Equity: Section 3 [1]

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[1] Exercises inherited from Vinicio De Sola

Pset 1 review

1.
   a. $V = \begin{bmatrix} 0.09 & 0.06 \\ 0.06 & 0.16 \end{bmatrix}$
   b. $\sigma_P = 0.2989 = 29.89\%$
   c. $\psi_P = 0.0180 = 1.80\%$
   d. $V^{-1} \cdot e = \begin{bmatrix} 9.26 \\ 2.78 \end{bmatrix}$, $e^\top \cdot V^{-1} \cdot e = 12.04$, $h_C = \begin{bmatrix} 0.77 \\ 0.23 \end{bmatrix}$,
   $\sigma_C = 0.29 = 28.83\%$
2. 3%
3.
   a. $b = (X^\top X)^{-1} X^\top r = \begin{bmatrix} \frac{1}{N} \sum_{i=1}^{N} r_i \\ \frac{1}{N} \left(\sum_{i=1}^{N/2} r_i - \sum_{i=N/2+1}^{N} r_i \right) \end{bmatrix}$
   b. $h_{mkt} = \begin{bmatrix} 1/n & \cdots & 1/n \end{bmatrix}^\top$
   c. $h_{size} = \begin{bmatrix} 1/n & \cdots & 1/n & -1/n & \cdots - 1/n \end{bmatrix}^\top$

Pset 1 review

4. $N \approx 19$ or 20
5.
   a. $\omega_P = 0.0208 = 2.08\%$
   b. $U = 0.0052 = 0.52\%$
6.
   a. $L = h_{PA}^\top \cdot \alpha - \lambda h_{PA}^\top \cdot V \cdot h_{PA} + c \cdot h_{PA}^\top \cdot e$
   b. $h_{PA} = \frac{1}{2\lambda}\left(V^{-1}\alpha - e^\top V^{-1}\alpha \cdot h_C\right)$, $e^\top V^{-1}\alpha = 0 \Rightarrow h_{PA} = \frac{V^{-1}\alpha}{2\lambda}$
   c. $\alpha_Q = \frac{\alpha^\top V^{-1}\alpha}{e^\top V^{-1}\alpha}$, $\omega_Q^2 = \frac{\alpha^\top V^{-1}\alpha}{e^\top V^{-1}\alpha}$, $IR_Q^2 = \alpha^\top V^{-1}\alpha$,
   $h_{PA} = \frac{e^\top V^{-1}\alpha}{2\lambda}\left(h_Q - h_C\right)$

$h_{PA} = \frac{1}{2\lambda}\left(V^{-1}\alpha - e^{T}V^{-1}\alpha \cdot h_c\right)$

Pset 1 Review

7.

a.

monthly means		monthly std dev	
aug	-0.000112	aug	0.010645
sep	-0.000039	sep	0.010518
oct	-0.000034	oct	0.010393
nov	-0.000254	nov	0.010245
dec	-0.000311	dec	0.010500
jan	-0.000260	jan	0.010232
feb	-0.000517	feb	0.010217
mar	-0.000653	mar	0.010219
apr	-0.000663	apr	0.010343
may	-0.000910	may	0.010547
jun	-0.000756	jun	0.010530
jul	-0.000707	jul	0.010681

b. Average breadth = 367

8. $\psi_P = 5.6\%$. If we accounted for the true capitalization weighting of the index, we should be able to do better than this. That's because with capitalization weights, some stocks have higher weights in the index than other stocks.

Pset 1 Review

9.

a. annual %

b. ☺

c.

stock volatility

	index	TICKER	SRISL	vol	specific_frac	factor_frac
16	135	JPM	20.383	37.324897	0.298222	0.701778
24	346	SLB	16.896	35.292517	0.229194	0.770806
21	477	AMZN	22.748	31.134311	0.533836	0.466164
6	212	GE	15.804	29.745258	0.282292	0.717708
0	97	AAPL	20.002	29.373444	0.463700	0.536300
12	303	WFC	13.373	28.706688	0.217016	0.782984
23	411	QCOM	17.499	28.655694	0.372911	0.627089
15	308	ORCL	17.452	27.679183	0.397543	0.602457
18	240	INTC	12.771	26.654558	0.229566	0.770434
7	136	CVX	12.343	26.411229	0.218406	0.781594
8	8	GOOG	17.727	26.121620	0.460543	0.539457
3	280	MSFT	13.839	24.386447	0.322042	0.677958
1	188	XOM	12.343	24.112597	0.262031	0.737969
17	279	MRK	17.212	24.037009	0.512745	0.487255
11	321	PFE	14.189	22.099406	0.412233	0.587767
14	71	PM	12.343	21.298026	0.335863	0.664137
5	359	T	12.343	21.125308	0.341378	0.658622
19	114	VZ	12.343	20.673251	0.356471	0.643529
2	398	WMT	13.955	20.450331	0.465649	0.534351
4	241	IBM	12.343	20.212299	0.372915	0.627085
22	72	ABT	12.618	19.513421	0.418133	0.581867
13	332	PG	12.343	19.354487	0.406704	0.593296
9	247	JNJ	12.343	18.462053	0.446973	0.553027
20	320	PEP	12.631	17.954640	0.494905	0.505095
10	145	KO	12.343	17.845016	0.478418	0.521582

d. The total risk of cap-weighted 25 stock portfolio is 16.4%. Its variance is 95% common factor and 5% specific.

Q 8

$$\psi_p^2 = h_{PA}^T V h_{PA} \leftarrow \psi_p$$

scalars

$$\psi_p^2 = Var(\delta_p) = Var(r_p - r_B) = Var(h_p^T \cdot r - r_B)$$

Fully-invested portfolios

$h_p^T \cdot e = 1$

$$= Var(h_p^T \cdot r - 1 \cdot r_B)$$

$$= Var(h_p^T \cdot r - h_p^T \cdot e \cdot r_B)$$

$$= Var(h_p^T (r - e \cdot r_B))$$

$$= Var(h_p^T \cdot \delta)$$

$$= \sum_{i=1}^{N} h_p^2(i) \cdot \psi_i^2$$

Active returns uncorrelated

\+ stocks having same active risk

\+ S&P 500 being equally-weighted

$\Rightarrow$ minimum active risk portfolio is also equally-weighted

20 stocks $\quad \psi_i = 25\% \ \forall i$

$$\psi_p^2 = (25\%)^2 \sum_{i=1}^{20} \left(\frac{1}{20}\right)^2 = \frac{(25\%)^2}{(20)^2} \Rightarrow \psi_p = 5.6\%$$

```python
### b)

# Identify 25 largest firms by market capitalization
largest = exp.nlargest(25, 'CAPT')

# Note that we need to convert industry exposures from percent to decimal.
# Convert industry exposures to match styles exposures
for x in range(15,72):
    largest.iloc[:,x] = largest.iloc[:,x]/100

# Create new factor covariance matrix
F = cov.to_numpy()

# Create asset covariance matrix
X = largest.iloc[:, 3:71].to_numpy()
S = np.diag(largest['SRISL'].to_numpy())
S = np.square(S)
#print('specific risk covariance matrix\n', S)

V = np.matmul(X, np.matmul(F, np.transpose(X))) + S
#print(V)
```

Exercise 1 – APT

Suppose you have the following set of factor forecasts and exposures

Factors	Forecasts	Exposures
Heavy electrical (industry)	6%	1
Growth	2%	-0.24
Bond beta	-1%	0.13
Size	-0.5%	1.56
ROE	1%	0.15

If the market's expected excess return is 6%, and the stock's beta w.r.t. the market is 1.1, what is the stock's alpha?

Exercise 1 – APT

$$r = X\beta + u$$

$$\Downarrow$$

$$E[r] = X\beta$$

$$= \underbrace{6\% \cdot 1}_{\text{ind.}} + \underbrace{2\% \cdot (-0.24)}_{\text{growth}} + \underbrace{(-1\%) \cdot 0.13}_{\text{bond beta}} + \underbrace{(-0.5\%) \cdot 1.56}_{\text{size}} + \underbrace{1\% \cdot 0.15}_{\text{ROE}}$$

$$= 4.76\%$$

$= 0\%$

$$E[r] = \alpha + \beta(r_m - r_f)$$

$$4.76\% = \alpha + 1.1(6\%) \implies \alpha = -1.84\%$$

Exercise 1 – APT

tldr; APT is a straightforward way of getting you the residual return given a factor model.

Information processing

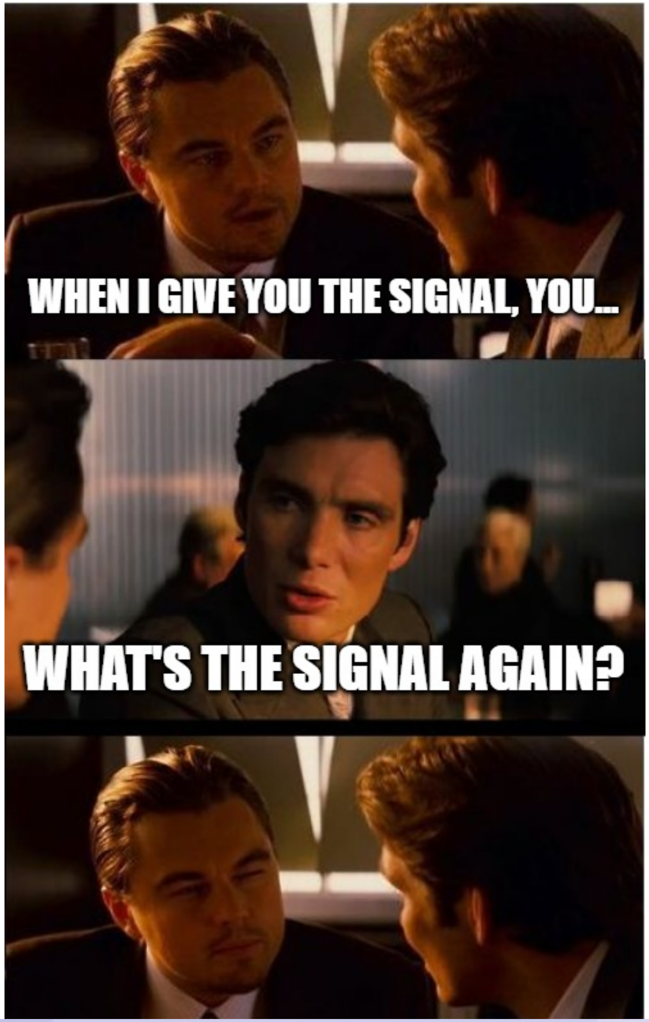
Raw information examples:

- Earnings estimates
- Measures of price momentum
- Brokers' buy recommendations

We use information to make forecasts:

- Naïve forecasts – the consesnsus expected return, informationless, leads to benchmark holdings.
- Raw forecasts – utilizes information in a raw form.
- Refined forecasts – transforms raw forecasts into *refined* forecasts

Signals



Exercise 2 – Signals

You model quarterly residual returns and information with the following binary model:

$$\theta = \sum_{i=1}^{100} \phi_i$$

$$g = \phi_3 + \phi_{76} + \sum_{i=1}^{23} \eta_i$$

Note: $\phi_i, \eta_i \sim N(0,1)$ and ϕ_i's and η_i's uncorrelated $\forall\, i$. Calculate:

a. Quarterly standard deviation of θ and g.

b. Covariance of θ and g.

c. The IC.

d. You receive information signal of 23. Forecast alpha.

observed/actual info. s is 23

$\phi_i, \eta_i \sim N(0,1)$ $\phi_i \perp \phi_j$ $\phi_i \perp \eta_i$

$\eta_i \perp \eta_j$

$\forall\, i,j$

$std(\theta) = 10$ $std(s) = 5$

a. $Var(\theta) = Var\left(\sum_{i=1}^{100} \phi_i\right) = Var(\phi_1 + \cdots + \phi_{100}) = 100 \Rightarrow$
$Var(g) = Var\left(\phi_3 + \phi_{76} + \sum_{i=1}^{23} \eta_i\right) = Var(\phi_3) + Var(\phi_{76}) + Var(\eta_1) + \cdots + Var(\eta_{23}) = 25 \Rightarrow$

b. $Cov(\theta, g) = 2$

c. $IC = corr(\theta, g) = \frac{Cov(\theta, s)}{\sigma_\theta \; \sigma_s} = \frac{2}{10 \cdot 5} = 0.04$

d. $\alpha = IC \cdot \omega \cdot z \leftarrow \frac{s - E[s]}{std(s)}$

$= 0.04 \cdot 10 \cdot \frac{(23 - 0)}{5} = 0.4 \cdot 4.6 = 1.84$ check

$= Cov\left(\sum_{i=1}^{100} \phi_i, \; \phi_3 + \phi_{76} + \sum_{i=1}^{23} \eta_i\right) = Cov(\phi_3 + \phi_{76}, \; \phi_3 + \phi_{76})$
$= Var(\phi_3) + Var(\phi_{76}) = 2$

Active bets

- How much risk are we taking on with each stock in our portfolio?
- Optimizing $U_P = h_{PA}^{\top}\alpha - \lambda h_{PA}^{\top} V h_{PA} \Rightarrow h_{PA}^{*}(i) = \frac{\alpha_i}{2\lambda\omega_i^2}$
- $\alpha_i = IC_i \cdot \omega_i \cdot z_i$ ← indiv. stock level
- $\Rightarrow h_{PA}^{*}(i) = \frac{IC_i \omega_i z_i}{2\lambda\omega_i^2} = \left(\frac{IC}{2\lambda}\right)\frac{z_i}{\omega_i}$
- Active bet $\equiv h_{PA}^{*}(i) \cdot \omega_i = \underbrace{\left(\frac{IC}{2\lambda}\right)} \cdot z_i$ *
- Active bet $\propto z_i$

$$z = \frac{s - E[s]}{std\{s\}}$$

> 0

< 0

Exercise 3 – Forecasting

$IC = corr(\theta, s) = corr(\theta, z)$

$\mathbb{P}(s_{ij.} = 1) = 1/5$

$\vdots$

$\mathbb{P}(s_{ij.} = 5) = 1/5$

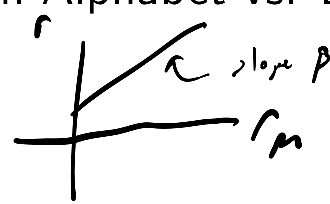
You have developed a model that forecasts stock returns. It provides stock ratings of 1 to 5 where 1 is a strong negative and 5 is a strong positive. The correlation of these ratings with subsequent residual return is 0.04. IC

a. The model currently gives a rating of 4 to Alphabet (residual risk of 30%). Translate that into a forecast α for Alphabet. $\alpha_i = IC_i \cdot \omega_i \cdot z_i$

b. The model gives a rating of 2 to Berkshire Hathaway (residual risk of 15%). Translate that into a forecast α for Berkshire Hathaway.

c. You now implement these alphas in an active strategy. Compare the size of the bet (active position times risk) on Alphabet vs. Berkshire Hathaway.

$$\alpha = IC \cdot \omega \cdot z$$



Exercise 3 – Forecasting

1, 2, 3, 4, 5 $\mathbb{P}(\text{sig.} = 1) = 1/5$
⋮
$\mathbb{P}(\text{sig.} = 5) = 1/5$ $E[s] = 3$
$\text{std}\{s\} = 1.414$

$\frac{s - E[s]}{\text{std}\{s\}}$

a. $\alpha_A = IC_A \cdot \omega_A \cdot z_A = 0.04 \cdot 0.3 \cdot \underbrace{\frac{4-3}{1.414}}_{z_A} \approx 0.008487$

b. $\alpha_B = IC_B \cdot \omega_B \cdot z_B = 0.04 \cdot 0.15 \cdot \underbrace{\frac{2-3}{1.414}}_{z_B} \approx -0.004243$

c. Active bet $= \left(\frac{IC}{2\lambda}\right) \cdot z$

$\left.\begin{array}{l}\lambda \text{ - constant} \\ IC \text{ - constant}\end{array}\right\}$ across stocks in our portfolio

$\Rightarrow$ Active bets are proportional to z

$z_A = -z_B$ from our signals

If we overweigh Alphabet, then we will underweigh BH

Pset 2 hints

There is a theme in this pset – combine our structure (equations) with what you are given

Q3.c – Recall Ron story from lecture desire 2% active risk

- $\psi_P^2 = h_{PA}^\top V h_{PA}$. From Pset 1: $h_{PA}^* = \frac{1}{2\lambda}\left(V^{-1}\alpha - e^\top V^{-1}\alpha h_C\right)$.

Q4 – Remember that your signals z_i are standardized.

Q6 – Don't think too hard about it.

Q9 – Ex ante and ex post IR.