MFE230I Section 1

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[1] Based on notes by Mohammad Rezaei, Paulo Manoel, David Echeverry, Jiakai Chen and Christoph Kröner. Errors are mine.

About this section

- Sections: Fridays 2pm-4pm
- OH: Thursday 2pm-3pm
- 2 GSIs: Yunbo Liu, and me.
- Reviewing key concepts and working out examples

Homework

- Use Python (suggested)
- Submit code and a write-up to Gradescope → please assign correct pages of your answer file to related questions.
- One submission for each group

Valuation

Basic idea: $PV = \Sigma\, Cash\ Flow_t \times DF_t$

First part of the course:

- Rates and discount factors
- Given rates (equivalently DF), figure out the cash flows of securities and price them.

Second part of this course:

- Model interest rates (equivalently DF)

Discount factor

Discount factor $Z(t, T)$

- Value at t of 1 unit of money in date T (so $t \leq T$)
- In fixed income markets (i.e. bonds), discount factors are the "real prices"
- Effective annual rates and annual percentage rates are only *representations* of discount factors.

Discount factor

- k times compounded interest rate - monthly rates, semi-annual rates, annual rates.

$$Z(t,T) = \frac{1}{\left(1 + \frac{r_k(t,T)}{k}\right)^{k\times(T-t)}}$$

- Continuously compounded interest rate

$$Z(t,T) = e^{r\times(T-t)}$$

Discount factor

Problem: If the semi-annually compounded rate is 8%, calculate the monthly compounded rate and the continuous rate.

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7.870%, 7.844%

Spot Rate & Forward Rate

- Rates varies with terms, time, and compounding rules, but $Z(t, T)$ is unique for given (t, T). So DF is the key to the calculation of interest rates.
- Always have functions converting rates to DF and vice versa when coding.
- Spot rate: $r(t, T) = -\frac{1}{T-t} \ln Z(t, T)$
- Forward rate: $f_k(t, T_1, T_2)$, $f(t, T_1, T_2)$
- Spot rate is the forward rate between today and T. i.e. $r(t, T) = f(t, t, T)$
- DF is the key.

Forward Rate

$$
\begin{aligned}
Z(t, T_2) &= e^{-r(t,T2)\times(T_2-t)} \\
&= e^{-r(t,T1)\times(T_1-t)} \times e^{-f(t,T_1,T2)\times(T_2-T_1)} \\
&= Z(t, T_1) \times e^{-f(t,T_1,T2)\times(T_2-T_1)} \\
\frac{Z(t, T_2)}{Z(t, T_1)} &= e^{-f(t,T_1,T2)\times(T_2-T_1)} \rightarrow Z(t, T_1, T_2)
\end{aligned}
$$

Instantaneous forward rate

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Instantaneous forward rate and forward rate.

$$f(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t, T) dT$$

Interpretation: forward rate between T_1 and T_2 is the average of instantaneous forward rates between T_1 and T_2.

Spot rates

The spot rate is the forward rate between today and T, so

$$r(t, T) = f(t, t, T) \tag{1}$$

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$$r(t,T) = f(t,t,T) \tag{1}$$

This implies that spot rates are also averages of forward rates:

$$r(t,T) = \frac{1}{T-t}\int_t^T f(t,\tau)d\tau \tag{2}$$

Interpretation: the spot rate is the average of instantaneous forward rates between t and T.