Equity: Section 1 [1]

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[1] Subset of slides inherited from Vinicio De Sola

Themes from the text[2]

Active management...

- is a process
- requires forecasting
- requires forecasting *often*
- acknowledges mathematics cannot overcome ignorance
  - i.e., bad info is useless ☹ we need *superior* information

[2] Grinold & Kahn 2000, pages 578-579

“The Process”

"The Process"

Select a sketdata

α

risk

MV

raw information → forecast → optimally & efficiently build portfolio, balancing risk & return forecasts

Mean-variance preferences

- We want to maximize active returns while minimizing risk.
- $U_P = \alpha_P - \lambda \cdot \omega_P^2 = h'_{PA} \cdot \alpha - \lambda \cdot h'_{PA} \cdot V \cdot h_{PA}$
  - Recall that we pretend we are institutional *equity* managers and ignore $\beta \neq 1$.
  - $(\beta_P - 1) \cdot E[r_B] - \lambda' \cdot (\beta_P - 1)^2 \cdot \sigma_B^2$ cancels out.
- Solving the maximization problem yields the optimal holdings:
  $\frac{\partial U}{\partial h'_{PA}} = \alpha - 2\lambda V h_{PA} = 0$
  $\Rightarrow h^*_{PA} = \frac{\alpha}{2\lambda V}$
- The homework will ask you to add additional constraints to this problem where you would be solving the Lagrangian.

Issues with mean-variance

- A mean-variance portfolio is not robust
  - Any changes in the parameters (e.g. σ – volatility) yields portfolio rebalancing
    - So high transaction costs
  - Mean-varaince only focuses on the 1st and 2nd moments, but other moments could matter
    - e.g. skewness, kurtosis, etc.
- Mean-variance is still used and sometimes gives useful information

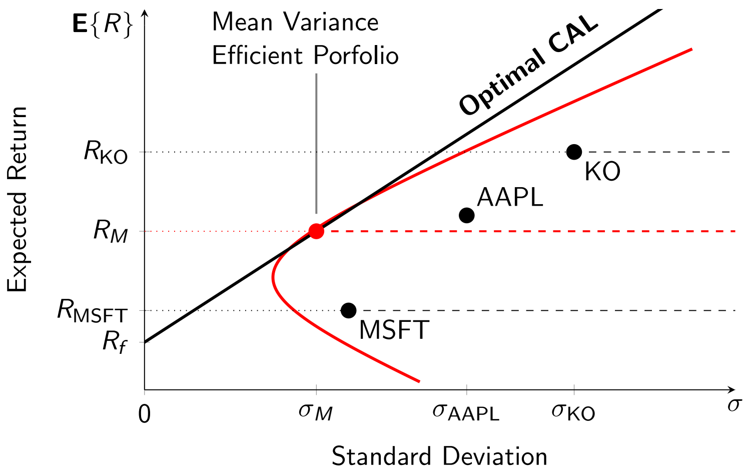
CAPM

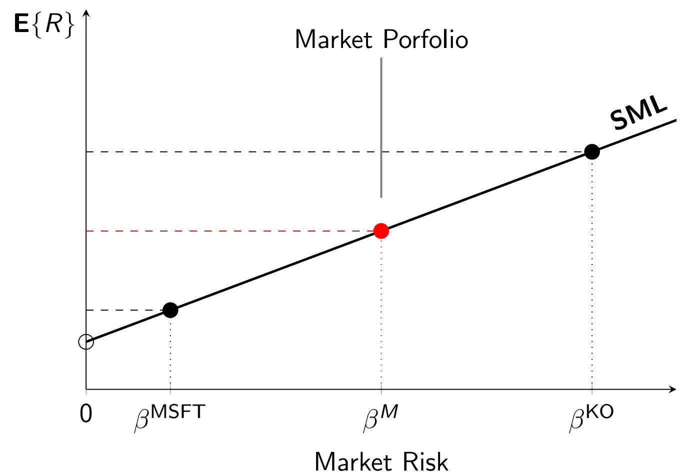
- Assumptions:
  - 1) Buying/selling at competitive market prices without taxes or transaction costs. Borrowing/lending at the risk-free rate.
  - 2) Investors hold portfolios that maximize expected returns given their level of volatility.
  - 3) Investors have homogenous expectations.
- $E[r_P] = r_f + (E[r_M] - r_f) \cdot \underbrace{\frac{Cov(r_P, r_M)}{Var(r_M)}}_{\beta_P} + \underbrace{E[\theta]}_{=0}$

CAPM

$y = m x + b$

$$\frac{R_M - r_f}{\sigma_M}$$





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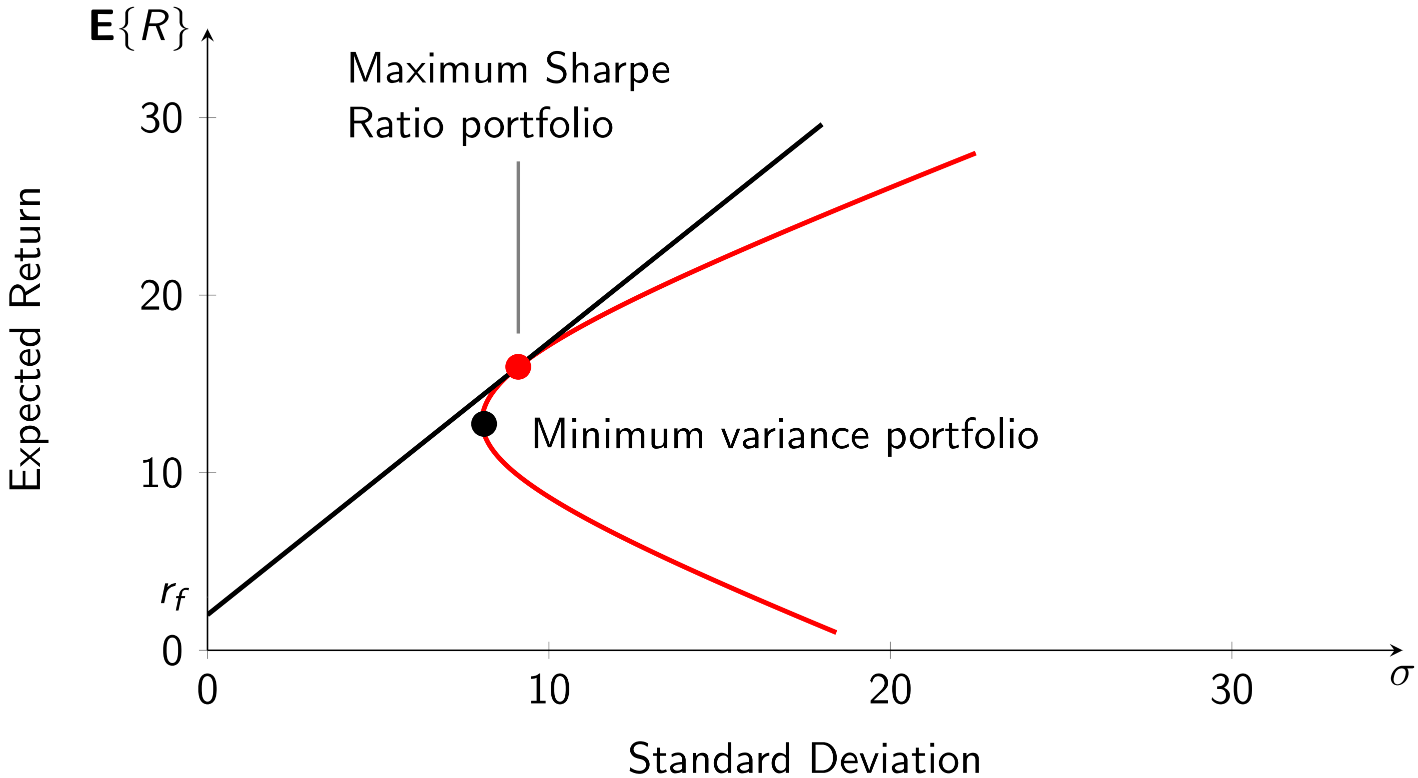
[3] Tikz code generously provided by Erik Loualiche

CAPM



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[4] Tikz code generously provided by ▸ Erik Loualiche

CAPM

- CAPM tells us that expected residual return on all stocks is zero.
  - $E[\theta] = 0$
- If we have superior information g not known to the market and we use that information,
  - $E[\theta_P|g] = \alpha_P$
  - $E[r_P|r_B] = \beta_P r_B + \alpha_P$
  - $E[\delta_P|r_B] = (\beta_P - 1)r_B + \alpha_P$

$= r_P - r_B$

$E[r_P - r_B | r_B] = \beta_P r_B + \alpha_P - r_B$

Framework:

Definitions:

Returns	**Variable**	**Mean**	**Variance**
Excess (above risk-free)	r	f	σ^2
Residual	θ	α	ω^2
Active	δ	$\alpha+(\beta-1)\cdot f_B$	$\omega^2+(\beta-1)^2\cdot\sigma_B^2$

Portfolios: $\mathbf{h}_P$ = portfolio holdings

$\mathbf{h}_B$ = benchmark holdings

$\mathbf{h}_{PA}$ = active holdings

Covariance Matrix: $\mathbf{V}$ = covariance matrix ($\mathrm{Cov}\{\mathbf{r},\mathbf{r}\}$)

Active Management Utility (for institutional equity managers with $\beta_P = 1$):

$$U_P = \alpha_P - \lambda\cdot\omega_P^2$$

Information Ratios:

$$IR_P = \frac{\alpha_P}{\omega_P}$$

Definitions: Framework

When is IR = Sharpe ratio = $\frac{E[R_a - R_f]}{\sigma_a}$?

Exercise

Homework

- Pset 1 due Tuesday, August 24th.
- Question 3: in regards to *size*, assume half are large and half are small.
- Python is encouraged (Excel acceptable), but please submit with Jupyter notebook output. If you only send me code and it doesn't run, you risk losing points on the problem.
- Submit your own pset (preferably typed pls).

Exercise 1

According to the Schwab website, the Fidelity Magellan Fund has a beta of 1.2 relative to the S&P 500. If over a quarter, the Magellan Fund is up by 5% and the S&P 500 is up by 3%:

- a) What is the active return of the Magellan fund over the quarter?
- b) What is the residual return of the Magellan fund over the quarter?

If we expect the S&P 500 to have an annual excess return of 5%, and we expect the Magellan Fund to have an alpha of 1%:

- c) What is the Magellan Fund's expected annual active return?

Exercise 1

- a) $r_p - r_B = 5 - 3 = 2\%$
- b) $r_p = \beta\, r_B + \theta_p \Rightarrow \theta_p = r_p - \beta\, r_B$
  $= 5 - 1.2 \cdot 3 = 1.4\%$
- c) Excess $\rightarrow E[r_p] = \beta_p \cdot E[r_m] + \alpha$
  $= 1.2 \cdot 5 + 1 = 7\%$

  Active $\rightarrow E[\delta_p] = E[r_p - r_B] = 7 - 5 = 2\%$

Breadth

- Say we have an investment process in equilibrium covering N stocks with old information decaying and new information arriving.
- In this eqm. breadth is: $BR = \gamma \cdot N$
- γ is the information turnover rate

Fundamental Law of Active Management

ex ante

ex post

$IR = \frac{\alpha_p}{\omega_p}$

- $IR = IC \cdot \sqrt{BR}$
- In words: Information Ratio depends on "skill" and independent bets per period (year).
- Recall in lecture we defined Information Ratio as an annualized ratio.
- Notice no constraints on portfolios, zero costs, and optimal implementation of ideas.
- Consequently, this is an upper bound of performance.
- Extended Fundamental Law: $IR = IC \cdot \sqrt{BR} \cdot TC$
- Multiple ICs: $IR_P^2 = \sum_{i \in P} BR_i \cdot IC_i^2$

N

M

Exercise 2

We are given 60 months of excess returns data. For an active equity strategy where the benchmark is the $S\&P$ 500, we can regress the excess returns on the benchmark

$$r_p(t) = \alpha_p + \beta_p r_B(t) + \varepsilon_p(t)$$

which yields estimates:

$$\hat{\alpha}_p = 0.1\%, \hat{\beta}_p = 0.95, RMSE = 0.5(\%)$$

where the data reveals

Benchmark excess return (monthly) of 0.67%

Benchmark risk (monthly) of 4.35%

Exercise 2

$$\hat{\alpha}_{ann} = 12 \cdot 0.1 = 1.2\%$$

- a) What's the annual residual return?
- b) What's the annual residual risk? $\omega_{ann}^2 = \omega_{jan}^2 + \omega_{feb}^2 + \dots + \omega_{dec}^2$
- c) What's the realized Information Ratio?
- d) What's the annual active return?
- e) Consider an active strategy of only going long with transfer coefficient 0.5. You receive new information of 200 stocks per quarter. Whats the average Information Coefficient for this strategy?

Exercise 2

- a)
- b) $\omega^2_{monthly} = MSE = 0.5^2 = 0.25$

  $\omega^2_{annual} = 12 \cdot \omega^2_{monthly} = 3$ $\quad\quad \omega_{annual} \approx 1.73\%$

- c) $IR_{expost} = \dfrac{\alpha_p}{\omega_p} = \dfrac{1.2}{1.73} \approx 0.69$ $\quad$ 80%

  !!

- d) $S_p = r_p - r_B = 0.1 + 0.95 \cdot 0.67 - 0.67$

  $r_p = \hat{\alpha}_p + \beta \bar{r}_B$

  $= 0.0665$

  $S_{ann} = S_p \cdot 12 = 0.798\%$

Exercise 2

0.69

200/qtr

800/year

0.5

- e) $IA = IC \cdot \sqrt{BR} \cdot TC$

$$IC = \frac{IA}{\sqrt{BR} \cdot TC} \approx 0.049$$

$IC = 0$

< 0

Exercise 3

Brad runs a domestic equity mutual fund. His team follows 250 stocks and updates their forecasts quarterly. Their forecasts are 5% correlated with future residual returns. The product is long-only, with a Transfer Coefficient of 35%.

IC

- a) What is the expected Information Ratio for the product?

E ex ante

$$IR = IC \cdot \sqrt{BR} \cdot TC$$

$$= 0.05 \cdot \sqrt{250 \cdot 4} \cdot 0.35 \approx 0.5534$$

Exercise 3

In the interest of improving performance, Brad is considering having his team follow an additional 250 stocks quarterly. He expects the forecasts for these stocks to be only 3% correlated with future residual returns, as he expects his team to know less about these stocks. IC_2

- b) What is the expected Information Ratio if he goes ahead and makes this change? You can assume that the same Transfer Coefficient applies to the updated strategy (i.e. find the upper bound of performance, and then adjust that number for the actual Transfer Coefficient of 0.35). $IR_p^2 = \left(\sum_i IC_i \sqrt{BR_i}\right) \cdot TC$

$$IR_p = \left[0.05^2 \cdot (250 \cdot 4) + 0.03^2 \cdot (250 \cdot 4)\right]^{1/2} \cdot 0.35$$

$$\approx 0.645$$

Exercise 4

You are researching a particular investment fund that has delivered an 0.1%
average residual return (relative to its benchmark) of 10 basis points per month with a monthly residual risk of 50 basis points per month.

- a) What is the fund's Information Ratio? Is it a top quartile Information Ratio? ex post
- b) The fund follows 500 stocks, receives new information on them quarterly, and develops forecasts that are 3% correlated with IC subsequent returns. What do you estimate is the Transfer Coefficient for this fund?

Exercise 4

$$\omega_{ann} = \sqrt{12 \cdot 0.005^2} \approx 0.0173$$

1,73%

- a) $IR = \dfrac{\alpha_p}{\omega_p} = \dfrac{12 \cdot 0.1}{1.73} = \dfrac{1.2}{1.73} \approx 0.69$

  top quartile

- b) $IR = IC \cdot \sqrt{BR} \cdot TC$

  $$\Rightarrow TC = \frac{IR}{IC \cdot \sqrt{BR}} = \frac{0.69}{0.03\sqrt{500 \cdot 4}} \approx 0.514$$

long only $TC \in (0, 0.5)$

long-short $TC \in (0.5, 1)$