Charge to Mass Ratio for Electron
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Abstract

In this variation of J. J. Thomson experiment, we aim to estimate the charge to mass ratio of the electron. As predicted by the theory, we observe the deflection of moving charges in a magnetic field. The electrons are deflected in such a way that they forms a circular trajectory, for which the radius can be measured. This radius can then be used to measure the charge to mass ratio for the electron, with the help of theoretical formulas relating the magnetic field to the radius of the orbit.

1 Introduction

A. Background Theory

An Electron moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$, experiences a force $\vec{F} = e\vec{v}\times\vec{V}$. In a constant magnetic field, with a velocity perpendicular to it, the electron moves in a circular orbit, such that $evB = m\frac{v^2}{r}$.

When the electrons are accelerated through the potential V in the electron gun, we have $eV = \frac{1}{2}mv^2$, which when combined with the former equation gives $\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{V}}$.

By measuring the potential, the magnetic field and the radius of the orbit, we can determine the charge to mass ratio of the electron with the above formula.

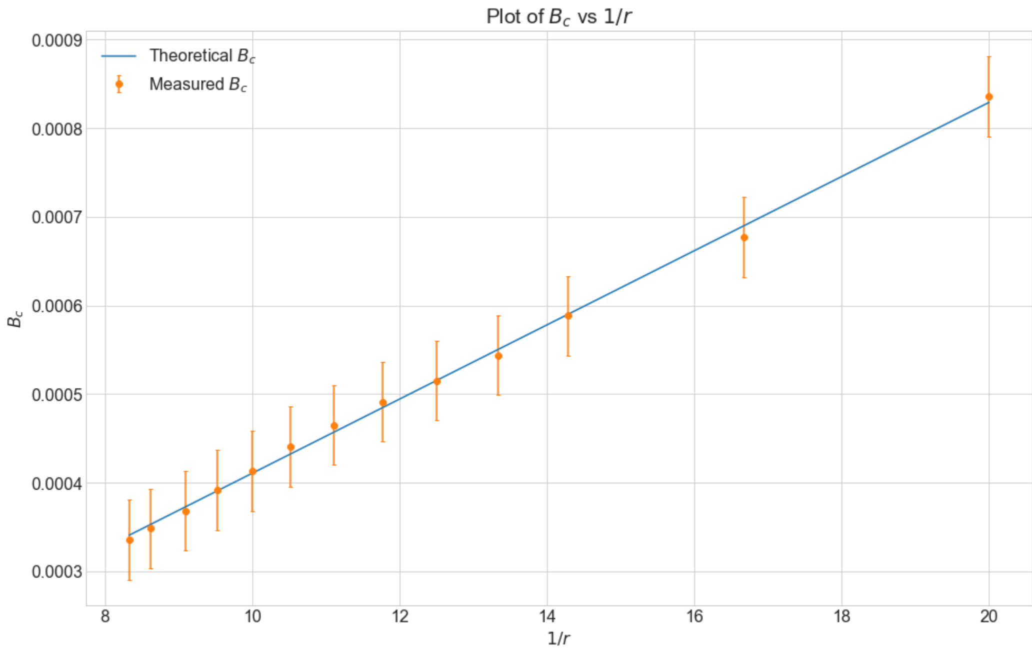
B. Materials and Methods

The main apparatus' for the experiment are the two Helmholtz coils, which produce the magnetic field that bend the electron beam, the vacuum tube, filled by a low density gas for minimal electron resistance, though which gives some rare collitions which emit light for visual inspection of the electron trajectory, and lastly the electron gun for creating the beam of free electrons with velocity v.

We followed the instructions given in the lab manual [1], and also the instructions given to us by the TA.

2 Results

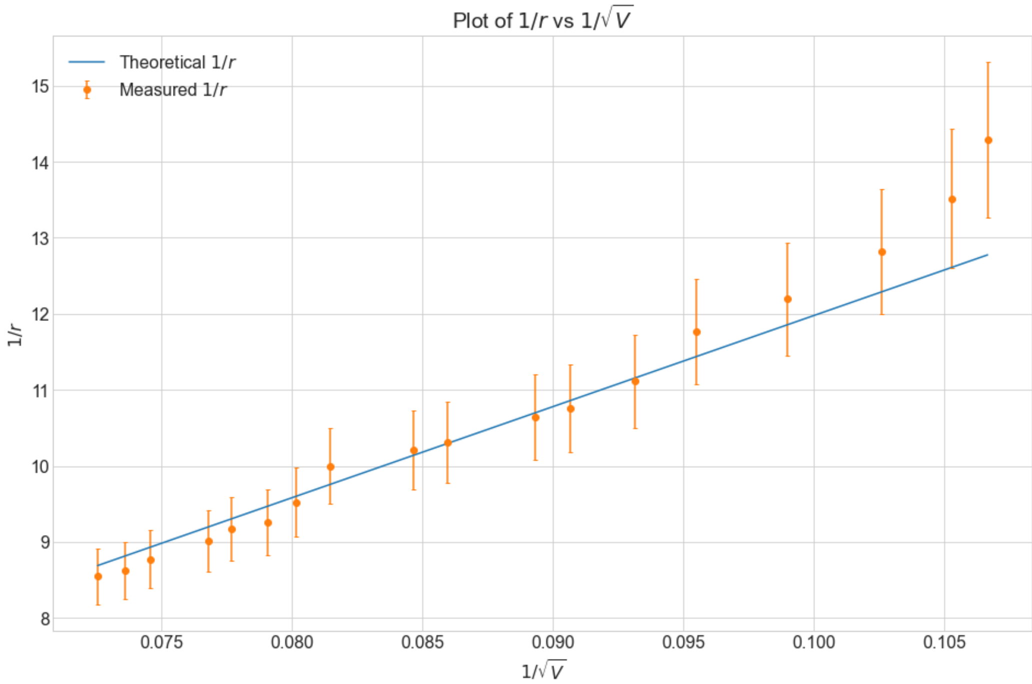
To compute the external magnetic field, we use $\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}(B_c + B_e) \implies B_c = (constant)\frac{1}{r} - B_e$. The constant factor is due to measurements done with constant voltage. The intercept of the fitted line gives $B_e = 0.00001 \pm 0.00005$ Tesla.



Fitting Equation $f(x) = ax + b$
$B_e = -b$

Figure 1: Computation of External Magnetic Field: B_c vs $1/r$

To compute the charge to mass ratio, we use $\frac{1}{r} = \sqrt{\frac{e}{m}} k \frac{I - I_0}{\sqrt{V}}$. For this case we use the data obtained keeping current constant. The charge to mass ratio is give by $(\frac{slope}{k(I-I_0)})^2$. The charge to mass ratio obtained in our case is $-(1.61 \pm 0.04) \times 10^{11}\ C/kg$.



Fitting Equation $f(x) = ax$
$a = 119.77$

Figure 2: Computation of e/m: $1/r$ vs $1/\sqrt{V}$

3 Uncertainty

For the calculation of B_e, the uncertainty is given by the fitting function. Hence $\Delta B_e = \Delta(y - intercept) = \pm 0.00005$ Tesla.

For the computation of the charge to mass ratio, we have $\frac{e}{m} = (\frac{slope}{k(I-I_0)})^2 \implies \Delta(\frac{e}{m}) = 2(slope)\frac{\Delta(slope)}{(k(I-I_0))^2}$. The uncertainty is then $\Delta(\frac{e}{m}) = \pm 0.04 \times 10^{11}\ C/kg$.

4 Discussion

Weak potential limit: The beam of electrons becomes visible when the electrons have enough kinetic energy to excite the gas by collision. However, when one lowers the potential below a certain threshold, for which we found to be around 90 Volts, the electrons do no longer have enough energy to emit a visible photon during collisions with the gas. Thus the electron beam becomes invisible when the electron beam is accelerated by a voltage below about 90 volt.

Strong potential, weak magnetic field limit: In this case, the electrons are accelerated to a velocity so high that the magnetic field bending of the electron beam becomes negligible. The electron beam then travels in a approximately straight line from the electron gun, perhaps weakly bent by the magnetic field, before the beam hits the wall on the other end of the vacuum chamber.

In general, one will observe that as long as the potential is strong enough to make the beam trajectory visible, only the relative strength of the potential and the magnetic field matters. Without loss of generality, if we fix the potential and tune the strength of the magnetic field from low to high, we will see that the electron beam goes from forming a straight line through the chamber, to being bent and hitting the wall of the vacuum chamber lower and lower, until the beam closes in on itself and forms a circle with a certain radius. As we continue to increase the strength of the magnetic field, the electrons are accelerated more and more. Therefore the radius of the beam circle shrinks and shrinks. In the infinite strength limit, the radius converges to zero.

In the case of low accelerating voltage and a strong magnetic field, we saw that the electron beam first is weakly bent downwards, close to the electron gun, and then gets more and more strongly bent as the beam approaches the centre of the vacuum chamber, and thus the maximum of the external magnetic field. This results in a non-circular electron beam trajectory. However this effect is small and not a significant source of error in our measurements, as we are still able to estimate the radius of the trajectory reasonably well. What does however introduces a significant source of error in our estimation of the electrons charge to mass ratio, is that in this situation the circular beam trajectory is not centered at the centre of the chamber. This is because the electrons velocity is so low that the electrons never reach far enough into the chamber before being bent into traveling in the other direction. Therefore the centre of the circle is significantly far away from the centre of the chamber. Our measurements in this situation therefore underestimate the charge to mass ratio of the electron, as we have assumed the magnetic field to be constant within the chamber and equal to the magnetic field strength at the centre of the chamber. However, in this situation, the magnetic field strength at the centre of the trajectory is weaker than at the centre of the chamber by an amount calculable by the formulas in the manual. This implies the solution to our problem: We can reduce this error in our charge to mass ratio estimate by estimating the actual magnetic field strength at the centre of the trajectory of the electron beam, and use this magnetic field strength in our calculations of charge to mass ratio.

When the light emitted by the electron beam hitting the gas reaches the curved walls of the vacuum chamber, the light is bent as it changes medium from low density gas, to glass and then to air outside the chamber. Also due to the curved walls of the chamber, the light is bent according to Snells law. Therefore it is difficult to measure the actual radius of the electron beam. To avoid this problem of parallax, we let the used a self illuminated scale with a plastic reflector. The light from the self illuminated scale hits the mirror and is reflected such that an image of the scale appears within the vacuum chamber by the electron beam. This is because the light from the scale is reflected by first the mirror, then travels to the glass wall of the vacuum chamber and from there is reflected to our eyes. Thus the light from the scale is bent in the same manner as the light from the beam, so by comparing the beam within the vacuum chamber to the scale hologram also appearing within the chamber, one can accurately measure the radius of the electron beam.

When bringing ferromagnetic materials close to the vacuum chamber with the circular electron beam, such as our computers or phones with metal casings, we see that we somewhat distort the magnetic field within the vacuum chamber. We know this because we can observe that the circular electron beam is weakly distorted into a different shape, for which depend on the placement of the ferromagnetic material in relation to the chamber. Depending on the size and amount of ferromagnetic material we brought close to the vacuum chamber, the magnetic field and thus our measurements ranged from being negligibly affected, to being significantly affected.

A Appendix

A.1 Experimental Data

A Constant Current

```
Current = 0.964 A
Radius (cm), Voltage (Volts)
7.0, 87.9
7.4, 90.2
7.8, 95.0
8.2, 102.1
8.5, 109.7
9.0, 115.3
9.3, 121.7
9.4, 125.3
9.7, 135.4
9.8, 139.6
10.0, 150.7
10.5, 155.6
10.8, 159.9
10.9, 165.7
11.1, 169.5
11.4, 179.9
11.6, 184.6
11.7, 190,4
```

B Constant Voltage

```
Voltage = 125.0 V
Radius (cm), Current (A)
12.0, 0.747
11.6, 0.775
11.0, 0.819
10.5, 0.872
10.0, 0.919
9.5, 0.980
9.0, 1.034
8.5, 1.093
8.0, 1.146
7.5, 1.210
7.0, 1.309
6.0, 1.506
5.0, 1.859
```

A.2 Python Code

The Python code for this exercise is divided into two files. statslab.py file contains utility methods which we will be frequently using in this course. lab_8.py file contains the code which analyzes the data.

A.2.1 statslab.py

```
import numpy as np
import scipy.optimize as optim
import matplotlib.pyplot as plt


################################################################################
# Utility Methods Library
#
# This file contains some utility method which are common to our data analysis.
# This library also contains customized plotting methods.
################################################################################

# use bigger font size for plots
plt.rcParams.update({'font.size': 16})

def chi2(y_measure,y_predict,errors):
    """Calculate the chi squared value given a measurement with errors and
    prediction"""
    return np.sum( np.power(y_measure - y_predict, 2) / np.power(errors, 2) )

def chi2reduced(y_measure, y_predict, errors, number_of_parameters):
    """Calculate the reduced chi squared value given a measurement with errors
    and prediction, and knowing the number of parameters in the model."""
    return chi2(y_measure, y_predict, errors)/ \
            (y_measure.size - number_of_parameters)

def read_data(filename, skiprows=1, usecols=(0,1), delimiter=","):
    """Load give\n file as csv with given parameters,
    returns the unpacked values"""
    return np.loadtxt(filename,
                      skiprows=skiprows,
                      usecols=usecols,
                      delimiter=delimiter,
                      unpack=True)

def fit_data(model_func, xdata, ydata, yerrors, guess=None):
    """Utility function to call curve_fit given x and y data with errors"""
    popt, pcov = optim.curve_fit(model_func,
                                 xdata,
                                 ydata,
```

```
                                absolute_sigma=True,
                                sigma=yerrors,
                                p0=guess)

    pstd = np.sqrt(np.diag(pcov))
    return popt, pstd

# y = ax+b
def linear_regression(xdata, ydata):
    """Simple linear regression model"""
    x_bar = np.average(xdata)
    y_bar = np.average(ydata)
    a_hat = np.sum( (xdata - x_bar) * (ydata - y_bar) ) / \
            np.sum( np.power((xdata - x_bar), 2) )
    b_hat = y_bar - a_hat  * x_bar
    return a_hat, b_hat
```

A.2.2 lab_8.py

```
#!/usr/bin/env python3
# @author: Pankaj
# -*- coding: utf-8 -*-

import statslab as utils
import matplotlib.pyplot as plt
import numpy as np

# constants
mu_0 = 4*np.pi*10**(-7)
n = 75 # number of turn of the coil.
R = 0.15 # 15 cm radius of the coil.
uncertainty_current = 0.1 # 0.1 Ampere
uncertainty_radius = 0.005 # 0.5 cm
V_constant = 125.0 # Volts
I_constant = 0.964 # Ampere

# characteristic of coil dimension
k = 1/np.sqrt(2)*(4/5)**(3/2)*mu_0*n/R

# computation of B_e
# work with constant voltage data
r, measured_currents = utils.read_data("../../data/Changing_current.csv",
                usecols=(0, 1),
                skiprows=2)

r = r / 100 # to meters
r_1 = 1/r # reciprocal of radius
B_c = (4/5)**(3/2)*mu_0*n/R*measured_currents # coil magnetic  field


# linear fitting equation
def model_function_Be(x, a, b):
    return  a*x + b

B_c_errors = np.ones_like(measured_currents) \
    * (4/5)**(3/2)*mu_0*n/R * uncertainty_current

popt, pstd = utils.fit_data(model_function_Be,
                            r_1,
                            B_c,
                            B_c_errors)

# get the y intercept for external magnetic field
B_e = -popt[1]
print("External Magnetic Field B_e = %.5f +/- %.5f Tesla" % (B_e, pstd[1]))
```

```
# plot the predicted and measured data
fig = plt.figure(figsize=(16,10))
fig.tight_layout()

xdata = np.linspace(np.min(r_1),
                    np.max(r_1), 1000)
ydata = model_function_Be(xdata, popt[0], popt[1])
plt.plot(xdata, ydata, label="Theoretical $B_c$")
plt.xlabel("$1/r$")
plt.ylabel("$B_c$")
plt.title("Plot of $B_c$ vs $1/r$")

# plot the measured data error bars
plt.errorbar(r_1,
             B_c,
             yerr=B_c_errors,
             marker="o",
             label="Measured $B_c$",
             capsize=2,
             ls="")

plt.legend()
plt.savefig("Coil B vs r_1.png", bbox_inches='tight')

# computation of e/m
# work with constant current data
r, measured_voltages = utils.read_data("../../data/Changing_voltage.csv",
                                       usecols=(0, 1),
                                       skiprows=2)

r = r / 100 # to meters
r_1 = 1/r # reciprocal of radius
r_1_errors = np.ones_like(r_1) * uncertainty_radius / (r ** 2)

I_0 = B_e / k

# linear fitting equation
def model_function(x, a):
    return  a*x

popt, pstd = utils.fit_data(model_function,
                            1 / np.sqrt(measured_voltages),
                            r_1,
                            r_1_errors)

slope = popt[0]

print("Slope of the line = %.2f" % slope)
```

```
# plot the predicted and measured data
fig = plt.figure(figsize=(16,10))
fig.tight_layout()

xdata = np.linspace(np.min(1 / np.sqrt(measured_voltages)),
                    np.max(1 / np.sqrt(measured_voltages)), 1000)
ydata = model_function(xdata, slope)
plt.plot(xdata, ydata, label="Theoretical $1/r$")
plt.xlabel("$1/\sqrt{V}$")
plt.ylabel("$1/r$")
plt.title("Plot of $1/r$ vs $1/\sqrt{V}$")

plt.errorbar(1 / np.sqrt(measured_voltages),
             r_1,
             yerr=r_1_errors,
             marker="o",
             label="Measured $1/r$",
             capsize=2,
             ls="")

plt.legend()
plt.savefig("Charge To Mass Ratio.png", bbox_inches='tight')


charge_to_mass_ratio = (slope / (k * (I_constant - I_0) ) ) ** 2
charge_to_mass_ratio_error = 2 * slope * pstd[0]/ \
    (k * (I_constant - I_0) ) ** 2
print("Charge to Mass Ratio for Electron = -%.2e +/- %.2e C/kg"\
      % (charge_to_mass_ratio, charge_to_mass_ratio_error))
```

References

[1] Charge to mass ratio for electron (e/m) - charge_to_mass.pdf (https://q.utoronto.ca/courses/235154/files/15436313/download?wrap=1).