(1) $a = 2+\sqrt{3}$

$a^3 = 8 + 3\sqrt{3} + 3\cdot 2^2\sqrt{3} + 3\cdot 2\cdot 3$

$a^3 = 8 + 3\sqrt{3} + 12\sqrt{3} + 18$

$\boxed{a^3 = 26 + 15\sqrt{3}}$

$$\frac{a^3}{a^6 + 3a^3 + 1}$$

$$= \frac{1}{a^3 + 3 + \frac{1}{a^3}}$$

$$\Rightarrow \frac{1}{26 + 15\sqrt{3} + 3 + \frac{1}{26 + 15\sqrt{3}}}$$

$$\Rightarrow \frac{26 + 15\sqrt{3}}{676 + 675 + 780\sqrt{3} + 78 + 45\sqrt{3} + 1}$$

$$\Rightarrow \frac{26 + 15\sqrt{3}}{1430 + 825\sqrt{3}}$$

$$\Rightarrow \frac{1}{55}\left(\frac{26 + 15\sqrt{3}}{26 + 15\sqrt{3}}\right)$$

$$\Rightarrow \frac{1}{55}$$ Ans

(2) $f(x) = 2x^3 + ax^2 + 11x + a + 3$, is divisible by $2x - 1$,

So, $(2x-1)$ will be factor of $f(x)$ →

$$2x - 1 = 0$$

$$\boxed{x = 1/2}$$ will Satisfy the function

$$2x^3 + ax^2 + 11x + a + 3 = 0, \text{ where } x = 1/2$$

$$2\left(\frac{1}{8}\right) + a\left(\frac{1}{4}\right) + 11\left(\frac{1}{2}\right) + a + 3 = 0$$

$$\frac{1}{4} + \frac{a}{4} + \frac{11}{2} + a + 3 = 0$$

$$\frac{1 + a + 22 + 4a + 12}{4} = 0$$

$$5a + 35 = 0$$

$$\boxed{a = -7}$$ Ans

(3) $p = 2 - a$, $a^3 + 6ap + p^3 - 8 = ?$

$p^3 = 8 - a^3 + 3.4.a + 3.2.a^2$

$$\boxed{p^3 = 8 - a^3 - 12a + 6a^2}$$

$\therefore$ $a^3 + 6a(2-a) + 8 - a^3 - 12a + 6a^2 - 8$

$\Rightarrow$ 0 Ans

(4) $x = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$x = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$ — (1)

$x^2 = 4 + 3 + 4\sqrt{3} \Rightarrow 7 + 4\sqrt{3}$ — (2)

$x^2 x = x^3 \Rightarrow 14 + 8\sqrt{3} + 7\sqrt{3} + 12$

$= 26 + 15\sqrt{3}$ — (3)

$\Rightarrow x^3 - 2x^2 - 7x + 5$

$= 26 + 15\sqrt{3} - 2(7 + 4\sqrt{3}) - 7(2 + \sqrt{3}) + 5$

$= 26 + 15\sqrt{3} - 14 - 8\sqrt{3} - 14 - 7\sqrt{3} + 5$

$= 31 - 28 = 3$ Ans

(5) let third is x ⇒

$$(x-y)^2 : (x^2-y^2)^2 : x$$

$$\frac{(x-y)^2}{(x^2-y^2)^2} = \frac{(x^2-y^2)^2}{x}$$

$$\frac{(x-y)^2}{(x+y)^2\,(x-y)^2} = \frac{(x^2-y^2)^2}{x}$$

$$\boxed{x = (x+y)^2\,(x^2-y^2)^2}$$

or

$$x = (x+y)^2\left\{(x+y)^2\,(x-y)^2\right\}$$

$$\boxed{x = (x+y)^4\,(x-y)^2}$$ (2) Ans