

Framework



Active Management Perspective

- Our focus is on building a portfolio, P, to outperform a benchmark, B. Our active return is $\delta_P \equiv r_P - r_B$
- What does the BLUE framework say about our expected return relative to that benchmark?

$$E\{r_P | r_B\} = E\{r_P\} + \underbrace{\text{Cov}\{r_P, r_B\} \cdot \text{Var}^{-1}\{r_B\}}_{\beta_P} \cdot (r_B - E\{r_B\})$$
$$\Rightarrow E\{r_P\} + \beta_P \cdot (r_B - E\{r_B\})$$

$$\beta_P = \frac{\text{Cov}\{r_P, r_B\}}{\text{Var}\{r_B\}}$$

$$\beta_P \equiv \text{Cov}\{r_P, r_B\} \cdot \text{Var}^{-1}\{r_B\}$$

Now add a Return Decomposition

- Decompose the portfolio return into a piece correlated with (explained by) the benchmark, and a piece uncorrelated (residual to) the benchmark:

$$r_P = \beta_P \cdot r_B + \theta_P$$

$$r_P - r_B = (\beta_P - 1) \cdot r_B + \theta_P$$

$$E\{r_P\} = \beta_P \cdot E\{r_B\} + E\{\theta_P\}$$

$$Var\{r_P - r_B\} = (\beta_P - 1)^2 \cdot \sigma_B^2 + \omega_P^2$$

- We can always do this. It allows us to update the prior result:

$$E\{r_P \mid r_B\} \Rightarrow \beta_P \cdot r_B + E\{\theta_P\} = \beta_P \cdot r_B + \alpha_P$$

- Also:

$$E\{r_P\} = \beta_P \cdot E\{r_B\} + \alpha_P = \beta_P \cdot f_B + \alpha_P$$

Add Economics and Information

- CAPM Equilibrium Model (Sharpe, Treynor, Lintner, Mossin):

$$E\{\theta\} = 0$$

$$E\{r_P | r_B\} \Rightarrow \beta_P \cdot r_B$$

$$E\{r_P - r_B | r_B\} = E\{\delta_P\} \Rightarrow (\beta_P - 1) \cdot r_B$$

- Later we will study an alternative—arbitrage based—approach called APT.

- What if we have information not widely known to the market:

$$E\{\theta_P | g\} \equiv \alpha_P$$

$$E\{r_P | r_B\} \Rightarrow \beta_P \cdot r_B + \alpha_P$$

$$E\{\delta_P | r_B\} \Rightarrow (\beta_P - 1) \cdot r_B + \alpha_P$$

Active Equity Management

- Two sources of active return:
 - Alpha $\alpha_P \equiv E\{\theta_P \mid g\}$ based on information.
 - Beta $(\beta_P - 1) \cdot r_B$ based on benchmark exposure $\neq 1$

- Two sources of active risk:

$$Var\{\delta_P\} \equiv \psi_P^2 = (\beta_P - 1)^2 \cdot \sigma_B^2 + \omega_P^2$$

- Institutional equity managers generally choose $\beta_P \approx 1$.
 - So we will focus on forecasting alpha and building active portfolios to capture alpha. This will lead to the utility function for active equity management.
 - This assumption doesn't hold as well for fixed income managers who sometimes take duration bets—equivalent to beta bets.

Active Management Utility

"Active Risk"
↓

Tracking Error

$= \text{Var} \{ \sigma_p \}$

$= (\beta_P - 1)^2 \sigma_B^2 + \omega_P^2$

- The natural utility function for active management is:

$$U_P = \left[\alpha_P - \lambda \cdot \omega_P^2 \right]$$

$$+ \left[(\beta_P - 1) \cdot E \{ r_B \} - \lambda' \cdot (\beta_P - 1)^2 \cdot \sigma_B^2 \right]$$

$\Rightarrow \boxed{\alpha_P - \lambda \cdot \omega_P^2}$ for institutional equity managers

- We trade off expected residual return against residual risk. The risk aversion parameter, λ , measures our individual preferences for that trade-off.
- Most institutional equity managers will ignore the $\beta \neq 1$ part of this and focus on residual return and risk. That part uses the expected benchmark return to drive an optimal portfolio beta.
- The book (Chapter 4) goes through a long hand-waving derivation of this. We will simply assert this result and move on.

Utility: Intuition

$$U_P = \alpha_P - \lambda \cdot \omega_P^2$$

$$\alpha_P = \mathbf{h}_P^T \cdot \boldsymbol{\alpha} \Rightarrow \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha}$$

$$\left(\mathbf{h}_{PA} \equiv \mathbf{h}_P - \mathbf{h}_B, \quad \mathbf{h}_B^T \cdot \boldsymbol{\alpha} = 0 \right)$$

$$\begin{aligned} \omega_P^2 &= \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA} - (\beta_P - 1)^2 \cdot \sigma_B^2 \\ &\Rightarrow \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA} \quad \textit{typically} \end{aligned}$$

$$U_P \Rightarrow \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

Utility Intuition

$$U_P = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

- As we change the positions in our actively managed portfolio, the portfolio alpha and active risk vary.
- There will be one set of positions that maximizes utility.
- If we also want to impose full-investment or long-only constraints, we must explicitly add these.

Simplified Portfolio Construction

- Goal: make some simplifying assumptions and then solve for optimal active holdings. How do these depend on alpha and risk?
- Key step: simplify residual risk:

$$\begin{aligned}\omega_P^2 &= Var\{\mathbf{h}_P^T \cdot \boldsymbol{\theta}\} = Var\{\mathbf{h}_{PA}^T \cdot \boldsymbol{\theta}\} \\ \Rightarrow Var\left\{\sum_{n=1}^N h_{PA}(n) \cdot [r_n - \beta_n \cdot r_B]\right\} &= Var\left\{\sum_{n=1}^N h_{PA}(n) \cdot \theta_n\right\}\end{aligned}$$

- Assume (just for now) that residual returns are uncorrelated*. Hence:

$$\omega_P^2 \Rightarrow \sum_{n=1}^N h_{PA}^2(n) \cdot \omega_n^2$$

*This idea goes back to Sharpe, as we will see, and predates the CAPM.

Simplified Portfolio Construction

- Under that simple assumption, the active utility function becomes:

$$U_P \Rightarrow \sum_{n=1}^N h_{PA}(n) \cdot \alpha_n - \lambda \cdot h_{PA}^2(n) \cdot \omega_n^2$$

- And the optimal holdings are:

$$h_{PA}^*(n) = \frac{\alpha_n}{2\lambda\omega_n^2}$$

$$\frac{\partial}{\partial h_{PA}(n)} \Rightarrow \alpha_n - 2\lambda h_{PA}(n) \omega_n^2 = 0$$

- So we take active positions in proportion to alphas and inverse proportion to variance. The positions scale inversely with risk aversion.
- More generally:

$$\mathbf{h}_{PA}^* = \left(\frac{1}{2\lambda} \right) \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}$$

Example: Simplified Portfolio Construction

- Consider two stocks, JP Morgan and Netflix. Assume high risk aversion ($\lambda=0.15$):

| | ω | α |
|-----------|----------|----------|
| JP Morgan | 23% | 2% |
| Netflix | 38% | 4% |

- In this case, the higher alpha stock has the smaller active position:

$$h_{PA}(JPM) = \frac{2\%}{0.3 \cdot (23\%)^2} = 1.26\%$$

$$h_{PA}(NFLX) = \frac{4\%}{0.3 \cdot (38\%)^2} = 0.92\%$$

- but it is a higher risk position (i.e. a bigger bet):

Active Bet

$$h_{PA}(JPM) \cdot \omega_{JPM} = \frac{2\% \cdot (23\%)}{0.3 \cdot (23\%)^2} = 0.29\%$$

h.w

$$h_{PA}(NFLX) \cdot \omega_{NFLX} = \frac{4\% \cdot (38\%)}{0.3 \cdot (38\%)^2} = 0.35\%$$

The Information Ratio

- Define IR as *annualized* ratio of expected residual return to residual risk:

$$IR_P \equiv \frac{\alpha_P}{\omega_P}$$

- Note that our approach to portfolio construction, maximizing:

$$U = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

leads to portfolios with the highest possible IR given $\boldsymbol{\alpha}$ and \mathbf{V} .

Fully-Invested vs Long-Short, IR s and SR s

$$U = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

- Fully-Invested Typical Case
 - Also called “benchmark-linked”.
 - Sometimes, but not always, long-only

$$\mathbf{h}_{PA} = \mathbf{h}_P - \mathbf{h}_B$$

$$\mathbf{h}_B^T \cdot \mathbf{e} = 1, \quad \mathbf{h}_P^T \cdot \mathbf{e} = 1, \quad \mathbf{h}_{PA}^T \cdot \mathbf{e} = 0$$

$$\text{Typically: } \beta_P = 1, \quad \beta_{PA} = 0$$

$$\sigma_P^2 \equiv \text{Var}\{r_P\}, \quad f_P \equiv E\{r_P\}$$

$$\psi_P^2 \equiv \text{Var}\{r_P - r_B\} = \text{Var}\{\delta_P\} \Rightarrow \text{Var}\{\theta_P\} \text{ if } \beta_P = 1$$

$$SR = \frac{f_P}{\sigma_P}, \quad IR = \frac{\alpha_P}{\omega_P}$$

- The portfolio is fully-invested.
- The active portfolio is long-short
- The Sharpe Ratio (comparing the portfolio to cash) differs from the Information Ratio (comparing the portfolio to its benchmark).

Fully-Invested vs Long-Short, IR s and SR s

$$U = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

- Long-Short Typical Case
 - Sometimes, but not always, market neutral

$\mathbf{h}_B = 0$ The benchmark is cash.

$$\mathbf{h}_{PA} = \mathbf{h}_P - \mathbf{h}_B \Rightarrow \mathbf{h}_P$$

Typically $\mathbf{h}_P^T \cdot \mathbf{e} = 0$, $\beta_P = 0$ (and hence $r_P = \theta_P$)

$$\sigma_P^2 = \omega_P^2 = \psi_P^2, \quad f_P = \alpha_P = E\{\delta_P\}$$

$$SR = IR$$

- The portfolio is long-short
- The Sharpe Ratio (comparing the portfolio to cash) equals the Information Ratio (comparing the portfolio to its benchmark), since the benchmark is cash.

The *IR* and Optimal Portfolios

- The set of optimal portfolios: $U = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \mathbf{h}_{PA}^T \mathbf{V} \mathbf{h}_{PA}$

$$\mathbf{h}_{PA}^* = \frac{\mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}{2\lambda}$$

← optimize

- differ only in overall risk and alpha:

$$\omega_P^2 = \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA} \Rightarrow \frac{\boldsymbol{\alpha}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}{4\lambda^2}$$

$$\alpha_P = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} = \frac{\boldsymbol{\alpha}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}{2\lambda}$$

with active positions scaling with λ .

- In fact, the *IR* depends only on $\boldsymbol{\alpha}$ and \mathbf{V} and is independent of λ :

$$IR^2 = \boldsymbol{\alpha}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}$$

Annualization

Risk \equiv Standard Deviation

- To annualize monthly alphas, multiply by 12.
- To annualize monthly risk, multiply by $\sqrt{12}$.
- To annualize monthly IR , multiply by $\sqrt{12}$.

$$\theta_{annual} = \theta_{Jan} + \theta_{Feb} + \dots + \theta_{Dec}$$

$$\alpha_{annual} = E\{\theta_{annual}\} = \alpha_{Jan} + \alpha_{Feb} + \dots + \alpha_{Dec}$$

$$\Rightarrow 12 \cdot \alpha_{Jan}$$

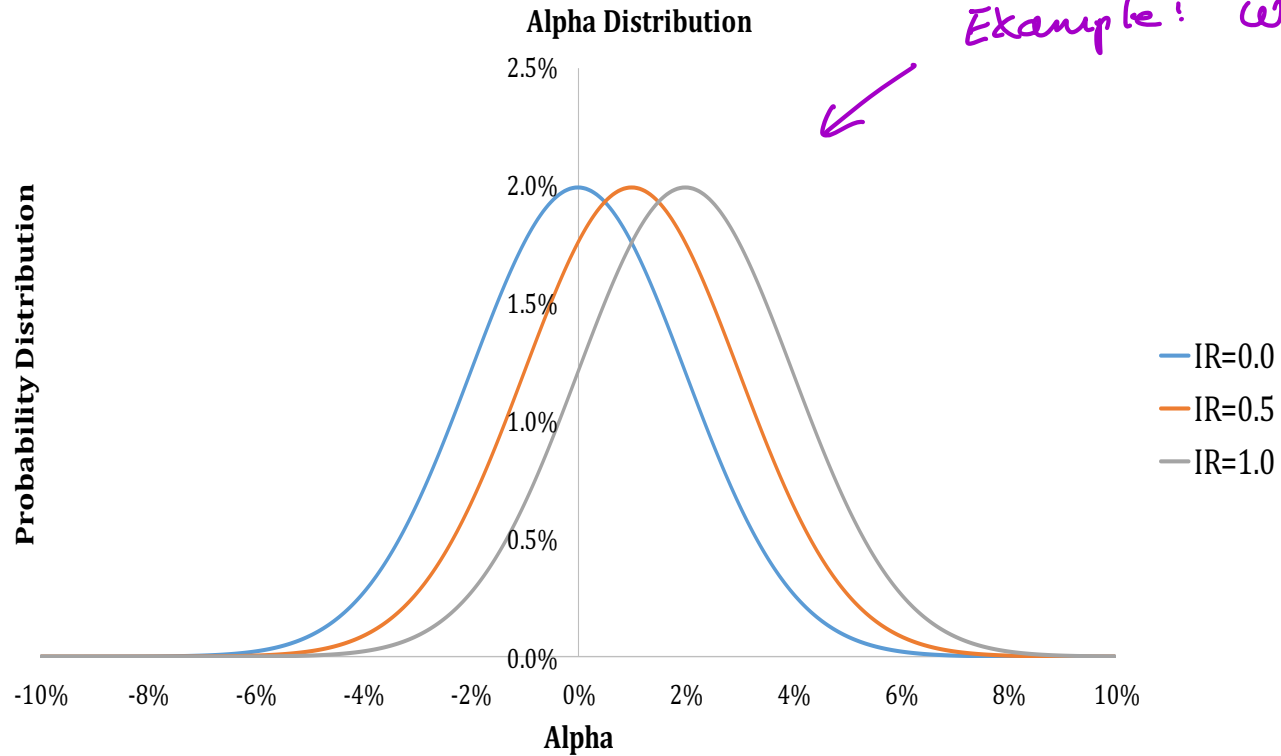
$$\omega_{annual}^2 = \omega_{Jan}^2 + \omega_{Feb}^2 + \dots + \omega_{Dec}^2$$

$$\Rightarrow 12 \cdot \omega_{Jan}^2$$

$$IR_{annual} = \sqrt{12} \cdot IR_{monthly}$$

Alpha
should be
theta

Information Ratios Measure Consistency of Performance

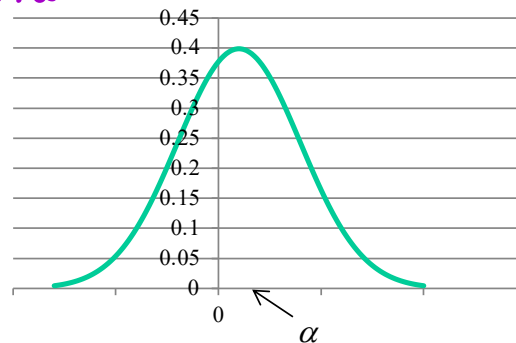


The Normal Distribution Result

- What is the probability that the annual residual return is positive, given the IR , and assuming residual returns are normally distributed?

Φ = cumulative normal distribution

$$\begin{aligned} prob &= 1 - \Phi \left\{ \frac{0 - \alpha}{\omega} \right\} = 1 - \Phi \{-IR\} \\ &= \Phi \{IR\} \end{aligned}$$



- This is our measure of consistency, and it is a monotonically increasing function of IR .

Information Ratios

- Typical Numbers:

| Percentile | IR | prob $\delta > 0$ |
|------------|------|-------------------|
| 90 | 1.0 | 84% |
| 75 | 0.5 | 69% |
| 50 | 0.0 | 50% |
| 25 | -0.5 | 31% |
| 10 | -1.0 | 16% |

Top Quartile

IR = .5

Top Decile

IR = 1

- Older Study Results*

| Information Ratios | | | | | |
|--------------------|--------------|-----------------|------------------|---------------|---------|
| Percentile | Equity | | | Fixed Income | Average |
| | Mutual Funds | Long-only Inst. | Long-Short Inst. | Institutional | |
| 90 | 1.04 | 0.77 | 1.17 | 0.96 | 0.99 |
| 75 | 0.64 | 0.42 | 0.57 | 0.50 | 0.53 |
| 50 | 0.20 | 0.02 | 0.25 | 0.01 | 0.12 |
| 25 | -0.21 | -0.38 | -0.22 | -0.45 | -0.32 |
| 10 | -0.62 | -0.77 | -0.58 | -0.90 | -0.72 |

*All US data 1/03 – 12/07. Studies include 338 equity mutual funds, 1679 equity long-only funds, 56 equity long-short funds, and 537 fixed income funds.

- More recent study (Morningstar data)

| Date | Top Quartile IR : Before Fees | |
|--------------|-------------------------------|-----------------------|
| | US Largecap Equity | US Broad Fixed Income |
| 10/97 - 9/02 | 0.42 | 0.14 |
| 10/02 - 9/07 | 0.43 | 1.40 |
| 10/07 - 9/12 | 0.34 | 0.60 |
| 10/12 - 9/17 | 0.26 | 1.32 |

IR as Budget Constraint

- Convert the definition of the Information Ratio into a budget constraint:

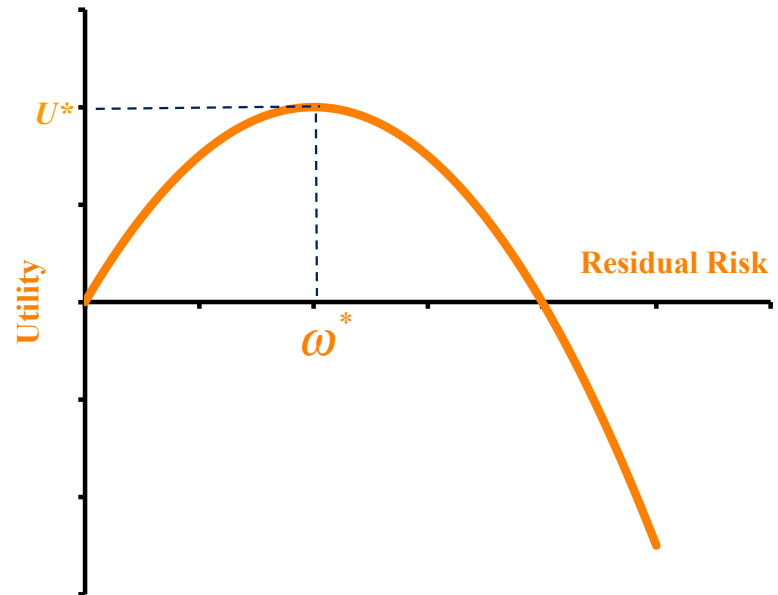
$$\alpha_P = IR \cdot \omega_P$$

- This works for the optimal portfolios because we increase risk by scaling positions. It breaks down when we can no longer do that—e.g. due to the long-only constraint.

Maximum Utility

$$U = \alpha - \lambda \cdot \omega^2$$

$$= IR \cdot \omega - \lambda \cdot \omega^2$$



Optimal Risk and Utility

- We can solve for the optimal risk level:

$$\omega_P^* = \frac{IR}{2\lambda}$$

- And the maximum achievable utility:

$$U_P^* = \frac{(IR)^2}{4\lambda}$$

Active Risk Levels

- Older Studies

| Percentile | Mutual Funds | Active Risk | | Fixed Income |
|------------|--------------|-----------------|------------------|--------------|
| | | Equity | | |
| | | Long-only Inst. | Long-short Inst. | Inst. |
| 90 | 6.85% | 6.80% | 10.18% | 1.64% |
| 75 | 5.52% | 5.33% | 6.99% | 0.94% |
| 50 | 4.05% | 4.03% | 4.76% | 0.63% |
| 25 | 3.08% | 3.01% | 3.67% | 0.45% |
| 10 | 2.35% | 2.21% | 2.97% | 0.33% |

- More Recent Studies (Morningstar data: 10/97-9/17)

| Percentile | Active Risk Levels | |
|------------|--------------------|-----------------------|
| | US Largecap Equity | US Broad Fixed Income |
| 90 | 7.63% | 2.56% |
| 75 | 5.79% | 1.69% |
| 50 | 4.13% | 1.06% |
| 25 | 2.86% | 0.75% |
| 10 | 1.68% | 0.58% |

Risk Aversion Parameter

- Prior result provides intuition for risk aversion.

$$\omega_P^* = \frac{IR}{2\lambda}, \quad \Rightarrow \lambda = \frac{IR}{2\omega_P^*}$$

- Intuition 1: Risk aversion depends on whether we measure residual risk as percent or decimal.
- Intuition 2: Typical levels:

| Risk Aversion | λ |
|---------------|-----------|
| low | 0.05 |
| medium | 0.10 |
| high | 0.15 |

} measuring ω_P
in percent.

Fundamental Law of Active Management

- We have seen that IR is critical measure for active management.
- The fundamental law can provide intuition into the IR .
- We will prove the simple case. Chapter 6 in the book proves the general case.

Plan of Attack

- Start with a snapshot of “information” about N stocks.
- Turn this information into an optimal portfolio.
 - What is the expected alpha, risk, and information ratio of that portfolio?
- Calculate expectation of the IR over all possible sets of information.

Information

- In principle, this could be broker BUY/SELL recommendations, P/E ratios, recent stock returns...
 - Any set of numbers we think is correlated with future active returns.
- We will generically refer to these numbers using the variable g
- How do we convert those numbers into alphas?

Use the BLUE approach

- Alpha will be the best linear unbiased estimate of θ conditional on g :

$$E\{\theta | g\} \equiv \alpha = E\{\theta\} + Cov\{\theta, g\} \cdot Var^{-1}\{g\} \cdot [g - E\{g\}]$$

$$\Rightarrow 0 + Corr\{\theta, g\} \cdot \omega \cdot \left[\frac{g - E\{g\}}{StDev\{g\}} \right]$$

$$\alpha = Corr\{\theta, g\} \cdot \omega \cdot z = IC \cdot \omega \cdot z$$

- In this result:
 - We convert the information, g , to a *z-score*.
 - We define the *IC* as the *Information Coefficient*. It is the correlation of θ with z (or equivalently with g).

Understanding ICs

- The Information Coefficient is the correlation of z-scores with future residual returns. It is also the correlation of the α and the underlying information, g , with future residual returns, since all three $\{z, \alpha, g\}$ are linearly related.
- One way to understand ICs is to connect them to something perhaps more intuitive: the fraction fr of forecasts that correctly predict the sign of θ .
- If residual returns are normally distributed, and the forecast alpha is a linear combination of information and noise (i.e. $\alpha_n = IC \cdot [IC \cdot \theta_n + \sqrt{1-IC^2} \cdot \omega_n \cdot Z_n]$), we can show (with lots of normal distribution integrals) that:

$$fr = \frac{1}{2} + \left(\frac{1}{\pi} \right) \cdot \arctan \left\{ \frac{IC}{\sqrt{1-IC^2}} \right\}$$

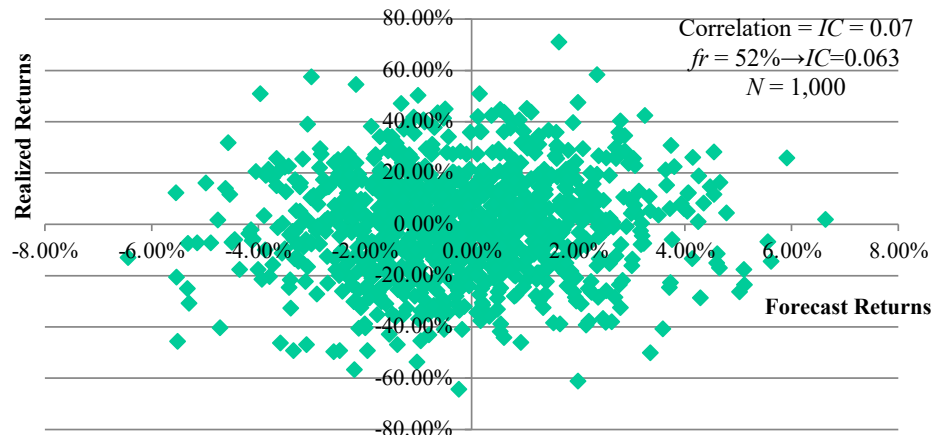
$$\approx \frac{1}{2} + \left(\frac{IC}{\pi} \right) \quad \text{for } IC \ll 1$$

Typical *ICs*

| Skill | IC | <i>fr</i> |
|---------|------|-----------|
| great | 0.10 | 53.2% |
| good | 0.05 | 51.6% |
| average | 0.00 | 50.0% |

- Note that this simple model is for *intuition only*. If we want to calculate the *IC*, we calculate the correlation.

Realized versus Forecast Returns



Back to the Fundamental Law

- Assume N assets and N z-scores (one for each asset), residual returns uncorrelated, residual risks identical, and correlations between signals and returns identical.*
- Optimal holdings:

$$h_{PA}(n) = \frac{\alpha_n(z)}{2\lambda \cdot \omega^2}$$

- What are predicted alpha and risk?

*The treatment in Chapter 6 relaxes these assumptions.

Portfolio Alpha and Risk

- Optimal conditional holdings:

$$h_{PA}(n) = \frac{IC \cdot z_n}{2\lambda\omega}$$

- Portfolio Alpha:

$$\alpha_P = \sum_{n=1}^N h_{PA}(n) \cdot \alpha_n = \left(\frac{IC^2}{2\lambda} \right) \cdot \sum_{n=1}^N z_n^2$$

- Portfolio Risk:

$$\omega_P^2 = \sum_{n=1}^N \left(h_{PA}(n) \cdot \omega_n \right)^2 = \left(\frac{IC^2}{4\lambda^2} \right) \cdot \sum_{n=1}^N z_n^2$$

Conditional Information Ratio

- We can now combine previous results to calculate the information ratio conditional on the signals $\{z\}$:

$$IR_P^2(z) = \frac{\alpha_P^2(z)}{\omega_P^2(z)} = IC^2 \cdot \sum_{n=1}^N z_n^2$$

- Taking the unconditional expectation of the above, and noting that:

$$E\left\{\frac{1}{N} \cdot \sum_{n=1}^N z_n^2\right\} = 1 \quad \text{since scores have mean 0, std 1}$$

– we find:

$$E\left\{IR_P^2(z)\right\} = IR_P^2 = IC^2 \cdot N$$

$$IR_P = IC \cdot \sqrt{N}$$

- The more detailed derivation, which accounts for correlations between z-scores, leads to a generalized version of this.

Fundamental Law

- Information Ratio depends on skill and breadth:


$$IR = IC \cdot \sqrt{BR}$$

- We measure skill by the information coefficient.
- To consistently outperform requires skill and breadth. We measure breadth as the number of independent bets per year.
- This result assumes that we use information to build optimal portfolios. It is an upper bound on what we can achieve.

Understanding Breadth

- Independent bets per year.
 - A rate, not a number. So we expect twice as many bets over two years.
 - We could implement a beta bet with positions in ~ 300 stocks, but that's one bet, not 300 bets.
 - So breadth is not as simple as the number of holdings.
- Consider an investment process in equilibrium.
 - Old information decays.
 - New information arrives.
 - The two are in balance, so an information turnover rate, γ , captures both.

Forecasts in Equilibrium

$\gamma = \text{information turnover rate}$

$$\alpha_n(t) = \underbrace{e^{-\gamma \cdot \Delta t} \cdot \alpha_n(t - \Delta t)}_{\text{Old information decaying}} + \underbrace{\tilde{s}_n(t) \cdot \sqrt{\Delta t}}_{\text{New information arriving}}$$

Note: This is a way of understanding what is going on “under the hood” with our alphas. It is not an operational statement. Every period, we forecast alphas using the latest available information. The above statement says that the value of that information decays over time, and that eventually (and often episodically) we receive new information. As George Box noted, “All models are wrong, but some are useful.”

$$BR = \gamma \cdot N$$

We can estimate γ from a history of forecasts.

Breadth Example

- Follow 300 stocks. Every week, receive new information on 12 stocks. (We can't predict which 12 stocks.)

$$BR = 12 \cdot 52 = 624$$

Here's the investment process analysis:

$$\alpha_n(t) = \left\{ \begin{array}{l} \text{No change, } p = \left(\frac{288}{300} \right) \\ \text{New information, } p = \left(\frac{12}{300} \right) \end{array} \right\}$$

$$E\{\alpha_n(t)\} \Rightarrow \left(\frac{288}{300} \right) \cdot \alpha_n(t - \Delta t)$$

$$e^{-\gamma \cdot \Delta t} = \left(\frac{288}{300} \right) = 1 - \left(\frac{12}{300} \right)$$

$$\gamma \cdot \Delta t \approx \left(\frac{12}{300} \right)$$

$$\gamma \cdot N \Rightarrow 12 \cdot 52 = 624$$

Investment Applications of the Fundamental Law

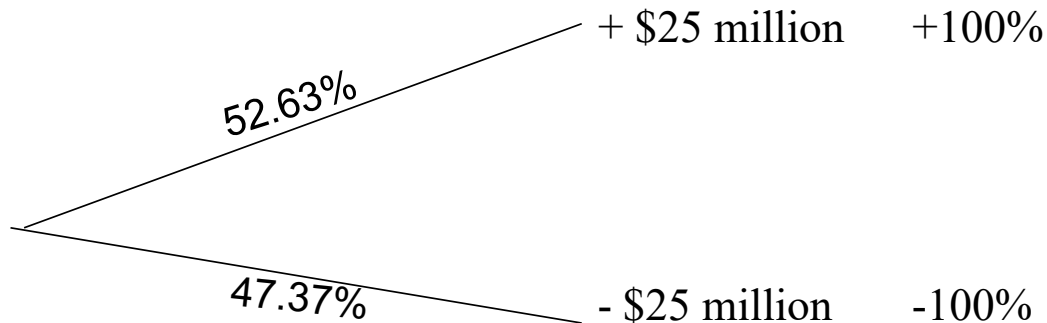
- More important for intuition and general direction than exact calculations.
- Example: stock picking versus market timing.
 - Stock picker follows 200 stocks, with quarterly rebalances based on new information. Correlation of forecasts with realizations is 0.02. This leads to IR of $0.02 * \sqrt{800} = 0.57$. This is top quartile.
 - Market timer makes 4 independent calls per year and is more accurate than the stock picker ($IC=0.1$). This leads to IR of $0.1 * \sqrt{4} = 0.20$. This is just above the median.
- Lower IR doesn't mean market timer will not have some great years.

Non-investment Example: The Roulette Wheel

- Consider an American roulette wheel, with numbers from 1-36, plus 0 and 00. There are 38 numbers in all. We are the casino.
- Bet on Evens. This pays off if the ball lands on an even number from 2, ... 36.
- Assume that gamblers bet \$25 million on such bets on this wheel over the course of a year. The casino has, therefore, \$25 million at risk.
- Two cases:
 - One \$25 million bet.
 - One million \$25 bets.

One \$25 Million Bet

- Casino wins with $20/38=52.63\%$ probability.
- We can view the forecasts as ± 1 and the realizations as also ± 1 .



One Bet Analysis

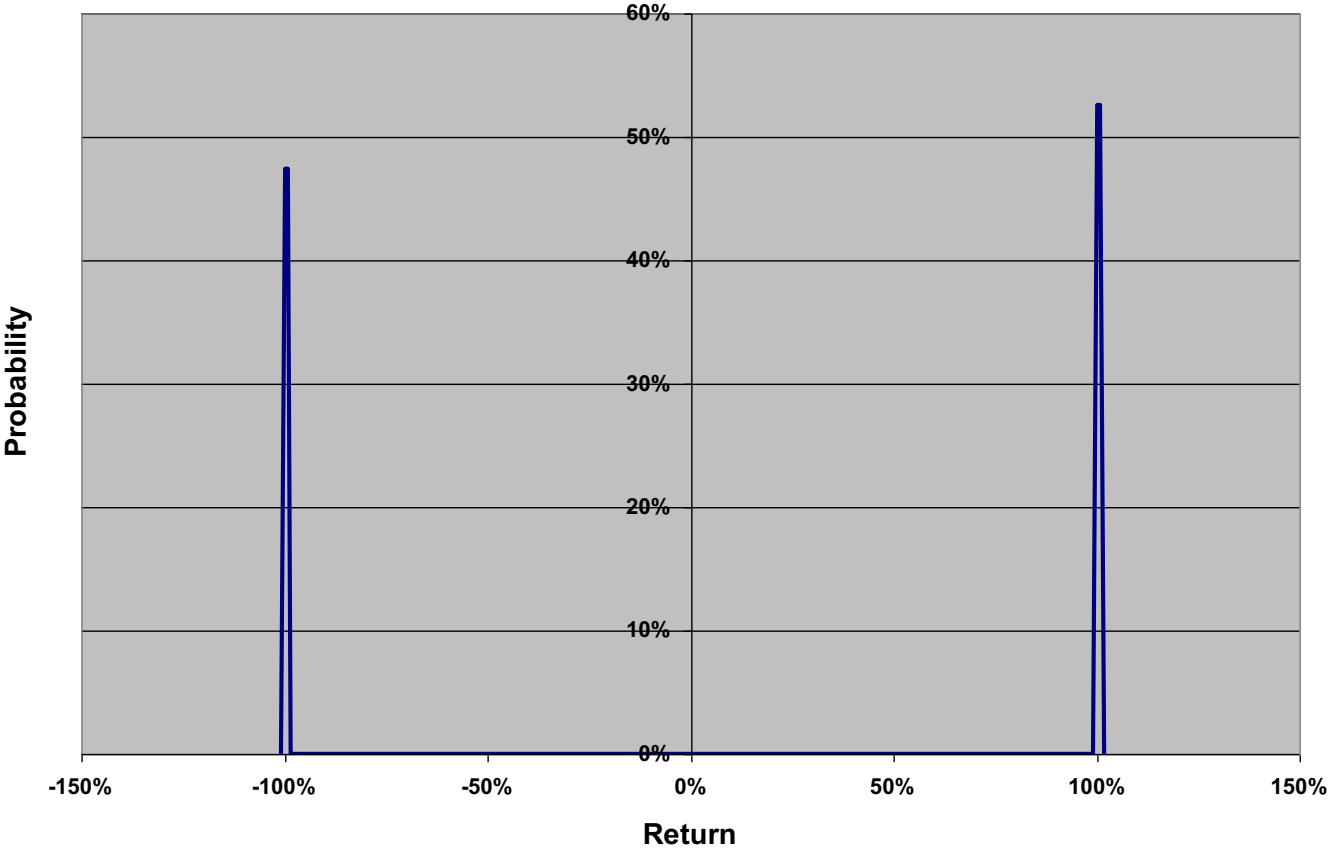
$$IC = Corr\{\alpha, \theta\} \Rightarrow \frac{\left(\frac{20}{38}\right) - \left(\frac{18}{38}\right)}{1} = 5.26\%$$

$$\begin{aligned} E\{r\} &= .5263(100\%) + .4737(-100\%) \\ &= 5.26\% \end{aligned}$$

$$\begin{aligned} Std\{r\} &= \sqrt{.5263(100\% - 5.26\%)^2 + .4737(-100\% - 5.26\%)^2} \\ &= 99.85\% \end{aligned}$$

$$IR = \frac{5.26\%}{99.85\%} = 0.0527 \approx IC \cdot \sqrt{BR}$$

Return Distribution: One Bet



Million Bet Analysis

- Invest 1 one-millionth of capital on each bet:

$$r = \sum_{n=1}^N \left(\frac{1}{N} \right) \cdot r_n$$

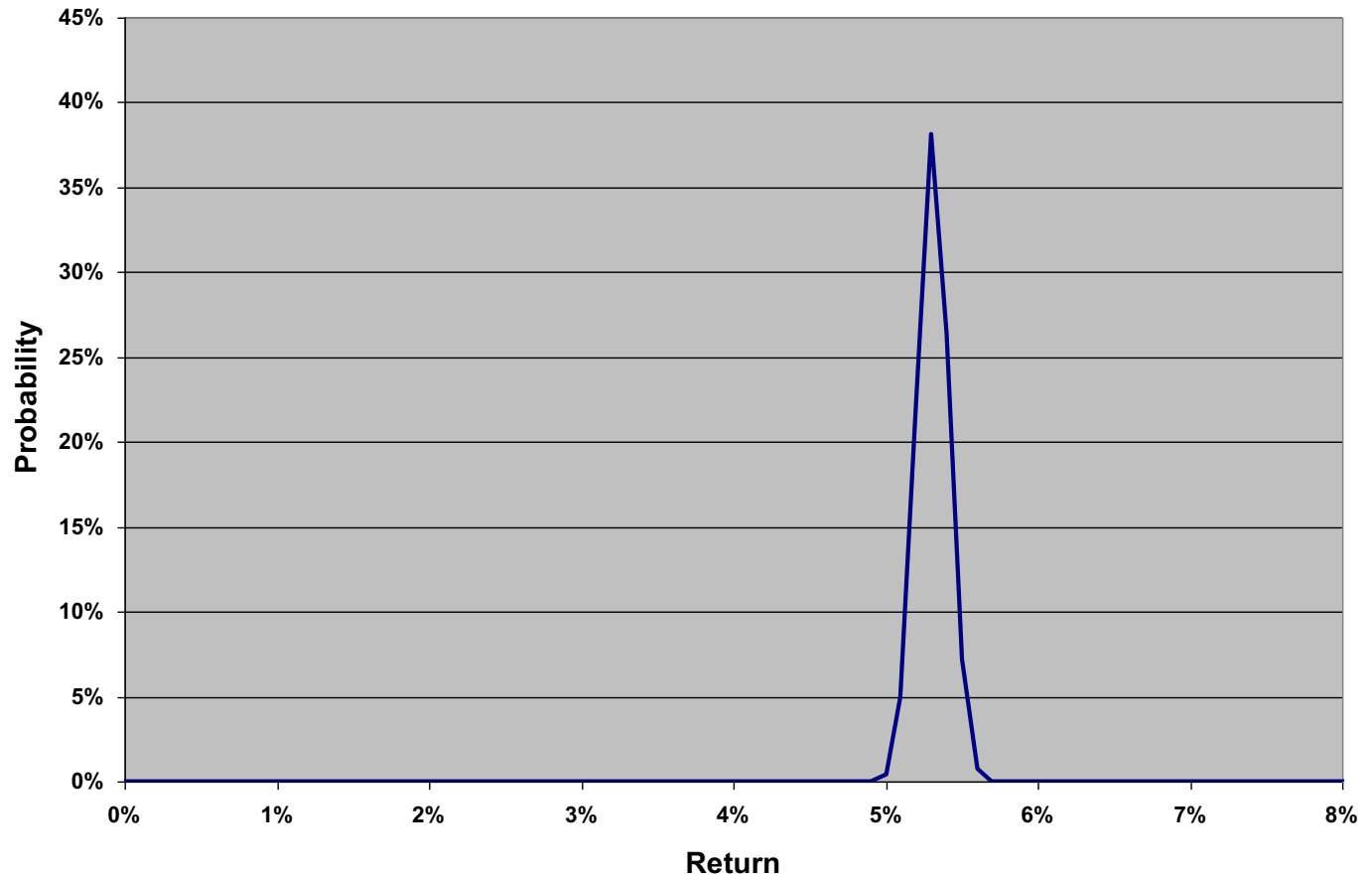
$$E\{r\} = \sum_{n=1}^N \left(\frac{1}{N} \right) \cdot E\{r_n\} \Rightarrow 5.26\%$$

$$Var\{r\} = \sum_{n=1}^N \left(\frac{1}{N} \right)^2 \cdot Var\{r_n\} \Rightarrow \frac{Var\{r_n\}}{N}$$

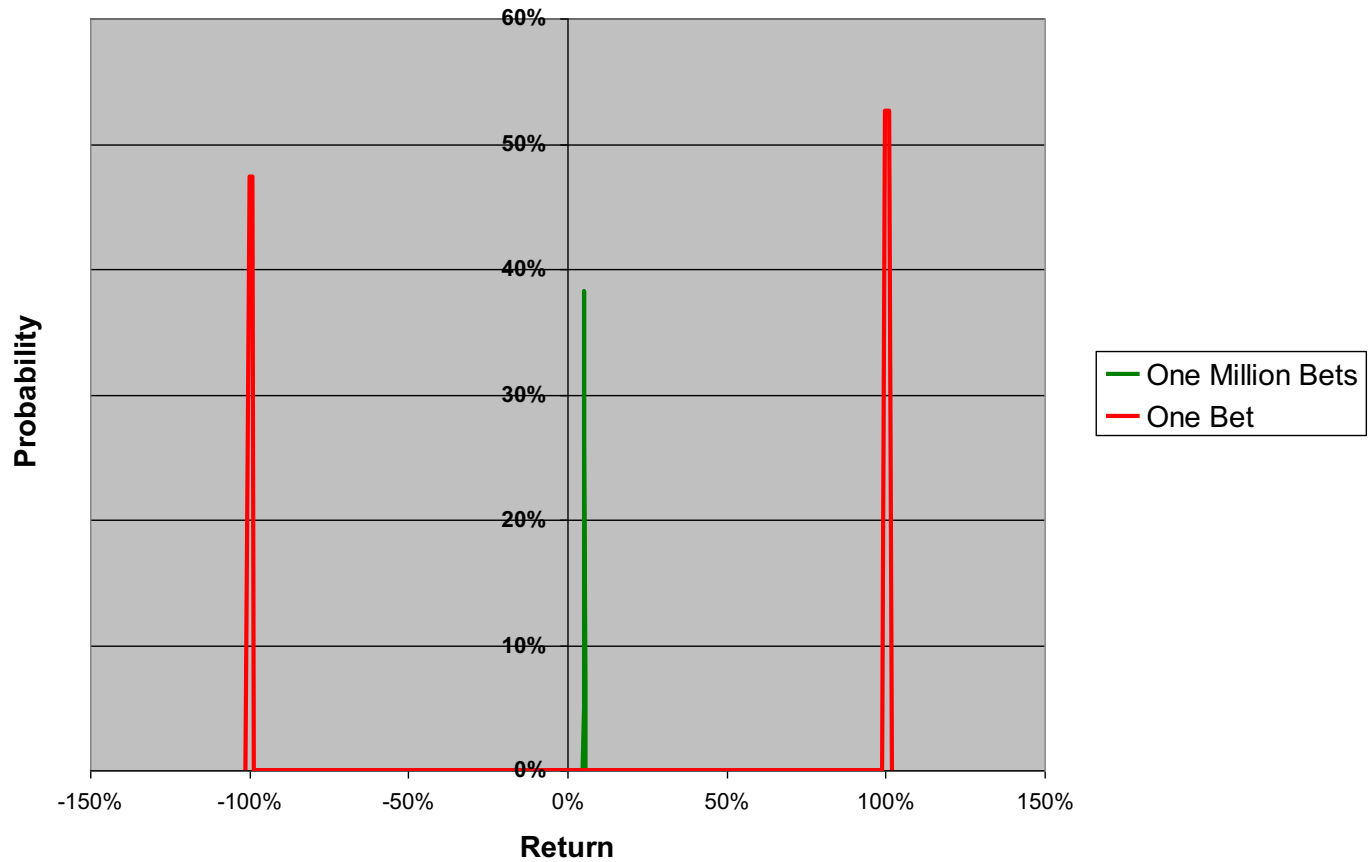
$$Std\{r\} = \frac{Std\{r_n\}}{\sqrt{N}} \Rightarrow 0.09985\%$$

$$IR = \frac{5.26\%}{0.09985\%} = 52.68 \approx IC \cdot \sqrt{BR}$$

Return Distribution: One Million Bets



Comparative Return Distributions



Handling different IC s for different stocks

- Assume strength IC_1 for BR_1 of the stocks, and IC_2 for BR_2 of the stocks. Then:

$$IR_p^2 = BR_1 \cdot IC_1^2 + BR_2 \cdot IC_2^2$$

- This is generally how IR 's add.
- Example:
 - 500 Largecap stocks, $IC=0.05$, $IR \rightarrow 1.12$
 - 2,000 Smallcap stocks, $IC=0.01$, $IR \rightarrow 0.45$
 - Optimal combination: $IR \rightarrow 1.21$

Implementation Efficiency

- What happens if constraints and costs keep us from the optimal tradeoff between return and risk,
Portfolio Q:

$$U = \mathbf{h}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}^T \cdot \mathbf{V} \cdot \mathbf{h}$$
$$\Rightarrow \boldsymbol{\alpha} - 2\lambda \mathbf{V} \cdot \mathbf{h}_Q = 0$$

- It's also true that:

$$\alpha_Q = \mathbf{h}_Q^T \cdot \boldsymbol{\alpha} \Rightarrow 2\lambda \cdot \mathbf{h}_Q^T \cdot \mathbf{V} \cdot \mathbf{h}_Q = 2\lambda \cdot \omega_Q^2$$
$$IR_Q = 2\lambda \omega_Q$$

But we own Portfolio P

- What is its information ratio?

$$\alpha_P = \mathbf{h}_P^T \cdot \boldsymbol{\alpha} \Rightarrow 2\lambda \cdot \mathbf{h}_P^T \cdot \mathbf{V} \cdot \mathbf{h}_Q = 2\lambda \omega_P \cdot \omega_Q \cdot \rho_{PQ}$$

$$IR_P = 2\lambda \omega_Q \cdot \rho_{PQ} = IR_Q \cdot \rho_{PQ}$$

- We call this correlation the *transfer coefficient*, *TC*. It measures the efficiency of our implementation, and has a maximum value of 1.
- Our extended fundamental law is:

$$IR = IC \cdot \sqrt{BR} \cdot TC$$

Understanding the Transfer Coefficient

- The Transfer Coefficient, TC , is the correlation of Portfolio Q with the portfolio we actually own, P.

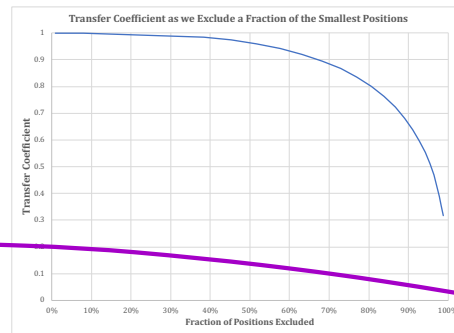
$$TC = \frac{\mathbf{h}_P^T \cdot \mathbf{V} \cdot \mathbf{h}_Q}{\omega_P \cdot \omega_Q}$$

- Some simple examples assuming uncorrelated residual returns, equal residual risk for all assets, and normally distributed scores:

- Portfolio P is long-short and equal-weight all the assets with positive alphas and short and equal-weight all the assets with negative alphas. In other words, Portfolio P uses the sign of the alpha and ignores any information about its magnitude

$$TC \Rightarrow \sqrt{\frac{2}{\pi}} \approx 0.8$$

- Portfolio P ignores a fraction x of the smallest magnitude alphas. How does the transfer coefficient vary as x increases toward 1:



- Typical TC numbers:

- Long-only equity funds: $TC < 0.5$ with lower numbers for higher active risk funds
- Long-short funds: $1 > TC > 0.5$

Transfer Coefficients

- Prior example showed benefits of stock-picking over market-timing.
- Now let's compare long-only stock picking with $IC=0.05$, $BR=500 \times 4$, $TC=0.3$; to asset allocation with $IC=0.10$, $BR=12 \times 4$, $TC=0.3$:

$$IR_{sp} = 0.05\sqrt{2000} \cdot 0.3 \Rightarrow 0.67$$

$$IR_{aa} = 0.10\sqrt{48} \cdot 0.3 \Rightarrow 0.21$$

- What happens if we convert that asset allocation fund to a global macro hedge fund. We follow exactly the same markets, but invest long-short using (low trading cost) futures contracts. Our TC goes from 0.3 to 0.95:

$$IR_{aa} = 0.10\sqrt{48} \cdot 0.95 \Rightarrow 0.66$$