Framework

Active Management Perspective

- Our focus is on building a portfolio, P, to outperform a benchmark, B. Our active return is $\delta_P \equiv r_P - r_B$
- What does the BLUE framework say about our expected return relative to that benchmark?

$$E\left\{r_P \mid r_B\right\} = E\left\{r_P\right\} + Cov\left\{r_P, r_B\right\} \cdot Var^{-1}\left\{r_B\right\} \cdot \left(r_B - E\left\{r_B\right\}\right)$$

$$\Rightarrow E\left\{r_P\right\} + \beta_P \cdot \left(r_B - E\left\{r_B\right\}\right)$$

$$\beta_P \equiv Cov\left\{r_P, r_B\right\} \cdot Var^{-1}\left\{r_B\right\}$$

Now add a Return Decomposition

- Decompose the portfolio return into a piece correlated with (explained by) the benchmark, and a piece uncorrelated (residual to) the benchmark:

$$r_P = \beta_P \cdot r_B + \theta_P$$

$$r_P - r_B = (\beta_P - 1) \cdot r_B + \theta_P$$

$$E\{r_P\} = \beta_P \cdot E\{r_B\} + E\{\theta_P\}$$

$$Var\{r_P - r_B\} = (\beta_P - 1)^2 \cdot \sigma_B^2 + \omega_P^2$$

- We can always do this. It allows us to update the prior result:

$$E\{r_P \mid r_B\} \Rightarrow \beta_P \cdot r_B + E\{\theta_P\} = \beta_P \cdot r_B + \alpha_P$$

- Also:

$$E\{r_P\} = \beta_P \cdot E\{r_B\} + \alpha_P = \beta_P \cdot f_B + \alpha_P$$

Add Economics and Information

- CAPM Equilibrium Model (Sharpe, Treynor, Lintner, Mossin):

$$E\{\theta\} = 0$$

$$E\{r_P \mid r_B\} \Rightarrow \beta_P \cdot r_B$$

$$E\{r_P - r_B \mid r_B\} = E\{\delta_P\} \Rightarrow (\beta_P - 1) \cdot r_B$$

  - Later we will study an alternative—arbitrage based—approach called APT.

- What if we have information not widely known to the market:

$$E\{\theta_P \mid g\} \equiv \alpha_P$$

$$E\{r_P \mid r_B\} \Rightarrow \beta_P \cdot r_B + \alpha_P$$

$$E\{\delta_P \mid r_B\} \Rightarrow (\beta_P - 1) \cdot r_B + \alpha_P$$

Active Equity Management

- Two sources of active return:
  - Alpha $\alpha_P \equiv E\{\theta_P \mid g\}$ based on information.
  - Beta $(\beta_P - 1)\cdot r_B$ based on benchmark exposure $\neq 1$
- Two sources of active risk:

$$Var\{\delta_P\} \equiv \psi_P^2 = (\beta_P - 1)^2 \cdot \sigma_B^2 + \omega_P^2$$

- Institutional equity managers generally choose $\beta_P \approx 1$.
  - So we will focus on forecasting alpha and building active portfolios to capture alpha. This will lead to the utility function for active equity management.
  - This assumption doesn't hold as well for fixed income managers who sometimes take duration bets—equivalent to beta bets.

Active Management Utility

- The natural utility function for active management is:

$$U_P = \left[\alpha_P - \lambda \cdot \omega_P^2 \right]$$
$$+ \left[(\beta_P - 1) \cdot E\{r_B\} - \lambda' \cdot (\beta_P - 1)^2 \cdot \sigma_B^2 \right]$$
$$\Rightarrow \alpha_P - \lambda \cdot \omega_P^2 \quad \text{for institutional equity managers}$$

- We trade off expected residual return against residual risk. The risk aversion parameter, λ, measures our individual preferences for that trade-off.
- Most institutional equity managers will ignore the $\beta \neq 1$ part of this and focus on residual return and risk. That part uses the expected benchmark return to drive an optimal portfolio beta.
- The book (Chapter 4) goes through a long hand-waving derivation of this. We will simply assert this result and move on.

Utility: Intuition

$$\boxed{U_P = \alpha_P - \lambda \cdot \omega_P^2}$$

$$\alpha_P = \mathbf{h}_P^T \cdot \boldsymbol{\alpha} \Rightarrow \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha}$$

$$\left(\mathbf{h}_{PA} \equiv \mathbf{h}_P - \mathbf{h}_B, \quad \mathbf{h}_B^T \cdot \boldsymbol{\alpha} = 0 \right)$$

$$\omega_P^2 = \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA} - (\beta_P - 1)^2 \cdot \sigma_B^2$$

$$\Rightarrow \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA} \quad \textit{typically}$$

$$U_P \Rightarrow \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

Utility Intuition

$$U_P = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

- As we change the positions in our actively managed portfolio, the portfolio alpha and active risk vary.
- There will be one set of positions that maximizes utility.
- If we also want to impose full-investment or long-only constraints, we must explicitly add these.

Simplified Portfolio Construction

- Goal: make some simplifying assumptions and then solve for optimal active holdings. How do these depend on alpha and risk?
- Key step: simplify residual risk:

$$\omega_P^2 = Var\left\{\mathbf{h}_P^T \cdot \boldsymbol{\theta}\right\} = Var\left\{\mathbf{h}_{PA}^T \cdot \boldsymbol{\theta}\right\}$$

$$\Rightarrow Var\left\{\sum_{n=1}^{N} h_{PA}(n)\cdot\left[r_n - \beta_n \cdot r_B\right]\right\} = Var\left\{\sum_{n=1}^{N} h_{PA}(n)\cdot\theta_n\right\}$$

- Assume (just for now) that residual returns are uncorrelated*. Hence:

$$\omega_P^2 \Rightarrow \sum_{n=1}^{N} h_{PA}^2(n)\cdot\omega_n^2$$

*This idea goes back to Sharpe, as we will see, and predates the CAPM.

Simplified Portfolio Construction

- Under that simple assumption, the active utility function becomes:

$$U_P \Rightarrow \sum_{n=1}^{N} h_{PA}(n) \cdot \alpha_n - \lambda \cdot h_{PA}^2(n) \cdot \omega_n^2$$

- And the optimal holdings are:

$$h_{PA}^*(n) = \frac{\alpha_n}{2\lambda\omega_n^2}$$

- So we take active positions in proportion to alphas and inverse proportion to variance. The positions scale inversely with risk aversion.
- More generally:

$$\mathbf{h}_{PA}^* = \left(\frac{1}{2\lambda}\right) \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}$$

Example: Simplified Portfolio Construction

- Consider two stocks, JP Morgan and Netflix. Assume high risk aversion (λ=0.15):

	ω	α
JP Morgan	23%	2%
Netflix	38%	4%

- In this case, the higher alpha stock has the smaller active position:

$$h_{PA}\left(JPM\right)=\frac{2\%}{0.3\cdot\left(23\%\right)^2}=1.26\%$$

$$h_{PA}\left(NFLX\right)=\frac{4\%}{0.3\cdot\left(38\%\right)^2}=0.92\%$$

- but it is a higher risk position (i.e. a bigger bet):

$$h_{PA}\left(JPM\right)\cdot\omega_{JPM}=\frac{2\%\cdot\left(23\%\right)}{0.3\cdot\left(23\%\right)^2}=0.29\%$$

$$h_{PA}\left(NFLX\right)\cdot\omega_{NFLX}=\frac{4\%\cdot\left(38\%\right)}{0.3\cdot\left(38\%\right)^2}=0.35\%$$

The Information Ratio

- Define *IR* as *annualized* ratio of expected residual return to residual risk:

$$IR_P \equiv \frac{\alpha_P}{\omega_P}$$

- Note that our approach to portfolio construction, maximizing:

$$U = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}$$

leads to portfolios with the highest possible *IR* given $\boldsymbol{\alpha}$ and $\mathbf{V}$.

Fully-Invested vs Long-Short, *IR*s and *SR*s

$$\boxed{U = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}}$$

- Fully-Invested Typical Case
  - Also called "benchmark-linked".
  - Sometimes, but not always, long-only

$$\mathbf{h}_{PA} = \mathbf{h}_P - \mathbf{h}_B$$

$$\mathbf{h}_B^T \cdot \mathbf{e} = 1, \quad \mathbf{h}_P^T \cdot \mathbf{e} = 1, \quad \mathbf{h}_{PA}^T \cdot \mathbf{e} = 0$$

Typically: $\beta_P = 1, \quad \beta_{PA} = 0$

$$\sigma_P^2 \equiv Var\{r_P\}, \quad f_P \equiv E\{r_P\}$$

$$\psi_P^2 \equiv Var\{r_P - r_B\} = Var\{\delta_P\} \Rightarrow Var\{\theta_P\} \; if \; \beta_P = 1$$

$$SR = \frac{f_P}{\sigma_P}, \quad IR = \frac{\alpha_P}{\omega_P}$$

- The portfolio is fully-invested.
- The active portfolio is long-short
- The Sharpe Ratio (comparing the portfolio to cash) differs from the Information Ratio (comparing the portfolio to its benchmark).

Fully-Invested vs Long-Short, *IR*s and *SR*s

$$\boxed{U = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA}}$$

- Long-Short Typical Case
  - Sometimes, but not always, market neutral

$\mathbf{h}_B = 0$ The benchmark is cash.

$\mathbf{h}_{PA} = \mathbf{h}_P - \mathbf{h}_B \Rightarrow \mathbf{h}_P$

Typically $\mathbf{h}_P^T \cdot \mathbf{e} = 0, \quad \beta_P = 0 \left(\text{and hence } r_P = \theta_P\right)$

$\sigma_P^2 = \omega_P^2 = \psi_P^2, \quad f_P = \alpha_P = E\left\{\delta_P\right\}$

$SR = IR$

- The portfolio is long-short
- The Sharpe Ratio (comparing the portfolio to cash) equals the Information Ratio (comparing the portfolio to its benchmark), since the benchmark is cash.

The *IR* and Optimal Portfolios

- The set of optimal portfolios:

$$\mathbf{h}_{PA}^{*} = \frac{\mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}{2\lambda}$$

- differ only in overall risk and alpha:

$$\omega_P^2 = \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA} \Rightarrow \frac{\boldsymbol{\alpha}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}{4\lambda^2}$$

$$\alpha_P = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} = \frac{\boldsymbol{\alpha}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}{2\lambda}$$

with active positions scaling with λ.

- In fact, the *IR* depends only on α and **V** and is independent of λ :

$$IR^2 = \boldsymbol{\alpha}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}$$

Annualization

- To annualize monthly alphas, multiply by 12.
- To annualize monthly risk, multiply by sqrt(12).
- To annualize monthly *IR*, multiply by sqrt(12).

$$\theta_{annual} = \theta_{Jan} + \theta_{Feb} + \ldots + \theta_{Dec}$$

$$\alpha_{annual} = E\left\{\theta_{annual}\right\} = \alpha_{Jan} + \alpha_{Feb} + \ldots + \alpha_{Dec}$$

$$\Rightarrow 12 \cdot \alpha_{Jan}$$

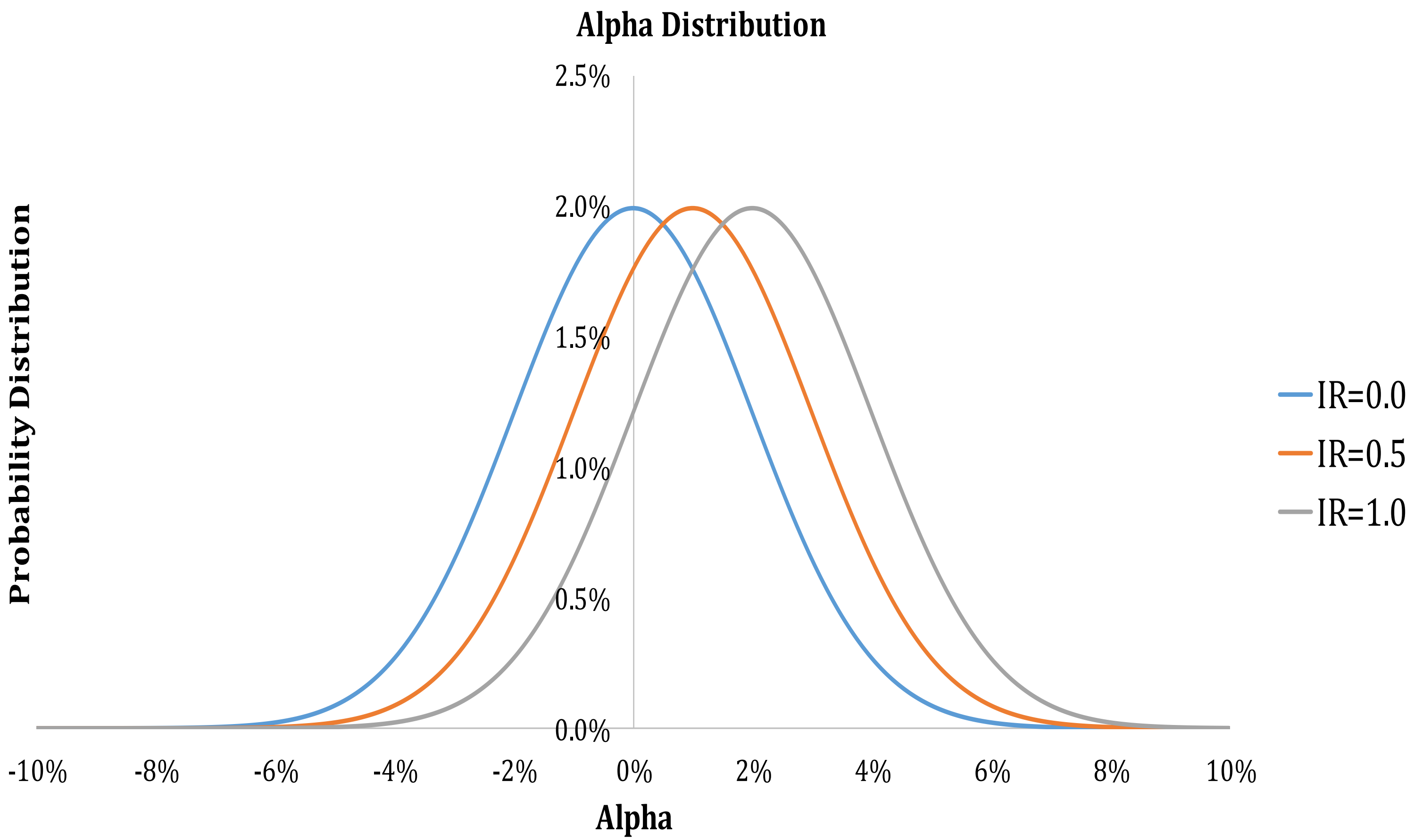
$$\omega^2_{annual} = \omega^2_{Jan} + \omega^2_{Feb} + \ldots + \omega^2_{Dec}$$

$$\Rightarrow 12 \cdot \omega^2_{Jan}$$

$$IR_{annual} = \sqrt{12} \cdot IR_{monthly}$$

Alpha should be theta

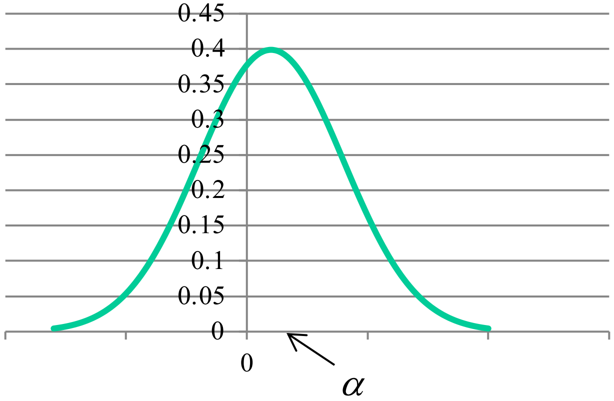
Information Ratios Measure Consistency of Performance



The Normal Distribution Result

- What is the probability that the annual residual return is positive, given the *IR*, and assuming residual returns are normally distributed?

$$prob = 1 - \Phi\left\{\frac{0-\alpha}{\omega}\right\} = 1 - \Phi\{-IR\}$$
$$= \Phi\{IR\}$$



- This is our measure of consistency, and it is a monotonically increasing function of *IR*.

Information Ratios

- Typical Numbers:

Percentile	*IR*	*prob* δ *>0*
90	1.0	84%
75	0.5	69%
50	0.0	50%
25	-0.5	31%
10	-1.0	16%

- Older Study Results*

Information Ratios					
	Equity			Fixed Income	
Percentile	Mutual Funds	Long-only Inst.	Long-Short Inst.	Institutional	Average
90	1.04	0.77	1.17	0.96	0.99
75	0.64	0.42	0.57	0.50	0.53
50	0.20	0.02	0.25	0.01	0.12
25	-0.21	-0.38	-0.22	-0.45	-0.32
10	-0.62	-0.77	-0.58	-0.90	-0.72

*All US data 1/03 – 12/07. Studies include 338 equity mutual funds, 1679 equity long-only funds, 56 equity long-short funds, and 537 fixed income funds.

- More recent study (Morningstar data)

	Top Quartile *IR*: Before Fees	
Date	US Largecap Equity	US Broad Fixed Income
10/97 - 9/02	0.42	0.14
10/02 - 9/07	0.43	1.40
10/07 - 9/12	0.34	0.60
10/12 - 9/17	0.26	1.32

IR as Budget Constraint

- Convert the definition of the Information Ratio into a budget constraint:

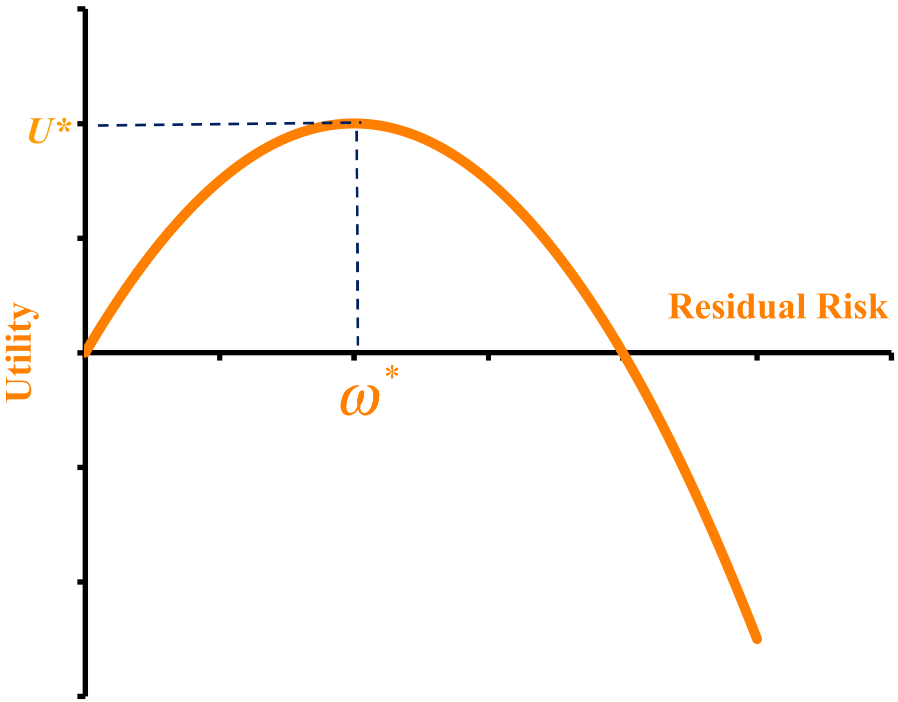
$$\alpha_P = IR \cdot \omega_P$$

- This works for the optimal portfolios because we increase risk by scaling positions. It breaks down when we can no longer do that—e.g. due to the long-only constraint.

Maximum Utility

$$U = \alpha - \lambda \cdot \omega^2$$

$$= IR \cdot \omega - \lambda \cdot \omega^2$$



Optimal Risk and Utility

- We can solve for the optimal risk level:

$$\omega_P^* = \frac{IR}{2\lambda}$$

- And the maximum achievable utility:

$$\boxed{U_P^* = \frac{(IR)^2}{4\lambda}}$$

Active Risk Levels

- Older Studies

	Active Risk			
	Equity			Fixed Income
Percentile	Mutual Funds	Long-only Inst.	Long-short Inst.	Inst.
90	6.85%	6.80%	10.18%	1.64%
75	5.52%	5.33%	6.99%	0.94%
50	4.05%	4.03%	4.76%	0.63%
25	3.08%	3.01%	3.67%	0.45%
10	2.35%	2.21%	2.97%	0.33%

- More Recent Studies (Morningstar data: 10/97-9/17)

	Active Risk Levels	
Percentile	US Largecap Equity	US Broad Fixed Income
90	7.63%	2.56%
75	5.79%	1.69%
50	4.13%	1.06%
25	2.86%	0.75%
10	1.68%	0.58%

Risk Aversion Parameter

- Prior result provides intuition for risk aversion.

$$\omega_P^* = \frac{IR}{2\lambda}, \quad \Rightarrow \lambda = \frac{IR}{2\omega_P^*}$$

- Intuition 1: Risk aversion depends on whether we measure residual risk as percent or decimal.
- Intuition 2: Typical levels:

Risk Aversion	λ
low	0.05
medium	0.10
high	0.15

Fundamental Law of Active Management

- We have seen that *IR* is critical measure for active management.
- The fundamental law can provide intuition into the *IR*.
- We will prove the simple case. Chapter 6 in the book proves the general case.

Plan of Attack

- Start with a snapshot of "information" about N stocks.
- Turn this information into an optimal portfolio.
  - What is the expected alpha, risk, and information ratio of that portfolio?
- Calculate expectation of the IR over all possible sets of information.

Information

- In principle, this could be broker BUY/SELL recommendations, P/E ratios, recent stock returns…
  - Any set of numbers we think is correlated with future active returns.
- We will generically refer to these numbers using the variable g

- How do we convert those numbers into alphas?

Use the BLUE approach

- Alpha will be the best linear unbiased estimate of θ conditional on g:

$$E\{\theta \mid g\} \equiv \alpha = E\{\theta\} + Cov\{\theta, g\} \cdot Var^{-1}\{g\} \cdot \left[g - E\{g\}\right]$$

$$\Rightarrow 0 + Corr\{\theta, g\} \cdot \omega \cdot \left[\frac{g - E\{g\}}{StDev\{g\}}\right]$$

$$\alpha = Corr\{\theta, g\} \cdot \omega \cdot z = IC \cdot \omega \cdot z$$

- In this result:
  - We convert the information, g, to a *z-score*.
  - We define the *IC* as the *Information Coefficient*. It is the correlation of θ with z (or equivalently with g).

Understanding *IC*s

- The Information Coefficient is the correlation of z-scores with future residual returns. It is also the correlation of the α and the underlying information, g, with future residual returns, since all three $\{z, \alpha, g\}$ are linearly related.

- One way to understand *IC*s is to connect them to something perhaps more intuitive: the fraction fr of forecasts that correctly predict the sign of θ.

- If residual returns are normally distributed, and the forecast alpha is a linear combination of information and noise (i.e. $\alpha_n = IC \cdot \left[IC \cdot \theta_n + \sqrt{1 - IC^2} \cdot \omega_n \cdot Z_n \right]$), we can show (with lots of normal distribution integrals) that:

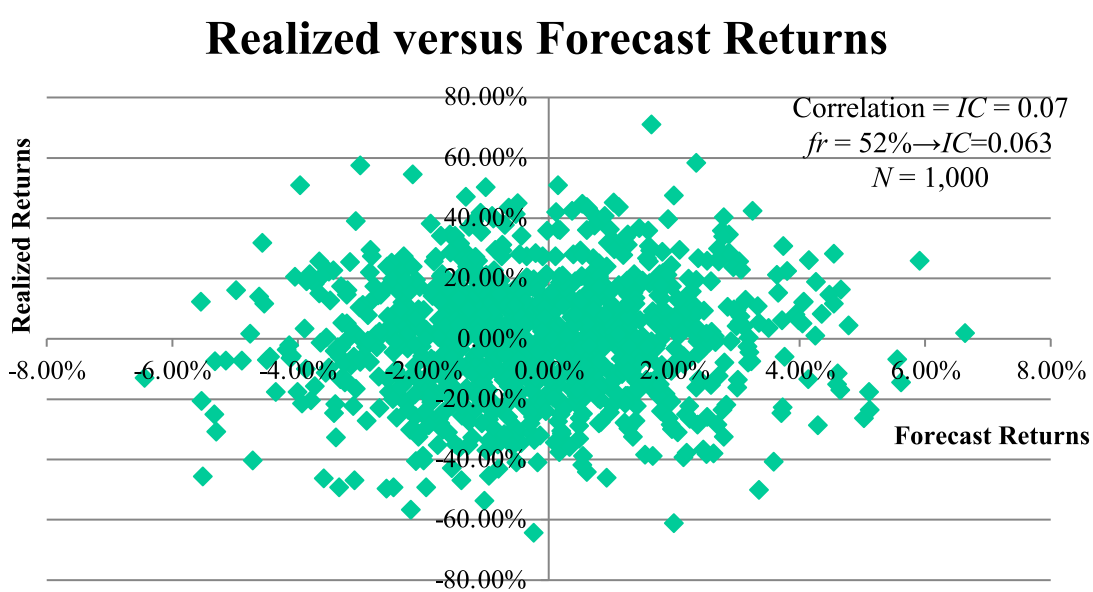
$$fr = \frac{1}{2} + \left(\frac{1}{\pi} \right) \cdot \arctan \left\{ \frac{IC}{\sqrt{1 - IC^2}} \right\}$$

$$\approx \frac{1}{2} + \left(\frac{IC}{\pi} \right) \quad \text{for } IC << 1$$

Typical *IC*s

Skill	IC	*fr*
great	0.10	53.2%
good	0.05	51.6%
average	0.00	50.0%

- Note that this simple model is for *intuition only*. If we want to calculate the *IC*, we calculate the correlation.



Back to the Fundamental Law

- Assume N assets and N z-scores (one for each asset), residual returns uncorrelated, residual risks identical, and correlations between signals and returns identical.*
- Optimal holdings:

$$h_{PA}(n) = \frac{\alpha_n(z)}{2\lambda \cdot \omega^2}$$

- What are predicted alpha and risk?

*The treatment in Chapter 6 relaxes these assumptions.

Portfolio Alpha and Risk

- Optimal conditional holdings:

$$h_{PA}(n) = \frac{IC \cdot z_n}{2\lambda\omega}$$

- Portfolio Alpha:

$$\alpha_P = \sum_{n=1}^{N} h_{PA}(n) \cdot \alpha_n = \left(\frac{IC^2}{2\lambda} \right) \cdot \sum_{n=1}^{N} z_n^2$$

- Portfolio Risk:

$$\omega_P^2 = \sum_{n=1}^{N} \left(h_{PA}(n) \cdot \omega_n \right)^2 = \left(\frac{IC^2}{4\lambda^2} \right) \cdot \sum_{n=1}^{N} z_n^2$$

Conditional Information Ratio

- We can now combine previous results to calculate the information ratio conditional on the signals $\{z\}$:

$$IR_P^2(z) = \frac{\alpha_P^2(z)}{\omega_P^2(z)} = IC^2 \cdot \sum_{n=1}^{N} z_n^2$$

- Taking the unconditional expectation of the above, and noting that:

$$E\left\{\frac{1}{N} \cdot \sum_{n=1}^{N} z_n^2\right\} = 1 \quad \text{since scores have mean 0, std 1}$$

  - we find:

$$E\left\{IR_P^2(z)\right\} = IR_P^2 = IC^2 \cdot N$$

- The more detailed derivation, which accounts for correlations between z-scores, leads to a generalized version of this.

Fundamental Law

- Information Ratio depends on skill and breadth:

$$IR = IC \cdot \sqrt{BR}$$

- We measure skill by the information coefficient.
- To consistently outperform requires skill and breadth. We measure breadth as the number of independent bets per year.
- This result assumes that we use information to build optimal portfolios. It is an upper bound on what we can achieve.

Understanding Breadth

- Independent bets per year.
  - A rate, not a number. So we expect twice as many bets over two years.
  - We could implement a beta bet with positions in ~300 stocks, but that's one bet, not 300 bets.
  - So breadth is not as simple as the number of holdings.
- Consider an investment process in equilibrium.
  - Old information decays.
  - New information arrives.
  - The two are in balance, so an information turnover rate, γ, captures both.

Forecasts in Equilibrium

$$\alpha_n(t) = \underbrace{e^{-\gamma\cdot\Delta t}\cdot\alpha_n(t-\Delta t)}_{\text{Old information decaying}} + \underbrace{\tilde{s}_n(t)\cdot\sqrt{\Delta t}}_{\text{New information arriving}}$$

Note: This is a way of understanding what is going on "under the hood" with our alphas. It is not an operational statement. Every period, we forecast alphas using the latest available information. The above statement says that the value of that information decays over time, and that eventually (and often episodically) we receive new information. As George Box noted, "All models are wrong, but some are useful."

$$BR = \gamma\cdot N$$

We can estimate γ from a history of forecasts.

Breadth Example

- Follow 300 stocks. Every week, receive new information on 12 stocks. (We can't predict which 12 stocks.)

$$BR = 12 \cdot 52 = 624$$

Here's the investment process analysis:

$$\alpha_n(t) = \left\{ \begin{matrix} \text{No change, } p = \left(\dfrac{288}{300}\right) \\ \\ \text{New information, } p = \left(\dfrac{12}{300}\right) \end{matrix} \right\}$$

$$E\{\alpha_n(t)\} \Rightarrow \left(\frac{288}{300}\right) \cdot \alpha_n(t - \Delta t)$$

$$e^{-\gamma \cdot \Delta t} = \left(\frac{288}{300}\right) = 1 - \left(\frac{12}{300}\right)$$

$$\gamma \cdot \Delta t \approx \left(\frac{12}{300}\right)$$

$$\gamma \cdot N \Rightarrow 12 \cdot 52 = 624$$

Investment Applications of the Fundamental Law

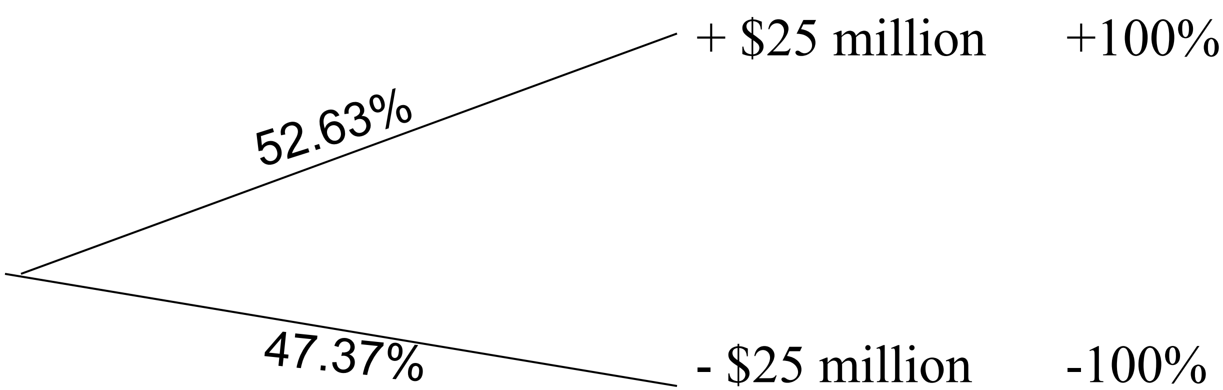
- More important for intuition and general direction than exact calculations.
- Example: stock picking versus market timing.
  - Stock picker follows 200 stocks, with quarterly rebalances based on new information. Correlation of forecasts with realizations is 0.02. This leads to *IR* of 0.02*sqrt(800)=0.57. This is top quartile.
  - Market timer makes 4 independent calls per year and is more accurate than the stock picker (*IC*=0.1). This leads to *IR* of 0.1*sqrt(4)=0.20. This is just above the median.
- Lower *IR* doesn't mean market timer will not have some great years.

Non-investment Example: The Roulette Wheel

- Consider an American roulette wheel, with numbers from 1-36, plus 0 and 00. There are 38 numbers in all. We are the casino.
- Bet on Evens. This pays off if the ball lands on an even number from 2, … 36.
- Assume that gamblers bet $25 million on such bets on this wheel over the course of a year. The casino has, therefore, $25 million at risk.
- Two cases:
  - One $25 million bet.
  - One million $25 bets.

One $25 Million Bet

- Casino wins with 20/38=52.63% probability.
- We can view the forecasts as +/-1 and the realizations as also +/-1.



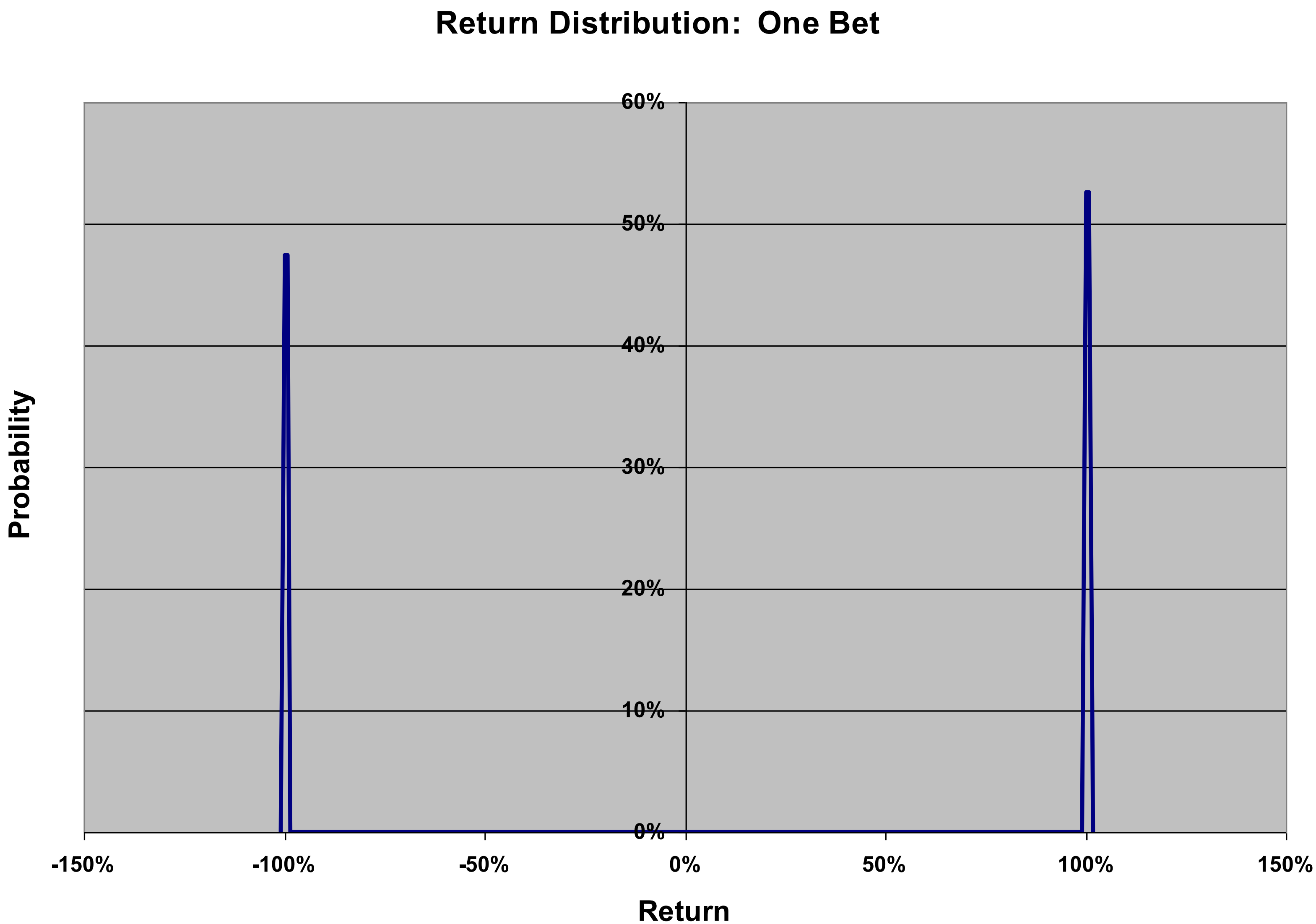
One Bet Analysis

$$IC = Corr\{\alpha, \theta\} \Rightarrow \frac{\left(\frac{20}{38}\right) - \left(\frac{18}{38}\right)}{1} = 5.26\%$$

$$E\{r\} = .5263(100\%) + .4737(-100\%)$$
$$= 5.26\%$$

$$Std\{r\} = \sqrt{.5263(100\% - 5.26\%)^2 + .4737(-100\% - 5.26\%)^2}$$
$$= 99.85\%$$

$$IR = \frac{5.26\%}{99.85\%} = 0.0527 \approx IC \cdot \sqrt{BR}$$



Return Distribution: One Bet
Probability
60%
50%
40%
30%
20%
10%
0%
-150%
-100%
-50%
0%
50%
100%
150%
Return

Million Bet Analysis

- Invest 1 one-millionth of capital on each bet:

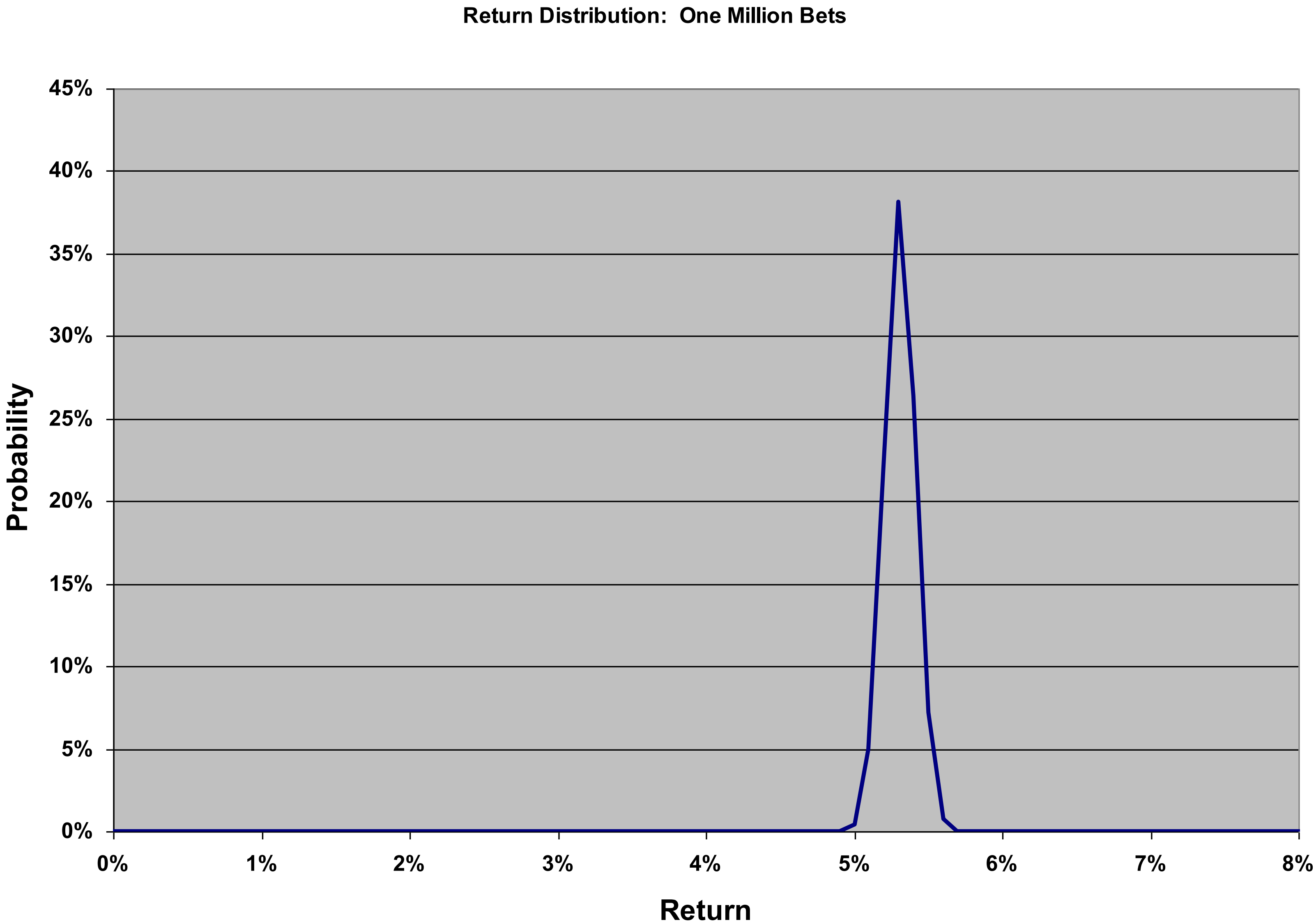
$$r = \sum_{n=1}^{N} \left(\frac{1}{N} \right) \cdot r_n$$

$$E\{r\} = \sum_{n=1}^{N} \left(\frac{1}{N} \right) \cdot E\{r_n\} \Rightarrow 5.26\%$$

$$Var\{r\} = \sum_{n=1}^{N} \left(\frac{1}{N} \right)^2 \cdot Var\{r_n\} \Rightarrow \frac{Var\{r_n\}}{N}$$

$$Std\{r\} = \frac{Std\{r_n\}}{\sqrt{N}} \Rightarrow 0.09985\%$$

$$IR = \frac{5.26\%}{0.09985\%} = 52.68 \approx IC \cdot \sqrt{BR}$$



Return Distribution: One Million Bets
Probability
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
0%
1%
2%
3%
4%
5%
6%
7%
8%
Return

Comparative Return Distributions
Probability
60%
50%
40%
30%
20%
10%
0%
-150%
-100%
-50%
0%
50%
100%
150%
Return
One Million Bets
One Bet

Handling different *IC*s for different stocks

- Assume strength IC_1 for BR_1 of the stocks, and IC_2 for BR_2 of the stocks. Then:

$$IR_P^2 = BR_1 \cdot IC_1^2 + BR_2 \cdot IC_2^2$$

- This is generally how *IR*'s add.
- Example:
  - 500 Largecap stocks, IC=0.05, $IR \rightarrow 1.12$
  - 2,000 Smallcap stocks, IC=0.01, $IR \rightarrow 0.45$
  - Optimal combination: $IR \rightarrow 1.21$

Implementation Efficiency

- What happens if constraints and costs keep us from the optimal tradeoff between return and risk, Portfolio Q:

$$U = \mathbf{h}^T \cdot \boldsymbol{\alpha} - \lambda \cdot \mathbf{h}^T \cdot \mathbf{V} \cdot \mathbf{h}$$

$$\Rightarrow \boldsymbol{\alpha} - 2\lambda \mathbf{V} \cdot \mathbf{h}_Q = 0$$

- It's also true that:

$$\alpha_Q = \mathbf{h}_Q^T \cdot \boldsymbol{\alpha} \Rightarrow 2\lambda \cdot \mathbf{h}_Q^T \cdot \mathbf{V} \cdot \mathbf{h}_Q = 2\lambda \cdot \omega_Q^2$$

$$IR_Q = 2\lambda \omega_Q$$

But we own Portfolio P

- What is its information ratio?

$$\alpha_P = \mathbf{h}_P^T \cdot \boldsymbol{\alpha} \Rightarrow 2\lambda \cdot \mathbf{h}_P^T \cdot \mathbf{V} \cdot \mathbf{h}_Q = 2\lambda\omega_P \cdot \omega_Q \cdot \rho_{PQ}$$

$$IR_P = 2\lambda\omega_Q \cdot \rho_{PQ} = IR_Q \cdot \rho_{PQ}$$

- We call this correlation the *transfer coefficient*, *TC*. It measures the efficiency of our implementation, and has a maximum value of 1.
- Our extended fundamental law is:

$$\boxed{IR = IC \cdot \sqrt{BR} \cdot TC}$$

Understanding the Transfer Coefficient

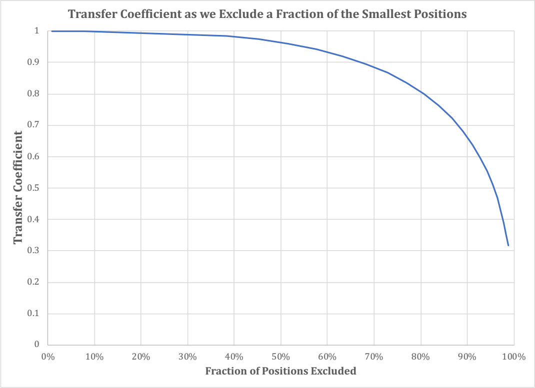
- The Transfer Coefficient, *TC*, is the correlation of Portfolio Q with the portfolio we actually own, P.

$$TC = \frac{\mathbf{h}_P^T \cdot \mathbf{V} \cdot \mathbf{h}_Q}{\omega_P \cdot \omega_q}$$

- Some simple examples assuming uncorrelated residual returns, equal residual risk for all assets, and normally distributed scores:
  - Portfolio P is long-short and equal-weight all the assets with positive alphas and short and equal-weight all the assets with negative alphas. In other words, Portfolio P uses the sign of the alpha and ignores any information about its magnitude

$$TC \Rightarrow \sqrt{\frac{2}{\pi}} \approx 0.8$$

  - Portfolio P ignores a fraction x of the smallest magnitude alphas. How does the transfer coefficient vary as x increases toward 1:



- Typical *TC* numbers:
  - Long-only equity funds: *TC*<0.5 with lower numbers for higher active risk funds
  - Long-short funds: 1>*TC*>0.5

Transfer Coefficients

- Prior example showed benefits of stock-picking over market-timing.
- Now let's compare long-only stock picking with IC=0.05, BR=500 x 4, TC=0.3; to asset allocation with IC=0.10, BR=12 x 4, TC=0.3:

$$IR_{sp} = 0.05\sqrt{2000} \cdot 0.3 \Rightarrow 0.67$$

$$IR_{aa} = 0.10\sqrt{48} \cdot 0.3 \Rightarrow 0.21$$

- What happens if we convert that asset allocation fund to a global macro hedge fund. We follow exactly the same markets, but invest long-short using (low trading cost) futures contracts. Our TC goes from 0.3 to 0.95:

$$IR_{aa} = 0.10\sqrt{48} \cdot 0.95 \Rightarrow 0.66$$