Forecasting

“Forecasting is very difficult, especially if it’s about the future.”

Niels Bohr

Introduction

- How do we move from signals to forecast returns?
- How do we combine signals?
- What is our intuition about the magnitude of our forecasts?
- What “real world” issues must we take into account?

From signals to forecasts

- Signals don't even have the right units
- Basic forecasting formula (as we have seen)

$$\boldsymbol{\alpha} \equiv E\{\boldsymbol{\theta} \mid \mathbf{g}\} = E\{\boldsymbol{\theta}\} + Cov\{\boldsymbol{\theta}, \mathbf{g}\} \cdot Var^{-1}\{\mathbf{g}\} \cdot \left(\mathbf{g} - \mathbf{E}\{\mathbf{g}\}\right)$$

- Best linear unbiased estimator (BLUE)

$$\mathbf{q} \equiv \boldsymbol{\theta} - E\{\boldsymbol{\theta} \mid \mathbf{g}\}$$

$$E\{\mathbf{q}\} = 0$$

$$Min\{\mathbf{q}^T \cdot \mathbf{q}\}$$

Prior Result (one signal)

- Remember that:

$$\alpha \equiv E\{\theta \mid g\} = E\{\theta\} + Cov\{\theta, g\} \cdot Var^{-1}\{g\} \cdot [g - E\{g\}]$$

$$\Rightarrow 0 + Corr\{\theta, g\} \cdot \omega \cdot \left[\frac{g - E\{g\}}{StDev\{g\}} \right]$$

mean zero
stdev 1

$$\alpha = Corr\{\theta, z\} \cdot \omega \cdot z = IC \cdot \omega \cdot z$$

$IC = Corr\{\theta, z\}$
$= Corr\{\theta, g\}$
$= Corr\{\theta, \alpha\}$

- Rule of Thumb:

“Alpha is *IC* times Volatility times Score”

Forecast controls for:

$\alpha = IC \cdot \omega \cdot z$

- Skill
  - If IC=0, alpha=0
- Expectations
  - If g=$E\{g\}$, alpha=0
- Volatility
  - If two stocks have the same score, we expect the more volatile stock to rise more.

Portfolio Construction: The Active Bet

- Previously, we calculated optimal active positions (assuming residual returns uncorrelated) as:

$$h_{PA}^{*}(n) = \frac{\alpha_n}{2\lambda\omega_n^2} = \frac{IC \cdot \omega_n \cdot z_n}{2\lambda\, \omega_n^2}$$

- Now we can see that:

$$h_{PA}^{*}(n) \Rightarrow \left(\frac{IC}{2\lambda}\right)\frac{z_n}{\omega_n}$$

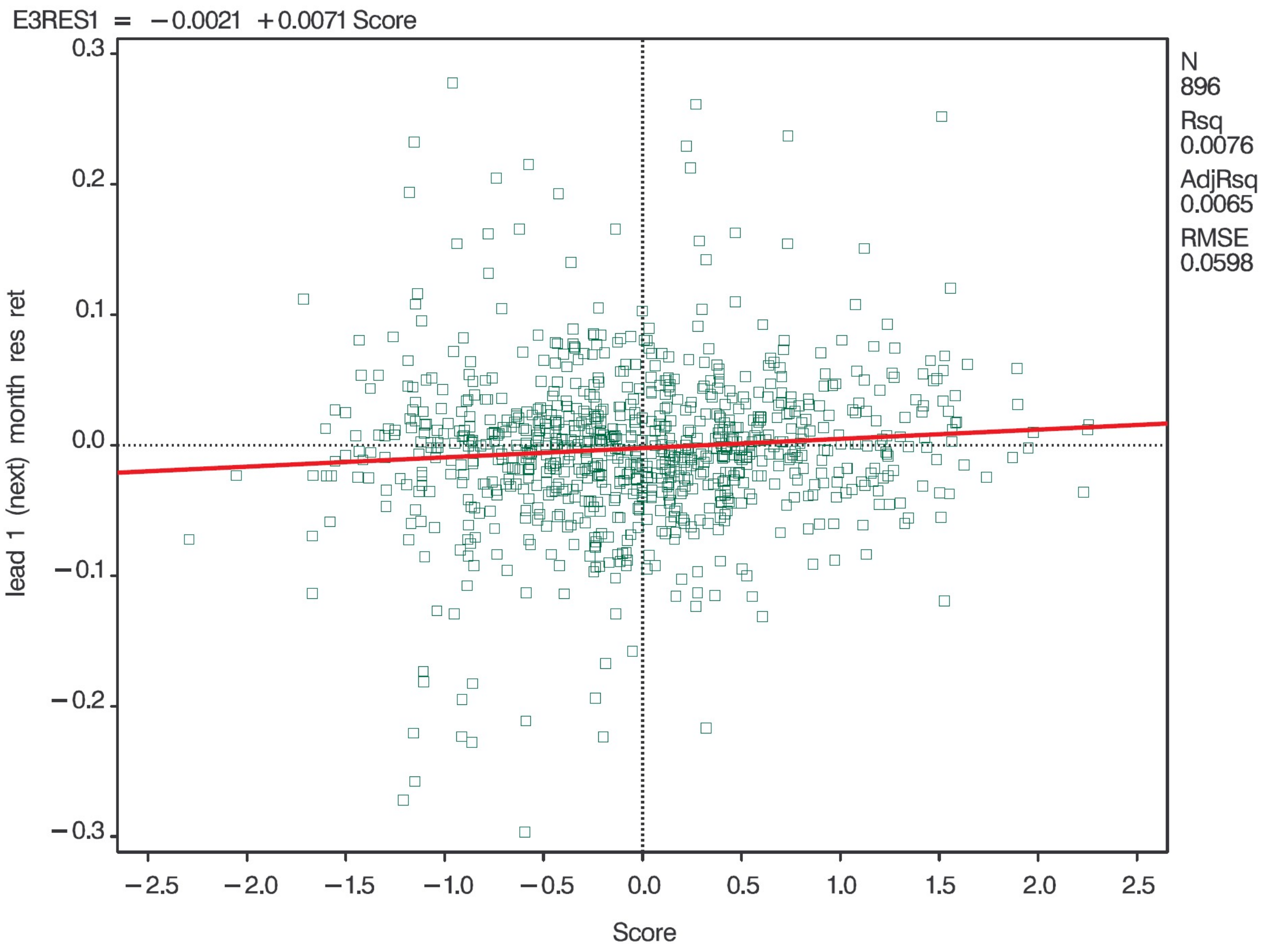
$$Active\ Bet \equiv h_{PA}^{*}(n) \cdot \omega_n \Rightarrow \left(\frac{IC}{2\lambda}\right) \cdot z_n$$

$$Active\ Bet \sim z_n$$

IC Intuition

- What do these *ICs* look like?
- An IC=0.1 implies a regression R^2 of 0.01
- Let's look at an example of a promising idea.

Information Coefficient = 8.7%



Binary Model: Another Stab at Intuition

- Intuitive view of signals and returns.
- Binary elements, ϕ_n, are ± 1 variables:

$$E\left\{\phi_n\right\} = 0$$

$$Var\left\{\phi_n\right\} = 1$$

$$Cov\left\{\phi_n, \phi_m\right\} = 0 \ if \ i \neq j$$

- The elements are the building blocks of signals and returns

Example

- Monthly residual returns

$$\theta = \sum_{i=1}^{64} \phi_i$$

$$E\{\theta\} = 0$$

$$Var\{\theta\} = Var\left\{\sum_{i=1}^{64} \phi_i\right\} \Rightarrow \sum_{i=1}^{64} Var\{\phi_i\} = 64$$

- Monthly signal

$$g = 1 + \phi_1 + \sum_{j=1}^{15} \eta_j$$

$$E\{g\} = 1$$

$$Var\{g\} = Var\left\{1 + \phi_1 + \sum_{j=1}^{15} \eta_j\right\} \Rightarrow 16$$

Monthly St Dev of $\theta = 8$

(remember that typical $\omega \sim 25\%$ - annual

monthly residual risk $\sim 8\%$

Variances and Covariances

- We can see that monthly residual volatility is 8%, monthly signal volatility is 4%, and expected signal value is 1.
- What is the covariance of the residual return with the signal?

$$Cov\{\theta, g\} = Cov\left\{\sum_{i=1}^{64}\phi_i, 1+\phi_1+\sum_{j=1}^{15}\eta_j\right\} \Rightarrow Var\{\phi_1\} = 1$$

  – In the binary model, covariances are simply *counts* of the number of binary elements in common.

Building the Alpha

- First the *IC*:

$$IC = \frac{Cov\{\theta, g\}}{\omega \cdot Std\{g\}} = \frac{1}{8 \cdot 4} \Rightarrow 0.03$$

<u>Intuition:</u>
Out of 64 things we could know about a return, we know only 1. And our knowledge of that 1 is masked by 15 bits of noise. That's what a decent (IC=0.03) signal looks like.

- Then the Alpha:

$$\alpha = IC \cdot \omega \cdot Score$$

$$= 0.03 \cdot 8 \cdot \left(\frac{g-1}{4} \right) = 0.06 \cdot (g-1)$$

Rule of Thumb Examples

$\alpha = IC \cdot \omega \cdot z$

- Stock Tip
- Stock residual volatility is 20%

	Score	
IC	**1**	**2**
0%	0%	0%
5%	1%	2%
10%	2%	4%

- Rule of thumb provides structure in classic unstructured situation

Rule of Thumb Examples

- Broker BUY/SELL recommendations
- Assume overall *IC*=0.05

Stock	ω	Recommendation	Score	Alpha
Exxon Mobil	21%	BUY	1	1.05%
IBM	28%	BUY	1	1.40%
GE	17%	SELL	-1	-0.85%

- Note the substantial scaling to account for skill.

How do we handle multiple signals?

- Go back to basic forecasting formula.

$$Cov\{\theta,\mathbf{g}\} = \left[Cov\{\theta,g_1\} \quad Cov\{\theta,g_2\} \quad Cov\{\theta,g_3\} \ldots \right]$$

$$= \omega \cdot \left[IC_1 \quad IC_2 \quad IC_3 \ldots \right] \cdot \underbrace{\begin{bmatrix} StDev\{g_1\} & 0 & 0 & \ldots \\ 0 & StDev\{g_2\} & 0 & \ldots \\ 0 & 0 & StDev\{g_3\} & \ldots \\ \vdots & & & \ddots \end{bmatrix}}$$

S = diagonal matrix of standard deviations of g

$$= \omega \cdot \mathbf{IC}^T \cdot \mathbf{S}$$

$$Var\{\mathbf{g}\} = \begin{bmatrix} Var\{g_1\} & Cov\{g_1,g_2\} & \cdots \\ Cov\{g_1,g_2\} & Var\{g_2\} & \\ \vdots & & \ddots \end{bmatrix}$$

$$= \mathbf{S} \cdot \boldsymbol{\rho} \cdot \mathbf{S}$$

ρ = correlation matrix of g

Multiple signals

- Substitute into basic forecasting formula

$$\alpha = Cov\{\theta, \mathbf{g}\} \cdot Var^{-1}\{\mathbf{g}\} \cdot (\mathbf{g} - E\{\mathbf{g}\})$$
$$= \omega \cdot \mathbf{IC}^T \cdot \mathbf{S} \cdot [\mathbf{S} \cdot \boldsymbol{\rho} \cdot \mathbf{S}]^{-1} \cdot (\mathbf{g} - E\{\mathbf{g}\})$$
$$= \omega \cdot \mathbf{IC}^T \cdot \boldsymbol{\rho}^{-1} \cdot \mathbf{z}$$

- We need to know the *IC* of each signal, as well as the correlation of the signals.

Multiple Signals

- What happens with two signals?

$$\boldsymbol{\rho}^{-1} = \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix}^{-1} = \left(\frac{1}{1-\rho_{12}^2} \right) \cdot \begin{bmatrix} 1 & -\rho_{12} \\ -\rho_{12} & 1 \end{bmatrix}$$

$$\alpha = \omega \cdot \left(\mathbf{IC}^T \cdot \boldsymbol{\rho}^{-1} \right) \cdot \mathbf{z}$$

$$= \omega \cdot IC_1' \cdot z_1 + \omega \cdot IC_2' \cdot z_2$$

$$IC_1' = \frac{IC_1 - \rho_{12} \cdot IC_2}{1-\rho_{12}^2} \qquad IC_2' = \frac{IC_2 - \rho_{12} \cdot IC_1}{1-\rho_{12}^2}$$

Case 1

$\rho_{12} = 0$

$\alpha = \omega IC_1 z_1 + \omega IC_2 z_2$

$= \alpha_1 + \alpha_2$

Case 2

$\rho_{12} \Rightarrow 1$

$IC_2 \Rightarrow IC_1$

$\alpha = \alpha_1$

or

$\alpha = \alpha_2$

Multiple Signals

- Check limiting cases.
- This becomes more interesting with 3 signals.
  - Imagine 3 signals, each with the same *IC*.
  - Two cases:
    - All signals uncorrelated
    - Signals 1 and 2 highly correlated, but uncorrelated with signal 3.

Rule of Thumb confronts Real World: Cross-sectional Scores

- Our analysis so far implicitly focused on time-series analysis:
  - Residual returns for 1 stock over time
  - Signal(s) for 1 stock over time
  - Score, z, is a time-series score; involving time-series expectations and standard deviations
- But the typical active management problem involves multiple assets at one time, with managers looking to pick relative winners and losers.

$\alpha = IC \cdot \omega \cdot z_{\text{Time Series}}$

Cross-sectional Scores: Example

- Stocks in S&P 500
- Calculate FY1 e/p for each stock
- Calculate mean and standard deviation across stocks
- Cross-sectional score:

$$z_{cs} = \frac{\left(\frac{e}{p}\right) - Mean_{cs}\left\{\left(\frac{e}{p}\right)\right\}}{Std_{cs}\left\{\left(\frac{e}{p}\right)\right\}}$$

Dilemma

$\alpha = IC \cdot \omega \cdot z_{cs}$??

- Do we "volatility adjust" these scores?
- Is alpha proportional to the cross-sectional scores, or to the cross-sectional scores, times volatility?
- Or, to put it more generally:

$$\alpha \sim \omega^{\gamma} \cdot z_{cs}$$

- What is γ?

Analysis

$$Z_{TS}^{(n)} = \frac{g_n}{\mathrm{Std}_{TS}\{g_n\}}$$

$$Z_{CS} = \frac{g_n}{\mathrm{Std}_{CS}\{g_n\}}$$

- The rule of thumb provides alpha as a function of time-series scores.
- We need to relate these two statistics.
- Ignore mean signal, either time-series or cross-sectional (not a big assumption).
- Ignore signal correlations across stocks (potentially a big assumption).

- Note that time-series scores sometimes measure something quite different from cross-sectional scores. (Earnings-to-price ratios are a good example.) In this analysis, we are only asking whether we should volatility adjust cross-sectional scores. We are not suggesting that cross-sectional scores measure the same thing as time-series scores.

Two Cases

- Compare $Std_{TS}\{g\}$ across stocks
- Case 1: Same for each stock
- Case 2: Proportional to stock volatility

For each stock n

$z_{TS} = \frac{(E/P)_n}{\text{STDev}_{TS}\{E/P\}_n}$

$z_{CS} = \frac{(E/P)_n}{\text{STDev}_{CS}\{E/P_n\}}$

Case 1

$$Std_{TS}\{g_n\} = c$$

(i.e. c is independent of n)

$$z_{TS} = \frac{g_n}{Std_{TS}\{g_n\}} = \frac{g_n}{c}$$

$$z_{CS} = \frac{g_n}{Std_{CS}\{g_n\}} = \frac{g_n}{c'} = z_{TS} \cdot \left(\frac{c}{c'}\right)$$

$$\alpha = IC \cdot \omega \cdot z_{CS} \cdot \left(\frac{c'}{c}\right) \sim \underline{IC \cdot \omega \cdot z_{CS}}$$

Case 2

$$Std_{TS}\left\{g_n\right\} = c \cdot \omega_n$$

$$z_{TS} = \frac{g_n}{Std_{TS}\left\{g_n\right\}} = \frac{g_n}{c \cdot \omega_n}$$

$$z_{CS} = \frac{g_n}{Std_{CS}\left\{g_n\right\}} = \frac{g_n}{c'} = z_{TS} \cdot \omega_n \cdot \left(\frac{c}{c'}\right)$$

$$\alpha = IC \cdot z_{CS} \cdot \left(\frac{c'}{c}\right)$$

Practical Experience

- Anecdotal evidence: Case 2 is more prevalent than Case 1.
- Case 1 mainly happens with 0/1 signals.
- Empirically we do see the connection between volatility scaling and modeling the time-series signal standard deviations.

Summary

- Forecasting analysis controls raw signals for expectations, skill, and volatility.
- It also tells us how to combine signals.
- Rule of thumb provides intuition, and structure in unstructured situations.
- Skill level is low in stock selection.
- Don’t confuse cross-sectional with time-series scores.