MFE230E Problem Set 1

Due March 31 6:00 pm PST via bCourses

1. Ruppert-Matteson ch. 12 Question 7
2. Ruppert-Matteson ch. 12 Question 14
3. Show that the ARMA(2,1) process

$$x_t = 1.5x_{t-1} - 0.5x_{t-2} + e_t - 0.5e_{t-1}$$

   is equivalent to a random walk.

4. Consider two AR(1) processes

$$x_t = \phi x_{t-1} + e_t$$
$$y_t = \psi y_{t-1} + w_t$$

   where $|\phi|, |\psi| < 1$.

   (a) What kind of ARMA process is $z_t = x_t + y_t$ if $\phi = \psi$?

   (b) What kind of ARMA process is $z_t = x_t + y_t$ if $\phi \neq \psi$?

5. Simulate and plot five sample paths of simulated AR(1) processes $x_t = \phi x_{t-1} + e_t, e_t \sim NWN(0,1)$ for sample sizes $T = 100, 1000, 10000$ and

   (a) $\phi = 0.9$.

   (b) $\phi = 0.99$.

   (c) $\phi = 0.99999$.

   (d) Random walk

   Use the same draws of the e_t's to construct the AR(1) with different ϕ's.

   Use the same y-scale for plots with the same sample size. Compute the sample means and standard deviations and compare them to the population moments. Compare the behavior of the processes.

   Note: Do not use built-in `statsmodels` routines, such as the `statsmodels.tsa.arima_process.arma_generate_sample` command. Draw shocks using a random number generator and then construct the processes recursively.

6. Consider an AR(2) process:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t$$

   Under what conditions for ϕ_1 and ϕ_2 is the AR(2) stationary and ergodic? Plot the stationarity region in a diagram with ϕ_1 on the x-axis and ϕ_2 on the y-axis. Also indicate the regions for which the roots are real and imaginary.