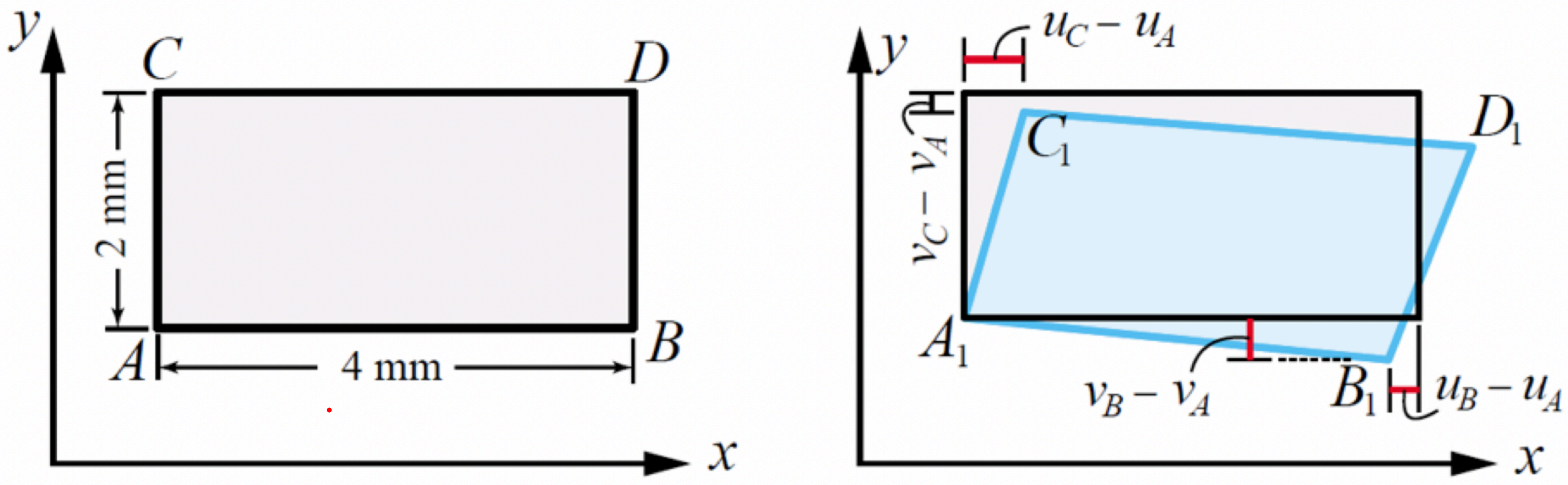


Figure 1: Shape to be analyzed for problem 1.

Displacements *u* and *v* in the *x* and *y* dimensions, respectively, were measured at points *A, B, C,* and *D* of the two-dimensional rectangular body shown above. The gage lengths along *x* and *y* are 4 mm and 2 mm, respectively. Upon deformation, the displacement data is $u_a = -0.0100\ mm$, $v_a = 0.0100\ mm$, $u_b = -0.0050\ mm$, $v_b = -0.0112\ mm$,$u_c = 0.0050\ mm$, $v_c = 0.0068\ mm$, $u_d = 0.0100\ mm$, and $v_d = 0.0080\ mm$. [The displacements of *D* are irrelevant for the computation of the strains at *A*.]

Find the Lagrangian small strains ε_{xx}, ε_{yy}, and γ_{xy} at point A using displacement differences. All answers should be in micros (μ); [one micro is $0.000001 = 0.0001\% = 10^{-6}$].

Solution 2

$$\varepsilon_{xx} = \frac{\Delta u_{BA}}{\Delta x_{BA}} = \frac{u_B - u_A}{x_B - x_A} = \frac{0.0050}{4} = 0.00125 = 1250\,\mu$$

$$\varepsilon_{yy} = \frac{\Delta v_{CA}}{\Delta y_{CA}} = \frac{v_C - v_A}{y_c - y_A} = \frac{-0.0032}{2} = -0.0016 = -1600\,\mu$$

$$\gamma_{xy} = \frac{\Delta v_{BA}}{\Delta x_{BA}} + \frac{\Delta u_{CA}}{\Delta y_{CA}} = \frac{v_B - v_A}{x_B - x_A} + \frac{u_C - u_A}{y_C - y_A} = \frac{-0.0212}{4} + \frac{0.0150}{2} = 0.0022 = 2200\,\mu$$

Problem 3: About 30 minutes

Figure 2: Rubber balloon to be analyzed for problem 2.

A rubber party balloon (see Figure above) is inflated from an initial diameter $D0$ = 50 mm (the reference state) to a final diameter Df = 150 mm (the deformed or final state). Two easy items and a tougher one:

a) Find the average circumferential strain in the deformed state as given by the Lagrangian measure ϵ_{circ}^{L} and the Eulerian measure ϵ_{circ}^{E}. Express both in %. Are these values close or far apart? Justify your answer (why is it close or far apart).

b) Initial wall thickness is t_0 = 0.18 mm. The rubber is incompressible (When rubber deforms, it maintains its volume). Find the thickness t_f in the deformed state.

c) (Tougher) For simplicity assume: (I) rubber is linearly elastic, (II) wall is in plane stress, (III) average Young's modulus for the Eulerian strain measure is $E_{rubber} \approx 1.9$ GPa, (IV) Poisson's ratio is $\nu = 1/2$. Under those assumptions, find the inflation pressure p.

Solution 3

(a) $\epsilon_{circ}^{L} = \frac{D_f - D_0}{D_0} = 2 = 200\%$; $\epsilon_{circ}^{E} = \frac{D_f - D_0}{D_f} = 0.667 = 66.7\%$. The values are far apart as the difference between initial and final diameter(s) are large relative to original diameter.

(b) Volume of the balloon (assuming it to be a spherical shell and taking $\frac{t_0}{D_0} \ll 1$) is $4\pi R_0^2 t_0$. Since we have assumed incompressibility, the volume remains constant. Final deformed volume is $4\pi R_f^2 t_f$. On solving, we obtain $t_f = 0.18/9 = 0.02\ mm$

(c) At an arbitrary wall point, choose *z* along thickness and *x*, *y* on tangent plane to sphere. Then $\sigma = \sigma_{xx} = \sigma_{yy} = pR_f /(2t_f)$ with $R_f = D_f /2$ (pressurized spherical vessel formula), $\epsilon_{circ}^{E} = \epsilon_{xx} = \epsilon_{yy} = (1 - \nu)\, \sigma/E_{rubber}$. Solving: $p = 2E_{rubber}\, t_f\ \epsilon_{circ}^{E}/((1 - \nu)R_f)$.
Substituting (careful with units!) gives $p \approx 1.35\ MPa$.

Problem 4: About 30 minutes

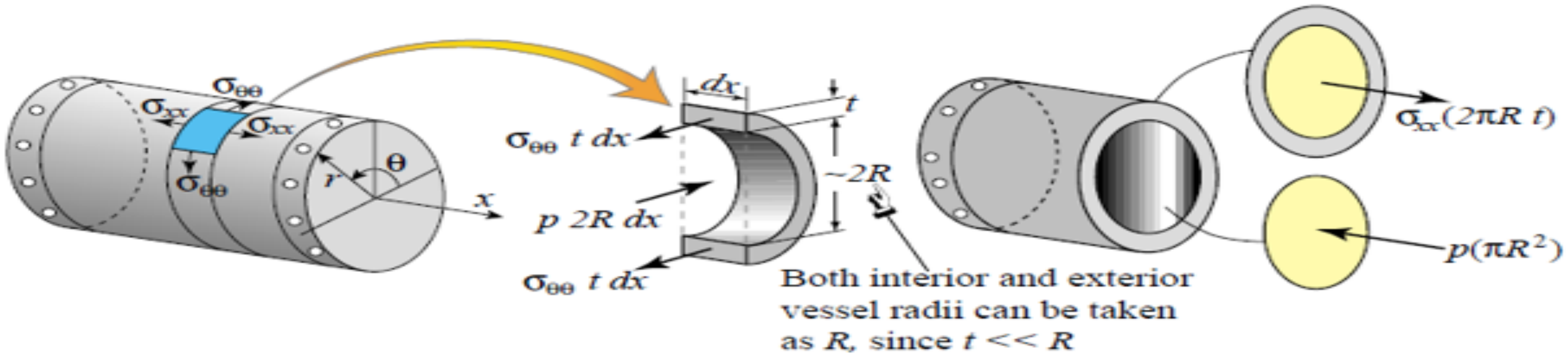


Figure 3: Vessel to be analyzed for problem 3.

$$\sigma_{xx} = \frac{pR}{2t} \quad \text{(axial stress)}, \qquad \sigma_{\theta\theta} = \frac{pR}{t} \quad \text{(hoop stress, also called circumferential stress)}. \qquad (1)$$

(a) For $p = 3$ psi, $R = 10$ in, $t = 0.05$ in, and compute σ_{xx} and $\sigma_{\theta\theta}$

(b) For the plane stress state obtained for the cylindrical pressure vessel above, and using the numerical values determined in part (a), determine the in-plane stresses. Find:

- The principal stresses σ_1 and σ_2, in which $\sigma_1 \geq \sigma_2$.
- The principal planes angles ϕ_1 and ϕ_2 (φ is measured from the x axis, positive CCW). Note: symbol ϕ is used in lieu of θ, as in the Notes, because ϑ is used here to denote the circumferential direction.
- The maximum in-plane shear stress τ_p

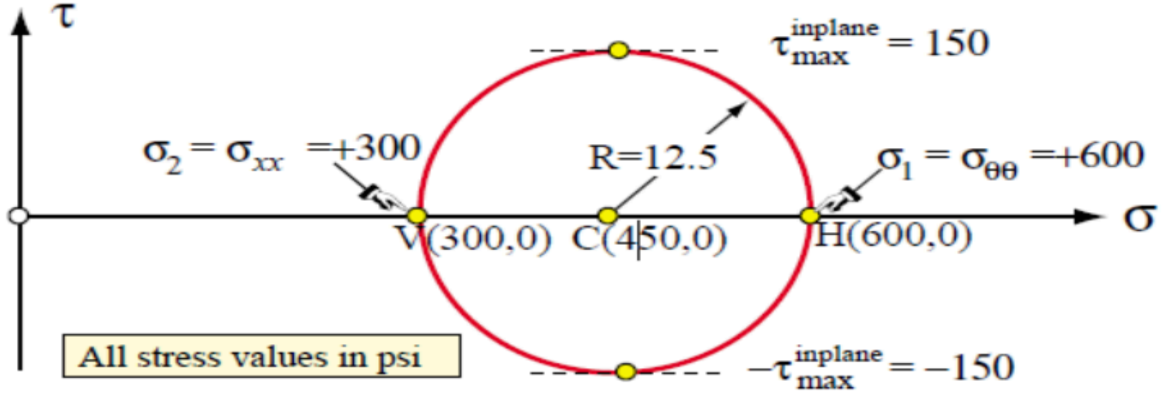
(c) Consider next the pressure vessel wall as a *three-dimensional body*. Use the fact that the radial (normal-to-the-wall) stress $\sigma_{rr} \approx 0$ is a zero principal stress, and that σ_{xx} and $\sigma_{\theta\theta}$ are principal stresses. Find the *overall* maximum shear stress $\tau_{max} = max(\frac{1}{2}|\sigma_{xx} - \sigma_{\theta\theta}|, \frac{1}{2}|\sigma_{xx}|, \frac{1}{2}|\sigma_{\theta\theta}|)$, which follows from eqn (6.18) of text.

(d) Compare τ_{max} to the maximum in-plane shear stress τ_p found in (b) above. Are they the same? What can you conclude about failure of the vessel if it is fabricated of a ductile material such a steel or aluminum?

Solution 4

(a) On substituting the values in the formulae given in eqn (1)

$$\sigma_{xx} = 300\text{psi}, \sigma_{\theta\theta} = 600 \text{ psi}$$



(b) Mohr's circle for in-plane stresses shown in above figure. Principal stresses: $\sigma_1 = \sigma_{\theta\theta} = \frac{pR}{t} =$ 600 psi. $\sigma_2 = \sigma_{xx} = pR/(2t) = 300$ psi. Principal plane angles: 0° to $\sigma_1 = \sigma_{\theta\theta}$ and 90° to $\sigma_2 = \sigma_{xx}$. Maximum in plane shear $\tau_p = (\sigma_1 - \sigma_2)/2 = pR/(4t) = 150$ psi,

(c) Overall maximum shear stress is the maximum of $|300 - 600|/2, |600|/2, |300|/2 = 300$ psi, which is twice $\tau_p = 150$ psi.

(d) For a ductile material, failure will occur under the overall maximum shear stress τ_{max}. This acts at $\pm 45°$ across the thickness.