Performance Analysis

Identifying Skill

- Basics
  - Alphas, t-statistics, and Information Ratios
  - We know that:

t-stat = t-stat of our realized alpha

$$t-stat = IR \cdot \sqrt{Y}$$

measure of statistical significance

measure of investment significance

  - Where Y measures the number of years of observation.
  - So t-statistics grow with time.
- What is the difference between a t-statistic and an Information Ratio?

Statistically Significant *IR*'s

- The standard error of a sample *IR* is:

$$SE\left\{IR\right\} = \frac{1}{\sqrt{Y}}$$

- How many years of data do we require to measure a top quartile *IR* with 95% confidence?
- Does the above result imply that we could more accurately measure monthly *IR*s?

Top Quantile $IR = .5$

(Top Decile $IR = 1$)

Standard Error of estimated $IR = \frac{1}{\sqrt{Y}}$

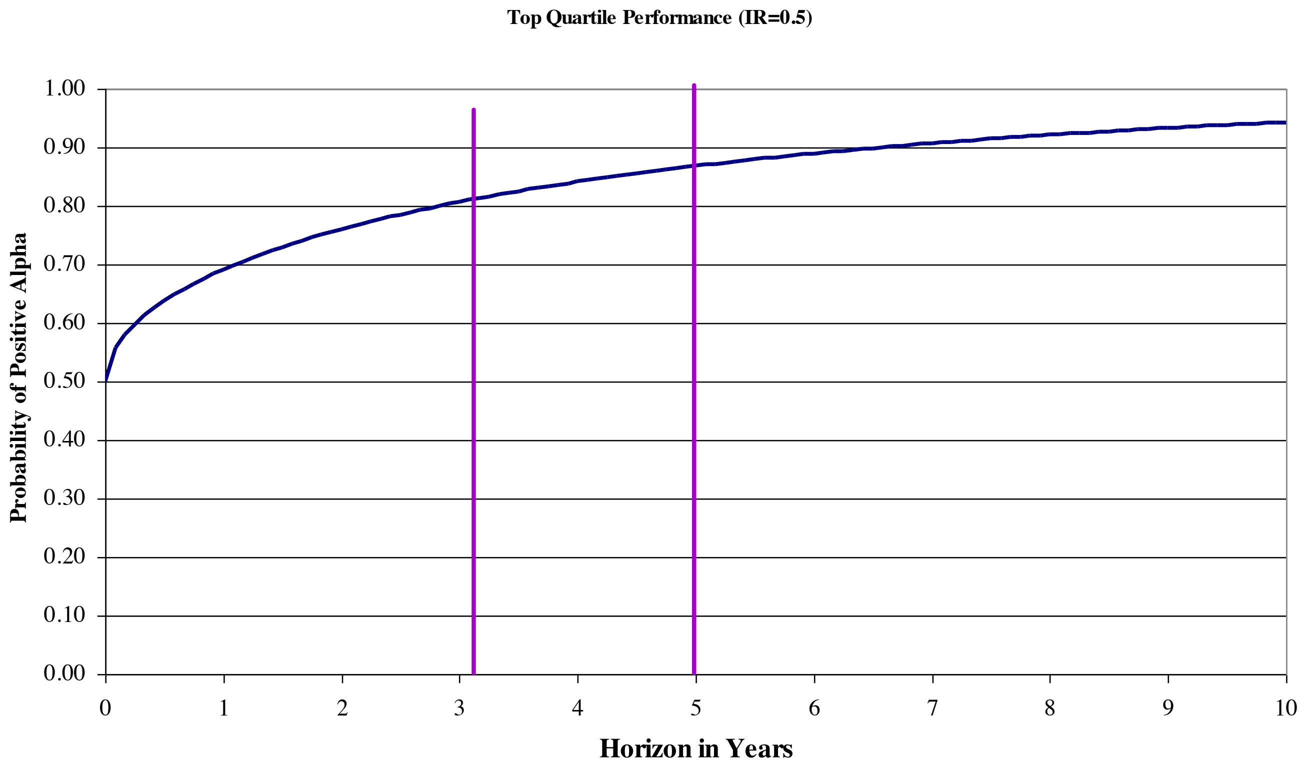
$$\frac{IR}{SE\{IR\}} = 2$$

$$\frac{.5}{1/\sqrt{Y}} = 2$$

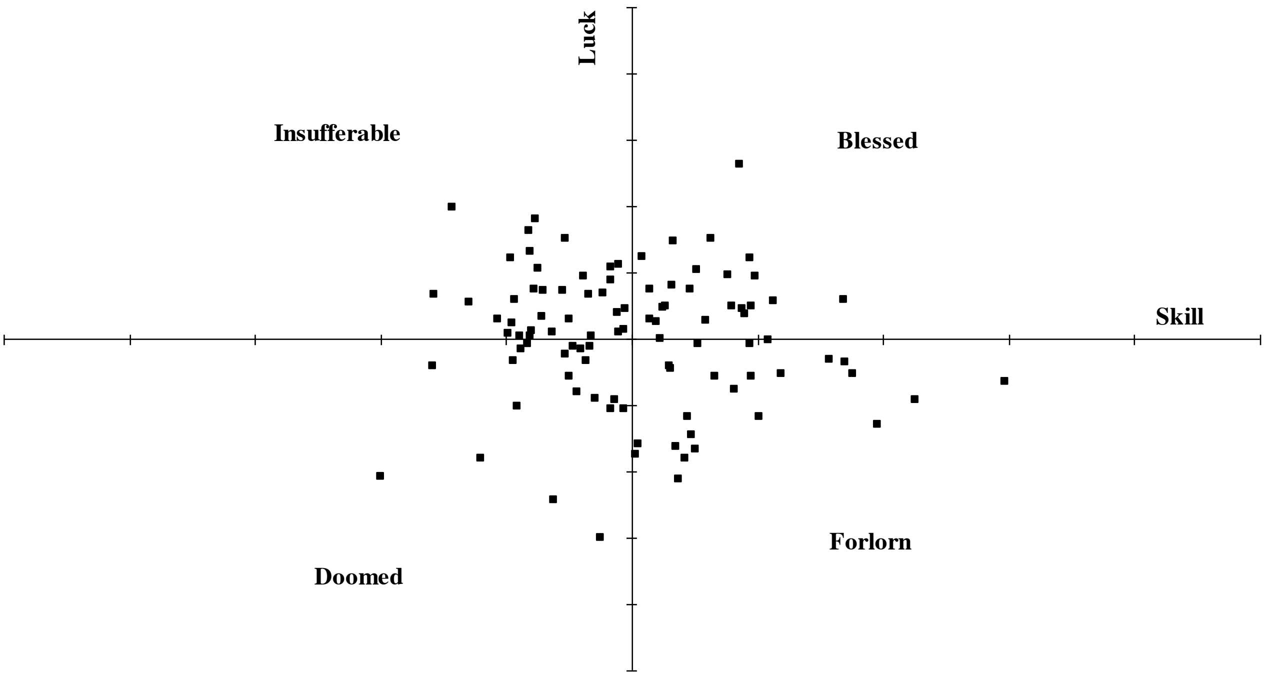
$$\sqrt{Y} = 4$$

$Y = 16$ years

Confidence Levels: *IR*=0.5



Skill and Luck



Performance Attribution and Analysis

- This comes in two flavors:
  - Returns-based
  - Portfolio-based
- The returns-based approach requires less information, and provides less detailed analysis.
- We will start with that.

Returns-based Analysis

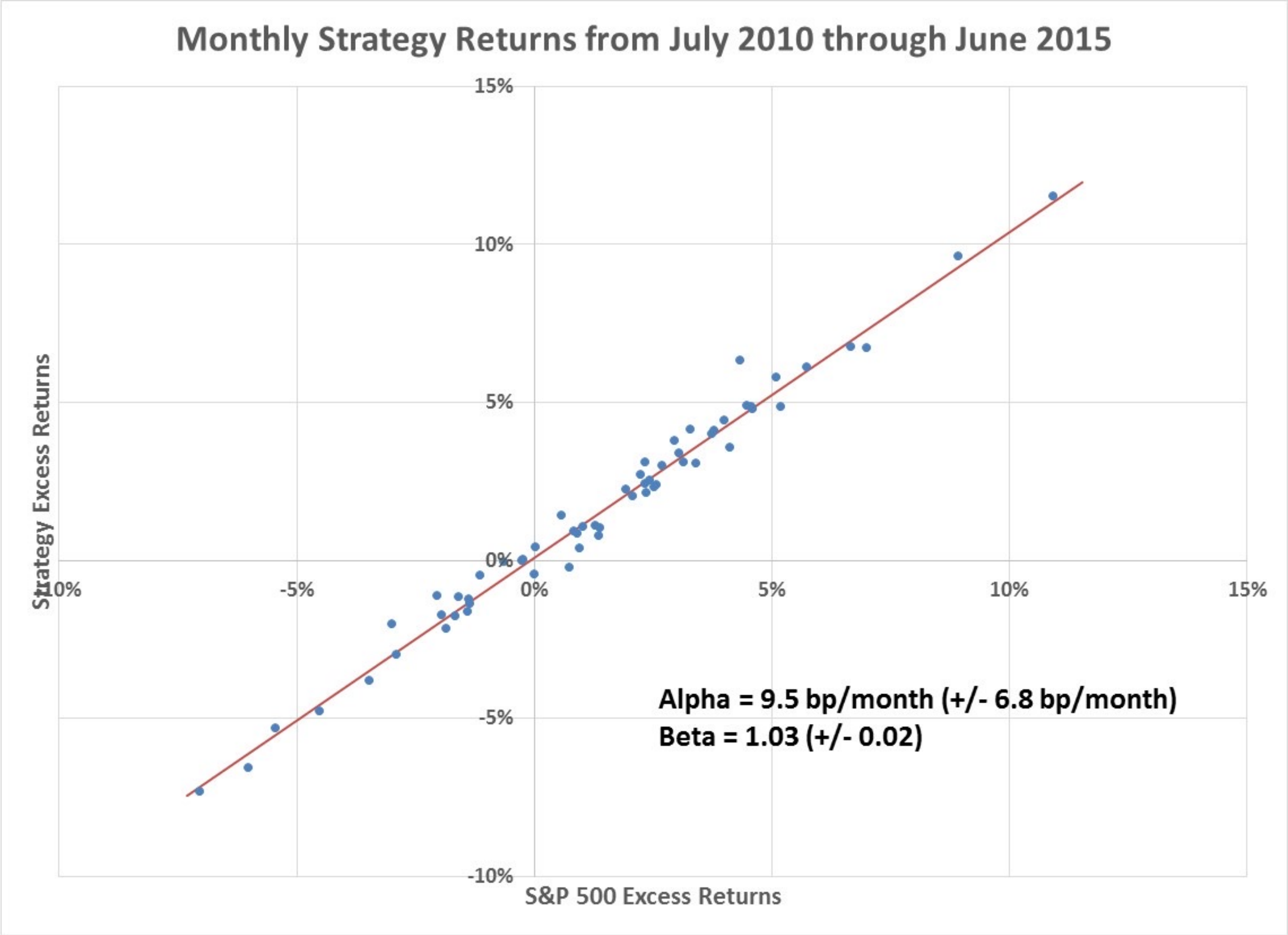
- One of the initial applications of CAPM. The analysis began with Jensen*.
- Take excess portfolio returns over time. Regress against market returns. Ask whether alpha is statistically significant.

"Jensen Alpha"

$$r_P(t) = \alpha_P + \beta_P \cdot r_B(t) + \varepsilon_P(t)$$

*Sharpe also developed an approach to performance analysis. His compared Sharpe Ratios for portfolios and benchmarks. He didn't call them Sharpe Ratios.

Returns-based Analysis: Example

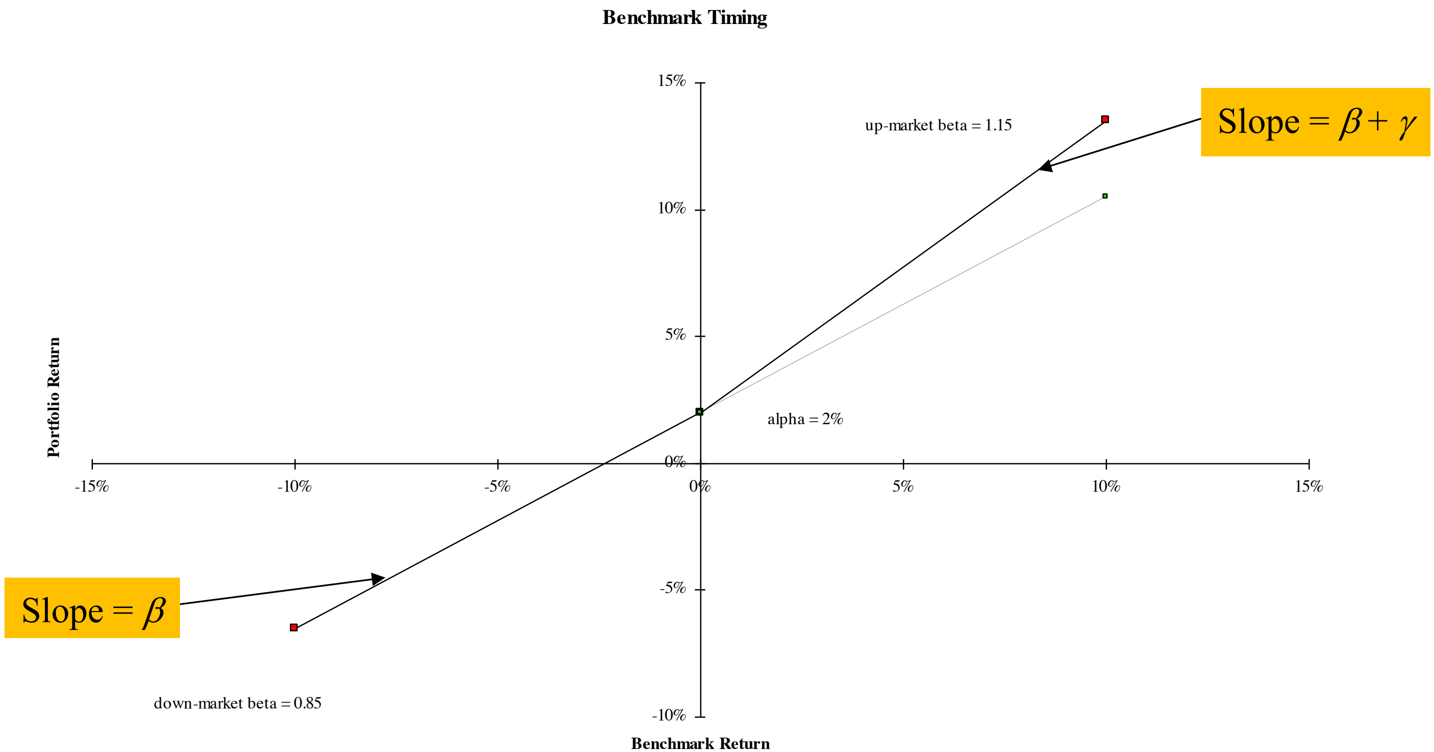


Returns-based Analysis

- There have been many embellishments of the basic approach. We will discuss three:
  - Benchmark timing
  - Style Analysis
  - Fama-French approach
- To analyze benchmark timing ability, expand the return regression to:

$$r_P(t) = \alpha_P + \beta_P \cdot r_B(t) + \underbrace{\gamma_P \cdot Max\left[0, r_B(t)\right]}_{\text{new term}} + \varepsilon_P(t)$$

Benchmark Timing Analysis



Style Analysis

- Another approach due to Sharpe.
- Basic Idea:
  - Decompose portfolio returns into a style component (based on styles like value, growth, large, small) and a selection component.
  - The style component captures the manager's effective benchmark.
  - Skillful active management should show up in the selection component.

Style Analysis

- The analysis is similar to regression, but with some added constraints:

selection

$$r_P(t) = \overbrace{\sum_{j=1}^{J} h_{Pj} \cdot r_j(t)}^{\text{style}} + u_P(t), \quad \textit{with} \sum_{j=1}^{J} h_{Pj} = 1 \quad \textit{and } h_{Pj} \geq 0 \forall j$$

- We choose h_{Pj} to minimize $Var\{u\}$.
- Style analysis has several uses:
  - Performance Analysis
  - Style identification
  - Risk Analysis

$r_j(t) =$ return to a "style factor"

styles: Value
Small Size
Interest rate-related

Typical: Large Value
Large Growth
Small Value

small Growth

Fama-French Approach

- Similar in spirit to style analysis. They perform the regression:

market return

$$r_P(t) = \alpha_P + \beta_P \cdot r_B(t) + \beta_S \cdot SMB(t) + \beta_V \cdot HML(t) + \varepsilon_P(t)$$

- SMB measures the performance of small relative to large stocks and HML measures the performance of value relative to growth stocks.
- Fama and French prescribe rules for calculating SMB and HML portfolios from CRSP data.
  - These series available at: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
- Academic papers will use this approach
  - One big advantage for academics is this approach does not require a covariance matrix. They can, e.g., sort stocks into deciles by signal exposure, calculate performance of the top decile relative to the bottom decile, and then regress that result against these factors.
  - But there are issues. See Cremers, Petajisto, and Zitzewitz, "Should Benchmark Indices Have Alpha?"

Holdings-based Performance Analysis

- Prior approaches did not require (or else ignored) information contained in portfolio holdings.
- This information allows for a much richer attribution of returns.
- The starting point is often a factor model

$$\mathbf{r} = \mathbf{X} \cdot \mathbf{b} + \mathbf{u}$$

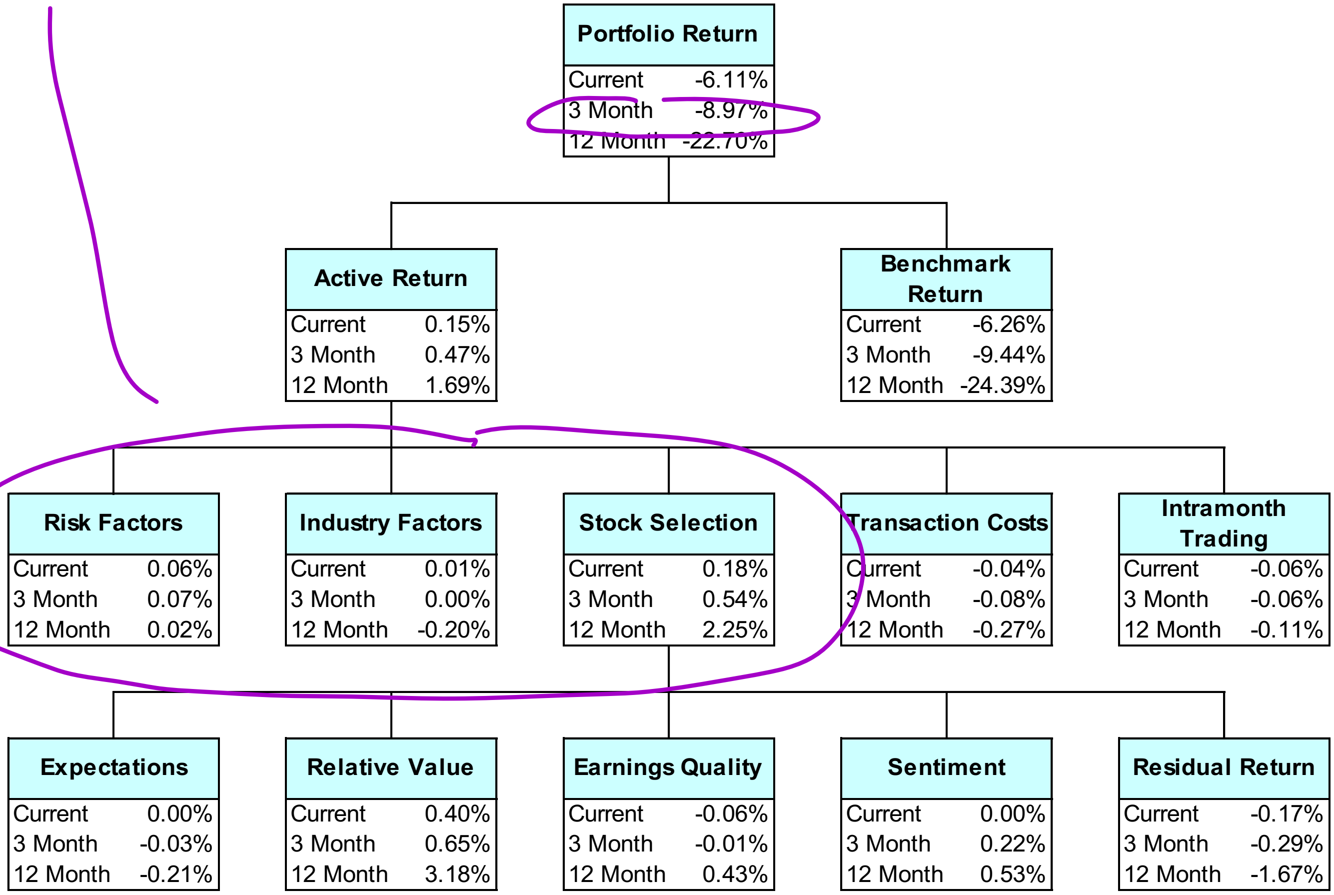
Using a Factor Model for Attribution

- Given the portfolio and the factor model, we can attribute each monthly return:

$$r_P(t) = \mathbf{h}_P^T \cdot \mathbf{r} = \sum_{j=1}^{J} x_{Pj} \cdot b_j(t) + u_P(t)$$

- We can use the same factor model we use for risk, or we can add factors (e.g. return signals). We still need to keep econometric issues in mind when adding factors.

From Barra model



Attribution to Constraints

- An advanced topic.
- Optimizer-defined source portfolios
  - Constraints (e.g., long-only) represented as portfolios
  - This requires more than just the holdings, it requires the optimization set-up and output.

“Optimizer-defined” Sources

A simple constrained optimization

$\max_h h'\alpha - \lambda h' V h$

subject to the long-only constraint that

$h_i \geq -h_{B,i}, i = 1, \cdots, N$

and the budget constraint

$\sum_{i=1}^{N} h_i = 0$

First-order conditions

$\alpha - 2\lambda V h - \theta_{LOC} - \theta_{bud} e = 0$

Implicit impact on alpha

$\alpha_{LOC,i} = -\theta_{LOC,i}, i = 1, \cdots, N$

$\alpha_{bud} = -\theta_{bud}$

Optimal portfolio

$h_{LO}^* = h_{LS}^* + h_{LOC} + h_{bud}$

$h_{LS}^* = \frac{1}{2\lambda} V^{-1}\alpha$ $\quad h_{LOC} = \frac{1}{2\lambda} V^{-1}\alpha_{LOC}$ $\quad h_{bud} = \frac{1}{2\lambda} V^{-1}\alpha_{bud} e$

See Grinold and Easton (1998) for a more detailed discussion

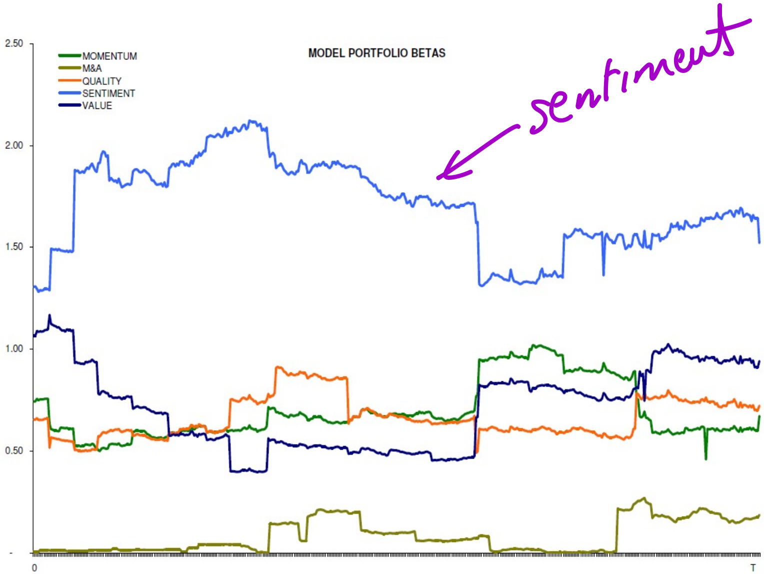
Performance Analysis

- Now for each factor, we have a monthly exposure and a monthly factor return.
- How do we analyze performance over time?
- Here is a simplified view:

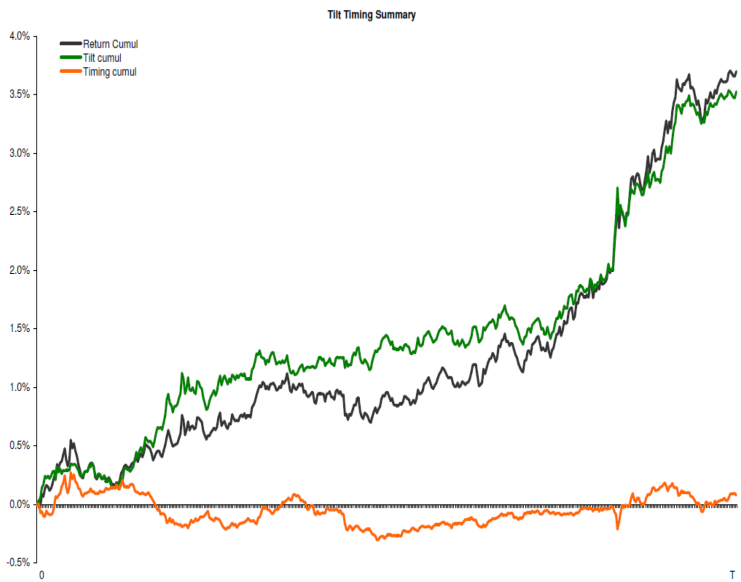
$$
\begin{aligned}
r_{Pj}(0,T) &= \sum_{t=0}^{T} x_{Pj}(t)\cdot b_j(t) \\
&= \underbrace{T\cdot \bar{x}_{Pj}\cdot \bar{b}_j}_{\text{“Tilt”}} + \underbrace{\sum_{t=0}^{T}\left[x_{Pj}(t)-\bar{x}_{Pj}\right]\cdot\left[b_j(t)-\bar{b}\right]}_{\text{“Timing”}}
\end{aligned}
$$

An Example of Tilt/Timing

Factor Exposures Over Time



Cumulative Return Attributed to Tilt and Timing



Performance Analysis

- The detailed attribution allows us to calculate alphas and *IR*s by factor.
- The question becomes not only is the *IR* statistically significant, but did the manager add value where he claimed to have skill? Did he avoid taking risks where he did not claim any skill.

One Page Summary:
The Process of Active Equity Management

- The active management process:
  Efficiently utilizing superior information

Research: Search for superior information
Better than consensus
Avoid datamining
The more signals the better
Goal: high IR

Goal: Effective Forecasts of Returns

Refinement: Information processing

Implementation & Trading:
Subtract as little value as possible

Performance Analysis

Five Things You Can Now Do

- Research and test equity investment strategies
- Combine signals
- Build optimal portfolios
- Analyze portfolio risk
- Attribute performance back to sources