SOLVING GEOMETRY PROBLEMS

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OVERALL OBJECTIVES

To develop a expert system that

1. **Tutors the user on geometry construction problems**
2. **Also address the population speaking languages other than English**

OBJECTIVES

To develop an expert system that

1. **Draws simple geometric figures, given the construction steps**
2. **Supports multiple natural languages for expressing the input steps**

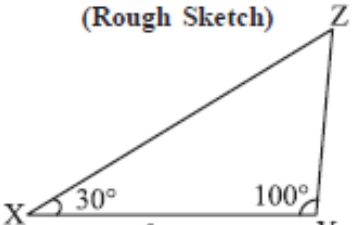
SAMPLE PROBLEMS

Example 3 Construct ΔXYZ if it is given that $XY = 6$ cm, $m\angle ZXY = 30°$ and $m\angle XYZ = 100°$.

Solution

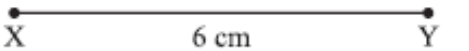
Step 1 Before actual construction, we draw a rough sketch with measures marked on it. (This is just to get an idea as how to proceed)
[Fig 10.6(i)].

(Rough Sketch)



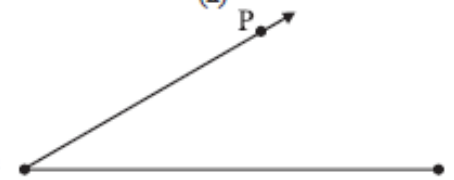
(i)

Step 2 Draw XY of length 6 cm.



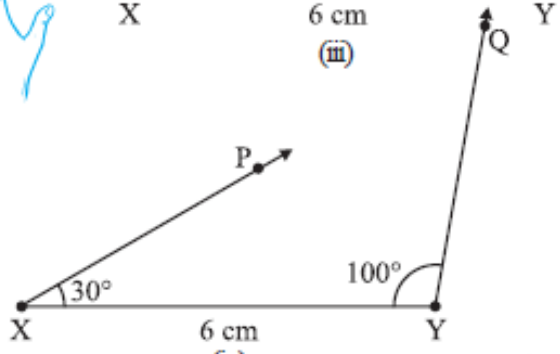
(ii)

Step 3 At X, draw a ray XP making an angle of 30° with XY. By the given condition Z must be somewhere on the XP.



(iii)

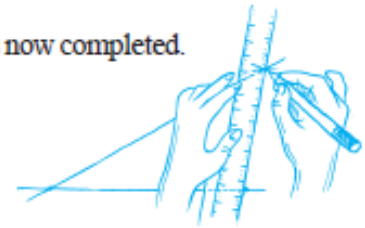
Step 4 At Y, draw a ray YQ making an angle of 100° with YX. By the given condition, Z must be on the ray YQ also.

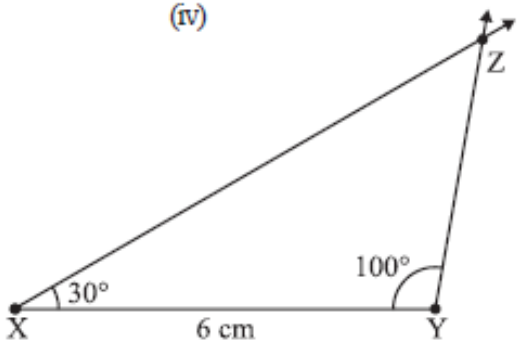


(iv)

Step 5 Z has to lie on both the rays XP and YQ. So, the point of intersection of the two rays is Z.

ΔXYZ is now completed.





(v)

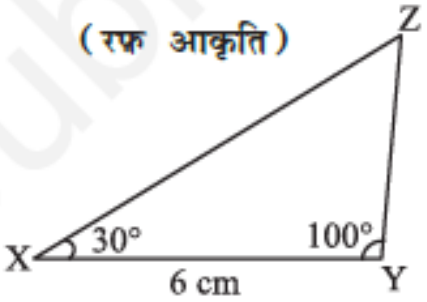
Fig 10.6 (i) – (v)

SAMPLE PROBLEMS

उदाहरण 3 ΔXYZ की रचना कीजिए, यदि, $XY = 6$ cm, $m\angle ZXY = 30°$ और $m\angle XYZ = 100°$ है।

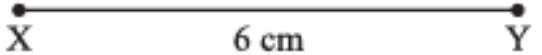
हल

चरण 1 वास्तविक रचना से पहले, हम इस पर अंकित मापों के अनुसार एक रफ़ आकृति खींचते हैं। (इससे कुछ अनुमान लग जाता है कि कैसे रचना की जाए) [आकृति 10.6(i)]।



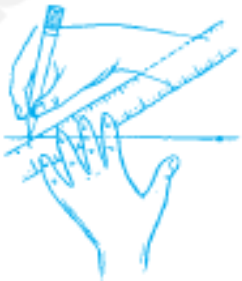
(i)

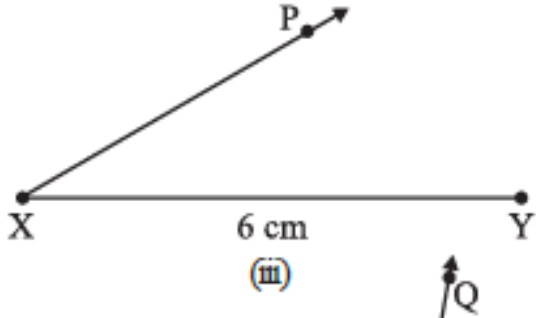
चरण 2 6 cm लंबाई का रेखाखंड XY खींचिए [आकृति 10.6(ii)]।



(ii)

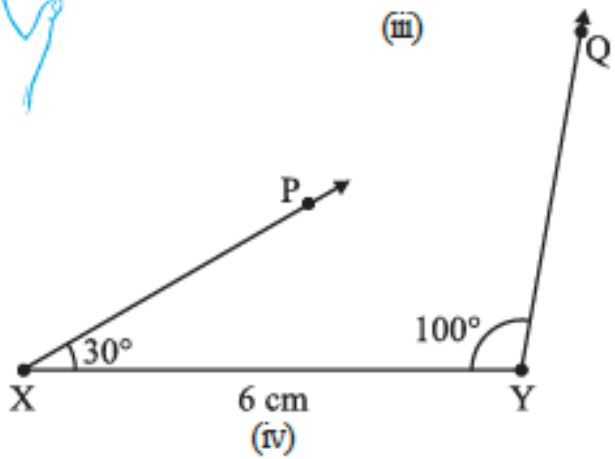
चरण 3 X पर एक किरण XP खींचिए जो XY से 30° का कोण बनाए। दिए हुए प्रतिबंध के अनुसार बिंदु Z किरण XP पर कहीं स्थित होना चाहिए [आकृति 10.6(iii)]।





(iii)

चरण 4 Y पर एक किरण YQ खींचिए, जो YX से 100° का कोण बनाए। दिए हुए प्रतिबंध के अनुसार Z किरण YQ पर भी अवश्य स्थित होना चाहिए [आकृति 10.6(iv)]।



(iv)

RELATED WORKS

Authors	Work
Gulwani et. al. [2]	Uses goal-based heuristic to simulate backward deduction; solves problem expressed in terms of predefined logical constructs
Schreck et. al.[5]	Uses CAD methods to deal with constrants
Itshaky et al.[3]	Uses number of nondeterministic choices as a measure of good solution
Ahmed, Umair et. Al.[1]	Uses domain specific measures to minimize parser errors and augment the geometry problem solver, GeoSynth

OBSERVATIONS

Authors	Uses domain knowledge	Assumes linguistic clues already translated into logical constructs	Uses parse knowledge
Gulwani et. al. [2]	YES	YES	NA
Schreck et. al.[5]	YES	YES	NA
Itshaky et al.[3]	YES	YES	NA
Ahmed, Umair et. Al.[1]	YES	NO	YES

CHARACTERISTIC FEATURES

- **No assumption language of the domain**
- **No assumption about availability of parser**
- **Scalable to any number of input languages**

Cross-lingual Alignment

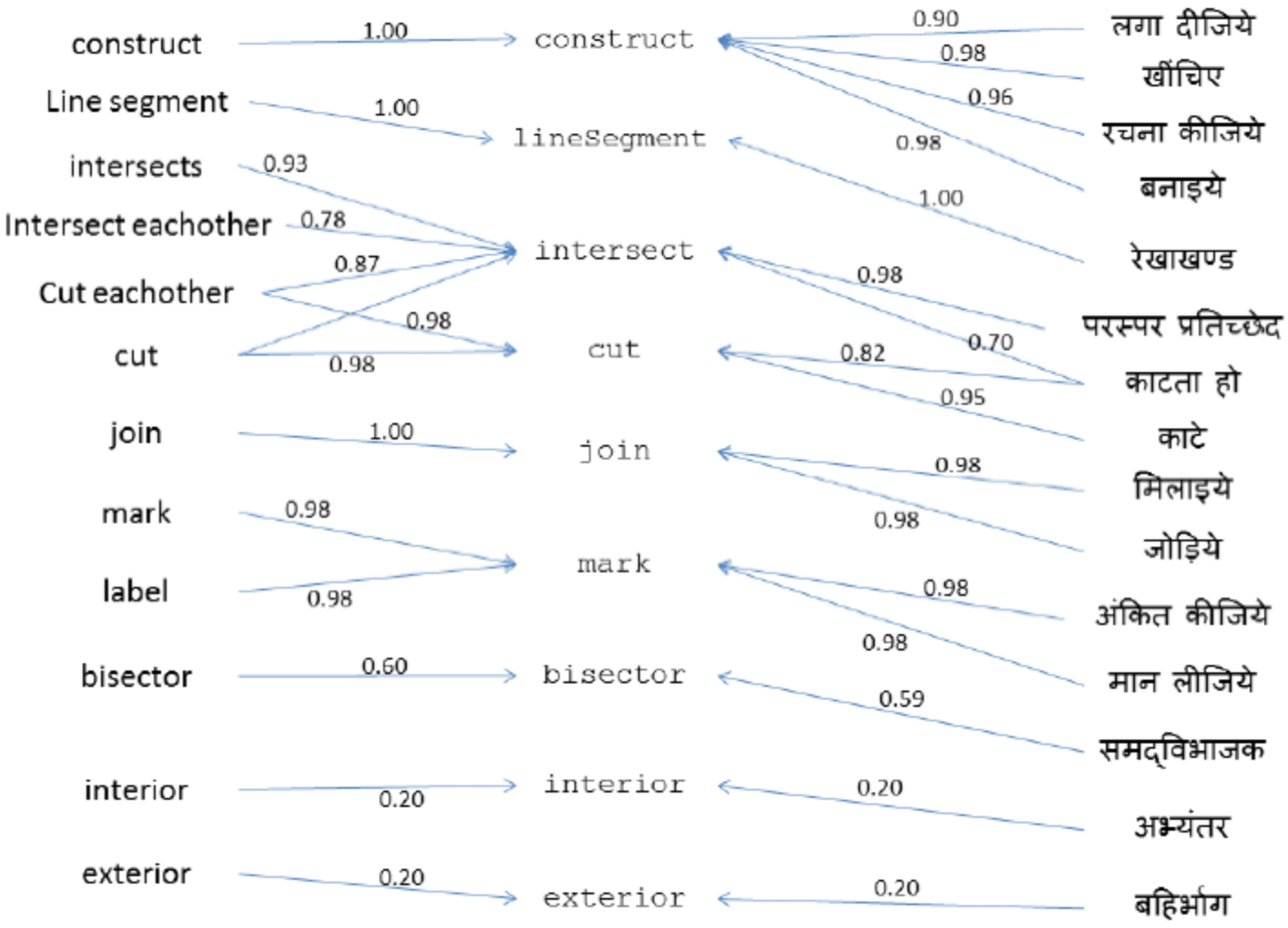
WHAT IS CROSS-LINGUAL ALIGNMENT?

- **Assigns probability to the event that a particular source language token corresponds to a particular target language token**

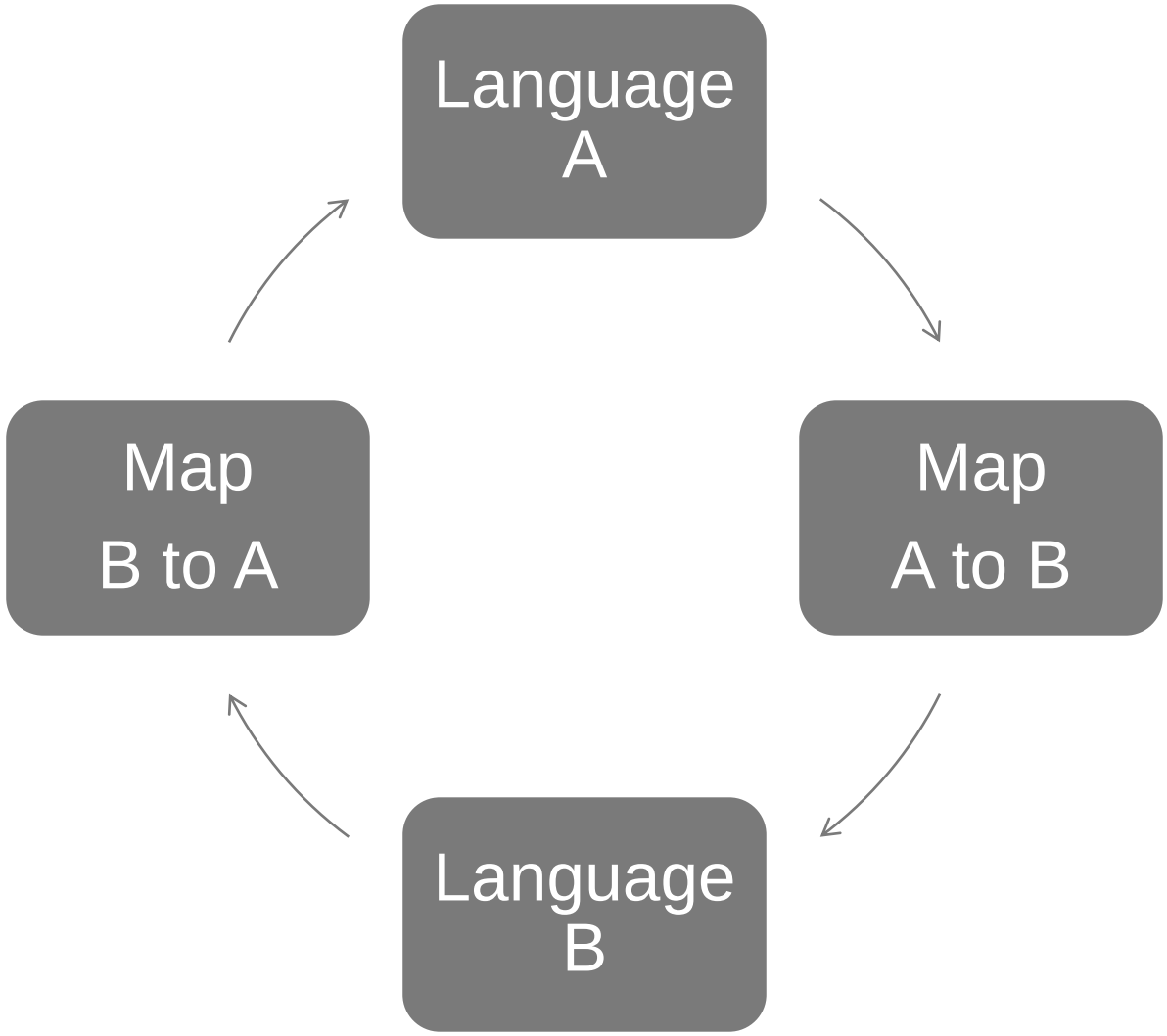
SAMPLE CORPUS

English	Hindi	Meta Language
Construct a line AB of length 4 cm	4 सेमी लम्बाई का एक रेखाखण्ड AB खींचिए	`construct lineSegment AB length 4 cm`
With A as center and radius 3 cm, draw an arc	केंद्र A और त्रिज्या 3 सेमी लेकर एक चाप खींचिए	`constrcut arc center A radius 3 cm`
With B as center and radius 5 cm, draw an arc cutting the previously drawn arc at C	केंद्र B और त्रिज्या 5 सेमी लेकर एक चाप खींचिए जो पहले खींची चाप को C काटता हो	`construct intersectingArc center C radius 5 cm cuts arc previous at C`

SAMPLE ALIGNMENT

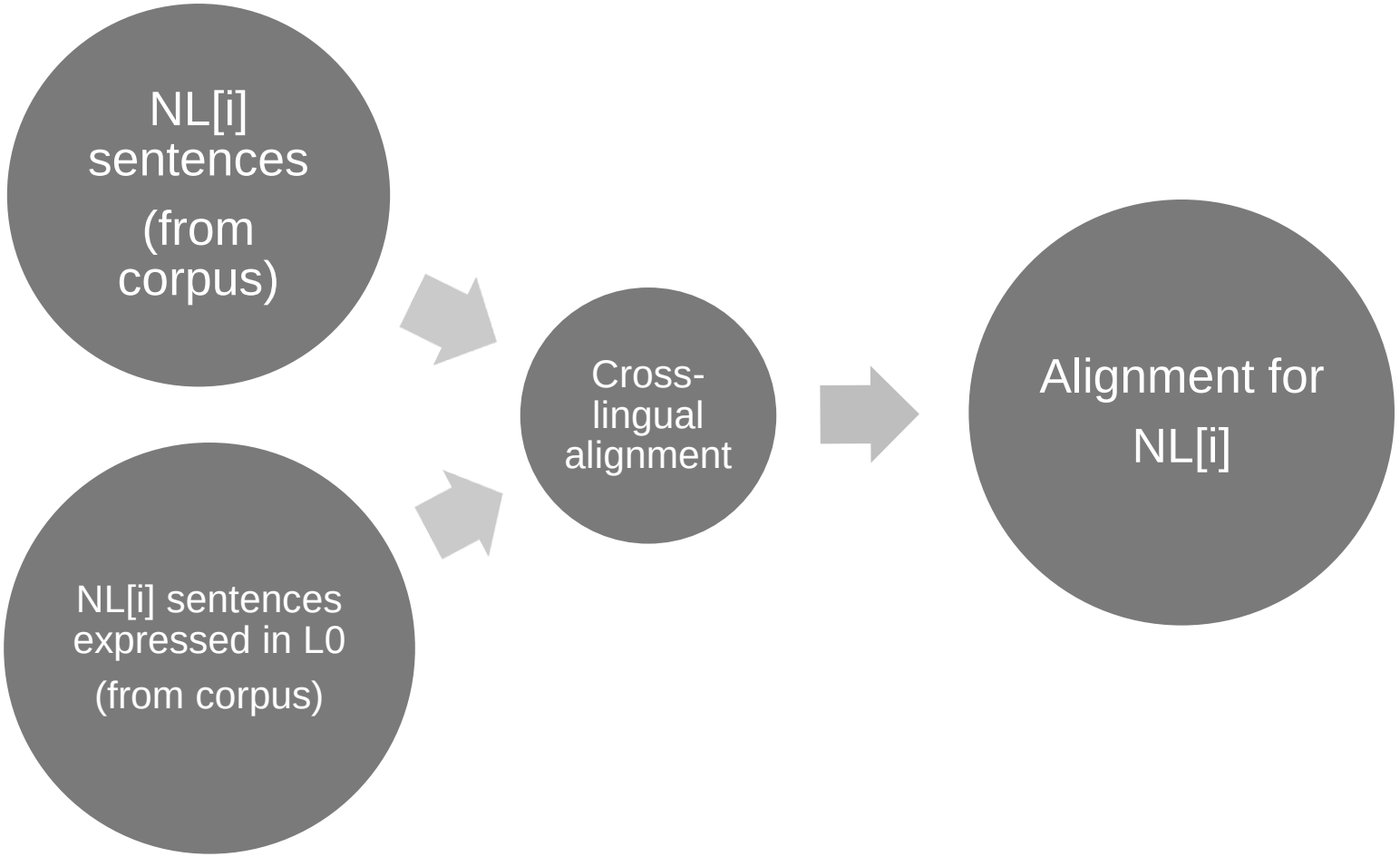


ALIGNMENTS



INITIAL IDEAS

L0: Fixed predicate language (carefully designed)
NL[i]: ith natural language, 1<= i <= n
A[i]: word alignment between NL[i] and L0



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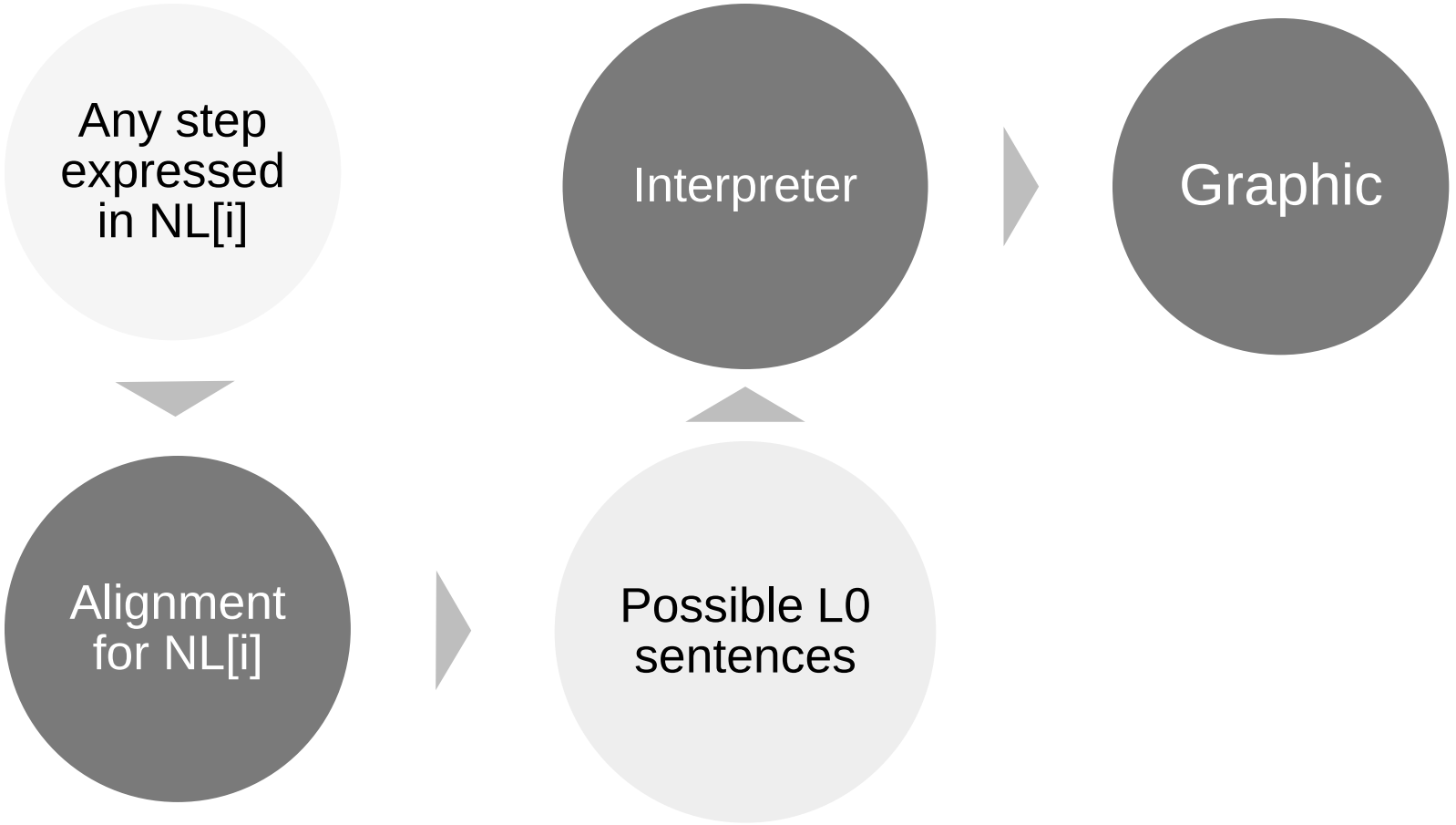
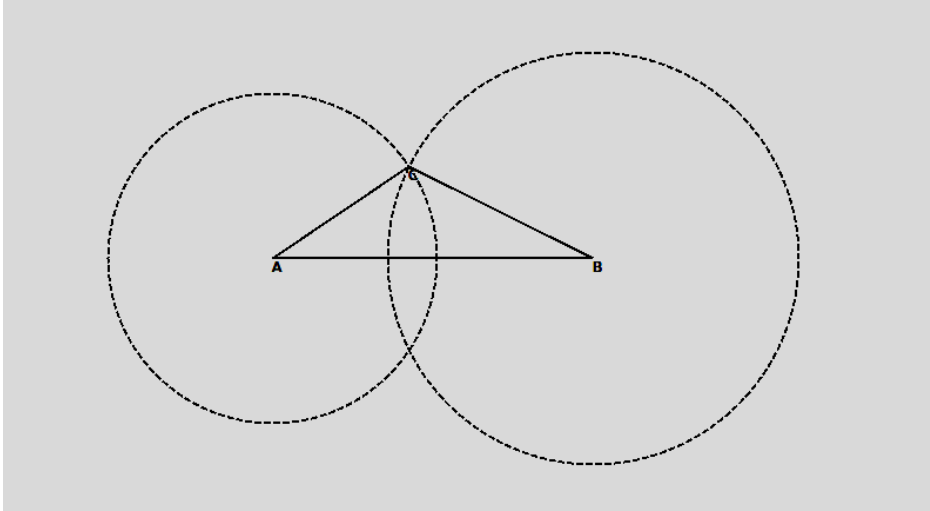


SAMPLE RUN

Construct line segment AB of length 7.8 cm
With A and B as centers and radius 4 and 5 cm draw two arcs intersecting each other at C
Join AC
Join BC



7.8 सेमी लम्बाई का एक रेखाखण्ड AB खींचिए

DIFFICULTIES

- **Anaphoras**

"एक सुविधाजनक त्रिज्या लेकर पिछले चरण वाले चाप को बिंदु A पर काटें"

- **Underspecified Parameters**

"With A and B as centers and a suitable radius, draw two arcs intersecting each other at point C"

- **Probabilistic Mapping**

Mapped metalanguage sentence	Probability
Construct AB any length 7.8 cm	0.71683
Construct AB lineSegment length 7.8 cm	0.21081
Construct AB angle length 7.8 cm	0.07232
Construct AB center length 7.8 cm	1.90645e-06

FURTHER WORK

- **Augment the corpus**
- **Provide for bigger set of construction steps**
- **Capture colloquial ways of expressions**
- **Integrate into the bigger context of tutoring**

REFERENCES

[1] Umair Z Ahmed, Arpit Kumar, Monojit Choudhury, and Kalika Bali. Can modern statistical parsers lead to better natural language understanding for education? In Computational Linguistics and Intelligent Text Processing, pages 415427. Springer, 2012

[2] Sumit Gulwani, Vijay Anand Korthikanti, and Ashish Tiwari. Synthesizing geometry constructions. In ACM SIGPLAN Notices, volume 46, pages 5061. ACM, 2011.

[3] Shachar Itzhaky, Sumit Gulwani, Neil Immerman, and Mooly Sagiv. Solving geometry problems using a combination of symbolic and numerical reasoning. Technical report, Technical report, Tel Aviv University, 2012.

[4] Franz Josef Och and Hermann Ney. A systematic comparison of various statistical alignment models. Computational linguistics, 29(1):1951, 2003.

[5] Pascal Schreck, Pascal Mathis, and Julien Narboux. Geometric construction problem solving in computer-aided learning. In Tools with Articial Intelligence (ICTAI), 2012 IEEE 24th International Conference on, volume 1, pages 11391144. IEEE, 2012.

[6] Luke S Zettlemoyer and Michael Collins. Learning to map sentences to logical form: Structured classication with probabilistic categorial grammars. arXiv preprint arXiv:1207.1420, 2012.

THANK YOU!

GIZA++

- **GIZA++ is a statistical machine translation toolkit**
- **Used to train IBM Models 1-5 and an HMM word alignment model.**
- **http://code.google.com/p/giza-pp/**
- **Franz Josef Och, Hermann Ney. "A Systematic Comparison of Various Statistical Alignment Models", *Computational Linguistics*, volume 29, number 1, pp. 19-51 March 2003.**