MFE230E Problem Set 3

Due April 14 10:00am via bCourses

You may NOT use built-in regressions routines for problem sets, i.e. construct the $\mathbf{Y}$ and $\mathbf{X}$ matrices and compute $(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$, etc. yourself.

1. Suppose the true process for x_t is an ARMA(1,1):

$$x_{t+1} = 0.8x_t + \epsilon_{t+1} + 0.7\epsilon_t \quad \epsilon \sim NWN(0,1)$$

   (a) Create 10,000 simulations with sample size of 1,000 of the ARMA(1,1) process. For each simulation, estimate an AR(1) process by OLS.

$$x_{t+1} = \phi x_t + e_{t+1}$$

   Plot the histogram of the 10,000 estimated $\hat{\phi}$'s. Describe the properties of the OLS estimator.

   (b) Compute the autocorrelation of the OLS errors.

   (c) For each simulation compute the IV estimator with using an appropriate instrument. Plot the histogram of the 10,000 estimated $\hat{\phi}_{IV}$'s. Describe the properties of the IV estimator.

2. Suppose the true process for x_t is an AR(3) with $\lambda_1 = 0.95, \lambda_2 = 0.9, \lambda_3 = 0.8$.

   (a) Create 10,000 simulations with a sample size of 1,000 of the AR(3) process. For each simulation, estimate an AR(1) process by OLS.

$$x_{t+1} = \phi x_t + e_{t+1}$$

   and compute the autocorrelation function of the errors. What do you learn from the error ACF? Assess the properties of the estimated parameters.

   (b) Repeat (a) for an AR(2), AR(3) and AR(4).

   (c) What do you learn from this exercise about estimation of AR models?

3. Suppose the true model is:

$$y_i = x_{1i} + x_{2i} + \epsilon_i \quad \epsilon \sim NWN(0,1)$$

   and $x_{1,i} \sim N(0,1), x_{2,i} \sim N(0,1)$ and $\mathsf{Cov}(x_{1t}, x_{2t}) = 0$.

   (a) Create 10,000 simulations with sample size of 1,000 of the above process. For each simulation, estimate the following model by OLS.

$$y_i = \beta x_{1i} + e_i$$

Plot the histogram of the 10,000 estimated $\hat{\beta}$'s. Describe the properties of the OLS estimator.

(b) Repeat (a) when the correlation of x_{1i} and x_{2i} is set to 0.5 instead of zero.

(c) Repeat (a) when the correlation of x_{1i} and x_{2i} is set to -0.5 instead of zero.

(d) Describe your results.

4. Is the following bivariate process stationary?

$$\mathbf{Y}_t = \begin{pmatrix} .5 & .1 \\ .4 & .5 \end{pmatrix} \mathbf{Y}_{t-1} + \begin{pmatrix} 0 & 0 \\ .25 & 0 \end{pmatrix} \mathbf{Y}_{t-2} + \boldsymbol{\epsilon}_t$$

5. For this question, you may **not** use any MLE or ARIMA packages (such as `statsmodels.tsa.arima_model.ARMA`). However, you may use a numerical optimization package (e.g., `scipy.optimize`).

Consider an MA(1) process:

$$z_t = \mu + e_t + \theta e_{t-1} \quad e_t \sim N(0, \sigma^2).$$

(a) Derive the likelihood function as a function of $\boldsymbol{\omega} = (\mu, \theta, \sigma)'$.

(b) Simulate a sample of length 100 of the following MA(1) process:

$$z_t = 1 + e_t + 0.5 e_{t-1} \quad e_t \sim N(0, 1).$$

Using the data from your simulation, maximize the likelihood function numerically to obtain the MLE estimate of $\boldsymbol{\omega}$.

(c) Compute the standard errors of $\widehat{\boldsymbol{\omega}}$ numerically.

(d) Compare your results to the results using the statsmodels estimation (`statsmodels.tsa.arima_model.ARMA`).