Equity: Section 4 [1]

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[1] Exercises inherited from Vinicio De Sola

Currency course

230G	Kermani	Fr	9/3/2021	1:30-3:30pm	F320
230G	Kermani	W	9/8/2021	4:30pm-6:30pm	F320
230G	Kermani	Fr	9/10/2021	1:30-3:30pm	F320

- Pset break (after equity pset 3) – currency pset 1 due Sep. 21
- Next week's section will start at 9:30am instead of 10:30am

@ Berkeley

Pset 2 review

$\frac{p}{e} = \frac{p(0)}{e(1)}$ or $\frac{p(1)}{e(1)}$

forward p/e

trailing p/e

1.
   a. 7.5%
   b. p/e = 10
   c. 11.67% – this is greater than 10% b/c we are buying below fair price
2.
   a. p/e = 12.5
   b. 78.12%
   c. 16.25% Although mispricing gap declines over time, it does not disappear
3.
   a. See below.
   b. $\alpha = 0.7\% \neq 0$ Intuition: In this case, the factor exposures to Earnings Yield, Value, and Momentum are z-scores defined to be zero on average across the estimation universe (which is about 1,500 stocks). Since we are looking at only the top 25 stocks, the average z-scores aren't necessarily zero.

Pset 2 review

$\psi_P^2 = h_{PA}^T V h_{PA}$ ← only λ not known

20% known

Solve for lambda

3.

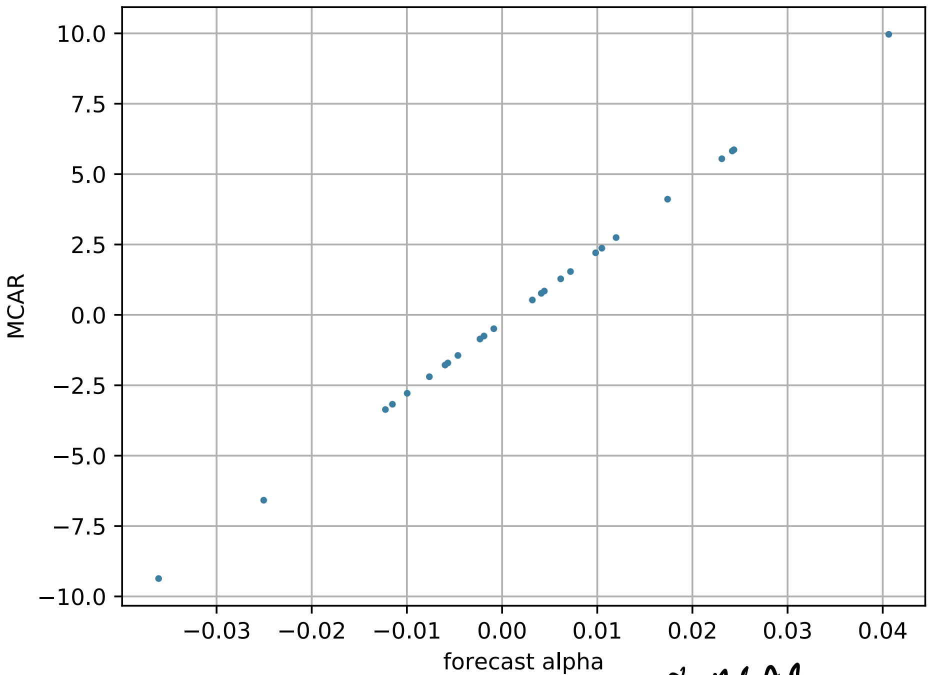
TICKER	alpha	h_PA	MCAR
AAPL	0.04063	[0.04081767737407186]	[9.965057245115673]
XOM	0.00412	[-0.002595257267614718]	[0.7662746137817873]
WMT	0.01739	[0.027803697222393384]	[4.109683146178408]
MSFT	0.00318	[0.0025998641456864728]	[0.5294393447197798]
IBM	-0.00087	[-0.006359673247260788]	[-0.4909679315580169]
T	0.00444	[0.005372095645928525]	[0.8468993862284258]
GE	-0.00232	[-0.012799536565771081]	[-0.856298931706851]
CVX	0.02309	[0.053982381049966145]	[5.545811905384193]
GOOG	-0.01226	[-0.010526046907993606]	[-3.360705925830623]
JNJ	-0.00599	[-0.018582668048538635]	[-1.7809642907042624]
KO	-0.00464	[-0.010306026410403388]	[-1.4408285319449954]
PFE	0.01198	[0.013053263350385785]	[2.7466205870023916]
WFC	0.02436	[0.02320055397204908]	[5.865791471031793]
PG	-0.01152	[-0.030386967799308977]	[-3.1742611395477676]
PM	0.00616	[0.006978411654309261]	[1.2802575381291188]
ORCL	-0.00570	[-0.01202134629628337]	[-1.7078980906744938]
JPM	0.02419	[0.01249319379240721]	[5.822959560669519]
MRK	0.01048	[0.0070822458280136]	[2.3686919661587647]
INTC	0.00983	[0.01070317187457647]	[2.2049228971265276]
VZ	-0.00190	[-0.0109970981717076]	[-0.7504789178706375]
PEP	-0.00997	[-0.025033559302832447]	[-2.7837348980093544]
AMZN	-0.02505	[-0.01911940949773415]	[-6.583177299557288]
ABT	0.00719	[0.012642014197479626]	[1.5397685244417412]
QCOM	-0.00764	[-0.013832538195703922]	[-2.196685773632254]
SLB	-0.03609	[-0.04416844239611474]	[-9.364731948966373]

c.

Pset 2 review

3.

$$MCAR = \frac{V\cdot h_{PA}}{\sigma_{PA}}$$



c.

$$U = h_{PA}^{\top}\cdot\alpha - \lambda h_{PA}^{\top}\cdot V\cdot h_{PA} + c h_{PA}^{\top}\cdot e \Rightarrow 2\lambda \overbrace{V\cdot h_{PA}}^{\propto MCAR} = \alpha - \underbrace{\left(\frac{\alpha^{\top}\cdot V\cdot e}{e^{\top}\cdot V\cdot e}\right)}_{0.11\%}\cdot e$$

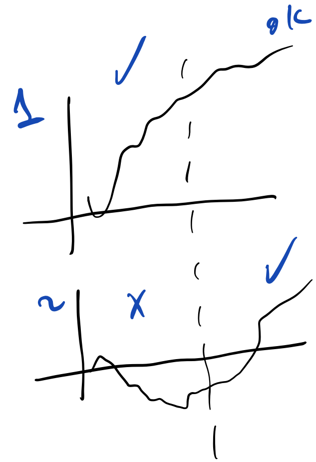
Pset 2 review

4.

a. $\alpha = \omega\left(0.058z_1 + 0.058z_2 + 0.078z_3\right)$

b. $IC^2 = IC^\top \rho^{-1} IC = 0.1363$

c. 0.1344



5.

a. $c_1 = 0.0025$, $c_2 = 0.01248$

b. $IC = \frac{Cov(\alpha,\theta)}{\sqrt{Var(\alpha)Var(\theta)}}$

c. $Var(\alpha) = IC^2 \cdot \omega^2$

6. Signal 1 was ☺ then 😐. Signal 2 was ☹ then ☺. But I'll take pretty much anything ¯_(ツ)_/¯

Pset 2 review

7. P&G largest active holding (3.09%) with lowest forecast active risk and largest active bet (0.35%)
8. P&G largest active holding (1.41%). All the BUYs represent 0.16% bets, while the SELLs represent -0.16% bets. This is what volatility scaling achieves.

Company Name	Active Risk	Broker Rating	Research Service Alpha	alpha	h_PA	Active bet	alpha2	h_PA2	Active bet2
American Express	0.193450	SELL	0.001652	-0.01	-0.008271	-0.206772	-0.000077	-0.008271	-0.0016
Boeing	0.136949	BUY	0.024995	0.01	0.011683	0.292079	0.000055	0.011683	0.0016
Chevron	0.125926	BUY	0.019838	0.01	0.012706	0.317646	0.000050	0.012706	0.0016
Coca-Cola	0.120848	SELL	-0.006561	-0.01	-0.013240	-0.330994	-0.000048	-0.013240	-0.0016
Walt Disney	0.117339	BUY	-0.001328	0.01	0.013636	0.340892	0.000047	0.013636	0.0016
Dow Chemical	0.215148	SELL	-0.022159	-0.01	-0.007437	-0.185919	-0.000086	-0.007437	-0.0016
Du Pont	0.130345	BUY	-0.019568	0.01	0.012275	0.306878	0.000052	0.012275	0.0016
Exxon Mobil	0.117544	BUY	-0.019310	0.01	0.013612	0.340299	0.000047	0.013612	0.0016
General Electric	0.151949	SELL	-0.035266	-0.01	-0.010530	-0.263247	-0.000061	-0.010530	-0.0016
Hewlett-Packard	0.127986	SELL	0.015947	-0.01	-0.012501	-0.312535	-0.000051	-0.012501	-0.0016
IBM	0.119767	SELL	0.004079	-0.01	-0.013359	-0.333982	-0.000048	-0.013359	-0.0016
Johnson & Johnson	0.120931	BUY	-0.049155	0.01	0.013231	0.330768	0.000048	0.013231	0.0016
JP Morgan	0.153596	SELL	0.021143	-0.01	-0.010417	-0.260424	-0.000061	-0.010417	-0.0016
McDonalds	0.119144	BUY	0.005549	0.01	0.013429	0.335727	0.000048	0.013429	0.0016
Merck	0.147363	BUY	0.003295	0.01	0.010858	0.271438	0.000059	0.010858	0.0016
Microsoft	0.122588	BUY	-0.013407	0.01	0.013052	0.326295	0.000049	0.013052	0.0016
3M	0.116459	BUY	-0.000974	0.01	0.013739	0.343470	0.000047	0.013739	0.0016
Proctor & Gamble	0.113771	BUY	0.010943	0.01	0.014063	0.351584	0.000046	0.014063	0.0016
Walmart	0.117697	SELL	0.019890	-0.01	-0.013594	-0.339856	-0.000047	-0.013594	-0.0016
Wells-Fargo	0.176851	SELL	0.024876	-0.01	-0.009047	-0.226180	-0.000071	-0.009047	-0.0016

9. $IR^2 = \alpha^\top \cdot V^{-1} \cdot \alpha, StDev\{\alpha\} \approx IC \cdot \bar{\omega}$ with Fundamental Law of Active Management $\Rightarrow IC \approx 0.15, IR \approx 0.67$

Testing

Ex-ante ratio of positive to negative results: pos 25%, neg 75%

$$R_{pn} = \frac{pos}{neg} = \frac{25\%}{75\%} \approx 0.33$$

$\mathbb{P}$(false negative)

$$\underbrace{\text{Positive predictive value}}_{PPV} = \frac{R_{pn}\,(1 - f_{fn})}{R_{pn}(1 - f_{fn}) + f_{fp}}$$

$\mathbb{P}$(false positive)

$$= \underbrace{\frac{R_{pn} \cdot \mathbb{P}\,(\text{no false negative})}{R_{pn} \cdot \mathbb{P}\,(\text{no false negative}) + \mathbb{P}\,(\text{false positive})}}_{\text{observed positive outcome states}}$$

Measuring transactions costs: Implementation shortfall

$n(\cdot)$ – # shares paper portfolio, $m(\cdot)$ – # shares actually traded

$$\Delta Value_P - \Delta Value_R = \underbrace{\sum_{i=1}^{N} [n_i(T) - m_i(T)] \cdot [p_i(T) - p_i(0)]}_{\text{opportunity cost}}$$

implicit

$$+ \underbrace{\sum_{i=1}^{N} \sum_{j=1}^{J} [p_i(t_j) - p_i(0)] \cdot \overbrace{\delta m_j(t_j)}^{\text{shares traded at } t_j}}_{\text{execution cost}}$$

explicit

Exercise 1 – Shortfall analysis

Suppose as a portfolio manager you instruct your trader to sell 100 shares of stock A, which closed at $1 per share. Trades take place the next day at the following prices:

Time	Trade	Price
10:00am	sell -25	0.99
11:00am	-50	0.98
12:00pm	-25	0.97

we actually sold 100 shares

Compute $\underbrace{\text{opportunity costs}}_{\text{implicit}}$ and $\underbrace{\text{execution costs}}_{\text{explicit}}$.

0

Exercise 1 – Shortfall analysis

$p(0) = \$1.00$,
$n(\cdot)$ – # shares paper portfolio, $m(\cdot)$ – # shares actually traded
Notice we wanted to sell 100 shares of A and we sold 100 shares of A so

$$
\begin{aligned}
OppCost &= \sum \underbrace{(n_i - m_i(T))}_{100-100=0} [p_i(T) - p_i(0)] \\
&= 0 \\
ExecCost &= \cancel{m_i(p_i(T) - p_i(0))} \text{ Insert formula here} \\
&= (-25) \cdot (\$0.99 - \$1.00) + (-50) \cdot (\$0.98 - \$1.00) \\
&+ (-25) \cdot (\$0.97 - \$1.00) \\
&= \$2.00
\end{aligned}
$$

Market impact model

- Price impact proportional to volatility & $\sqrt{\text{volume}}$
- $\frac{\Delta p}{p} \propto \sigma \cdot \sqrt{\frac{V}{\bar{V}_{daily}}}$
- Overall cost:
  - $\text{Cost} = \text{commission} + \frac{\text{spread}}{p} + c \cdot \sigma \cdot \sqrt{\frac{V_{trade}}{\bar{V}_{daily}}}$
- Cost-aware portfolio construction:

$$\underbrace{U = h_{PA}^{\top} \cdot \alpha - \lambda h_{PA}^{\top} \cdot V \cdot h_{PA}}_{} - \frac{Cost\{h_i, h\}}{\tau_h}$$

- where τ_h is the **transactions cost amortization horizon**

Dynamic portfolio management[2]

A quadratic model (quadratic dynamic model for portfolio management) with a quadratic structure on transaction costs:

$$c(h, h_i) = \frac{\hat{\eta}}{2} \cdot (h - h_i)^\top \cdot V \cdot (h - h_i)$$

- $\hat{\eta}$ – positive, sets the relative level of costs
- Similar to a market impact model
- Convenient because it provides an analytic solution

[2] Richard Grinold, 2006. "A Dynamic Model of Portfolio Management." *Journal of Investment Management*, Vol. 4.2, pp. 5-22.

Dynamic portfolio management

You want to maximize lifetime utility today

Optimal dynamic policy derived via the maximization problem:

- g – steady-state utility per unit of time
- $U(h, \alpha)$ – utility impact (relative to g) given portfolio h and α
- $u = h'_{PA}\alpha - \lambda h_{PA}' \cdot V \cdot h_{PA}$
- c – transaction costs with quadratic structure

$$g \cdot \Delta t + U(\underset{\text{initial }\alpha}{\overset{\text{initial holdings}}{h_i, \alpha}}) = \max_h \left\{ u \cdot \Delta t - c + \underbrace{E\left[U(h, \tilde{\alpha}) | \alpha\right]}_{\substack{\text{EV of "next starting} \\ \text{point" given } \alpha \text{ (initial } \alpha\text{)}}} \right\}$$

Dynamic portfolio management

The maximization problem yields optimal dynamic policy:

$$h^* = (1-\delta)\psi h_Q^* + \delta h_i$$

initial portfolio

portfolio we want

where we have

- Effective risk aversion – $\hat{\lambda} \equiv \lambda \cdot \Delta t \cdot \left[1 + \sqrt{1 + \frac{2\hat{\eta}}{\lambda \cdot \Delta t}}\right]$
- $\delta = \frac{\hat{\eta}}{\hat{\eta}+\hat{\lambda}}$
  - $0 \leq \delta \leq 1$
- $\psi = \frac{1-\delta}{1-e^{-\gamma\Delta}\cdot\delta}$
  - $1-\delta \leq \psi \leq 1$

Dynamic portfolio management

- The really cool part is that the optimal policy solution can be adjusted to a single-period problem 😎
- The single-period problem gives us what we want in expectation

$$\max_h \left\{ h^\top \cdot \psi \cdot \alpha - \lambda \cdot h^\top \cdot V \cdot h - \underbrace{\left(\frac{2\lambda}{\hat{\lambda}}\right)}_{\substack{\text{transactions} \\ \text{cost} \\ \text{amortizations} \\ \text{according} \\ \text{to this}}} \cdot c \right\}$$

$$\tau = 1 \Big/ \left(\frac{2\lambda}{\hat{\lambda}}\right) = \frac{\hat{\lambda}}{2\lambda}$$

Exercise 2 – Sharpe ratios and alpha (Chp. 17.9)

$$\frac{r_p - r_f}{\sigma_p} > \frac{r_B - r_f}{\sigma_p} \Rightarrow \alpha > 0$$

Show that:

a. a portfolio Sharpe ratio above the benchmark Sharpe ratio implies a positive alpha for the portfolio,

b. but that a positive alpha does not necessarily imply a Sharpe ratio above the benchmark Sharpe ratio

i.e.,

$$\text{Higher Sharpe ratio} \Rightarrow \alpha > 0$$

$$\alpha > 0 \not\Rightarrow \text{ higher Sharpe ratio}$$

Exercise 2 – Sharpe ratios and alpha (Chp. 17.9)

a. $\frac{r_P - r_f}{\sigma_P} > \frac{r_B - r_f}{\sigma_B}$

Let's assume $r_f = 0$. Recall $r_P = \alpha + \beta_P r_B$.

$\Rightarrow \frac{\alpha + \beta_p r_B}{\sigma_P} > \frac{r_B}{\sigma_B}$

$\Rightarrow \frac{\alpha}{\sigma_P} > \frac{r_B}{\sigma_B} - \frac{\beta_P r_B}{\sigma_P}$

Recall $\beta_P = corr(r_P, r_B) \cdot \frac{\sigma_P}{\sigma_B}$.

$= \frac{r_B}{\sigma_B} - \frac{\rho_{P,B} r_B}{\sigma_B}$

$= \frac{r_B}{\sigma_B} \underbrace{\left(1 - \rho_{P,B}\right)}_{<1}$ $r_B > 0$

$\Rightarrow \alpha > 0$

b. $\alpha > 0$

$$\Rightarrow \underbrace{\frac{\alpha+\beta_p r_B}{\sigma_P}}_{=r_P/\sigma_P} > \frac{\beta_P r_B}{\sigma_B}$$

$$= \rho_{P,B} \cdot \frac{\cancel{\sigma_P}}{\sigma_B} \cdot \frac{r_B}{\cancel{\sigma_P}}$$

$$= \rho_{P,B} \cdot \frac{r_B}{\sigma_B}$$

$$\geq \frac{r_B}{\sigma_B}$$

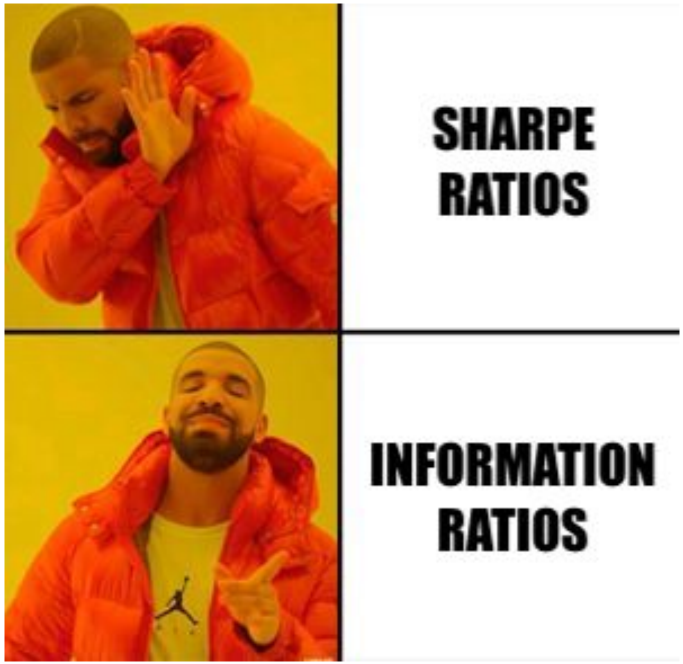
$\rho_{P,B} \leq 1$

$\frac{r_P}{\sigma_P} \geq \frac{r_B}{\sigma_B}$

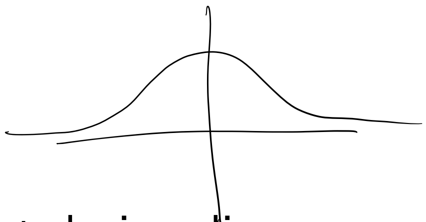
$$\Rightarrow \frac{r_P}{\sigma_P} \text{ not necessarily} > \frac{r_B}{\sigma_B}$$

strictly greater than

insert meme here LOL



Exercise 3 – Forecasting and credibility



20

$z = \frac{s - E[s]}{StDev[s]}$

One night during dinner, you receive a phone call from a stock broker recommending that you immediately buy shares in Countrywide Financial. The broker describes how the recent turmoil in the subprime mortgage markets have overly punished Countrywide, and then predicts that Countrywide's stock price will soon increase from its current \$20 to \$30. Knowing that Countrywide's active return volatility is 50%, estimating that this stock broker's IC is 0.05, and ignoring any possible overall market move, what can you say about this forecast for Countrywide?

$\alpha = \frac{30-20}{20} = 0.5$

$\alpha = IC \cdot \omega \cdot z$

predicted

$$\boxed{z = \frac{\alpha}{IC \cdot \omega} = \frac{0.5}{0.05 \cdot 0.5} = 20}$$

Exercise 3 – Forecasting and credibility

This is a situational question.
What is our α? – 50%
We have a predicted α, IC, and ω. So we can find the signal z.

$$
\begin{aligned}
\alpha &= IC \cdot \omega \cdot z \\
\Rightarrow z &= \frac{\alpha}{IC \cdot \omega} \\
&= \frac{0.5}{0.05 \cdot 0.5} \\
&= 20
\end{aligned}
$$

Recall z is standardized $\Rightarrow z = 20$ is basically impossible. So we don't really believe this signal, not very credible.

Exercise 4 – Review

As your first job after graduation, you are managing a multi-strategy fund that invests in five underlying long-short funds. For each underlying fund, the returns are given by:

$$r_n(t) = g(t) + o_n(t)$$

Each funds' return is the sum of a *generic* return (in common across all five funds) and an *orthogonal* return unique to each fund. We know that:

$$Corr(o_n, o_m) = 0 \text{ if } n \neq m$$

$$\sigma_n = StDev\{r_n\} = 5\%$$

$$\sigma_g = StDev\{g\} = 3.2\%$$

Exercise 4 – Review

a. What is $\sigma_o = StDev\{o_n\}$?

b. What fraction of each underlying fund's risk budget is generic and orthogonal risk? In other words, what are σ_g^2/σ_n^2 and σ_o^2/σ_n^2?

c. What are the pairwise correlations between underlying fund returns?

d. What is the volatility of the multi-strategy fund, assuming it invests equally in these five funds?

Exercise 4 – Review

After a few months running the multi-strategy fund, we enter a market crisis characterized by greatly increased generic risk. In fact, the generic risk rises to 11.2%, while the orthogonal risk remains unchanged.

e. Now what fraction of each underlying fund's risk budget is generic and orthogonal risk?

f. What are the pairwise correlations between underlying fund returns now?

g. What is the volatility of the multi-strategy fund now?

h. Compare the increase in volatility of the underlying funds to the increase in volatility of the multi-strategy fund.

Pset 3 hints

Q2 – Open ended. See if you can translate the paper into a signal.

Q5 –

- Information decay at equilibrium $\alpha(t) = \underbrace{e^{-\gamma\Delta t}}_{\text{half life}}\alpha(t - \Delta t) + \tilde{s}(t)\sqrt{\Delta t}$
- $Cov(h_1, h_2) = h_1^\top \cdot V \cdot h_2$
- Low half-life $e^{-\gamma\Delta t} \to 0 \Rightarrow \gamma \to \infty$

Q6 – Check out the Key formulas doc. $e^{-\gamma\Delta t} \equiv \frac{1}{2}^{\Delta t/HL}$

Q7-9 – Check out the Key formulas doc.

Chp. 12

$$tstat = \frac{\hat{\beta} - \beta_0}{s.e.\{\beta\}} = \frac{\alpha}{\omega/\sqrt{T}}$$

$$tstat = IR\sqrt{T}$$

$$\approx \frac{\alpha}{\omega}$$