Dynamic Portfolio Management

What dynamics?

- So far, we have considered portfolio management as a 1-period problem:
  - Given α, risk, and costs, what is the optimal portfolio?
- Our discussion of transaction cost amortization is symptomatic of a larger issue.

Dynamics

- Actual portfolio management is a dynamic problem.
- Alpha change over time:
  - New information arrives.
  - Old information becomes stale.
  - Trading occurs to close the gap between what we hold and what we would like to hold (a moving target).
- In this section, we will introduce a simple model* to illustrate dynamic portfolio management.
  - Simple enough to solve analytically, but with just enough complexity to exhibit interesting behavior.
- *Advances in Active Portfolio Management* (Grinold & Kahn, 2019) covers this topic in considerable depth.

*Grinold, Richard. "A Dynamic Model of Portfolio Management," *Journal of Investment Management*, Vol. 4, No. 2, pp. 5-22. (2006).

Grinold, Richard. "Dynamic Portfolio Analysis." *Journal of Portfolio Management*, pp. 12-26. (Fall 2007).

Alpha Dynamics

$$\underbrace{\boldsymbol{\alpha}(t)}_{\text{New } \alpha} = \underbrace{e^{-\gamma\cdot\Delta t}\cdot\boldsymbol{\alpha}(t-\Delta t)}_{\text{Proportion of old } \alpha \text{ (old information) relevant in current period}} + \underbrace{\tilde{\mathbf{s}}(t)\cdot\sqrt{\Delta t}}_{\text{New information}}$$

- Note that:

$$E\{\tilde{\mathbf{s}}(t)\} = 0$$

$$Corr\{\tilde{\mathbf{s}}(t), \boldsymbol{\alpha}(t-\Delta t)\} = 0$$

This is what we mean by "new information"

Return and Risk

- Zero-cost utility (flow, i.e. utility/time):

$$u = \mathbf{h}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}^T \cdot \mathbf{V} \cdot \mathbf{h}$$

- Optimizing this leads to a familiar result:

$$\mathbf{h}_Q = \frac{\mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}{2\lambda}$$

Assuming a long short portfolio / no constraints.

Transactions Costs

- We will use a *quadratic* model. (Hence we will sometimes call this the quadratic dynamic model for portfolio management.)

constant of proportionality

$$c(\mathbf{h}, \mathbf{h}_{old}) = \left(\frac{\hat{\eta}}{2}\right) \cdot (\mathbf{h} - \mathbf{h}_{old})^T \cdot \mathbf{V} \cdot (\mathbf{h} - \mathbf{h}_{old})$$

- This model is similar to previously discussed market impact model, though this is based on trade portfolio risk.
- The model ignores linear costs, commissions, spreads, etc.
- This assumed model of transactions costs is critical for developing a simple, analytically solvable model, that exhibits interesting behavior.

Optimization

- We want to set up a dynamic programming problem.
  - Goal: maximize steady-state performance.
  - No constraints (i.e. long-short portfolios)
- Define *g* as the steady-state utility per unit of time, and $U(\mathbf{h},\alpha)$ as the utility impact (relative to *g*) of having particular portfolio **h** and particular alpha α.
- We want to choose the optimal dynamic policy such that:

$$g \cdot \Delta t + U\left(\mathbf{h}_{old}, \boldsymbol{\alpha}\right) = Max_{\mathbf{h}} \left\{ u \cdot \Delta t - c + E\left\{ U\left(\mathbf{h}, \tilde{\boldsymbol{\alpha}}\right) \mid \boldsymbol{\alpha} \right\} \right\}$$

Optimal Policy

$$h_Q = \frac{V^{-1}\cdot\alpha}{2\lambda}$$

- Analytical solution is simple and intuitive:

$$\boxed{\mathbf{h}^{(t)} = (1-\delta)\cdot\psi\cdot\mathbf{h}_Q^{(t)} + \delta\cdot\mathbf{h}_{old}} \rightarrow h(t-1)$$

- Optimal choice is a weighted combination of the old portfolio and a scaled back version of Portfolio Q.
  – Weight and scaling parameters and constants that depend on costs, risk aversion, and halflife.

Parameters

- For convenience, we define an effective risk aversion:

$$\hat{\lambda} \equiv \lambda \cdot \Delta t \cdot \left[1 + \sqrt{1 + \frac{2\hat{\eta}}{\lambda \cdot \Delta t}} \right]$$

- Then we have simple formulas for δ and ψ:

$$\delta = \frac{\hat{\eta}}{\hat{\eta} + \hat{\lambda}} \qquad \psi = \frac{1-\delta}{1 - e^{-\gamma \Delta t} \cdot \delta}$$

$$0 \le \delta \le 1 \qquad 1 - \delta \le \psi \le 1$$

Limiting Cases

$$\mathbf{h}^{(t)} = (1-\delta)\cdot\psi\cdot\mathbf{h}_Q^{(t)} + \delta\cdot\mathbf{h}_{\cancel{old}}^{(t-1)}$$

- In the case of zero costs, $\hat{\eta} = 0,\ \delta \Rightarrow 0,\ \psi \Rightarrow 1.$ Then $\mathbf{h} \Rightarrow \mathbf{h}_Q$. Without costs, we don't have to worry about the dynamic problem.

- When costs dominate risk aversion, $\delta \Rightarrow 1,\ \psi \Rightarrow 0.$ Then $\mathbf{h} \Rightarrow \mathbf{h}_{old}$ $\hat{\eta} \gg \hat{\lambda}$

- When the halflife gets very long, $e^{-\gamma\Delta t} \Rightarrow 1,\ \psi \Rightarrow 1.$
  - With long halflife, there is no scaling back of $\mathbf{h}_Q$.

Equivalent Single Period Optimization

$h = (1-\delta)\psi h_\alpha + \delta h(t-1)$

- The optimal policy is the solution to an adjusted single period problem:

$$Max_{\mathbf{h}}\left\{\mathbf{h}^T\cdot\psi\cdot\boldsymbol{\alpha}-\lambda\cdot\mathbf{h}^T\cdot\mathbf{V}\cdot\mathbf{h}-\left(\frac{2\lambda}{\hat{\lambda}}\right)\cdot c\right\}$$

tcost amortization according to this quadratic model

- The adjustments include scaling back the alpha and amortizing the transactions costs.
- In this simple model, we can still justify single period optimization.

Garleanu and Pedersen [2013]

- "Dynamic Trading with Predictable Returns and Transactions Costs," *Journal of Finance*, December 2013.
- Extends the Grinold framework to handle multiple expected return factors with different mean reversion parameters, i.e. a combination of *fast* and *slow* signals.

$$Grinold: \quad \tilde{\boldsymbol{\alpha}}(t) = e^{-\gamma \cdot \Delta t} \cdot \tilde{\boldsymbol{\alpha}}(t - \Delta t) + \tilde{\mathbf{s}} \cdot \sqrt{\Delta t}$$

$$Garleanu, Pedersen: \quad \tilde{\mathbf{r}}(t) = \mathbf{B} \cdot \mathbf{f}(t - \Delta t) + \tilde{\mathbf{s}} \cdot \sqrt{\Delta t}$$

$$\Delta \mathbf{f}(t) \equiv \mathbf{f}(t) - \mathbf{f}(t - \Delta t) = -\boldsymbol{\Phi} \cdot \mathbf{f}(t - \Delta t) + \boldsymbol{\varepsilon}(t)$$

- A set of *K* factors **f** predict returns for *N* stocks. These factors mean-revert over time according to the *K*x*K* matrix Φ. The *N*x*K* matrix **B** captures stock exposures to the factors.

Garleanu and Pedersen

- Assuming that transactions costs are proportional to risk, and solving a dynamic programming problem, they find that:

$$\mathbf{h} = \left(1 - \frac{a}{\lambda}\right) \cdot \mathbf{h}_{old} + \left(\frac{a}{\lambda}\right) \cdot \mathbf{h}_{aim}$$

- The "aim" portfolio accounts for the mean reversion speeds of the different factors. If Φ is diagonal, then this becomes:

$$\mathbf{h}_{aim} = \left(\frac{1}{2\lambda}\right) \cdot \mathbf{V}^{-1} \cdot \mathbf{B} \cdot \begin{bmatrix} \left(\dfrac{f_1}{1 + \dfrac{a \cdot \phi_1}{\lambda}}\right) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \left(\dfrac{f_K}{1 + \dfrac{a \cdot \phi_K}{\lambda}}\right) \end{bmatrix}$$

- If all factors decay at the same rate, we are back at the Grinold result. But if they decay at different rates, then the more persistent factors have a bigger impact on the aim portfolio.