MFE 230E Problem Set 7 - Solutions

Spring 2020

Note: This document only presents answers. Cf. the accompanying Jupyter notebook for details and the corresponding code. Please email mfe230e@gmail.com if there are any corrections you would like to make to these solutions.

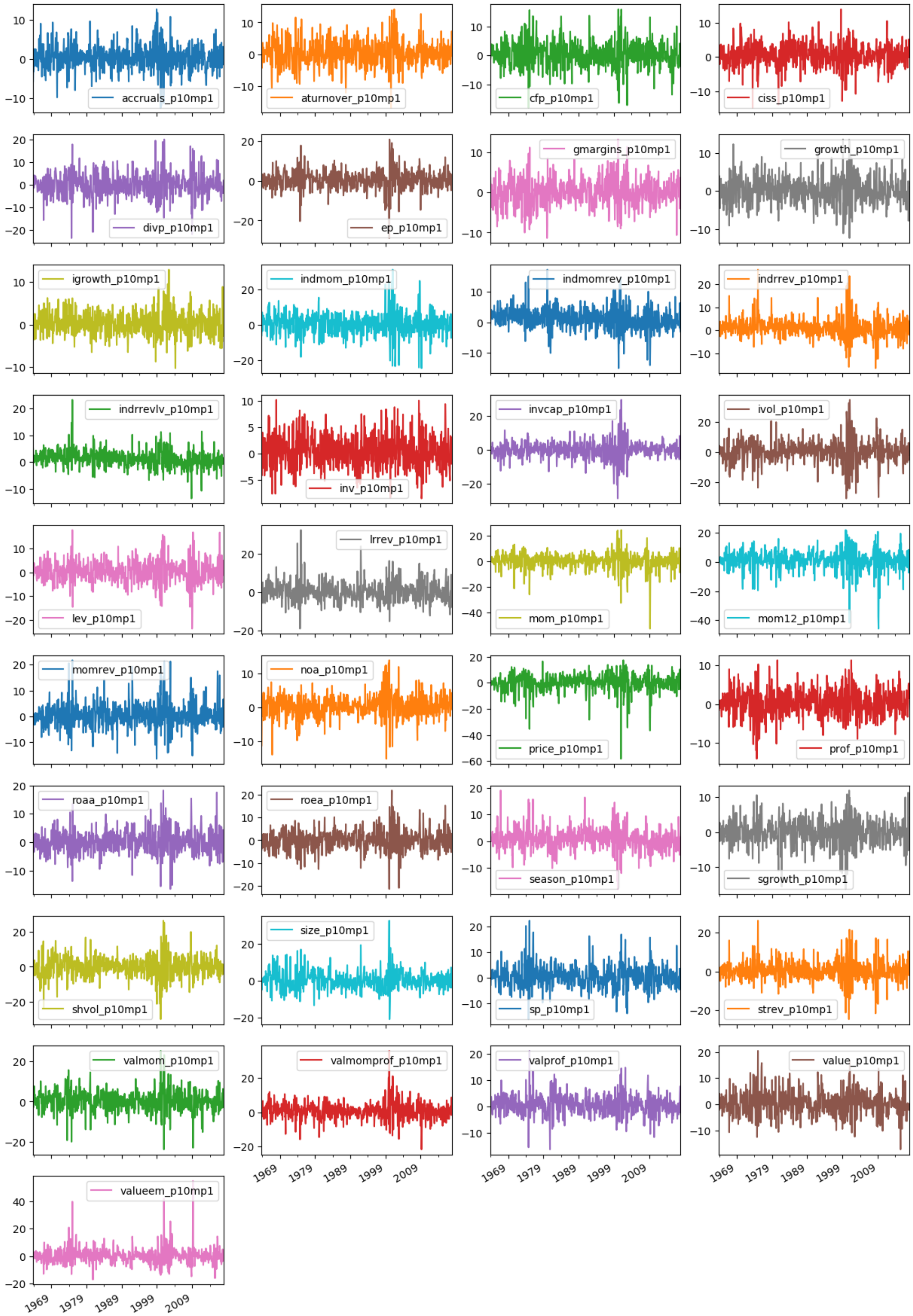
Problem 1

(a) You have many ways to summarize the results. An example is to show means, variances, and Sharpe Ratios for simple long-short portfolios, which are obtained by subtracting the returns on the lowest decile (p1) from the returns on the highest decile (p10), for each signal. The result is shown in Table 1. The time-series of excess returns on those long-short portfolios are also plotted in Figure 1.

	accruals	aturnover	cfp	ciss	divp	ep	gmargins	growth
mean	0.35%	0.41%	0.43%	0.48%	0.18%	0.57%	0.01%	0.29%
std	3.14%	3.83%	4.37%	3.30%	5.08%	4.69%	3.37%	3.46%
SR	0.11	0.11	0.10	0.14	0.04	0.12	0.00	0.08
	igrowth	indmom	indmomrev	indrrev	indrrevlv	inv	invcap	ivol
	0.38%	0.47%	1.17%	1.01%	1.28%	0.47%	0.12%	0.54%
	2.71%	6.21%	3.44%	4.12%	3.05%	3.06%	5.01%	7.16%
	0.14	0.08	0.34	0.24	0.42	0.15	0.02	0.08
	lev	lrrev	mom	mom12	momrev	noa	price	prof
	0.26%	0.41%	0.35%	1.25%	0.46%	0.38%	-0.01%	0.36%
	4.63%	5.07%	6.25%	6.88%	4.84%	3.28%	6.80%	3.38%
	0.06	0.08	0.06	0.18	0.10	0.12	0.00	0.11
	roaa	roea	season	sgrowth	shvol	size	sp	strev
	0.21%	0.08%	0.79%	-0.05%	-0.02%	0.29%	0.52%	0.36%
	4.08%	4.39%	3.94%	3.64%	5.96%	4.80%	4.28%	5.27%
	0.05	0.02	0.20	-0.01	0.00	0.06	0.12	0.07
	valmom	valmomprof	valprof	value	valueem			
	0.51%	0.83%	0.75%	0.48%	0.40%			
	5.05%	4.83%	3.81%	4.56%	5.86%			
	0.10	0.17	0.20	0.11	0.07			

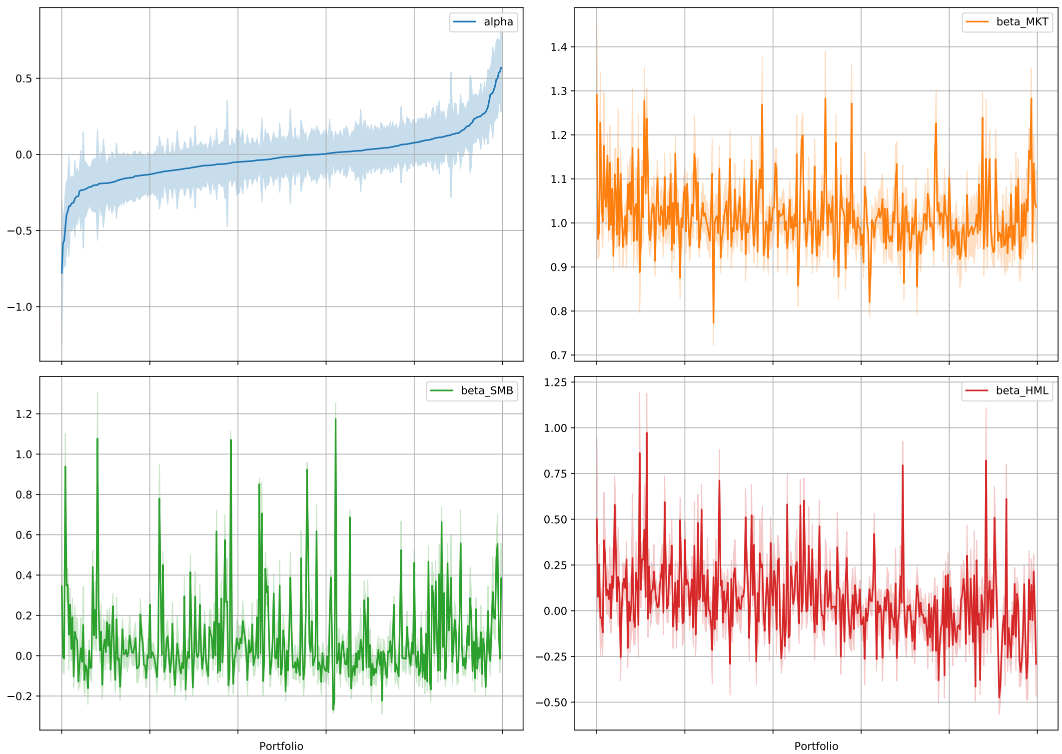
Table 1: Summary statistics for long-short portfolios (p10-p1)

Figure 1: Excess returns on long-short portfolios.



(b) The pricing error α_i (in %) and the factor loadings $\boldsymbol{\beta_i}$ for the FF3 and FF5 models are plotted on Figures 2 and 3. They are obtained from time-series regressions, given that the factors are excess returns. Those results are summarized in Tables 2 and 3. We obtain RMSEs for α of 0.18% and 0.16%, respectively, which does not seem too bad given the number of test assets. We could also run a GRS test, which would most likely reject the model. Note that the adjusted-R^2 are still quite large, even with all those tests, as shown in Figure 4. Finally, Figure 5 shows predicted vs. actual excess returns using FF3 and FF5 factors. This suggests that even though the Fama-French factors do capture an important part of the variations in excess returns across all assets, there still understandably remains somewhat large asset pricing errors for some of them.

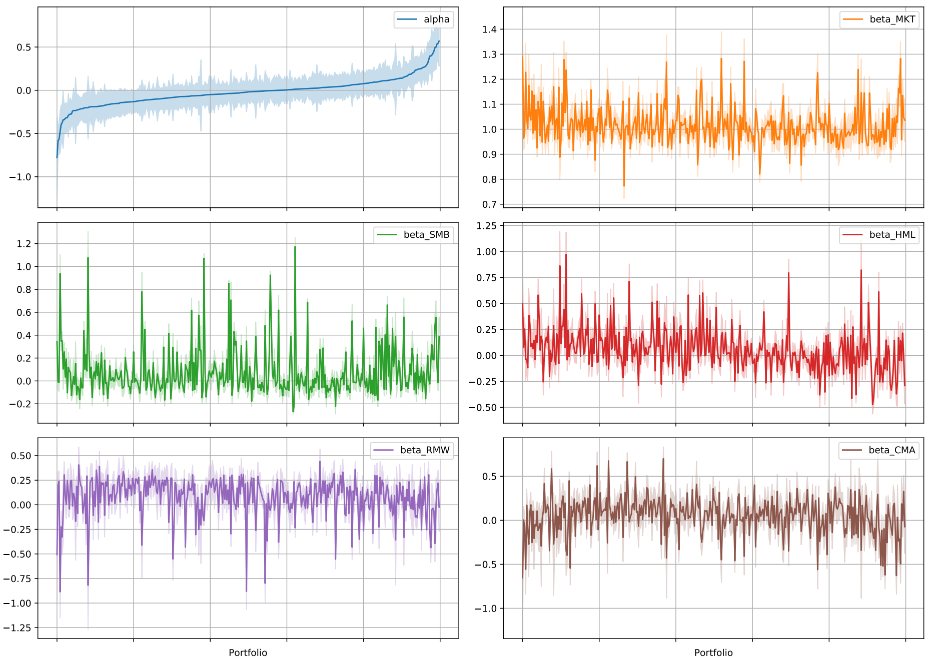
Figure 2: Pricing errors and risk exposure with FF3 factors



	α	std(α)	β_{MKT}	std(β_{MKT})	β_{SMB}	std(β_{SMB})	β_{HML}	std(β_{HML})
count	370	370	370	370	370	370	370	370
mean	0.02%	0.08%	1.01	0.03	0.06	0.04	0.08	0.06
std	0.18%	0.02%	0.09	0.01	0.23	0.02	0.23	0.02
min	-1.08%	0.02%	0.74	0.00	-0.28	0.01	-0.66	0.01
25%	-0.06%	0.06%	0.95	0.02	-0.08	0.03	-0.04	0.04
50%	0.03%	0.07%	0.98	0.02	-0.02	0.04	0.09	0.05
75%	0.11%	0.08%	1.04	0.03	0.11	0.05	0.20	0.07
max	0.68%	0.17%	1.39	0.10	1.26	0.12	0.93	0.18

Table 2: Summary statistics for FF3

Figure 3: Pricing errors and risk exposures with FF5 factors



	α	std(α)	β_{MKT}	std(β_{MKT})	β_{SML}	std(β_{SML})	β_{HML}	std(β_{HML})
count	370	370	370	370	370	370	370	370
mean	-0.02%	0.08%	1.02	0.02	0.08	0.04	0.06	0.06
std	0.16%	0.03%	0.08	0.01	0.21	0.02	0.21	0.03
min	-0.78%	0.02%	0.77	0.01	-0.27	0.01	-0.47	0.01
25%	-0.11%	0.06%	0.97	0.02	-0.05	0.03	-0.07	0.04
50%	-0.02%	0.07%	1.00	0.02	0.02	0.03	0.04	0.05
75%	0.05%	0.09%	1.05	0.03	0.13	0.04	0.15	0.07
max	0.57%	0.24%	1.29	0.08	1.17	0.12	0.97	0.23

	β_{RMW}	std(β_{RMW})	β_{CMA}	std(β_{CMA})
count	370	370	370	370
mean	0.08	0.07	0.05	0.08
std	0.20	0.03	0.21	0.04
min	-0.89	0.01	-0.65	0.02
25%	-0.01	0.05	-0.06	0.06
50%	0.12	0.06	0.07	0.07
75%	0.22	0.07	0.18	0.09
max	0.44	0.24	0.70	0.30

Table 3: Summary statistics for FF3

Figure 4: Adjusted-R^2

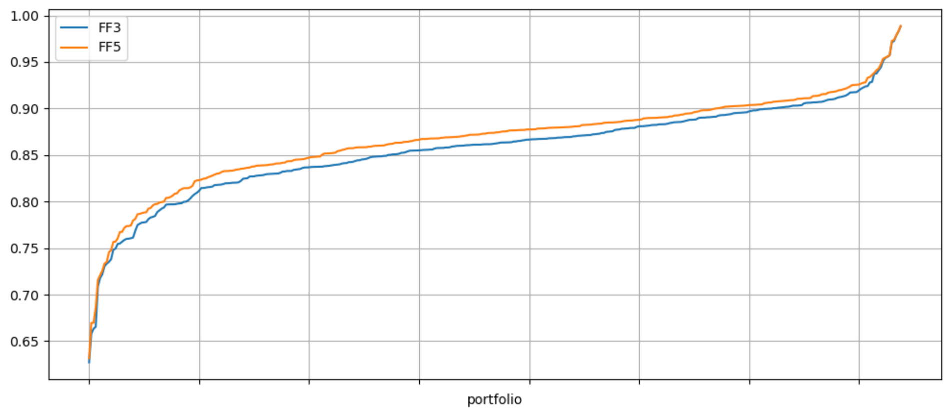
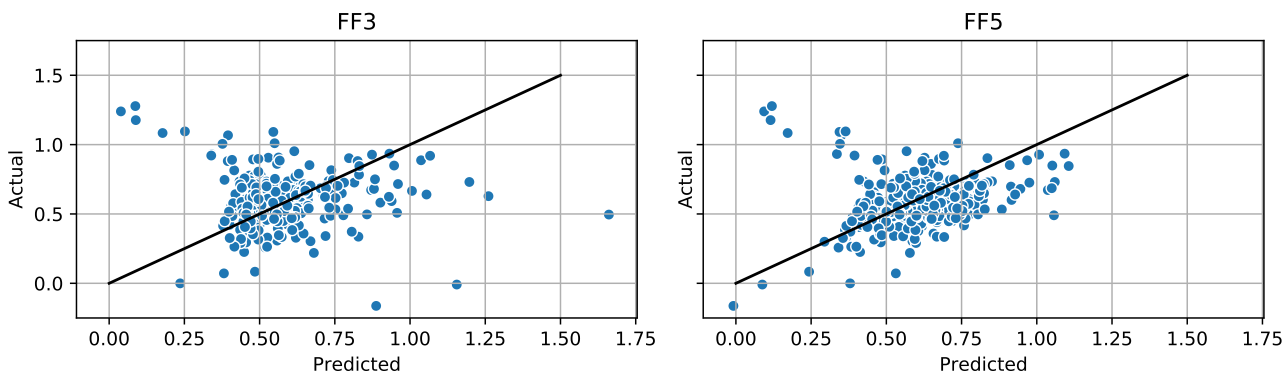


Figure 5: Actual vs. predicted average excess returns (% per month)



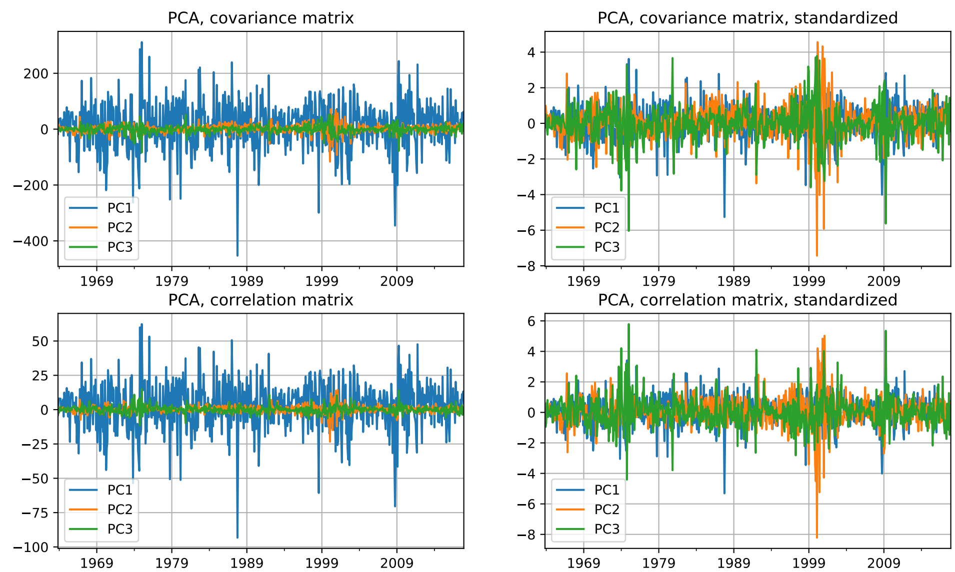
(c) The resulting principal components (PCs) are shown on Figure 6. The figure shows both the PCs themselves, and those obtained after standardizing each by its standard deviation. To get a better sense of how they compare, Figure 7 shows scatter plots of the principal components obtained from the correlation matrix as a function of those obtained from the covariance matrix. Three remarks:

1. The overall evolutions of the principal components in both versions are very similar.

2. If we do not standardize the principal components, those obtained from the covariance matrix are of course much more volatile. This is also confirmed in Table 4, which shows summary statistics for the first three principal components. Note that this can be taken care of by standardizing the resulting principal components, and is therefore not necessarily a big issue. Instead, the main reason behind standardizing the data itself is that without it, the PCs would mostly explain the variations in the most volatile variables.

3. For PC3, the sign is reversed between the two versions. This points to an important fact: principal components are identified only up to their sign. This is because if a given vector v is an eigenvector, then so is $-v$. As a result, we are free to normalize the sign as we see fit. Here, we had not normalized the sign yet to show this point, but in what follows, after the comparison with *statsmodels*, we will take the convention that we want the mean of each principal component to be positive. This means that if the resulting PC has a negative mean, we will simply multiply it by -1. In the case at hand here, the third principal component obtained from the covariance matrix would then be flipped, and its evolution would match that of the one obtained from the correlation matrix.

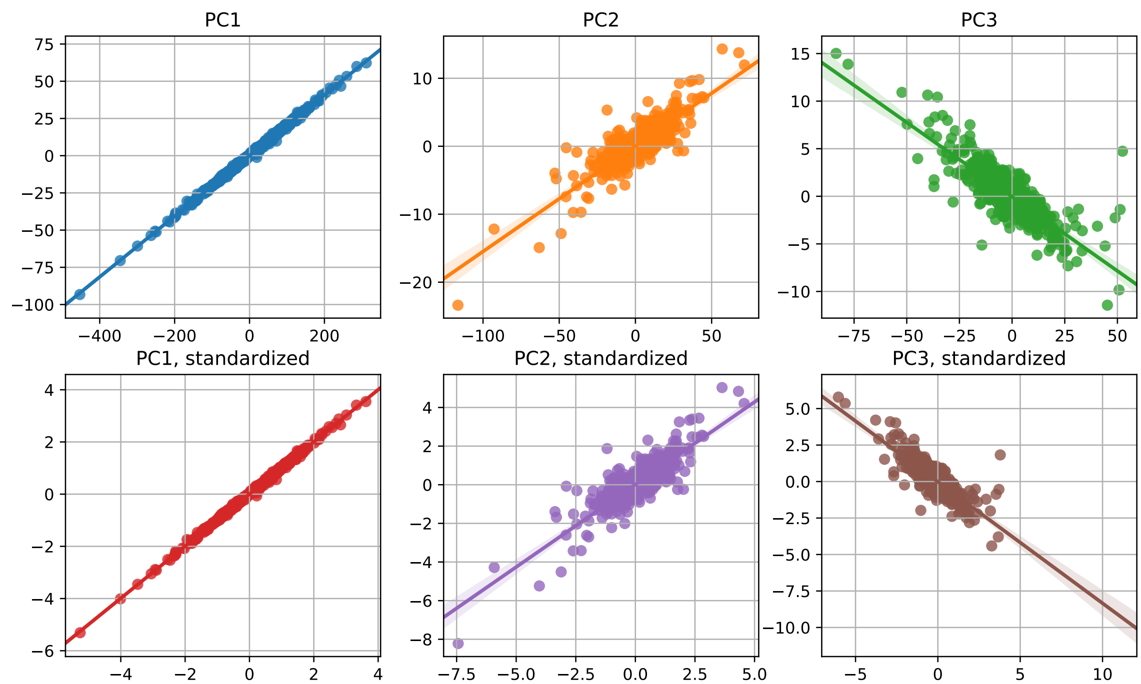
Figure 6: PCA decomposition computed manually



	From covariance matrix			From correlation matrix		
	PC1	PC2	PC3	PC1	PC2	PC3
count	650	650	650	650	650	650
mean	11.21%	1.53%	-0.62%	2.37%	0.27%	0.05%
std	86.11%	15.67%	13.83%	17.57%	2.85%	2.59%
min	-453.15%	-116.47%	-83.55%	-93.29%	-23.39%	-11.44%
25%	-36.88%	-6.20%	-7.11%	-7.47%	-1.11%	-1.44%
50%	17.75%	1.53%	-0.24%	3.61%	0.19%	-0.01%
75%	66.39%	9.53%	6.54%	13.69%	1.54%	1.30%
max	311.66%	71.49%	52.41%	62.31%	14.31%	15.02%

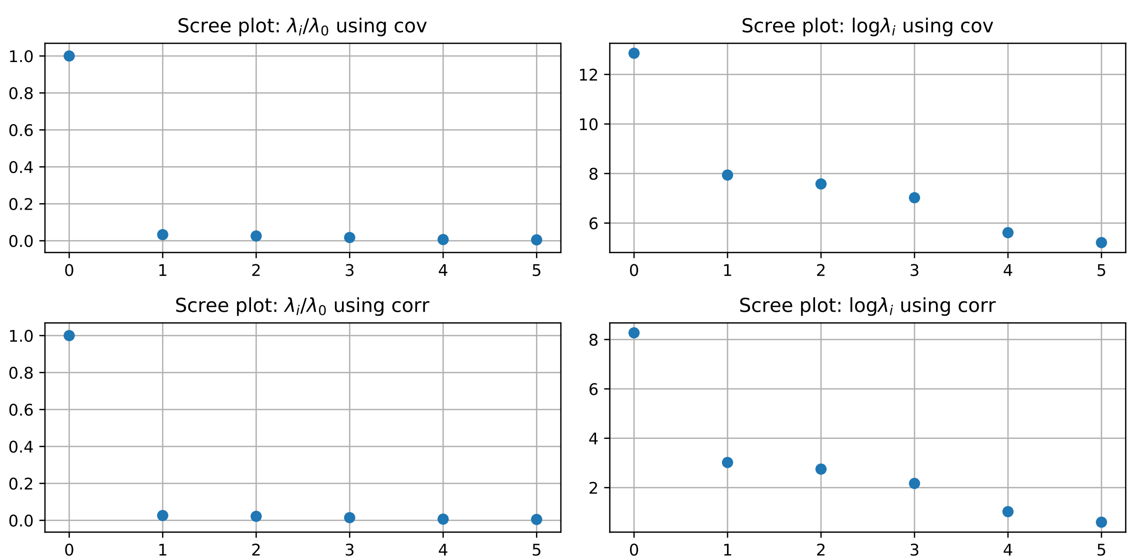
Table 4: Summary statistics

Figure 7: PCA on correlation as a function of PCA on covariance



Finally, Figure 8 shows the scree plots. The ratio of eigenvalues are equal in both cases, even though the values of the eigenvalues themselves are lower when computed on the correlation matrix, as expected.

Figure 8:



(d) The principal components obtained from *statsmodels* are identical to those obtained manually, provided that we set options appropriately. Graphs and statistics are omitted here in the interest of space, but details are available in the attached Jupyter notebook. Here are the main points:

1. By default, *statsmodels* standardizes and demeans the data. To avoid this behavior and deal with the data ourselves: both *standardize* and *demean* should be set to **False**. Note that it is not possible to let *statsmodels* only standardize the data (i.e. *demean* = **False** is ignored if *standardize* is **True**). Once this is done, we can feed either directly the excess returns (to get the equivalent of the manual PCA from the covariance matrix), or the standardized excess returns (to get the equivalent of the manual PCA from the correlation matrix).

2. In addition, *statsmodels* has a *normalize* option, which is set to **True** by default. If this is the case, the factors themselves are normalized to have unit inner product, instead of the weights. To be able to compare the results from *statsmodels* directly to ours, we therefore have to set *normalize* to **False**. Note that even if we do not, the obtained principal components as still perfectly correlated with the ones we obtained, but their scales are different.

3. Once the above is set appropriately: *statsmodels* yield the same principal components as the manual method, but again up to the sign that is not identified. This can be seen in the Jupyter notebook for instance with the correlations between both being always 1 or -1. If we follow the convention mentioned above to normalize the sign, then the results are the same.

In what follows, we focus on the principal components obtained from the standardized excess returns using *statsmodels* with *standardize* = **False**, *demean* = **False**, and *normalize* = **True** (default). We standardize them by their standard deviations, and set the sign of each so that its mean is positive.

(e) To get a sense of which principal components are "significant", let us a look at a few different metrics. First, Table 5 shows how much variance is explained as we increase the number of principal components. Those results suggest that PC1 explains the vast majority of variance (83.6%) with PC2, PC3, and perhaps PC4, helping explaining some more of the variation, marginally. This is confirmed in Figure 9, which shows that the individual R^2 in the individual regression of each data series on an increasing number of principal components, and suggests that PC1 is by far the most important, with PC2 and PC3 increasing the median R^2 some, and the R^2 increasing only slowly and gradually thereafter.

To confirm the picture, and decide whether we should include any principal component beyond PC3, Figure 10 plots the absolute value of the t-statistic for the coefficient on the last included principal component. The t-statistic for the coefficient on PC1 is not included to make the graph readable, but is very large (median around 40, cf. Jupyter notebook). The figure suggests that even though t-statistics are substantially smaller for PC2 and PC3, they are still noticeably higher than for PC4 and beyond. This is also confirmed if we plot the distribution of p-values for the coefficient on the last included coefficient (cf. Jupyter notebook).

In conclusion: our preferred model includes PC1, PC2, and PC3. We will still also experiment with including a few more principal components as a robustness check, and to see whether those might have an interesting economic interpretation.

Last included	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
Share	83.6%	2.2%	1.8%	1.2%	0.5%	0.4%	0.4%	0.3%	0.3%	0.2%
Cum. share	83.6%	85.7%	87.5%	88.8%	89.3%	89.7%	90.1%	90.4%	90.6%	90.9%

Last included	PC11	PC12	PC13	PC14	PC15	PC16	PC17	PC18	PC19	PC20
Share	0.2%	0.2%	0.2%	0.2%	0.2%	0.1%	0.1%	0.1%	0.1%	0.1%
Cum. share	91.1%	91.3%	91.5%	91.6%	91.8%	91.9%	92.1%	92.2%	92.4%	92.5%

Table 5: Variance explained by principal components

Figure 9: R^2 for individual regressions

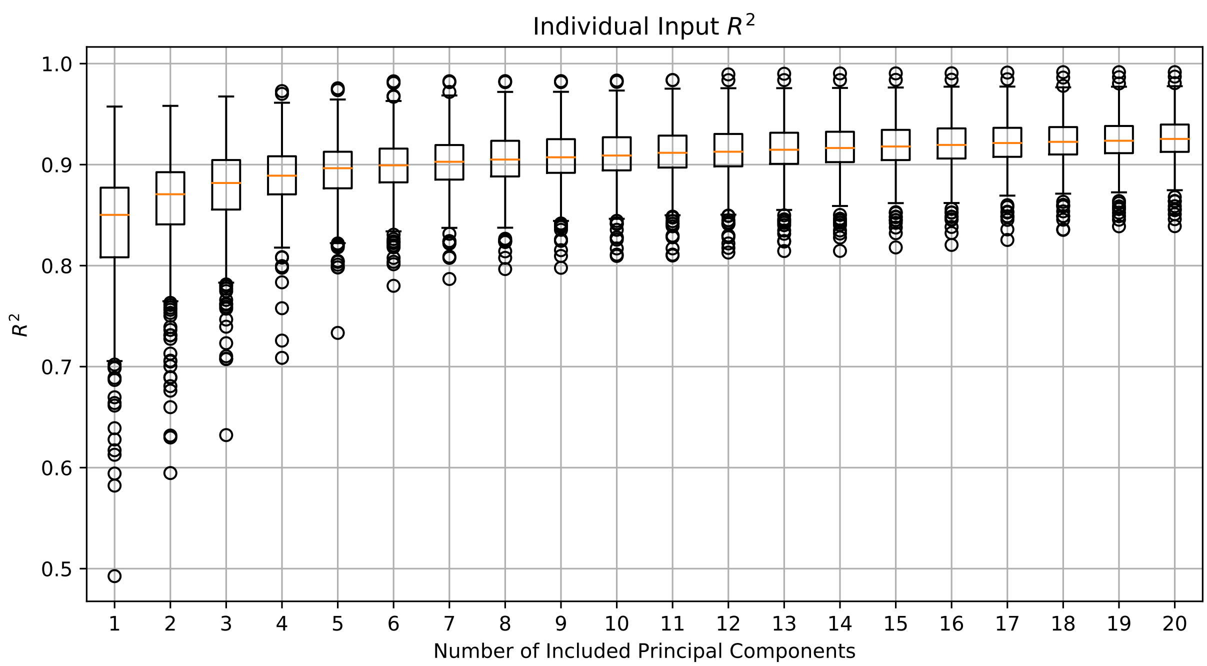
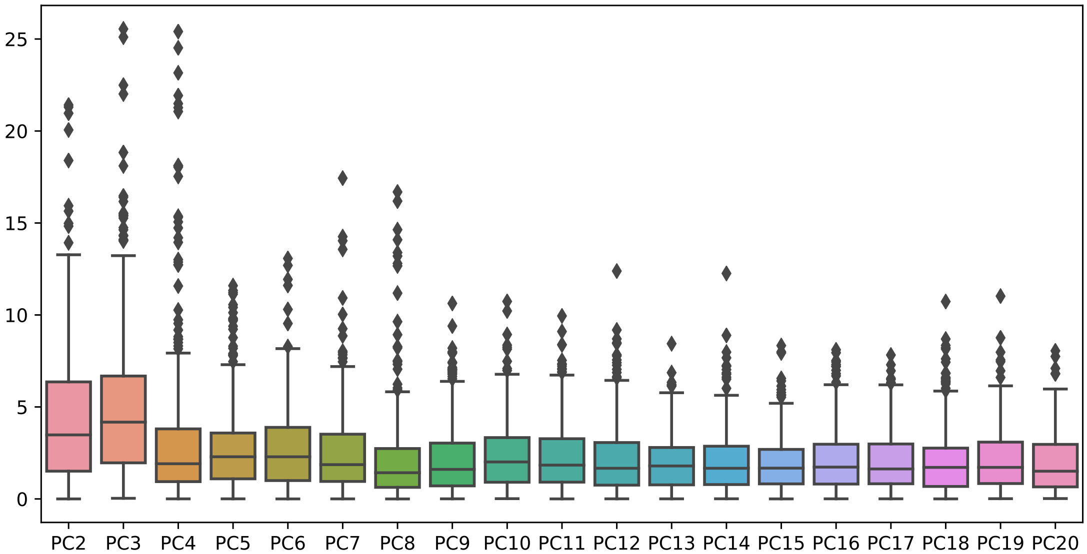


Figure 10: Absolute value of t-statistic on last included principal component



Economic interpretation Finally, let us discuss whether those factors can be given an economic interpretation. As a note of caution, keep in mind that even though we can attach particular apparent economic meaning to some of those principal components, they remain a mere statistical decomposition of the data, and nothing guarantees that the interpretation that we find will remain true moving forward or that they are in fact related on a fundamental level to those underlying economic channels.

With that caveat in mind, Figures 11 to 15 show the loadings of each of the first five principal components on the underlying assets/portfolios, grouped by factor. These suggest the following:

- PC1 loads relatively uniformly on each asset, so that it captures the broad **level** of excess risk premia on all of them. This is reminiscent of the first principal component that we obtained from the Treasury yield curve.

- PC2 loads on average negatively on the portfolios at the two extreme, i.e. as we get closer to p1 and p10 for each factor, but does not load much on median portfolios (around p5). This suggests that PC2 tends to capture the **heterogeneity** in excess returns within each factor with respect to the median portfolio. Note that the loadings of PC2 also increase monotonically with volatility, and tend to rise (although while keeping the inverted-U shape) for factors related to value (*divp*, *value*, *etc.*). This underlines how difficult and imperfect the mapping necessarily is between principal components obtained from a statistical decomposition, and economic variables.

- The patterns for PC3 start showing increasing heterogeneity across factors, but we can note that loadings increase significantly and monotonically with **value**-type factors, and decrease monotonically with **volatility**, **price** and **momentum**.

- Interestingly, the patterns for PC4 are somewhat clear and this principal component seems to capture **momentum** quite sharply. This is evident from the monotonically increasing loadings on any factor including momentum vs. broadly zero loadings on anything else.

- PC5 appears to capture **profitability** in a broad sense, with monotonically increasing loadings on *prof*, *aturnover*, and other related factors. It is also impacted by leverage, with a U-shape pattern that is tilted towards p10.

To get a further sense of the interpretation of the principal components, Table 6 shows their correlation with the main factors used in the literature, namely FF5 and momentum. On one hand, this confirms some of the patterns discussed above. For instance, PC1 is most correlated with the overall level of excess returns captured by Mkt-RF, while PC3 is quite correlated with HML, PC4 with MOM, and PC5 with RMW (which captures robust minus weak profitability). On the other, those results confirm the difficulty of attributing unique and clean interpretation to each. For instance, PC1 and PC2 are also correlated with all other factors, PC3 is in fact even more correlated with SMB, PC4 is also correlated with SMB and CMA although to a smaller extent, and PC5 is almost as correlated with SMB as it is with RMW. Those difficulties are compounded by the fact that the Fama-French factors together with momentum do not necessarily have clear and agreed upon economic underpinnings either, and are themselves to some extent statistical in nature.

Overall, we can attach some economic interpretation to some of the principal factors, but it is absolutely crucial to keep in mind that the mapping is far from perfect, and that there are still first and foremost statistical decompositions of the data over a given period. This could also very well evolve as more data is gathered, underlying the fact the mapping is in no way “structural” or fundamental.

	Mkt-RF	SMB	HML	RMW	CMA	MOM
PC1	0.995	0.294	-0.205	-0.202	-0.340	-0.156
PC2	-0.067	-0.436	0.669	0.398	0.557	-0.095
PC3	-0.050	0.615	0.519	-0.416	0.367	-0.452
PC4	-0.028	0.253	0.116	-0.094	0.200	0.744
PC5	-0.024	0.266	0.096	0.346	-0.065	0.080

Table 6: Correlation between principal components and FF5 + momentum

Figure 11: Loadings of first principal component

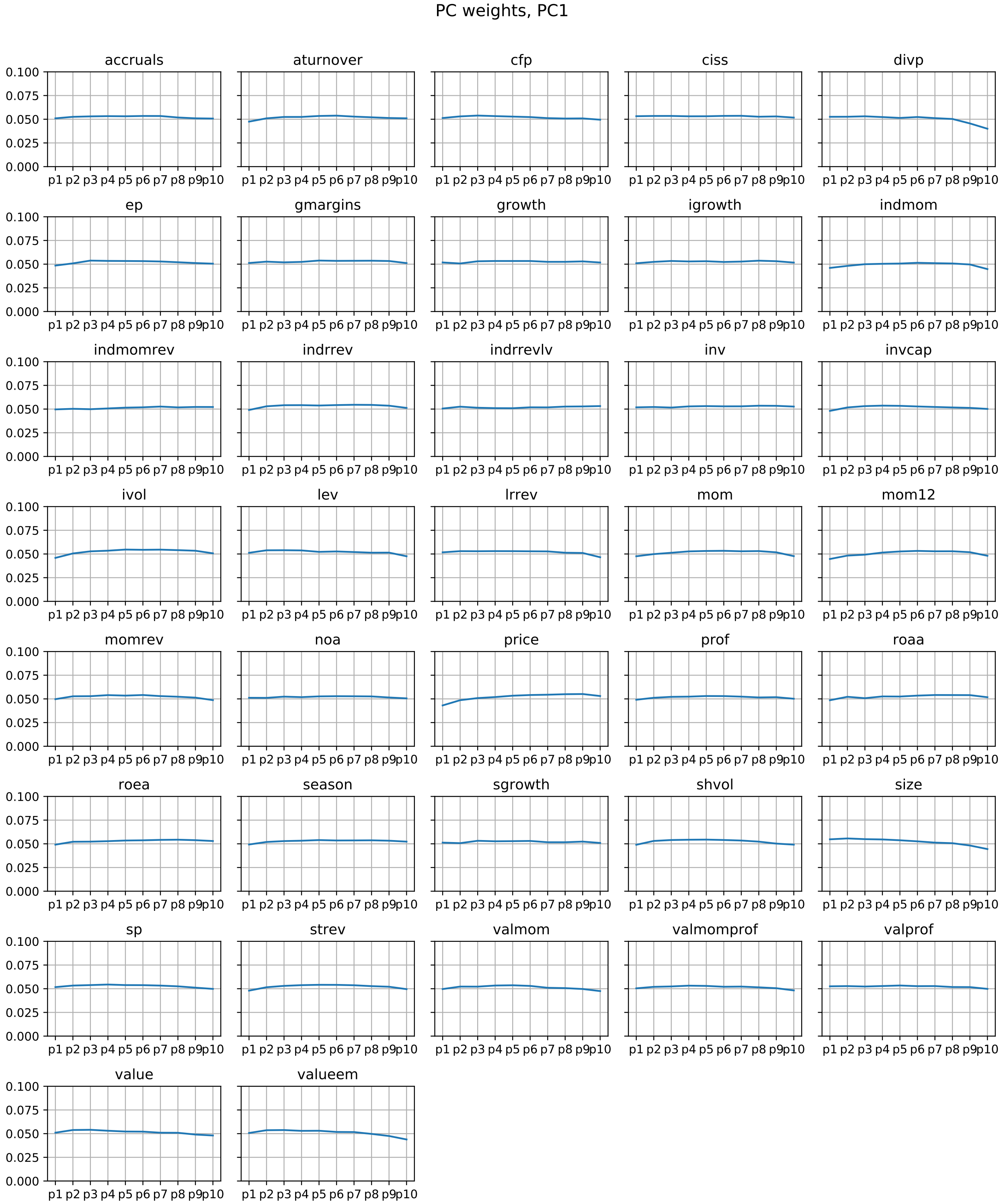


Figure 12: Loadings of second principal component

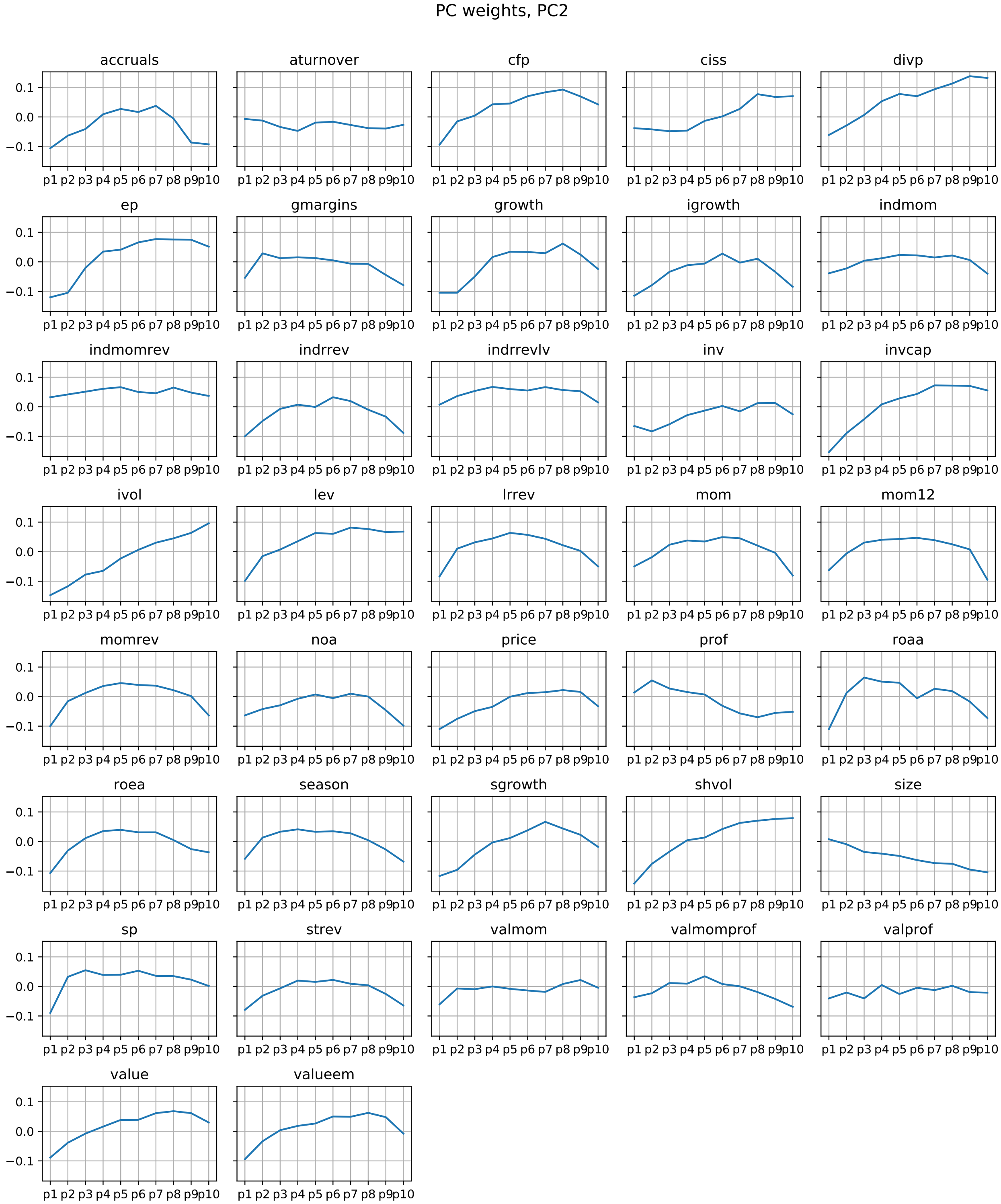


Figure 13: Loadings of third principal component

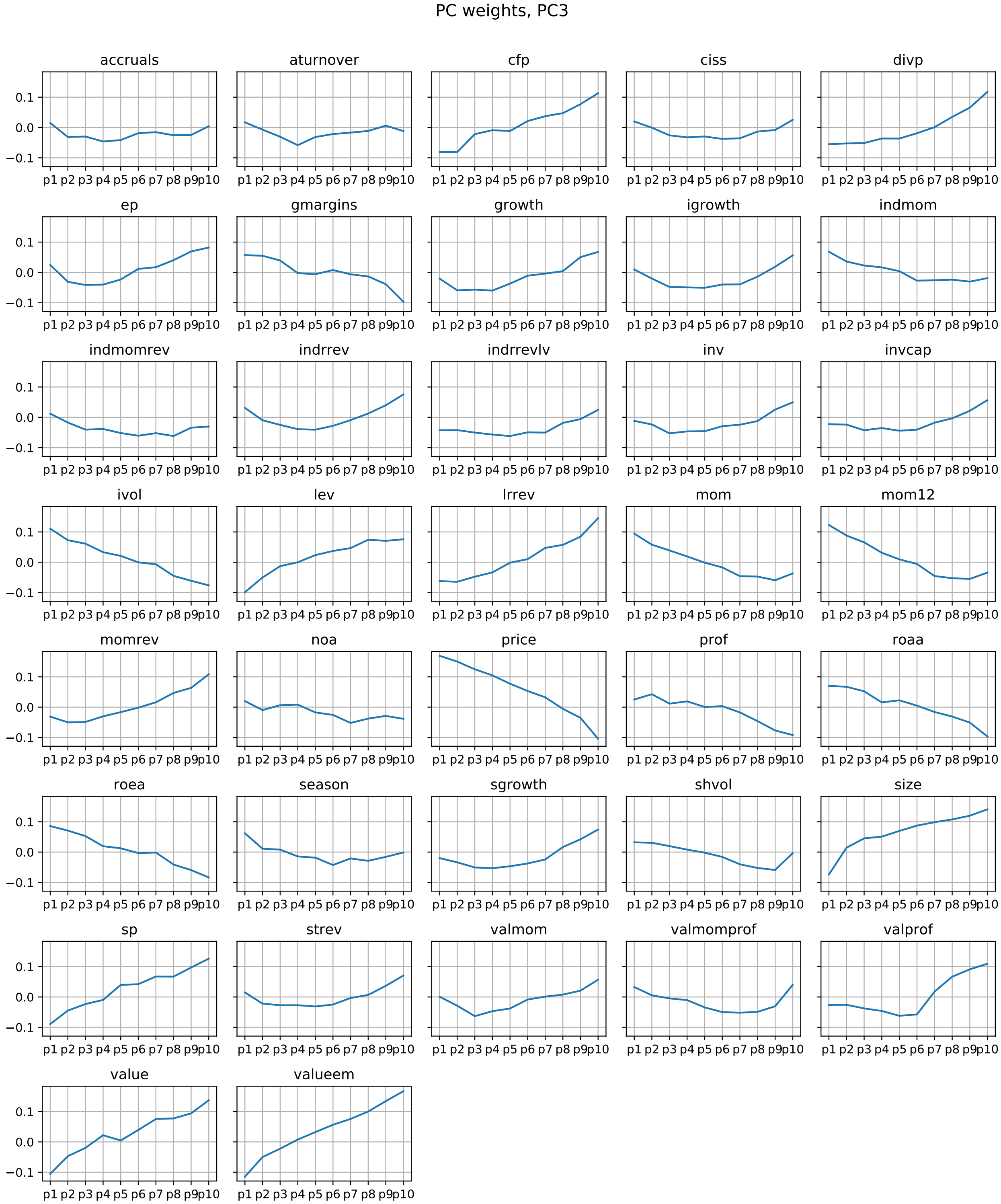


Figure 14: Loadings of fourth principal component

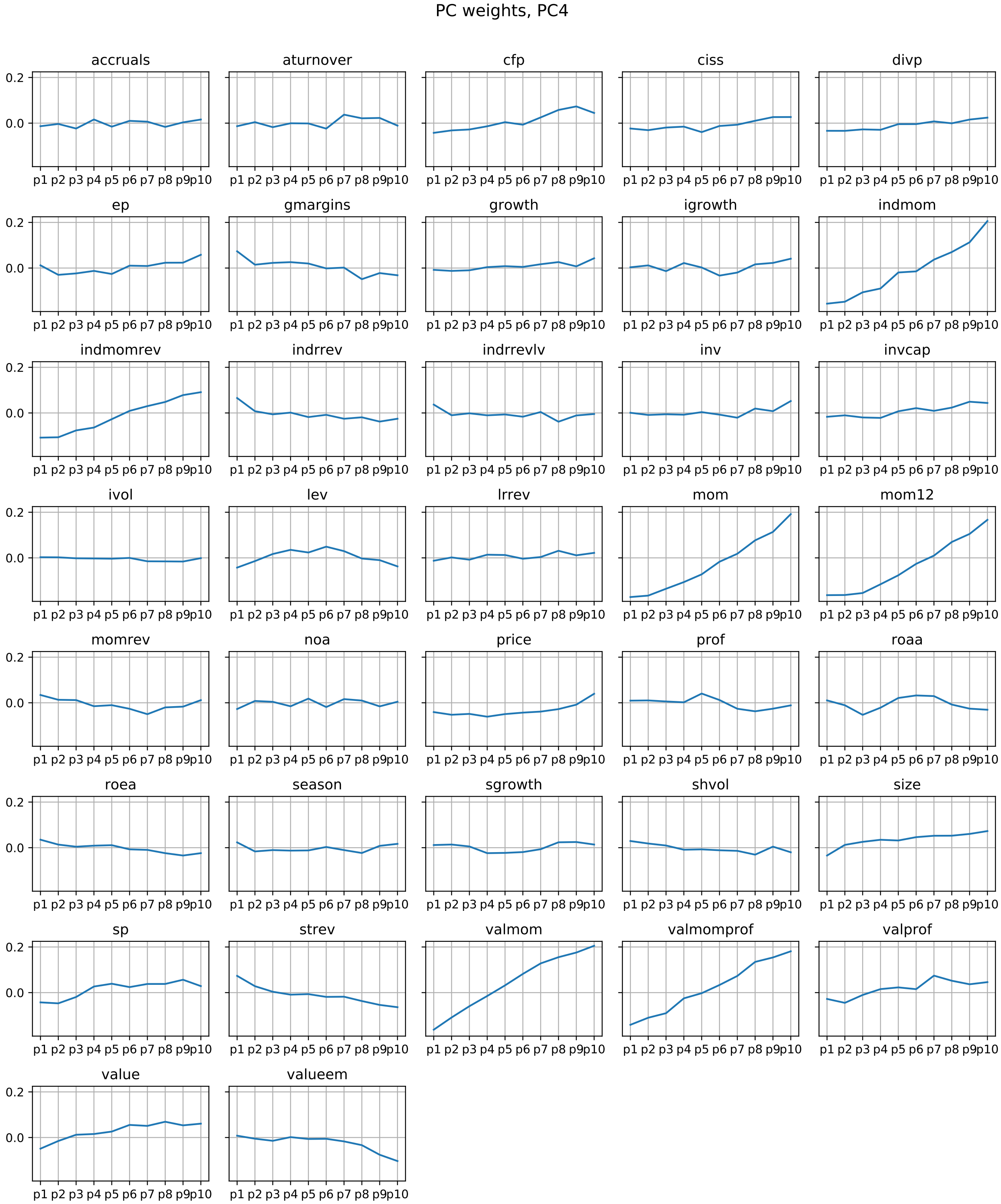
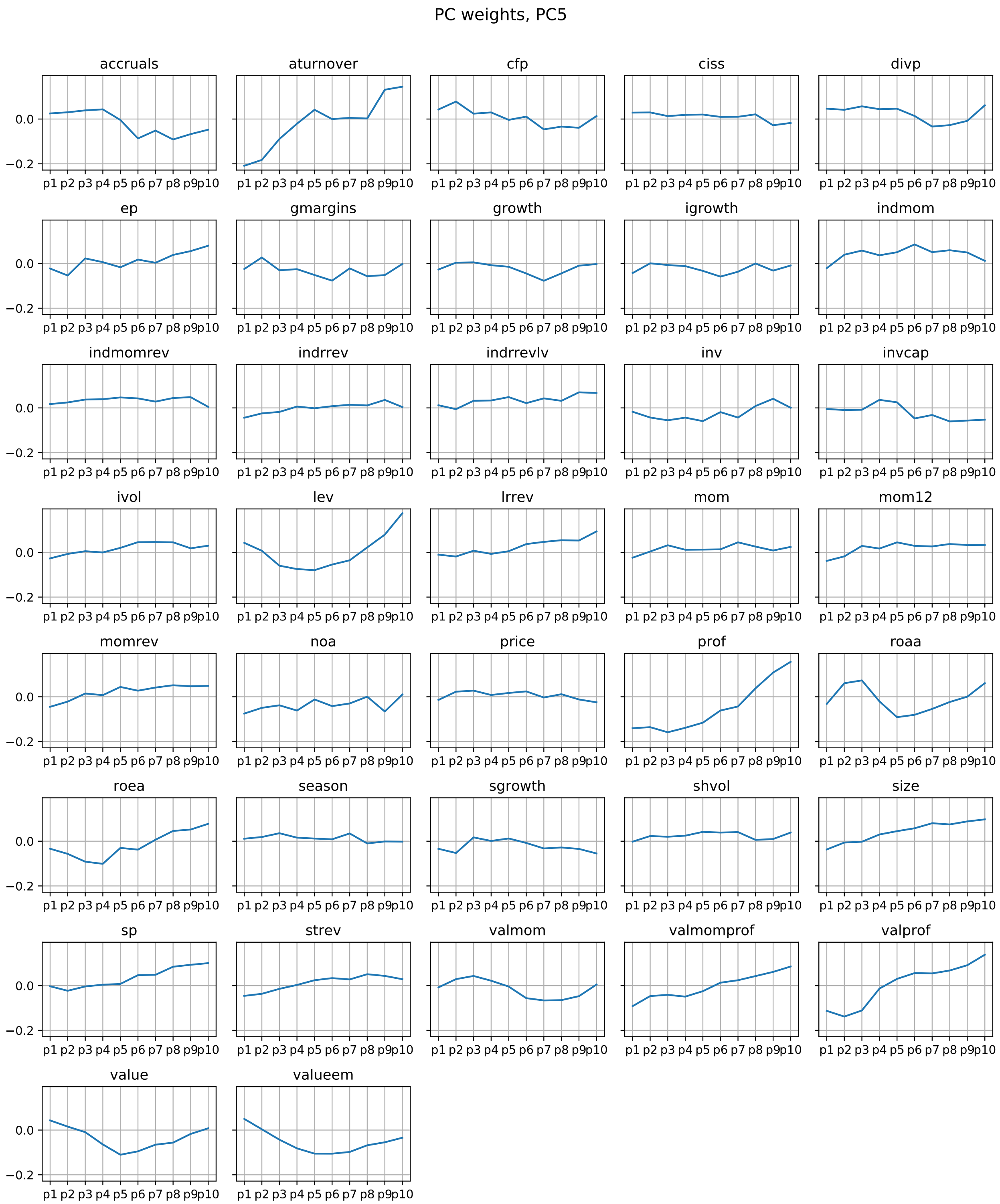


Figure 15: Loadings of fifth principal component



(f) We now turn to using principal components as linear asset pricing factors. To get an overview, Table 7 shows the root mean square of the pricing error (α) as we increase the number of principal components in the model. The RMSE with three and five principal components (0.17%, 0.14%) are relatively small given the number of assets, and broadly comparable although slightly smaller to those obtained using the FF3 and FF5 factors (0.18%, 0.16%). Figure 16 shows pricing error across all portfolios as we increase the number of principal components (sorted by pricing errors). The picture is more refined but mostly confirms what is shown by the RMSEs. We also observe that even with one principal component, the pricing error are small for a substantial set of portfolios, even though they remain large for portfolios at the extreme. The figure broadly confirms the conclusion from Question **(e)**: the first few principal components (PC1, PC2, PC3) are responsible for a large part of most of the explained variance in excess returns, even though adding more principal components thereafter continues to gradually decrease the average α.

Last included	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
RMSE	0.19%	0.17%	0.17%	0.15%	0.14%	0.14%	0.13%	0.12%	0.12%	0.12%

Last included	PC11	PC12	PC13	PC14	PC15	PC16	PC17	PC18	PC19	PC20
RMSE	0.12%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%	0.10%	0.10%

Table 7: Root mean square pricing error (α, % per month)

Figure 17 provides another way to assess the performance of using principal components as pricing factors. The figure shows the average actual excess returns on each of the asset as a function of the average excess returns predicted by the model as we increase the number of included PCs. The 45-degree line shows where returns should line up if the model is a good description of actual returns. Visually, we observe that a model with 3 principal components does "well", even though the addition of a fourth principal component could be appropriate. This figure broadly confirms the results above, but also that even though each additional principal component explains an increasingly smaller share of the variance, its use in terms of getting better predictions for excess returns is not necessarily negligible. This is consistent with the number of assets being quite large here, especially compared to the case we are used to studying of 25 portfolios for which 1 to 3 principal components might be mostly enough.

The attached Jupyter notebook provides a few additional graphs, for instance that of the distribution of p-values or t-statistics for α, but the emerging picture is the same and they are omitted in the interest of space.

Figure 16: Pricing errors for different sets of PCs as factors

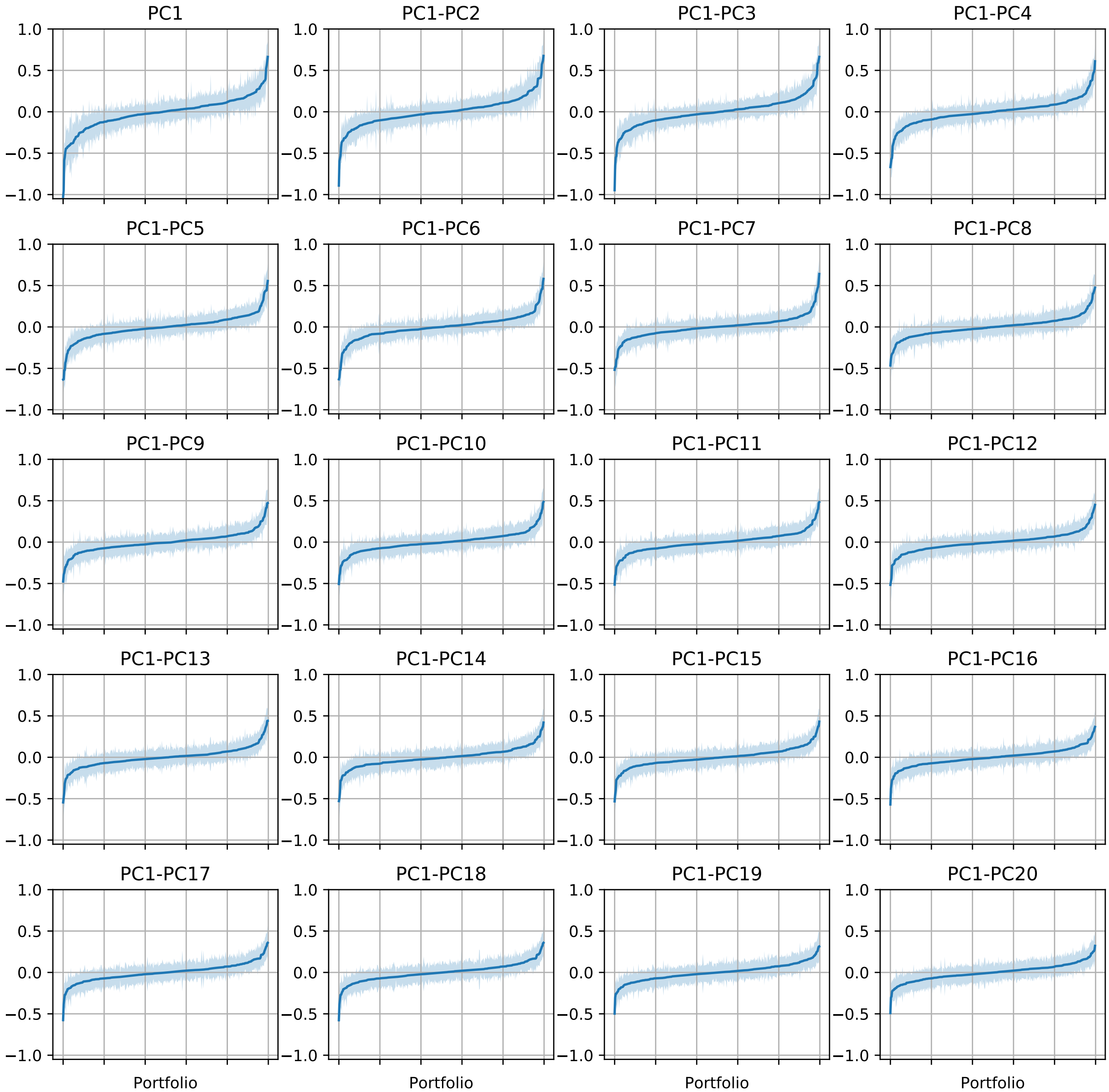
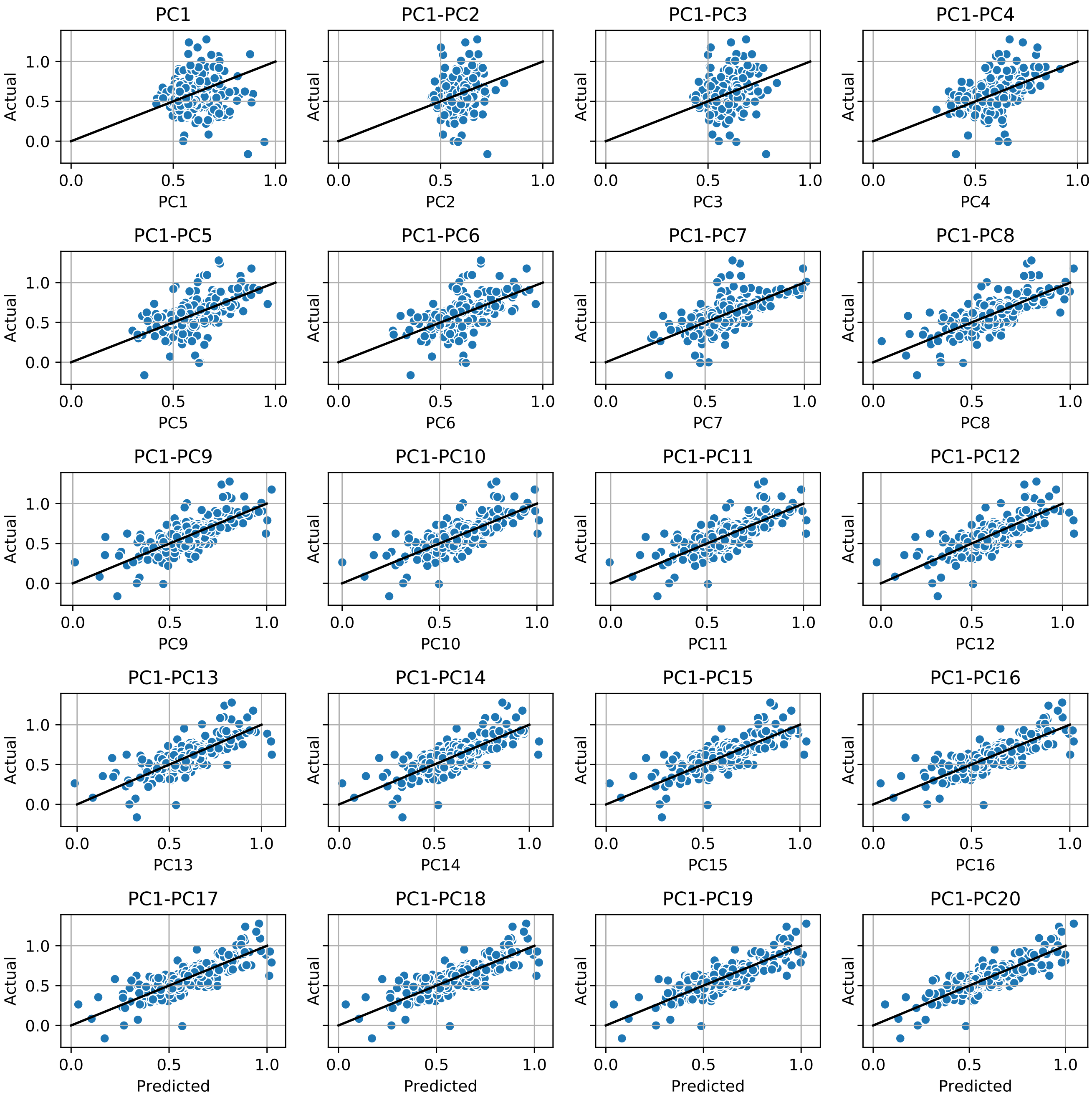


Figure 17: Actual vs. predicted average excess returns (% per month)



(g) The relationship between principal components and Fama-French factors was already discussed at length in previous questions (especially **(e)** and **(f)**) so we will not provide much more details here. Instead, let us provide a brief summary of our results:

- Our preferred specification in terms of explained variance would be to include PC1, PC2 and PC3. Those three principal components explain around 87.5% of the variance of excess returns, with the lion's share attributable to PC1 with 83.6%.

- Accordingly, those three principal components would be the most important to include as factors in a linear asset pricing model and together, they yield pricing errors that appear at first glance to be "not too bad" on average given the number of assets (RMSE = 0.17%). Of course, those pricing errors still remain significant.

- Because we have many more assets than the usual 25 Fama-French portfolios, and even though they explain a small share of the variance, adding a few more principle components can help in terms of getting better predictions of excess returns.

- The principal components appear to be quite related to the Fama-French factors. PC1 mostly captures the average level of excess returns, the same way Mkt-RF does. PC3 is somewhat related to HML, while PC4 and PC5 seem related to momentum and profitability (RMW). The mapping between the two sets of factors is of course far from perfect, which is expected given that principal components are computed purely statistically from the data with weights being obtained optimally, while Fama-French factors are based on somewhat ad-hoc weights set by the two researchers. Still, despite those differences, the dimensions captured by the principal components are in the spirit as those captured the Fama-French factors. At the end of the day, the factors in each set is broadly a linear combinations of the factors in the other.

- Note that newer methods are being introduced to try to do better than standard PCA. For instance, among others, Bryan Kelly (Yale SOM/AQR) has worked on "instrumented" PCA where the exposure to each of the factors extracted from the data are time-varying. The time variation is assumed to be captured by several individual characteristics of each traded asset and can take a linear or more general function form (e.g. neural/autoencoder networks). Professor Lettau, in his work with Markus Pelger (Stanford), also proposes a way to extract better pricing factors from the data by introducing a penalty on the cross-sectional pricing error of the factors. Their method is called Risk-Premium-PCA ("RP-PCA"), and the dataset that we used in this Problem is one of those on which they test the method.

Problem 2

(a) We estimate the following model using the *pykalman* package[1]:

$$\begin{aligned} r^e_{i,t} &= \alpha_t + \beta_t r^e_{M,t} + w_t \\ \alpha_{t+1} &= \alpha_t + u_t \\ \beta_{t+1} &= \beta_t + v_t \end{aligned}$$

Note that α_t and β_t are the state variables in this state-space representation, and follow a random-walk. This specification has become a bit of a standard, in part because its estimation converges quite well for most assets. These could be changed to AR(1) or other processes, but the estimation is sometimes less stable or might converge less easily.

Figures 18 and 19 show the results when allowing both parameters to be time-varying, while Figure 20 shows time-varying pricing errors (α_t) when β is assumed to be constant. We focus on the four corner portfolios but the interested reader can check the enclosed Jupyter notebook for the evolution on all 25 portfolios. Here are the main observations.

- When both α_t, β_t are allowed to be time-varying, the pricing errors decrease significantly over the sample (even though they remain somewhat large for value stocks, i.e. high book-to-market). This could be consistent with some of those αs having been arbitraged away over the years since the discovery and wide discussion of the CAPM.
- There is evidence for time variation in exposure to market risk (β_t). Even though those differences are not necessarily massive, they are precisely estimated and appear to be statistically and economically significant.
- When only α_t is allowed to vary, its evolution are much choppier and much more volatile, consistent with the fact that it must account for some of the time variation that is in fact due to β_t. In this case, pricing errors appear to get closer to zero only in the last part of the sample from around 2000.
- Although omitted here in the interest of space, we observe that the pattern obtained from the Kalman Filter are broadly consistent with those obtained from simpler rolling regressions. Cf. the Jupyter notebook for details. Note that if there are indeed evidence of time variations, a model formally accounting for them like the one we estimate here is more appropriate and precisely estimated than an ad-hoc rolling regression.

[1] Another possibility would be to use MCMC. In this case, the model takes the form:

$$\begin{aligned} r_t &= \alpha_t + \beta_t \; r_{M,t} + e_t \\ \alpha_{t+1} &\sim N(\alpha_{t-1}, \sigma^2_\alpha) \\ \beta_{t+1} &\sim N(\beta_{t-1}, \sigma^2_\beta) \end{aligned}$$

For this, we need to define hyper-priors for σ^2_α and σ^2_β. This parameter can be interpreted as the volatility in the regression coefficients.

$$\sigma^2_\alpha \sim \exp\{\alpha_0\} \quad \text{and} \quad \sigma^2_\beta \sim \exp\{\beta_0\}$$

All this is performed in lab 8, and is omitted here. The interested reader is referred there for details.

Figure 18: Pricing errors (α) for CAPM with time-varying α_t, β_t

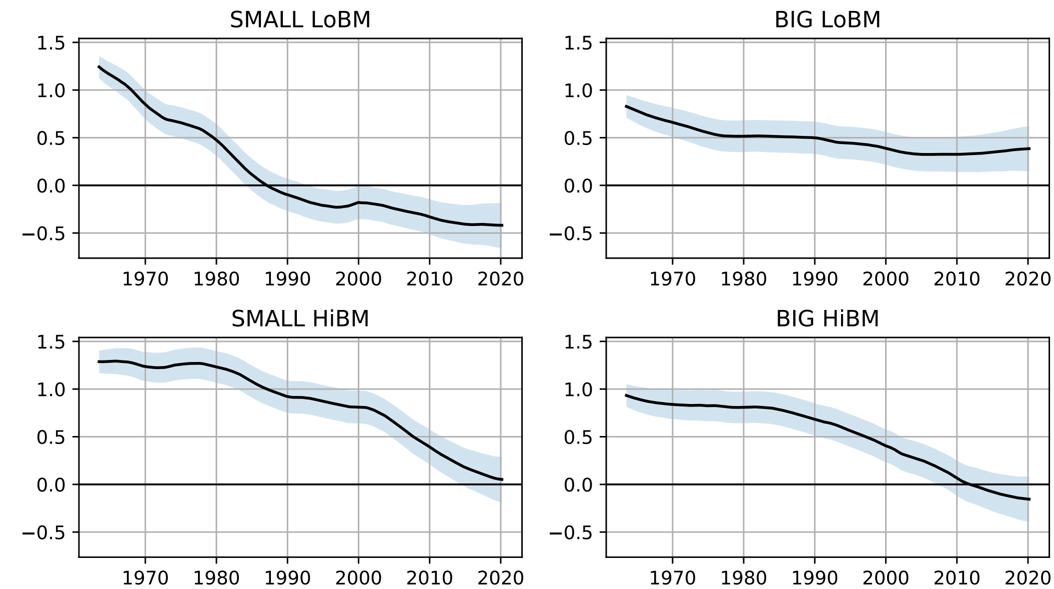


Figure 19: Exposure to market risk (β_{MKT}) for CAPM with time-varying α_t, β_t

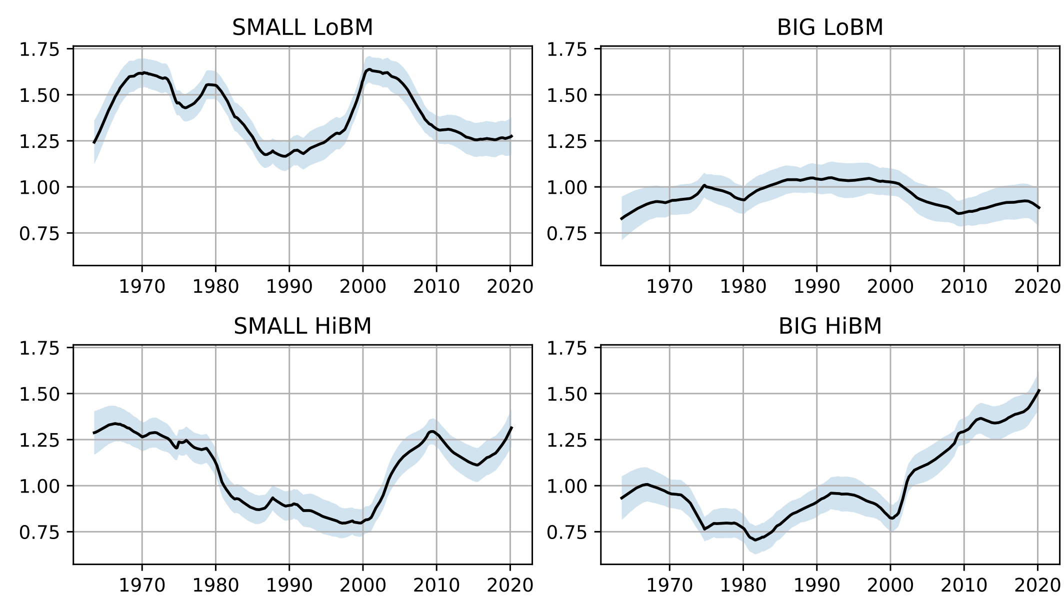
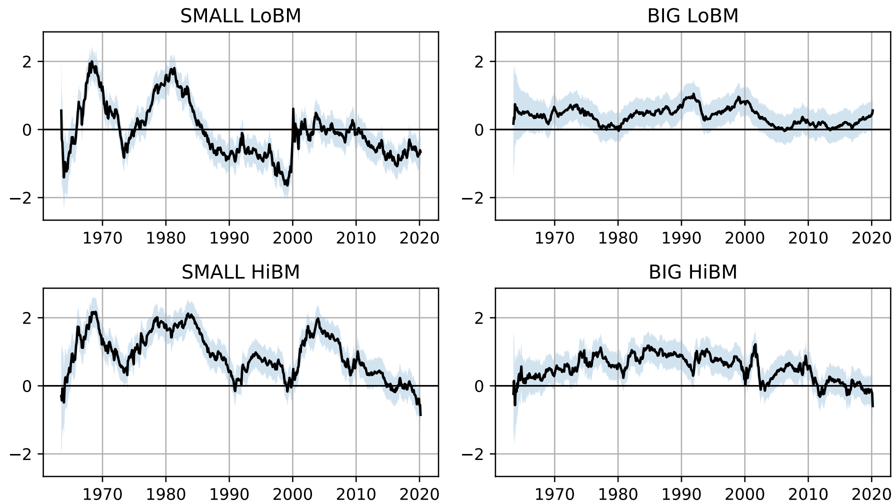


Figure 20: Pricing errors (α) for CAPM with time-varying α_t and constant β



(b) Figures 21, 22, 23 and 24 show α_t and $\boldsymbol{\beta}_t$ when using the three Fama-French factors where all parameters are allowed to vary, while Figure 25 shows pricing errors when $\boldsymbol{\beta}$s are assumed to be constant.

Observations are broadly similar to the ones for the CAPM in **(a)**. Note that, as expected, pricing errors α_t are on average smaller. There is evidence of significant time variation in α_t. For the risk exposures ($\boldsymbol{\beta}_t$), time variation is particularly marked at the beginning of the sample, which could partly be due to the initialization of the Kalman Filter, but there is still significant evidence of some time variation throughout, both statistically and economically.

Figure 21: Pricing errors (α) for FF3 with time-varying $\alpha_t, \boldsymbol{\beta}_t$

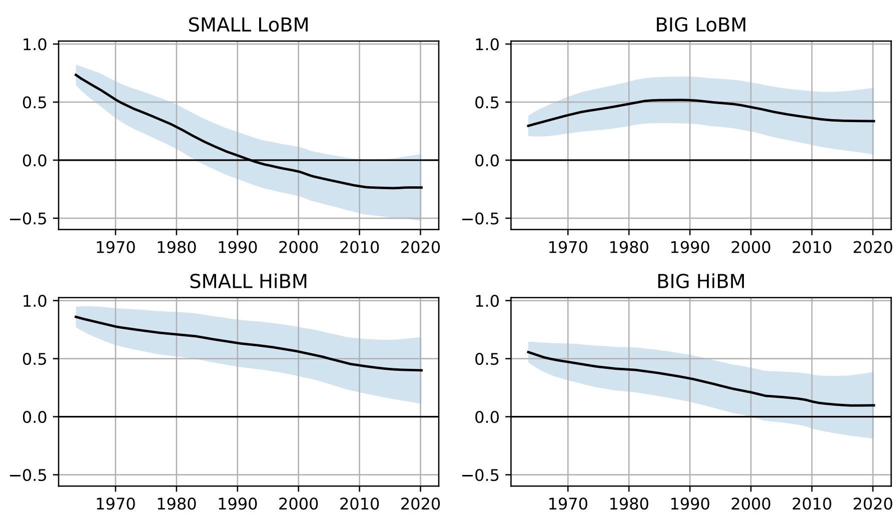


Figure 22: Exposure to market risk (β_{MKT}) for FF3 with time-varying $\alpha_t, \boldsymbol{\beta}_t$

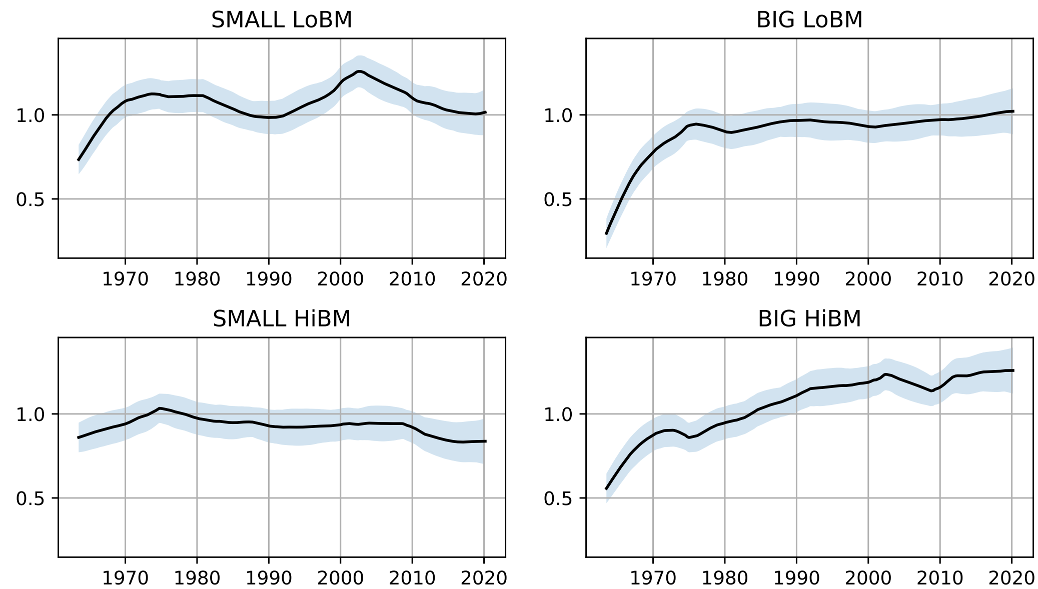


Figure 23: Exposure to SMB risk (β_{SMB}) for FF3 with time-varying $\alpha_t, \boldsymbol{\beta}_t$

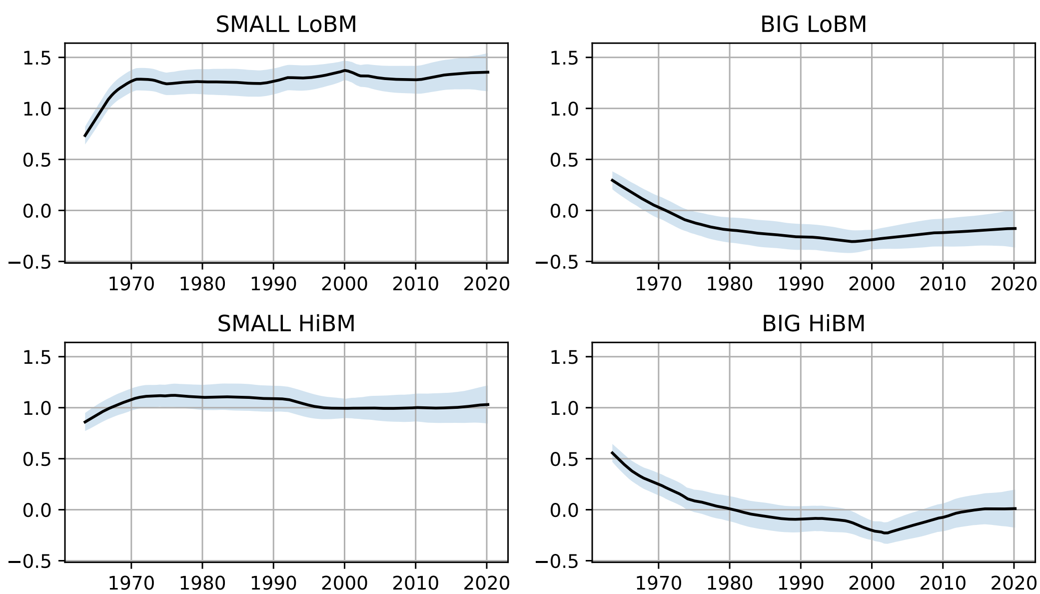


Figure 24: Exposure to HML risk (β_{HML}) for FF3 with time-varying $\alpha_t, \boldsymbol{\beta}_t$

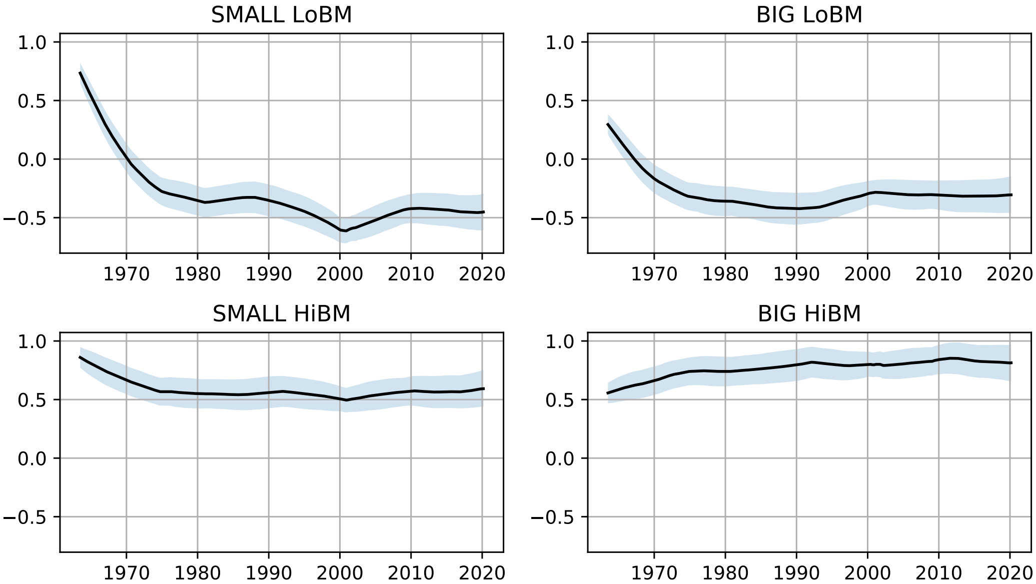
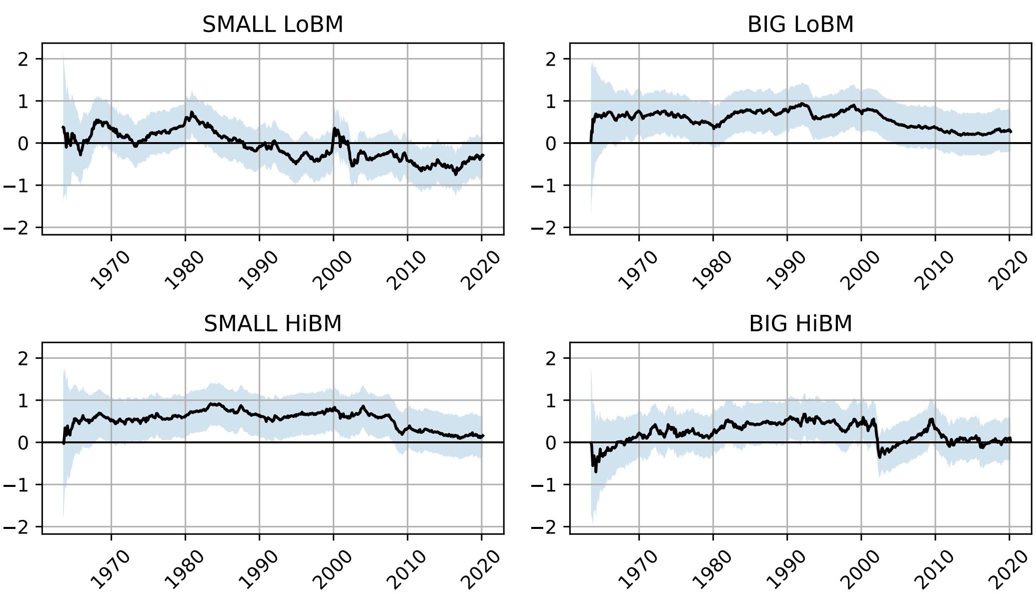


Figure 25: Pricing errors (α) for FF3 with time-varying α_t and constant $\boldsymbol{\beta}$s



(c) We now turn to the results for individual stocks. Figures 21, 22, 23 and 24 show α_t and $\boldsymbol{\beta}_t$ when using the Fama-French factors where all parameters are allowed to vary, while Figure 25 shows pricing errors when $\boldsymbol{\beta}$s are assumed to be constant.

Figure 26: Pricing errors (α) for FF3 with time-varying $\alpha_t, \boldsymbol{\beta}_t$

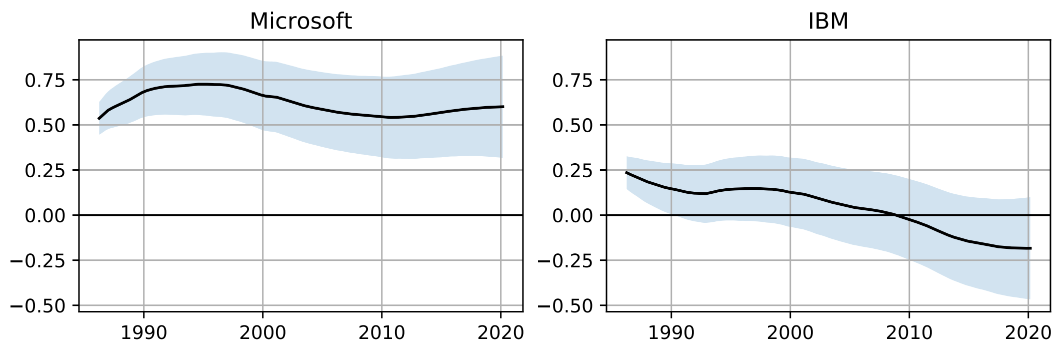


Figure 27: Exposure to market risk (β_{MKT}) for FF3 with time-varying $\alpha_t, \boldsymbol{\beta}_t$

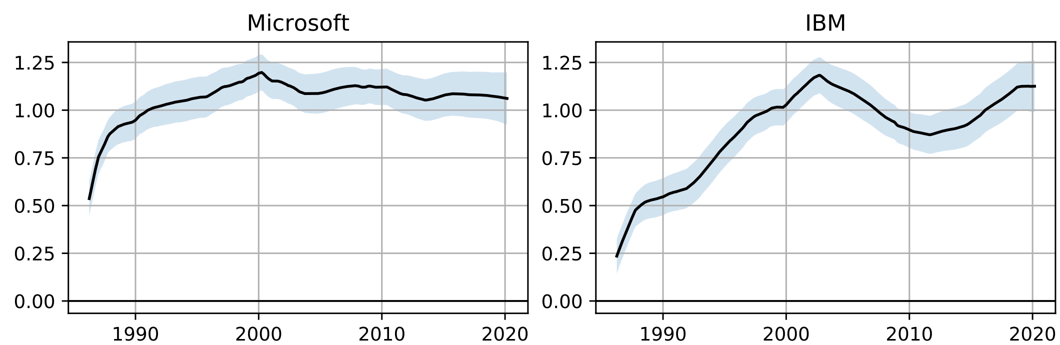


Figure 28: Exposure to SMB risk (β_{SMB}) for FF3 with time-varying $\alpha_t, \boldsymbol{\beta}_t$

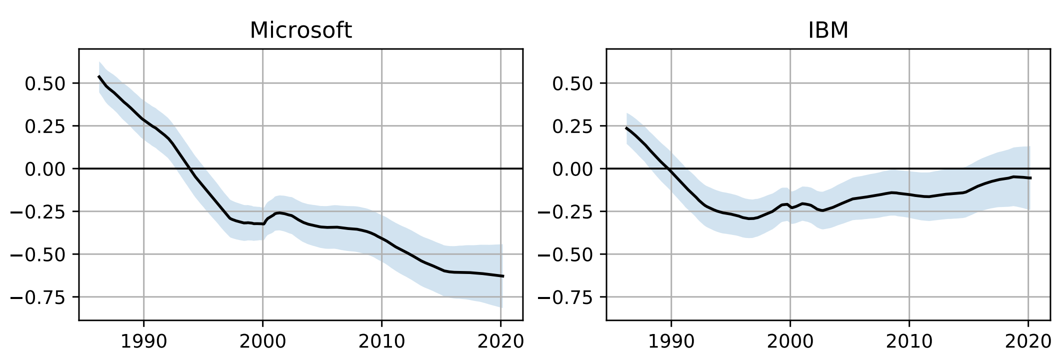


Figure 29: Exposure to HML risk (β_{HML}) for FF3 with time-varying $\alpha_t, \boldsymbol{\beta}_t$

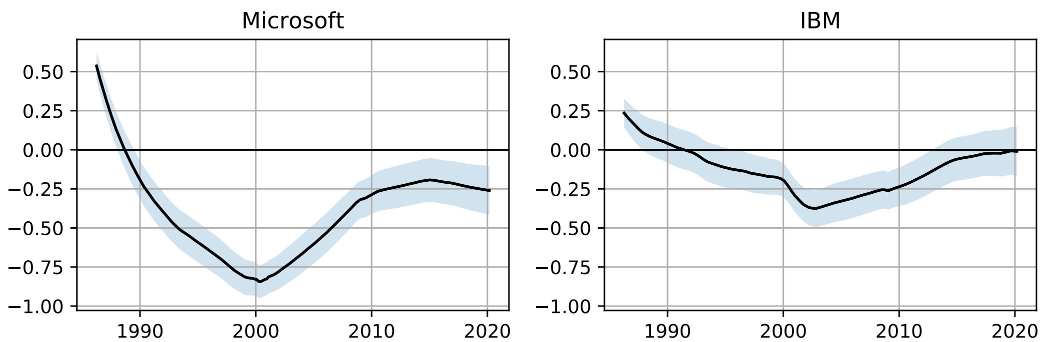
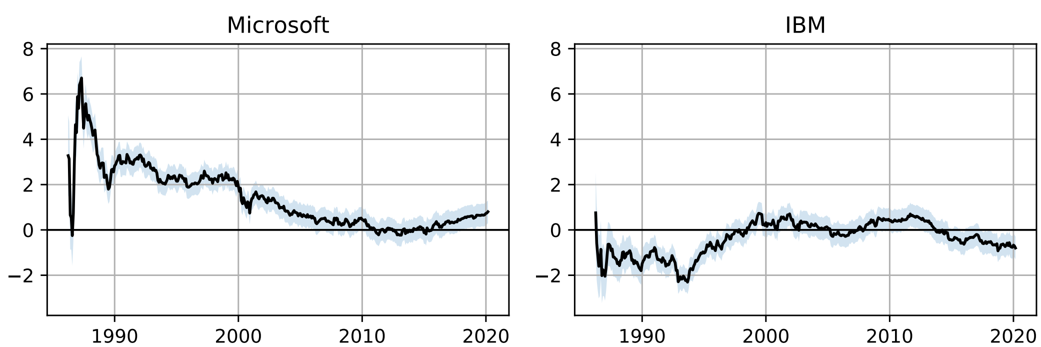


Figure 30: Pricing errors (α) for FF3 with time-varying α_t and constant $\boldsymbol{\beta}$s



(d) Let us compare the results of **(b)** and **(c)**:

- When we allow for time-varying $\boldsymbol{\beta}_t$, the pricing error for the Microsoft stock appear to be larger than for portfolios, and to have broadly stayed constant and large (≈ 0.5) throughout the sample. For IBM, the pricing error appears to have decreased, but it is difficult to conclude because α_t is estimated with somewhat less precision.

- When we do not allow for time-varying $\boldsymbol{\beta}_t$ however, the pricing error appears to have decrease substantially since the late 1980s. This could hint at the fact that investors have focused on arbitraging away αs obtained from regressions with constant $\boldsymbol{\beta}_t$ such as with the original CAPM or Fama-French factors.

- When risk exposures are allowed to vary over time, we find evidence of substantial changes in the β_t over each of the three Fama-French factors. This is much more so the case here with individual stocks than in Questions **(a)** and **(b)** where we focused on the 25 Fama-French portfolios. For instance, for the Microsoft and IBM stocks, both $\beta_{SMB,t}$ and $\beta_{HML,t}$ flip sign over the sample with very wide variations. In addition, $\beta_{MKT,t}$ also varies a lot, as we had already discussed in Lecture.

- Overall, this shows one of the reasons why constructing portfolios has long bee very popular: it allows us to group similar assets and to some extent to diversify away some of the idiosyncratic changes in them so that we end up with more stable exposures $\boldsymbol{\beta}_t$.
- This should not hide the fact that portfolios are often formed in somewhat arbitrary ways, or at least with somewhat ad-hoc weights and rules. Ultimately, understanding the variations in individual α_t and β_t is clearly a goal to thrive for, and it has been the subject of considerable research in recent years, including by Professor Lettau.