Name: ______________________

MFE 230I Sample Final Exam

Important: Show all your work in detail to allow us to provide as much credit as possible. Just writing the solution, with no explanation, will earn few points. If you cannot answer one part of a question, you can still get full marks for the remaining parts, so don't leave anything blank. All answers should be written below and on the page facing each question (use the blank pages at the end if you need more space, making it clear that you've done so). Points (out of 180) are roughly equal to the suggested number of minutes for each question. There are **6** questions in total.

Note: Unless otherwise specified in a particular question, assume that all coupon bonds make semi-annual payments, all interest rates are quoted as APRs (compounded semi-annually), and all bonds have a face value of \$100.

1. (30 points) The yield curve is currently flat at 4% for all maturities.

   (a) Calculate the modified duration of a 5% coupon bond with maturity 2 years.

   (b) Calculate the convexity of this bond.

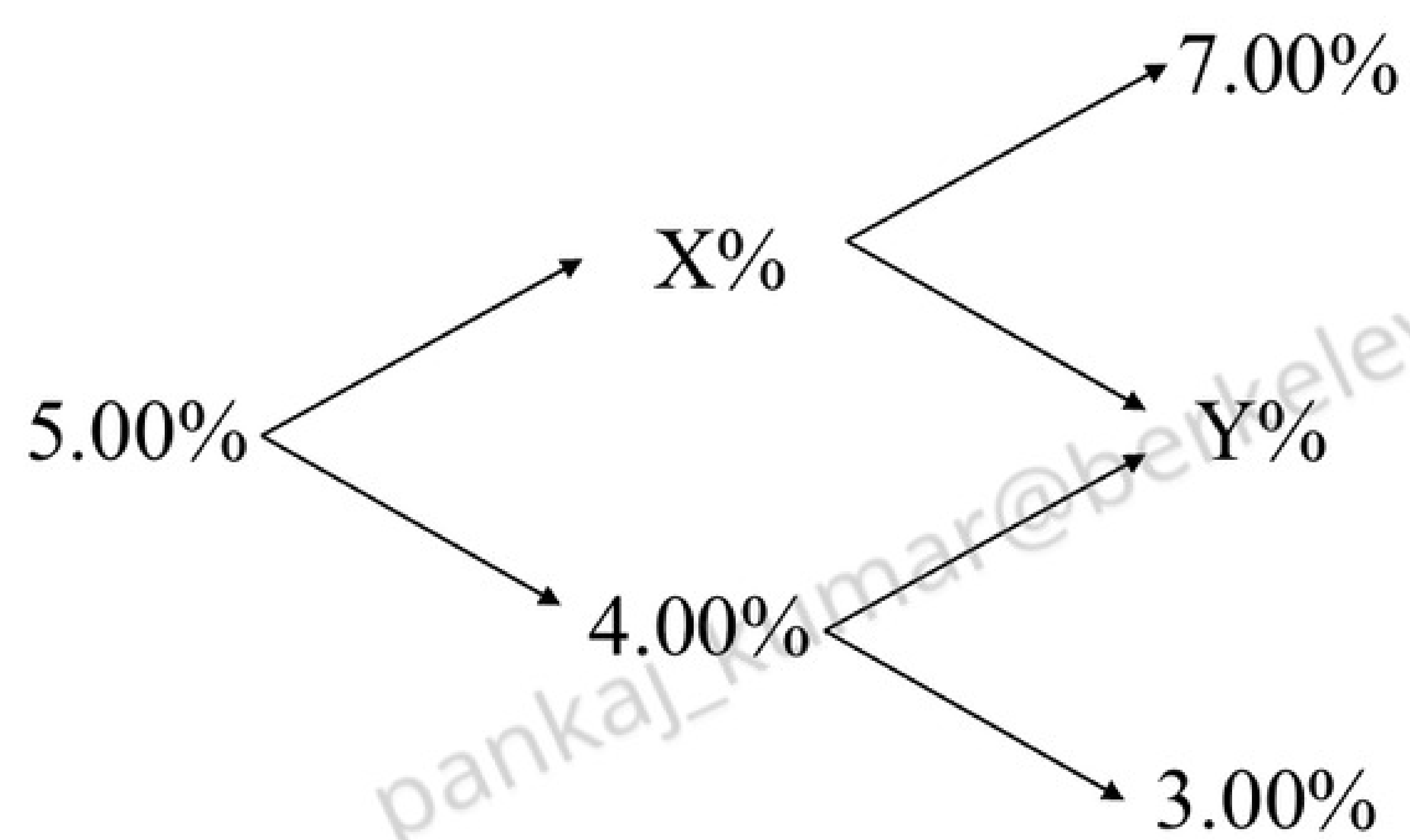
   (c) Using the duration and convexity measures you just calculated, estimate the percentage change in price of the 2 year bond if interest rates increase by 0.25%.

   (d) You own one of the 2-year bonds, and want to hedge it as well as possible against (parallel) interest rate movements. The assets available for hedging are zero-coupon bonds with maturities 0.5, 1.0 and 1.5 years. What hedging position (in some or all of the three zeros) would you set up, given that you'd like to rebalance your portfolio as little as possible over the next two years?

   (e) What is the Macaulay duration of a two-year "double inverse floater", whose coupon rate each period is set equal to 20% minus *double* the current 6-month rate? I.e.,

$$\text{Coupon payment}_i = 0.5 \times 100 \times (20\% - 2r_{i-1})\,.$$

2. (40 points) The following tree shows the possible movements in the 6-month interest rate over the next year (each period in the tree = 6 months):



The risk-neutral probability of an upward shift in interest rates is 0.42 in the first period and 0.75 in the second period. You are given the following zero-coupon bond prices (face value = \$100):

Maturity	Price
6 months	\$Z
1 year	\$95.198
18 month	\$92.768.

(a) Calculate the values X, Y and Z.

(b) What is the value of an 18 month bond with coupon rate 5%, callable at par any time between now and maturity?

(c) A 12-month Asian futures option has terminal price at date 12m

$$\$1,000,000 \times \max\left(\overline{r} - K, 0\right).$$

where $K = 0.05$, and $\overline{r}$ is the average of the 6 month rate at dates 0, 6m and 12m. This is a futures contract, and as with any futures contract, no cash initially changes hands. The contract is marked to market every 6 months. What is the current futures price of this contract?

(d) What position do you need to take today in this futures contract to hedge the interest rate risk of the callable bond in (b)?

3. (30 points) Interest rates are described by the Vasicek model, with risk-neutral dynamics for the continuously compounded instantaneous rate given by

$$dr = \kappa\left(\theta - r\right)\, dt + \sigma\, dZ.$$

Parameter values: $\kappa = 0.5$, $\theta = 0.06$, $\sigma = 0.015$. Current instantaneous rate: $r = .05$.

(a) Calculate the forward rate for a 3-year loan beginning in 2 years.

(b) What is the futures price for a 3-year zero-coupon bond to be delivered in 2 years? What is the implied interest rate (i.e., the yield on a 3-year zero-coupon bond with price equal to the value you just calculated)? Explain why this rate is different from that calculated in (a).

4. (30 points) For each of these questions, is the statement true or false? Explain your answer (most points will be for the explanation).

(a) You possess a callable bond issued by a corporation with zero risk of default. A severe market event causes a sudden rise in interest rates (parallel shift) combined with a sudden rise in interest rate volatility. The overall effect of this event may be either to increase or to decrease the value of the bond, depending on the relative size of the interest rate shift versus that of the volatility shift.

(b) The effective duration of an IO (Interest Only) stripped mortgage-backed security can be either positive or negative, depending on the level of interest rates.

(c) Holding all else equal, and ignoring default, an increase in interest rate volatility will increase the expected life of a callable bond (i.e., the expected time until the bond either matures or is called, whichever comes first).

(d) Long-maturity coupon bonds can be more or less volatile than short-maturity coupon bonds.

(e) Long-maturity zero-coupon bonds can be more or less volatile than short-maturity zero-coupon bonds.

5. (30 points) Assume interest rates are governed by a one-factor LIBOR market model (LMM). You are given the following information about bond yields and forward rate volatilities:

i	T_i	$Z(0, T_i)$	S_i (%)
1	0.25	99.20	30%
2	0.50	98.50	33%
3	0.75	98.00	35%
4	1.00	97.50	40%
5	1.25	96.90	42%
6	1.50	96.20	45%

In other words, the instantaneous proportional volatility of forward LIBOR between dates T_{i-1} and T_i is 30% if t is between T_{i-1} and T_i, 33% if if t is between T_{i-2} and T_{i-1}, etc.

What is the value of a call option, with expiration date 1.25 years from today, on a zero-coupon bond with maturity date 1.5 years from today? The strike price of the option equals the forward price for delivery of the underlying bond 1.25 years from today.

Note: If you have trouble answering this question, you may "purchase" a hint for 8 points. If you purchase the hint, the score you receive for this question will be 8 points lower than if you had written the same answer without the hint. This applies even if the hint turns out not to help you (but note that you will never receive less than 0 points for the question overall).

6. (20 points) Discuss the pros and cons of using the Heath, Jarrow and Morton model versus BDT? Give as many pros and cons as possible for each model, including as many examples as you can of types of securities/problems that are particularly well- or ill-suited to each of the two models.