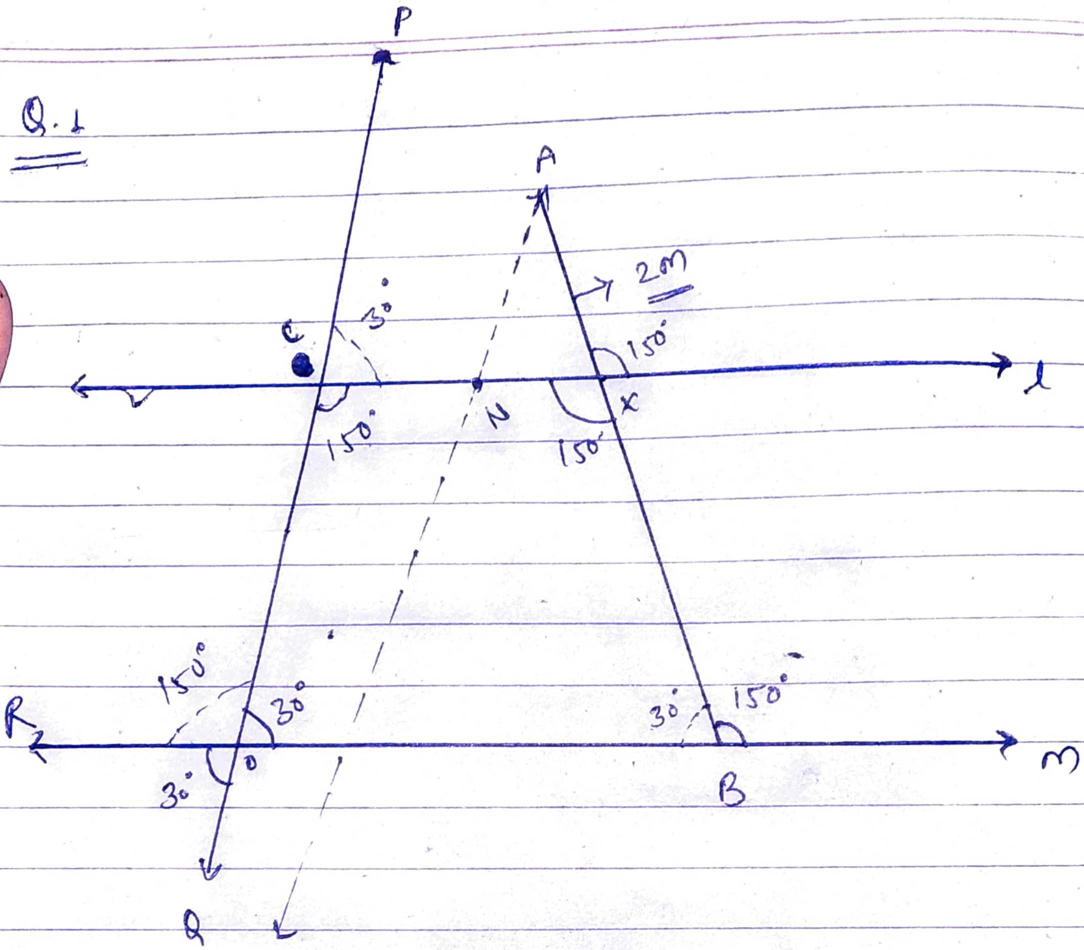
CHALLENGE

Q.1



Given that :-

- $AX = 2m$
- $l \parallel m$
- $\angle XBM = 150°$
- $\angle QOR = 30°$

$AN = ?$ Where N lies on l.

Such that,

$AN \parallel PQ$

Let N be the point which lies on the line l. Such that $AN \parallel PQ$.

Now, Since $\angle QOR = 30^\circ$ then

$\angle QOR = \angle ROB = 30^\circ$ { vertically opposite angle (शीर्षाभिमुख कोण) }

$\because \angle XBM + \angle XBO = 180^\circ$

$= 150^\circ + \angle XBO = 180$

$\boxed{\angle XBO = 30^\circ}$

Simmillarly $\Rightarrow \angle ROC + \angle COB = 180^\circ$

$\angle ROC + 30 = 180$

$\boxed{\angle ROC = 150^\circ}$

Then, $\angle ROC = 150^\circ = \angle NCO$

$\angle MBX = \angle NXB = 150^\circ$ { Alternate angle }

and, $\angle AXl = \angle XBM = 150^\circ$

Then $\angle AXN + \angle AXl = 180$

$\boxed{\angle AXN = 30}$

$\angle PCN = \angle ANX = 30^\circ$ { $\because PQ \parallel AN$ }

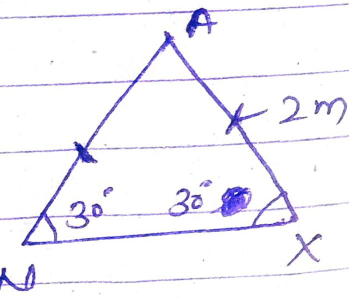
In $\triangle ANX$, ~~$\angle N = \angle X = 30$~~

$\boxed{\angle ANX = \angle AXN = 30^\circ}$ — (**)

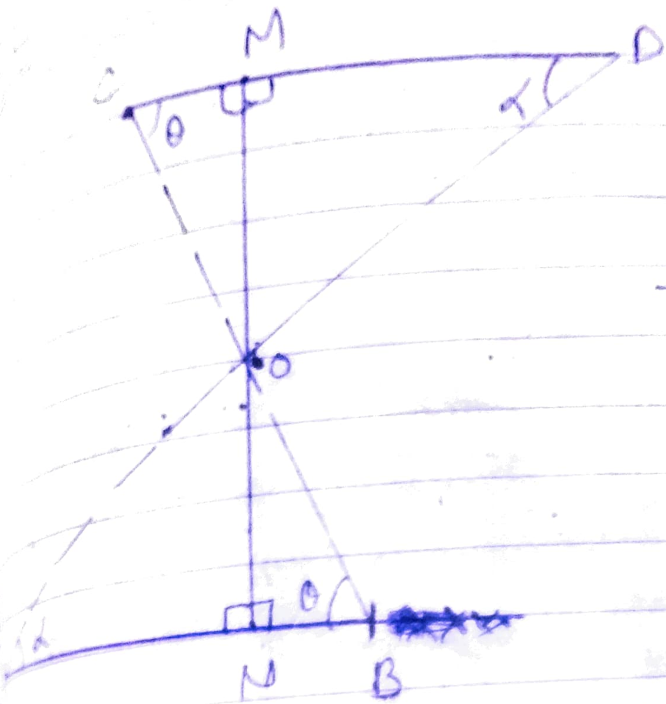
We know that if two angles are same in a triangle, then, their respective edges will be same. because that triangle will called isosceles triangle.

Hence, $\triangle ANX$ is an isosceles triangle

So, $AN = AX = 2m$.



$\boxed{AN = 2m}$ Answer



Given that:

$NN \perp AB \perp CD$

$CD \parallel AB$

$AB = MN = CD$

$CM : MD = 1 : (2\sqrt{3}-1)$

$AN : NB = (2\sqrt{3}-1) : 1$

Since $AB = CD$, then

$$\boxed{\frac{AB}{CD} = 1}$$

$$\Rightarrow \frac{AN + NB}{CM + MD} = 1$$

$$\frac{NB}{MD}\left\{\frac{\frac{AN}{NB}+1}{\frac{CM}{MD}+1}\right\} = 1$$

$$\frac{NB}{MD}\left\{\frac{2\sqrt{3}-\cancel{1}+\cancel{1}}{\frac{1}{2\sqrt{3}-1}+1}\right\} = 1$$

$$\frac{NB}{MD}\left\{\frac{2\sqrt{3}}{\frac{1+2\sqrt{3}-1}{2\sqrt{3}-1}}\right\} = 1$$

$$\frac{NB}{MD}\left\{2\sqrt{3}-1\right\}=1$$

$$\frac{NB}{MD}=\frac{1}{2\sqrt{3}-1}$$

$\boxed{NB=1}$, $\boxed{MD=2\sqrt{3}-1}$

In ΔMCO,

$$\tan\theta=\frac{MO}{CM} \quad -(1)$$

$\because AB=CD$

$\because \frac{CM}{MD}=\frac{NB}{AN}=\frac{1}{2\sqrt{3}-1}$

In ΔOBN,

$$\tan\theta=\frac{ON}{NB} \quad -(2)$$

Eq^n (1) + Eq^n (2)

$$2\tan\theta=\frac{MO+ON}{1}$$

$\because CM=NB=1$

$$2\tan\theta=MN=AB$$

$$2\tan\theta=2\sqrt{3}-1+1$$

$$2\tan\theta=2\sqrt{3}$$

$$\tan\theta=\sqrt{3}$$

$\boxed{\theta=60^\circ}$ Ans

Now In ΔOAN,

$$\tan\alpha = \frac{ON}{AN} \quad \text{--- (3)}$$

In, ΔMDO.

$$\tan\alpha = \frac{MO}{MD} \quad \text{--- (4)}$$

Eqⁿ (3) + Eqⁿ (4)

$$2\tan\alpha = \frac{ON}{2\sqrt{3}-1} + \frac{MO}{2\sqrt{3}-1}$$

$$2\tan\alpha = \frac{ON + ON}{2\sqrt{3}-1} = \frac{CD}{2\sqrt{3}-1}$$

$$2\tan\alpha = \frac{2\sqrt{3}-1+1}{2\sqrt{3}-1} = \frac{2\sqrt{3}}{2\sqrt{3}-1}$$

$$\boxed{\tan\alpha = \frac{\sqrt{3}}{2\sqrt{3}-1}}$$

$$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{2\sqrt{3}-1}\right) = 35.01 \simeq 35$$

$$\boxed{\alpha = 35^\circ}$$

Ans