

Interpreter for Geometric Constructions

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ABSTRACT

In this project, we take up the problem of designing and implementing a language-independent interpreter for drawing geometric diagrams, focusing mainly on ruler and compass based construction problems. We start with the use cases and motivations on why such a system would be useful and what places deploying it would be fruitful. We give a brief tabulated summary review of related research work done on geometry related problems and point out the common trait that they lack the ability to decipher any problem/constraint expressed in a natural language. We, then, move to describing the design of the interpreter and the usage of cross-lingual technique to provide language-independent interpretation ability. We briefly mention how is the alignment model exploited to realize the powerful translation feature. This is followed by accounts about the work done during this semester, results obtained, difficulties faced and plan for the next semester.

1 Objective

To design and implement interpreter that

- is language-independent (works for English, Hindi at present)
- Receives steps for a geometric construction as input e.g. "Draw a line segment AB of length 4 cm", "केंद्र B और त्रिज्या 5 सेमी लेकर एक चाप खींचिए जो पहले खींची चाप को C पर काटता हो" etc
- Outputs the geometric figure obtained on executing the given sequence of steps

2 Introduction

It is often the case that students, teachers, architects and artists need to draw complex diagrams manually using simple geometric instruments like ruler, compass, set squares, dividers etc. This demands labor, time as well as expertise. Resorting to sophisticated graphics applications requires knowhow of application-specific details as well as expertise in coordinating hand micro-movements.

In order to save these resources required in drawing geometric diagrams as well as to reduce dependence on complex graphics applications, this project introduces an interpreter for diagram construction steps expressed in a suitable natural language. This not only simplifies the construction, but also leads to easily understood natural language ‘programs’.

3 Related work

One of the most common techniques today to perform effective parsing is to use Part-of-Speech tagged language banks. Only few languages (about 8-10 of the 600+ languages in wide use) have the privilege of being resource-rich in the sense of having Penn-tree banks and other large-sized standardised corpus. An important observation is that fixed grammars, POS tags etc used in traditional parsers are not always the best way to break up a language. Taking a note of these shortcomings, we take a statistical approach to learn a language.

There has been a significant amount of work done on solving geometry construction problems. Gulwani et. al. [2] propose a method that uses goal-based heuristic to simulate backward deduction to solve a problem expressed in a predefined logical construct. Schreck et. al. [5] talk about the same problem but use CAD methods to deal with constraints. At the same time, Itzhaky et. al. [3] use the number of nondeterministic choices as a measure of a good solution. Ahmed, Umair et. al. [1] look more into using domain-specific measures to minimize parser errors and augment the geometry problem solver, GeoSynth. All these papers have provided us with valuable insights into selecting important and expressive constructs for our intermediate language.

We summarize our observations in terms of following 3 parameters:

- Uses other linguistic/domain knowledge
- Assumes linguistic cues are already translated into logical forms
- Uses Parse knowledge

Paper	Uses other linguistic/domain knowledge	Assumes linguistic clues already translated into logical forms	Uses parse knowledge
Geometric Construction Problem Solving in Computer-Aided Learning [5]	YES	YES	NA
Synthesizing geometry constructions [2]	YES	YES	NA
Solving geometry problems using a combination of symbolic and numerical reasoning [3]	YES	YES	NA
Can modern statistical parsers lead to better natural language understanding for education? [1]	YES	NO	YES
Learning to map sentences to logical form: Structured classification with probabilistic categorical grammars [6]	YES	NO	YES

Table 1: Related Works

Unlike all previous attempts, we do not assume that the logical forms of the sentences are available, nor we assume the availability of a parser. We use cross-lingual alignment to map the given sentence in natural language to metalanguage. It is therefore applicable to a large range of languages. We demonstrate this for two widely known languages belonging to very different families - Hindi and English.

4 Design

The interpreter consists of two main components:

- Aligner
- Metalanguage Interpreter

4.1 Aligner

Cross-lingual alignment is the heart of the learning mechanism in this project. It is used to obtain mapping/alignment between a natural language and our carefully designed and structured imperative metalanguage, L0. Since this alignment can be obtained for any natural language, the application is scalable to any number of natural languages.

We have used GIZA++ [4] as the cross-lingual aligner. GIZA++ is a statical machine translation toolkit that is used to train IBM Models 1-5 and an HMM word alignment model.

4.2 Metalanguage Interpreter

The interpreter component exploits the alignment model obtained from the Aligner component. Given a sentence expressed in a natural language, we use the alignment model to get statistically most probable interpretation in L0. A command in L0 is then used to output the desired figure.

5 Cross-Lingual alignment

Cross-Lingual alignment is a technique to statistically align the words of a given pair of languages. It is close to translating one language into another, without any syntactic or semantic knowledge of any of the languages. As an example, for a pair of languages, given sufficiently many sentences as illustrated in the table 2, a cross-lingual alignment assigns probabilities to the event that a particular source-language token is mapped to a target-language token.

English	Hindi	Meta Language
Construct a line AB of length 4 cm	4 सेमी लम्बाई का एक रेखाखण्ड AB खींचिए	construct lineSegment AB length 4 cm
With A as center and radius 3 cm, draw an arc	केंद्र A और त्रिज्या 3 सेमी लेकर एक चाप खींचिए	construct arc center A radius 3 cm
With B as center and radius 5 cm, draw an arc cutting the previously drawn arc at C	केंद्र B और त्रिज्या 5 सेमी लेकर एक चाप खींचिए जो पहले खींची चाप को C काटता हो	construct intersectingArc center C radius 5 cm cuts arc previous at C

Table 2: Sample Corpus

English	Meta Language	Probability
Construct	construct	1.00
Line segment	lineSegment	1.00
intersects	intersect	0.93
intersect eachother	intersect	0.78
cut eachother	intersect	0.87
cut	cut	0.98
join	join	1.00
mark	mark	0.98
label	mark	0.98
bisector	bisector	0.60
interior	interior	0.20
exterior	exterior	0.20

Table 3: Sample alignment between English and Metalanguage

Hindi	Meta Language	Probability
लगा दीजिये	construct	0.90
खींचिए	construct	0.98
रचना कीजिये	construct	0.96
बनाइये	construct	0.98
रेखाखण्ड	lineSegment	1.00
परस्पर प्रतिच्छेद	intersect	0.98
काटता हो	cut	0.82
काटे	cut	0.95
मिलाइये	join	0.98
जोड़िये	join	0.98
अंकित कीजिये	mark	0.98
मान लीजिये	mark	0.98
समद्विभाजक	bisector	0.59
अभ्यन्तर	interior	0.20
बहिर्भाग	exterior	0.20

Table 4: Sample alignment between Hindi and Metalanguage

6 What has been done

6.1 Corpus

The corpus contains around 350 sentences from the geometry construction field in both the languages. These were collected from the NCERT textbooks of standards 6th to 9th. English-to-Metalanguage and Hindi-to-Metalanguage sentence maps were generated manually which were used to train GIZA++.

This approach has been shown to work for simple construction steps. Output diagram for the steps above is shown in Figure 2.

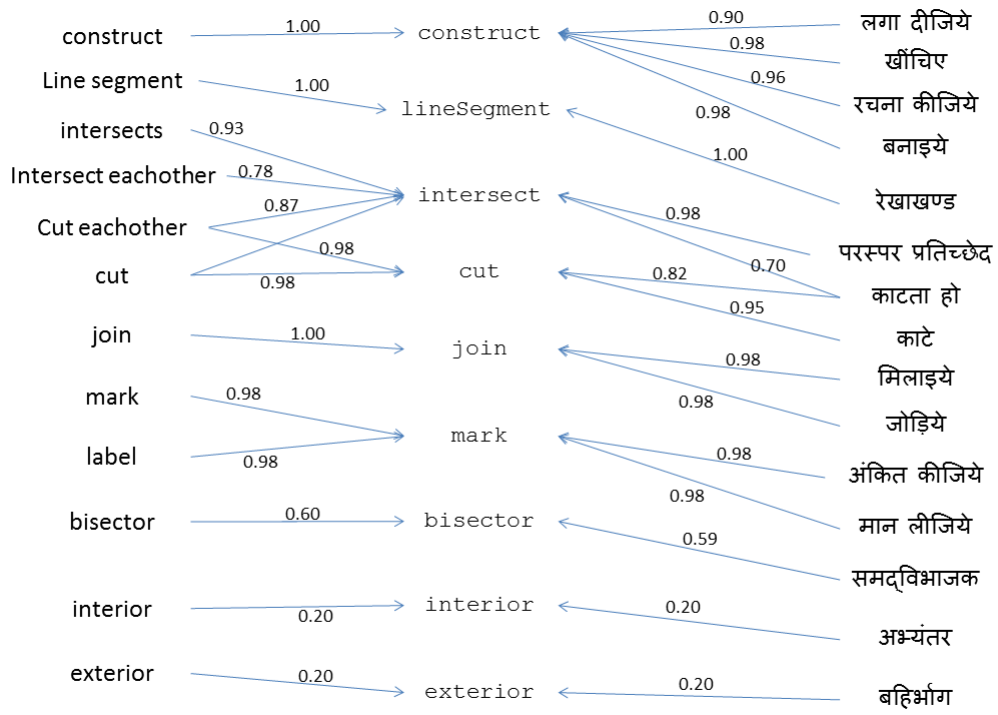


Figure 1: Graphical visualization of the sample alignment

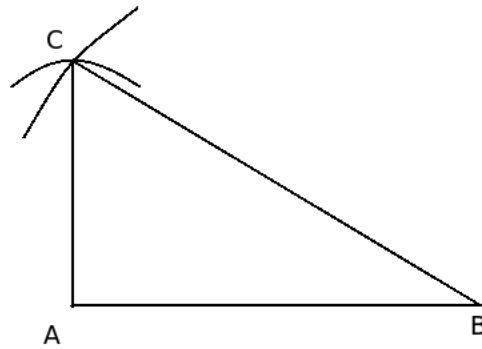


Figure 2: Constructed Triangle

7 Results

1. Number of sentences in the corpus: 360 each in English-to-Metalanguage and Hindi-to-Metalanguage corpus
2. A sample prototype implementation that demonstrates the performance of the system for simple construction steps of drawing line segments and arcs.
3. Number of unique tokens in each of the three languages:

Language	Number of unique tokens
English	181+
Hindi	169+
Metalanguage	110+

Table 5: Unique tokens in different languages

7.1 Difficulties

- **Anaphoras** Dealing with anaphoras like “this”, “it”, “previous”, “last” etc in sentences like “Mark any point M on it”, “एक सुविधाजनक त्रिज्या लेकर पिछले चरण वाले चाप को बिन्दु A पर काटें” etc
- **Underspecified Parameters** We also need to provide for unspecified radii, lengths and angle measures e.g. “With A and B as centers and a suitable radius, draw two arcs intersecting each other at point C”
- **Probabilistic Mapping** Since the mapping is statistical in nature, there might be various close alignments for the same source-language word. This forces us to resort to try out different possible alignments and look for the one that fits e.g. for the sentence “Construct AB of length 7.8cm”, we get the following alignments with their corresponding probabilities (we assume that probabilities of word-wise alignment are independent of each other)

Mapped meta-language sentence	Probability
construct AB any length 7.8cm	0.716853
construct AB lineSegment length 7.8cm	0.21081
construct AB angle length 7.8cm	0.0723206
construct AB center length 7.8cm	1.90645e-06

Table 6: Map to possible metalanguage sentences alongwith their probabilities

8 What needs to be done

1. Enrich the corpus to cover larger domains of expressions e.g. having larger number of primitive steps, providing for higher-level primitives, specifying relative distances, dealing with anaphoras and references
2. Expand the functionality of the system to work for all kinds of construction steps claimed so far
3. Deploy the system on a webserver to demonstrate its functionality and work towards making it more robust
4. In order to capture colloquial (non-academic) ways of expressing construction steps, we need to develop a user interface that augments the corpus with input sentences

References

- [1] Umair Z Ahmed, Arpit Kumar, Monojit Choudhury, and Kalika Bali. Can modern statistical parsers lead to better natural language understanding for education? In *Computational Linguistics and Intelligent Text Processing*, pages 415–427. Springer, 2012.

Focusing on the educational domain, this paper seeks to build robust domain specific natural language understanding modules by suitable pre-processing of the text and use of domain knowledge to deal with parser errors. The paper starts with showing that even off-the-shelf parsers do not perform up to the mark on texts with domain-specific vocabulary and long sentences. The paper also talks about GeoSynth, built using state-of-the-art parsing technology. GeoSynth is a system which can automatically solve geometric constructions word problems.

- [2] Sumit Gulwani, Vijay Anand Korthikanti, and Ashish Tiwari. Synthesizing geometry constructions. In *ACM SIGPLAN Notices*, volume 46, pages 50–61. ACM, 2011.

Aiming to solve ruler and compass based geometry construction problems, the tool described in this paper claims to have synthesized construction steps for problems posed from high-school textbooks and examination papers. It focuses on three important insights viz. reduction of symbolic reasoning to concrete reasoning using verification based on random testing, augmenting the instruction set with higher level constructs and simulating backward deduction used by humans by using a goal-based heuristic.

- [3] Shachar Itzhaky, Sumit Gulwani, Neil Immerman, and Mooly Sagiv. Solving geometry problems using a combination of symbolic and numerical reasoning. Technical report, Technical report, Tel Aviv University, 2012.

This paper combines deductive, numeric and inductive reasoning to solve geometric problems. Using a mixture of these methods, several problems are solved which, otherwise, wouldn't have been solved using the methods individually. The paper uses the number of nondeterministic choices as a measure of how close a partial program is to the solution, claiming this to be useful in grading and providing hints to the students. The implementation takes an average of few seconds on 18 Scholastic Aptitude Test geometry problems and 11 other ruler and compass-based geometry construction problems.

- [4] Franz Josef Och and Hermann Ney. A systematic comparison of various statistical alignment models. *Computational linguistics*, 29(1):19–51, 2003.
- [5] Pascal Schreck, Pascal Mathis, and Julien Narboux. Geometric construction problem solving in computer-aided learning. In *Tools with Artificial Intelligence (ICTAI), 2012 IEEE 24th International Conference on*, volume 1, pages 1139–1144. IEEE, 2012.

This paper introduces CAD methods to deal with Constraint Satisfaction Problems related to geometry, at the same time claims to have met the requirement met by the educational domain. The methods show that results must be construction programs to take into account particular cases. A knowledge-based system implemented in Prolog has also been presented.

- [6] Luke S Zettlemoyer and Michael Collins. Learning to map sentences to logical form: Structured classification with probabilistic categorial grammars. *arXiv preprint arXiv:1207.1420*, 2012.

With the objective of mapping natural language sentences to their counterpart lambda-calculus encoding, this paper introduces an algorithm which learns from the input set of natural language sentences labeled with lambda calculus expressions. The algorithm attempts to infer the underlying distribution over semantic and syntactic analyses conditioned on the input sentences and outputs a log-linear model.