- Definition of duration
 D_{Mac} , D_{mod} , $D_{\$}$ and DV01 (Dollar Value of 1 Basis Point)
- Assuming annual coupon and compounding, the yield on a bond solves the following equation

$$P = \sum_{t=1}^{T} \frac{e^{du}c}{(1+y)^{t}} + \frac{1}{(1+y)^{T}}$$

 Macaulay duration: The average time of payment weighted by PV of each payment, then normalized by price

$$D_{Mac} \equiv \frac{1}{P} \left[\sum_{t=1}^{T} t \times \frac{c}{(1+y)^t} + T \times \frac{1}{(1+y)^T} \right]$$
$$= -\frac{1+y}{P} \frac{dP}{dy} = -\frac{dP/P}{d(1+y)/(1+y)}$$

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pankaj_kumar@berkeleyMacaulay duration

$$P(T) = \frac{c}{y} \left[1 - \frac{1}{(1+y)^T} \right] + \left[\frac{1}{(1+y)^T} \right]$$

By the formula of power series:
$$P(T) = \frac{c}{y} \left[1 - \frac{1}{(1+y)^T} \right] + \left[\frac{1}{(1+y)^T} \right]$$

$$D_{Mac} = \frac{(1+\frac{1}{y})c[(1+y)^T - 1] + T(y-c)}{c[(1+y)^T - 1] + y}$$

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Modified Duration & Dollar Duration pankaj_kumar@berk

Modified Duration & Dollar Duration
$$D_{\text{mod}} = \begin{cases} \frac{D_{\text{mac}}}{(1+y)} & \text{(annual compounding)} \\ \frac{D_{\text{mac}}}{(1+y/n)} & \text{(rotinuous compounding)} \end{cases}$$

$$D_{\$} = D_{\text{mod}} \times \frac{P}{100} \approx -\frac{1}{100} \times \frac{\Delta P}{\Delta y}$$

$$D_{\text{eff}} = -\frac{1}{P} \frac{dP}{dy}$$

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Convexity

Interest rate sensitivity of duration.

$$C = \frac{1}{P} \frac{d^2 P}{dy^2}$$

$$\frac{\Delta P}{P} \approx -D_{\text{mod}} \times \Delta + \frac{1}{2}C\Delta y^2$$

$$\frac{\Delta P}{P} \approx -D_{\text{mod}} \times \Delta + \frac{1}{2}C\Delta y^2$$

$$\frac{\Delta P}{P} \approx -D_{\text{mod}} \times \Delta + \frac{1}{2}C\Delta y^2$$

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Applications: immunization and hedging W.edu-May 2, 2022, 1:49:08 AM PDT pankaj_kumar@berl

Immunization - insurance

pankaj_kumar@berkeley.edu - May 2, 2022, 1:49:08 AM Pr match interest rate risk of assets and liabilities

Two ways of thinking:

- Make the portfolio duration zero.
- Match the dollar duration.

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pankaj kumar@berkeley Hedging Example

- Semi-annual coupon bond: 2 years, 5% coupon, 4% flat yield curve.

 Hedging by zero-coupon bonds with mathematical to the second secon Hedging by zero-coupon bonds with maturities 0.5, 1 and
 1.5 years , and 1.49:08 pankaj kumar@berkeley.edu - May 2, 2022, 1.49:08
 - How to design your position?

See excel.

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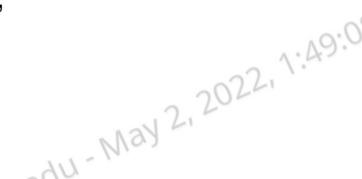
Calculate duration by forming portfolio 2022, 1:49:08 AM PDT

Sample Midterm Q5(3)

- 2-year 6% annual coupon bond. Discount rate 8%
- What is the dollar duration of a newly issued forward pankaj_kumar@berkeley.edu - May 2, 2022, 1:49:08 AM Pr contract to buy the bond in 5a in one year? [To clarify, when you buy the bond, it will have 1yr remaining]

Sample Midterm Q5(3)

- 2-year 6% annual coupon bond. Discount rate 8%
- What is the dollar duration of a newly issued forward contract to buy the bond in 5a in one year? [To clarify, when you buy the bond, it will have 1yr remaining]
- forward contract == long a 2-year zero coupon bond with face value N(1+6%) and short 1-year zero coupon bond to make $CF_0 = 0$
- $ZCP_2 = 90.88$, $D_{\$2} = 2/1.08 \times 90.88/100 = 1.68$, $D_{\$1} = 1/1.08 \times -90.88/100 = -0.84$ $D_{\$} = 1.68 - 0.84 = 0.84$

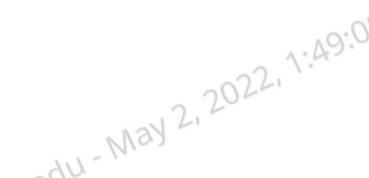


Duration

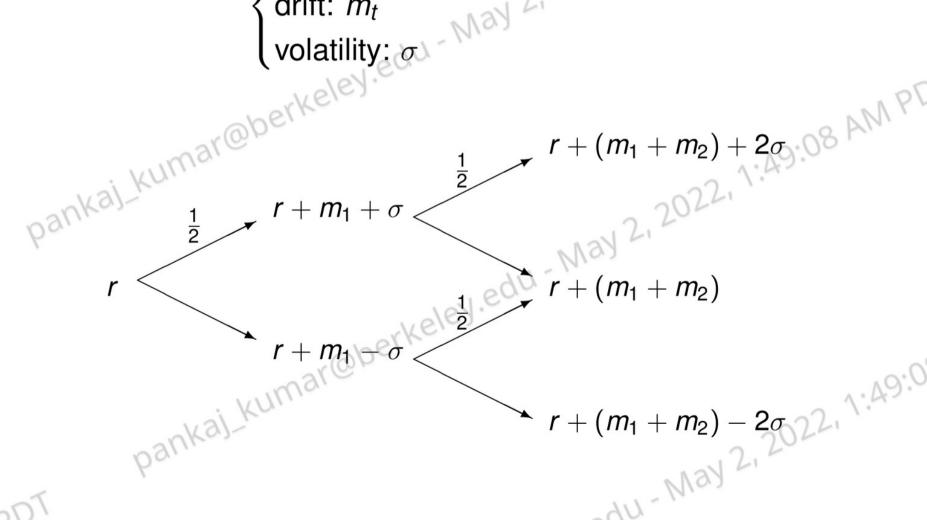
Ho and Lee Binomial Model 2022, 1:49:08 AM PDT

Ho and Lee propose a binomial model for interest rate dynamics where

- interest rate volatility is constant over time (measured in basis points),
- the drift of the short rate evolves over time (but not across
- transition probability under the constant $\frac{1}{2}$ for all times and states. the transition probability under the risk-neutral measure is



Pankaj kumar@berkel Binomial Ho-Lee tree $\begin{cases} \text{initial one year rate: } r \\ \text{drift: } m_t \\ \text{volatility: } \sigma \end{cases}$



Calibration exercise for Ho-Lee

1:49:08 AM PDT Calibration is choosing the model's free parameters, typically in a way to match observed prices

Calibration is choosing a way to match observe		free parameters, typically in
ZCB prices:	redu-N	
ark.	Maturity	Price
- ar@per	1 year	95
i Kuma	2 years	90
pankaj_kumar@berr	3 years	85 2022
Par	4 years	Price 95 90 85 80 Nay 21 2022
• valetility of the (open	م مرمور می درالم در در	aughtal) and water to

- volatility of the (annually compounded) one-year rate is pankaj_kumar@berkele 1.5% per year MAN 7. 2022. 1:49:0
- See excel.

Pricing with interest rate trees

After calibration of the model, pricing is similar to pricing derivatives on stocks

- start from payoff and work backwards,
- only difference is that discounting is always with the

For the following exercises suppose the calibrated interest rate tree is

year	0	1	2	321
r	5%	4%	3%	2%
		6%	5%	4%
	rope	111	7%	6%
Knws	//			8%



Zero coupon bonds

Construct the tree for a 4 year ZCB.

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Construct t	he tree f	or a 4 ye	ar ZCB.		2211:49	
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		orkeley	83.99	90.71	96.15	MP
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Zero coupon bonds

Construct the tree for a 4 year ZCB.

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Construct the tree	for a 4 year z	ZCB.	022, 1:49.	
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Z(t,4) pankaj_kumar@ pankaj_k pankaj_k	98.04	$=\frac{100}{1.02}$	May	
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$$98.04 = \frac{100}{1.02}$$

$$94.27 = \frac{1}{1.03} \frac{1}{2} (98.04 + 96.15)$$
aj kumar $\frac{1}{1.03} \frac{1}{2} (98.04 + 96.15)$

Options & Callable bonds

What is the price of the 4 year zero coupon bond if the issuer has the right to buy it back 2 years from now for 92? Is the price bigger or smaller?

- a long zero coupon bond and a short two year call option on that bond
- price of callable bond = price of ZCB value of call option

What is the price of a 2-year American put option on this bond?

What is the price of a 2 year American put option but instead of trading the 4 year bond when exercising, it gives a bond with 2 May 2.2022, 1:49:0 years of maturity left independently of the exercise date.



pankaj_kumar@berkele\$wap and Swaption

Sample Midterm Q6 erkeley.edu - May 2, 2022, 1:49:08 AM PDT

pankaj_kumar@berkeley.edu - May 2, 2022, 1:49:08 AM Pr AII - NAZII 7 - 2022, 1:49:0