## ✅ DATASET (Very Small Example — 6 points)

We will cluster these 6 points into **K = 2 clusters**.

| Point | Coordinates (x, y) |
|---|---|
| A | (1, 1) |
| B | (2, 1) |
| C | (4, 3) |
| D | (5, 4) |
| E | (3, 2) |
| F | (4, 2) |

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## ✅ STEP 1 — Initialize Centroids (Random selection)

Let:

- **Centroid C1 = A = (1,1)**
- **Centroid C2 = C = (4,3)**

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## ✅ STEP 2 — Assignment Step (Find nearest centroid)

We compute **Euclidean distance** from each point to C1 and C2.

### 📌 Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

## 📍 Distance Table

| Point | (x,y) | d to C1 (1,1) | d to C2 (4,3) | Assigned Cluster |
|---|---|---|---|---|
| A | (1,1) | 0 | $\sqrt{[(1-4)^2 + (1-3)^2]} = \sqrt{13} = 3.60$ | C1 |
| B | (2,1) | $\sqrt{[(2-1)^2 + (1-1)^2]} = 1$ | $\sqrt{[(2-4)^2 + (1-3)^2]} = \sqrt{8} = 2.83$ | C1 |
| C | (4,3) | 3.60 | 0 | C2 |
| D | (5,4) | $\sqrt{[(5-1)^2 + (4-1)^2]} = \sqrt{25} = 5$ | $\sqrt{[(5-4)^2 + (4-3)^2]} = \sqrt{2} = 1.41$ | C2 |
| E | (3,2) | $\sqrt{[(3-1)^2 + (2-1)^2]} = \sqrt{5} = 2.23$ | $\sqrt{[(3-4)^2 + (2-3)^2]} = \sqrt{2} = 1.41$ | C2 |
| F | (4,2) | $\sqrt{[(4-1)^2 + (2-1)^2]} = \sqrt{10} = 3.16$ | $\sqrt{[(4-4)^2 + (2-3)^2]} = 1$ | C2 |

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### 🎯 Clusters after Iteration 1

- **Cluster 1 (C1): A, B**
- **Cluster 2 (C2): C, D, E, F**

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# ✅ STEP 3 — Update Centroids

### New C1

Mean of A(1,1) & B(2,1):

$$C1 = (\frac{1+2}{2}, \frac{1+1}{2}) = (1.5, 1)$$

**New C2**

Mean of C(4,3), D(5,4), E(3,2), F(4,2):

$$C2 = (\frac{4+5+3+4}{4}, \frac{3+4+2+2}{4}) = (4, 2.75)$$

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## ✅ STEP 4 — Reassign Points (Iteration 2)

| Point | Dist to New C1 (1.5,1) | Dist to New C2 (4,2.75) | Cluster |
|---|---|---|---|
| A | √[(1−1.5)² + (1−1)²] = 0.5 | √[(1−4)² + (1−2.75)²] = 3.43 | C1 |
| B | √[(2−1.5)² + (1−1)²] = 0.5 | √[(2−4)² + (1−2.75)²] = 2.69 | C1 |
| C | √[(4−1.5)² + (3−1)²] = 3.20 | √[(4−4)² + (3−2.75)²] = 0.25 | C2 |
| D | 4.61 | √[(5−4)² + (4−2.75)²] = 1.25 | C2 |
| E | 2.06 | √[(3−4)² + (2−2.75)²] = 1.30 | C2 |
| F | 2.69 | √[(4−4)² + (2−2.75)²] = 0.75 | C2 |

🎯 **Same clusters → Algorithm converged**

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## 🎉 FINAL CLUSTERS

**Cluster 1 → A, B**

**Cluster 2 → C, D, E, F**

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## ✅ NOW CALCULATE SILHOUETTE SCORE (DRY RUN)

Silhouette score for each point:

$$s = \frac{b - a}{\max(a, b)}$$

Where:

- **a** = **intra-cluster distance** (average distance to points in same cluster)
- **b** = **nearest-cluster distance** (average distance to other cluster)

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## ✳ CLUSTER 1: Points A, B

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### Point A (1,1)

#### Step 1: Compute a (intra cluster)

Only other point in cluster: B(2,1)

$$a = d(A, B) = 1$$

#### Step 2: Compute b (nearest other cluster)

Distances to C,D,E,F:

- dA-C = 3.60
- dA-D = 5
- dA-E = 2.23
- dA-F = 3.16

$$b = \frac{3.60 + 5 + 2.23 + 3.16}{4} = 3.49$$

#### Step 3: Silhouette

$$s_A = \frac{3.49 - 1}{3.49} = 0.713$$

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### Point B (2,1)

**a**

Only A:

$$a = d(B, A) = 1$$

### b

Distances to C,D,E,F:

- 2.83
- 3.61
- 1.41
- 2.24

$$b = \frac{2.83 + 3.61 + 1.41 + 2.24}{4} = 2.52$$

### Silhouette

$$s_B = \frac{2.52 - 1}{2.52} = 0.595$$

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## ✳ CLUSTER 2: Points C, D, E, F

We compute **all pair distances.**

### Distances among cluster 2 points

| Pair | Distance |
|---|---|
| C–D | 1.41 |
| C–E | 1.41 |
| C–F | 1 |
| D–E | 2.23 |
| D–F | 2.23 |
| E–F | 1 |

## Point C (4,3)

### a (intra)

$$a = \frac{1.41 + 1.41 + 1}{3} = 1.27$$

### b (nearest other cluster)

Distances to A,B:

- 3.60
- 2.83

$$b = \frac{3.60 + 2.83}{2} = 3.215$$

### silhouette

$$s_C = \frac{3.215 - 1.27}{3.215} = 0.60$$

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## Point D (5,4)

### a

$$a = \frac{1.41 + 2.23 + 2.23}{3} = 1.96$$

### b

Distances to A,B:

- 5
- 3.61

$$b = \frac{5 + 3.61}{2} = 4.305$$

### silhouette

$$s_D = \frac{4.305 - 1.96}{4.305} = 0.54$$

## Point E (3,2)

### a

$$a = \frac{1.41 + 2.23 + 1}{3} = 1.55$$

### b

Distances to A,B:

- 2.23
- 1.41

$$b = \frac{2.23 + 1.41}{2} = 1.82$$

### silhouette

$$s_E = \frac{1.82 - 1.55}{1.82} = 0.15$$

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## Point F (4,2)

### a

$$a = \frac{1 + 2.23 + 1}{3} = 1.41$$

### b

Distances to A,B:

- 3.16
- 2.24

$$b = \frac{3.16 + 2.24}{2} = 2.70$$

### silhouette

$$s_F = \frac{2.70 - 1.41}{2.70} = 0.48$$

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## ⭐ FINAL AVERAGE SILHOUETTE SCORE

$$S = \frac{s_A + s_B + s_C + s_D + s_E + s_F}{6}$$

$$= \frac{0.713 + 0.595 + 0.60 + 0.54 + 0.15 + 0.48}{6}$$

$$= \frac{3.078}{6} = 0.513$$

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## 🎉 FINAL RESULT

✔ **K-Means Clustering Completed**

✔ **Silhouette Score Fully Dry-Run**

| Metric | Value |
|---|---|
| Final Clusters | {A,B} and {C,D,E,F} |
| Final Centroids | (1.5,1) and (4,2.75) |
| Average Silhouette Score | **0.51** |