A). let $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}_{2\times1}$ be mean vector.

& $\Sigma = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix}_{2\times2}$ be covariance matrix

where $\sigma_x = 1$, $\sigma_y = 1$, $\rho = a$.

$$f_{x_2|x_1}(x_2|x_1=x_1) = \frac{f_{x_1,x_2}(x_1,x_2)}{f_{x_1}(x_1)}$$

Conditional distribution of x_2 given $x_1 = x_1$

here $f_{X_1}(x_1)$ = ~~Marg~~ Marginal PDF of X_1.

& $f_{x_1,x_2}(x_1,x_2)$ = Joint PDF of X_1 & X_2.

for bivariate distribution

$$f_{x_2|x_1}(x_2|x_1=x_1) \Rightarrow \frac{\dfrac{e^{-\frac{1}{2(1-a^2)}\left[\frac{(x_1-\mu_1)^2}{1} + \frac{(x_2-\mu_2)^2}{1} - 2a(x_1-\mu_1)(x_2-\mu_2)\right]}}{2\pi\sqrt{1-a^2}}}{\dfrac{e^{-\frac{1}{2}(x_1-\mu_1)^2}}{\sqrt{2\pi}}}$$

$\Rightarrow$ rearranging the terms & simplifying them:-

we get:-

$$\frac{e^{\left[\frac{1}{2(1-a^2)}\left[(x_1-\mu_1)^2+(x_2-\mu_2)^2-2a(x_1+\mu_1)(x_2+\mu_2)\right]+\frac{(x_1-\mu_1)^2}{2}\right]}}{\sqrt{2\pi}\sqrt{1-a^2}}$$

$$\Rightarrow \frac{e^{-\frac{1}{2(1-a^2)}\left[x_2-\mu_2-a[x_1-\mu_1]\right]^2}}{\sqrt{2\pi}\sqrt{1-a^2}}$$

here we can see it is a normal distribution

$$N\left[\mu_2+a(x_1-\mu_1),\ (1-a^2)\right]$$

hence $f(x_2|x_1) = N\left[\mu_2+a(x_1-\mu_1),(1-a^2)\right]$

similarly this eqn will apply for $f(x_1|x_2)$

$$f(x_1|x_2) = N\left[\mu_1+a(x_2-\mu_2),\ (1-a^2)\right]$$