Progressive Stochastic Search

Application to solving sudoku

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Sudoku

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- Gained mainstream popularity in Japan in 1980s

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Sudoku: Problem Definition

A Sudoku square of order n consists of n^4 variables formed into a n^2 x n^2 grid such that:

1. Each row of cells contains the integers 1 through n^2 exactly once.
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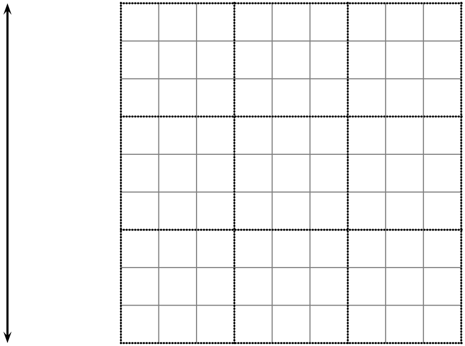
A simple illustration of PSS



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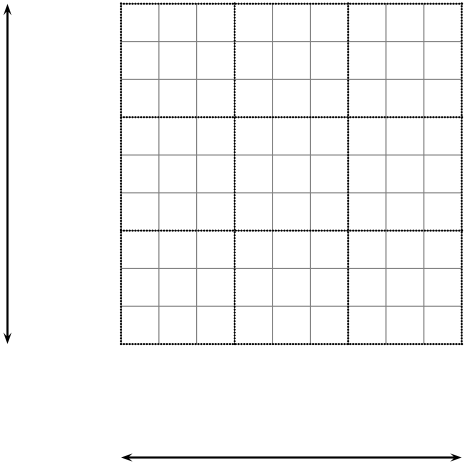

Motivation

- All Sudoku puzzles are not logic-solvable; many involve guesswork
- Proved to be an NP-complete problem: a polynomially bound algorithm for solving all problem instances is not possible
- It is claimed that even for order-3 Sudoku puzzles, there are 6,670,903,752,021,072,936,960 valid arrangements (!)
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Stochastic Search

- Typical SS algorithms first generate a complete initial variable assignment and repair the assignment by heuristic local search with reference to a cost function until a solution is found.
  - Problem: Might get stuck on a plateau or in a local optima
  - Repairs:
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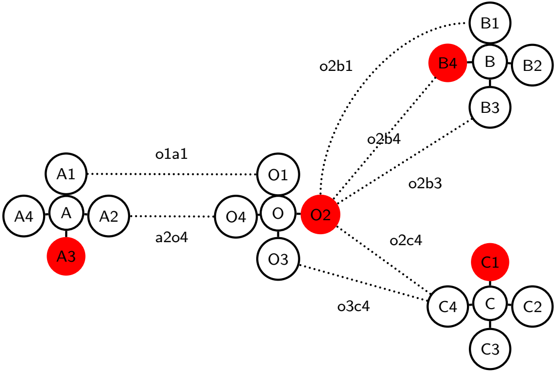

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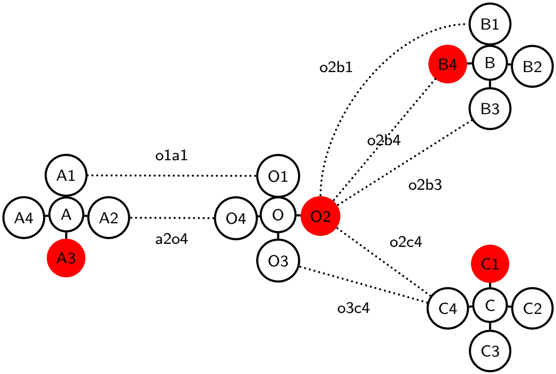
Step 1

Pick up the head cluster O of queueQ.



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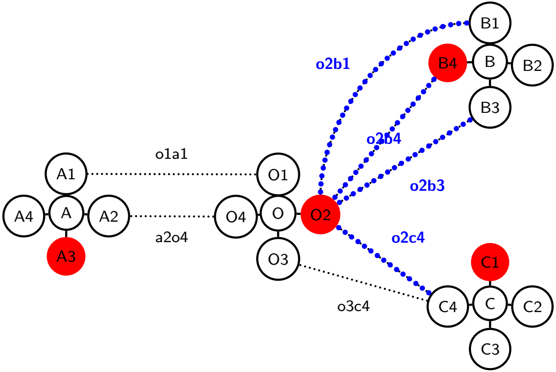
Step 2

$conflicts_{O2} = o2b1 + o2b3 + o2b4 + o2c4 \neq 0$



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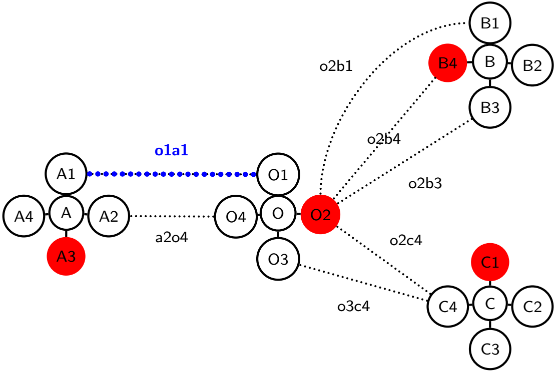
Step 3

$conflicts_{O1} = o1a1$



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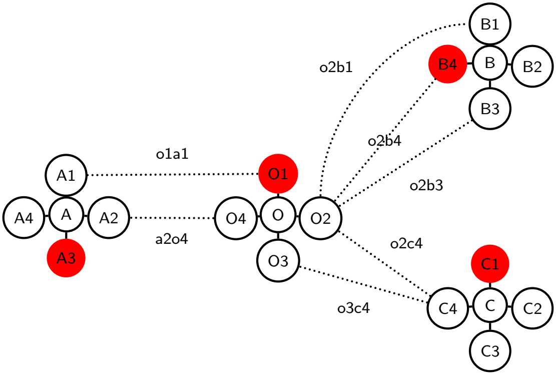
Step 4

$value_O := O_1$



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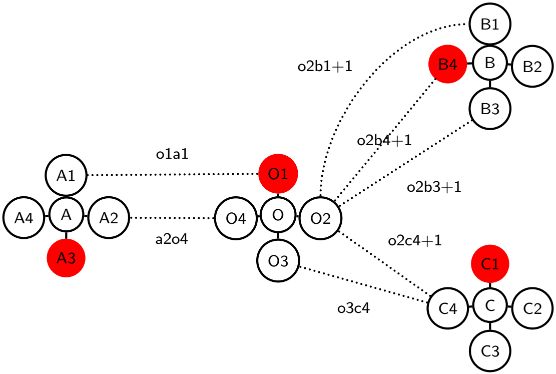
Step 5

$o2b1 := o2b1 + 1$ $o2b3 := o2b3 + 1$
$o2b4 := o2b4 + 1$ $o2c4 := o2c4 + 1$



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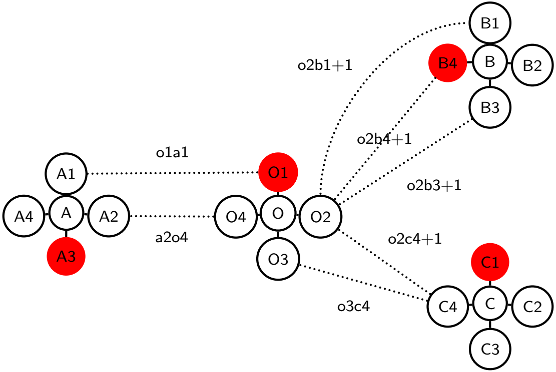
Step 7

Append A_1 to the queue



Progressive Stochastic Search

- Maintains a list of variables, which dictate the sequence of variables to be repaired
- When a variable is designated to be repaired in PSS, it always chooses a new value even if its original value gives a better cost
- Search paths slightly marked by worsening at every point on the paths
- Intuitively, it's driven by a "force" so that the search is able to "rush through" the local minima and plateaus

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Modelling Sudoku as a CSP

- The Sudoku puzzle will be modelled as a grid of n^4 cells - each of which represents an integer variable which initially will have a domain of 1 through n^2
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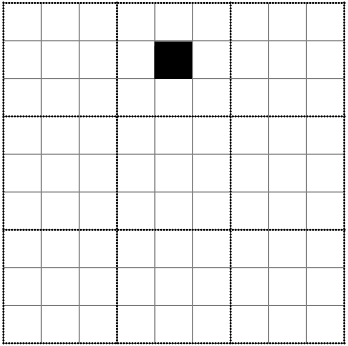

$Domain = Domain - v$

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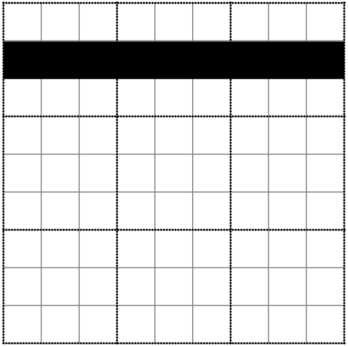

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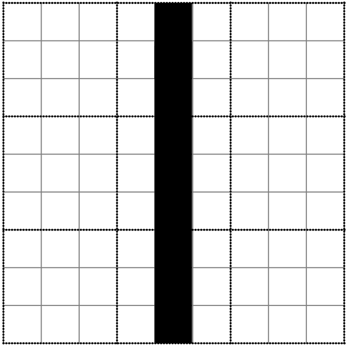

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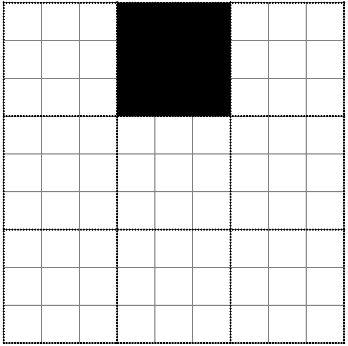

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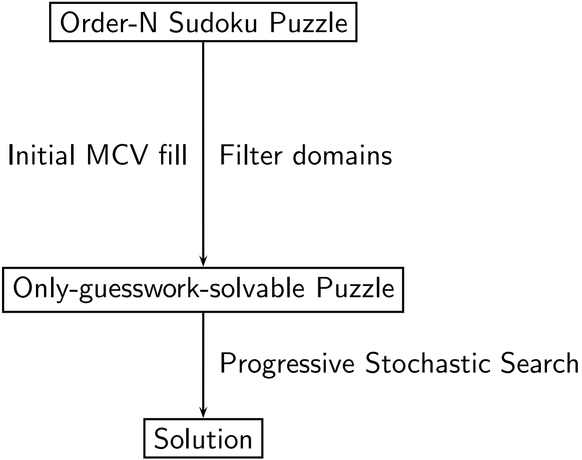

Implementation

1. typedef struct cluster{
   bool domain[N*N]; domain[i]==true means i is in the domain of this cell
   num value; value is the current value of this cell
   bool given; given==true means that this cell was already given in the original problem
   bool queued; queued==true means that this cell is currently queued
   }cell;
2. cell matrix[N*N] The main array acting as the workplace
3. long long int rwt[N*N][N*N][N*N][N*N]; rwt[r][i][j][v] is the weight of constraint between ith and jth cell in rth row for the value v
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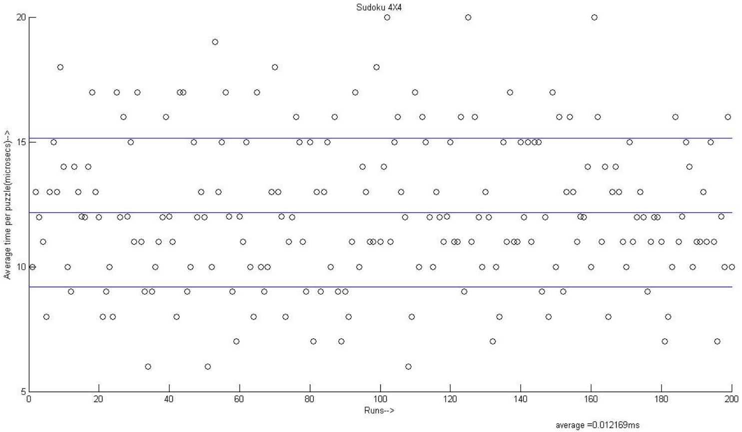
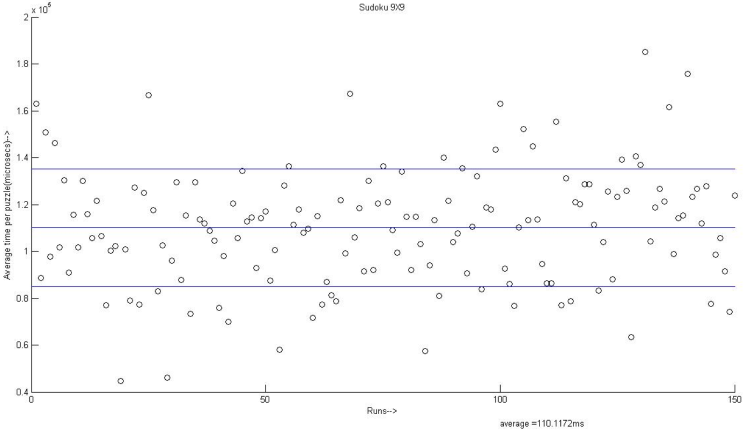
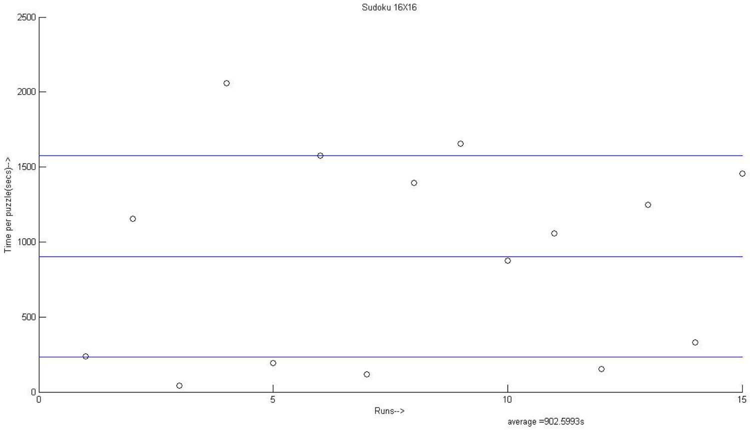

Sudoku 4X4
Average time per puzzle(microsecs)-->
Runs-->
5
10
15
20
0
20
40
60
80
100
120
140
160
180
200
average =0.012169ms


Sudoku 9X9
x 10^5
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
Average time per puzzle(microsecs)-->
0
50
100
150
Runs-->
average =110.1172ms


Sudoku 16X16
2500
2000
1500
1000
500
0
Time per puzzle(secs)-->
0
5
10
15
Runs-->
average =902.5993s

Acknowledgements

- Bryan Chi-ho Lam and Ho-fung Leung, 2003. Progressive Stochastic Search for Solving Constraint Satisfaction Problems. Proceedings of the 15th IEEE International Conference on Tools with Artificial Intelligence (ICTAI03) 1082-3409/03.
- Rhydian Lewis, 2007. On the Combination of Constraint Programming and Stochastic Search. The Sudoku CaseProceedings of the 4th international conference on Hybrid metaheuristics.
- Helmut Simonis, 2005, Sudoku as a Constraint Problem, IC-Parc, Imperial College, London