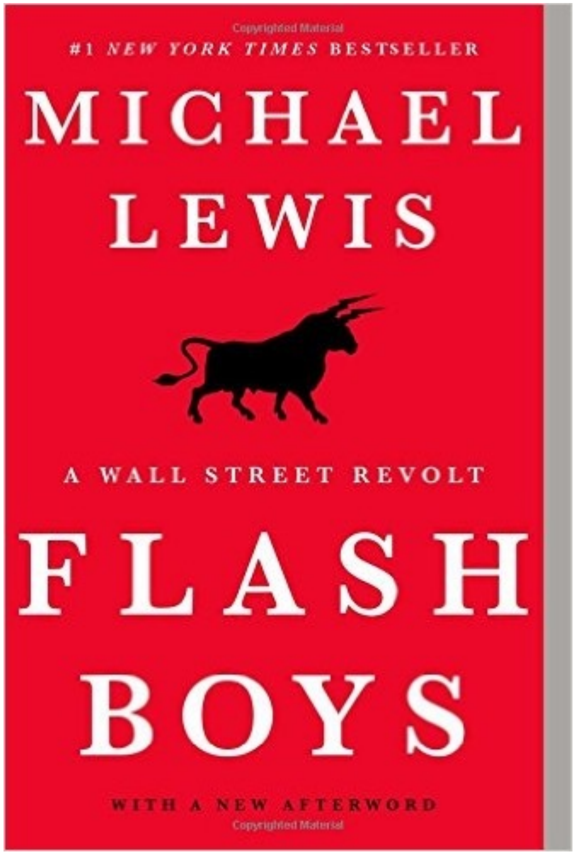
# Transactions Costs

# Important Topic

- Average manager underperforms by average amount of trading costs and fees.
- Portfolio management trades off *expected* returns against costs incurred with certainty.
- Traditionally, the importance of transactions costs has far exceeded the amount of attention paid.
  - The death of a thousand cuts.

# Attention to Transactions Costs Has Increased




"In Michael Lewis's game-changing bestseller, a small group of Wall Street iconoclasts realize that the U.S. stock market has been rigged for the benefit of insiders. They band together—some of them walking away from seven-figure salaries—to investigate, expose, and reform the insidious new ways that Wall Street generates profits. If you have any contact with the market, even a retirement account, this story is happening to you."

– From the back cover

About High Frequency Trading

# Explicit and Implicit Costs

- Transactions costs have explicit and implicit components:
- Explicit components:
  - Commissions
  - Taxes
- Implicit components:
  - Spreads, market impact, opportunity costs
  - These show up in the prices. We don't separately pay them.

# Defining and Measuring Costs

- Implementation Shortfall Approach.
- Treynor and Perold
- Good overall framework for understanding components of costs.
- Basic idea:
  - Compare real portfolio to a paper portfolio. Both portfolios start with same value. Real portfolio trades over interval, while paper portfolio does not. Compare final values.
  - Implementation shortfall is difference in value between the two portfolios.

# Implementation Shortfall

- Note: we will find it useful to work with numbers of shares, rather than our usual fractional portfolio holdings.
- Paper Portfolio
  - $n_i$, $p_i(0)$, $p_i(T)$
- Real Portfolio
  - $m_i(t_j)$, $p_i(t_j)$
  - Trades occur at times $\{t_j\}$ between 0 and $T$.
  - Prices are net of commissions and taxes.

# Self-financing Condition

- Trades are self-financing:

$$\delta m_i\left(t_j\right) = m_i\left(t_j\right) - m_i\left(t_{j-1}\right) = \text{shares of } i \text{ traded at } t_j$$

$$\sum_{i=1}^{N} p_i\left(t_j\right) \cdot \delta m_i\left(t_j\right) = 0$$

- Note that one of the assets can be cash.

# Implementation Shortfall

- Paper portfolio performance over period:

$$\Delta Value_P = \sum_{i=1}^{N} n_i \cdot \left[ p_i(T) - p_i(0) \right]$$

All trades happen at $t=0$, $p(0)$ for the paper portfolio

- Real portfolio performance over period:

$$\Delta Value_R = \sum_{i=1}^{N} m_i(T) \cdot p_i(T) - m_i(0) \cdot p_i(0)$$

$$\Rightarrow \sum_{i=1}^{N} m_i(T) \cdot \left[ p_i(T) - p_i(0) \right] - p_i(0) \cdot \left[ m_i(0) - m_i(T) \right]$$

$$= \sum_{i=1}^{N} m_i(T) \cdot \left[ p_i(T) - p_i(0) \right] + p_i(0) \cdot \left[ m_i(T) - m_i(0) \right]$$

# Shortfall Calculation

- We can rewrite trade term as:

$$\sum_{i=1}^{N} p_i(0)\cdot\left[m_i(T)-m_i(0)\right] \Rightarrow \sum_{i=1}^{N} p_i(0)\cdot\sum_{j=1}^{J}\delta m_i(t_j)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{J} p_i(0)\cdot\delta m_i(t_j) - \underbrace{\sum_{i=1}^{N}\sum_{j=1}^{J} p_i(t_j)\cdot\delta m_i(t_j)}_{=0 \text{ due to self financing}}$$

- Hence, the shortfall becomes:

$$\Delta Value_P - \Delta Value_R = \overbrace{\sum_{i=1}^{N}\left[n_i(T)-m_i(T)\right]\cdot\left[p_i(T)-p_i(0)\right]}^{\text{opportunity cost}} + \underbrace{\sum_{i=1}^{N}\sum_{j=1}^{J}\left[p_i(t_j)-p_i(0)\right]\cdot\delta m_i(t_j)}_{\text{execution cost}}$$

= opportunity cost+execution cost

<u>Example</u>

trade list developed between mkt close + next market open

# Cost Components

- Execution Cost
  - Adds up shares traded times the difference between the traded price and the reference price (which is the $t$=0 price).
- Opportunity Cost
  - Adds up overall price moves times the discrepancy between final shares in the real and paper portfolios.
- Possible set-up:
  - Receive new information at $t$=0 and immediately rebalance paper portfolio. Allow real portfolio until $t$=$T$ to implement new information. Compare using implementation shortfall methodology.

# Transactions Costs Forecasting

- Commissions and other explicit costs
  - Easy to model
  - Typically small for institutional investors.
- Spreads
  - Also easy to observe and model.
- Market Impact
  - Extra cost of trading more than one share.
  - Liquidity provider assessment of counterparty:
    - Sheep?
    - Wolf?
    - Probabilities vary with urgency of trading.

# Market Impact Model

- Inventory Risk
- Liquidity provider receives offer to sell $V_{trade}$.
- How long before the liquidity provider might receive similar orders (in aggregate) to buy $V_{trade}$?

$$\tau_{clear} \sim \frac{V_{trade}}{\overline{V}_{daily}}$$

- Risk of holding the position over that time:

$$\sigma_{inventory} = \sigma \cdot \sqrt{\frac{\tau_{clear}}{250}}$$

# Market Impact

- Liquidity provider demands price concession proportional to risk:

$$\frac{\Delta p}{p} = c' \cdot \sigma_{inventory} \Rightarrow c \cdot \sigma \cdot \sqrt{\frac{V_{trade}}{\bar{V}_{daily}}}$$

  – Price impact proportional to square root of shares traded

- Market Impact cost proportional to shares traded times cost per share
  – Proportional to shares traded to power 3/2.

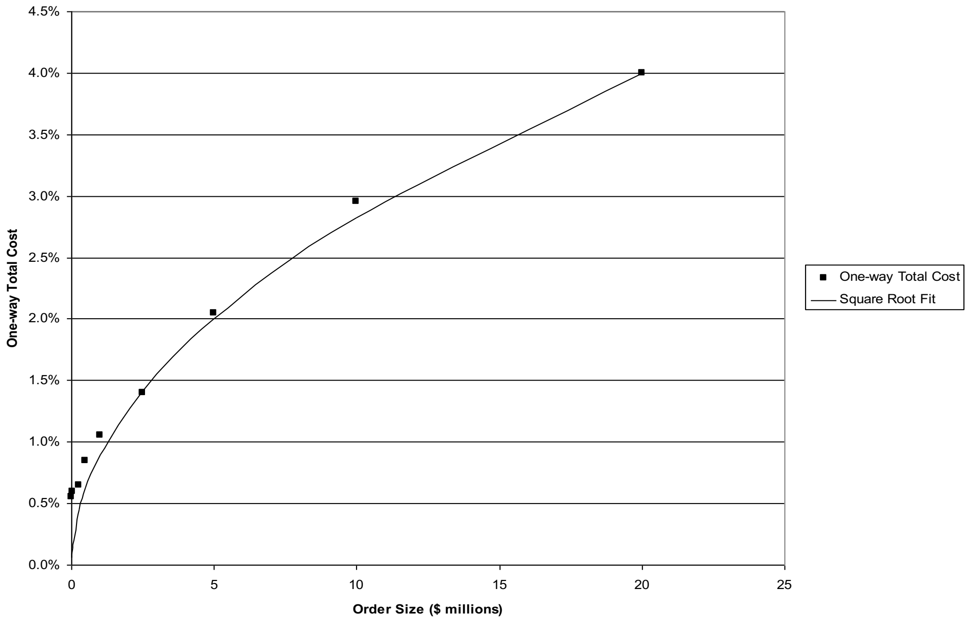
# Example

- Trade 10,000 shares:
  - Move price from \$50 to \$50.25
  - Price impact = 0.50%
  - Market impact cost = 10,000 * (\$0.25) = \$2,500
- Trade 20,000 shares:
  - Price impact $\Rightarrow 0.50\% \cdot \sqrt{2} = 0.71\%$
  - Move price to \$50.35
  - Market impact cost = 20,000 * (\$0.35) = \$7,000

$$\left(\frac{7,000}{2,500}\right) = 2.8 = 2\sqrt{2} = 2^{\frac{3}{2}}$$

# Overall Cost Model

$$Cost = commission + \frac{spread}{p} + c \cdot \sigma \cdot \sqrt{\frac{V_{trade}}{\overline{V}_{daily}}}$$




Thomas F. Loeb, "Trading Costs: The Critical Link between Investment Information and Results." FAJ, 1983

# Rule of Thumb Calibration

Need coefficient "c" to use then model

- It costs one day's volatility to trade one day's volume.
  - This sets coefficient *c* in model.
  - Calibration with trade data can refine this further.
- Improvements on this simple model
  - Same framework, add structure.
  - Better forecasts of daily volume and volatility.
  - Elasticity:  as prices move, how does this change the mix of buy and sell orders?

# Cost-aware Portfolio Construction

$h_I$ = initial portfolio

- We need to adjust our utility function:

$h$ = final portfolio

$$U = \mathbf{h}_{PA}^T \cdot \boldsymbol{\alpha} - \lambda \mathbf{h}_{PA}^T \cdot \mathbf{V} \cdot \mathbf{h}_{PA} - \frac{Cost\{\mathbf{h}_I, \mathbf{h}\}}{\tau_h}$$

$h_{PA}$ = final portfolio active positions

- Notes:
  - Costs effectively lower portfolio $\alpha$.
  - Utility depends on initial position.
  - Cost is zero in absence of trading.
  - $\tau_h$ is the *tcost amortization horizon*

# Transaction Cost Amortization

- The quantities $\alpha$ and $\omega$ are forecasts over time (i.e. flow variables).
- The quantity *Cost* forecasts costs at the moment of trading.

$h_s \Rightarrow h$

- We need to convert *Cost* for comparability with $\alpha$ and $\omega$ .
  - The section on "Dynamic Portfolio Management" provides a framework to handle these different constructs together.
- This number can have a large impact on portfolio construction, yet receives little attention.

# Trading and Turnover

- We define turnover in terms of trading.

purchases $$TO_P = \sum_{n=1}^{N} Max\{0, h^*(n) - h(n)\}$$

$h^*$ = new holdings

sales $$TO_S = \sum_{n=1}^{N} Max\{0, h(n) - h^*(n)\}$$

$h$ = old holdings

$$TO = Min\{TO_P, TO_S\}$$

- Canonical example:
  – Go from 100% stock A to 100% stock B.
  – We count that as 100% turnover.

$$\tau_{clear} \sim \frac{V_{traded}}{\bar{V}_{daily}}$$