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Comprehensive Capital Calculation using Stochastic Dominance Approach

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Abstract

Banks and other Financial Institutions are required to communicate their daily risk forecasts to the appropriate monetary authorities at the beginning of each trading day, using any risk model to forecast Value-at-Risk (VaR) by the Basel III Accord. The risk estimates from the models are used to determine the daily capital charges (DCC). In this project, we define risk by the optimal selection of the risk models. We consider robust uniform rankings of models over cumulative distribution functions. The rankings we give to each model are based on statistical tests of stochastic dominance (SD).

Introduction

Value at Risk (VaR)

In its most general form, the Value at Risk measures the potential loss in value of a risky asset or portfolio over a defined period for a given confidence interval.

When using VaR measure, we are interested in making a statement of the following form:

‘We are P% certain that we will not lose more than $V in the next N days’

Here, V is the VaR of the portfolio. It is a function of two parameters*:* N, the time horizon, and, the confidence level. It is the loss level over N days that the manager is P% certain will not be exceeded. In general, when an N day is the time horizon, and P% is the confidence level, VaR is the loss corresponding to the $(100\text{-}P)^{th}$ percentile of the percentile of the distribution of the change in the value of the portfolio over the next N days.

VaR is an attractive measure for risk measurement because it is easy to understand.

The VaR threshold for Y_t can be calculated as:

$$VaR_t = \mathrm{E}(Y_t \mid F_{t-1}) - \alpha\sigma_t$$

where α is the critical value from the distribution of ε_t, which is the random component, to obtain the appropriate confidence level. It is possible for σ_t to be replaced by alternative estimates of the conditional standard deviation to obtain an appropriate VaR.

Daily Capital Charges (DCC)

Daily Capital Charges are the capital requirements and cost to be maintained by the financial institutions to meet the Basel III requirements, and this is calculated from various VaR models.

The Basel III Accord requires that banks and other Financial Institutions communicate their daily risk forecasts to the appropriate monetary authorities at the beginning of each trading day, using one of a range of alternative financial risk models to forecast Value-at-Risk (VaR). The risk estimates from these models are used to determine the daily capital charges (DCC) and associated capital costs of Financial Institutions, depending in part on the number of previous violations (k), whereby realized losses exceed the estimated VaR.

Data and DCC Evaluation Framework

Data Description

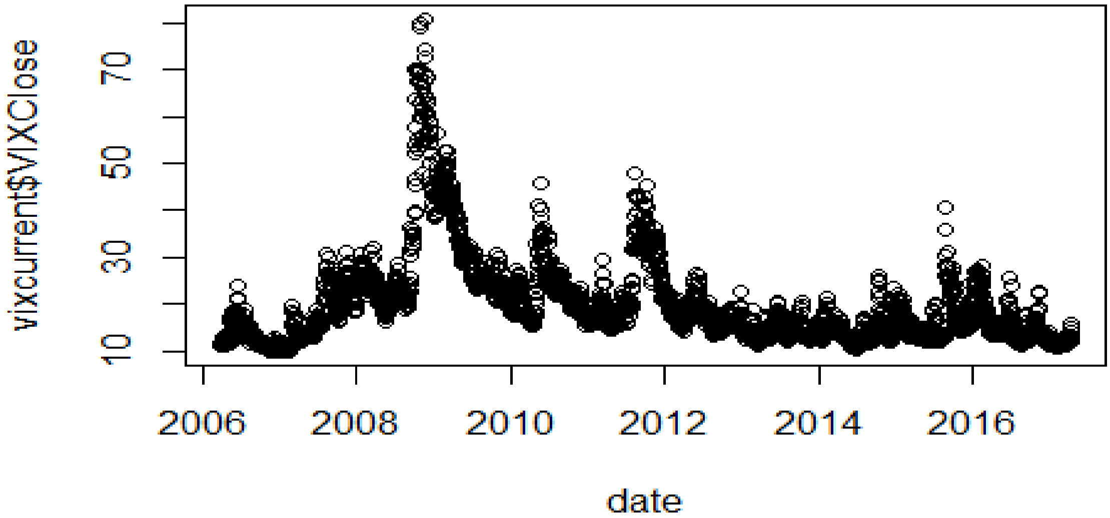
The data used for estimation and forecasting are closing daily prices (settlement prices) for the 30-day maturity CBOE VIX volatility index futures (ticker name VX).

The CBOE Volatility Index® (VIX® Index®) is a key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices. We can also define it as an index that measures the market’s perceived future volatility. More specifically, the VIX measures the market’s expectation of future volatility implied by the S&P 500 stock index (SPX) option prices.

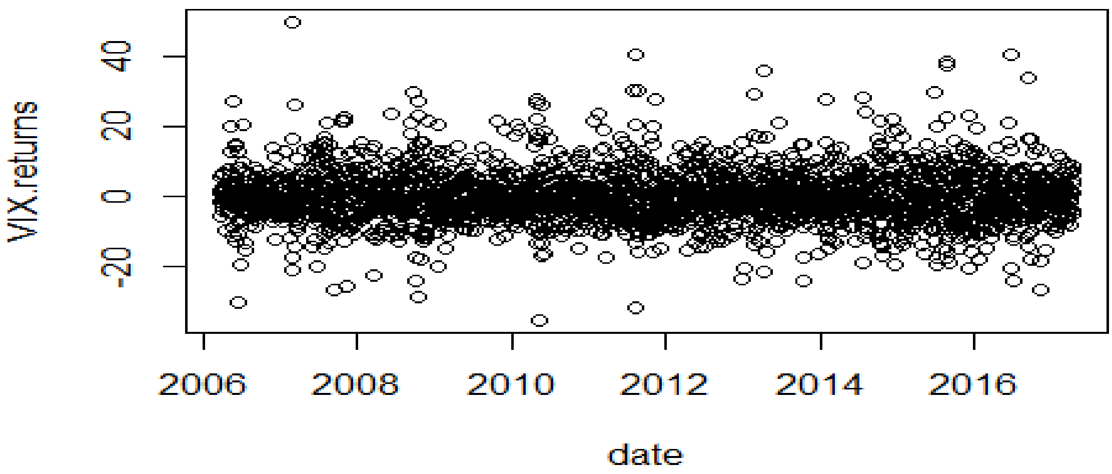
Since its introduction in 1993, the VIX Index has been considered by many to be the world's premier barometer of investor sentiment and market volatility. Several investors expressed interest in trading instruments related to the market's expectation of future volatility, and so

VIX futures were introduced in 2004, and VIX options were introduced in 2006. The VIX is updated continuously throughout the trading day.

Although VIX represents a measure of the expected volatility of the S&P500 over the next 30-days, the prices of VIX futures are based on the current expectation of what the expected 30-day volatility will be at a time in the future (on the expiration date). Although the VIX futures should converge to the spot at expiration, it is possible to have significant disparities between the spot VIX and VIX futures prior to expiration. The figure below shows the daily VIX futures index together with the 30-day maturity VIX futures closing prices.



If Pt denotes the closing prices of the VIX futures contract at time t, the returns at time t, (Rt) are defined as: **Rt = 100* log (Pt/Pt−1)**. The figure below shows the daily VIX futures returns,



Characteristics of VIX futures data

- Non-Stationary
- Time varying volatility
- No clear trend
- Liquid

The appeal to use VIX data comes from the above characteristics of it.

DCC Evaluation Framework

We are considering the calculation of daily capital charges (DCC) as a basic criterion for choosing between risk models. The Basel II Accord stipulates that daily capital charges (DCC) must be set at the higher of the previous day's VaR or the average VaR over the last 60 business days, multiplied by a factor (3 + k) for a violation penalty, where a violation occurs when the actual negative returns exceed the VaR forecast negative returns for a given day.

Using the models, we get CDF and then using stochastic dominance we will give out the results.

The primary objective is to evaluate each of the alternative conditional volatility models with respect to the DCC function. Each model will imply different values of DCC as they will produce different VaR forecasts. The stochastic dominance concept is applied in this context to determine which model should be used for producing a lower DCC for a given amount invested. The main point of the analysis is to help risk managers to choose between alternative models.

$$DCC_t = \sup\{-(3 + k)\, VaR_{60},\ -VaR_{t-1}\}$$

where
DCC_t = daily capital charges,
$VaR_t = Y_t - z_t \cdot \sigma_t$, the Value-at-Risk for day t,
VaR_{60} = mean VaR over the previous 60 working days,
Y_t = estimated return at time t,
z_t = 1% critical value of the distribution of returns at time t,
σ_t = estimated risk (or square root of volatility) at time t,
$0 \le k \le 1$ is the Basel II violation penalty; the table below shows the value of k with respect to the number of violations.

Models for forecasting VaR

The basic version of the least squares model assumes that the expected value of all error terms, when squared, is the same at any given point. This assumption is called homoscedasticity, and it is this assumption that is the focus of ARCH/ GARCH models. Data in which the variances of the error terms are not equal, in which the error terms may reasonably be expected to be larger for some points or ranges of the data than for others, are said to suffer from heteroscedasticity.

The standard warning is that in the presence of heteroscedasticity, the regression coefficients for an ordinary least squares regression are still unbiased, but the standard errors and confidence intervals estimated by conventional procedures will be too narrow, giving a false sense of precision. Instead of considering this as a problem to be corrected, ARCH and GARCH models treat heteroscedasticity as a variance to be modeled. As a result, not only are the deficiencies of least squares corrected, but a prediction is computed for the variance of each error term. This prediction turns out often to be of interest, particularly in applications in finance. Considering all of this, we use the GARCH, EGARCH and GJR models to make the prediction.

There are alternative time series models for estimating conditional volatility. We have used GARCH, GJR and EGARCH which are well-known conditional volatility models that can be used to evaluate strategic market risk disclosure.

Let's now look at each of the models in detail.

GARCH Model

For a wide range of financial data series, time-varying conditional variances can be explained empirically through the autoregressive conditional heteroscedasticity (ARCH) model, which was proposed by Engle. When the time-varying conditional variance has both autoregressive and moving average components, this leads to the generalized ARCH(p, q), or GARCH(p, q). It is very common in practice to impose the widely estimated GARCH(1, 1) specification in advance.
A natural generalization of ARCH (GARCH)

$$a_t = \sigma_t \varepsilon_t$$

$$\sigma^2_t = \alpha_0 + \Sigma^p_{i=1}\alpha_i a^2_{t-i} + \Sigma^q_{j=1}\beta_j \sigma^2_{t-j}$$

where ε_t is identically and independently distributed with mean 0, standard deviation 1 and independent of a_{t-1}... Also $\alpha > 0$, α_i and $\beta_j \geq 0$, and $\Sigma^p_{i=1}\alpha_i + \Sigma^q_{j=1}\beta_j < 1$. Such a model is called GARCH(p,q) model. σ^2_t is weighted average of past volatilities and squared residuals.

Properties of GARCH model:

1. The strength of the GARCH model is that it manages to model the volatility clustering and the mean reverting characteristics. In other words, large and small shocks tend to be followed by large and small shocks respectively of either sign.
2. The GARCH model also imposes restrictions on the parameters to have a finite fourth moment as was the case for the ARCH model

GJR-GARCH Model

An alternative way of modeling the asymmetric effects of positive and negative asset returns, and resulted in the GJR-GARCH model. In the GJR-GARCH(p,q) model Z_t is assumed to have the same representation as before; the conditional variance now given by

$$\hbar_t^2 = \propto_0 + \Sigma^p_{i=1} (\alpha_i Z_{t-i}{}^2 (1 - I[Z_{t-i} > 0]) + \gamma_i Z_{t-i}{}^2 I[Z_{t-i} > 0]) + \Sigma^q_{i=1}\beta_j \hbar_{t-j}{}^2$$

where $\propto_0 > 0$, $\propto_i \geq 0$, $\beta_j \geq 0$ and $\gamma_i \geq 0$ to guarantee that the conditional variance is nonnegative.

The indicator variable differentiates between positive and negative shocks, so that asymmetric effects in the data are captured by the coefficient γ.

As in the case of the EGARCH the asymmetrical effect of shocks can be seen by considering the function

$$g(Z_t) \equiv \alpha_1 Z_{t-1} 2 (1 - I[Z_{t-1} > 0]) + \gamma_1 Z_{t-1}{}^2 I[Z_{t-1} > 0]$$

Positive shocks thus have an impact γ_1 on the logarithm of the conditional variance while negative shocks have an impact $\propto_1$. Typically, $\propto_1 > \gamma_1$ which imposes a larger weight for negative shocks than for positive shocks in line with the leverage effect.

Properties of GJR-GARCH model:
The properties of the GJR-GARCH model are very much like the EGARCH model which are both able to capture the asymmetric effect of positive and negative shocks. In the symmetric GARCH model, the effects of positive shocks (or upward movements in daily returns) on the conditional variance, h_t, are assumed to be the same as the effect of negative shocks (or downward movements in daily returns) of equal magnitude.

First Order Stochastic Dominance

The stochastic dominance concept is applied to determine which model should be used to produce the lowest DCC for a given investment, while not considering the number of violations since the primary purpose of the analysis is to assist risk managers in choosing among alternative models. Stochastic dominance theory is appealing because of its non-parametric orientation. Stochastic dominance criteria require minimal assumptions about returns distribution and preferences. For example, returns can display time series dependence and conform to any distribution.

Let X and Y be two random variables with cumulative distribution functions (CDF) F_X and F_Y, respectively. For first-order stochastic dominance (FSD), Y FSD X, if $F_Y(z) \leq F_X(z)$ for all $z \in R$.

Let $W_U(F)$ denote an evaluation function of the form $W_U(F) = U(z)dF(z)$, where F is the distribution of an underlying variable, and U is any ‘‘utility’’ function. FSD is defined over monotonically increasing utility functions, that is, $W_U(F_Y) \geq W_U(F_X)$ for all U(z) such that $U'(z) \geq 0$.

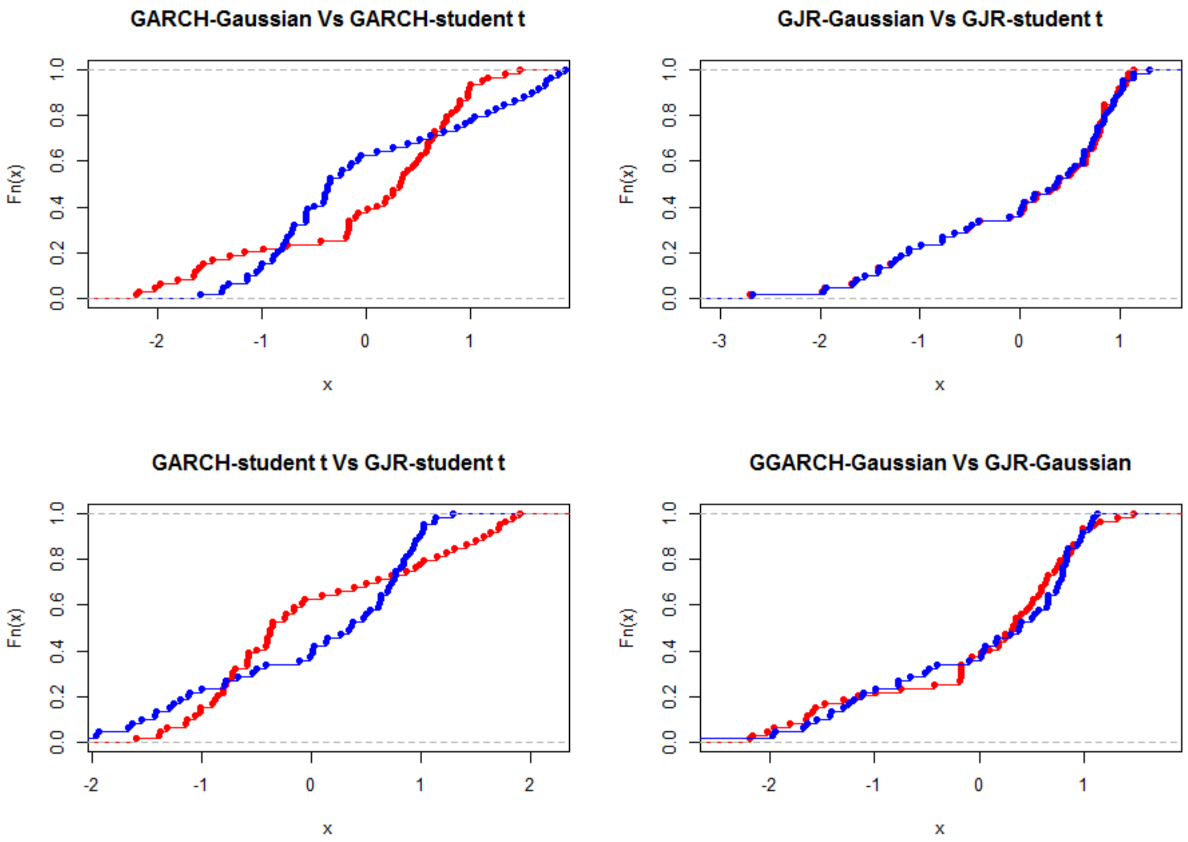
The way that SD may be used to choose the best risk management strategy is as follows. For notational simplicity, we write Model 1 FSD Model 2 whenever Model 1 dominates Model 2 according to first order stochastic. Let Y and X be the DCC produced using model 1 and model 2, respectively. Based on the previous definition, if Y first-order stochastically dominates X, then Y will involve higher DCC than X in the sense that it has a higher probability of producing higher values of DCC.

Therefore, Y must be associated with a higher DCC than X if both X and Y require the same initial investment. In this context, Model 1 FSD Model 2 would mean that the risk manager would prefer Model 2 because the probability is higher that the bank will have to set aside less money for covering losses. In summary, the decision rule would be as follows:

Null Hypothesis	Decision Rule	Inference	Decision of a risk manager
H_0: Y FSD X	If H_0 not rejected, Y dominates X	DCC of Model 1 is likely to be higher than of Model 2	Risk manager prefers Model 2 to Model 1

Conclusion

In this project, we proposed robust risk forecasts that use combinations of several conditional volatility models for forecasting VaR. The idea of combining different VaR forecasting models is entirely within the spirit of the Basel II Accord, although its use may require approval by the regulatory authorities, as for any forecasting model. This approach is not computationally demanding, even though several models should be specified and estimated over time. Further research is needed to compute the standard errors of the forecasts of the combination models, including the median forecast, using numerical methods.



- In general, a stochastic dominance criterion can be used to rank different models of VIX futures and distributions, as illustrated in the previous empirical results. Even in cases of no FSD, the tests provide additional information about the entire distribution over specific ranges.
- The graphs of the CDFs of each pair of models allow a comparison globally for the whole distribution, and locally for a given range of DCC values and probabilities. This allows more specific comparisons than previously afforded based on the mean and other moments of the relevant distributions.
- On plotting the CDFs, we found that they cross, and hence first order SD cannot be established. However, high crossing point SSD is likely.

References

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Appendix

Volatility Clustering

Although volatility is not directly observable, it has some characteristics that are commonly seen in asset returns.

- There exist volatility clusters (i.e., volatility may be high for certain time periods and low for other periods).
- Volatility evolves over time in a continuous manner—that is, volatility jumps are rare.
- Volatility does not diverge to infinity—that is, volatility varies within some fixed range. Statistically speaking, this means that volatility is often stationary.
- Volatility seems to react differently to a big price increase or a big price drop.

These properties play an important role in the development of volatility models. Some volatility models were proposed specifically to correct the weaknesses of the existing ones for their inability to capture the characteristics mentioned earlier.

About VIX

In the spectrum of financial assets, VIX futures prices are a relatively new product. As with any financial asset, VIX futures are subject to risk. In this project, we analyzed the performance of a variety of strategies for managing the risk, through forecasting VaR, of VIX futures under the Basel II Accord. The alternative strategies for forecasting VaR of VIX futures, and for managing financial risk under the Basel II Accord, are several univariate conditional volatility models, specifically GARCH, EGARCH and GJR, with each based on either the Gaussian and Student t distributions.

The main criterion for choosing among the alternative strategies was minimizing average daily capital charges. In this project, we used a methodology based on stochastic dominance that permits partial ordering of strategies by accommodating the entire distribution of DCC values. This methodology provides a search for uniformly higher ranked volatility models, based on large classes of evaluation functions and the entire DCC distribution.

EGARCH Model

One of the main weaknesses of the GARCH models is that the conditional variance responds symmetrically to "positive" and "negative" past innovations. On the other hand, in finance literature, there is evidence that current return is negatively correlated with the future return volatility. To remedy such weaknesses, the EGARCH models was proposed.

EGARCH model allows for asymmetric effects for positive and negative asset returns. The following weighted innovation is used

$$g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - E|\varepsilon_t|]$$

The general EGARCH(p,q) is

$$a_t = \sigma_t\varepsilon_t$$

$$\ln(\sigma^2_t) = \alpha_0 + ((1 + \beta_1 B + \cdots + \beta_{p-1}B^{p-1}) / (1 - \alpha_1 B - \cdots - \alpha_q B^q))\, g(\varepsilon_{t-1}) \text{ (or)}$$

$$\ln(\sigma^2_t) = \alpha_0 + ((1-\omega B) / (1-\delta B))\, g(\varepsilon_{t-1})$$

Properties of EGARCH model:

1. Relaxation of positive constraint of model coefficients
2. The $g(\varepsilon_t)$ enables the model to respond asymmetrically to positive and negative a_t

Code

```
install.packages(c("tseries", "forecast", "zoo", "FinTS", "rugarch"))

library("tseries")

library("zoo")

library("forecast")

library("FinTS")

library("rugarch")

setwd("C:/Users/manal/Desktop/FTS project")

price=as.data.frame(vixcurrent$VIXClose)

date = as.Date(vixcurrent$Date, "%m/%d/%Y")

plot(date,vixcurrent$VIXClose)

N=length(price[,1])

VIX.returns=100*(log(price[2:N,])-log(price[1:(N-1),]))

VIX.returns

VIX.returns=c(VIX.returns,1)

plot(date,VIX.returns)

VaR5n=NULL

logret=VIX.returns

for (i in 1:59)
```

```
{
model=garchFit(~garch(1,1),data=logret,trace=F,cond.dist='norm')
model
x=as.data.frame(predict(model,5));
x
VaR5n =append(VaR5n,x$meanForecast[1]- qnorm(1-0.05)*(x$meanError[1]))
VaR5n
logret=logret[-1]
}
VaR5n
meanVaR=NULL
for (i in 1:59)
{
  meanVaR=append(meanVaR,mean(VaR5n[1:i]))
}
meanVaR
summary(meanVaR)
Gaussian_DCC=NULL
for (i in 1:59)
{
  Gaussian_DCC=append(Gaussian_DCC,min(meanVaR[i],VaR5n[i]))


}
Gaussian_DCC
norm_Gaussian_DCC=normalized(Gaussian_DCC)
summary(norm_Gaussian_DCC)
#Calculating Daily Capital Charge Using Forecasted VaR Calculated by
GARCH(1,1)DCC=max(-VaR5n60day,-VaR5n)


#prediction of VaR using GARCH(1,1)-student t distribution
tdist_logret=VIX.returns
```

```
tdist_VaR5n=NULL
for (i in 1:59)
{
tdist_model=garchFit(~garch(1,1),data=tdist_logret,trace=F,cond.dist=c("std"))
x=as.data.frame(predict(tdist_model,5))
tdist_VaR5n =  append(tdist_VaR5n,x$meanForecast[1]- qnorm(1-0.05)*(x$meanError[1]))
#VaR5nt
tdist_logret=tdist_logret[-1]
}
tdist_VaR5n

tdist_meanVaR=NULL
for (i in 1:59)
{
  tdist_meanVaR=append(tdist_meanVaR,mean(tdist_VaR5n[1:i]))
}
tdist_meanVaR
summary(tdist_meanVaR)
#Calculating Daily Capital Charge Using Forecasted VaR Calculated by
GARCH(1,1)DCC=max(-VaR5n60day,-VaR5n)
tdist_DCC=NULL
for (i in 1:59)
{
  tdist_DCC=append(tdist_DCC,min(tdist_meanVaR[i],tdist_VaR5n[i]))


}
tdist_DCC
norm_tdist_DCC=normalized(tdist_DCC)
summary(norm_tdist_DCC)


gjr_logret=VIX.returns
```

```
spec.gjrGARCH = ugarchspec(variance.model=list(model="gjrGARCH", garchOrder=c(1,1)), mean.model=list(armaOrder=c(1,1), include.mean=TRUE), distribution.model="norm")

gjrGARCH <- ugarchfit(gjr_logret, spec=spec.gjrGARCH)

gjrGARCH

spec2=spec.gjrGARCH

setfixed(spec2)=as.list(coef(gjrGARCH))

filt=ugarchfilter(spec2,gjr_logret)

gjr_VaR=fitted(filt)+sigma(filt)+qdist("norm",p=0.05,mu=0,sigma=1,skew=coef(gjrGARCH)["skew"],shape=coef(gjrGARCH)["shape"])

gjr_VaR[(length(gjr_VaR) -60):length(gjr_VaR)]

gjr_meanVaR=NULL

for (i in 1:59)

{

  gjr_meanVaR=append(gjr_meanVaR,mean(gjr_VaR[1:i]))

}

gjr_meanVaR

summary(gjr_meanVaR)

gjrGaussian_DCC=NULL

for (i in 1:59)

{

  gjrGaussian_DCC=append(gjrGaussian_DCC,min(gjr_meanVaR[i],gjr_VaR[i]))


}

gjrGaussian_DCC

norm_gjrGaussian_DCC=normalized(gjrGaussian_DCC)

summary(norm_gjrGaussian_DCC)

gjr_logret=VIX.returns

spec.gjrGARCH = ugarchspec(variance.model=list(model="gjrGARCH", garchOrder=c(1,1)), mean.model=list(armaOrder=c(1,1), include.mean=TRUE), distribution.model="std")

gjrGARCH <- ugarchfit(gjr_logret, spec=spec.gjrGARCH)
```

```
gjrGARCH

spec2=spec.gjrGARCH

setfixed(spec2)=as.list(coef(gjrGARCH))

filt=ugarchfilter(spec2,gjr_logret)

fitted(filt)

sigma(filt)

qdist("sstd",p=0.05,mu=0,sigma=1,skew=coef(gjrGARCH)["skew"],shape=coef(gjrGARCH)
["shape"])

gjr_tVaR=fitted(filt)+sigma(filt)+qdist("sstd",p=0.05,mu=0,sigma=1,skew=coef(gjrGARCH)
["skew"],shape=coef(gjrGARCH)["shape"])

gjr_tVaR[(length(gjr_VaR) -60):length(gjr_VaR)]

gjr_tmeanVaR=NULL

for (i in 1:59)

{

  gjr_tmeanVaR=append(gjr_tmeanVaR,mean(gjr_tVaR[1:i]))

}

gjr_tmeanVaR

summary(gjr_tmeanVaR)

gjrtdist_DCC=NULL

for (i in 1:59)

{

  gjrtdist_DCC=append(gjrtdist_DCC,min(gjr_tmeanVaR[i],gjr_VaR[i]))


}

gjrtdist_DCC

norm_gjrtdist_DCC=normalized(gjrtdist_DCC)

summary(norm_gjrtdist_DCC)


normalized=function(x)

{

  z=(x-mean(x))/stdev(x)
```

```
  return (z)
}
par(mfrow=c(2,2))
plot(ecdf(norm_Gaussian_DCC), col = "red",main="GARCH-Gaussian Vs GARCH-student t")
lines(ecdf(norm_tdist_DCC), col = "blue")


plot(ecdf(norm_gjrGaussian_DCC), col = "red" ,main="GJR-Gaussian Vs GJR-student t")
lines(ecdf(norm_gjrtdist_DCC), col = "blue")

plot(ecdf(norm_tdist_DCC), col = "red" , main="GARCH-student t Vs GJR-student t")
lines(ecdf(norm_gjrtdist_DCC), col = "blue")

plot(ecdf(norm_Gaussian_DCC), col = "red", main="GARCH-Gaussian Vs GJR-Gaussian")
lines(ecdf(norm_gjrGaussian_DCC), col = "blue")
```