Name: ______________________

<u>**MFE 230I Sample Midterm Exam**</u>

Note: Unless otherwise specified in a particular question, assume that all coupon bonds make semi-annual payments, all interest rates are quoted as APRs (compounded semi-annually), and all bonds have a face value of \$100.

1. (Short answer questions) **Assume annual coupon payments and annual compounding in this question, unless specified otherwise.**

   (a) A 2-year bond with a 3% coupon rate has a yield to maturity of 6%. What is its price?

   (b) Suppose that in addition to the bond in 1a, there is also another 2-year bond, this one with a 5% coupon rate and a yield of 6.2%. What is the 2-year spot rate?

   (c) A colleague tells you, "If the forward curve is always upward sloping, the spot curve must also be upward sloping." Is this statement true or false? Explain your answer.

   (d) Your colleague also says, "If the spot curve is always upward sloping, the forward curve must also be upward sloping." Is this statement true or false? Explain your answer.

   (e) Assume continuously compounded zero-coupon bond yields at a particular date t are given, for maturities $T - t \leq 5$, by

   $$r(t,T) = 4\% + [(T-t) \times 0.3]\,\%.$$

   What is the instantaneous 3-year forward rate, $f(t, t+3)$?

   (f) If the 6m, 12m and 18m par yields are 5%, 6% and 7% respectively, what are the 6m, 12m and 18m spot rates?

   (g) Does the swap rate for a swap between two riskless counterparties equal the (riskless) Treasury par rate? Explain your answer.

2. (Duration/convexity) The yield curve is currently flat at 4% for all maturities.

   (a) Calculate the modified duration of a 5% coupon bond with maturity 2 years.

   (b) Calculate the convexity of this bond.

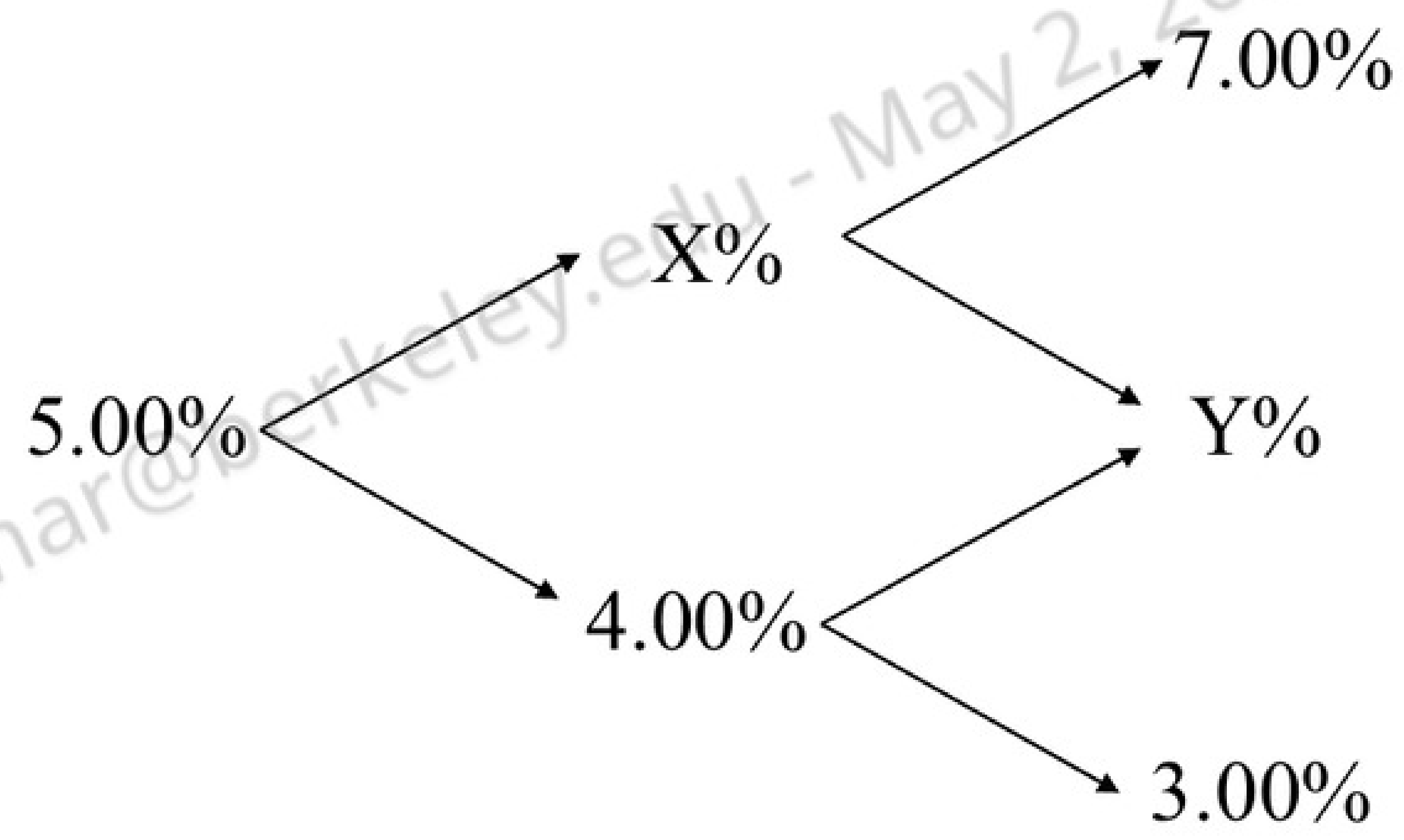
   (c) Using the duration and convexity measures you just calculated, estimate the percentage change in price of the 2 year bond if interest rates increase by 0.25%.

   (d) You own one of the 2-year bonds, and want to hedge it as well as possible against (parallel) interest rate movements. The assets available for hedging are zero-coupon bonds with maturities 0.5, 1.0 and 1.5 years. What hedging position (in some or all of the three zeros) would you set up, given that you'd like to rebalance your portfolio as little as possible over the next two years?

(e) What is the Macaulay duration of a two-year "double inverse floater", whose coupon rate each period is set equal to 20% minus *double* the current 6-month rate? I.e.,

$$\text{Coupon payment}_i = 0.5 \times 100 \times (20\% - 2r_{i-1})\,.$$

3. (Binomial models) The following tree shows the possible movements in the 6-month interest rate over the next year (each period in the tree = 6 months):



The risk-neutral probability of an upward shift in interest rates is 0.42 in the first period and 0.75 in the second period. You are given the following zero-coupon bond prices (face value = \$100):

Maturity	Price
6 months	\$Z
1 year	\$95.198
18 month	\$92.768.

(a) Calculate the values X, Y and Z.

(b) What is the value of an 18 month bond with coupon rate 5%, callable at par any time between now and maturity?

(c) A 12-month Asian futures option has terminal price at date 12m

$$\$1,000,000 \times \max(\bar{r} - K, 0)\,.$$

where $K = 0.05$, and $\bar{r}$ is the average of the 6 month rate at dates 0, 6m and 12m. This is a futures contract, and as with any futures contract, no cash initially changes hands. The contract is marked to market every 6 months. What is the current futures price of this contract?

(d) What position do you need to take today in this futures contract to hedge the interest rate risk of the callable bond in (b)?

4. (True/false) For each of these questions, is the statement true or false? Explain your answer (most points will be for the explanation).

   (a) You possess a callable bond issued by a corporation with zero risk of default. A severe market event causes a sudden rise in interest rates (parallel shift) combined with a sudden rise in interest rate volatility. The overall effect of this event may be either to increase or to decrease the value of the bond, depending on the relative size of the interest rate shift versus that of the volatility shift.

   (b) The effective duration of an IO (Interest Only) stripped mortgage-backed security can be either positive or negative, depending on the level of interest rates.

   (c) Holding all else equal, and ignoring default, an increase in interest rate volatility will increase the expected life of a callable bond (i.e., the expected time until the bond either matures or is called, whichever comes first).

   (d) Long-maturity coupon bonds can be more or less volatile than short-maturity coupon bonds.

   (e) Long-maturity zero-coupon bonds can be more or less volatile than short-maturity zero-coupon bonds.

5. (Duration/convexity). **Assume annual coupon payments and annual compounding in this question, unless specified otherwise.**

   In parts 5a–5c, assume yields on bonds of all maturities are 8%.

   (a) What is the Macaulay duration of a 2-year 6% coupon bond?

   (b) What is the convexity of the bond in 5a?

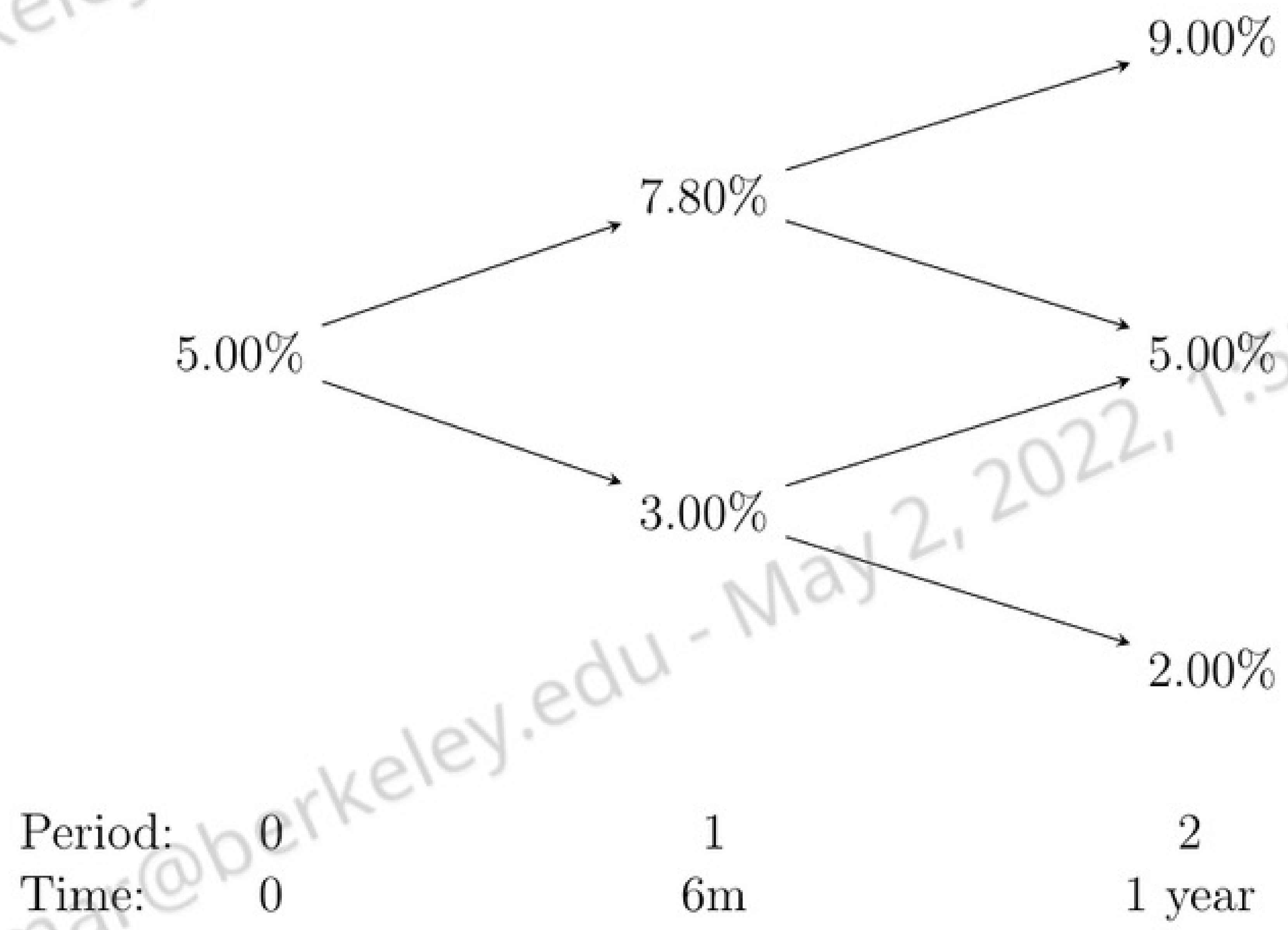
   (c) What is the dollar duration of a newly issued forward contract to buy the bond in 5a in one year? [To clarify, when you buy the bond, it will have 1yr remaining].

   (d) Your portfolio contains a PO (principal-only) stripped mortgage-backed security backed by a pool of fixed-rate mortgages. It also contains a coupon bond whose (fixed) payments are exactly equal to the payments that the PO security would make in the absence of any prepayment on the underlying mortgage pool. Which of these two securities has a higher effective duration?

   (e) A *growing annuity* will pay \$1 in one year, $\$(1+g)$ in two years, $\$(1+g)^2$ in three years, ..., $\$(1+g)^{T-1}$ in T years. Derive a closed-form expression (i.e., one that does not involve summing one term for each cash flow) for the Macaulay duration of this security if its yield (annually compounded) is y.

   (f) What is the limit of the expression you calculated in 5e as $g \to y$?

6. (Valuation) The following tree shows the possible movements in the 6-month interest rate over the next year. The risk-neutral probability of an upward shift in interest rates is 0.5 in each period:



(a) Consider a puttable bond. The bond has a face value of \$100, a 6% semi-annual coupon, and a stated maturity of 1.5 years. Investors have the option to put (sell) the bond to the issuer for par value on any coupon date (immediately after the coupon is paid). What is the value of the bond today?

(b) What is the time 0 value of the fixed receiver's position in a 1-year, 6%, plain vanilla, semi-annual interest rate swap with \$100 notional par amount?

(c) Suppose the swap is cancelable, at no cost, at the option of the party paying fixed, at either time 0 or time 0.5 (immediately after the swap payment). What is the value of this cancelable swap from the viewpoint of the party receiving fixed?

(d) Suppose instead that the swap is cancelable, at no cost, at the option of the party *receiving* fixed, at either time 0 or time 0.5 (immediately after the swap payment). What then is the value of this cancelable swap from the viewpoint of the party receiving fixed?