Equity: Section 2 [1]

Leonel Drukker

University of California - Berkeley

[1] Subset of slides inherited from Vinicio De Sola

Pset 1 hints

Q6

- Constraint 1: fully invested portfolio – $h_P^\top \cdot e = 1$ $(h_{PA} = h_P - h_B)$
- Constraint 2: minimize portfolio risk with **known** V
- Part c: use what you found in part b.

Pset 1 hints

Q7.b

- We assume equilibrium $\Rightarrow \alpha_n(t) = e^{-\gamma \cdot \Delta t} \cdot \alpha_n(t - \Delta t) + \varepsilon_n(t)$ is a valid model.
- You can use a regression or assume think of information decay as simply a cross-sectional correlation between monthly return.
- Then compute monthly breadth and give the average.

Recall from lecture

- Forecasting risk is hard, but we think it's doable.
- Forecasting alphas is a completely different situation.

lol part 2


ACTIVE MANAGER
Come closer...
I need...
I need...
SUPERIOR INFORMATION

Factor models of risk

- Single factor
  - $r = \beta r_{mkt} + \theta$
  - $V = Cov(r, r) = Var(\beta r_{mkt} + \theta) = Var(\beta r_{mkt}) + Var(\theta) = \beta \cdot \beta^\top \sigma^2_{mkt} + \Delta$
  - $\Delta = \begin{bmatrix} \omega_1^2 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & & \\ \vdots & & \ddots & \\ 0 & & & \omega_N^2 \end{bmatrix}$

Factor models of risk

In general, a factor model represents excess returns as

- $r = Xb + u$
  - $X_{N \times K}$ – stock exposures to the factors.
  - $b_{K \times 1}$ – vector of factor returns.
  - $u_{K \times 1}$ – specific return vector.

With asset covariance matrix

- $V = Cov(r, r) = XFX^{\top} + \Delta$
  - $F_{K \times K}$ – The covariance of factors
  - $\Delta_{N \times N}$ – Diagonal covariance matrix of "specific risk".

Portfolio exposure and risk

Additionally,

- Portfolio factor exposures – $x_P = X^\top \cdot h_P$.
- Total risk – $\sigma_P^2 = h_P^\top \cdot V \cdot h_P = x_P^\top \cdot F \cdot x_P + h_P^\top \cdot \Delta \cdot h_P$.
- Active risk – $\psi_P^2 = h_{PA}^\top \cdot V \cdot h_{PA} = x_{PA}^\top \cdot F \cdot x_{PA} + h_{PA}^\top \cdot \Delta \cdot h_{PA}$.

Fundamental models

- GLS regression: $r = \Delta X b + u$
- "Calculate X, estimate b"
- $b = \underbrace{(X^\top \Delta^{-1} X)^{-1} X^\top \Delta^{-1}}_{H^\top} r$
- H is the factor-mimicking portfolio.
- The columns of H correspond to portfolio weights for each factor.
- How can we quickly check that H is a factor-mimicking portfolio?

Putting this into practice

It is *relatively* easy to build a factor model using historical returns and factor exposures. In order to predict future excess returns with a factor model, you need factor forecasts. This is hard!
Some structure helps.

1. Taking exposures (X) as given, estimate factor returns (β).
   - PCA
   - ML – LASSO
2. Taking factor returns (β) as given, estimate exposures (X).
   - Maximum likelihood factor analysis
3. Combine the two.

Exercise 1 – Fundamental risk model

Stocks A and B have excess returns of 6% and 5%, respectively. Stock A's specific variance and stock B's specific variance are 0.01. The two stocks are in the same industry. Stock A has a loading of 0.5 on a size factor while stock B has a loading of 0.25 on the size factor.

a. Assuming a two-factor (size and industry) risk model, compute the returns to factor-mimicking portfolios.
b. What are the weights of A and B in the factor portfolios you have computed?

Exercise 1 – Fundamental risk model

a. $b = (X^\top \Delta^{-1} X)^{-1} X^\top \Delta^{-1} r$, factor returns from GLS regression via Risk Modeling slides 12, 13

Exercise 1 – Fundamental risk model

b. $b = \underbrace{(X^\top \Delta^{-1} X)^{-1} X^\top \Delta^{-1}}_{H^\top} r$

Recall the columns of H correspond to portfolio weights for each factor

Exercise 1 – Fundamental risk model

Now assume that the factor variance-covariance matrix is:

$$F = \begin{bmatrix} 0.004 & 0.002 \\ 0.002 & 0.006 \end{bmatrix}$$

Note: 1st column corresponds to the size factor, 2nd column corresponds to industry factor.

c. Compute the variance-covariance matrix of asset returns.

Exercise 1 – Fundamental risk model

c. $X = \begin{bmatrix} 0.5 & 1 \\ 0.25 & 1 \end{bmatrix}, \Delta = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$

$$V = XFX^\top + \Delta$$

Exercise 2 – Beta and MCTR/MCAR

Given the following data:

$$h_P = \begin{bmatrix} 0.45 \\ 0.55 \end{bmatrix}, h_B = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, V = \begin{bmatrix} 0.02 & 0.01 \\ 0.01 & 0.04 \end{bmatrix}$$

a. Compute the beta of the portfolio w.r.t. the benchmark.
b. Approximate the impact to total and active risk of a rebalancing trade resulting in: $h_{P2} = \begin{bmatrix} 0.46 \\ 0.54 \end{bmatrix}$.

Exercise 2 – Beta and MCTR/MCAR

a. Standard definition for beta $\beta = \frac{Cov(r_P, r_B)}{Var(r_B)} = \frac{h_P^\top V h_B}{h_B^\top V h_B}$ (see Basic Math slides)

Exercise 2 – Beta and MCTR/MCAR

b. $h_{P2} = \begin{bmatrix} 0.46 \\ 0.54 \end{bmatrix}, MCTR = \frac{\partial \sigma_P}{\partial h_P^\top} = \frac{Vh_P}{\sigma_P}$

Exercise 2 – Beta and MCTR/MCAR

b. $h_{P2} = \begin{bmatrix} 0.46 \\ 0.54 \end{bmatrix}, MCAR = \frac{\partial \psi_P}{\partial \boldsymbol{h}_P^\top} = \frac{Vh_{PA}}{\sigma_P}$

Exercise 3 – Covariance matrix

You are given the following estimated covariance matrix (annual, percent^2) for GE and Amazon

$$\begin{bmatrix} 900 & 600 \\ 600 & 1600 \end{bmatrix} = \begin{bmatrix} \sigma_{GE}^2 & \sigma_{GE,A} \\ \sigma_{A,GE} & \sigma_A^2 \end{bmatrix}$$

a. What is GE's annual volatility?
b. What is the correlation between GE and Amazon?
c. What is the annual volatility of a portfolio that invests 50% in GE and 50% in Amazon?

Exercise 3 – Covariance matrix

a. $\sigma_{GE}^2 = 900 \Rightarrow \sigma_{GE} = 30\%$

b. $\rho = \frac{\sigma_{GE,A}}{\sigma_A \sigma_{GE}} = \frac{600}{30 \cdot 40} = 0.5$

c. $h_P = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

$$\sigma_P^2 = h_P^\top V h_P = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 900 & 600 \\ 600 & 1600 \end{bmatrix} \cdot \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = 925$$

$$\Rightarrow \sigma_P \approx 30.41\%$$

Valuation

- Discount dividend model – there is uncertainty

$$p(0) = \sum_t \frac{E\left[d(t)\right]}{(1+y)^t} \tag{1}$$

- Constant-growth dividend discount model – assume a growth rate

$$d(t) = d(1) \cdot (1+g)^{t-1} \tag{2}$$

- Eq. (1) + Eq. (2) $\Rightarrow$

$$p(0) = \frac{d(1)}{y-g}$$

- Constant dividends and discount rate imply total return is

$$y = \frac{p(1) - p(0) + d(1)}{p(0)}$$

More equations

- $e(t) = d(t) + I(t)$
- $d(t) = \kappa \cdot e(t)$
  - $\Rightarrow I(t) = (1 - \kappa) \cdot e(t)$
- $e(t+1) = e(t) + ROE \cdot I(t) = (1 + ROE \cdot (1 - \kappa)) \cdot e(t)$
  - $\Rightarrow g = ROE \cdot (1 - \kappa)$
- $y = i_f + \beta f_B + \alpha$
  - $+CGDDM \Rightarrow g = \alpha + \beta f_B + i_f - \frac{d}{P}$
- Observe p_{mkt} and estimate: $p_{model}(t) = \frac{\kappa \cdot e(t+1)}{y-g}$
  - Multiples: $m_{mkt} = \frac{p_{mkt}}{e}$, $m_{model} = \frac{\kappa}{y-g}$
  - Relative mispricing: $rmp = \frac{p_{model}}{p_{mkt}} - 1 = \frac{m_{model}}{m_{mkt}} - 1$

Exercise 4 – Valuation

You are analyzing a particular stock, and observe or estimate the following:

- Payout ratio $= 0.75$
- ROE $= 10\%$
- Dividend in one year $= \$1.00$
- Expected annual return $= 5\%$

a. What do you estimate is the stock's growth rate?

b. What is your estimated price under the constant growth dividend discount model?

c. The current market price is \$25. If the market multiple doesn't change, and growth is as forecasted, what will be the estimated stock price in one year?

Exercise 4 – Valuation

a. $g = ROE \cdot (1 - \kappa) = 10\% \cdot 0.25 = 2.5\%$

b. $p(0) = \frac{d(1)}{y-g} = \frac{1}{0.05-0.025} = \$40$

c. $d(t) = \kappa \cdot e(t) \Rightarrow e(t) = \frac{d(t)}{\kappa} = \frac{1.00}{0.75} \approx \$1.33$
$m_{mkt} = \frac{p_{mkt}}{e} = \frac{\$25}{\$1.33} = 18.77$
If m_{mkt} constant $\Rightarrow p_{mkt}/e$ constant,
$\Rightarrow p(1) = p(0) \cdot (1 + g) = \$25 \cdot (1.025) = \$25.625$

Exercise 5 – DDM & Gordon Growth Model

A quick search of Yahoo Finance reveals the following data for General Electric (GE)

- Price = \$26.15
- Market cap = \$263 Billion
- Volume = 28,400,000
- Dividend per share = \$0.88
- Earnings per share = \$1.22
- Return on equity = 11%
- Beta = 1.15

a. Based on these data, estimate GE's growth rate from these data.

b. Using the constant growth DDM, and assuming a risk-free rate of zero and an expected benchmark excess return of 5%, estimate a fair price for GE.

Exercise 5 – DDM & Gordon Growth Model

a. $\kappa = \frac{d(t)}{e(t)} = \frac{\$0.88}{\$1.22} = 0.7213$
$g = ROE \cdot (1 - \kappa) = 11\% \cdot 0.2787 \approx 3.07\%$

b. $y = i_f + \beta_{GE} \cdot r_b$
CCDDM $\Rightarrow p(0) = \frac{d(1)}{y-g} \Rightarrow y = g + \frac{d(1)}{p(0)}$
$\Rightarrow g + \frac{d(1)}{p(0)} = i_f + \beta_{GE} \cdot r_b$
$\Rightarrow 0.0307\% + \frac{0.88}{p} = 0 + 1.15 \cdot 0.05\%$
$\Rightarrow p = \frac{0.88}{1.15 \cdot 0.05 - 0.0307} = \$32.84$

Exercise 6 – Sharpe market model

You are given monthly return data for all stocks in the S & P 500 over the past five years.

a. Describe two problems you would confront with using a historical covariance matrix estimated from these data.

Exercise 6 – Sharpe market model

Given your concern with a historical covariance matrix, you decide to use Sharpe's market model:

$$Var(r_n) = \beta_n^2 \cdot \sigma_{mkt}^2 + \omega_n^2$$
$$Cov(r_n, r_{mkt}) = \beta_n \cdot \beta_{mkt} \cdot \sigma_{mkt}^2$$

to estimate risk. Using the available data, you estimate

Stock	Beta (w.r.t. S& P 500)	Residual risk
GE	1.15	20.30%
Microsoft	0.73	18.70%

From the same data, you also estimate that the S & P 500 has an annual volatility of 13.2%. Using these data:

b. What are the volatilities of GE and Microsoft?

c. What is the correlation of GE with Microsoft?

Exercise 6 – Sharpe market model

b. $Var(r_{GE}) = \beta_{GE}^2 \cdot \sigma_{mkt}^2 + \omega_n^2 = (1.15)^2 \cdot (0.132)^2 + (0.203)^2 \approx 0.0643$
$\Rightarrow \text{vol}_{GE} \approx 0.2535$
$Var(r_{MSFT}) = (.73)^2 \cdot (0.132)^2 + (0.187)^2 \approx 0.0443$
$\Rightarrow \text{vol}_{MSFT} \approx 0.2107$

c. $\rho = \frac{Cov(r_{GE}, r_{MSFT})}{\sigma_{GE}\sigma_{MSFT}} = \frac{\beta_{GE}\beta_{MSFT}\sigma_{mkt}^2}{\sigma_{GE}\sigma_{MSFT}} = \frac{1.15 \cdot 0.73 \cdot (0.132)^2}{0.2535 \cdot 0.2107} \approx 0.2739$