MFE 230G: Homework 1
Due: August 24, 2021

1. Consider the following two-stock universe. Stock A has volatility 30%. Stock B has volatility 40%. The two stocks are 50% correlated:

$$\sigma_A = 30\%$$
$$\sigma_B = 40\%$$
$$\rho_{AB} = 0.5$$

This two-stock universe contains a benchmark, $\mathbf{h}_B$, and a managed portfolio, $\mathbf{h}_P$:

$$\mathbf{h}_B = \begin{bmatrix} 50\% \\ 50\% \end{bmatrix}, \quad \mathbf{h}_P = \begin{bmatrix} 55\% \\ 45\% \end{bmatrix}$$

a) What is the covariance matrix, $\mathbf{V}$?
b) What is the volatility, σ_P, of the managed portfolio?
c) What is the active risk, ψ_P, of the managed portfolio?
d) We denote the vector of 1's as $\mathbf{e}$. In this problem with 2 assets:

$$\mathbf{e} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Calculate the vector $\mathbf{V}^{-1} \cdot \mathbf{e}$, and the number $\mathbf{e}^T \cdot \mathbf{V}^{-1} \cdot \mathbf{e}$. What is the volatility, σ_C, of the portfolio:

$$\mathbf{h}_C = \frac{\mathbf{V}^{-1} \cdot \mathbf{e}}{\mathbf{e}^T \cdot \mathbf{V}^{-1} \cdot \mathbf{e}}$$

This is the minimum variance, fully-invested portfolio.

2. The market strategist at a leading investment bank believes (based on long-term data) that the stock and bond markets are 30% correlated. She also forecasts bond market volatility of 8% and stock market volatility of 20%. While she expects the bond market over the long-run to deliver about 2% excess return, over the next 12 months she has forecast that the bond market will drop by 2% (excess return, not return relative to its long-run average). Based only on that forecast, and on her long-run expected stock market excess return of 6%, what should she forecast for the stock market's excess return over the next 12 months?

3. The “Basic Math” section introduced a two-factor model:

$$r_n(t) = b_{Market}(t) + X_{Size} \cdot b_{Size}(t) + u_n(t).$$

We can write this as:

$$\mathbf{r} = \mathbf{X} \cdot \mathbf{b} + \mathbf{u}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \\ 1 & -1 \\ \vdots & \vdots \\ 1 & -1 \end{bmatrix}$$

$$\mathbf{b} = \left(\mathbf{X}^T \cdot \mathbf{X}\right)^{-1} \cdot \mathbf{X}^T \cdot \mathbf{r}$$

a) Working through the details, what are the formulas to estimate b_{Market} and b_{Size}?
b) What is the market portfolio? Or in other words, what set of portfolio weights will guarantee that the portfolio’s return equals b_{Market}? Does this look like a cap-weighted index?
c) Similarly, what set of portfolio weights will generate b_{Size}? What is this portfolio’s exposure to the market?

4. Fischer runs a long-only equity strategy following 500 stocks per quarter, with an information coefficient of 0.05 and a transfer coefficient of 30%. Myron is considering starting up an asset allocation strategy. He appears to have an information coefficient of 0.08, and plans to invest long-short, and only in asset classes with liquid futures contracts. That way he expects to achieve a transfer coefficient of 95%. Into how many asset classes must he invest quarterly to match Fischer’s information ratio?

5. You invest to maximize utility:

$$U = \alpha_P - \lambda \cdot \omega_P^2.$$

You have an information ratio of 0.5, and a risk aversion parameter, λ, of 12 (in decimal units per year). So, for example, an alpha of 3%, and a risk of 5%, would lead to utility of zero.

a. What is your optimal level of risk?
b. What utility can you achieve at that risk level?

6. Through your research program, you have developed a set of active return forecasts, $\{\boldsymbol{\alpha}\}$. You purchase a forecast covariance matrix, $\mathbf{V}$. You now want to construct a fully-invested portfolio designed to outperform your benchmark, $\mathbf{h}_B$.

   a. Write out the utility function, including Lagrange multiplier, in terms of the active portfolio, $\mathbf{h}_{PA}$.
   b. Solve for the optimal $\mathbf{h}_{PA}$. Use the definition of Portfolio C (in problem 1 above) to simplify your result. Under what condition does the optimal active portfolio reduce to the result you would find in the absence of the Lagrange multiplier?
   c. We can define the optimal alpha portfolio as:

$$\mathbf{h}_Q \equiv \frac{\mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}{\mathbf{e}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}$$

   Calculate α_Q, ω_Q^2, and the Information Ratio of Q. What is the optimal $\mathbf{h}_{PA}$ in terms of $\mathbf{h}_C$ and $\mathbf{h}_Q$?

7. You will have access on the course website to the file "MFE230G Homework 1 Problem 7." This file includes annualized alpha forecasts for selected US largecap stocks monthly from August 2017 through July 2018.
   a. What is the average forecast alpha each month? What is the cross-sectional standard deviation of the alphas each month?
   b. Estimate the breadth of these forecasts, assuming the underlying process is in equilibrium.

8. Assuming that active returns are uncorrelated, what would you estimate as the minimum active risk achievable with a 20 stock portfolio relative to the S&P 500? Assume that each stock has active risk of 25%, and that the S&P 500 is an equal weighted benchmark. Would your answer increase or decrease if you correctly accounted for the true capitalization-weighting of the index?

Problem 9 relies on the following data. We will post two excel spreadsheets. One, labeled Factor Covariance Matrix-0712 contains a factor covariance matrix (F) for US equities as of the end of July 2012. It includes 68 factors, consisting of 13 styles and 55 industries. The other, labeled US Asset Data-0712-parse contains the factor exposures for 500 US equities, along with other useful information:

- The factor exposures are labeled in the spreadsheet.

- The spreadsheet also includes data on specific risk (srisl), price, market cap (capt) and dividend yield (yld).

9. Basic Covariance Matrix questions:

    a. Based on an examination of the numbers in the factor covariance matrix, what are the units? Are they percent or decimal, monthly or annual? (This has been the source of more than one big error in asset management.)

    b. Now focus on the 25 largest stocks (by market capitalization). Calculate (using python, matlab, EXCEL, or your favorite data analysis tool) the 25 x 25 asset covariance matrix. This will require calculating the 25 x 68 exposure matrix, as well as the specific variance matrix. Keep track separately of the factor and specific pieces.

    c. What are the 3 highest total risk stocks? What are the 3 lowest total risk stock? For each of those 6 stocks, calculate what fraction of total variance comes from factor risk, and what fraction from specific risk.

    d. Assume that we have a capitalization-weighted benchmark of these 25 stocks. What is the total risk of that portfolio? What fraction of its variance is common factor and what fraction is specific?