MFE230E Problem Set 6

Due May 5 6:00pm via bCourses

You may use the `arch` package (https://pypi.org/project/arch). It is not included in Anaconda and you need to install it yourself.

1. Download daily prices of the S&P500 index and GE from starting in 1/1/1970 and construct daily, monthly, and annual log returns from prices.

   (a) Compute the autocorrelations of daily, monthly, and annual log returns up to lag 5.

   (b) Compute the average volatility over the sample using the following volatility measures (all expressed in annual units):

      i. Average annualized volatility of *daily* returns.

      ii. Average annualized volatility of *monthly* returns.

      iii. Average volatility of *annual* returns.

   Why are the sample averages different?

   (c) Next, compare the time series of

      i. Volatility of *monthly* returns.

      ii. Monthly volatility based on daily returns in each month.

   Plot these monthly series and compute their means, variances and correlation.

   (d) Estimate an AR(1) model for realized monthly volatility. What features of the data and model are noteworthy?

   (e) Using the AR(1) model, compute one-month-ahead forecasts of the realized volatility. Plot realized volatility and forecasted realized volatility. (For simplicity, us the forecasts from the in-sample AR(1) regression instead of true out-of-sample forecasts.)

   (f) Compute the Mean-Square-Error (MSE) of the AR(1) forecasts $E\left[\left(\sigma_{t+1} - \hat{\sigma}_{t+1|t}\right]^2\right)$, where σ_{t+1} is the realized value of the volatility at $t+1$ and $\hat{\sigma}_{t+1|t}$ is its forecast given information at time t.

2. (a) Estimate a GARCH(1,1) model for monthly returns. What features of the data and model are noteworthy?

   (b) Regress realized monthly volatility on conditional GARCH(1,1) volatility. Plot realized volatility and conditional volatility.

   (c) Compute the RMSE of the GARCH(1,1) forecasts.

   (d) Find the "best" GARCH(p, q) model.

3. Repeat the above exercise for the GJR-GARCH(1,1) model

$$\sigma_t^2 = \omega + \delta\, \sigma_{t-1}^2 + \alpha\, \epsilon_{t-1}^2 + \phi\, \epsilon_{t-1}^2 I_{\epsilon_{t-1}<0}.$$

4. Compare the RMSE and MAE of the AR(1), GARCH(1,1) and GJR-GARCH(1,1) models.

Optional: The `arch` package includes other xGARCH specifications. Find the "best" model among all applicable specifications that are part of the `arch` package.