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MFE230I Section 4

Can Li

June 25, 2021 - May 2, 2022, 1:49:16 AMP 7.49:1 NAW 7.2022. 1:49:1

# Forward and futures contracts

1:49:16 AM PDT Main difference is that futures are exchange traded while forwards are bilateral agreements

	Future	Forward
Flexibility	standardized	more flexible
Trading	more liquid	unique
Settlement	marked-to-market	end of contract, A9

 When interest rates correlated with the underlying asset price, then forward and futures prices are different!

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- $f(t) = \frac{Z(t, T_2)}{Z(t, T_1)}$   $F(t) = E^{Q}[F(T_1)]$

May Li What is the initial forward and futures price for a contract on the 2.1:49:16 AM PDT previous 4 year bond, that expires after 2 years? Suppose the calibrated interest rate tree is

pankaj_kumi	year	0	1	2	3
	r	5%	4%	3%	2%
			6%	5%	4%
			gu	7%	6%
	. ork	elex.			8%
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i KUMI					
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Po					May

pankaj\_kumar@berkeley.edu - May 2, 2022, 1:49:16 AM Pr NAN 2022. 1:49:1 What is the initial forward and futures price for a contract on the previous 4 year bond, that expires after 2 years?

Suppose the calibrated interest rate tree is

aj-kuma	year	0	1	2	3	1.49.
23-	r	5%	4%	3%	2%	- ·
			6%/	5%	4%	
		10V.E	90	7%	6%	
	-herk	ele)	P4		8%	NNP
pankaj_kumar	$f = \frac{Z}{Z}$	$\frac{7(0,4)}{7(0,2)}$	× 10	0 = 9	0.78	2,2022, 1:49:16 AM P
Par	year	0		Lee ,-	2	21
	F =	90.76		49	94.27	
	В		89.	.03	90.71	
		"Ope	( ,		87.35	
panka	KUMC	94.27	= Z(	2, 4)		27.49.
nanka	1-					2.2022
T	92.49	= 0.5	(94.2	7 + 90	0.71)	U-May 2, 2022, 1:49:
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# Hedging using Forward and Future

Class example: Use forward or future to hedge the interest risk Calculate forward delta by replication.

Calculate Future delta of 5-year zero-coupon bond.

- 2. Calculate Future delta by tree.



## Hedging using Forward and Future

Class example: Use forward or future to hedge the interest risk Calculate forward delta by replication.
 Calculate Fig. of 5-year zero-coupon bond.

- 2. Calculate Future delta by tree.

3. We can also calculate forward delta by tree.  $P_{2}(0)$   $P_{2}(1)$   $P_{3}(0)$   $P_{4}(1)$   $P_{5}(0)$   $P_{5}(1)$ P, (t), P, (t)  $\sigma$   $\pi(2) = P_{5}(2) - f$ PV, (TT(2)) = PV, (Ps(21)) - PV, (f)  $PV_o(\pi(2)) = PV_o(P_s(2)) - PV_o(f)$  $= P_{s}(0) - f \cdot 2(0,2) = 0$   $= P_{s}(1) - f \cdot 2(1,2)$   $= P_{s}(1) - f \cdot 2(1,2)$   $= P_{s}(1) - f \cdot P_{s}(1) / 10$   $= P_{s}(1) - f \cdot P_{s}(1) / 10$ =  $P_{5}^{4}(1) - f^{4} \cdot P_{2}^{4}(1)/100$  $PV_{1}^{d}(T(21) = P_{2}^{d}(1) - f \cdot P_{2}^{d}(1)/100$ 

Quality option

What is the futures price for a contract where the short side can deliver either the 4 or the 3 year bond (with 2 or 1 year remaining)? Computation similar as before:

- first construct futures prices at expiration (year 2),
- then take expectation under risk-neutral measure.

## Conversion factors

• for 3 year bond (1 year remaining at expiration):  $\frac{1+c}{1+y}$ 

$$C_3 = \frac{1}{1.06} = 0.94$$

 $\frac{c}{1+ty} + \frac{c+1}{(1+ty)^2}$  2022• for 4 year bond (2 year remaining at expiration):  $C_4 = \frac{1}{1.06^2} = 0.89$ 

# Pricing tree with conversion factors 3 year bond: 2/2022, 1:49:16 AM PDT

year	0	May	2	
Р	100.97	101.93	102.91	$\checkmark$
	rkeley.	100.01	100.95	V
, abe	3/10		99.07	7:49:16 AM PI
uar@p.				1.A9.
				20221
year	0	1	22	

## 4 year bond:

year	0	1	223	
Р	92.52	99.92	105.92	
	- V	94.37	101.92	
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# Futures price with quality option

Short party can choose which bond to deliver, thus in each state she pays  $min(Z(2,3) - FC_3, Z(2,4) - FC_4)$ state she pays

$$\min(Z(2,3) - FC_3, Z(2,4) - FC_4)$$

. This leads to

. This leads to		veley.ec			
	year	O	1	2	20.16 AM
. vum	a'F	100.74	101.93	102.91	1.A9.
ankaj-ka			99.55	100.95	20221
bar				98.15	

$$102.91 = \min\left(\frac{Z(2,3)}{C_3}, \frac{Z(2,4)}{C_4}\right) = \min\left(\frac{97.09}{0.94}, \frac{94.27}{0.89}\right)$$

$$101.93 = \frac{1}{2}(102.91 + 100.95)$$

$$\frac{C_{3}}{C_{4}} = \frac{1}{2} (102.91 + 100.95)$$

(a) 
$$100 \text{ r}_{1}(t-1,t)$$
 ,  $t=1$  , 3, ....

$$(a)$$
 100  $r_1(t-1,t)$  ,  $t=1$  , 3, ....

(a) 
$$loor_{1}(t-1,t) = 1, t=1, 3, ...$$

[Price]
$$R = \frac{loor}{ltr} + \frac{loor}{(ltr)^{3}} + ...$$

Price
$$P_{\text{even}} = \frac{100r}{(1+r)^2} + \frac{100r}{(1+r)^4} + \dots$$

$$P_{\text{even}} = \frac{P}{1+r}$$

$$P + P_{\text{even}} = 100$$

$$P + \frac{P}{1+r} = 100$$

$$P + \frac{P}{1+r} = 100$$

$$P = 100 \times \frac{1+r}{2+r}$$

$$=\frac{1}{(1+r)^2}+\frac{1}{(1+r)^4}+$$

$$P_{even} = \frac{P}{1+r}$$

$$P = 100 \times \frac{1+r}{2+r}$$

## Duration

$$\Delta D_{mac} = -\frac{1+r}{p} \times \frac{dP}{dr} \qquad r, y$$

P + Peven. Fulbor

$$P = |Bo \times \frac{I+\Gamma}{2+\Gamma}$$

Duration

$$D_{mac} = -\frac{I+\Gamma}{P_{max}} \times \frac{dP}{d\Gamma}$$

$$D_{mac} = -\frac{I+V}{I+V} + \left(|Bo| \frac{I+V}{2+V} - \frac{IBOV}{I+\Gamma}\right)$$

$$D_{mac} = -\frac{I+V}{P_{max}} \times \frac{dP}{dV} = \frac{IBOV}{I+\Gamma}$$

$$D_{mac} = -\frac{I+V}{P_{max}} \times \frac{dP}{dV} = \frac{IBOV}{I+\Gamma}$$

$$D_{mac} = -\frac{I+V}{P_{max}} \times \frac{dP}{dV} = \frac{IBOV}{I+\Gamma}$$

$$D_{mac} = -\frac{I+V}{P_{max}} \times \frac{dP}{V} = \frac{IBOV}{V} = \frac{IBOV}{V}$$

$$D_{mac} = -\frac{I+V}{P_{max}} \times \frac{dP}{V} = \frac{IBOV}{V} = \frac{IBOV}{V}$$

$$D_{mac} = -\frac{I+V}{P_{max}} \times \frac{dP}{V} = \frac{IBOV}{V} = \frac{IBO$$

$$(b) \qquad P, D \qquad (P, D) \qquad P, D \qquad (A) + (b) - P \qquad (Containt)$$

$$\Rightarrow P_b = P_p - P_a$$

 $V_{mac,p} = \sum_{odd} \frac{1}{(1+r)^{t}} \sqrt{p_{n}a_{n}}^{t}$   $\Rightarrow \frac{P_{a}}{P_{p}} \times D_{a} + \frac{P_{b}}{P_{p}} \times D_{b} = D_{p}$   $= (C \otimes d)$ pankaj\_kumar@berkeley.edu - May 2, 2022, 1:49:16 AM PDT

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