Numerical Analysis Of Ideological Outreach In Social Media Networks

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Problem Statement

Understanding the dynamics of ideological outreach on social media networks is crucial for comprehending the influence of these platforms on society and politics. The aim of this project is to develop a numerical simulation model that analyses the process of ideological outreach within social media networks. By employing numerical methods, this study seeks to uncover the mechanisms and patterns through which various ideologies spread across digital platforms, enabling the formulation of informed strategies to understand and engage with these dissemination processes effectively.

Ideological Outreach refers to the deliberate dissemination and promotion of specific beliefs, ideologies, or perspectives through social media networks. This report analyses ideological outreach and the most important governing attributes in the following scenario -

1. Small Network
2. Community Type Network
3. Twitter-like network
4. Election Campaigning Analysis

Objectives are presented below:

I. Network Model

A synthetic social media network is created where individuals are represented as nodes and associated with vectors of relevant characteristics quantifying content engagement, social interaction, receptiveness, and network position.

II. Assumptions

Make necessary assumptions to analyse the network.

III. Governing Equations And Parameters

Formulate equations governing the spread of ideas within the network. Integrate parameters that capture the influence of interactions, exposure, and resistance to new ideas.

IV. Initial Conditions

Define realistic initial conditions that mirror social media communities' varying ideological stances and user preferences. Distribute characteristics to individuals based on suitable probability distribution curves.

V. Numerical Simulation Algorithm

Implement suitable numerical methods to simulate the propagation of ideology based on the equations of the information propagation model.

VI. Scenario-Specific Sensitivity Analysis

Conduct sensitivity analysis to identify critical parameters influencing the speed and extent of ideology spread in various scenarios.

Social Network Model

The NetworkX library of Python is used to create a synthetic social media network. In this network, the nodes represent people, and the edges represent connections between people on the platform. The following procedure has been used to generate the graph –

1. Using the NetworkX library in Python, a graph with a fixed number of nodes is created.

2. The nodes are assigned a random number of connections (degree) based on the Gaussian distribution.

3. Each node is then connected to N other nodes selected randomly among all the nodes, where N is the number of connections for that particular node.

4. This graph representation is stored as an adjacency list, a dictionary in Python that gives information about the nodes to which any particular node is connected.

5. Each person (node) is assigned attributes quantifying content engagement, social interaction, receptiveness, and network position.

Assumptions

1. A network of 1000 people has been analysed so as to keep the algorithm's runtime within limits.
2. The distribution of properties of individuals, like activity and number of connections, has been considered to be Gaussian in nature.

Attributes And Parameters

The following characteristics characterise each person(node)-

1. Ideology (i) – This parameter demarcates the current level of acceptance of the idea by the person. A negative value indicates opposition to the idea. The scale is not linear in nature. Instead, the growth rate is logarithmic in nature.

   Ideology is signified by numerical value, where positive value signifies positive alignment towards the idea, and negative value signifies opposition to the idea. A zero value means the person has a neutral perspective towards the idea.

   In more detail, in the General Equations section.

2. Receptiveness (R) – Refers to an individual's openness and willingness to accept and engage with new ideologies presented within the social media environment.
3. Activity (A) - Activity signifies the level of engagement and participation of an individual within the social media network, which sums up actions such as posting, commenting, sharing, and interacting with other users and content.

4. Persuasiveness (P) - Persuasion power measures an individual's ability to influence and convince others regarding specific ideologies or viewpoints through their communication and persuasive skills within the social media space.
5. Suggestibility (S) - Suggestibility represents the degree to which an individual is prone to adopting or being influenced by external ideologies presented within the social media network, indicating their susceptibility to external influence.
6. Exponential Factor(E) - Represents the size of the network. Also, the exponential factor is in the governing equation.
7. Acceptance value - The ideological value upon reaching which an individual is considered to have accepted the idea is the acceptance value.
8. Single Sect Model - No opposing ideology is considered. The values of ideologies are zero or positive.
9. Dual Sect Model - Opposing ideologies are considered. The value of ideologies can be negative, zero, or positive.

Governing Equations Proportionality Analysis

1. Derivation for persuasion power (P)

Consider a person A in the social network.

Then i_a, A_a, R_a, P_a and S_a represent the ideology, activity, receptiveness, persuasion power, and suggestibility of A, respectively.

The persuasive power of A is in direct proportionality to his own ideology.

$$P_a \propto i_a \tag{1}$$

It is also directly proportional to his activity level on the social media network.

$$P_a \propto A_a \tag{2}$$

Hence, from (1) and (2), the persuasive power of A can be written down as:

$$P_a \propto i_a A_a \tag{3}$$

The above proportionality can be rewritten in equation form as:

$$P_a = k_1 i_a A_a + c_1 \quad (4)$$

where k_1 is a proportionality constant and c_1 is some real constant.

2. Derivation for Suggestibility (S)

Consider there are only two persons in a network, and there is a connection between the two persons, A and B.

Let i_a, A_a, R_a, P_a and S_a represent the ideology, activity, receptiveness, persuasion power and suggestibility of A, respectively. Similarly i_b, A_b, R_b, P_b and S_b represent the ideology, activity, receptiveness, persuasion power, and suggestibility of B, respectively. Considering the influence of A on B, the suggestibility of B is directly proportional to the ideology of B.

$$S_b \propto i_b \quad (5)$$

Also, the receptiveness of B has a major role in deciding how easily he can be influenced. Hence, suggestibility is considered to be directly proportional to the square of receptiveness.

$$S_b \propto R_b^2 \quad (6)$$

We can say that the suggestibility of B is directly proportional to its activeness on the social media platform.

$$S_b \propto A_b \quad (7)$$

Hence, from (5), (6) and (7), the suggestibility of B can be written as:

$$S_b \propto i_b A_b R_b^2 \quad (8)$$

The above proportionality can be rewritten in equation form as:

$$S_b = k_2 i_b A_b R_b^2 + c_2 \quad (9)$$

where k_2 is a proportionality constant and c_2 is some real constant.

3. Ideological Evolution

Considering the same scenario of a system of two individuals A and B.

We will derive the rate of change of ideological alignment of B due to the influence of A . We know that the rate of change in the ideology of B is dependent on the persuasive power of A as well as the suggestibility of B. Let $f(P_a)$ be the function which accounts for the effect of the persuasive power of A and $g(S_B)$ be the function which accounts for the effect of suggestibility of B in change of ideology of B. So, we can say:

$$\frac{di_b}{dt} = f(P_a) * g(S_b) \tag{10}$$

The rate of change of ideological alignment of B due to the influence of A has an exponential relationship to the persuasive power of A. Therefore,

$$f(P_a) \propto E^{P_a} \tag{11}$$

This proportionality can be written as:

$$f(P_a) = k_3 E^{P_a} + c_3 \tag{12}$$

where k_3 is the proportionality constant and c_3 is some real constant.

For getting $g(S_B)$, there are two possible cases which are discussed below.

Case I) Both individuals have the same ideological alignment. This means that they both agree to the idea, but the degree of alignment may differ.

The rate of change of ideological alignment of B due to the influence of A has an exponentially decaying relationship to the suggestibility of B. This means that a person with a closer to neutral perspective will have a larger growth in his ideological alignment due to influence compared to a person who already strongly believes (relatively) in the idea. Hence,

$$g_1(S_b) \propto E^{-S_b} \tag{13}$$

This proportionality can be expressed in the equation form as below.

$$g_1(S_b) = k_4 E^{-S_b} \quad (14)$$

where k_4 is the proportionality constant.

Case II) The two individuals had opposing ideologies.

Consider the ideological alignment of A to be demarcated by the '+' sign and B by the '-' sign. In the situation of A influencing B, the magnitude i_b will decrease. Hence, the rate of change of ideological alignment of B due to the influence of A has an exponential relationship to suggestibility.

$$g_2(S_b) \propto E^{S_b} \quad (15)$$

The above proportionality can be written in equation form as:

$$g_2(S_b) = E^{k_5 S_b} \quad (16)$$

where k_5 is the proportionality constant.

4. Extension to Multi-Nodal Model

For a large network consisting of more than two persons, a certain person A is impacted by all of his connections, that is A_1, A_2, A_3, …, A_m. Let the rate of change in the ideological alignment of $A\ by\ A_k$ be denoted by $\frac{di_k}{dt}$.

In this situation, considering a finite time length, the following equations are used to calculate the final change in ideological value -

Case-1: If an individual is impacted by individuals of the same alignment only -

$$\frac{di}{dt} = max(\frac{di_1}{dt}, \frac{di_2}{dt}, \ldots, \frac{di_m}{dt}) \quad (17)$$

Case-2: If an individual is impacted by individuals of opposite alignment only -

$$\frac{di}{dt} = min(\frac{di_1}{dt}, \frac{di_2}{dt}, \ldots, \frac{di_m}{dt}) \quad (18)$$

Case-3: If an individual is impacted by individuals of the same as well as opposite alignment -

$$\frac{di}{dt} = avg(\frac{di_1}{dt}, \frac{di_2}{dt}, \ldots, \frac{di_m}{dt}) \quad (19)$$

Quantification of Attributes

The exponential factor E is equal to the size of the social network, that is, the number of nodes in the network model. This is to ensure that the values i do not explode on conducting a large number of iterations. This is also in sync with the individual's reduction in suggestibility and gains in persuasion power with the increase in the network size. Hence,

$$E = 1000$$

The acceptance value for all the scenarios discussed has been taken to be 0.6.

From (1) and (5), we can see Suggestibility and Persuasive Power to have direct proportionality to the ideology value. A higher activity suggests an increased S as well as P and vice versa. Hence, for activity(A) -

$$A \in [0.9, 1.1]$$

Using the same argument, the value of receptiveness(R) -

$$R \in [0.9, 1.1]$$

Then, the values of suggestibility and persuasive power are calculated using the above values.

Boundary Conditions And Final Equations Derivation

Again, consider the binary model in which person A and B are the only connection. Also, i_a, A_a, R_a, P_a and S_a represent the ideology, activity, receptiveness, persuasion power and suggestibility of A, respectively. Similarly i_b, A_b, R_b, P_b and S_b represent the ideology, activity, receptiveness, persuasion power, and suggestibility of B, respectively.

From equation (4), we know:

$$P_a = k_1 i_a A_a + c_1$$

If person A has 0 as ideological value, then he has null persuasion power. Hence at i_a = 0, P_a = 0. Therefore, c_1= 0. Also, at A_a = 1, P_a = i_a. Therefore, k_1=1.

Hence, the final equation is

$$P_a = i_a A_a \tag{20}$$

Similarly, using the same argument, considering equation (9) for person A, the equation becomes:

$$S_a = i_a A_a R_a^2 \tag{21}$$

Now, from equation (12), we have

$$f(P_a) = k_3 E^{P_a} + c_3$$

In the situation where A has null persuasive power, the change in ideological alignment 0 is nullified. That is, at P_a = 0, $f(P_a)$ = 0. On putting these in the above equation, we get $c_3 = (-k_3)$.

The saturation limit of persuasive power is taken as 1. In such a case, $f(P_a)$ = 1. On applying these boundary conditions in the above equation, we get: $k_3 E + c_3 = 1$.

Now, putting $c_3 = (-k_3)$ in this equation, we get: $k_3 = \frac{1}{E-1}$

On putting the above value of k_3 and c_3 in equation (12), we get:

$$f(P_a) = \frac{E^{P_a}-1}{E-1} \tag{22}$$

Consider the equation (14), for case-(I) that is both individuals have the same ideological alignment.

$$g_1(S_b) = k_4 e^{-S_b}$$

A person with 0 suggestibility means he has a neutral ideology. In this case, he must have a maximum change in ideological alignment in reference to other individuals. Hence, at $S = 0$, $g_1(S_b) = 1$. On applying this boundary condition to the above equation, we get $k_4 = 1$. Therefore, the above equation becomes:

$$g_1(S_b) = e^{-S_b} \tag{23}$$

Now consider case-(II) that is, the two individuals had opposing ideologies. From equation (16), we know:

$$g_2(S_b) = E^{k_5 S_b}$$

Considering the ideological alignment of A to be demarcated by the '+' sign and B by the '-' sign. In this case, a person with a neutral perspective must have the maximum change in ideological alignment in reference to others (as others have an opposing opinion).

However, the magnitude of change in the ideological alignment of a person due to influence by a person of opposing views should be lesser than the magnitude of change generated for a person of the same alignment.

Considering that factor to be 10%, we get $k_5 = (1/10)$. So, the above equation becomes:

$$g_2(S_b) = E^{(\frac{S_b}{10})} \tag{24}$$

From equation (10), we know:

$$\frac{di_b}{dt} = f(P_a) * g(S_b)$$

Now, using equation (22), (23) and (24), the final equation for ideological evolution becomes:

For case (I): Both individuals have the same ideological alignment.

$$\frac{di_b}{dt} = \left(\frac{E^{P_a}-1}{E-1}\right)\left(E^{-S_b}\right) \tag{25}$$

For case (II): The two individuals had opposing ideologies.

$$\frac{di_b}{dt} = \left(\frac{E^{P_a}-1}{E-1}\right)\left(E^{\frac{S_b}{10}}\right) \tag{26}$$

Initial Conditions

Truncated Gaussian distribution has been used to distribute attribute values to people (nodes) because Gauss normal distribution expands from -∞ to ∞.

$$f(x;\mu,\sigma,a,b) = \frac{1}{\sigma}\frac{\psi\left(\frac{x-\mu}{\sigma}\right)}{\phi\left(\frac{b-\mu}{\sigma}\right)-\phi\left(\frac{a-\mu}{\sigma}\right)} \tag{27}$$

where, $$\psi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma\sqrt{2\pi}}e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

and, $$\phi(x) = \frac{1}{2}\left(1 + erf\left(\frac{x}{\sqrt{2}}\right)\right)$$

σ = mean, μ = standard deviation

a,b = lower and upper truncation limits, respectively.

Activity and Receptiveness - The general distribution of activity has the following parameters -

$$\sigma = 1,\ \mu = 0.1,\ a = 0.9,\ b = 1.1$$

Initial Ideology Values - Relevant initial ideological values have been considered in respective scenarios.

Duration - The algorithm is run for a period of 100 days, that is, 100 iterations.

Numerical Solution

We want to perform the analysis over a period of several days.

Hence, we can take dt = 1 day.

Using the ODE equations (25) and (26), we can then apply the Eulers method to perform the iterations. By Euler's first-order approximation, we know:

$$\frac{dy}{dx} = f(x)$$

can be approximated to:

$$y_{j+1} = y_j + (x_{j+1} - x_j) f(x_j) \tag{28}$$

where $(x_{j+1} - x_j)$ is known as step-size. In our case, step-size $(dt) = 1$.

On applying Euler's first-order approximation to the differential equations (25) and (26), we get:

For case-(I):

$$(i_b)_{j+1} = (i_b)_j + \left(\frac{E^{P_a} - 1}{E - 1}\right)\left(E^{-S_b}\right) \tag{29}$$

For case-(II):

$$(i_b)_{j+1} = (i_b)_j + \left(\frac{E^{P_a} - 1}{E - 1}\right)\left(E^{\frac{S_b}{10}}\right) \tag{30}$$

Numerical Simulation Algorithm

The algorithm is as follows -

1. Consider a network of E nodes (people) and distribute the number of connections each person has randomly using truncated Gaussian distribution.
2. Now, create an adjacency list based on the number of connections of each node computed above. This connection of a node to other nodes is again random based on truncated Gaussian distribution.

3. Now, for each node, create a vector ($NodeValues$ in our code) that stores the attributes of that node and initiates the values of ideology, activity, and receptiveness for all nodes. This distribution is also a truncated Gaussian distribution.
4. Calculate the attributes of persuasion power and suggestibility for each individual using equations (20) and (21), respectively.
5. Now, for updating the attributes of each node after a day (in each iteration), create a dictionary ($TempIData$ in our code) with nodes as keys and an empty list as values.
6. Traverse over each node and calculate what its updated ideologies will be after interacting with its direct connections and store it in the list (key is the node, and value is the list in our dictionary $TempIData$).
7. For updating the final ideology of nodes after a day (end of an iteration), we follow the below algorithm. For a node N with the current ideology I if the list is L ($TempIData[N] = L$):
    a. If all values in L are greater than I, (i.e. each connection favours increasing the ideology of N), we update I with maximum value in L $max(L)$).
    b. If all values in L are smaller than I, (i.e. each connection favours to decrease the ideology of N), we update I with minimum value in L $min(L)$).
    c. Otherwise, we update I with the average of values in the list. (i.e. $avg(L)$).
8. The value of ideology is updated now at the end of the day.
9. The new values of persuasion power and suggestibility are calculated using the new ideology values and updated in $NodeValues$.
10. The above algorithm is run for the required number of iterations, where one iteration corresponds to one day.

Scenario-Specific Sensitivity Analysis

1. Small Network Analysis (Single Sect Model)

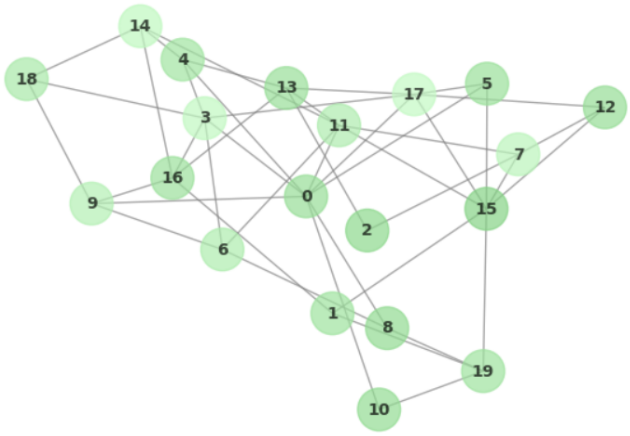
To elucidate the understanding of the mathematical model and the simulation algorithm, as well as verify the accuracy of the model, we will first analyse a small network of 20 people. Here, we have considered a single-sect model (there is no opposing view).

The initial conditions are as follows -

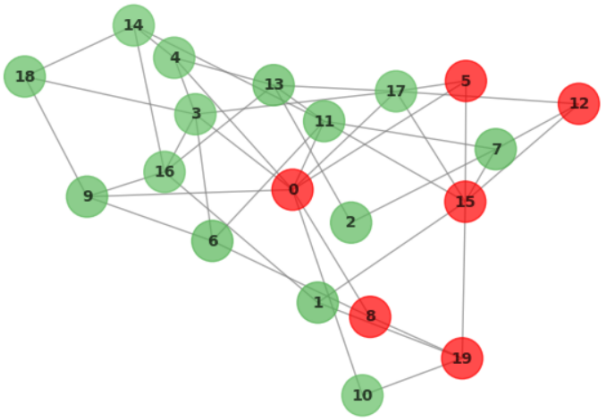
- Ideology: Truncated Gaussian distribution: $\sigma = 0.2$ and $\mu = 0.1$
- Connection distribution: count =20, σ =3, μ =2, a =1, b =5

The intensity of the colour of the node demarcates how strongly aligned a person is to the ideology.

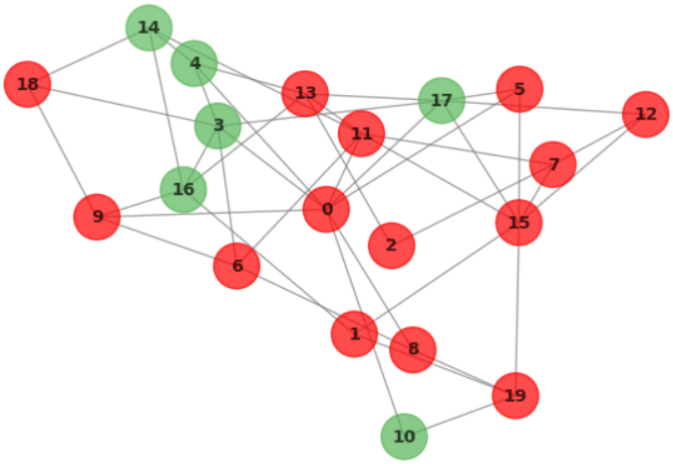
A person having an ideology of 0.6 or above is considered to have accepted the ideology. The node colour is red for such a case. The initial network and its evolution are as follows -



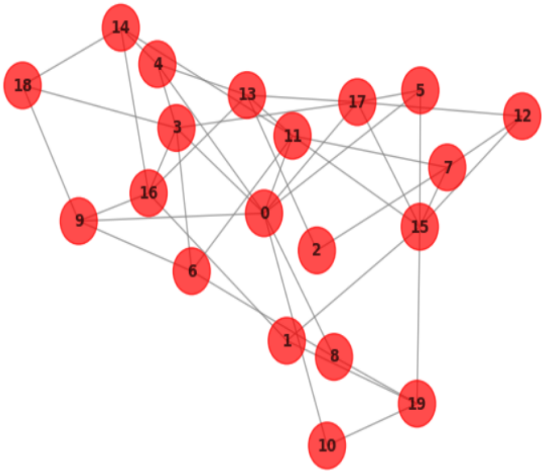
Initial Network



Network after 200 days



Network after 300 days



Network after 400 days

Result And Discussion

The above 4 graphs show the increasing number of individuals who have accepted the ideology. This gives the idea of the working of the model.

2. Community Type Network Analysis (Single Sect Model)

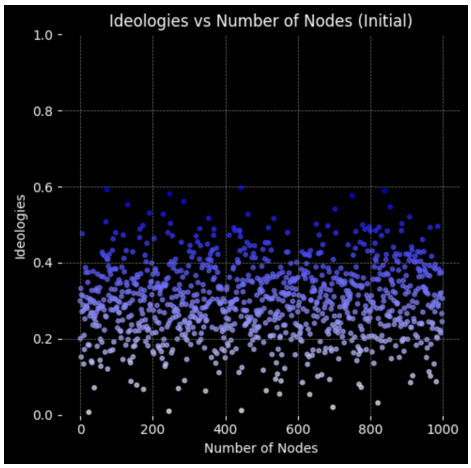
Aim - To analyse the relationship between network's overall ideological acceptance with how strongly connected the network is.

A community type network refers to a network with identically distributed properties for all the individuals in the network, including number of connections, activity and receptiveness.

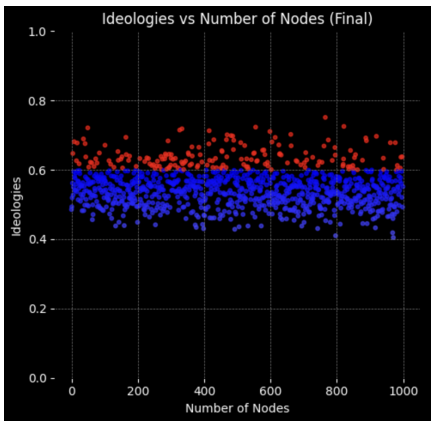
Initial conditions are as follows -

- Ideology - Truncated Gaussian distribution - $\sigma = 0.3$ and $\mu = 0.1$
- Connection distribution - count =20, σ =3, μ =2, a =1, b =5

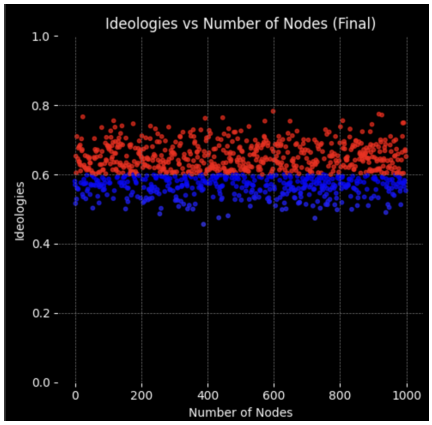
The below added graph shows the initial network's ideological alignment -



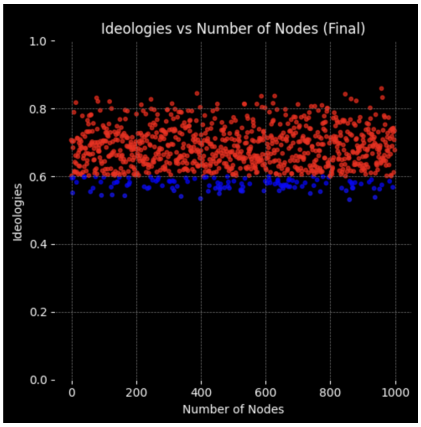
The number of connections of an individual is varied from 10 to 200 to understand how the ideological alignment changes as the network becomes more strongly connected.The below pictures illustrate apon this -



At 10 connections



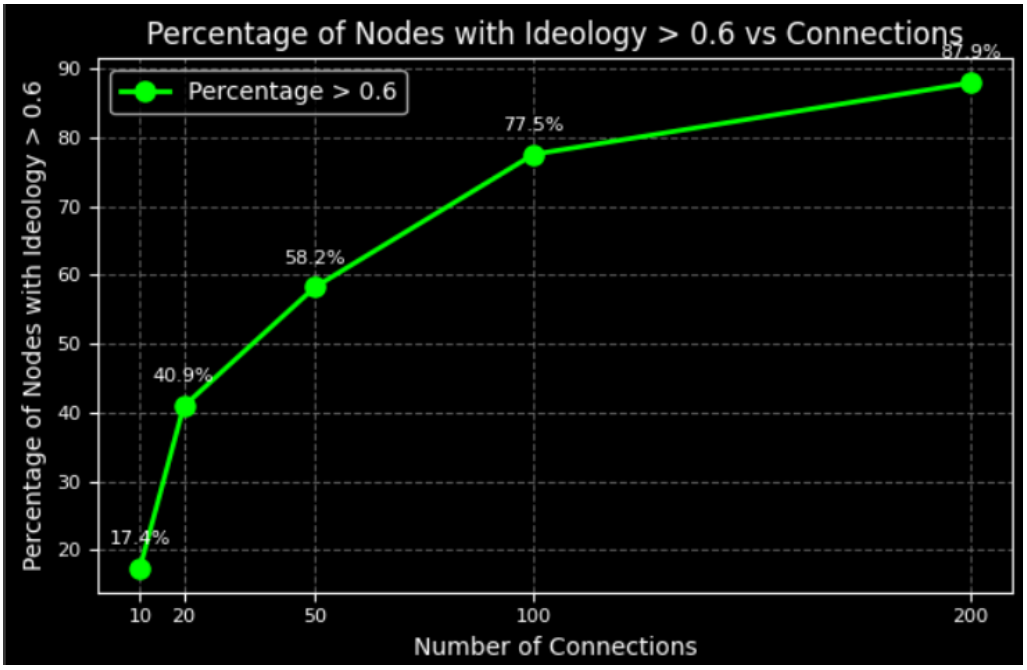
At 50 connections



At 200 connections

Result And Discussion

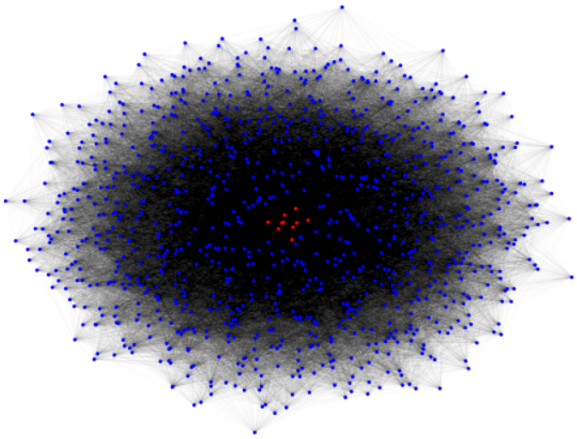
The following graph is obtained that shows that as the network becomes more strongly connected, a higher percentage of people accept the ideology in the same time period of 100 days.



3. Twitter Type Network Analysis (Influencer Model)

Aim - The aim is to understand how powerfully a highly connected individual can impact the overall ideological alignment of the network.

A twitter type network refers to a model that resembles a real like social media network with a few individuals (influencers) who are highly connected and hence have large influencing power. The below image shows the influencers in red colour -

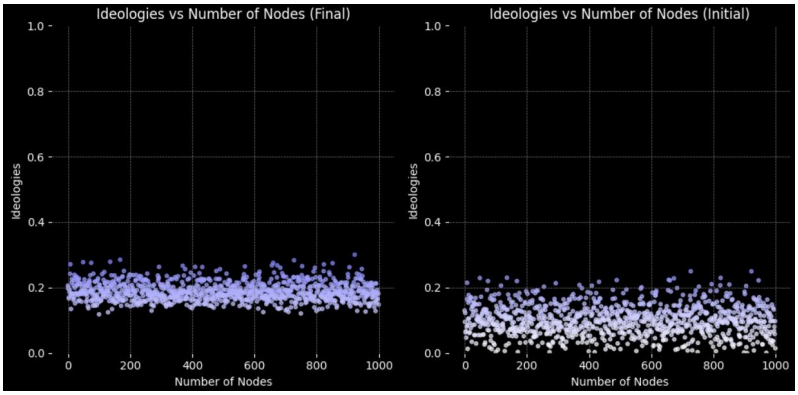


Visualisation of the network model

Initial conditions are as follows -

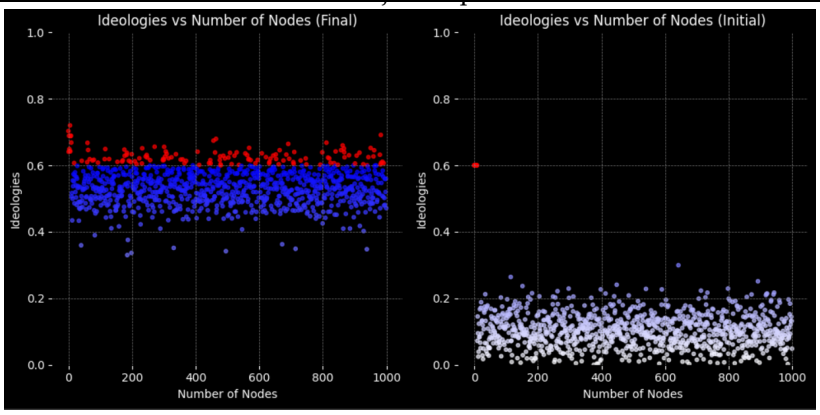
- Ideology - Truncated Gaussian distribution - $\sigma = 0.1$ and $\mu = 0.05$
- Connection distribution - $\sigma =$, $\mu = 2$, $a = 10$, $b = 100$
- 10 influencers with an initial ideological value of 0.6 and a high number of connections (300) are induced in the network.
- A comparison is drawn in the situation of the network with and without them.

The following graphs show the network after 100 days without the influencers -



After 100 days Initial situation

The following graphs show the network after 100 days with the influencers -



After 100 days Initial situation

Result And Discussion

We see that the influencers were able to convince people much more of there ideologies in comparison to a normal community-type network.

4. Election Campaigning Analysis (Dual Sect Model)

Aim - To analyze the most important factors that influence people's choice in an election.

Consider two parties A and B. Positive ideology value indicates support for party A and negative ideology value indicates support for party B.

Initial conditions are as follows -

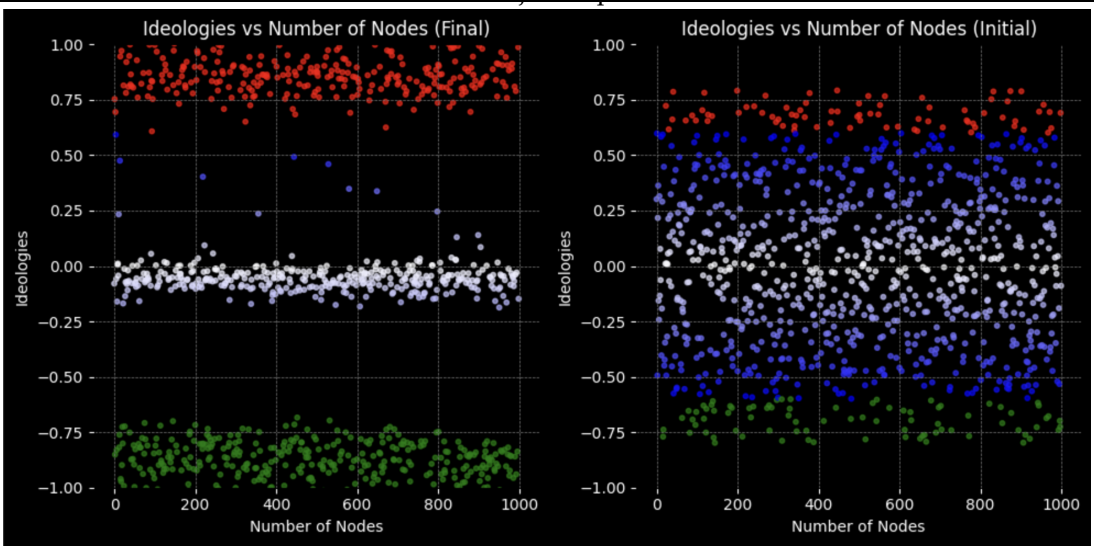
- Ideology - Truncated Gaussian distribution - $\sigma = 0$ and $\mu = 0.2$
- Connection distribution - per person
- A base condition is created to compare later with other factors.

Base Condition -

After 100 days -

Party A - 33.3 %

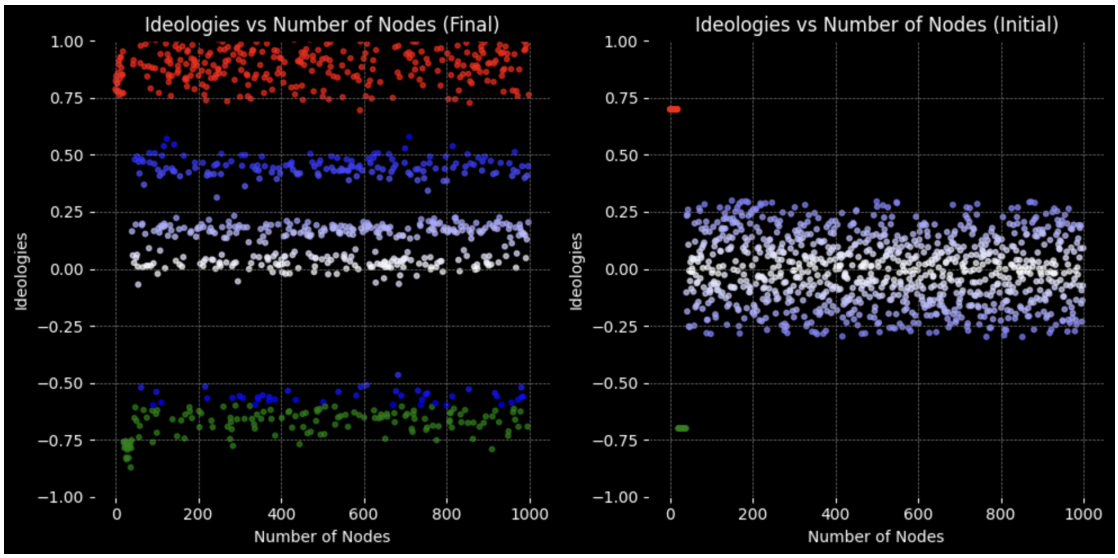
Party B - 14.4 %



Base condition

Impact Of High Connections -

Party A is given 20 people with 200 connections each.



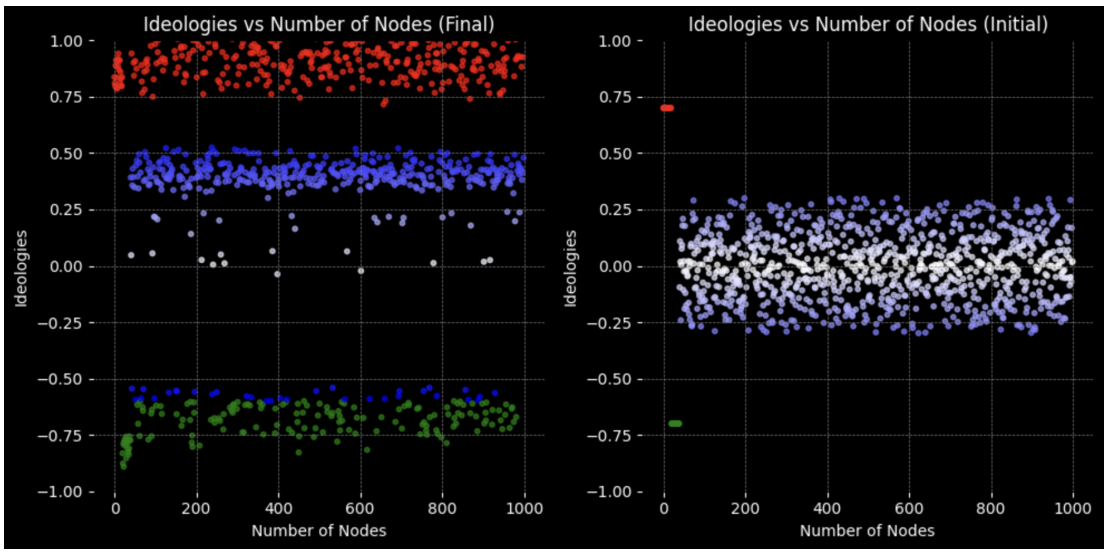
After 100 days -

Party A - 39.8%

Party B - 16.2%

High Connections versus high count -

Party A is given 4 people with 400 connections and Party B is given 20 people with 50 connections.



After 100 days -

Party A - 39.2%

Party B - 7%

Result And Discussion

Highly influential individuals win over large number of less connected party.

References

[1] ‘Truncated Normal Distribution’, Wikipedia.org, Link

[2] ‘Inside Twitter: An In-Depth Look Inside the Twitter World’, Sysomos, April 2014.