Portfolio Construction

Introduction

- Information analysis ignored real world issues.
- We now confront those issues directly, especially:
  - Constraints
  - Transactions costs
  - Things are much easier analytically if these issues didn't exist.
- We use portfolio construction in actual implementation, as well as backtesting of new investment strategies.

Objective Function/Approach

- We have focused on the objective function:

$$U = \alpha_P - \lambda \cdot \omega_P^2 = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

- Some investors have raised issues with this approach:
  - Sensitivity to errors in inputs: "error maximization"
- We will briefly discuss alternatives

Optimization and Data Errors

- How do estimation errors in α and **V** impact portfolio construction?

$$\alpha_{forecast} = \alpha_{true} + \varepsilon_{\alpha}$$

$$\sigma^2_{forecast} = \sigma^2_{true} + \varepsilon_{\sigma}$$

true $\sim \dfrac{\alpha_{true}}{\sigma^2_{true}}$

portfolio $\sim \dfrac{\alpha_{forecast}}{\sigma^2_{forecast}}$

- Relative to the "true" solutions, we will:
  - Overweight stocks with $\varepsilon_\alpha > 0$, $\varepsilon_\sigma < 0$.
  - Forecast higher alpha and lower risk than justified.
- This may still be the best construction approach.
- We can adjust expectations for size of the bias.

Muller's Portfolio Construction Test

- Muller (1993) tested four alternative methods in the following lab setting:
  - Follow S&P 500 stocks monthly from 1984 through 1987, with annual rebalance. (1987 treatment unusual)
  - Generate panels of alphas with fixed *IC* (0.10)

noise
$N(0,1)$

$$\alpha_n(t) = IC \cdot \left[IC \cdot \theta_n(t) + \sqrt{1 - IC^2} \cdot \omega_n \cdot Z_n(t) \right]$$

  - What does this do?

Combination of the actual residual returns and noise

    - Forecast alpha is a combination of the actual residual return plus noise.
    - What is the correlation of this forecast with the residual return?
    - What is the volatility of the alpha forecast?

Muller's Portfolio Construction Test

$$\alpha_n(t) = IC \cdot \left[IC \cdot \theta_n(t) + \sqrt{1 - IC^2} \cdot \omega_n \cdot Z_n(t) \right]$$

- The four alternative methods:
  - Screen 1: Top *N* stocks, equal weighted
  - Screen 2: Top *N* stocks, cap weighted
  - Stratification: Top *J* stocks in each BARRA industry, match weight
  - Quadratic programming, long-only, no position>10%.

mean-variance optimization

Test Results: Observed IR's

Date	Risk Aversion	SCREEN I	SCREEN II	STRAT	QP
	High	1.10	1.30	0.63	2.16
Jan-84	Medium	0.95	2.24	0.64	1.89
	Low	0.73	1.31	0.69	1.75
	High	0.78	1.47	1.98	0.98
Jan-85	Medium	0.74	-0.53	1.29	1.68
	Low	0.50	-0.15	0.83	1.49
	High	1.17	0.91	0.69	2.08
Jan-86	Medium	0.69	0.98	0.33	2.29
	Low	0.60	0.99	0.51	2.51
	High	1.43	2.04	2.82	2.14
May-87	Medium	1.01	1.48	2.60	1.76
	Low	0.66	1.17	2.17	1.82
Average		0.86	1.10	1.27	1.88
Standard Deviation		0.27	0.79	0.89	0.40
Maximum		1.43	2.24	2.82	2.51
Minimum		0.50	-0.53	0.33	0.98

Compare *IR* results to theory (2.24 in absence of constraints)

Bias in Optimization

- Muller's analysis shows quadratic optimization superior to alternative methods in generating high realized *IR* portfolios.
- In addition, he estimated a typical risk bias of ~20% for his optimized portfolios. So if the predicted risk was 3%, the realized risk averaged about 3.6%.
- This depends on the portfolios of interest, and on the accuracy of the risk model.

Back to Quadratic Optimization

- While prior tests show this approach superior to others on average, it is still susceptible to errors in input variables.
- We can treat this problem in two ways:
  - Controlling alphas
  - Adding constraints.
- We tend to prefer controlling alphas over adding constraints, as this provides more transparency into what we are doing. Both approaches can lead to the same answer.

Controlling Alphas

- Trimming
  - Winsorization
- Scale
  - We can check the scale of the alphas by examining the intrinsic *IR*

$$IR = \sqrt{\boldsymbol{\alpha}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}$$

  - If input alphas are inconsistent with our expectations for *IR*, we can rescale them.

- Note: We can get a rough estimate for the scale of the alphas by using:

$$\alpha_n = IC \cdot \omega_n \cdot z_n$$

$$Std\{\alpha\} \sim IC \cdot \omega$$

If $\omega \sim 25\%$
$IC \sim .05$
$std\{\alpha\} \sim 1.25\%$

And ignoring cross-sectional correlations and the variation of residual risk across stocks.

Neutralization

- Remove unintentional industry, sector, or market timing bets.
- This is different from scaling and trimming. Neutralization isn't always the right answer.
- What economic information does the signal contain?

Example: B/P ratios as alphas

Software – very low B/P

Utilities very high B/P

(on average)

Insights into Neutralization

- We can think about neutralization in terms of alphas or portfolios. The connection:

$$\boldsymbol{\alpha} = 2\lambda \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

- Examples

	Perspective	
	Alpha	**Portfolio**
Benchmark Neutrality	$\boxed{\alpha_B = \mathbf{h}_B^T \cdot \boldsymbol{\alpha} = 0}$	$\boxed{\beta_{PA} = \beta^T \cdot \mathbf{h}_{PA} = 0}$
Industry Neutrality	$\boxed{\alpha_j = \mathbf{H}_j^T \cdot \boldsymbol{\alpha} = 0}$	$\boxed{x_{PA}(j) = \mathbf{X}_j^T \cdot \mathbf{h}_{PA} = 0}$

Insight into Neutralization

- Benchmark Neutrality: Here are two possible choices:

Goal $\alpha_B \Rightarrow 0$

$$\boldsymbol{\alpha} \Rightarrow \boldsymbol{\alpha} - \alpha_B \cdot \mathbf{e}$$

$$\boldsymbol{\alpha} \Rightarrow \boldsymbol{\alpha} - \alpha_B \cdot \boldsymbol{\beta}$$

- Both transformations guarantee that the benchmark has zero alpha.
  - How do they differ?
  - What is happening to the optimal portfolio in each case?

Understanding Neutralization

- Start with the relationship between alphas and portfolios:

$$\boldsymbol{\alpha} = 2\lambda \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

- Now, what does this tell us about the hedging portfolio for one choice for benchmark neutrality?

$$\boldsymbol{\alpha} - \alpha_B \cdot \mathbf{e} = 2\lambda \cdot \mathbf{V} \cdot \left(\mathbf{h}_{PA} - \mathbf{h}_{hedge} \right)$$

$$\alpha_B \cdot \mathbf{e} = 2\lambda \cdot \mathbf{V} \cdot \mathbf{h}_{hedge}$$

$$\mathbf{h}_{hedge} = \left(\frac{\alpha_B}{2\lambda} \right) \cdot \mathbf{V}^{-1} \cdot \mathbf{e}$$

Aside: An Interesting Portfolio

- The hedging portfolio just described is related to Portfolio C, the minimum variance, fully-invested portfolio:

$$Min\left\{\mathbf{h}_C^T \cdot \mathbf{V} \cdot \mathbf{h}_C\right\} \quad subject\ to\ \mathbf{h}_C^T \cdot \mathbf{e} = 1$$

$$h_C \sim V^{-1} \cdot e \qquad \mathbf{h}_C \Rightarrow \frac{\mathbf{V}^{-1} \cdot \mathbf{e}}{\mathbf{e}^T \cdot \mathbf{V}^{-1} \cdot \mathbf{e}}$$

- So we are hedging the beta of our original portfolio with a portfolio that has $\beta_C \neq 0$
- There has been considerable interest in "minimum variance" strategies over the past few years. They have delivered market-like returns with less than market risk.

Benchmark Neutralization

- Two choices: $\boldsymbol{\alpha} \Rightarrow \boldsymbol{\alpha} - \alpha_B \cdot \mathbf{e}$

  $$\boldsymbol{\alpha} \Rightarrow \boldsymbol{\alpha} - \alpha_B \cdot \boldsymbol{\beta}$$

- The first hedges beta with Portfolio C. The second hedges with the benchmark.
  - The benchmark seems a more intuitive portfolio to use for hedging beta.
  - And in fact, it leads to the lowest total risk for the hedged portfolio.
  - But any portfolio with non-zero beta can hedge beta.

Factor Neutralization

- Start with a factor model and factor portfolios:

$$\mathbf{r} = \mathbf{X} \cdot \mathbf{b} + \mathbf{u}$$

$$\mathbf{b} = \left[\mathbf{X}^T \cdot \boldsymbol{\Delta}^{-1} \cdot \mathbf{X} \right]^{-1} \cdot \mathbf{X}^T \cdot \boldsymbol{\Delta}^{-1} \cdot \mathbf{r} = \mathbf{H}^T \cdot \mathbf{r}$$

- Plus our optimal connection between alphas and portfolios:

$$\boldsymbol{\alpha} = 2\lambda \cdot \mathbf{V} \cdot \mathbf{h}_{PA} = 2\lambda \cdot \mathbf{X} \cdot \mathbf{F} \cdot \mathbf{x}_{PA} + 2\lambda \cdot \Delta \cdot \mathbf{h}_{PA}$$

Goal

- Separate $\boldsymbol{\alpha}$ and $\mathbf{h}$ into common factor and specific components:

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_{CF} + \boldsymbol{\alpha}_{SP} \qquad \mathbf{h}_{PA} = \mathbf{h}_{CF} + \mathbf{h}_{SP}$$

- Such that:

$$\boldsymbol{\alpha}_{CF} = 2\lambda \cdot \mathbf{V} \cdot \mathbf{h}_{CF} = 2\lambda \cdot \mathbf{X} \cdot \mathbf{F} \cdot \mathbf{x}_{PA}$$

$$\boldsymbol{\alpha}_{SP} = 2\lambda \cdot \mathbf{V} \cdot \mathbf{h}_{SP} = 2\lambda \cdot \Delta \cdot \mathbf{h}_{PA}$$

- Challenge: This separation isn't unique. There are an infinite number of portfolios with active factor exposures $\mathbf{x}_{PA}$.

Separating the Portfolio

- To uniquely define a separation, we stipulate that $\mathbf{h}_{CF}$ is the minimum risk portfolio with factor exposures $\mathbf{x}_{PA}$. We construct this then from factor portfolios:

$$\mathbf{h}_{CF} = \mathbf{H} \cdot \mathbf{x}_{PA}$$

- We can then show that:

$$\mathbf{h}_{SP} = \boldsymbol{\Delta}^{-1} \cdot \left[\mathbf{I} - \mathbf{X} \cdot \mathbf{H} \right] \cdot \frac{\boldsymbol{\alpha}}{2\lambda}$$

- This approach can lead to pinpoint control over portfolio factor exposures.

Portfolio Construction

- Actual optimization isn't as simple as:

$$U = \mathbf{h}_{PA}^{T} \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^{T} \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

- We also have constraints:
  - Full investment: $\mathbf{h}_{P}^{T} \cdot \mathbf{e} = 1, \quad \mathbf{h}_{PA}^{T} \cdot \mathbf{e} = 0$
  - Long-only $h_P(n) \geq 0, \forall n$
  - Position size: $\left|h_{PA}(n)\right| \leq h_{PA}(n, \max)$
  - Factor exposure: $\left|x_{PA}(j)\right| \leq x_{PA}(j, \max)$
- And transactions costs.

Observations on Constraints and Costs

- These are equivalent to alpha adjustments.
- Some can significantly impact the portfolio.
- Now we will develop our intuition for constraints by analyzing the surprisingly large impact of the long-only constraint.

Long-only Constraint

- This constraint involves the benchmark:

$$h_P(n) \geq 0 \Rightarrow h_{PA}(n) \geq -h_B(n)$$

- Let's develop our intuition with a simple model:

$$\alpha_n = IC \cdot \omega_n \cdot z_n = \frac{IR \cdot \omega_n \cdot z_n}{\sqrt{N}}$$

$IR = IC \cdot \sqrt{N}$

$$h_{PA}(n) = \frac{IR \cdot z_n}{2\lambda \cdot \omega_n \cdot \sqrt{N}}$$

- We also know that: $\omega_P = \frac{IR}{2\lambda}$

Simple Model of Long-Only Constraint

- Hence we have:

$$h_{PA}(n) = \frac{\omega_P \cdot z_n}{\omega_n \cdot \sqrt{N}}$$

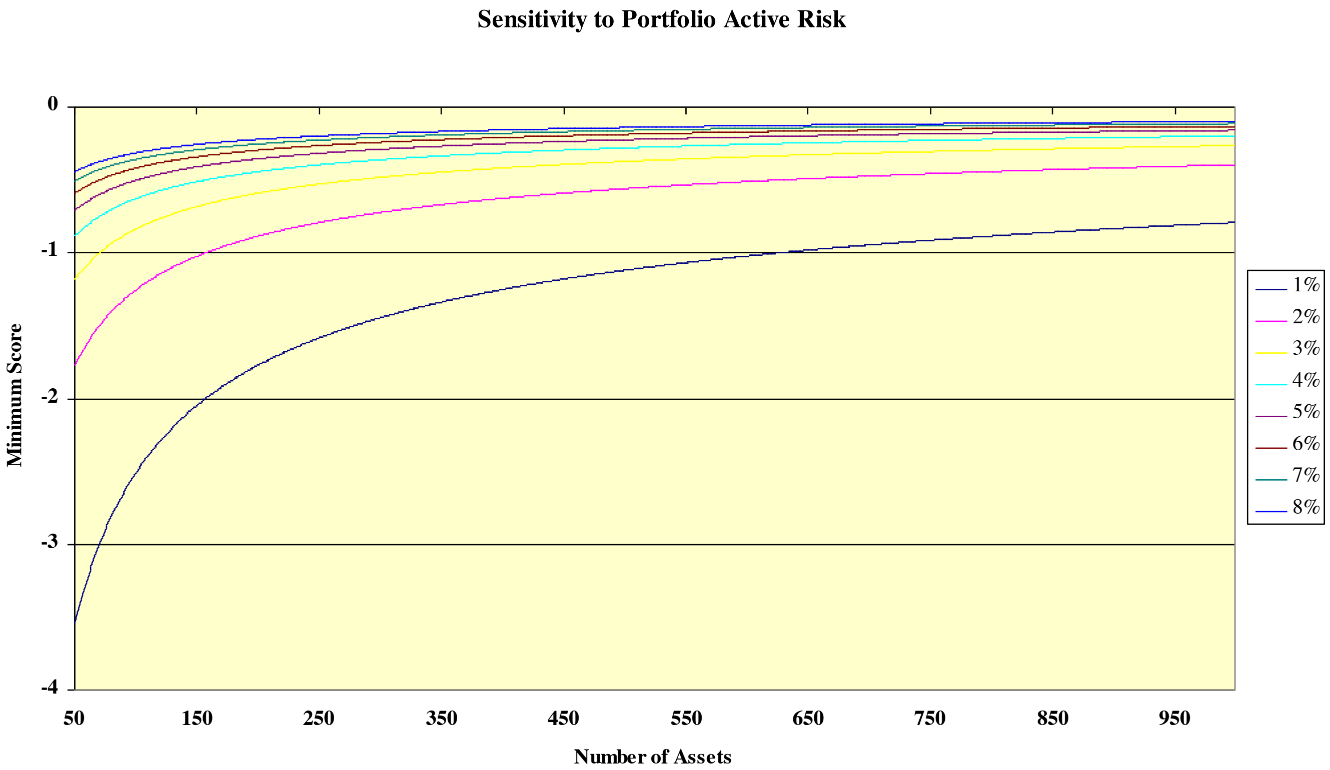
- So for an *equally-weighted* benchmark:

$h_B(n) = \frac{1}{N}$

$$h_{PA}(n) \geq -\frac{1}{N} \Rightarrow z_n \geq \frac{-\omega_n}{\omega_P \cdot \sqrt{N}}$$

- The constraint is more binding, the higher the portfolio active risk, the lower the stock residual risk, and the more stocks in the benchmark

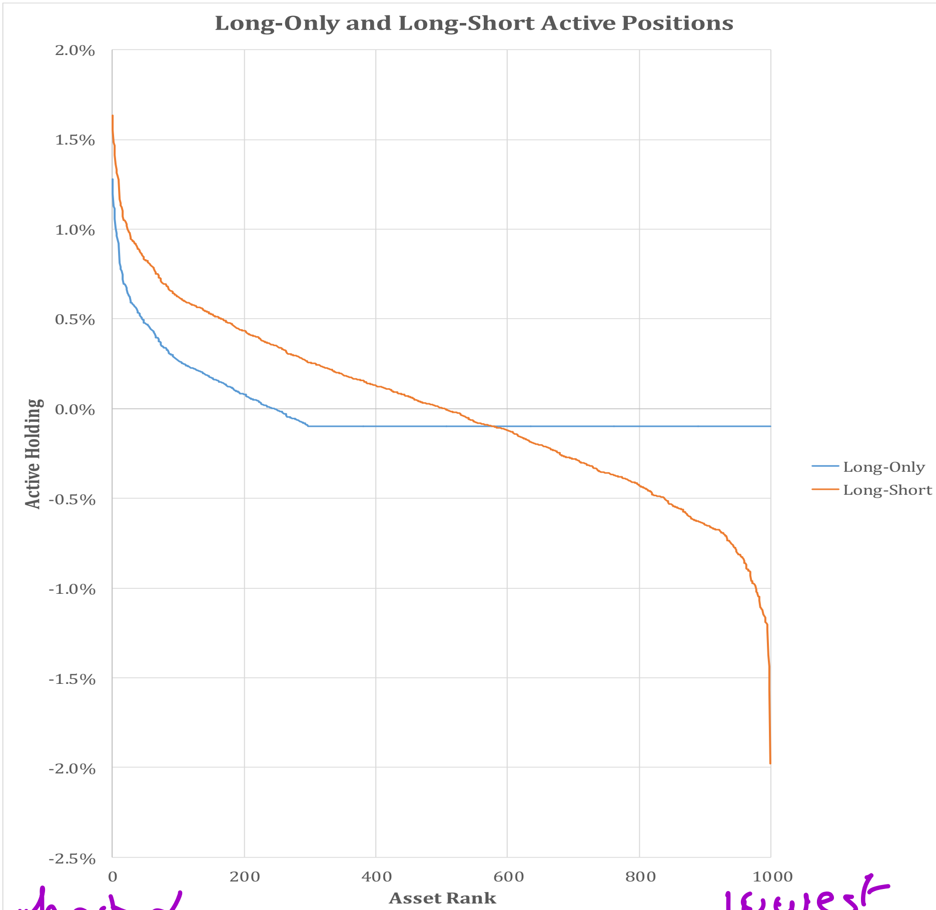
Minimum *z*-score



This isn't just about negative z-scores

- The long-only constraint interacts with the full investment constraint.
  - Active short positions *fund* active long positions.
- If we limit short positions, this will impact long positions too.
- Example: N=1,000
  - Compare long-only and long-short implementations.

Implications for Long Positions



highest α lowest alphas

Estimating More Realistic Impact

- We can only derive analytic results in overly simplified examples.
  - Inequality constraints don't lend themselves to analytic results.
  - The results depend on benchmark weights. Assuming equal weighting is not realistic.
  - We will now analyze the long-only constraint in more realistic detail. This will require simulations.

The Framework

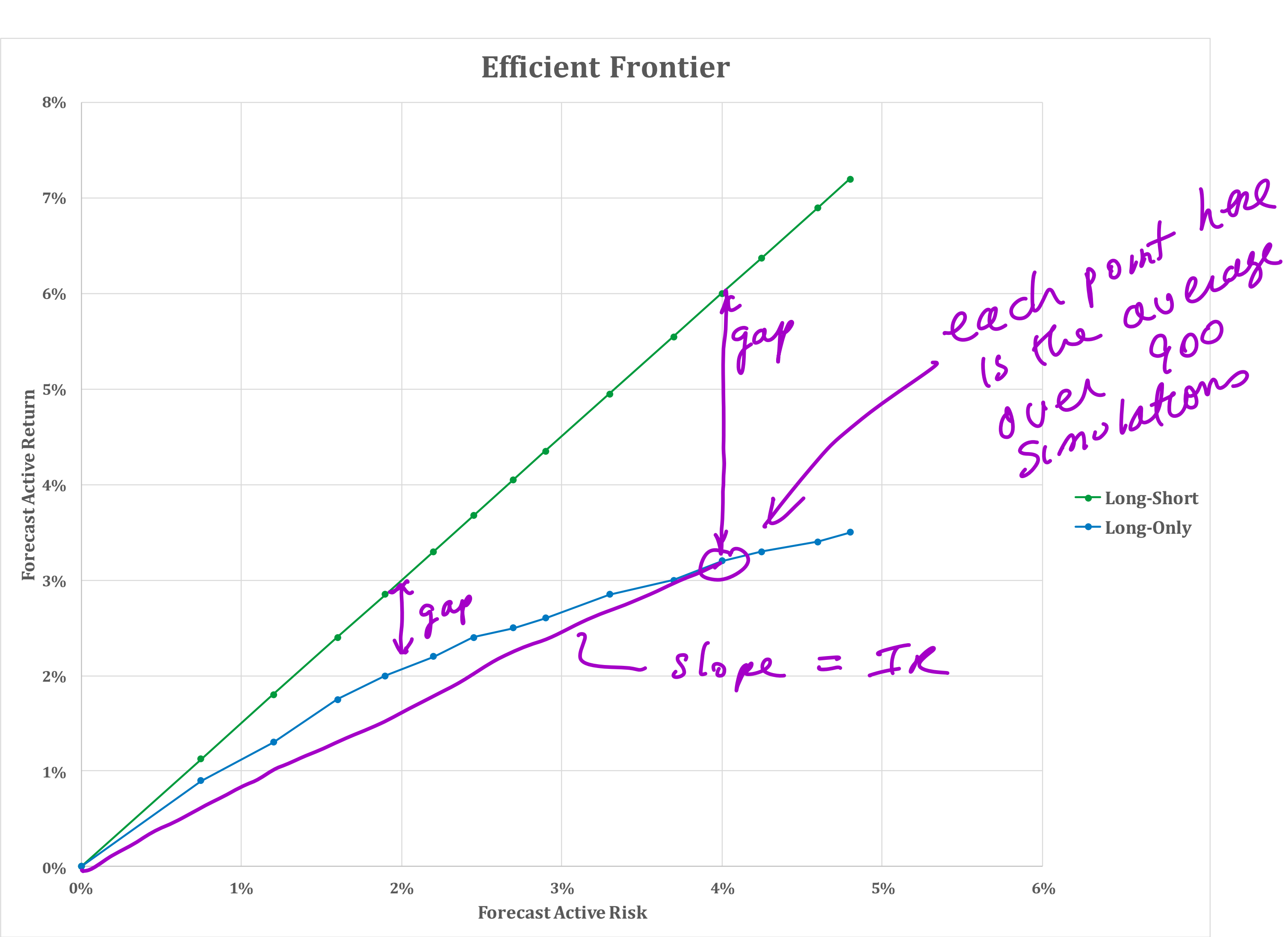
- We will start with a more realistic N stock cap-weighted benchmark.
  - More on this shortly.
- We will generate random alphas according to:

$$\alpha_n = \frac{IR \cdot \omega \cdot z_n}{\sqrt{N}}$$

- for many choices of N and ω_P. For each such choice, we will generate 900 panels of alphas. We will then analyze subsequent ex ante portfolio alpha and risk on average over those 900 samples.

Realistic Benchmarks

- Capitalization Weighted
- Every one is different (S&P 500, Russell 1000, MSCI EAFE, …)
- But they are typically not far from log-normally distributed.
- We will use the log-normal distribution, fit to typical benchmarks.



Efficient Frontier
8%
7%
6%
5%
4%
3%
2%
1%
0%
0%
1%
2%
3%
4%
5%
6%
Forecast Active Return
Forecast Active Risk
Long-Short
Long-Only
gap
gap
slope = IR
each point here is the average over 900 simulations

$$TC = \frac{IR}{IR_{int.}}$$

130/30 portfolio

Constraint: $\sum_{\text{short positions}} \leq 30\%$

$$\sum h_p = 100\%$$