

**Scientific Computation  
Homework-2**

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Q1.

Q1. Since we have linear system  $Ax = b$  and  $L, K$   
 $z \in \mathbb{R}^{n \times n}$  and  $B \in \mathbb{R}^{n \times n}$   
 and  $L, K$  are lower triangular matrices.  
 Now we have to find  $X \in \mathbb{R}^{n \times n}$  and  $L \times K = B$   
 here is the algorithm for calculating  $Y$  and  $X$ .

```

set  $Y(L, B)$  {
  for  $j$  iterate from  $[1, n]$ 
     $j++$ 
    for  $i$  iterate from  $[1, n]$ 
       $i++$ 
       $sum\_y = B[i][j]$ 
      for  $m$  iterate from  $[1, i-1]$ 
         $m++$ 
         $sum\_y = sum\_y - L[i][m] * Y[m][j]$ 
       $Y[i][j] = sum\_y / L[i][i]$ 
  return  $Y$ 
}

set  $X(K, Y)$ 
  for  $i$  iterate  $[1, n]$ 
     $i++$ 
    for  $j$  iterate  $[1, 1]$ 
       $j--$ 
       $sum\_x = Y[i][j]$ 
      for  $m$  iterate  $[j+1, n]$ 
         $m++$ 
         $sum\_x = sum\_x - K[m][j] * X[i][m]$ 
       $X[i][j] = sum\_x / K[j][j]$ 
  return
  
```



Now since we have algorithm to calculate  
Y and X.

we get Y by the Forward Substitution  
and then by Backward Substitution  
 $XK = Y$ , we can easily get X.

$$\therefore LK = B \text{ and } XK = Y$$

$$\therefore LY = B, \quad XK = Y$$

## Q2

a) Done in the problem\_2.py

b) Done in the problem\_2.py

c) By observing the outputs we can clearly see some of the cases

1. We saw that matrix has a large condition no and there could be large error if the data has small error.
2. We saw that for the good approximation residual is not a good.
3. There could be small residual even if the errors are pretty high in the matrix. As we saw in case of Hilbert Matrix.

Random Matrix- We observe that both error and residual without pivoting is less than pivoting one.

Hilbert Matrix - It's clearly observable that it is a ill condition linear system as it's error is pretty high.

Given Matrix - I observe that error as well as residual is 0 because there is no computation in floating points ,instead of that there is arithmetic computation which result in 0 residual and error.

## Q3

a)

```
#-----Q3-----  
A=np.array([[1,1,2e9],[2,-1,1e9],[1,2,0]])  
B=np.array([1.0,1.0,1.0])  
print(GE(A,B))
```

Output

```
[5.55555556e-01 2.22222222e-01 1.11111111e-10]
```

b)

```
A=np.array([[1,1,2e9],[2,-1,1e9],[1,2,0]] )
B=np.array([1.0,1.0,1.0])
for i in range(len(A)) :
    maxValue = abs(max(A[i], key = abs))
    B[i] =B[i]/ maxValue
    A[i] =A[i]/ maxValue
print(GE_pp(A,B))
```

Output

```
[5.55555556e-01 2.22222222e-01 1.11111111e-10]
```

**Yes, the answer is the same as in the part-a**

c) Yes the answer in both case as in the part-a because they ended up follow the same system to compute the output, thus because of floating point precision. i.e absolute difference of two vectors are less than the machine epsilon, Since there is very small difference which is very less than machine epsilon itself therefore ,we ended up having the same result.