HANDLING ANAPHORAS IN MULTIPLE LANGUAGES IN THE FRAMEWORK OF GEOMETRY CONSTRUCTIONS

PANKAJ PRATEEK

JEETESH MANGWANI

OVERALL OBJECTIVE

To develop a system that tutors the user on geometry construction problems

OBJECTIVE

To design and implement an interpreter for geometric construction sentences that

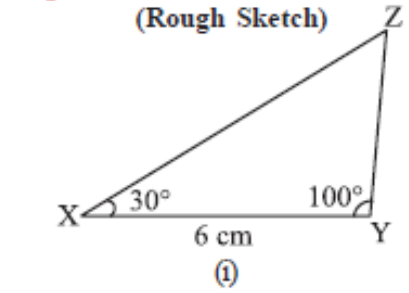
- **Is language-independent (works for English, Hindi at present)**
- **Receives steps for a geometric construction as input e.g. "Draw a line segment AB of length 4 cm", "केंद्र B और त्रिज्या 5 cm लेकर एक चाप खींचिए जो पहले खींची चाप को C पर काटता हो" etc**
- **Handles anaphoras using the task context**
- **Outputs the geometric figure obtained on executing the given sequence of steps**

SAMPLE PROBLEMS

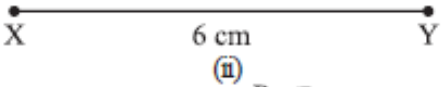
Example 3 Construct ΔXYZ if it is given that $XY = 6$ cm, $m\angle ZXY = 30°$ and $m\angle XYZ = 100°$.

Solution

Step 1 Before actual construction, we draw a rough sketch with measures marked on it. (This is just to get an idea as how to proceed)
[Fig 10.6(i)].



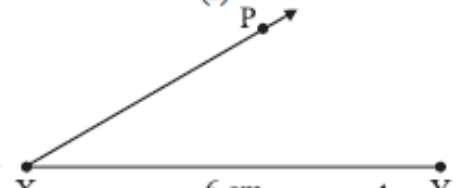
(i)



(ii)

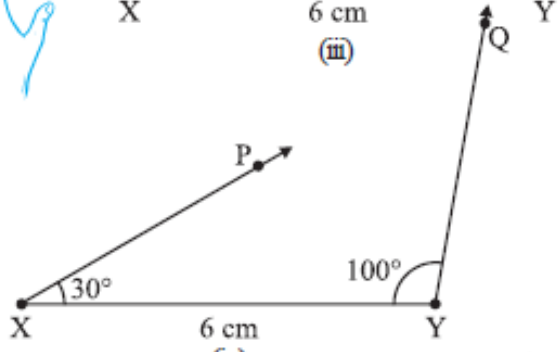
Step 2 Draw XY of length 6 cm.

Step 3 At X, draw a ray XP making an angle of 30° with XY. By the given condition Z must be somewhere on the XP.



(iii)

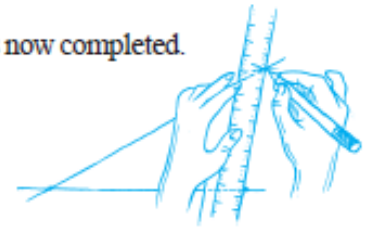
Step 4 At Y, draw a ray YQ making an angle of 100° with YX. By the given condition, Z must be on the ray YQ also.

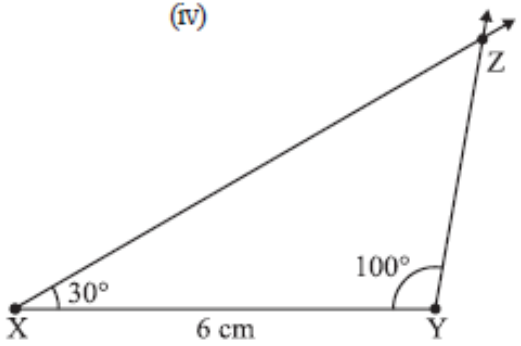


(iv)

Step 5 Z has to lie on both the rays XP and YQ. So, the point of intersection of the two rays is Z.

ΔXYZ is now completed.





(v)

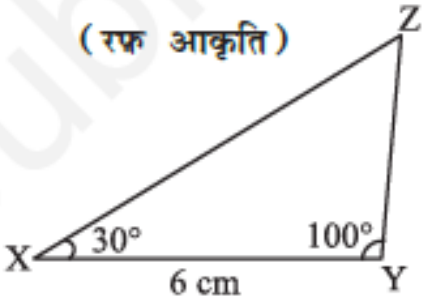
Fig 10.6 (i)–(v)

SAMPLE PROBLEMS

उदाहरण 3 ΔXYZ की रचना कीजिए, यदि, $XY = 6$ cm, $m\angle ZXY = 30°$ और $m\angle XYZ = 100°$ है।

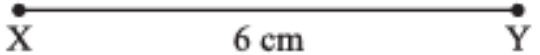
हल

चरण 1 वास्तविक रचना से पहले, हम इस पर अंकित मापों के अनुसार एक रफ़ आकृति खींचते हैं। (इससे कुछ अनुमान लग जाता है कि कैसे रचना की जाए) [आकृति 10.6(i)]।



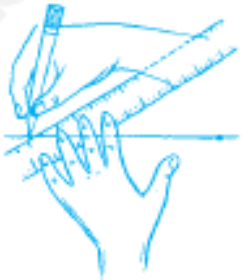
(i)

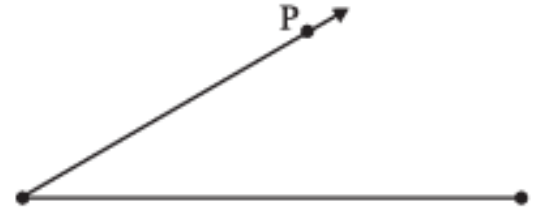
चरण 2 6 cm लंबाई का रेखाखंड XY खींचिए [आकृति 10.6(ii)]।



(ii)

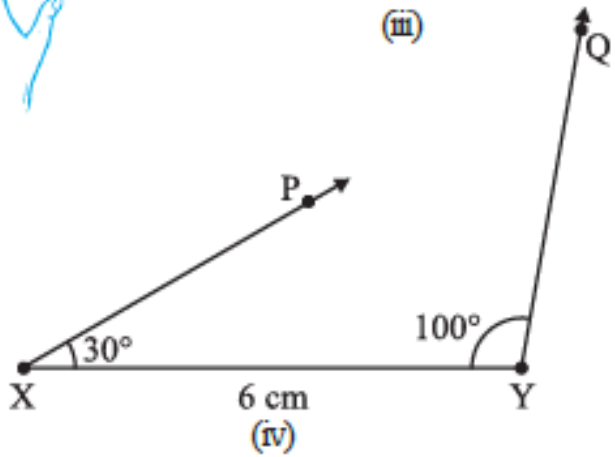
चरण 3 X पर एक किरण XP खींचिए जो XY से 30° का कोण बनाए। दिए हुए प्रतिबंध के अनुसार बिंदु Z किरण XP पर कहीं स्थित होना चाहिए [आकृति 10.6(iii)]।





(iii)

चरण 4 Y पर एक किरण YQ खींचिए, जो YX से 100° का कोण बनाए। दिए हुए प्रतिबंध के अनुसार Z किरण YQ पर भी अवश्य स्थित होना चाहिए [आकृति 10.6(iv)]।



(iv)

PROPOSED APPROACH

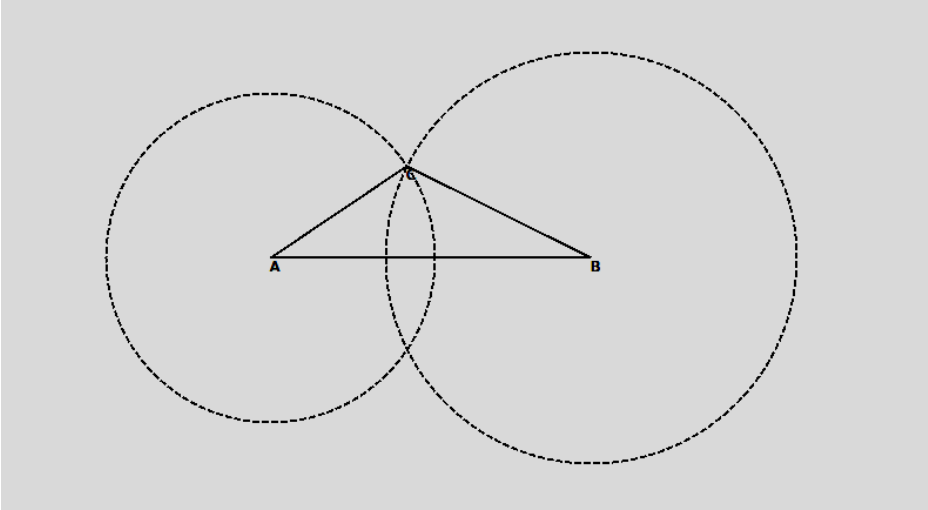
Cross-lingual Alignment

+

Heuristic based parsing

DEMONSTRATION

Construct line segment AB of length 7.8 cm
With A and B as centers and radius 4 and 5 cm draw two arcs intersecting each other at C
Join AC
Join BC



7.8 सेमी लम्बाई का एक रेखाखण्ड AB खींचिए
A और B को केंद्र मानकर 4 सेमी और 5 सेमी त्रिज्या लेकर दो चाप बनाइये जो परस्पर C पर काटें
AC को जोड़िये
BC को जोड़िये

SAMPLE PROBLEM

ENGLISH

Example 1:

Construct a triangle ABC, given that AB=5 cm, BC=6 cm and AC=7 cm.

Solution

1. Draw line segment BC of length 6 cm.

2. With B as center, draw an arc of radius 5 cm.

3. With C as center, draw an arc of radius 7 cm.

4. Mark an intersection point of these arcs as A.

5. Join AB.

6. Join AC.

HINDI

उदहारण 1:

एक त्रिभुज ABC की रचना कीजिये जबकि AB=5 सेमी, BC=6 सेमी और AC=7 सेमी दिया है

हल

1. 6 सेमी लम्बाई का एक रेखाखण्ड BC खींचिए

2. B को केंद्र मानकर और 5 सेमी त्रिज्या लेकर एक चाप खींचिए

3. B को केंद्र मानकर और 7 सेमी त्रिज्या लेकर एक चाप खींचिए

4. इन चापों के प्रतिछेद बिंदु को A से अंकित कीजिये

5. AB को जोड़िये

6. AC को जोड़िये

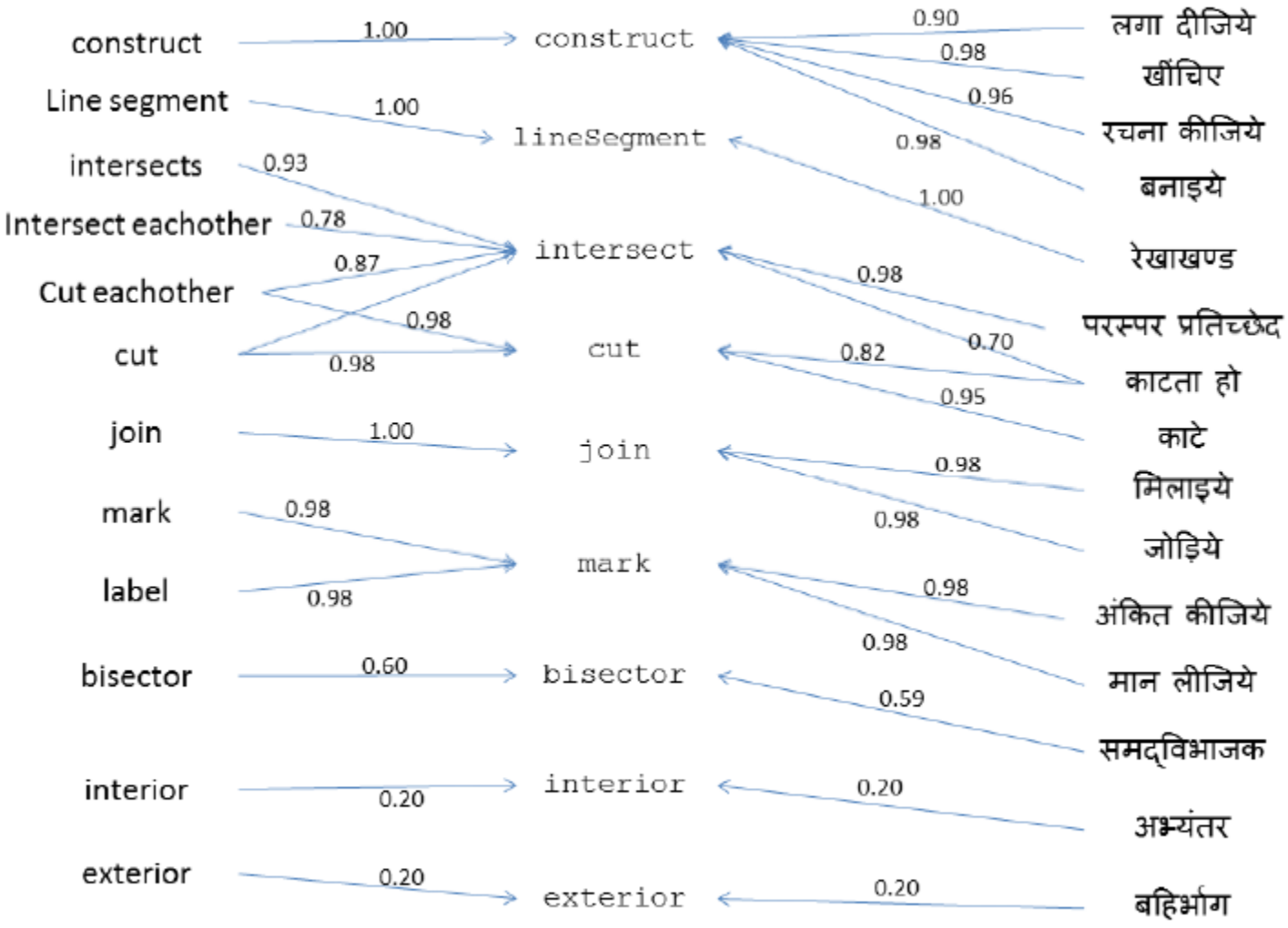
WHAT IS CROSS-LINGUAL ALIGNMENT?

- **Assigns probability to the event that a particular source language token corresponds to a particular target language token**

PARALLEL CORPUS

English	Hindi	Meta Language
Construct a line AB of length 4 cm	4 सेमी लम्बाई का एक रेखाखण्ड AB खींचिए	`construct lineSegment AB length 4 cm`
With A as center and radius 3 cm, draw an arc	केंद्र A और त्रिज्या 3 सेमी लेकर एक चाप खींचिए	`constrcut arc center A radius 3 cm`
With B as center and radius 5 cm, draw an arc cutting the previously drawn arc at C	केंद्र B और त्रिज्या 5 सेमी लेकर एक चाप खींचिए जो पहले खींची चाप को C काटता हो	`construct intersectingArc center C radius 5 cm cuts arc previous at C`

SAMPLE ALIGNMENT



HEURISTICS BASED PARSING

1. Type System
2. Proximity
3. Validity of parse

INTERPRETER

English Input

With A as center and same radius, draw an arc which cuts AB at G

A center same radius construct arc intersecting AB at G

Translation to Metalanguage

Order differs with language!

A center same radius AB G at intersecting arc construct

Hindi Input

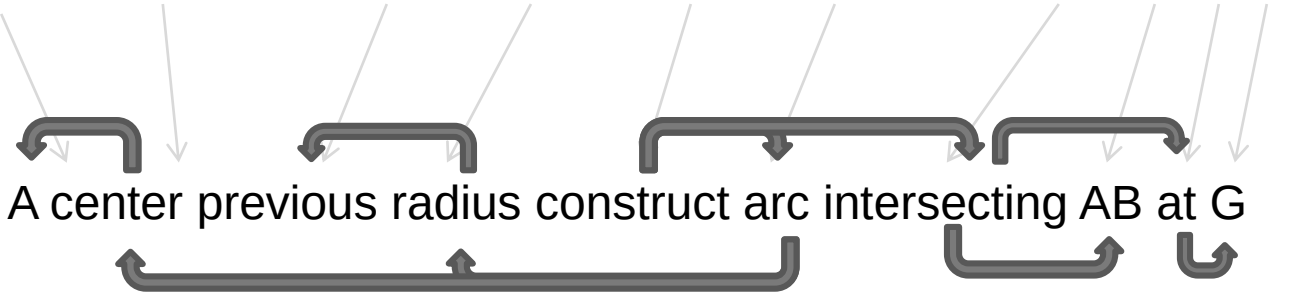
A को केंद्र मानकर और वही चाप लेकर, AB को G पर काटता हुआ एक चाप लगाइये

Stack-based parsing cannot be applied here

INTERPRETER

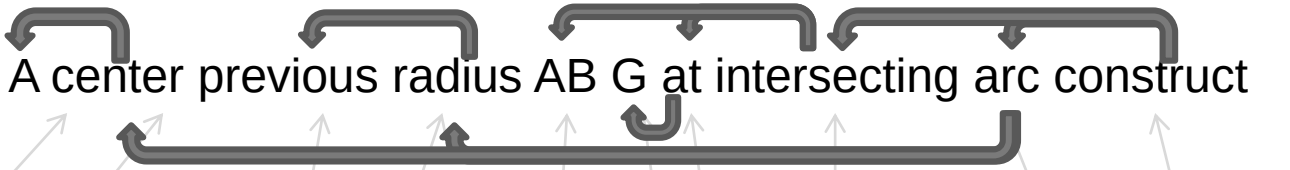
English Input

With A as center and same radius, draw an arc which cuts AB at G



A center previous radius construct arc intersecting AB at G

Translation to Metalanguage

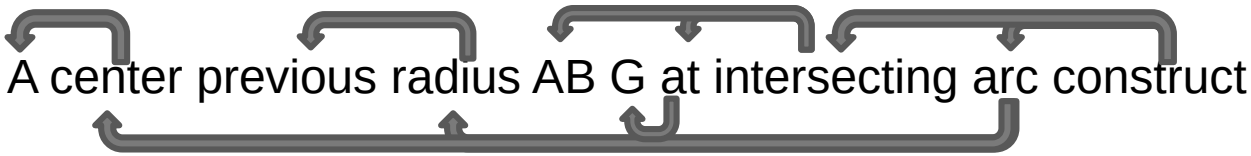


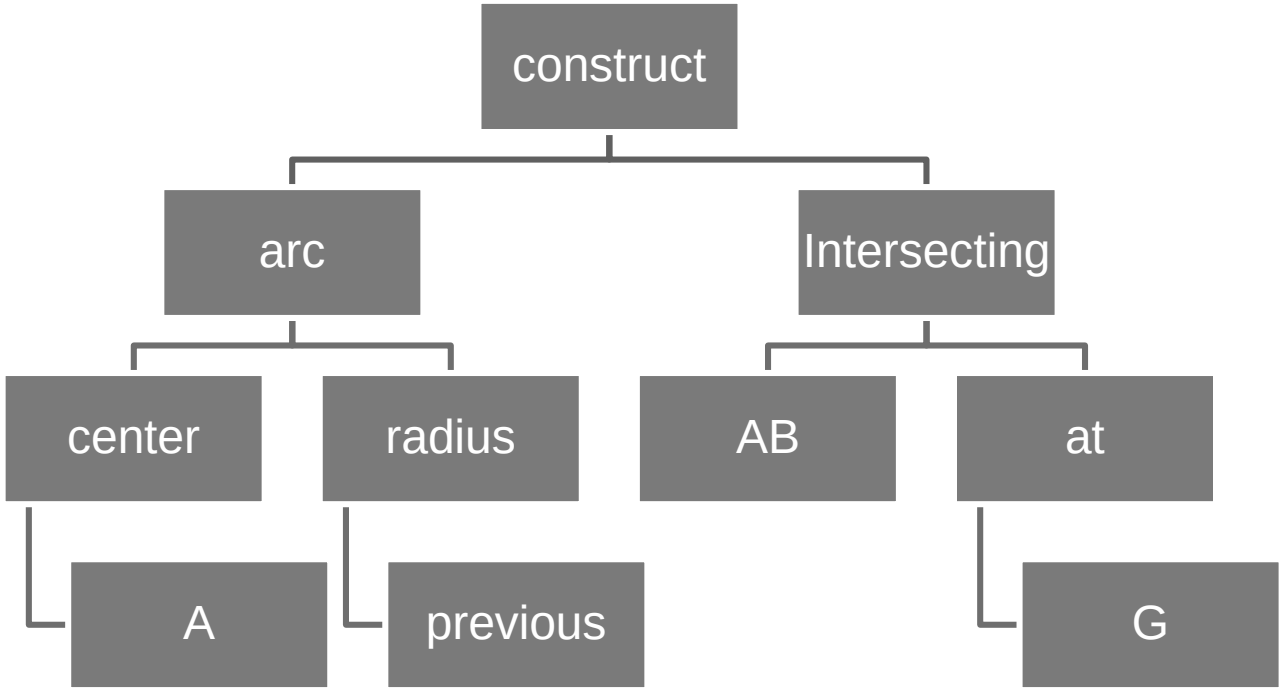
A center previous radius AB G at intersecting arc construct

Hindi Input

A को केंद्र मानकर और वही चाप लेकर, AB को G पर काटता हुआ एक चाप लगाइये

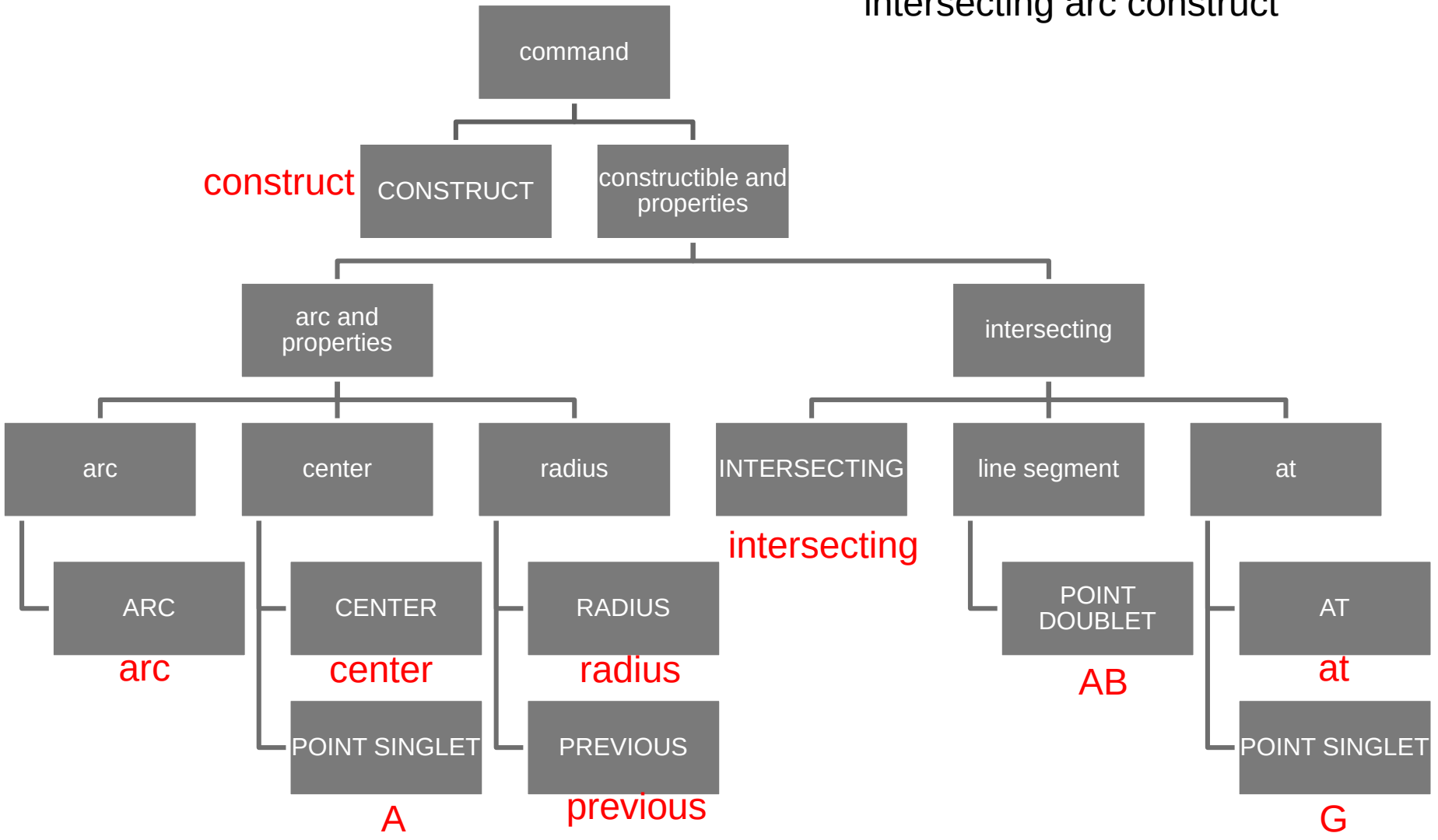
DESIRED TREE STRUCTURE





METALANGUAGE GRAMMAR

A center previous radius AB G at intersecting arc construct



RESOLVING ANAPHORAS

…these arcs…

…those line segments…

Bisect it.

…cutting the previous arc at P.

…its perpendicular bisector.

RESOLVING ANAPHORAS: USING CONTEXT

Points	Line Segments	Arcs	Objects
• A(xa,ya) • B(xb,yb) • C(xc,yc)	• AB	• A(ra) • B(rb)	• LineSegment AB • Arc at A • Arc at B • Point at C

getLastObject();

getLastIntersectableObject();

getLastBisectableObject();

RESULTS

1. The implementation demonstrates the performance of the system for atomic construction statements like “construct a line segment AB of length 5 cm” etc.

2. Statements like “Construct an equilateral triangle of side length 5 cm”, which involve more than one atomic step, have not been implemented.

RESULTS

3. The chapter on Geometry from CBSE NCERT Mathematics textbook for 6th standard contains only the atomic construction steps, while chapters from 7th, 8th and 9th standard contain construction steps which require two or more applications of simple ruler-compass based (atomic) construction step.

Standard/ Grade	Corpus Size	No of Successful Parses	Percentage
6^{th}	84	77	91.67
6^{th} to 9^{th}	225	173	76.44

IMMEDIATE WORK

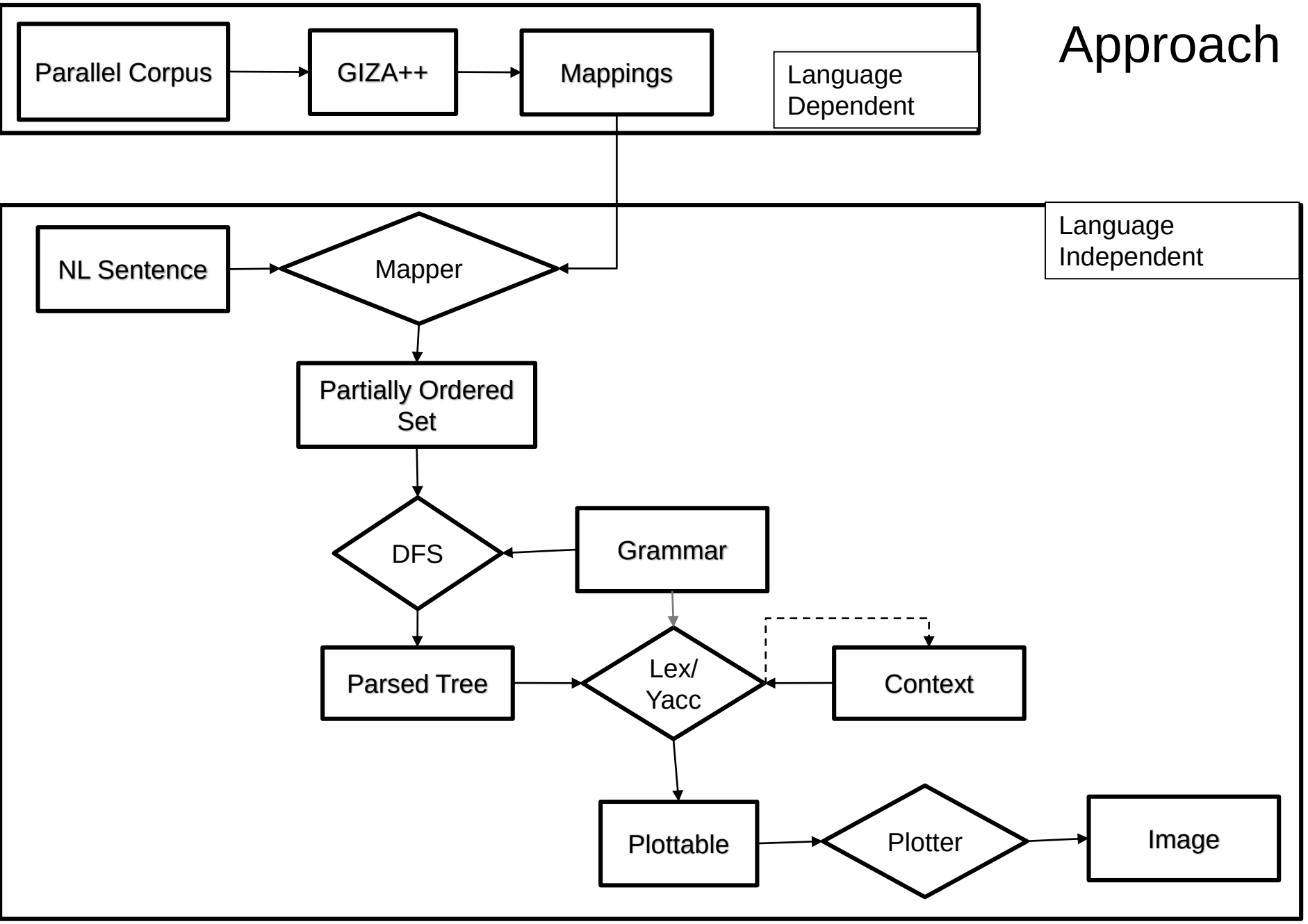
1. Extend as a tutoring system wherein: user presents his solution to the posed problem; the system checks for the correctness of the solution, finds out the place where the proof is wrong, then poses problems based on the concepts he did not use correctly.

2. Use the system to generate problems of various difficulty levels

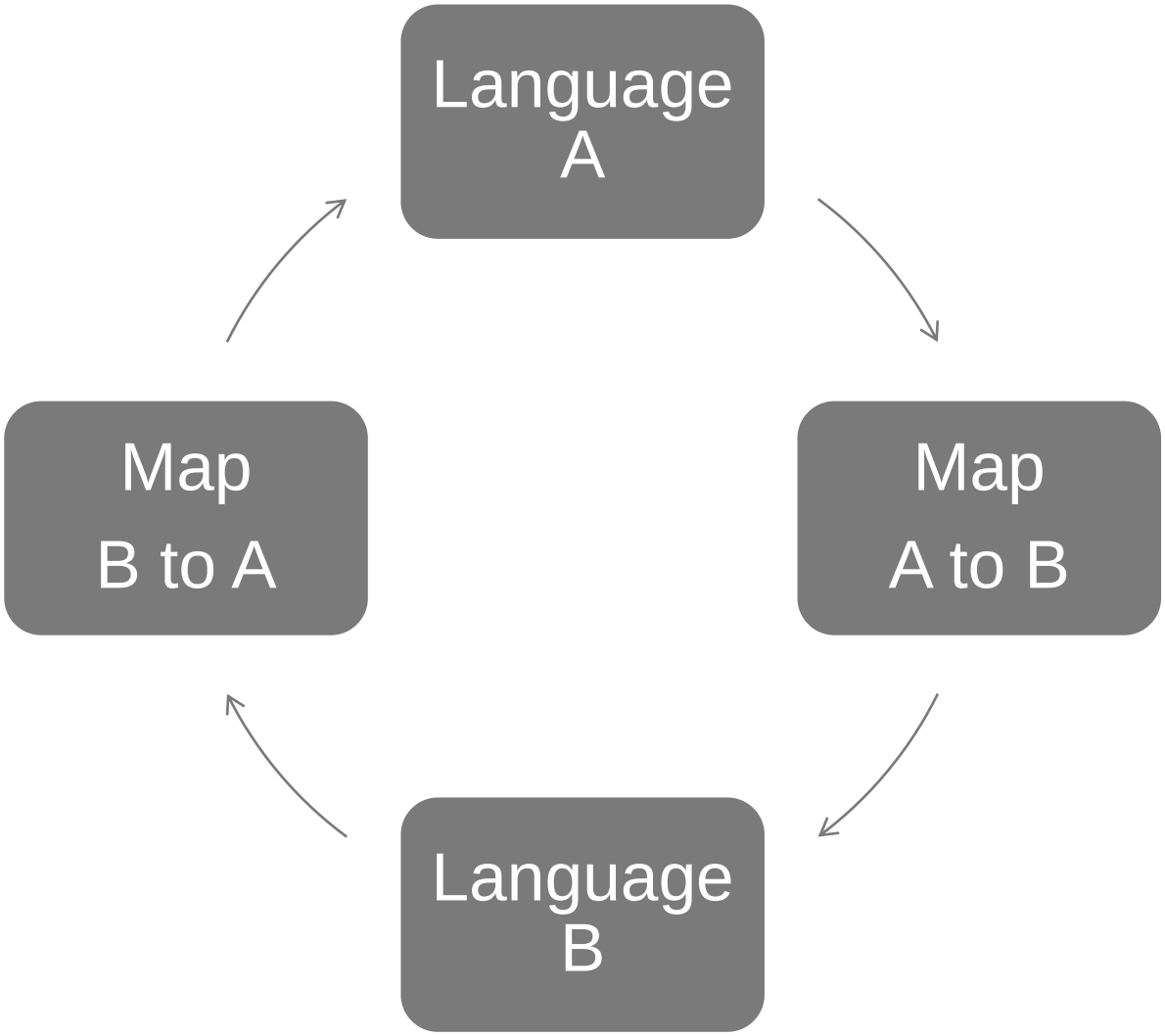
3. The system can be extended to various domains, especially to arithmetic/geometric proving: output the meta-language generated by the system to some existing automated theorem provers (e.g. Coq, Why3) to prove the problems presented by the users

THANK YOU!

Approach

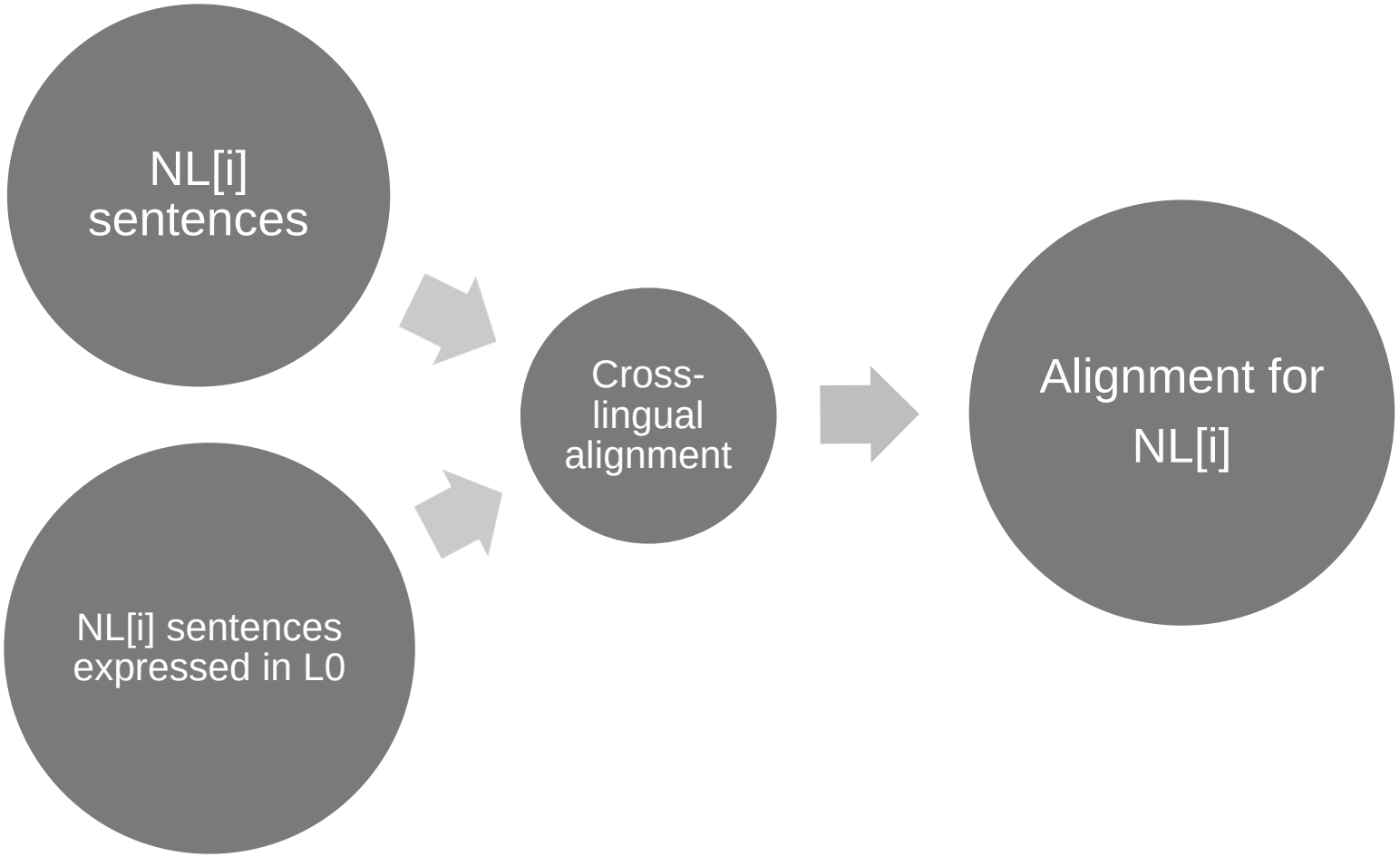

Parallel Corpus
GIZA++
Mappings
Language Dependent
Language Independent
NL Sentence
Mapper
Partially Ordered Set
DFS
Grammar
Parsed Tree
Lex/ Yacc
Context
Plottable
Plotter
Image

ALIGNMENTS

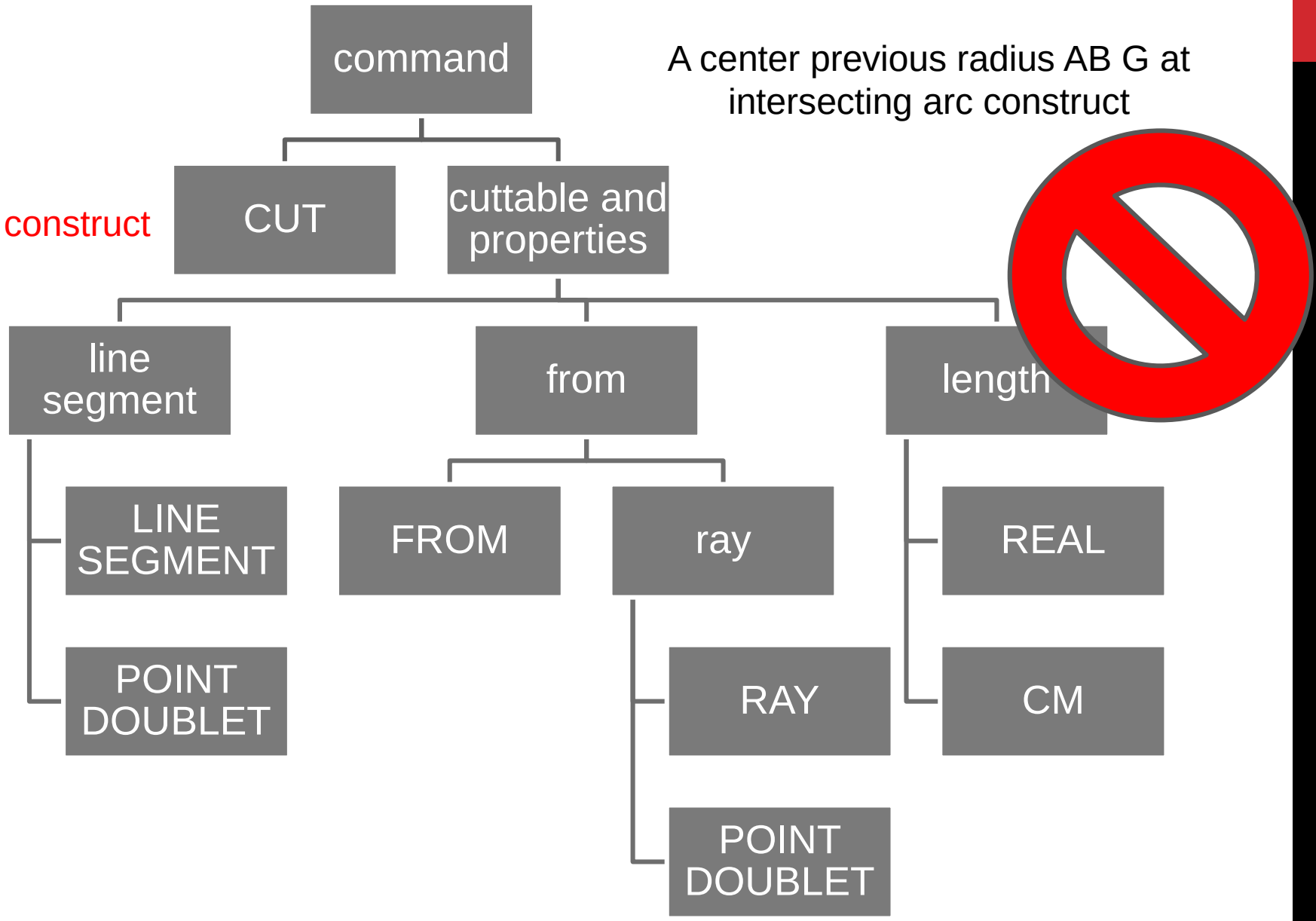


ALIGNING LANGUAGES

L0: Fixed predicate language (carefully designed)
NL[i]: ith natural language, 1<= i <= n
A[i]: word alignment between NL[i] and L0

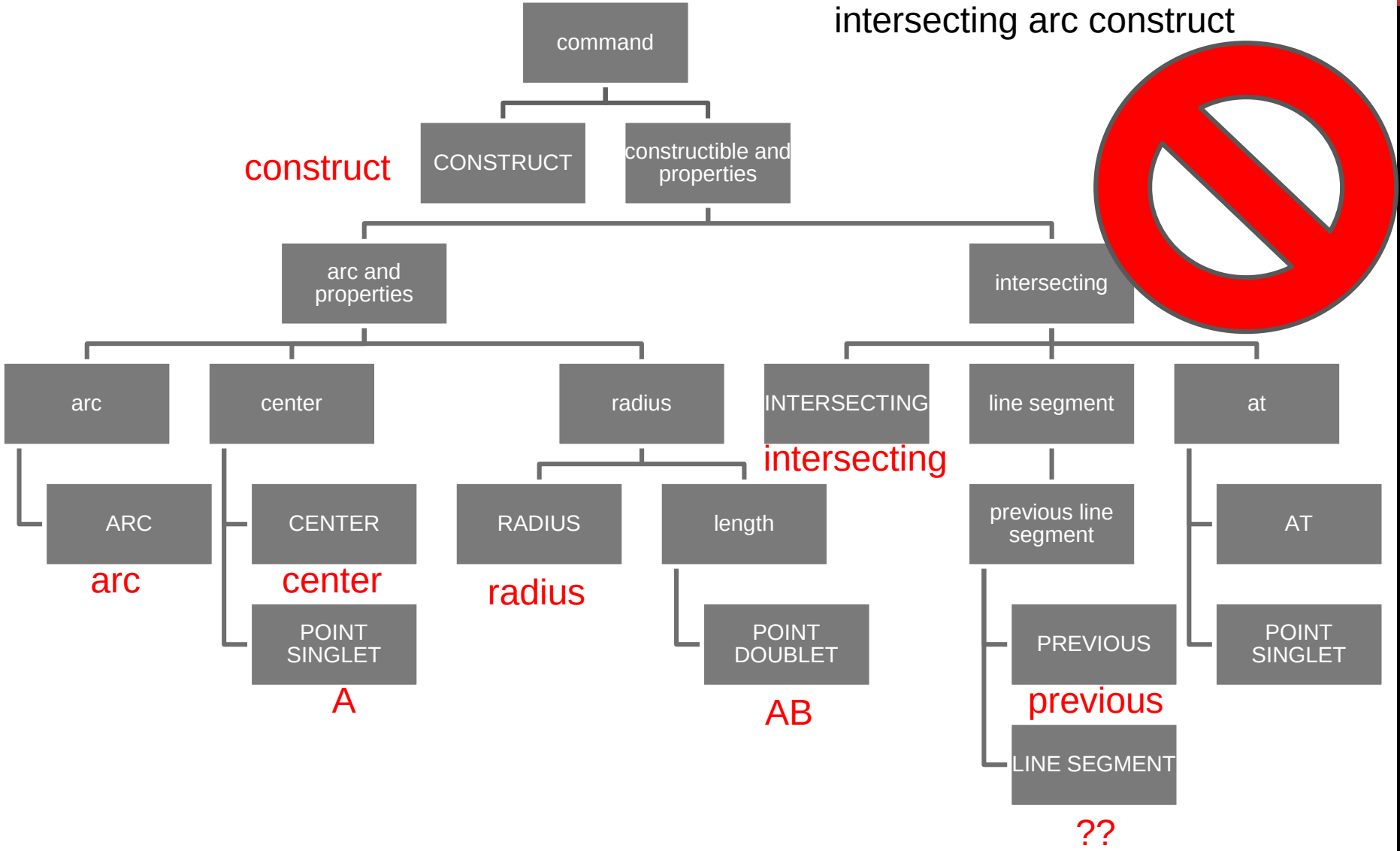


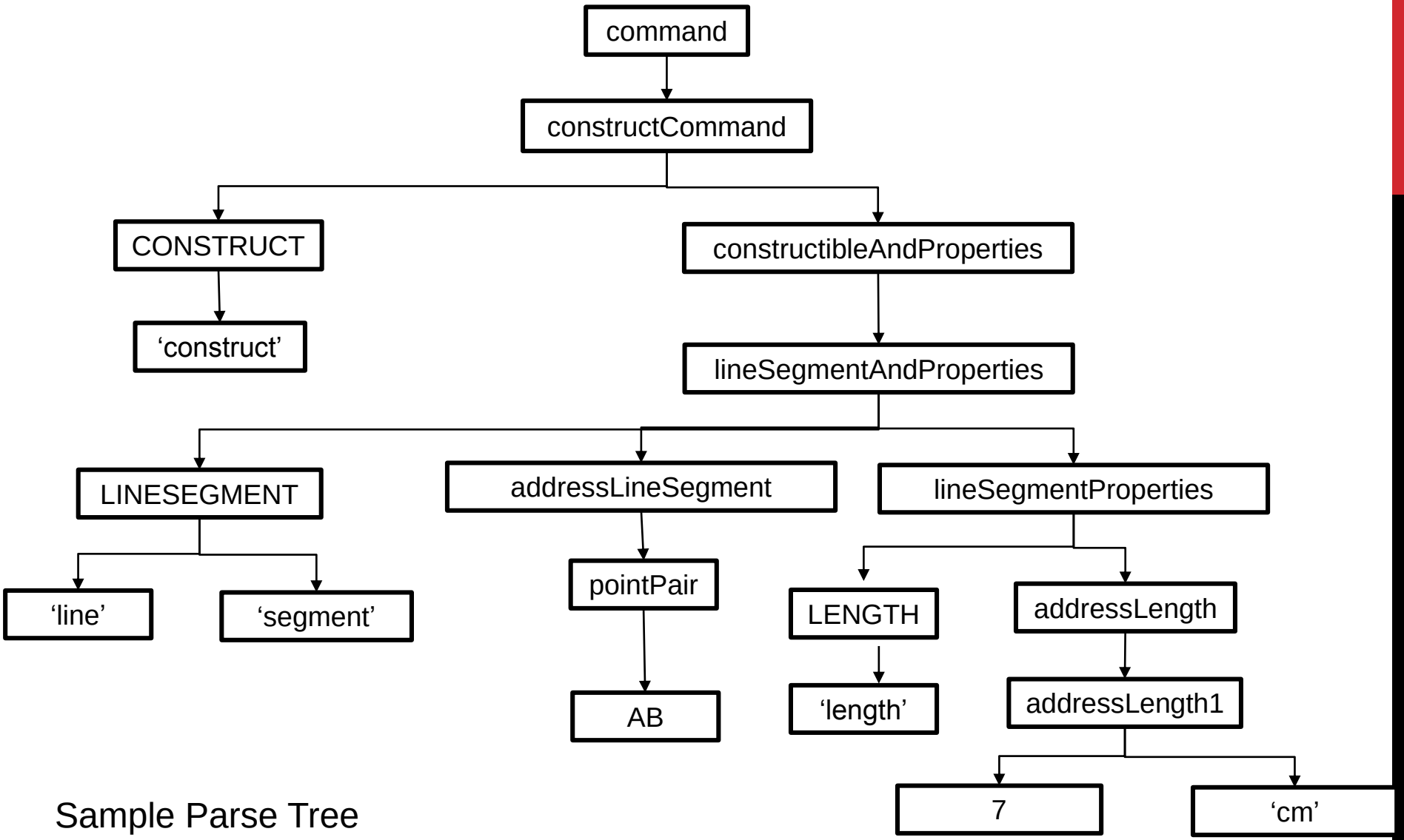
METALANGUAGE GRAMMAR



METALANGUAGE GRAMMAR

A center previous radius AB G at intersecting arc construct





Sample Parse Tree

USING CONTEXT TO HANDLE ANAPHORA

- An anaphora is a word or phrase which points back to a previously referred linguistic or semantic object. Thus the anaphora and its antecedent both corefer to the same referent.

The semantics analyzer uses the context (a list of objects which were plotted in the previous construction steps) to resolve the anaphora. For example, consider the sentence “Mark a point M on it”. Here “it” would refer to the most recently plotted markable object (objects on which a point can be marked e.g. lines, line segments, arcs, circles etc.). We fetch such markable object from the context to resolve the anaphora.

DIFFICULTIES

- **Anaphoras**

"एक सुविधाजनक त्रिज्या लेकर पिछले चरण वाले चाप को बिंदु A पर काटें"

- **Underspecified Parameters**

"With A and B as centers and a suitable radius, draw two arcs intersecting each other at point C"

- **Probabilistic Mapping**

Mapped metalanguage sentence	Probability
Construct AB any length 7.8 cm	0.71683
Construct AB lineSegment length 7.8 cm	0.21081
Construct AB angle length 7.8 cm	0.07232
Construct AB center length 7.8 cm	1.90645e-06

CHARACTERISTIC FEATURES

- **Scalable to any number of input languages**
- **Uses an intermediate metalanguage to express intended construction steps**
- **Assumes availability of corpus corresponding to each input language**

IMPORTANT ASSUMPTION

Parameter names and their values are close to eachother in the metalanguage translation.

REFERENCES

[1] Umair Z Ahmed, Arpit Kumar, Monojit Choudhury, and Kalika Bali. Can modern statistical parsers lead to better natural language understanding for education? In Computational Linguistics and Intelligent Text Processing, pages 415427. Springer, 2012

[2] Sumit Gulwani, Vijay Anand Korthikanti, and Ashish Tiwari. Synthesizing geometry constructions. In ACM SIGPLAN Notices, volume 46, pages 5061. ACM, 2011.

[3] Shachar Itzhaky, Sumit Gulwani, Neil Immerman, and Mooly Sagiv. Solving geometry problems using a combination of symbolic and numerical reasoning. Technical report, Technical report, Tel Aviv University, 2012.

[4] Franz Josef Och and Hermann Ney. A systematic comparison of various statistical alignment models. Computational linguistics, 29(1):1951, 2003.

[5] Pascal Schreck, Pascal Mathis, and Julien Narboux. Geometric construction problem solving in computer-aided learning. In Tools with Articial Intelligence (ICTAI), 2012 IEEE 24th International Conference on, volume 1, pages 11391144. IEEE, 2012.

[6] Luke S Zettlemoyer and Michael Collins. Learning to map sentences to logical form: Structured classication with probabilistic categorial grammars. arXiv preprint arXiv:1207.1420, 2012.

GIZA++

- **GIZA++ is a statistical machine translation toolkit**
- **Used to train IBM Models 1-5 and an HMM word alignment model.**
- **http://code.google.com/p/giza-pp/**
- **Franz Josef Och, Hermann Ney. "A Systematic Comparison of Various Statistical Alignment Models", *Computational Linguistics*, volume 29, number 1, pp. 19-51 March 2003.**

OBSERVATIONS

Authors	Uses domain knowledge	Assumes linguistic clues already translated into logical constructs	Uses parse knowledge
Gulwani et. al. [2]	YES	YES	NA
Schreck et. al.[5]	YES	YES	NA
Itshaky et al.[3]	YES	YES	NA
Ahmed, Umair et. Al.[1]	YES	NO	YES

RELATED WORKS

Authors	Work
Gulwani et. al. [2]	Uses goal-based heuristic to simulate backward deduction; solves problem expressed in terms of predefined logical constructs
Schreck et. al.[5]	Uses CAD methods to deal with constrants
Itshaky et al.[3]	Uses number of nondeterministic choices as a measure of good solution
Ahmed, Umair et. Al.[1]	Uses domain specific measures to minimize parser errors and augment the geometry problem solver, GeoSynth