MFE230I Section 2

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Fixed Income Securities

Understand the cash flows and quoted price.

- Fixed-Rate Mortgage: Equal payment in each period.
  - Balance at t: The value of future payments at t.
- Treasury Bills: Zero-coupon bonds.
  - Price quoted as discount.

$$d = \frac{360}{n} \times \frac{100 - P}{100}$$

$$P = 100 \times \left(1 - \frac{nd}{360}\right)$$

Fixed Income Securities

- Single floating payment:
  - Cash flow: $N\Delta r_n(T-\Delta, T)$ at T
  - Price as if the forward rates are realized.

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- Forward Rate Agreement: Fixed payment c − Single floating payment.
  - The value of FRA at $t = 0$ is 0. $c = f(0, T_1, T_2)$
  - What is the value at $t \in (0, T_1)$?

FRA: valuation

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- What is the value V_t of FRA at t?
- Sell another FRA at t. (c_t will be $f(t, T_1, T_2)$)
- Value of two FRA: V_t
- Value of cash flows (Assume simple rate, N=1):

$$
\begin{aligned}
CF &= Z(t, T_2)\left(f(0, T_1, T_2)(T_2 - T_1) - f(t, T_1, T_2)(T_2 - T_1)\right) \\
&= Z(t, T_2) \times \left(\frac{Z(0, T_1)}{Z(0, T_2)} - \frac{Z(t, T_1)}{Z(t, T_2)}\right) \\
&= Z(t, T_2) \times \frac{Z(0, T_1)}{Z(0, T_2)} - Z(t, T_1)
\end{aligned}
$$

Floating Rate Note: pricing

FRN: bond with a coupon indexed to a benchmark interest rate

- In class, we have seen that the value of a FRN is $P^{FI} = 100$ at every reset date

What if we are not at a reset date?

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What if we are not at a reset date?

- In general for $T_i < t < T_{i+1}$, the formula (assuming semiannual compounding) is

$$\begin{aligned} P^{FI}(t, T) &= Z(t, T_{i+1}) \times 100 \times \left(1 + \frac{r_2(T_i, T_{i+1})}{2}\right) \\ &= 100 \times \frac{Z(t, T_{i+1})}{Z(T_i, T_{i+1})} \end{aligned}$$

Fixed Income Securities

- Swap: Bond - floater
  - The value of the floater $= N$.
  - The value of the bond:

$$P = N \times Z(0, T) + N \times \frac{c}{n} \sum_{i+1}^{n_c} Z\left(0, T_i\right)$$

Calculate Discount Factor from Swap Rate

Sample problem:

- Spot date is 6/6/18
- 2 year swap, payment frequency = 2. swap rate = 2.77%
- Discount factors for 12/6/18, 6/6/19, 12/6/19 are 0.987, 0.974, 0.960
- Find the discount factor at maturity date.

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- $\text{DF} = \frac{1-0.5\times2.77\%\times(0.987+0.974+0.960)}{1+\frac{182}{360}\times2.77\%} = 0.946$

Securities for Bloomberg 23 Curve

- Forward Rate Agreements
    - Effective dates: Third Wednesday of the month.
    - Quoted rates $f_s(t, T_1, T_2)$
- Swap
    - Pays semiannually. If it is not a business day, pay on the next business day.
    - Hint: the time different between two payments are not always 0.5.

Calculate Yield Curve

- Compute the discount factor at maturity date for each given security
- Compute simple zero rate from discount factor by

$$DF(d_0, d) = \frac{1}{1 + rs(t) \times t}$$

- Simple zero rate fits a piece-wise linear continuous function. Calculate the simple zero rates for the wanted dates and turn them into discount factor.
- Turn discount factors into semi-annually compounded zero rates and simple forward rates.
- See example in Excel.

Nelson-Seigel model

Consider the Nelson-Seigel model for spot rates:

$$r(t, t+T) = \beta_0 + \beta_1 * \left(\frac{1 - e^{-\tau_1 T}}{\tau_1 T}\right) + \beta_2 * \left(\frac{1 - e^{-\tau_2 T}}{\tau_2 T} - e^{-\tau_2 T}\right)$$

Assume that you observe spot rates $r(t, T)$ for several (short and very long) maturities.

- **(A)** How would you estimate βs given τs?
- **(B)** Estimate all the parameters of the model, but optimizing over only two parameters.

Nelson-Seigel model

- There are many numerical complexities in seemingly simple fitting
- There may be no guarantee of getting the global minimum
- Try a variety of parameters to see the convergence result
- Interpret the parameters. Generate a sensible grid of parameters to explore.