# ASEN 3112

# Spring 2020

# Lecture 24

## Whiteboard

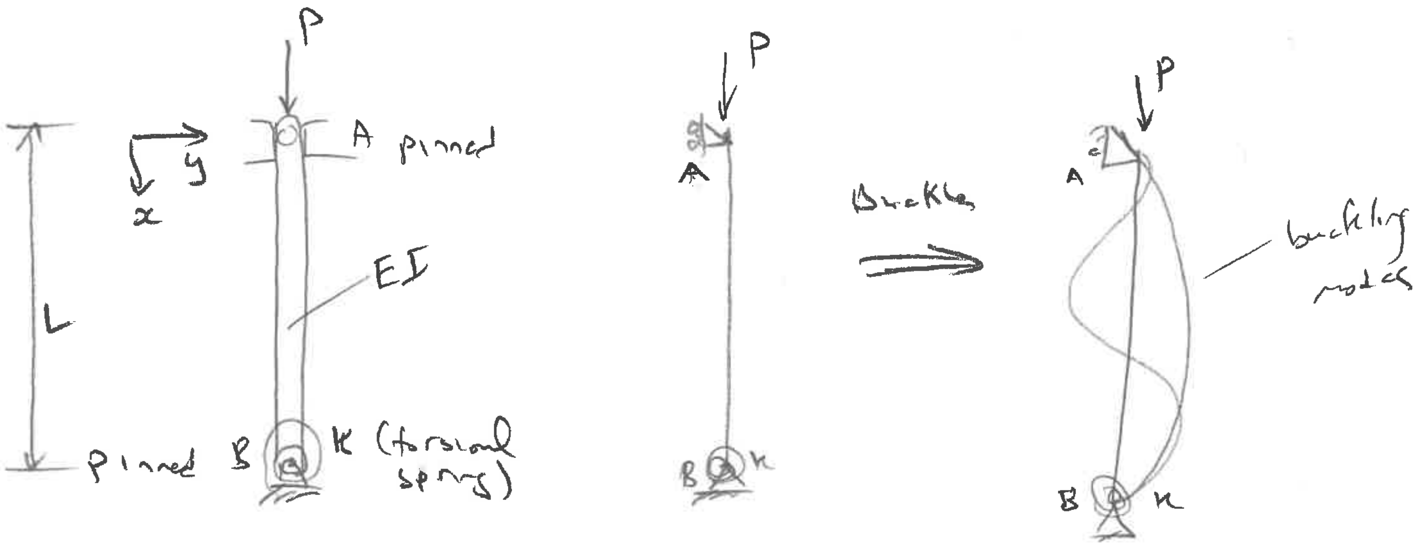
April 21, 2020

L24

1

Consider elastically restrained column

Consider slight modification to the classical Euler column problem (1st example)



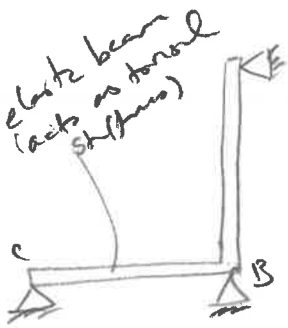

$I = I_{zz}$ : minimum moment of inertia

$\Rightarrow$ of the cross-section that buckles

$$I = \frac{bt^2}{12}$$

Torsional spring at B makes the system more realistic i.e. less idealized.

corresponding "real" system



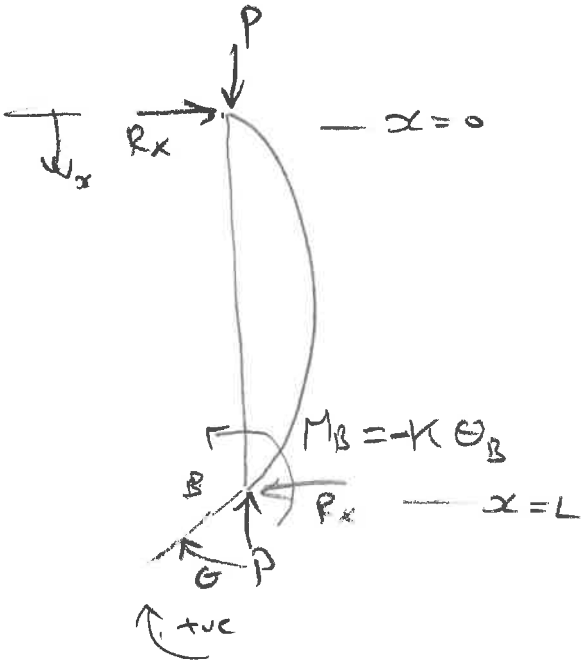

See Ch 28.4 for more details

2/21/2020

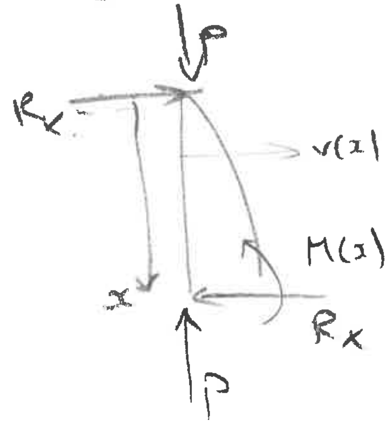
$\frac{L24}{2}$

FBD

Whole column




Cut section




$$\Sigma M_x = 0$$

$$M(x) + Pv - R_A x = 0$$

Notation

$$\theta_B = v'(L)$$

$$K \equiv \beta \frac{EI}{L}$$

defined

β: rotational coefficient

Critical load Analysis

Whole column: $M_B = R_A L$

$$R_A = \frac{M_B}{L} = -\frac{K\theta_B}{L} = -\frac{\beta EI \theta_B}{L^2}$$

Cut column: $M(x) + Pv(x) - R_A x = 0$

+ve

$$EIv'' + Pv = R_A x = -\frac{\beta EI\theta_B\, x}{L^2}$$

Divide by EI, $\lambda^2 = \frac{P}{EI}$

$$v'' + \lambda^2 v = -\frac{\beta\theta_B\, x}{L^2}$$

Assume:

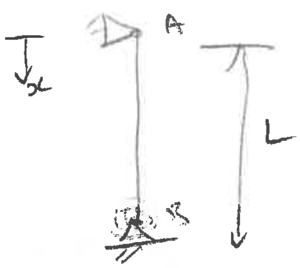
$$v(x) = \underbrace{A\sin\lambda x + B\cos\lambda x}_{v_h} + \underbrace{Cx}_{v_p}$$

Assume $v_p = Cx$

$$\lambda^2 C x = -\frac{\beta\,\theta_B}{L^2}\,x \implies C = -\frac{\beta\,\theta_B}{\lambda^2 L^2}$$

$$v(x) = A\sin\lambda x + \cancel{B\cos\lambda x} - \frac{\beta\,\theta_B}{\lambda^2 L^2}\,x$$

Use BC's




At A $\quad v(0) = 0$ (pinned) $\implies B = 0$

$$\theta(x) = v'(x) = A\lambda\cos\lambda x - \frac{\beta\,\theta_B}{\lambda^2 L^2}$$

At B $\quad \theta(L) = v'(L) = A\lambda\cos\lambda L - \dfrac{\beta\,\theta_B}{\lambda^2 L^2} = \theta_B$

$$\implies \theta_B = A\,\frac{\lambda^3 L^2}{\beta + \lambda^2 L^2}\cos\lambda L$$

$$v(x) = A\sin\lambda x - \frac{A\beta\lambda x}{(\beta+\lambda^2 L^2)}\cos\lambda L$$

At B $\quad v(L) = 0 = A\left(\sin\lambda L - \dfrac{\beta\lambda L}{\beta+\lambda^2 L^2}\cos\lambda L\right) = 0$

For non-trivial solution, $A \neq 0 \implies (\text{———}) = 0$

For convenience $\alpha = \lambda L$

$$\sin\alpha = \frac{\beta\alpha}{\beta+\alpha^2}\cos\alpha$$

$$\implies \boxed{\tan\alpha = \frac{\beta\alpha}{\alpha^2+\beta}}$$ transcendental equation.

$$\tan\alpha = \frac{\alpha\beta}{\alpha^2+\beta}$$

For $\beta \geq 0$, we seek a solution $\alpha_{cr} > 0$ closest to zero

$\Rightarrow$ cannot be solved analytically

$\Rightarrow$ need to solve numerically

Good initial guess:

for $\beta = [0, \infty]$, $\pi \leq \alpha_{cr} < 4.5$

So $\alpha \approx 4$ is a good first guess.

Use root finding scheme to converge to value of $\alpha_{cr}$ for a given choice of $\beta$.

See chapter 28.4.2 (lecture 24) for details.

$$P_{cr} = \frac{\alpha_{cr}^2 EI}{L^2} = \frac{\pi^2 EI}{L_e^2}$$

$L_e$: effective length (differs from one problem to the other)

$$L_e = \frac{\pi}{\alpha_{cr}} L.$$

Table provided for $\alpha_{cr}$, $\frac{L_e}{L}$ for various values of $\beta$

For example

$\beta = 0$ $\quad \alpha_{cr} = \pi$, $\frac{L_e}{L} = 1$ (see Example 1, pinned-pinned)

$\beta = \infty$ $\quad \alpha_{cr} = 4.4934 = \sqrt{20.19}$, $\frac{L_e}{L} \approx 0.7$ (see Example 2, pinned-fixed)

$\beta = 1$, $\alpha_{cr} = 3.4056$, $\frac{L_e}{L} = 0.9224$