① $f(x) = 2x^3 + ax^2 + 11x + a + 3$, is divisible by $2x - 1$,

So, $(2x-1)$ will be factor of $f(x)$ →

$2x - 1 = 0$

$\boxed{x = 1/2}$ will satisfy the function →

$2x^3 + ax^2 + 11x + a + 3 = 0$, where $x = 1/2$

$2\left(\frac{1}{8}\right) + a\left(\frac{1}{4}\right) + 11\left(\frac{1}{2}\right) + a + 3 = 0$

$\frac{1}{4} + \frac{a}{4} + \frac{11}{2} + a + 3 = 0$

$\frac{1 + a + 22 + 4a + 12}{4} = 0$

$5a + 35 = 0$

$\boxed{a = -7}$ Ans

---

③ Given that:- $x + y + z = 120$

$x = y + 20$

$x = z - 20$ → $\boxed{y + 40 = z}$

$y+20+y+y+40=120$

$3y+60=120$

$3y=60$, $\boxed{y=20}$ Answer

(4) $x^2-3x+1=0$, $\boxed{x^3+\frac{1}{x^3}=?}$

$x^2+1=3x$

$\Rightarrow \boxed{x+\frac{1}{x}=3}$

$(a+b)^3=a^3+b^3+3a^2b+3ab^2$

$\left(x+\frac{1}{x}\right)^3=27$

$x^3+\frac{1}{x^3}+3x^2\left(\frac{1}{x}\right)+3x\left(\frac{1}{x^2}\right)=27$

$x^3+\frac{1}{x^3}+3x+\frac{3}{x}=27$

$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=27$

$x^3+\frac{1}{x^3}+3(3)=27$

$x^3+\frac{1}{x^3}=27-9=18$

$\boxed{x^3+\frac{1}{x^3}=18}$ Ans

(5) $x^4 + \frac{1}{x^4} = 47$, $\boxed{x^3 + \frac{1}{x^3} = ?}$

We know that,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2(x^2)\left(\frac{1}{x^2}\right)$$

$$= 47 + 2$$

$$= 49$$

$$\boxed{x^2 + \frac{1}{x^2} = 7}$$

Similarly $\rightarrow$ $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right)$

$$= 7 + 2$$

$$= 9$$

$$\boxed{\left(x + \frac{1}{x}\right) = 3}$$

$\because$ $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - \frac{x}{x}\right)$$

$$x^3 + \frac{1}{x^3} = 3(7-1) = 18 \text{ Ans}$$

(2) $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$

$\therefore \quad a+b+c = 3x$

$\therefore \quad a^3+b^3+c^3-3abc$

$= (a+b+c)(a^2+b^2+c^2-ab-ac-bc)$

$\Rightarrow (x-a+x-b+x-c)\{(x-a)^2+(x-b)^2+(x-c)^2 - (x-a)(x-b) - (x-a)(x-c) - (x-b)(x-c)\}$

$(3x-(a+b+c))\{ \text{———} \;'' \;\text{———} \}$

$3x$

$(3x-3x)\{ \text{———} \;'' \;\text{———} \}$

$0\{ \text{———} \;'' \;\text{———} \} = 0$

$\boxed{\text{Answer} = 0}$