

### Non-conjugate Models:

Assume,

$$y_i | \mu \stackrel{iid}{\sim} N(\mu, 1) \quad [\text{unknown } \mu \text{ and known } \sigma^2]$$

$$u \sim t(0, 1, 1)$$

$$\text{Posterior}(u|y_1, \dots, y_n) \propto \text{likelihood} * \text{prior}(u)$$

$$\propto \prod_{i=1}^n \left[ \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} (y_i - \mu)^2\right) \right] \times \frac{1}{\pi (1 + \mu^2)}$$

$\uparrow$                        $\uparrow$   
 [Density of Normal]    [Density of T]

$$\propto \exp \left[ -\frac{1}{2} \sum_{i=1}^n (y_i - \mu)^2 \right] * \frac{1}{(1 + \mu^2)}$$

$$\propto \exp \left\{ -\frac{1}{2} \left[ \sum_{i=1}^n y_i^2 - 2\mu \sum_{i=1}^n y_i + n\mu^2 \right] \right\} \cdot \frac{1}{1+\mu^2}$$

$$\propto \exp \left[ \frac{n(\bar{y}\mu - \mu^2/2)}{1 + \mu^2} \right]$$

Conclusion: looks like Normal but the denominator.  
so no known posterior distribution

let  $g(\mu) = \text{Posterior}(\mu | y_1, \dots, y_n)$

Taking log on both sides,

$$\log(q(u)) = n(\bar{y}\mu - u^2/2) - \log(1 + u^2)$$